# An Experimental Switching System Using New Electronic Techniques

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*This is the first of a series of articles describing an experimental electronic telephone switching system employing a number of new techniques. These include use of a stored program, a network employing gas tube crosspoints, time-division common control and large-capacity barrier grid tube and photographic storage systems.*

## I. INTRODUCTION

Following recent activities in switching research[1,2] and the development of new solid state devices, engineering studies indicate that a telephone switching system using new electronic techniques is practicable. An experimental electronic switching system designed and built for laboratory purposes is described in this paper. Other articles will present the various parts of the system in more detail (e.g., Refs. 3, 4 and 5).

Earlier papers have outlined many of the techniques which are available in electronic switching,[6,7] and the particular choices which were made for this experimental system have been publicized.[8] A brief review is given here.

The experimental system may be said to have a space-division electronic switching network for telephone connections, in which the connections themselves are made under the direction of time-division (one-at-a-time) common control. Large-capacity temporary and semipermanent electronic storage units fulfill most of the memory requirements of the system. The time-division control uses a program stored in the semipermanent memory. This program specifies the manner in which an office will function on various types of calls and provides various service features. Since a photographic memory medium is used in the semipermanent memory, the operating features of the office may be changed readily.

## II. ORGANIZATION OF THE EXPERIMENTAL SYSTEM

Fig. 1 is a block diagram of the major components of the system. The upper portion of the figure shows the section through which telephone connections are established. This is known as the switching network. It interconnects lines with other lines, lines with trunks, and lines or trunks with tone sources.

The remainder of the system is its time-division control. The scanner is directed by the central control to examine the current flow condition of lines and trunks (current on or off), with the results of this examination determining all actions in the information inputs to the system. The signal distributor seizes trunks and sends information over them to distant offices; it is directed to control the flow of current in the trunk circuit by the central control. The barrier grid store contains the temporary memory used to assemble the line and trunk call status information obtained from the scanner. The flying spot store holds in its permanent memory information about the service characteristics of the lines and trunks and a program to direct all the control operations

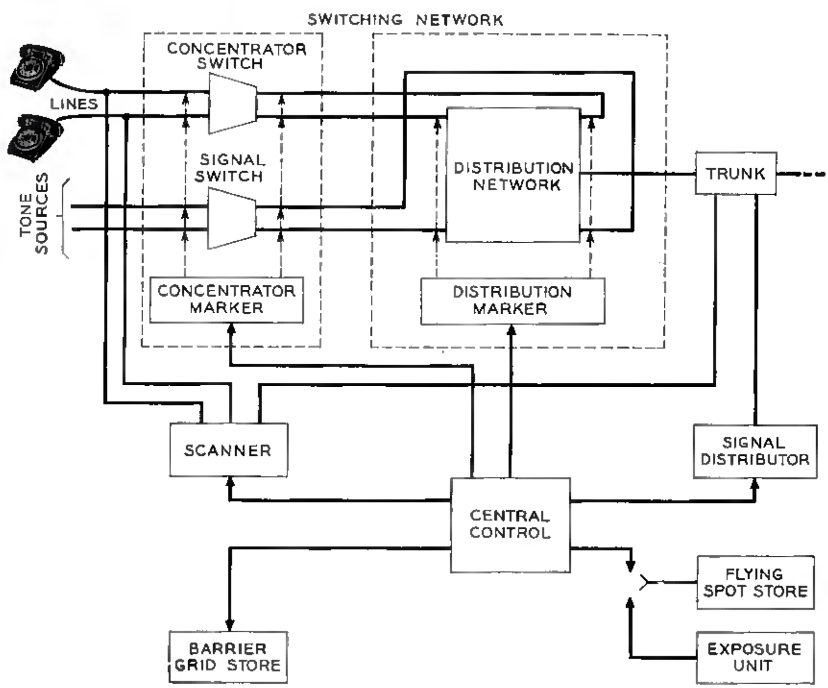


Fig. 1 — Block diagram of experimental system.

which take place in the system. As its name implies, the central control is the focal point for directing each of the other components of the system.

Obviously, the time-division control concept takes advantage of the speed of electronic devices. Its techniques will be described in more detail after the following discussion of the electronic space-division switching network, which illustrates the manner in which all major system components are controlled.

## III. SPACE-DIVISION SWITCHING NETWORK

A space-division network is one in which physical devices are taken into use by a particular call for the entire duration of the call. A switchable transmission circuit (in space) is established between the calling and called customers. All present-day electromechanical systems are of this type. Unlike previous space-division networks, however, this experimental system uses only electronic devices.

### 3.1 *Components*

The transmission path established through the experimental network has no metallic contacting elements like those in crossbar or step-by-step switches. Instead, each crosspoint is a two-element gas tube or gas diode (see Fig. 2) in which the gas breaks down when sufficient voltage is applied across the electrodes.[9,10] An electrical path is established through this gaseous discharge which transmits the speech current and controls the connection.

As in a crossbar switching network, cascade stages with concentration, distribution and expansion configurations are used. Each switch-

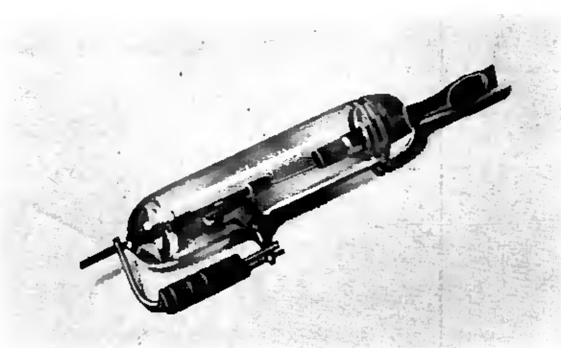

Fig. 2 — Gas tube crosspoint.

ing network stage has its own control, which is called a marker and has subdivisions called a concentrator marker and a distribution marker.

All network connections are made through several gas tubes in series (see Fig. 3). The number of tubes in series varies with the type of connection, eight being employed on an intra-office call and seven on calls between lines and trunks. Only one crosspoint tube is used in the concentrator stage, but six are employed in the distribution switching network.

### 3.2 *Transmission*

All talking paths through gas tubes are single-wire with ground return. Transformers are used to convert from line and trunk balanced pairs to the single-wire circuits in the sections of the switching network. Balanced pairs carrying only voice currents (dry circuits) also terminate in transformers between the concentrator and distribution switching sections of the network. In the present experimental system only in-office concentrators are used. However, this type of network and its controls can be readily adapted for use with remote concentrators.

Electronic crosspoints restrict the power level of the signals which may be transmitted through the switching network. Therefore, a new telephone ringing arrangement has been developed for use with electronic switching.[11] The customer station set (Fig. 4) is provided with a

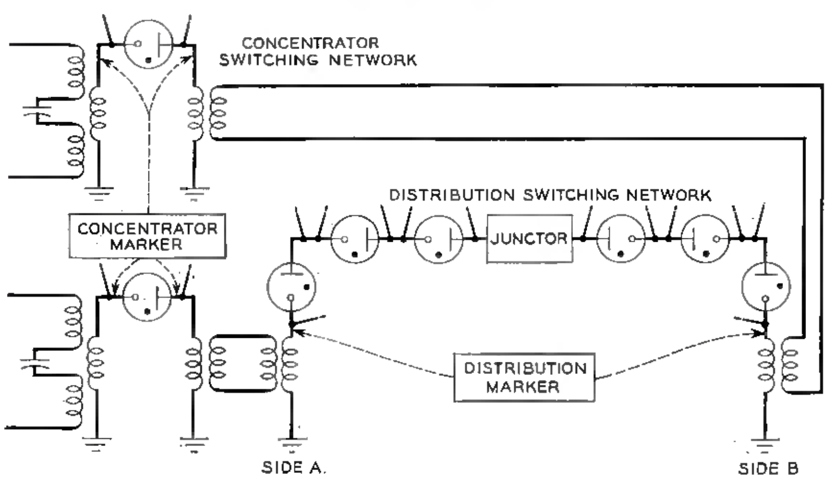


Fig. 3. — Switching network plan.

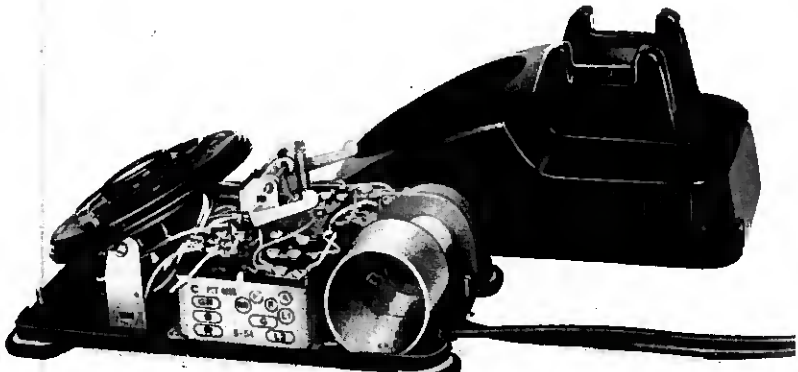

Fig. 4 — Telephone set with tone ringing.

transistorized tuned-circuit amplifier and a receiver element with an acoustic resonating chamber.

Once transistorized amplification is placed in the telephone set, it is an obvious step to use it to improve speech side-tone transmission. In the experimental system the use of a lower line current (approximately 10 milliamperes) permits a reduction in the size of the concentrator line transformers and the amplification in the telephone set helps overcome the insertion losses of the transformers.

### 3.3 *Ringing*

When the telephone receiver is on the switchhook the ringer is bridged across the line. A tone signal in the voice band that is sent through the switching network over the line to call a customer can be detected by the tuned circuit, amplified and radiated as sound energy. The tuned circuit responds to different frequencies for different parties, as in harmonic ringing. When the tone is transmitted the amplifier draws a small amount of line current, and this might look to the scanner like a receiver-off-hook or answer condition. For this reason, the scanner is directed to a line being rung only while its amplifier is drawing a minimum of current, during either the ringing cycle or the silent interval.

The ringing tones are applied through a concentrator-like network called the signal switching network, which is controlled by the concentrator marker. Two connections are set up through the signal switching network whenever a line is being rung. One of these is for sending tone ringing to the called line, the other is for sending audible ringing tone to the calling line (see Fig. 17). Should difficulty be encountered in setting up the talking connection when the call is answered, an alternative connection will be established through the signal switching network, using paths previously engaged for the ringing functions.

## IV. FUNCTIONAL CONCENTRATION

The control of the switching network serves to illustrate functional concentration, one of the principal features of the electronic switching system. To the rest of the system, the markers for the concentrator and signal and the distribution switching networks appear to be two black boxes. On command, they can set up or take down connections between trunks or between lines and trunks.

The commands are electrical control signals received in parallel in two parts, one part known as the "address" or "location" at which something is to be done, the other as the "instruction" or "order". The addresses are specific line or trunk numbers in coded form, usually binary digits (bits). The orders are to set up or release a connection at the designated address. The way in which this control is effected is briefly described in what follows.

In the marker, the address code is translated (from binary to one-out-of-$n$) and the electronic selector, in the form of a semiconductor diode matrix, applies a suitable voltage or "mark" potential to address the line terminal on the grounded side of the transformer. Potentials elsewhere in the network are controlled so that the set up or release order may be executed to the marked point. As a result, a connection is established from one marked address point to another marked address point, or a connection to a marked address point is released. This is the so-called "end-marking" principle which has been previously employed in the electronic control of relay crosspoints.[12] In this application, however, electronic crosspoints are used as the transmission medium as well as for the memory and control functions.

The switching network controls are designed so that, on release orders, the connection about to be released will be traced. At the end of a connection not being marked for release, the trunk will be identified; that is, its address in binary form will appear in the marker as the connection is released. This trace or identification operation occurs as part of each release order; it is used to process disconnect information for the trunk. Fig. 5 shows the switching network of the experimental system

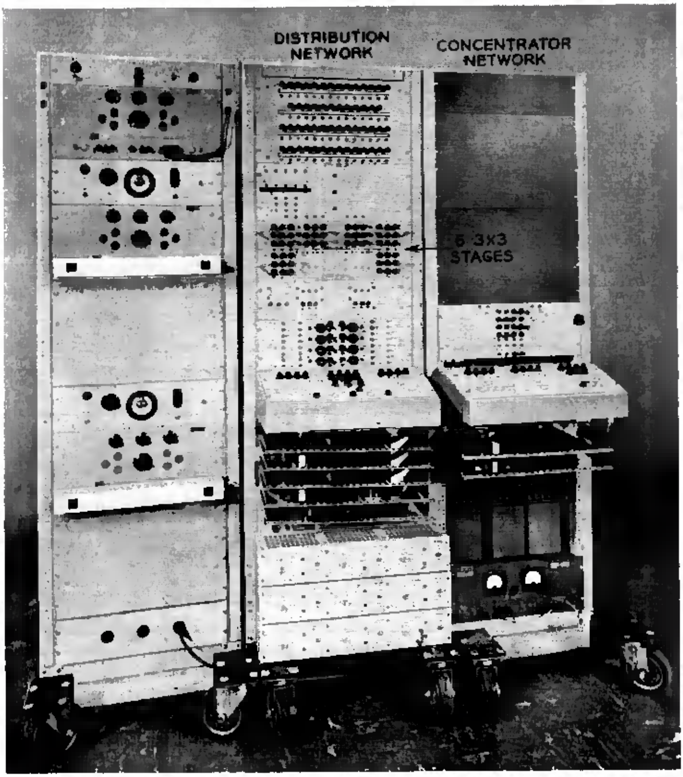


Fig. 5 — Experimental switching network.

setup, which uses 3 × 3 switches in the distribution switching network and 2 × 2 switches in the concentrator and signal switching networks. The marker circuits are located in the rear.

Most system components have a limited amount of output information. In the case of the switching network the output is a signal that a connection was either "set up" or " not set up" (due to blocking or trouble). As will be shown later, other major components of the system employ this same functional concentration method of control with address and order inputs and simple outputs.

Table I lists the "address", "instructions" and "outputs" of each of the major blocks of the system. With the functional concentration concept, each major unit may be viewed from a system standpoint as a black box to which may be assigned a simple but complete statement of what it can do. Here there is considerable contrast between electronic and modern electromechanical common control systems. In the latter

TABLE I — COMMANDS

| Block of System | Address | Instructions | Outputs |
|---|---|---|---|
| Concentrator | Line<br><br><br>Trunk | Connect Side A<br>Connect Side B<br>Trace<br>Release | Successful<br>Blocked<br>Busy |
| Distribution Network | Trunk, Side A<br>Trunk, Side B | Connect<br>Release | Successful<br>Blocked<br>Busy |
| Scanner | Line | Interrogate | 0<br>1 |
| Signal Distributor | Trunk (or Line) | Set<br>Reset | — |
| Barrier Grid Store | Spot Coordinate (X and Y) | Regenerate (Read)<br>Reverse (Read)<br>Write 1<br>Write 0 | 0<br>1 |
| Flying Spot Store | Spot Coordinate (X and Y) | Advance<br>Transfer<br>Expose<br>Off Plate | Command |

case, it is sometimes difficult to assign a broad system function to each of the inter-unit control paths.

## V. TIME-DIVISION CONTROL

### 5.1 *Speed Advantage*

A switching network employing gas tubes and controlled in the above manner is more than 10 times faster than conventional networks employing electromechanical devices. This speed advantage is utilized in several ways. First, "one-at-a-time" operation is made possible; that is, sufficient capacity is available so that, even for large offices, only one marker is required to handle all the traffic. Second, system functions which usually are performed in trunk and register circuits may be performed by temporarily establishing certain network connections. For example, incoming trunk circuits in many electromechanical systems ring the called party and return audible tone to the calling party [see Fig. 6(a)]. In this electronic system, each of these functions is accomplished by setting up separate network connections from the appropriate line to a signal source [see Fig. 6(b)].

But electronic crosspoints are slow compared with speeds which may be obtained with the other electronic elements in the experimental system, such as transistors and semiconductor diodes. The use of devices of this type enables the control portions of the system to process all the information concerning each call on a "one-at-a-time" or time-division basis. This is possible because these devices are from 1,000 to 10,000 times faster than electromechanical elements.

The experimental electronic system is of the common control type, with the control serving all calls during their establishment and release but not during conversation. Time-division common control means that the control as a whole is acting on only part of one call at any particular instant of time. Transistor and diode logic circuits, known collectively as the central control, process call information among the various functionally concentrated system components on a one-at-a-time basis.

For purposes of control, each second of time is subdivided into hundreds of thousands of information-processing cycles. During each of these cycles, one or more system components may be commanded to perform one of its functions for a single call. The information processed by the central control comes from the system inputs, the lines and trunks. To gather this information, another functionally concentrated system element, the scanner, is employed to convert information oc-

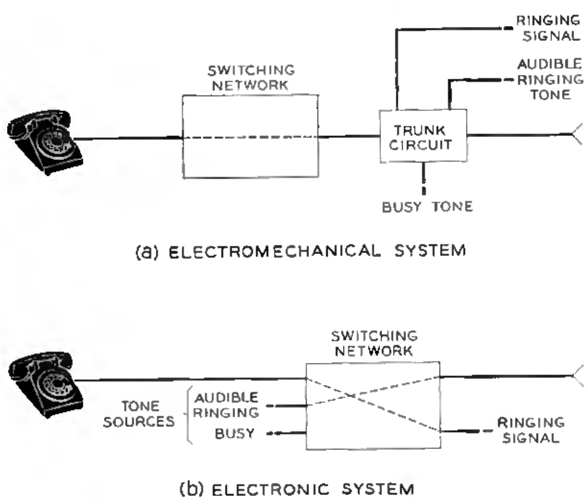


Fig. 6 — (a) Example of trunk circuit functions in electromechanical system; (b) example of use of switching network for signaling and supervisory functions.

TABLE II — LINE CONDITIONS

| | | Scanner Output Line Condition | |
|---|---|---|---|
| | | 0 | 1 |
| Barrier grid store output (line spot L1) | 0 | Idle or Dialing | Request Service or Dialing |
| | 1 | Disconnect | Talking |

curring simultaneously on all lines and trunks into a series of time-related events.

Two major system components, the scanner and signal distributor, play a most important role in the time-division control, since they permit the interchange of information signals to and from space-separated inputs and outputs (lines and trunks). The experimental system is further limited to one-at-a-time operation because the barrier grid store can only read, write or regenerate one memory bit at a time and the flying spot store can only read out one word at a time. The basic rate for all of these one-at-a-time information processing operations is in the range of 100 kilocycles per second.

### 5.2 *Scanner*

The scanner is a collection of transistor and diode logic circuits arranged to examine an addressed line or trunk and determine whether current is flowing in it. A telephone line normally has no current flowing, but, when the receiver is raised from the switchhook, current flows.

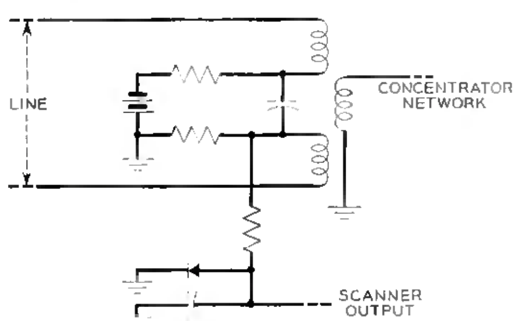


Fig. 7 — Line circuit.

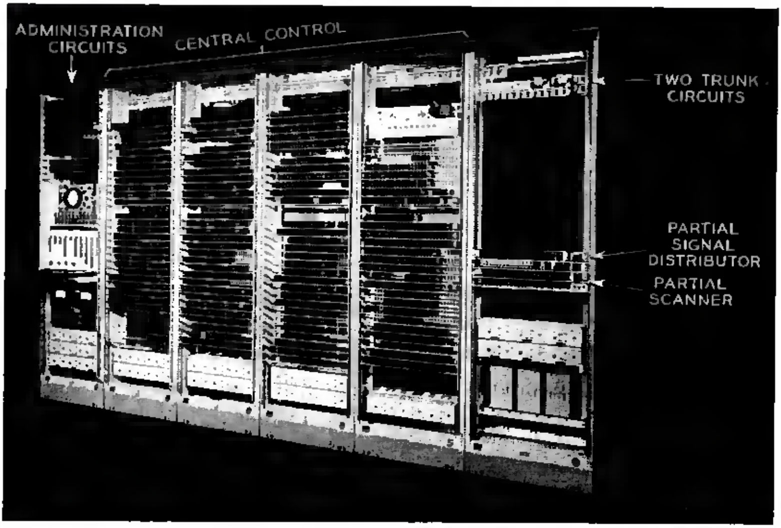


Fig. 8 — Experimental central control (including scanner and signal distributor).

This flow is interrupted during each dial pulse. If the scanner is addressed (directed) to the line during a receiver-on-hook condition or in the middle of a dial pulse, no scanner output results, but if it is addressed to the line when the receiver is off-hook and not in the middle of a dial pulse, an output signal is given by the scanner (see Table II). There are a few simple passive elements per line or trunk in the scanner. These, together with concentrator switching network line crosspoints and selector, are the only per-line units in the system (see Fig. 7). The experimental system includes a skeletonized version of the scanner since only a few lines and trunks are served. It is shown as part of the central control equipment in Fig. 8.

The scanner serves several different purposes. First, it is directed to each line every 0.1 second to determine if the line is requesting service to start a new call, if the line is in use or if the customer has hung up. Once a request for service is detected, the scanner is directed to the line every 0.01 second so as to be sure to recognize the shortest detectable open period that may occur on a line with a 20-pulse-per-second dial. Fig. 9 illustrates the two scanning rates as they occur on a calling line. The scanner is also directed to called lines approximately every 0.1 second during ringing to detect whether the call has been answered. If it has, the ringing of the called line is stopped by release of the connection from the ringing trunk.

### 5.3 *Signal Distributor*

The scanner gathers information when directed; the signal distributor is the component which performs the inverse function: "effecting." For the experimental system it is skeletonized as shown in Fig. 8. The signal distributor consists of a large number of output flip-flops (two-state electronic circuits) which are either set (operate) or reset (release) as determined by addresses and commands sent to it from the central control. The flip-flops respond rapidly, so that these commands may be given to the signal distributor at speeds in keeping with the logic of the central control. The flip-flops can then operate relays or other devices which must be used on individual lines or trunks for signal-control purposes.

An example of this is the sending of supervisory and dial pulse signals over trunks. The signal distributor is addressed to a device in the trunk circuit which will close the loop. Sufficient time must then elapse before the first dial pulse may be sent to allow relays at the distant end of the trunk to respond. During this time (hundreds of milliseconds), the high-speed electronic system controls are busy performing other functions, and the signal distributor may be ordered to set or reset other flip-flops for other slower-speed control needs, such as originating calls on other trunks. Dial pulses are sent (see Fig. 10) by repeatedly addressing set and reset orders at the required rate (10 or 20 pps) to the flip-flop associated with the trunk. During each of these periods the signal distributor may be sending pulses, as directed, over several trunks.

The signal distributor is also used for effecting other slow-speed output functions, such as returning called party supervision to originating offices over incoming trunks. The only slower-speed system functions not performed by the signal distributor are those which take place in the network markers.

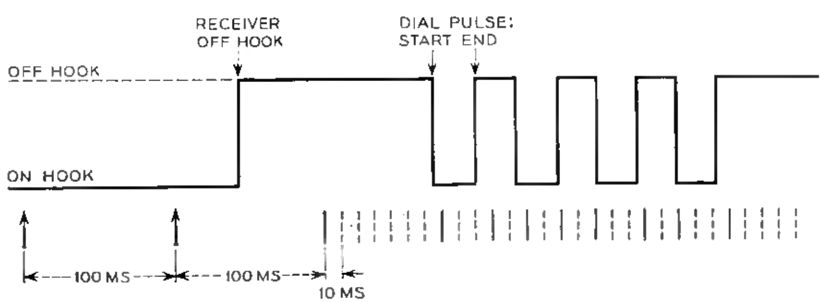


Fig. 9 — Two-rate scanning of calling lines.

## VI. LARGE-CAPACITY MEMORIES AND THEIR USE

### 6.1 *Barrier Grid Store*

There are two types of memory used at present in electromechanical systems: changeable memories, which store call status information such as called numbers as they are dialed, and translation memories, which store information that is needed for determining the routing of a call. Changeable memories usually are locking relays or selector switches with a number of positions (multistate locking relays); translation memories usually are nothing more than cross connections or relay contacts wired so that operating combinations will produce a desired action.

The memory functions of the experimental electronic office are functionally concentrated in new high-speed electronic memory media. The "call status" or "temporary" memory is in the form of a barrier grid or electrostatic storage tube.[13] Each tube is integrated into a complete set of electronic circuits to control the selective addressing, reading and order-writing of any one of the thousands of spots in a few microseconds. The complete assemblage is known as a barrier grid store. The store used in the experimental system is shown in Fig. 11 and described in detail elsewhere in this issue.[3] The barrier grid store is used to record all of the call status information for each line, trunk, register or sender, as relays do in electomechanical systems. The memory spot array of this temporary memory medium is subdivided for these functions.

Each line and trunk has two spots associated with it. The state (0 or 1) of one of these spots (L1) is related to the condition of the line the last time it was scanned. The other spot (L2) indicates whether the line or trunk is being served by the control portion of the system; usually, this means that a coded representation of the line number is stored in

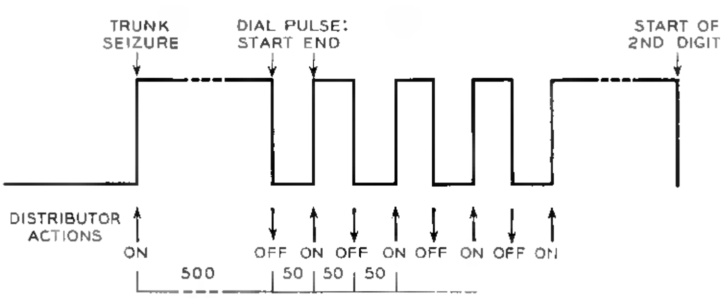


Fig. 10 — Outpulsing by signal distributor.

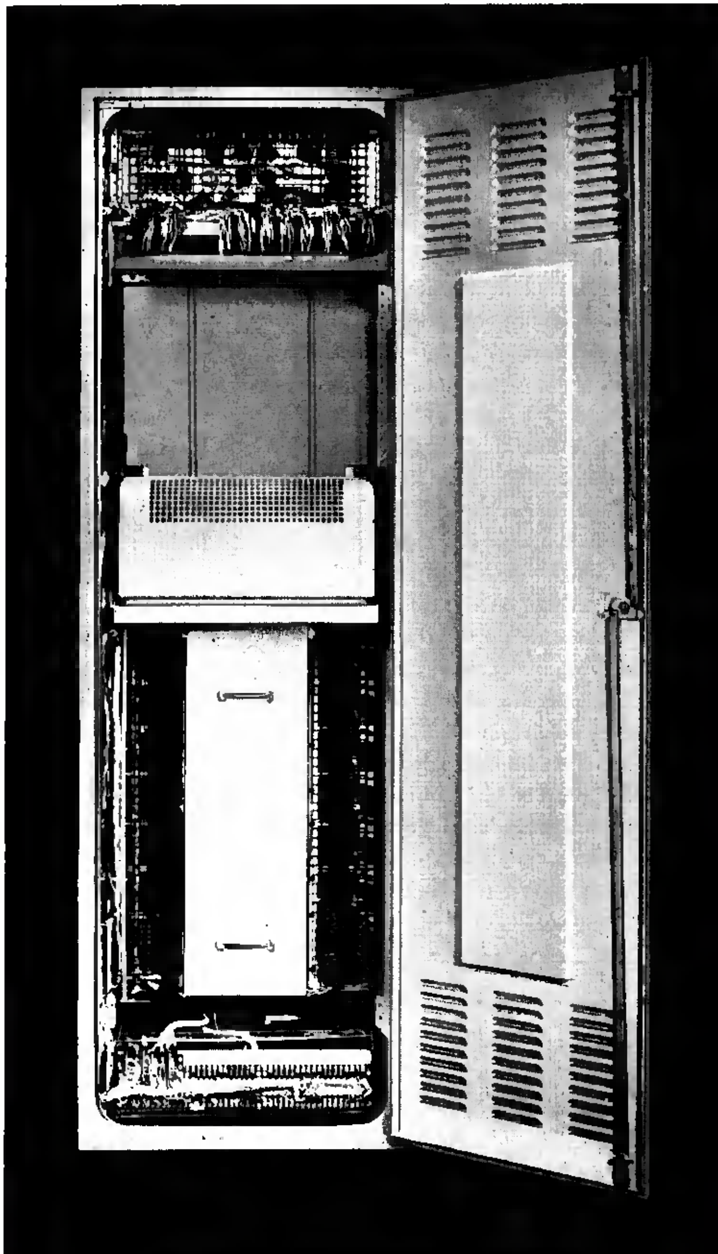

Fig. 11 — Experimental barrier grid store.

some group of spots in the barrier grid store. In addition, there is a spot (T3) assigned for each trunk of a group to aid in the hunt for an idle trunk. These spots are grouped so that the T3 spots of all trunks in the same group are adjacent.

Generally, all spots other than line or trunk spots are grouped by function and called "registers." Registers are assigned so that they comprise columns of spots; that is, the spots for any one register have the same x address coordinate. In the experimental system there are six types of registers:

1. Originating registers — to record dial-pulsed calls and control the completion of intra-office calls.

2. Outpulsing registers — to store information that controls the sending of called numbers over a trunk to a distant office on a dial-pulse basis.

3. Ringing registers — to store information that controls the ringing of the called line and the sending of audible ringing to the calling line.

4. Disconnect registers — to store information that controls the release of established connections.

5. Main program register — to store information relative to the identity of any particular 0.005-second division of time (the basic time subdivision of the system).

6. Network register — to store information concerning the type of call being handled by the network so that the appropriate program sequence can be referred to at the conclusion of a network action.

To enable adequate control by the central control and stored program, the amount and type of information stored in their spots varies among the register types. Table III lists this information for each of the register types. Note that spots are used much like relays in electromechanical switching systems to store coded line or trunk equipment numbers, dialed digits, number of digits dialed (digit location counter) and other numerical items. Other spots are used for call status indications such as "dialing completed" or "network requested". Fig. 12 shows a typical assignment of line, trunk and register spots in the square memory array of the barrier grid tube.

Registers of categories 1 through 4 above are considered as call-carrying registers. The number of each of these types provided is dependent upon the amount of traffic to be handled by the system. All registers of a given type are usually set up so that their corresponding spots have the same Y address. In this way, the barrier grid store is used so that the x address indicates the type of register and the Y address the particular spot in the register.

TABLE III — REGISTER SPOTS

| Originating Registers | Outpulsing Registers | Ringing Registers | Disconnect Registers | Main Program Register | Network Register |
|---|---|---|---|---|---|
| Activity | Activity | Activity | Activity | Ringing scan | Activity |
| Number assigned to calling line equipment | Number assigned to outgoing trunk | Number assigned to called line | Number assigned to line equipment | Timer function | Program address |
| Last look | | Ringing trunk used | | Interval counter | Register number |
| Call state | | Ringing code | | | |
| Abandoned interdigital timer | Interpulse interdigital timer | Don't answer timer | Disconnect timer | Timer | Type of register |
| Pulse counter | Pulse counter and fourth digit | | Trunk guard timer | | |
| Digit location counter | Digit location counter | | | | |
| First digit | Fifth digit | | | | |
| Second digit | Sixth digit | | | | |
| Third digit | Seventh digit | | | | |
| Fourth digit | Signal distributor address | | | | |
| Fifth digit | | | | | |
| Sixth digit | | | | | |
| Seventh digit | | | | | |
| Dial tone | | | | | |
| Permanent signal and partial dial | | | | | |
| Outgoing call | | | | | |
| Terminal line busy | | | | | |
| Network request | | | | | |
| Concentrator trunk used | | | | | |

Each of the call-carrying registers has at least one spot which is used to indicate its busy or idle condition. This spot is known as the "activity" spot and is used when a search is made for an idle register. This concept for register-busy tests is similar to that used for physical trunk (or line) facilities (T3 spots), but it applies only to an entire group of spots in a column.

Spots in the registers may be reused for different call functions at different periods during the progress of a call, thus reducing total memory requirements. For example, groups of four spots are used to record the numerals of the called directory number as dialed. When dialing is completed and it has been determined that the call is intraoffice, the equipment number of the called line, translated from the directory number in the flying spot store, is stored by the spots which formerly held the called directory number.

Some functions in electromechanical systems depend upon the dif-

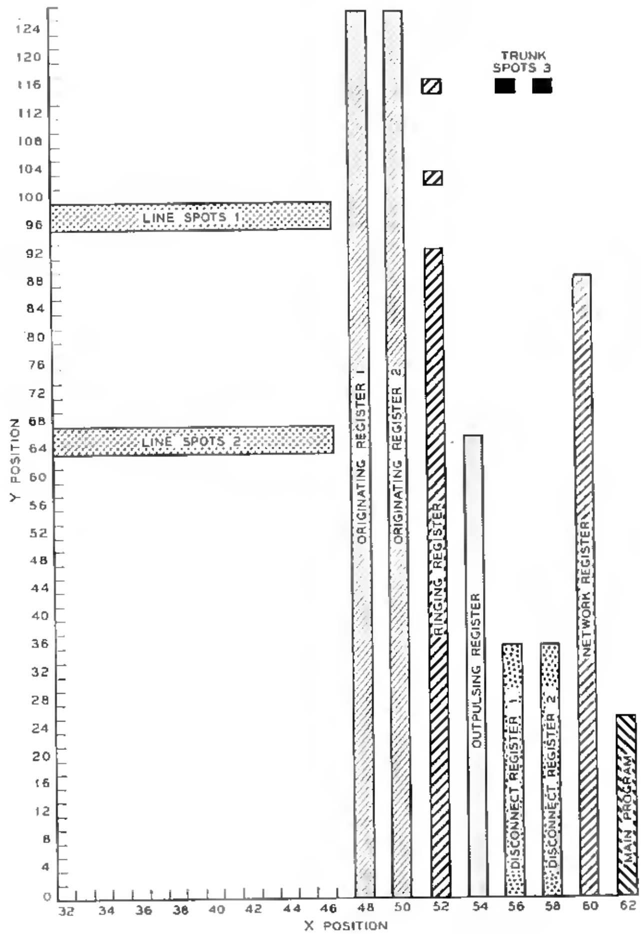


Fig. 12 — Barrier grid store spot layout.

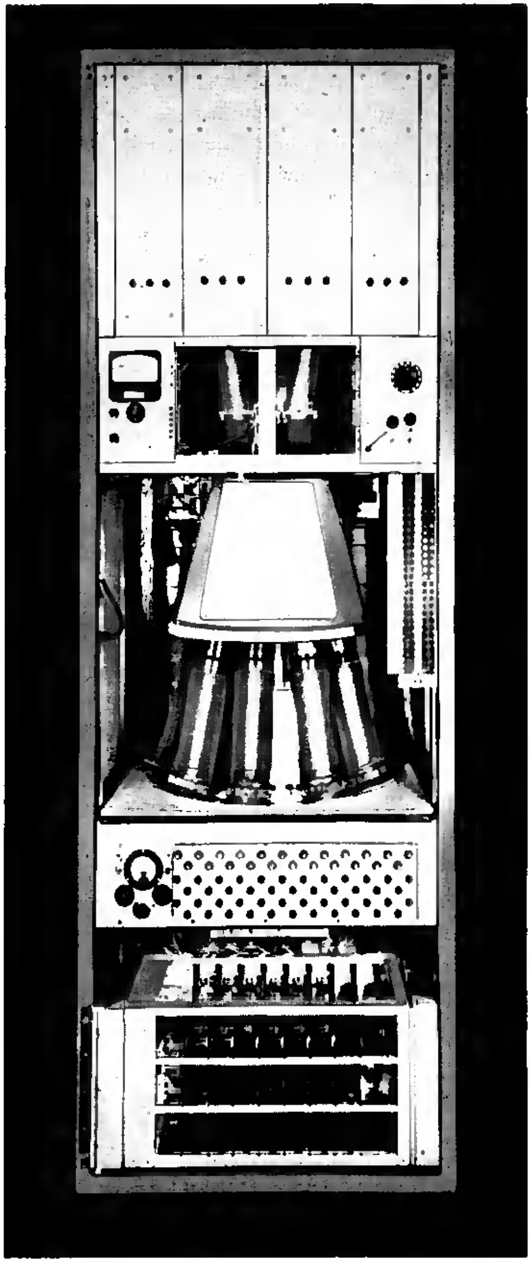

Fig. 13 — Experimental flying spot store.

ference in operate or release time of two or more relays. Such arrangements are used to distinguish a dial-pulse line opening from a call abandonment or a line closure between dial pulses from an interdigital interval. Slow operate or release relays are also used to determine time intervals such as those needed to distinguish a short accidental operation of the switchhook or a hit on the line from a disconnect. In the experimental electronic system, a time-counting technique employs barrier grid store spots which are part of the various registers. A counter consists of a group of spots. If a count is to be scored, the binary record in the spot group is changed by recording the next higher binary number. Scoring is under the control of the main program and may occur each 0.005 second or a multiple thereof.

### 6.2 *Flying Spot Store*

The second new type of memory provides the "semipermanent", "translation" or "wired-in" functions, using photographic plates as the memory medium, with digital information being recorded as transparent or opaque spots. These plates, together with an optical system, a cathode ray tube with digital-servo deflection control and photocells, are combined with electronic addressing and amplifier circuits in what is known as a "flying spot store". As described elsewhere in this issue,[4] stores of this type are capable of recording on photographic plates, selectively addressing and reading any word group of bits of stored information in a few microseconds. Fig. 13 shows the flying spot store used in the experimental system.

### 6.3 *Plate Exposure Process*

The information to be placed on the photographic plates is first keypunched into business machine cards, each card containing an address and the order to be stored at that address. These cards are then processed by punched card business machines which produce a magnetic tape with information arranged by successive flying spot store addresses, a channel at a time. This tape is then read by a portable exposure control unit, Fig. 14, which is temporarily connected to the flying spot store. Unexposed photographic plates are inserted into their channels one at a time while the exposure unit reads the tape, addresses the store and turns the cathode ray beam on or off as determined by the information read simultaneously from the tape. Each plate is exposed in a few seconds

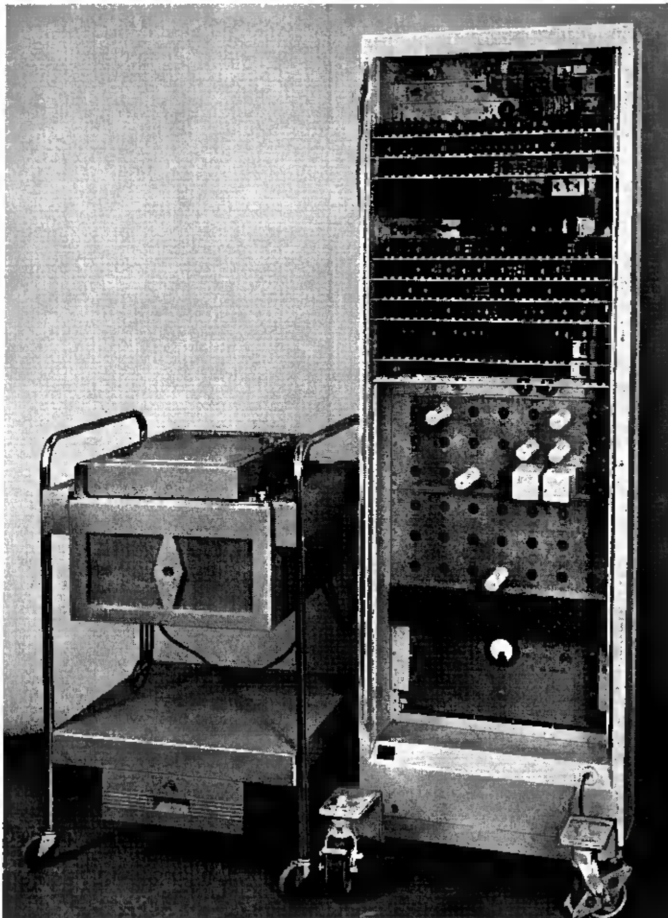

Fig. 14 — Experimental exposure unit.

and then removed to be developed. After being exposed and developed, all plates are inserted back into the proper channels. The magnetic tape may be rerun with the exposure unit in a "verifying" mode of operation, in which the channel outputs are compared, one at a time, with the magnetic tape which was used to control exposure. In this way, any exposure or developing errors may be detected before the plates are used in the system. Once verified, the store may be disconnected from the exposure unit and connected to the system central control.

## VII. SYSTEM TECHNIQUES

### 7.1 *Stored Program Technique*

One of the techniques which is basic to the design of flexible and general-purpose digital computers is also of great value to the present experimental electronic switching system. This is known as "the stored program," in which binary-coded information is stored and read out of the flying spot store as needed to control most of the system's actions.

In a sense, the stored program is a further effort to concentrate functionally. In electromechanical systems the determinations of sequences and consequences of decisions are based on the predetermined circuit wiring in many circuits, markers, registers, senders, trunks, lines, etc. In the electronic system the flying spot store contains sequences of coded commands for the various parts of the system. This memory is ideal for this function since it is permanent and can be changed only by changing photographic plates. Changes in the program are engendered only when a change in service or administration is necessary.

Each order readout of the flying spot store is first interpreted by the logic circuits of the central control in one cycle. Thus, the proper gating and forming of the command to another system component may be executed when the next order is read into the central control from the flying spot store. Sometimes the commands are combination orders for two system components, as discussed above for the supervisory line scan. Other commands for a system component may require more information than can be contained in a single output word from the flying spot store. An example is a command to a marker, in which the parts of the network command are assembled in transistor flip-flops in the central control during several word readouts from the flying spot store until the complete command is ready.

The stored program is designed to be self-perpetuating, so that the system will not stop for lack of knowing what to do. To achieve this, the addressing of the flying spot store is advanced automatically one step at a time in the x coordinate on each readout cycle. This automatic advance is internal to the store, but a duplicate record of the address at which the flying spot store is directed is maintained in the central control.

There are two ways of changing the Y (or X) coordinate of the address to jump the reading to a different portion of the photographic plate. The most important of these is the "decision" transfer, in which the flying spot store address is changed according to the interpretation of the outputs from one or more system components. The other type of transfer is of the "nondecision" type, which is used when jumping to addresses where translations appear, rather than the program. This type of order is also used when the end of one program section is reached, to permit advance to the next section which may be at a completely different address.

Use of the stored program technique permits the command sequences to be tested independently of the switching system. Simulation of the system is accomplished with modern high-speed electronic digital computers. Some 19 kinds of calls were successfully simulated using the stored program on a computer before the experimental system itself became operative.

### 7.2 *Main Program*

As implied above, the program is made up of many sections. Each section contains orders or instructions for handling a particular system routine or type of call. Flow or sequence charts are used extensively to make possible precise specification of required system actions. As may be observed from Fig. 15, which is a layout of the words stored in the flying spot store for the experimental system, some sections are related directly to the types of subdivisions of the barrier grid storage medium. Other sections are related to the frequency with which actions are performed in the system, such as the 100-millisecond line scan or the 10-millisecond dial pulse scan, and to the instructions for dealing with network actions and making translations from directory to equipment number. Overseeing all these routines is a main program which determines the sequence and time limitations, if any, for the individual routines and instructions. The end of each routine or sequence causes a transfer to the next point of the main program. To start the system into operation, the flying spot store is addressed to the beginning of the main program and thereafter the action should be self-perpetuating if the program is logically correct.

The main program is a timetable and priority list of the major functions which the system must perform. Successive cycles of the main program are usually not the same. In the experimental system the main program cycles are normally 5 milliseconds in length. Barrier grid store spots are assigned (see Fig. 12) and used to keep track of the cycle number so that the predetermined main program for that cycle may be carried out. Odd-numbered 5-millisecond cycles may scan lines for odd-numbered originating call registers and even-numbered cycles scan lines

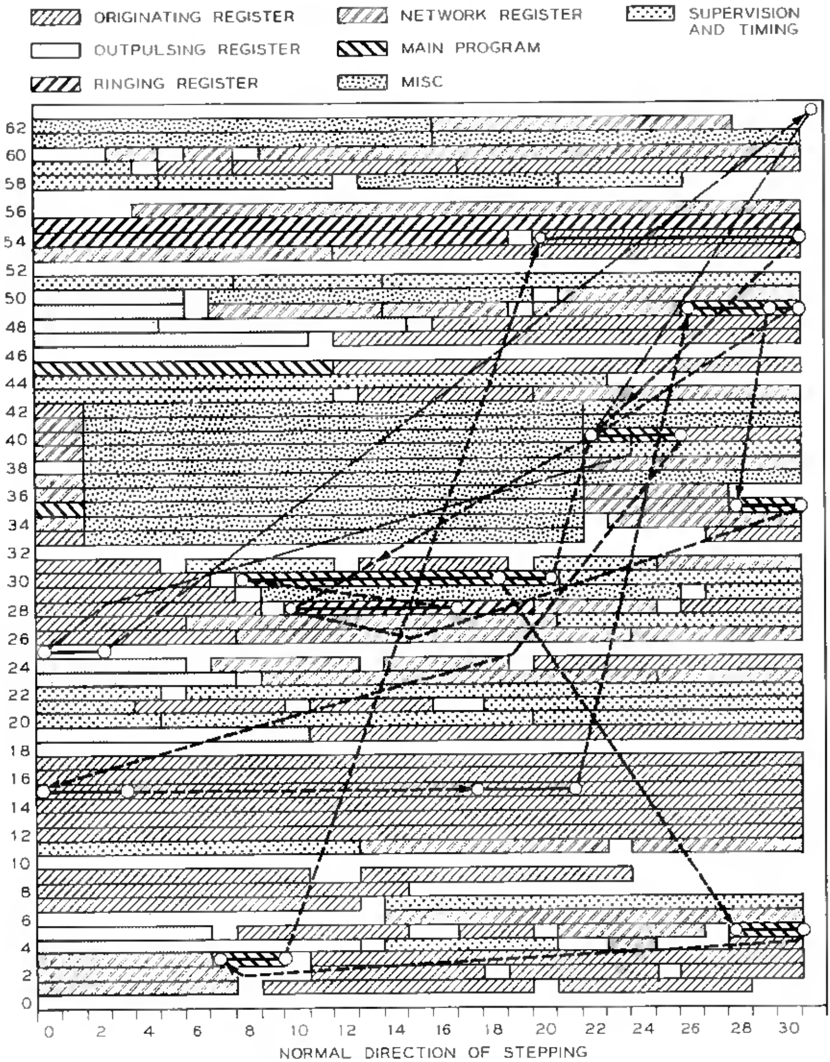


Fig. 15 — Flying spot store spot layout, with typical random access program sequence.

for even-numbered registers. Once during 20 cycles the main program shifts to various functions that must be performed each 0.1 second.

Most cycles provide intervals for commanding or acknowledging completion of network actions. A conventional busy test of the network is made during one central control cycle, as part of a main program sequence to determine whether the network may be addressed during this cycle of the main program. This characteristic is known as "network break-in". Similar break-ins alter the main program for synchronizing the scanning of lines for answer during modulation cycles of the ringing tone. Fig. 16 shows a typical main program cycle. It should be noted that the main program is another manifestation of the time-division form of system control.

Since each main program cycle may take a different length of time to carry out its intended functions, depending upon the amount of traffic and break-ins, there is an independent 5-millisecond timing circuit in the central control which the main program refers to after each routine or functional sequence has been completed. In this way, a cycle longer than 5 milliseconds is ended as soon as possible.

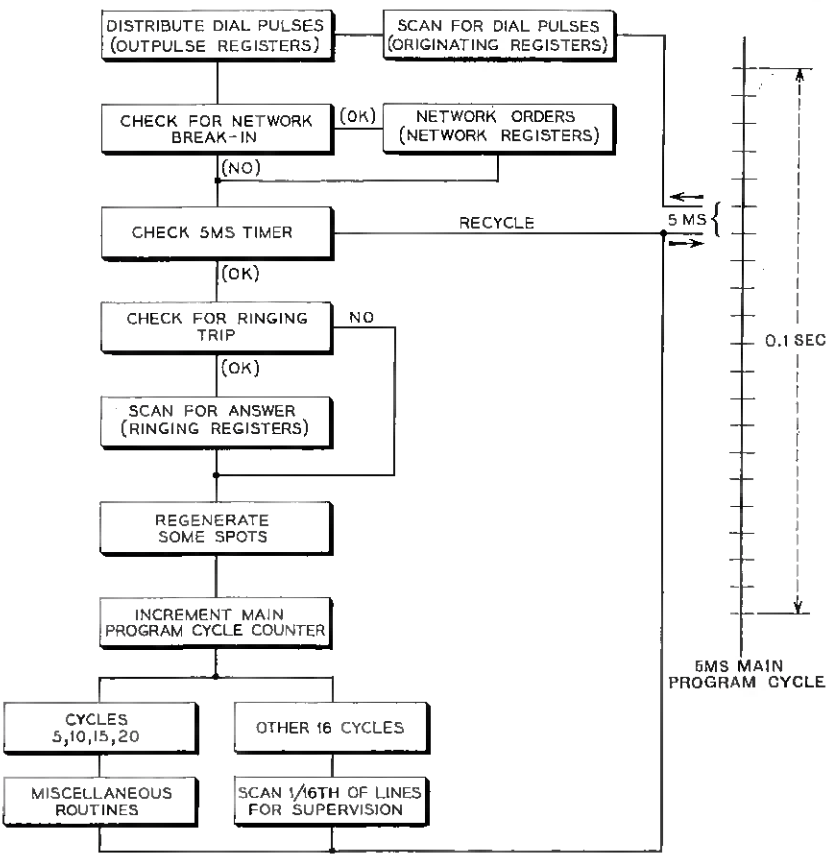


Fig. 16 — Main program.

### 7.3 *Central Control*

Fig. 1 shows the interrelation of the major components of the system described in the previous sections. Note that the commands for all the system components pass through central control. The central control, shown in Fig. 8, contains circuits which use 1500 germanium alloy junction transistors and 15,000 germanium point contact diodes. These circuits, which are combined to perform the required logic functions, are described in detail elsewhere in this issue.[5]

The central control makes all decisions which control the flow of commands to the various parts of the system.

None of the system's components can work alone or make decisions which will change the routing of a call or other system action. In many cases, the central control must command and receive output information from more than one component before a decision is made. For example, reading the scanner output alone is not enough to make a decision that a new request for service or a disconnect has originated. To do this, the call status store (the barrier grid store) must be consulted. In the store there is a memory spot associated with each line, which is examined at the same time the scanner is directed to the line circuit. This spot is kept up to date by commands from the central control indicating what the scanner saw the last time it was directed there. By comparing, in the central control, the output of the scanner and the barrier grid store at the address corresponding to each line, it is then possible to decide whether there is a request for service or disconnect. (See Table II).

Another example of this cooperation is in setting up a network connection to an idle trunk. The network addresses for the trunks to a particular destination are reached by addressing the flying spot store on the basis of the office or service code dialed. The word read out of the flying spot store gives translation information such as the class of the trunk, as well as the network address. Before the address is passed to the network marker with a command to set up the connection, a spot in the barrier grid store associated with the trunk is addressed to determine whether the trunk is idle.

In the first of these two examples of cooperation, the actions in the various system components occurred simultaneously. In the second, the cooperation occurred as a sequence of actions based on information received from one component and passed via the central control to another. This means that a certain amount of short-term memory, using transistor flip-flops, is needed in the central control. The information is stored temporarily for one or more cycles until this small step in the call has been completed or while it is being worked on by a slower part of the system, such as a network or trunk.

## VIII. CALL DESCRIPTION

Having described the various parts of the experimental electronic system and the new techniques employed, we may now trace the progress of a call through the system. The high-speed time-division type of control makes it necessary to describe more steps for a complete call than are necessary in an electromechanical system. It has been estimated that, at the level of detail with which a call will be described in this section, three times as many steps are required than for a comparable description of a call progressing through the most modern electromechanical system.

The description will assume an intra-office call to an idle line. No special situations such as call abandonments, all registers or trunks busy, permanent signal or partial dial, network blockage or busy, translation changes, reverting calls or overloads on the control will be covered. However, the experimental system is designed to care for all of these situations.

1. Detect Call Origination — With the system components functioning, the main program causes each line to be scanned every 0.1 second. Approximately one-twentieth of the lines are scanned on each 0.005-second main program cycle. Simultaneously, one of the line memory spots (L1) corresponding to each line being scanned is examined for a change in line state. When the calling party lifts the receiver from the hook, the change is detected on the next 0.1-second scan or, on the average, within 0.05 second.

2. Hunt For and Seize Idle Originating Register — After the call origination has been detected, the availability spots of originating registers are examined until an idle register is found. It is then made busy, and all information previously recorded, which might confuse this call, is erased. The calling line equipment number is written in the appropriate spots of the originating register. A register spot is marked to indicate that it desires to use the network to set up a connection to a dial tone trunk. The L2 spot of the line is marked to indicate a call is now being served by a register.

3. Dial Tone Connection — Assuming the concentrator marker is available, it is addressed with the originating line equipment number. The program order address, which will be required later for the next network action, is placed in the network register. The concentrator marker sets up a connection from the calling line through the concen-

trator to a trunk to the distribution switching network, preferably the A or left side. In this process the identity (address) of the trunk is recorded in the marker. The main program, in the meantime, continues to complete the 0.1-second scan of lines and other functions. Once in each 0.005-second main program cycle the network markers are examined to see if the network is ready for new action (network break-in).

To continue the network actions, the next flying spot store address is read from the network register in the barrier grid store. This program includes routines for the flying spot store to obtain dial trunk spot addresses in the barrier grid store. These spots are examined until an idle trunk is found and marked busy. Its address is then used in the flying spot store to find the address of this trunk on the distribution switching network. Together with the trunk number from the concentrator marker, the address is then placed in the distribution marker with an order to establish the dial tone trunk switching network connection. Once the connection has been successfully established, the calling line receives dial tone and the distribution marker awaits a main program break-in. When this occurs, the network register is made idle and the originating register is marked to indicate that the dial tone connection to the customer is complete and dialing may now start.

4. 10-Millisecond Dial Pulse Scan — At the beginning of each 0.005-second main program cycle, half of the originating registers are examined. If any are busy with a call in the dialing state, the originating line equipment number is read out into the central control, which then directs the scanner to the line to determine if there has been a change from the last time (0.010 second ago) the line was examined. This process continues every 0.010 second as long as dialing is expected.

5. Detect First Dial Pulse — To avoid changing the L1 line spot with each dial pulse, a "last look" LL spot is provided in the originating register, and this is read out along with the line number when the scanner is directed. This spot is compared with the scanner output and, if a change from closed to open is noticed, the beginning (open) of the first dial pulse is detected. As a result, the dial-pulse counter spots are changed to read binary 1 and the abandoned call-timer spot is erased to start a time-count cycle if this line open is longer than a dial pulse. Each succeeding change from closed to open of 0.010-second scans increases by one the binary number in the pulse counter spot.

6. Release Dial Tone Connection — The originating register spot indicating there is a dial tone connection is examined after the first pulse is detected. A test is then made to determine if the network is available. Assuming the distribution network marker is idle, the calling line equip-

ment number is read out of the originating register and transferred to the marker with an order to release the dial tone connection. The line number is coded so that it includes the number of the concentrator switch and the trunk. Only this part of the line number is read out to mark the end of the trunk in the distribution switching network which is to be released. When the connection is released, the identity of the dial tone trunk appears in the distribution marker and is passed to the central control on the next network break-in of the main program. With the dial tone trunk number known, the trunk busy spot T3 is changed to show that the trunk is now idle. It should be noted that the concentrator connection is maintained, so that later connections to the calling line will not block in the concentrator.

7. 100-Millisecond Examination of Originating Registers — Each 0.1 second the main program causes the abandoned call or interdigital time-count spots of the busy originating register to be changed. On each line change detected on the 0.010-second dial-pulse scans, these time counter spots are restored to zero. If a count of two is reached with the last look spot showing a line-open condition, an abandoned call is detected and the network is requested to release the concentrator connection. A count of two with the line showing a closed condition indicates the end of a digit. The binary count in pulse counter spots is read and stored in the central control and the counter cleared in anticipation of the next digit.

8. Move Dialed Digit to Digit Register Spots — A group of spots known as the "digit location counter" spots is read to indicate to the central control the Y address at which the dial-pulse count is to be stored. The binary number in the digit location counter is increased by one. The dialed digit is then written into the appropriate digit spots of the originating register. The main program then continues to make the 0.1 second examination of the time counter of next originating register.

9. Recognizing Local Central Office Code — When the digit location counter reads 3 and the third digit is transferred to the appropriate set of digit spots, the first three dialed digits are read out and referred as an address to a translation area of the flying spot store. The resulting output will indicate, among other things, that the call is intra-office, so an indication is made in the originating register that the call is not an outgoing or service code call.

10. After Registering Seven Digits — When the seventh digit has been counted and registered during a 0.1-second examination, the spots which indicate the state of the register are marked to show that dialing is completed. Thereafter the 0.010-second dial pulse scan is not made for this register.

11. Translation of Called Number — Since this is an intra-office call the four called numerical digits (16 bits) are read out into the central control. Some of these digits are referred to a special translation area of the flying spot store to compress the coding so that the line translation address will fit the address limitations of the store. The called line translation is then made from the compressed directory number to an equipment number and class of service of the called line, including ringing information.

12. Party Lines — Reverting Calls — If the ringing information indicates a party line has been called, the calling line equipment number is read out of the originating register and matched in the central control against the called line equipment number to determine if the call is for another party on the same line.

13. Place Called Equipment Number in Originating Register — The called equipment number in coded form is written in the same spots where the called directory number was accumulated. The interpretation to be placed on this information in the future is determined by the code recorded in the call status spots of the register.

14. Line Busy Test — The called line number is used by the central control to examine the line spots to determine whether or not it is idle. If it is idle, the L2 line spot is changed to indicate that the line is being served by the control portion of the system.

15. Establish Ringing Connection (Fig. 17 should aid in understanding these connections; refer to the circled numbers) — With called line and

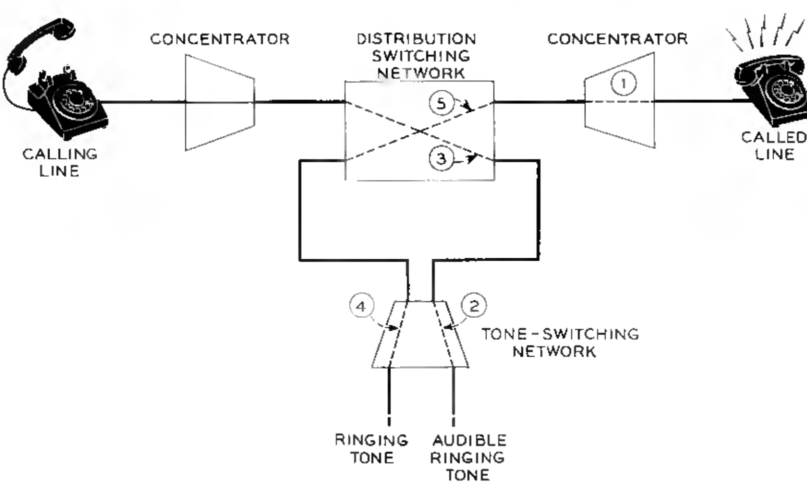


Fig. 17 — Ringing connections.

the network markers idle, the ringing function is started. ① The called equipment number is placed in the concentrator marker with a command to set up a connection between the called line and an idle trunk to the right or B side of the distribution switching network. This completes the actions taken during this 0.1-second register examination by the main program for this call. The number of the trunk to which a connection is established from the called line is read out of the concentrator marker and placed in the central control on the next network break-in operation. An idle ringing trunk is then hunted by examination of spots in the barrier grid store, and an address is obtained from the flying spot store in the same manner as for the dial tone trunk (item 3). The ringing trunk number and the address of the trunk from the concentrator connected to the called line are held in the central control. The former is used as an address for the next operation of the concentrator marker ②, when the audible ringing tone is applied to one of the tone switch output trunks to the right or B side of the distribution switching network. In the process, the particular trunk which is used from the tone switch is identified.

On the next network break-in, the distribution marker is given the address of the output trunk of the ringing switch through which the audible ringing tone is connected. The address of the trunk connected to the calling line, as interpreted by the central control on reading out the calling line equipment number from the originating register, is also read into the distribution marker, and the required distribution network connection ③ is set up. The calling party now hears ringing tone. The originating register number is held in the network register so that it knows what call it is handling.

The next network break-in is used to establish the ringing tone path to the distribution switching network through the signal network ④. The identity of the trunk from the ringing tone to the distribution switching network is passed to the central control on the next break-in. At this time the final link ⑤ for the ringing connection is established. Tone ringing current of the correct frequency now rings the called telephone.

16. Hunt For and Seize Idle Ringing Register — On the next network break-in of the main program the distribution marker reports the completion of the last link of the ringing connection. The barrier grid store is addressed, by the stored program referred to by the network register, to search for an idle ringing register. The selected register is then made busy. The called line equipment number and the address of the trunk connected to the ringing tone on the trunk switching network are written in the register.

17. Release of Originating Register — The calling line spots are now changed from the control condition and set to indicate talking. The orig-

inating register availability spots are changed to indicate that the register is now idle and available for use on a new call.

18. Scanning Called Line for Answer — Each 0.005 second during the silent period of the ringing cycle, or when the ringing modulation goes through zero, the called line number is read out of the ringing registers. The line number is used to address the scanner to detect the call answer. The register also has spots which time the ringing period so that a limit may be placed on unanswered time.

The answer is detected as a change from line open to closed during the appropriate 0.005-second main program cycle. At this time, the line spots of the called line are taken out of the control condition and put into the talking condition.

19. Release of Ringing Connections — Commands are issued simultaneously to the concentrator and distribution markers with information read from the ringing register to release the ringing switch and trunk switching connection to the called line.

On the next network break-in after these release functions have been successful, the trunk addresses identified in this process are held in the central control for use in establishing the talking connection. Release commands are now sent again to the network markers to take down the audible ringing connection to the calling party. This is also based on the knowledge of the ringing switch trunk address now stored in the ringing register.

20. Establish Talking Connection — On the next network break-in upon the successful release of the second half of the ringing connection, a command is given to the distribution marker to establish a talking connection.

On the next network break-in, when the successful establishment of the talking connection is reported, the availability spot of the ringing register is changed to make it available for another call.

The conversation between the two parties now takes place.

21. Detect Disconnect — The main program continues to cause each line to be scanned every 0.1 second. When, referring to the line spots, a change from line closed to line open is detected on either the calling or called line, a disconnect program is started.

22. Hunt For and Seize Idle Disconnect Register — A search is made immediately for an available disconnect register. The line number is recorded in the appropriate spots and the time-counting spots are set to zero.

23. Disconnect Timing — Each 0.1 second the main program incre-

ments the timer until at least 0.3 second has elapsed. At this time, a spot in the register is marked to indicate a request for the network when next it is idle on a break-in portion of the main program cycle. On each count scoring the timer, the line number is read out of the barrier grid store to address the scanner. This insures that the disconnect is bona fide, since a line hit might also have caused the line to appear open momentarily when the line open was originally detected. During this timing interval the other party's disconnect may also be detected. A different idle register is seized and timing started.

24. Release Talking Connection — If the network is idle, the line equipment number is read from the disconnect register and placed in the concentrator marker as the address part of a trace command. As on all network operations, the number of the disconnect register is written in the network register, together with the type of network action taking place.

On the next network break-in of the main program, the concentrator marker reports back to the central control with the number of the trunk being used by this line in the trunk switching network. The network markers may now proceed to accept release commands based on this trunk number.

The distribution marker identifies the address of the trunk connected to the other party's concentrator. On the next network break-in after successful execution of the previous commands, this address is given to the concentrator marker with a command to release the last link of the intra-office connection.

On the next network break-in, when the successful execution of this last command is reported, the disconnect register is again made available and the line spots of the line whose number is read out of the disconnect register are changed from the control to normal supervisory state.

Later, when the other party's disconnect is detected and timed, the concentrator marker, in response to the trace command, finds the concentrator connection already released and so reports this condition on the next network break-in. At that time, the disconnect register and line spots are restored to their normal states.

## IX. CONCLUSION

A number of new electronic switching techniques have been described and their applications in the first experimental laboratory system illustrated. This system is shown in Fig. 18 as it was assembled for proving the feasibility of these new approaches. In addition to the features described, the left bay of the central control (Fig. 8) contains key and

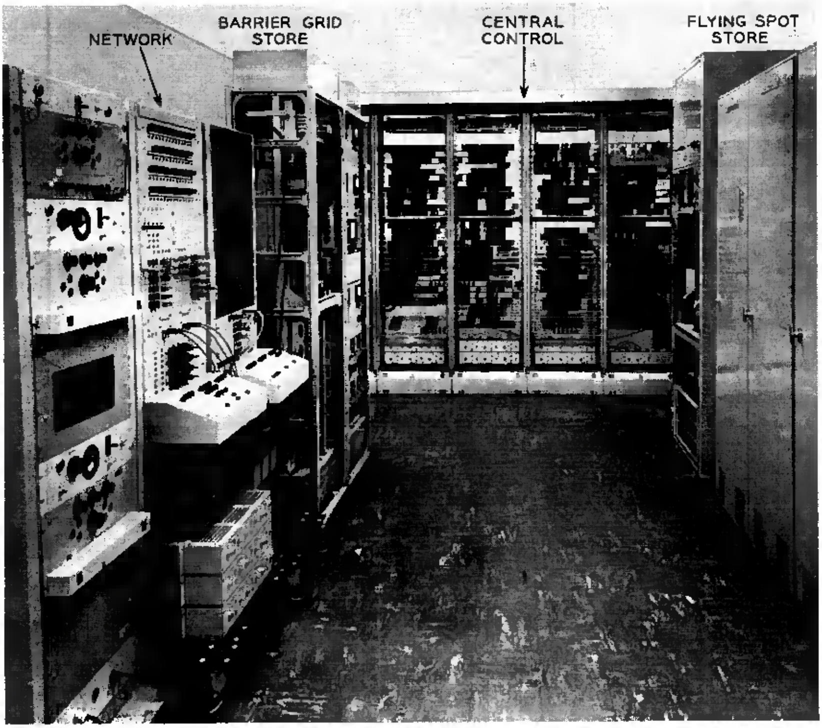


Fig. 18 — General view of experimental system.

indicating equipment to aid in the testing of the system. This includes means for regenerating the barrier grid store when the system program is not functioning during the location of trouble and means for inserting orders into the central control manually.

Descriptions of the circuit, apparatus, equipment and program details appear elsewhere in this issue and will be published in future issues of this publication.

Much has been done to develop the techniques of this experimental system to the point where they should be satisfactory for use with traffic from telephone customers. Other system considerations are being developed before the commercial and trial applications of these techniques take place. Maintenance facilities; system expansion beyond the memory capacities of the present stores; administrative procedures, including traffic recording, remote concentrators, power facilities and other services such as operator class calls are among the new system items which are being developed. At the same time, intensive study of component and system reliability is proceeding to insure the successful operation of this system.

## X. ACKNOWLEDGMENT

The philosophy of this system is an outgrowth of engineering studies made by C. E. Brooks.

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# Semiconductor Circuit Design Philosophy for the Central Control of an Electronic Switching System

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*The advent of electronic switching has necessitated a considerable number of changes in the circuit design philosophy employed in the electromechanical switching art. The invention of the transistor and the refinement of other semiconductor devices have made possible new techniques. This paper discusses the philosophy of circuit design for the central control of an electronic switching system. Primary emphasis in the designs has been low cost consistent with good margins, reliability and the meeting of systems requirements.*

## I. INTRODUCTION

The proper selection of semiconductor switching circuits to be employed in electronic switching systems is influenced by a large number of different factors. Some of these are the result of systems philosophy,[1] while others are concerned with device availability, required reliability, economy and ease of design, manufacture and maintenance, etc. We do not propose to go through an exhaustive design analysis of the circuits described herein, but we shall present the techniques which were developed.

The particular experimental electronic switching system of interest here operates with a stored program in real time and in a generally synchronous serial mode, utilizing direct-coupled (dc) logic. The largest concentration of semiconductor circuitry is in the central control of the system, which is the data and information processing center of the telephone central office. It is with the circuits in the central control that this paper is mainly concerned.

In most present-day electromechanical switching systems many operations take place in parallel, due to the speed limitations of relay circuits. Electronic switching systems also can be built using the parallel building block approach, but significant savings in quantities of apparatus are possible if serial operation is employed. The serial mode, however, makes it necessary for the processing equipment to operate at much higher speeds. In fact, for the particular electronic system under consideration, it is necessary for a program order to be executed in a few microseconds if the office is to handle the telephone traffic properly.

Although most parts of the electronic switching system operate in a serial mode, the central control itself operates in a parallel mode. The speed limitations of available semiconductor devices and circuits are a contributing factor to this choice. However, the program order structure, the manner in which information is stored in the system and the way in which the central control affects the operation of the other major system blocks in the office are of even greater importance in the selection of the parallel mode. It should be noted, however, that in receiving address words from the barrier grid store some serial-to-parallel conversion equipment is required, since this memory furnishes information only one bit at a time.

The central control operates in a generally synchronous manner, using a single-phase clock having a frequency of several hundred kilocycles. However, the synchronous circuit actions are controlled by some asynchronous logic circuits which can stop the clock when certain delays are encountered in system functions, and then restart it. After the clock has been restarted, it runs at a constant rate. The design of synchronous circuits is somewhat more economical in equipment and lends itself more readily to the automatic checking features employed in this system.

The logic circuitry of the central control is direct-coupled (dc), that is, signals are represented by steady-state voltages. Two levels are employed, one representing a binary 1 and the other a binary 0. The memory elements used are flip-flops with double-rail output signals to drive the logic circuits. A double-rail output is one in which both the logical 1 and 0 conditions are represented by active signals on separate leads. Only one of the two leads, however, has an active signal at any time. The use of dc circuitry avoids a design problem implicit in ac circuitry. In order to obtain the AND function in an ac system, precise coincidence is required of pulses applied to an AND gate. Since delays are encountered as a pulse progresses through the logic, the design of circuits which will cause all pulses to be propagated to the right place at the right time can be quite complex. In order to make signals coincide exactly in time, extensive use of precision delay apparatus and multiphase clocks is generally required. The ac system also lacks a certain amount of flexibility, since the addition of any function generally requires the overhauling of other sections of the logic in order to maintain the proper signal coordination. With dc circuits, on the other hand, unequal chains of logic may exist with no particular problems. Since both flexibility and economy are of importance in an electronic switching system, the dc mode was chosen.

To a major degree, the advent of electronic switching was made possible by the invention of the transistor and the refinement of other semiconductor devices. The required logical functions are highly complex and necessitate thousands of active devices in the central control. Before the development of the transistor, hot-cathode vacuum tubes were the only available active elements for use in high-speed switching systems. The inherently high-power-consuming vacuum tubes require relatively high-cost circuits for adequate reliability, and this makes the vacuum tube unsuitable for large-scale electronic switching applications. However, semiconductor devices of moderate cost have the high reliability, high speed and low power consumption required for large-quantity use in such systems. It is believed, from the evidence so far available, that the reliability of the transistor and semiconductor diode will be orders of magnitude better than that of vacuum tubes performing comparable functions.

After deciding upon the use of semiconductor devices and synchronous dc circuitry in the central control, several other general decisions remain to be made. It is possible, if one is not concerned with the amount of equipment involved, to reduce all information-processing circuits to two tandem stages, an AND and an OR. However, two-stage logic is generally nonminimal. The use of multistage logic results in a reduction in the equipment needed for simple circuits and in even greater savings in complex circuits. However, a longer time is required for the logic operation, since information is passed through more stages and the delay time of the signals will increase with the number of stages. Greater attenuation of the signals also will occur. Therefore, in multistage logic an economic balance must be achieved between equipment savings and the cost of additional time. This balance limits the minimization of the logic circuits.

Another approach which leads to a less costly design, although perhaps to more circuit elements, is the development of universal circuit packages. The use of such universal packages in the central control is made possible by distributing amplification throughout the logic. These subjects are more fully covered in Sections III and IV.

The use of these packages has allowed a separation between logical and circuit design. The building blocks can be treated as black boxes by following a few simple interconnection rules, thus considerably reducing the over-all design time. The circuits have also been designed to be consistent with marginal and diagnostic testing techniques developed for the system.

## II. SEMICONDUCTOR DEVICES

Transistors and semiconductor computer diodes were specifically developed for this electronic switching system. While the central control is the major user of these devices, they are also employed in large numbers in other portions of the system, such as the switching network and scanner. In order to develop units of high reliability and to obtain the economic benefits of high-level production, it was felt necessary to minimize the number of designs and codes for the system. Therefore, the device characterizations represent a compromise among all possible uses in the system and, as such, are not necessarily the best for any individual application. The over-all benefits obtained from a smaller number of codes, however, more than compensate for this disadvantage.

The transistors employed in this electronic switching system are a complementary pair — an n-p-n and a p-n-p designed for medium power and relatively high-speed switching applications. The transistors are capable of dissipating 450 milliwatts in free air at 25°C without the use of an external heat sink. In general, the logic circuits use the tran-

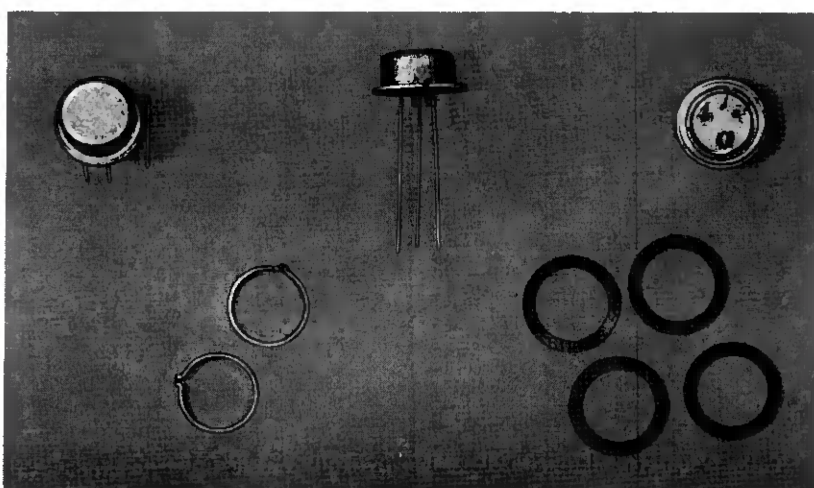


Fig. 1 — Germanium alloy junction transistor for electronic switching system central control.

TABLE I — TRANSISTOR CHARACTERISTICS

| Parameter | Limits | |
|---|---|---|
| | n-p-n Transistor | p-n-p Transistor |
| Collector breakdown voltage | 30 volts min. | 30 volts min. |
| Emitter breakdown voltage | 25 volts min. | 25 volts min. |
| $I_{co}$ at 15 volts (25°C) | 35 μa max. | 15 μa max. |
| Punch-through voltage | 25 volts min. | 25 volts min. |
| Common emitter current gain | | |
| $I_c = 25$ ma; $V_c = 1$ volt dc | 50 min. | 50 min. |
| $I_c = 150$ ma dc; $V_c = 1$ volt dc | 25 min. | 25 min. |
| Collector capacitance at 4.5 volts | 45 μμf max. | 45 μμf max. |
| Inverse gain bandwidth (IGB) | 0.25 μsec/cycle max. | 0.25 μsec/cycle max. |
| Power dissipation, free air 25°C ambient | 450 mw | 450 mw |

sistors well below their power rating in order to obtain satisfactory system reliability. Fig. 1 is a photograph of the transistor, and Table I lists its important electrical characteristics, for both n-p-n and p-n-p.

These characteristics are self-explanatory except for inverse gain bandwidth (IGB). This is a measure of switching speed and is related to the normal $f\alpha$ measurement for transmission applications. The main difference between the transmission and switching fields lies in the conditions of operation. In the transmission field (for other than power amplifier applications) a bias point is chosen and small excursions are made around this point. In the switching field, the whole operating region is traversed, and the effective values of the parameters are obtained by integration over this region. Although the parameter $\alpha_n\omega_n$, the gain-bandwidth product of the transistor, is useful in both fields, in transmission this parameter is a function of the chosen bias point, while in switching this parameter is a weighted average taken over the whole range of bias points traversed, and properly should be labeled $\overline{\alpha_n\omega_n}$.

A corresponding method of measurement must be chosen, such as putting the transistor in a standard switching circuit and arranging to measure $\overline{\alpha_n\omega_n}$. One way this can be done is to compare the output waveform in a sufficiently low-impedance circuit with a waveform derived from a simple $RC$ integrating circuit. The low-impedance circuit eliminates effects of collector capacitance; the use of the same voltage supply cancels the effect of voltage variations.

It can be shown that the quantity $\overline{\alpha_n\omega_n}$ is inversely proportional to the $RC$ product if the initial slopes of the rise portions of the pulses at the outputs of the transistor and $RC$ circuits are matched. In the actual measuring apparatus, a further factor of $2\pi$ has been introduced and

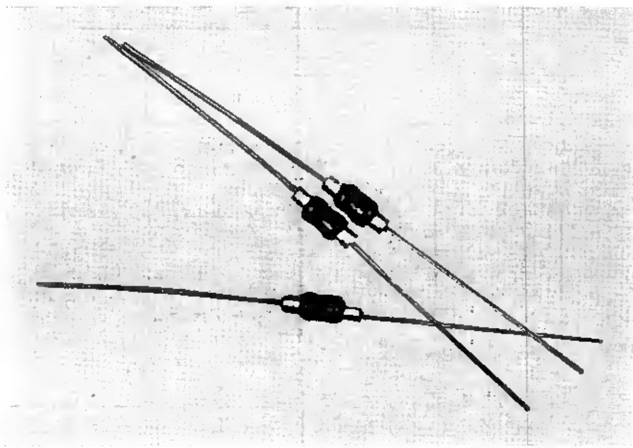


Fig. 2 — Germanium point-contact diode for electronic switching system central control.

the quantity $1/\overline{\alpha_n f_\alpha}$ is read directly. This is called the inverse gain-bandwidth product or IGB and is measured in microseconds per cycle. Where $\alpha_n \cong 1$, as it is for these transistors, IGB is approximately equal to $1/\overline{f_\alpha}$. Therefore, the upper limit of IGB, 0.25 microsecond per cycle, roughly corresponds to an average $f_\alpha$ over the active region of 4 mc minimum.

Two codes of semiconductor computer diodes have been developed for the logic and switching circuits. Both are glass-enclosed, hermetically sealed, point-contact germanium diodes (Fig. 2). One of these units has a higher forward conductance and is used in AND gates, where the series voltage drop is of more importance. The other diode has better reverse characteristics and is used in OR gate applications, where the effect of leakage currents has a greater effect on circuit operation. Both diodes have maximum ratings of 100 ma steady-state (dc) forward current with a peak surge-current limit of 500 ma. The maximum inverse working voltage is 100 volts. Maximum power dissipation at an ambient temperature of 25° C is 200 mw, with a derating factor of 3 mw per degree C for ambient temperatures above 25°C The other relevant characteristics of these diodes are given in Table II.

## III. TYPES OF SEMICONDUCTOR LOGIC CIRCUITS

There are two general requirements which must be placed on a logic gate. The first is that the state of the output of the gate must depend in

TABLE II — DIODE CHARACTERISTICS

| Electrical Requirements at 25°C | Code 1 | Code 2 |
|---|---|---|
| Max. forward voltage drop at 20 ma | 1.5 volts | 1.2 volts |
| Max. reverse current at −5 volts | 2.2 μa | 4.5 μa |
| Max. reverse recovery time | 0.06 μsec. | 0.06 μsec. |
| Max. inverse voltage | 100 volts | 100 volts |

some logical sense on the states of the inputs. The second is that each of the inputs must remain free to change state independently of the states of the other inputs, i.e., the logic gate must maintain isolation between the inputs.

It is convenient to classify the various methods of constructing physical embodiments of logic gates in terms of the devices used to obtain the isolation of inputs. These devices constitute a very large percentage of the equipment required for a logic system, since at least one device is required per input. There are two common methods of obtaining this isolation in semiconductor logic circuits, one using diodes and the other using transistors. These are referred to as diode logic and transistor logic circuits.

Diode logic circuits have been chosen for use in the central control. This decision was reached because semiconductor diodes at the present time are considerably less expensive than transistors, are capable of operating at higher speeds, and have achieved better reliability. The use of transistors in the central control has been restricted to amplification and memory functions, so that approximately seven times as many diodes as transistors are used.

## IV. DIODE GATES

Two-input conventional diode AND and OR gates are illustrated in Fig. 3. Additional inputs could, of course, be added. The logical function performed by each of these circuits depends on whether the high signal voltage ($V_1$) or the low signal voltage ($V_0$) is considered to be the active,

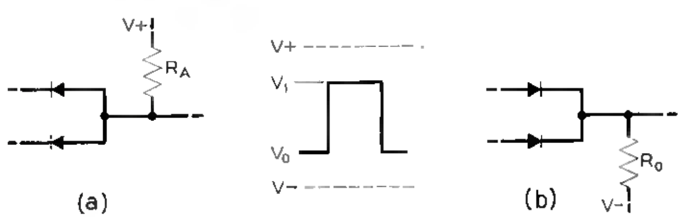



Fig. 3 — Conventional diode gates: (a) AND gate; (b) OR gate.

or 1 condition. These conventions are referred to as "positive" logic and "negative" logic respectively. In the central control, positive logic is used, i.e., the higher voltage condition is considered the 1 state. It is obvious that, if this convention is adopted, the circuit in Fig. 3(a) performs an AND function, since the output will be clamped to the lowest input voltage and, hence, a coincidence of high signals is necessary at the inputs to give a high signal at the output. The circuit in Fig. 3(b) can be seen to perform an OR function, since the output voltage will be high if any input voltage is high. Note that the inputs in both circuits can be at either level, independent of the states of the other inputs, because of the nonlinear characteristics of the diodes.

For reasons of economy, the central control uses multistage logic, i.e., many gates occur in tandem between the input and output of the system. Furthermore, the complicated logical functions to be performed often make it necessary for a gate to drive several other gates in parallel. This "branching" is commonly referred to as "fanout." As will be seen below, these features of the central control greatly complicate the use of diode gates.

A diode gate in a multistage logic circuit normally has its inputs driven by gates and its output used to drive a load consisting of other diode gates. In order to provide the desired output voltage and current levels when a gate is activated, it is necessary to select a resistor which will be the correct value for the particular load impedance which this gate must drive. In complex circuits such as the central control the selection of the proper resistor for each gate becomes a very complicated procedure. Also, many different types of gate designs are usually necessary.[2] This means that many different types of packages must be manufactured and many varieties of spare packages must be kept on hand for adequate maintenance. In addition, a single change may require redesign of substantial parts of the logic. Thus, such a system is difficult to design and has serious economic disadvantages.

Another factor makes the use of a multistage diode logic system undesirable. Since diode gates contain only passive elements, the logic signal is attenuated as it proceeds through tandem stages. To minimize this attenuation and to account for the signal power lost due to fanout, the impedance levels of the stages must be increased in successive stages. Thus, the problems of signal crosstalk and loss of high-frequency response due to stray wiring capacitance will increase in the later stages and can seriously limit the operating speeds of the system.

The use of transistor amplifiers in conjunction with the diode gates alleviates many of these problems. Such amplifiers are used in the central control and make possible the use of standard gates (gates whose resistor values need not be tailored to the application) to construct relatively low-impedance, high-fanout, multistage logic circuits.

Amplification can be inserted into the logic chains at various points. It is possible to provide amplifiers either for each logic stage or only at other selected locations. In the central control, amplifiers are provided after every OR gate. In this particular system, many more AND gates than OR gates are required and thus fewer amplifiers are necessary if they are associated with OR gates. The use of amplifiers with every OR gate, rather than only in selected spots, makes possible the design of very flexible diode gates, as will be described below. This choice of amplifier location results in two stages of logic (one AND-OR cycle) between amplifiers, although several AND gates can be used in series, as can several OR gates.

The electronic switching system's versions of the conventional diode gates of Fig. 3 are shown in Fig. 4(a). The resistor $R_0$ on the conventional OR gate has been included in the amplifier which always follows an OR gate. The resistor $R_A$ of the conventional AND gate has been removed and has been replaced by individual resistors on each OR input. These

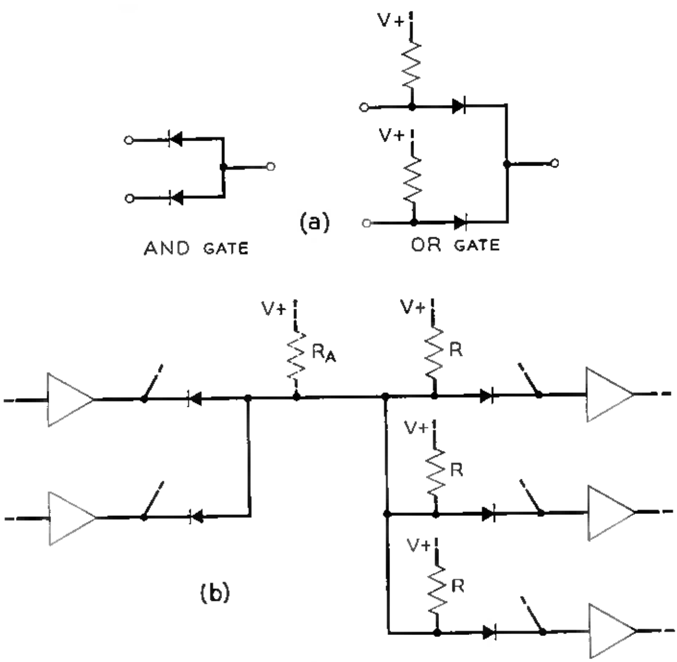



Fig. 4 — (a) Diode gates for central control; (b) interconnection pattern for diode gates.

individual resistors perform the same function as $R_A$ but automatically give the correct equivalent resistance value for driving the load regardless of the degree of fanout. This is illustrated in Fig. 4(b). If a single resistor $R_A$ were used, its value would necessarily be changed for different numbers of loads on the AND gate in order to provide efficient amplifier operation. The slight extra cost of the added resistors is more than compensated for by the increased ease of design, the reduction in the number of types of packages and the increased efficiency of the amplifiers.

The use of the individual OR input resistors also makes possible the use of a very efficient decoupling gate. The necessity for such a gate can be seen by considering Fig. 5(a). When two or more OR gate inputs are activated from a common point, the input current which each load will receive depends on the relative input impedances and threshold voltages of these loads. For instance, if $R_A < R_B$ and $V_A < V_B$, load $A$ will receive most (or all) of the current from both input resistors $A$ and $B$. The use of the decoupling gate [see Fig. 5(b)] prevents the flow of current between the input gate resistors $A$ and $B$, and thus insures that each load will get the total output current of its associated gate resistor regardless of its input impedance or threshold voltage. The OR gate loads are al-

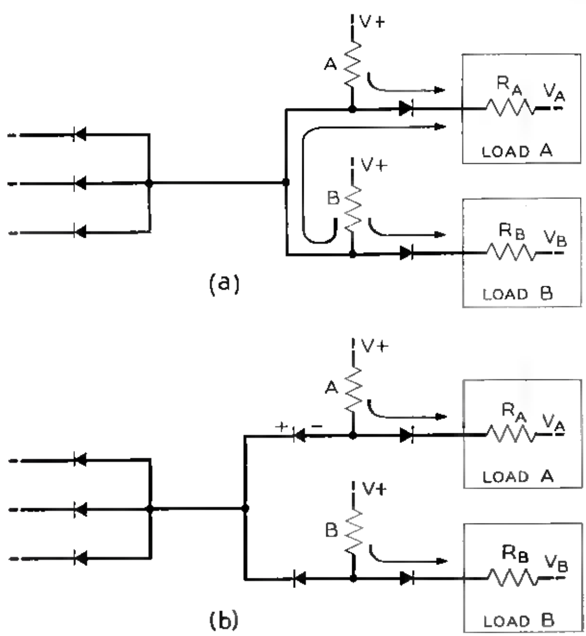



Fig. 5 — (a) Drive distribution problem; (b) use of decoupling gates.

ways transistorized packages in the central contol. The use of the decoupling gate allows the design of low-input impedance transistor circuits for maximum efficiency of operation and also allows the use of different input thresholds if desirable.

From Fig. 4(b) it can be seen that AND gates can be used in series without affecting the correct operation of the system. Also, OR gates can be used in series if input resistors are used only on the first stage. The use of gates in this fashion is often desirable to save diodes and is permitted in the central control, although it increases the logic signal voltage loss and, therefore, increases the voltage gain required of the amplifier.

The operating currents and voltages for the circuits must be determined before a decision can be made on the maximum number of OR inputs to be allowed. To be able to select operating currents and voltages, it is necessary to make engineering estimates of the effects of these levels on device and circuit reliability, operating speed and economy. In this system, the characteristics of the transistors and diodes are quite influential in the selection of operating levels. The OR gate currents are roughly 5 ma and the maximum transistor currents are 100 ma. The maximum reverse voltages applied to devices are less than 20 volts for diodes and well below the breakdown voltages for transistors. These levels allow the devices to be operated well within their ratings to insure long life. The resulting impedance levels are low enough to obtain sufficient switching speeds and tolerable crosstalk levels.

A limit of 20 inputs has been placed on the OR gate. This is necessary because the leakage currents through the diodes of all inactivated inputs will subtract from the current supplied to the amplifier by the activated

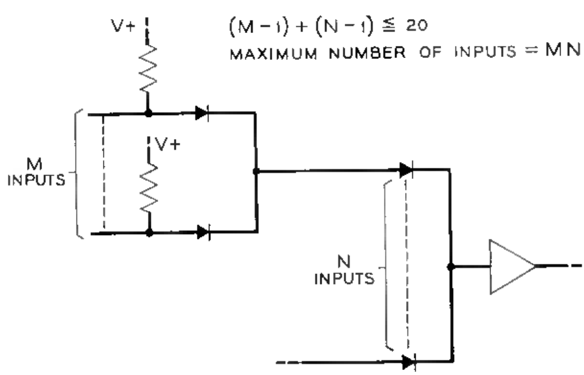



Fig. 6 — Use of series OR gates to extend the number of inputs.

input. By the use of OR gates in series, larger numbers of inputs can be obtained without increasing the number of shunting diodes. An example of how this can be done is shown in Fig. 6.

## V. FACTORS IN THE DESIGN OF TRANSISTOR CIRCUITS

In the design of dc transistor amplifiers for logic circuits, it is convenient to use the transistors as switches. By utilizing the nonlinear characteristics of a transistor switch, many of the adverse effects of device parameter variation on correct amplifier operation can be greatly reduced. Before proceeding with the description of the actual circuit designs, some of the basic large signal properties of junction transistors will be summarized.

The use of a transistor as a switch[3] is illustrated in Fig. 7. From the characteristic curves it can be seen that the transistor can operate in one

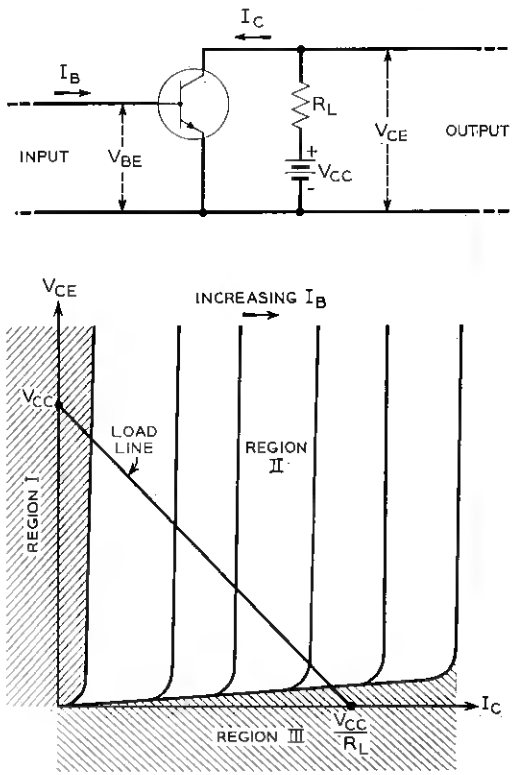
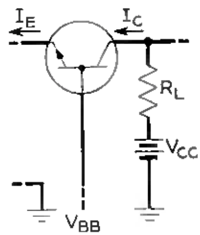
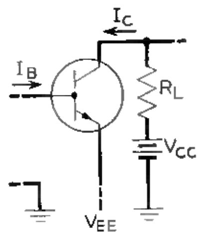
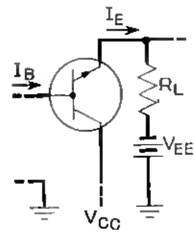



Fig. 7 — Transistor switch.

TABLE III — LARGE-SIGNAL PROPERTIES OF JUNCTION TRANSISTORS

| | Common Base | Common Emitter | Common Collector |
|---|---|---|---|
| Circuit |   |   |   |
| Current in Region II | $I_C = \alpha I_E + I_{C0}$ | $I_C = \frac{\alpha}{1-\alpha}\left(I_B + \frac{I_{C0}}{\alpha}\right)$ | $I_E = \frac{1}{1-\alpha}(I_B + I_{C0})$ |
| Current gain | No | Yes | Yes |
| Voltage gain | Yes | Yes | No |
| Inversion | No | Yes | No |

of three distinct regions. The region can be controlled by changing the input signal applied to the base. In region I, both collector and emitter junctions are reverse-biased and the transistor will be cut off, i.e., the impedance from collector to emitter will be very high. In region II, the active region, the emitter junction is forward-biased and the collector junction is reverse-biased. In region III, both junctions are forward-biased and the transistor switch will be closed. In this region the transistor is said to be in saturation and the impedance from collector to emitter will be very low.

The transistor switch described above is an example of the common emitter configuration. In this configuration the transistor exhibits both current and voltage gain and for this reason it is the most commonly used connection. The common emitter connection also provides signal inversion.

The general properties of the two other possible configurations are shown in Table III. The direction of currents and the polarity of the biasing voltages in Fig. 7 and Table III are for n-p-n transistors and should be reversed when p-n-p transistors are being considered.

In the design of dc transistor circuits, it is often useful to know the general relationships between the input voltage levels necessary to switch a transistor from cutoff to saturation and the resultant output voltage levels. These characteristics for the three configurations are shown in Fig. 8 for both n-p-n and p-n-p transistors. The waveforms represent the

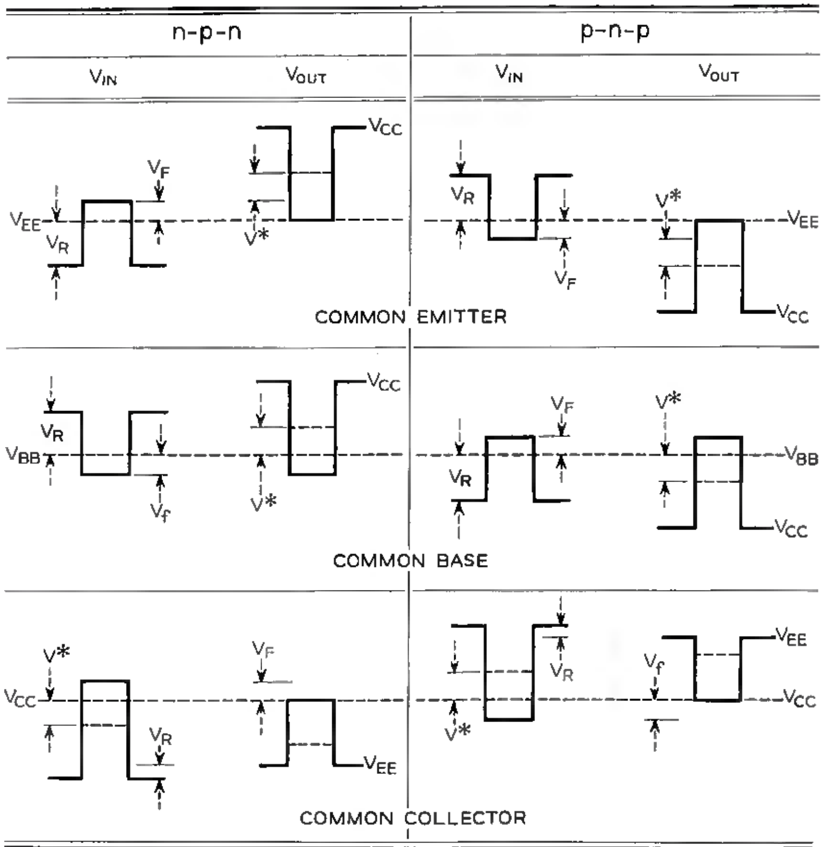



Fig. 8 — Input-output voltage relationships of various junction transistor configurations.

voltages which appear directly on the transistor terminals; $V_R$ and $V_F$ are the reverse and forward biases on the base-emitter junction and $V^*$ is the reverse bias voltage which will appear across the base-collector junction if the transistor is not allowed to saturate. The usefulness of these characteristics is illustrated in the design of the gate amplifier in Section VI.

The switching speeds of transistors are important in nearly all applications, especially in the circuits for the central control. While these speeds depend mainly on the transistor parameters, it is possible to maximize them by suitable circuit design. Analytical studies of the switching times of junction transistors have been made by Moll[4] and have been extended by Easley[5] to include the effect of collector capacitance. The results of these studies are quite useful in determining the relative effects of transistor and circuit parameters on speed. The discussion here will be limited to the common emitter configuration, but the results are quite similar for all configurations.

The basic common emitter circuit analyzed by Moll and Easley is shown in Fig. 9, together with waveforms illustrating the currents and switching times of interest. Here, $T_0$ is the rise time and $T_2$ is the fall time. The delay caused by carrier storage if the transistor is allowed to saturate, $T_1$, will be discussed below in more detail. The maximum value of the collector current, $I_{C1}$, will be approximately equal to $V_{CC}/R_L$ if the transistor is driven into saturation.

The above-mentioned studies[4, 5] show the expression for the rise time to be

$$T_0 = \frac{1}{1-\alpha}\left(\frac{1+\omega_\alpha R_L C_C}{\omega_\alpha}\right) \ln \left[\frac{I_{B1}}{I_{B1} - 0.9\left(\frac{1-\alpha}{\alpha}\right) I_{C1}}\right], \qquad (1)$$

where

$\alpha$ = common base short-circuit current gain,
$\omega_\alpha$ = alpha cutoff frequency in radians/sec,
$C_C$ = collector junction depletion layer capacitance.

This equation is plotted in Fig. 10 to illustrate more graphically the ef-

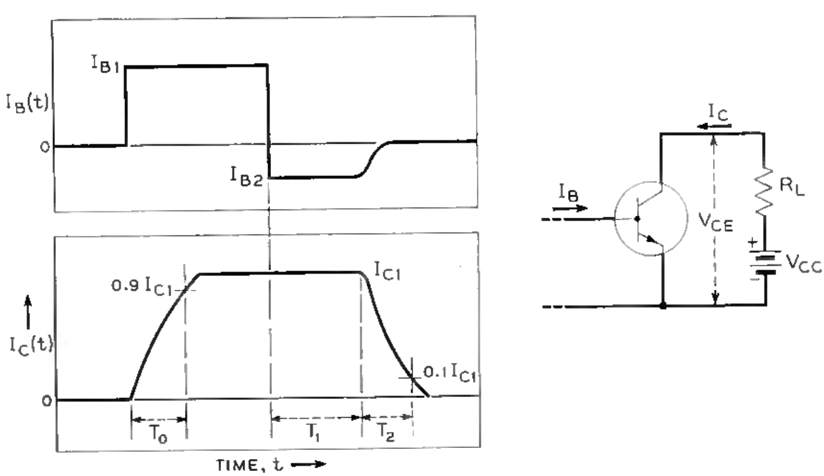



Fig. 9 — Transient behavior of common emitter junction transistor stage.

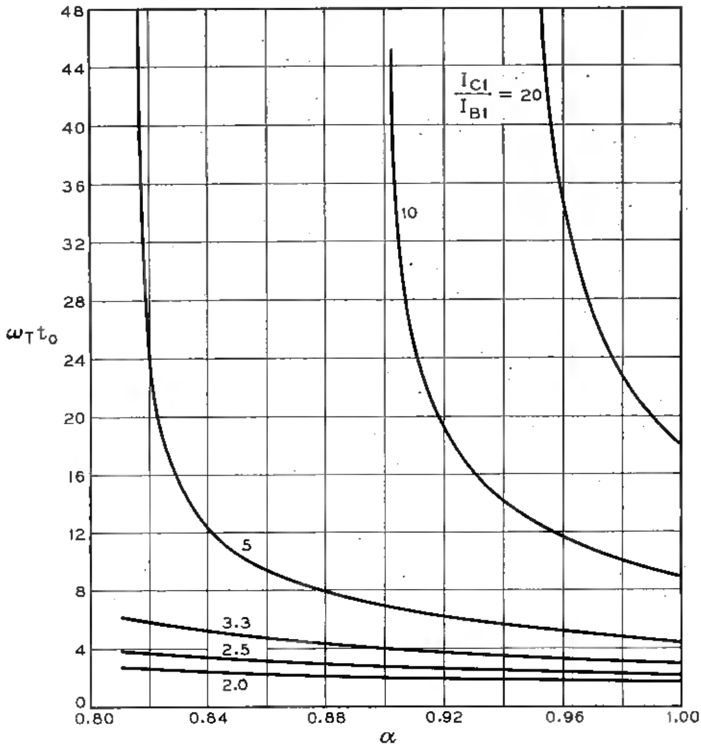



Fig. 10 — Plot of equation (1).

fects of variations in device and circuit parameters. The quantity $\omega_t$ in Fig. 10 represents the reciprocal of the effective time constant of the circuit, i.e.,

$$\omega_t = \frac{\omega_\alpha}{1 + \omega_\alpha R_L C_C}. \tag{2}$$

From Fig. 10, it can be seen that the rise time can be improved by increasing $\omega_t$ and $\alpha$ and by decreasing the actual circuit gain, $I_{C1}/I_{B1}$. Thus, given a specific transistor, the rise time can be improved by sacrificing circuit gain and by operating the circuit at low enough impedance levels to make $\omega_t \cong \omega_\alpha$. A further advantage of operating with low circuit gain is that the rise time will be little affected by changes in $\alpha$ which occur due to transistor aging. For circuit gains which are small compared to the possible gain, (1) can be reduced to

$$T_0 \cong 0.9 \frac{I_{C1}}{I_{B1}} \frac{(1 + \omega_\alpha R_L C_C)}{\omega_\alpha}. \tag{3}$$

Refs. 4 and 5 give the following expression for the fall time:

$$T_2 = \frac{1}{1-\alpha}\left(\frac{1+\omega_\alpha R_L C_C}{\omega_\alpha}\right) \ln \left[\frac{I_{C1} - \left(\frac{\alpha}{1-\alpha}\right) I_{B2}}{0.1 I_{C1} - \left(\frac{\alpha}{1-\alpha}\right) I_{B2}}\right]. \qquad (4)$$

A plot of this equation would be quite similar to that in Fig. 10, where the forward current gain $I_{C1}/I_{B1}$ is replaced by the reverse current gain

$$\frac{I_{C1}}{I_{B2} + I_{C1}\frac{(1-\alpha)}{\alpha}}.$$

Thus, the fall time can be improved in the same general way as the rise time. For the case where the transistor is heavily overdriven, equal rise and fall times are obtained when $I_{B1}$ and $I_{B2}$ are approximately equal.

When a transistor is operated in region III (saturation), more minority carriers are present in certain regions of the transistor than the number which must be present in these regions when the transistor is operated in region II. Before the transistor can be switched from region III to region II, these excess carriers must be removed. While they are being removed the transistor will continue to operate in region III and the output current will change only slightly. This effect, called storage, results in a true delay between the time the input current is changed and the time when the output current begins to respond. For transistors such as those described in Section II, this delay may be as long as microseconds—much too long to be tolerated in most circuits. Special antisaturating circuits have been developed which prevent the transistor from saturating but still permit overdrive for fast rise and fall times. These techniques are illustrated in Section VI.

## VI. BASIC BUILDING-BLOCK CIRCUITS

In the earlier sections the philosophy of using as few types of circuits as possible has been explained. The basic AND-OR diode circuits, which constitute a large percentage of the total number of circuits used, have already been discussed. The need for transistorized circuits to perform the memory function and to provide gain was also established. In this section transistor circuits will be discussed.

### 6.1 *Gate Amplifier*

The exact nature of the circuit used to provide the gain within the logic is determined by requirements and limitations imposed by the type of logic and the parameters of the available transistors. The major requirements of the amplifier are:

1. The amplifier must have sufficient current gain to satisfy the fanout occurring at most OR gates. To provide enough gain to satisfy the worst case would be costly, since the number of large fanout points is relatively small. An available gain of six is sufficient to satisfy approximately 85 per cent of the applications. This rather modest current gain requirement can drive fanouts which appear much larger, as can be understood by reference to Fig. 11. In Fig. 11 amplifier $G$ is shown driving inputs to $n$ AND gates which control, in turn, $m$ OR gates ($m$ being greater than $n$). Since the amplifier shares the load of any AND gate with all the other passive (low-voltage state) inputs, the load on $G$ is not equal to the sum of the currents in the AND gates. Therefore, where large fanout situations arise, known functional relationships between the inputs to the AND gates often make it possible to have effective fanouts of much greater than six. The nature of the logic in the system favors this condition.

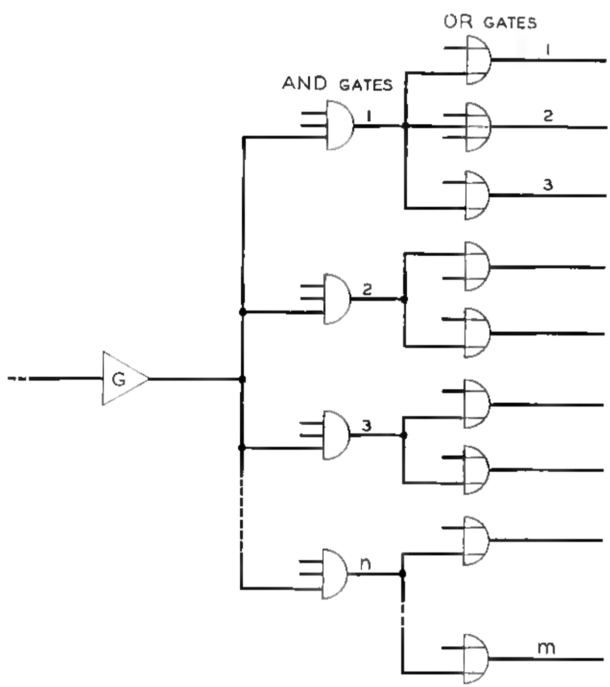



Fig. 11 — Fanout from an amplifier.

TABLE IV — PROPERTIES OF ONE- AND TWO-STAGE dc-COUPLED AMPLIFIERS

| Amplifier Requirement | Circuit Configuration | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | One Stage | | | Two Stages | | | | | |
| | CB | CC | CE | CB-CB | CB-CE | CB-CC | CE-CC | CE-CE | CC-CC |
| Current Gain | No | Yes | Yes | No | Yes | Yes | Yes | Yes | Yes |
| Voltage Gain | Yes | No | Yes | Yes | Yes | Yes | Yes | Yes | No |
| No Inversion | Yes | Yes | No | Yes | No | Yes | No | Yes | Yes |
| Voltage "Straddling" | No | Yes | No | Yes | Yes | Yes (?) | No | Yes | Yes |

CB = Common base
CE = Common emitter
CC = Common collector

2. The amplifier must have sufficient voltage gain to cancel the loss in the AND and OR gates and thus re-establish voltage levels. The AND gate diodes attenuate the passive-state signal level, while the OR gate diodes attenuate the active-state signal level. For these reasons the amplifier must provide gain about the center line of the dc input in both the positive and negative directions. Thus, the output voltage levels must "straddle" the input voltage levels.

3. The amplifier must be noninverting and dc-coupled.

4. The amplifier must be fast enough to allow the use of up to three amplifiers in a series logic chain. For the transistors employed, this requires nonsaturating, low-gain circuits.

In order to select the best circuit configuration to meet these requirements the properties of the three basic transistor configurations must be considered. These were discussed in Section V. The relatively modest current and voltage gain requirements of the amplifier make it evident that no more than two transistor stages should be necessary. The pertinent dc properties of all possible configurations of one- and two-stage amplifiers are listed in Table IV. From this table it may be seen that no one-transistor configuration meets the dc requirements of the amplifier. Of the two-transistor configurations only the common-base-common-collector and the common-emitter-common-emitter combinations meet all the requirements. A closer examination of the common-base-common-collector configuration reveals that some sort of passive voltage-shifting network would be necessary to meet the straddling requirement. In addition, in this configuration the common base transistor would have to furnish all the voltage gain and the common collector all the current gain. The two common emitter stages constitute the best configuration, since each provide both current and voltage amplification and the combination satisfies the remaining requirements. From Fig. 8 it can be seen that one transistor must be an n-p-n and the other a p-n-p to obtain voltage straddling. Since the signal is inverted in each stage, the two transistors will be either both ON or both OFF. An input to the first transistor of a polarity to turn it ON, for example, will, when inverted, turn ON the second stage of the amplifier.

The only decision remaining in determining the configuration is to establish the order in which the transistors should appear. The diode logic demands that the amplifier output provide a current path to approximately ground potential in the passive state and a high impedance in the active state. Two possible types of amplifiers exist, corresponding to the two possible methods of connecting the transistors. In the positive logic system, placing the n-p-n first and the p-n-p second leads to a "make" type amplifier; the reverse order produces a "break" type amplifier. These two types are shown symbolically in Fig. 12, along with the corresponding transistor circuit configurations which realize them. In the break amplifier both transistors would be ON in the passive state

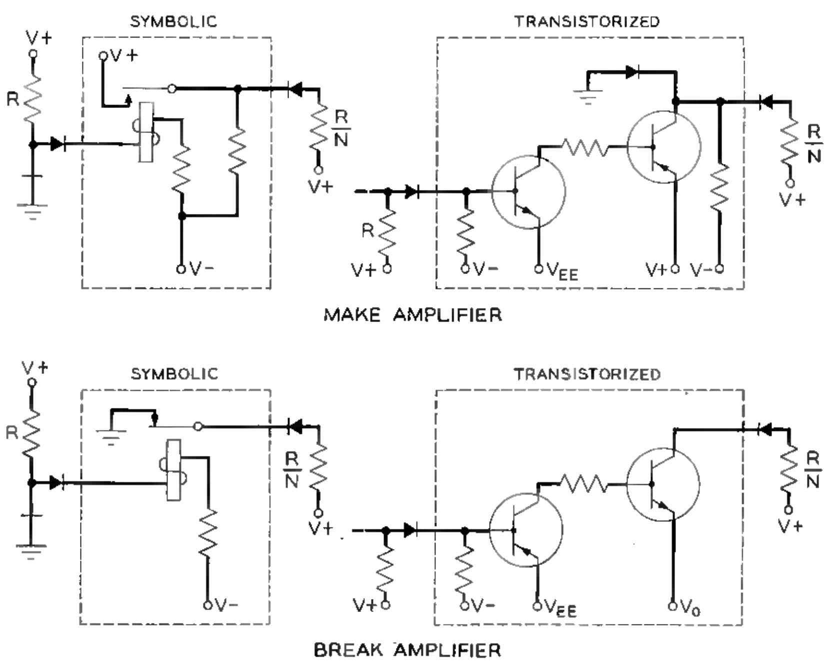



Fig. 12 — Possible amplifier configurations.

and OFF in the active state. The conditions would be reversed in the case of the make amplifier.

The choice between the two basic types of logic amplifiers above is not a simple one. The few apparent advantages of one over the other almost vanish when detailed final designs are approached. For example, the make amplifier appears much less efficient. In addition to supplying the full passive load current in the resistor to $-V$ at the output of the amplifier it must supply enough extra current to raise the output voltage to the active signal level. However, the need for dummy loading the break amplifier to compensate for stray wiring capacity in the system, as will be discussed below, almost nullifies this apparent advantage.

The decision as to which amplifier to choose must rest upon more subtle differences. The more important of these, leading to the choice of the break-type amplifier, are:

1. To stabilize the passive voltage level at the output of the make amplifier a diode clamp is required. This is a factor in reliability and introduces delay, since the current in the clamping diode must be replaced by current from the amplifier before any change in output voltage can occur during a transition from the passive to the active condition.

2. The make amplifier has a definite maximum current capacity and must switch this amount regardless of the load. (The diode clamp compensates for varying load conditions.) This necessitates an exact calculation of the maximum load on a particular amplifier. No such limitation exists in the case of the break amplifier, as will be developed later.

In addition to meeting the initial requirements in the system, the gate amplifier must have sufficient margin to function reliably as the components age. The most critical components in the circuit, and those for which the least aging information is presently available, are the transistors and diodes. A brief summary of the expected behavior of the most significant parameters is given below:

*Transistors*

1. Frequency cutoff of $\alpha$, ($f_\alpha$): This parameter is related almost exclusively to the internal geometry of the transistor and the physical constants related to transistor action. Therefore, no significant change should occur.

2. Transistor $\alpha$: Experience has shown that $\alpha$ tends to age downward initially after which time it stabilizes. In all the designs an $\alpha$ of 0.94 was assumed as a reasonable minimum.

3. Transistor saturation current, $I_{c0}$ : Almost without exception, $I_{c0}$

in transistors ages upward. In circuits which are sensitive to this parameter a maximum of 200 microamperes was assumed.

*Diodes*

1. Diode forward voltage drop: Previous experience with germanium point-contact diodes similar to those to be used in the system indicates little or no change with time.

2. Diode reverse leakage current: Diode reverse current behaves similarly to $I_{C0}$ in transistors except that its behavior is somewhat dependent upon the voltage at which it is measured. At low voltages (5 volts or less) it often ages downward. At higher voltages the trend is upward.

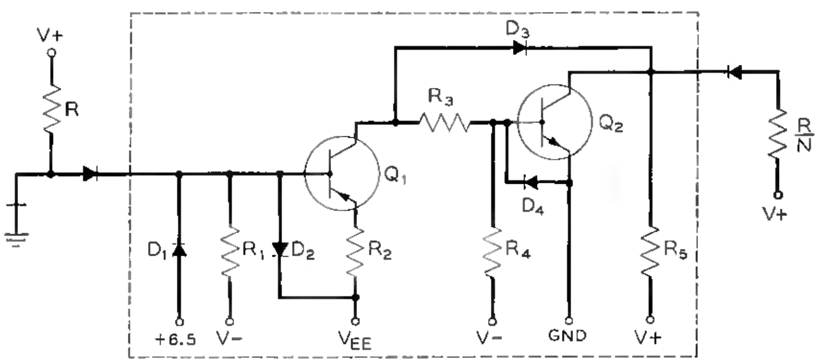



Fig. 13 — Nonsaturating amplifier.

In the discussion of the AND and OR gates in Section IV the current level in OR gates was stated as approximately 5 ma. In the passive state this current is shunted through one or more AND gates by one of the several types of amplifiers in the system. In the active state the shunt path through the AND gates is interrupted by the amplifier going OFF. Under this condition the OR gate current drives forward to turn OFF the succeeding amplifier stage. To operate properly the amplifier must, therefore, have an input voltage threshold higher than the total voltage drop in the AND gates preceding the OR gate plus any drop in the preceding amplifier in its passive state. From these considerations, the input threshold of the amplifier has been set at +6.5 volts. Having already established the output of the amplifier to be approximately ground in the passive state, we can choose the other voltages to provide the desired impedance level and current drives.

In Fig. 13 the break amplifier is redrawn with some complexity added to prevent the transistors from saturating and to establish the proper threshold. With no input to the amplifier (the input OR gate shunted down) the amplifier must assume its passive condition (both transistors ON). This requires that $V_{EE}$ be greater than the +6.5 volts which establishes the input threshold. The output current of $Q_1$ and thus the drive to $Q_2$, will be approximately

$$\frac{V_{EE} - 6.5}{R_2} \alpha_{Q_1} .$$

This current can be set by considering the input drive available and the required output current. Since the input must rise above $V_{EE}$ in the active state to turn off $Q_1$, the quantity ($V_{EE} - 6.5$) should be kept small. This leads to the choice of $V_{EE} = +8$ volts in the system.

In order to establish the internal current drives of the amplifier, the actual transistor gains must be calculated. The 5 ma current level in the OR gate is not all available to turn OFF the first transistor, $Q_1$. When the drive is removed from the input, the amplifier must return to its passive condition. Since both the active and passive states are equally important in the dc double-rail logic, this transition must be as fast as the transition from passive to active. A bias current drive is provided to establish the passive state when the input drive is removed. This bias reduces the forward drive when the amplifier is driven active. For equal turn-on and turn-off times, the bias current must be approximately one-half the input drive. This situation exists in both stages of the amplifier and, by itself, requires the combined transistor gain to be four times the apparent terminal gain of the amplifier.

Because of the large physical size of the central control, an amplifier often must drive loads located a considerable distance away from it. Stray capacity on these leads delays the transition from the passive to the active state. This effect can be reduced by the addition of $R_5$. Current in this dummy load is available to charge stray capacity at the expense of demanding more gain from the amplifier. A dummy-load current of 8 ma was chosen as a reasonable compromise. The dummy load also provides a well-defined active voltage level at the output of the amplifier.

By taking into account the current transfer loss of OR gates, the leakage currents of idle OR inputs and the dummy load, the combined transistor gain must be nearly 60 to realize the necessary terminal gain of six. This is accomplished by assigning a transistor gain of approximately 7.5 per stage, making the switching speeds relatively independent of $\alpha$ over the design range. (See Fig. 10).

By combining the information outlined above, the current level in $R_4$ can be set as $(I_{out\ max}/7.5) \doteq 5$ ma. This requires approximately 10 ma from $Q_1$ and a bias current to $Q_1$ of $(10/7.5) \doteq 1.3$ ma. From these basically simple considerations, the resistors $R_1$, $R_2$ and $R_4$ can be specified in terms of $-V$. The magnitude of $-V$ can be set by considering the breakdown voltage ratings of the transistors and the current transfer efficiency in $R_1$ and $R_4$. A value of $-4.5$ volts was selected.

The remaining components of the amplifier of Fig. 13 are $D_2$, $D_3$, $D_4$ and $R_3$. Diodes $D_2$ and $D_4$ limit the voltage swings at the transistor inputs and reduce the effect of collector capacity by lowering the transient

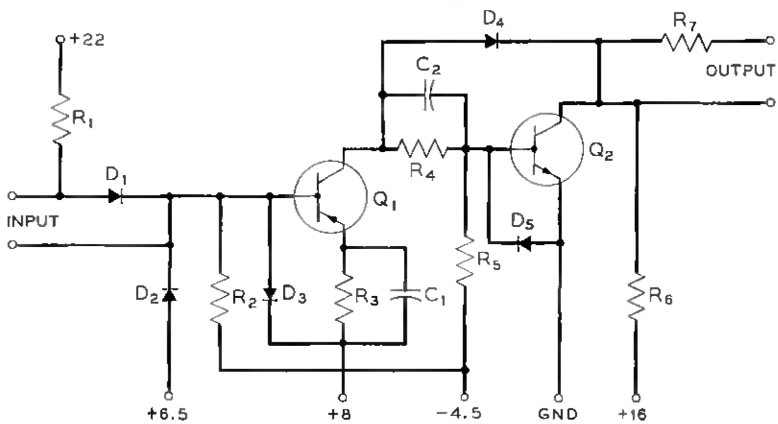



Fig. 14 — Gate amplifier.

base impedances. Resistor $R_3$ and diode $D_3$ combine to prevent saturation in transistor $Q_2$. Having established the currents in $Q_1$, $R_4$ and $Q_2$, and knowing the range of values of $\alpha$ and $I_{C0}$ in $Q_2$, we can determine the currents in $R_3$ and $D_3$ under all conditions. If $R_3$ is chosen so that its voltage drop is always greater than the voltage across $D_3$ when $Q_2$ is ON, $Q_2$ will never saturate. The advantage of this method of preventing saturation is that diode $D_3$ shunts the excess drive to $Q_2$ only after the output voltage has completed its transition to the passive state.

The final design of the amplifier is shown in Fig. 14. It differs from the previous drawing, Fig. 13, in the addition of capacitors $C_1$ and $C_2$ and resistors $R_1$ and $R_7$. The capacitors $C_1$ and $C_2$ do not affect the dc conditions but do improve the transient response. In particular, the delay in the amplifier is reduced.

Resistor $R_1$ and diode $D_1$ constitute a single input OR gate. This allows the gate amplifier to be driven by AND gates if desired. An option is also provided at the output of the amplifier for connecting loads to the collector of $Q_2$ directly or through $R_7$. Loads drawn through $R_7$ cause the voltage at the output to increase with load. Thus, if amplifiers are sharing a load (as in the case of AND gates), the load currents of the individual amplifiers will tend to equalize. This is an essential condition for realizing large fanouts, as was explained in conjunction with Fig. 11.

One of the advantages claimed for the break-type amplifier should be noted. Depending upon the $\alpha$'s of the transistors and the delay which can be tolerated in a particular logic chain, an amplifier may carry considerably more current than the design maximum of 30 ma. In engineering the logic of the central control, it becomes quite difficult to estimate the load current of each amplifier under all conditions of fanout. The break-type amplifier has margin for error when circuits are engineered to the design limit. This margin can also be of importance in locating a faulty amplifier under trouble conditions. Because of load-sharing between amplifiers, the failure of one causes the load on associated amplifiers to increase. If the associated amplifiers are capable of absorbing this added current, the trouble condition can be traced to the particular faulty amplifier. Amplifiers of the make type, however, by virtue of their inability to absorb current above a designed maximum, could indicate multiple troubles where only one actually existed.

### 6.2 *Flip-Flop and Associated Circuits*

The transistorized memory element of the central control must meet requirements which cannot be stated as explicitly as were those of the gate amplifier. The requirements can be generally stated as:

1. Stability: The flip-flop must maintain its bistability under all conditions of load, temperature and voltage encountered.

2. Compatibility: The flip-flop must trigger from pulses readily available in the system, must not respond to noise in the system and must be capable of driving into the standard dc double-rail diode logic.

The configuration of the transistorized flip-flop differs only slightly from the conventional Eccles-Jordan type. The development of the design is similar to that of the gate amplifier and the same considerations regarding gain, speed, etc., also apply. For this reason, the details of design will not be considered. Instead, emphasis will be placed upon how the flip-flop fits into the general system and upon some of its major characteristics.

The flip-flop used in the electronic switching system is basically a four-transistor circuit. The dc bistability is provided by two of the transistors, which are combined on a single package designated the flip-flop. The other transistors comprise two single-stage amplifiers which afford buffering between the 1 and 0 outputs of the flip-flop and the external dc logic that the composite flip-flop must drive. These amplifiers are designated the flip-flop to gate amplifiers. Special buffer amplifiers are provided for the cases where the flip-flop does not drive diode logic. All the amplifiers clamp the flip-flop output, this being essential in preventing saturation.

Fig. 15 shows the details of the flip-flop, with the buffer amplifiers simulated by resistor-diode clamps to ground. We shall consider the more important features of the circuit in turn.

The use of separate terminals to introduce the collector supply voltage (−16 volts) provides a method of predetermining the state a flip-flop will assume when power is applied. By connecting the terminals to separate supply buses and momentarily connecting one to +16 volts and then to the normal −16 volts, the system will assume the state represented by the wiring pattern.

The degeneration produced by the common emitter resistor stabilizes the voltage at the emitter node and the output current of the ON transistor. This voltage stabilization of the emitters (and therefore of the base of the ON transistor), combined with the clamping action of the amplifiers in the collector circuit of the ON transistor, prevents satura-

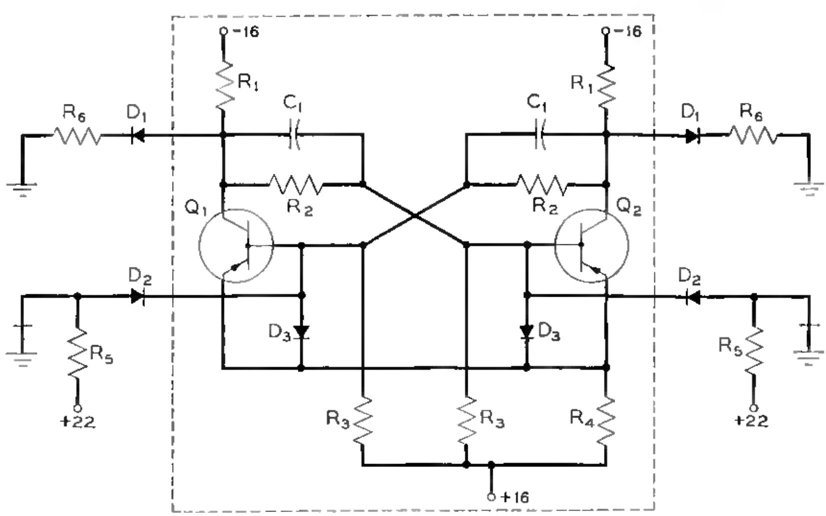



Fig. 15 — Flip-flop with simulated amplifier loads.

tion in the flip-flop. The voltage and current levels are designed to provide the same input voltage threshold as in the gate amplifier. By designing the output current to equal that of $Q_1$ in the gate amplifier, it was possible to make the flip-flop to gate amplifier essentially the same as the output stage of the amplifier of Fig. 14. Thus, when viewed from its terminals, the flip-flop and its amplifiers together are exactly equivalent to the gate amplifier insofar as dc conditions are concerned.

Fig. 15 shows the use of standard OR gates for triggering. To prevent race conditions from occurring in the system, all inputs to the flip-flop are gated by the master clock. This requires that a clock signal be an input on all AND gates preceding OR gates driving a flip-flop. A single-input OR gate is provided on each side of the flip-flop to allow driving from AND gates if desired.

Another feature of the flip-flop is the use of diodes between the bases and emitters of the transistors. These diodes limit the reverse bias on the transistors to a fraction of a volt, clamp the driving OR gate output and, most important, lower the transient impedance in the base circuit. The lower base impedance reduces the effect of collector capacity and improves triggering sensitivity.

The circuits of the two buffer amplifiers are shown in Fig. 16. The amplifier used to drive logic gates, the flip-flop to gate amplifier, needs no explanation since it is almost identical to the output stage of the gate amplifier previously described. The flip-flop to relay amplifier is designed to drive the highly inductive loads of wire-spring relays. The transistors are allowed to saturate in this circuit because of the slow speeds involved.

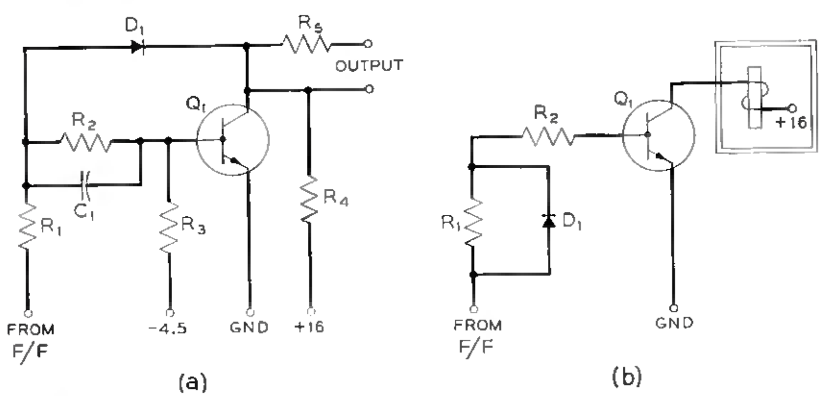



Fig. 16 — Flip-flop buffer amplifiers: (a) flip-flop to gate; (b) flip-flop to relay.

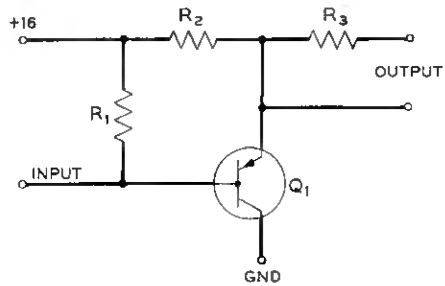



Fig. 17 — Emitter follower circuit.

### 6.3 *Emitter Follower*

In an earlier section, it was stated that the gate amplifier and the flip-flop to gate amplifier were not designed to have sufficient current gain to satisfy the points of largest fanout in the system. A special package, the emitter follower, is provided to supply this extra gain. One gate amplifier can drive a maximum of four emitter followers in parallel. For even greater fanouts, emitter followers can be cascaded. The inclusion of an emitter follower adds almost no delay to a logic chain. The circuit of the emitter follower is shown in Fig. 17.

### 6.4 *Inverter Amplifier*

An additional circuit, shown in Fig. 18, which saves equipment in many applications in the system is the inverter amplifier. The terminal conditions of the inverter amplifier are similar to those of the gate amplifier except for the logical inversion and a lower load capability.

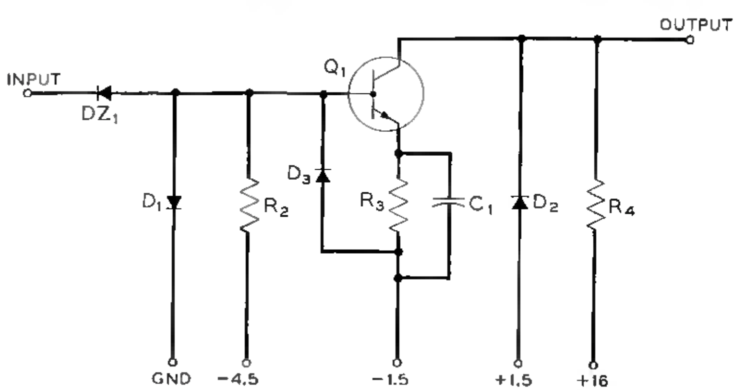



Fig. 18 — Inverter circuit.

The circuit is similar to the first stage of the gate amplifier, except for the use of an n-p-n transistor and different voltage levels. A Zener diode, $DZ_1$, raises the input voltage threshold to the same level as that of the other transistor circuits. Diode $D_2$ clamps the output of transistor $Q_1$ to prevent saturation.

### 6.5 *Cable Pulser*

The need for the central control to communicate with the electron tube circuits in other parts of the system requires special buffering amplifiers for transmitting high-speed signals over coaxial cable. These signals originate from OR gates in the central control. In addition to driving the cable, the OR gates must activate local flip-flops in the central control to retain a record of the information transmitted. The OR gate neither has sufficient current output to perform these two functions nor is its impedance compatible with that of the cable. The cable pulser circuit shown in Fig. 19 provides the required gain and achieves compatibility between the units.

The cable pulser is a two-stage amplifier. The first stage is a common collector configuration which provides sufficient current gain to drive a flip-flop and the second stage of the cable pulser. The second stage is a common emitter configuration which provides voltage and current gain to drive the cable. The output is coupled to the cable through an impedance-matching transformer. The input to the cable pulser is gated by the master clock.

### 6.6 *Special Circuits*

Although every attempt has been made to minimize the number of types of packages in the system, in some cases the use of special packages

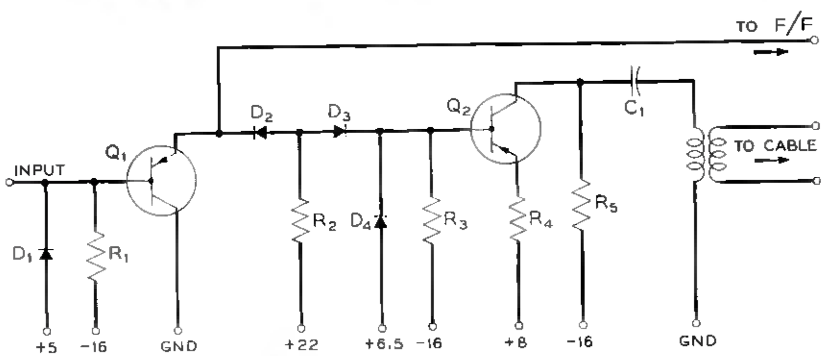



Fig. 19 — Cable pulser circuit.

is economically justifiable. In this system these special packages take the form of modifying networks which are used in conjunction with standard packages to realize special circuit functions. The circuits are shown in Fig. 20.

6.6.1. *Feedback Network*

Monostable flip-flops are required for timing applications in the system. This need has been satisfied by the design of a simple feedback network for use with the standard gate amplifier, consisting simply of a resistor and a capacitor in series. When connected between the output and input of a gate amplifier, the network provides positive feedback for a time determined by the values of the resistor and capacitor. The purpose of the extra resistor in the feedback network is to provide a voltage step at the output of the amplifier at the beginning of the timing cycle. If the load on the circuit is coupled through an AND gate, this step is sufficient to decouple the load during the timing cycle and, therefore, make the timing independent of load.

6.6.2. *Pulse-Shortening Network*

Occasionally the need arises in the system to derive a narrow (approximately 1-microsecond) pulse from the output of one of the standard amplifiers. This has been accomplished by a simple network consisting essentially of a shorted 0.5-microsecond delay line which can be connected to the output of an amplifier. As shown in Fig. 20(b), a resistance divider network is needed to derive a clamping voltage level approximately 2 volts above ground to prevent saturating the driving amplifier.

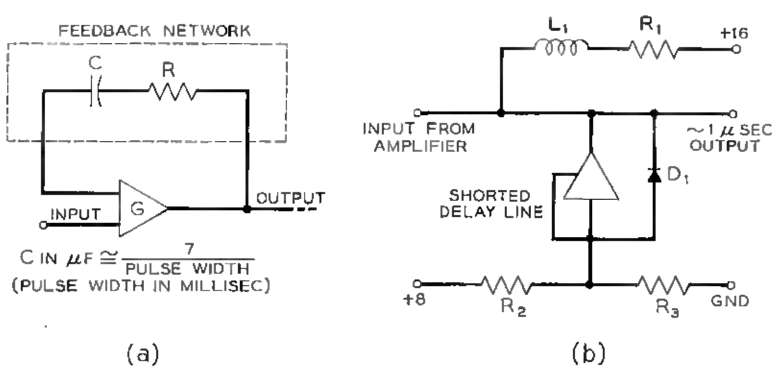



Fig. 20 — Special network circuits: (a) monostable feedback network; (b) pulse-shortening network.

The resistor $R_1$ and inductor $L_1$ provide a current step to the network, thus giving a better-shaped pulse in the output.

## VII. MECHANICAL DESIGN

In the semiconductor circuitry of the electronic switching system, a modular approach to mechanical design is used. This is in keeping with the universal building block approach to obtain economy in both manufacture and in the number of spare packages required in the telephone central office. It is also consistent with the marginal and diagnostic trouble-detecting techniques developed for the system, whereby faulty equipment can be easily replaced.

The semiconductor circuits used in this model of the electronic switching system are placed on printed wire boards. A board may contain one or more circuits, depending on the degree of complexity and the number of components employed in the individual circuits. Fig. 21 illustrates the basic board assemblies. On both sides of the transistor end of the board (and only on those boards containing transistors) is a series of terminals designated shorting plug terminations. These are provided so that the transistors may be tested, external to the circuit environment, without unsoldering any leads or physically disturbing the transistor in any way. In normal use the transistor is connected into the

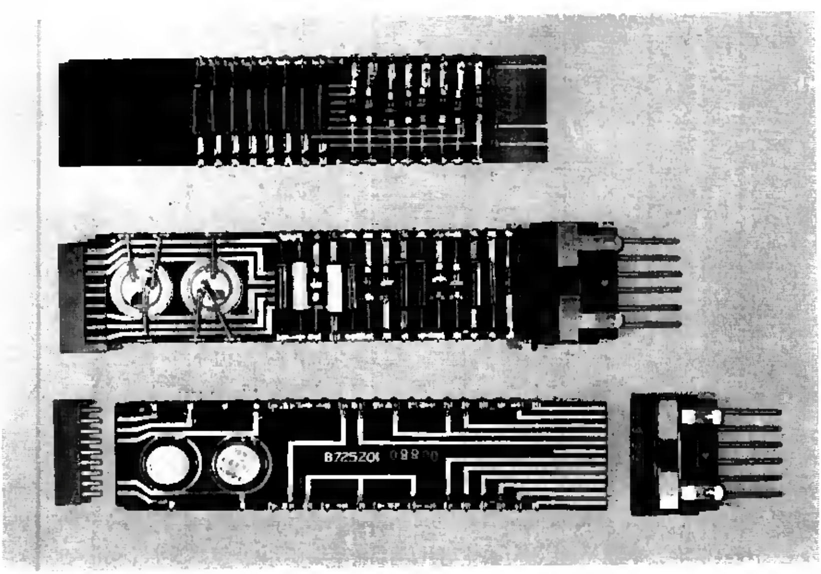


Fig. 21 — Printed circuit boards and components.

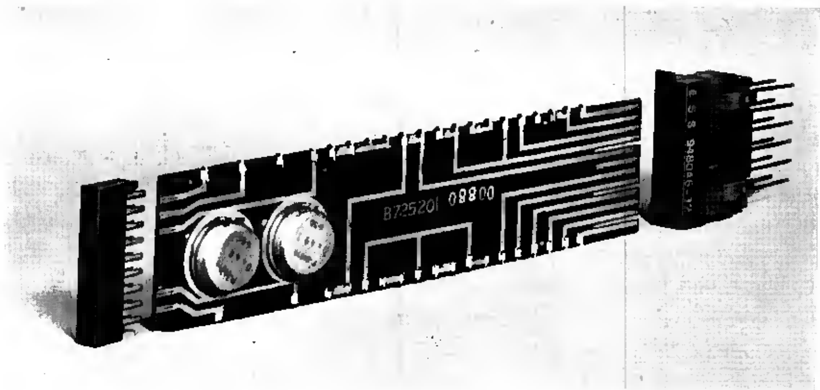



Fig. 22 — Printed board, shorting shoe and connector.

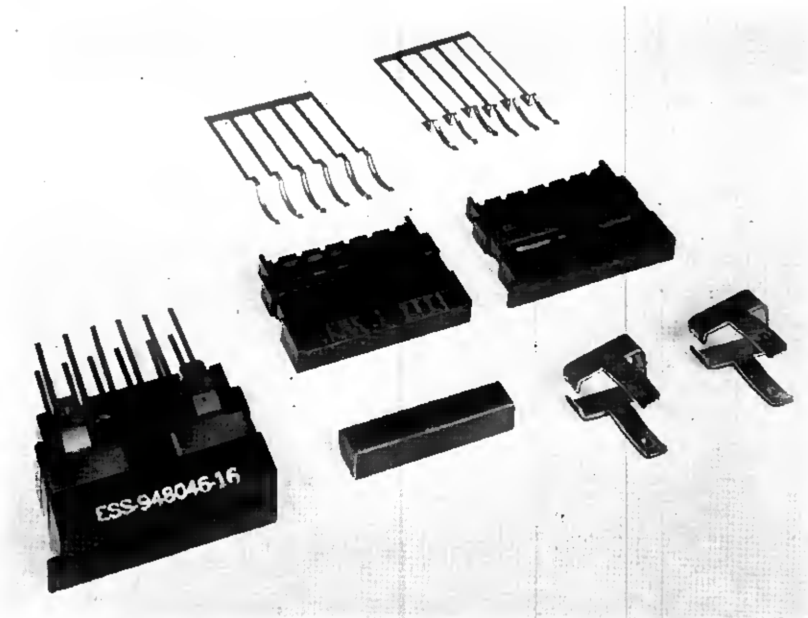



Fig. 23 — Coded connector for semiconductor package.

circuit by means of a shorting plug. Fig. 22 illustrates the use of the shorting plug and also shows the board terminations and connector. One can also see in Fig. 22 that each card assembly is coded on a 3-slot-out-of-a-possible-11 basis. The connector is similarly coded with barriers or teeth so that one particular type of card will fit into a companion coded

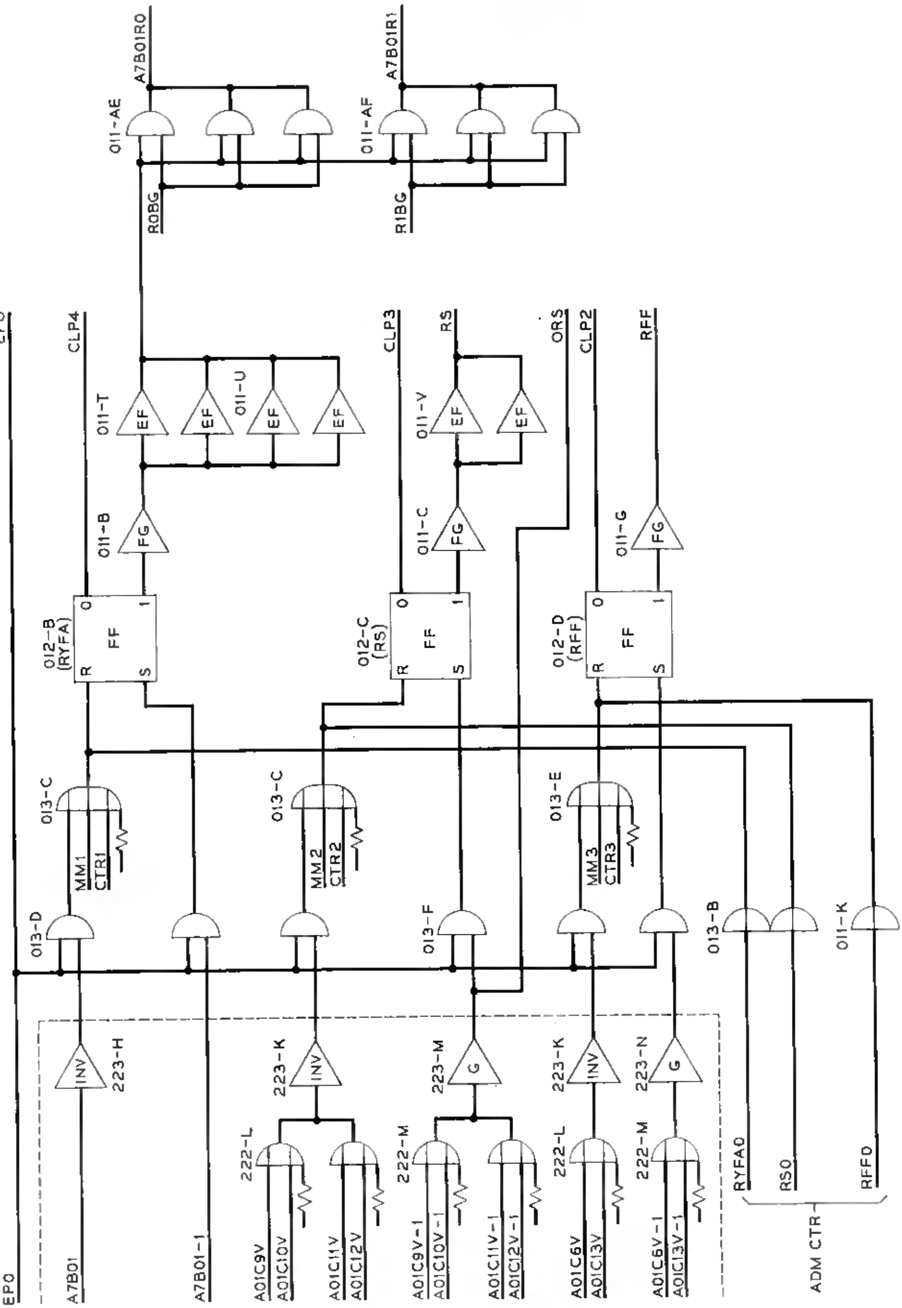



Fig. 24 — Representative portion of circuit schematic.

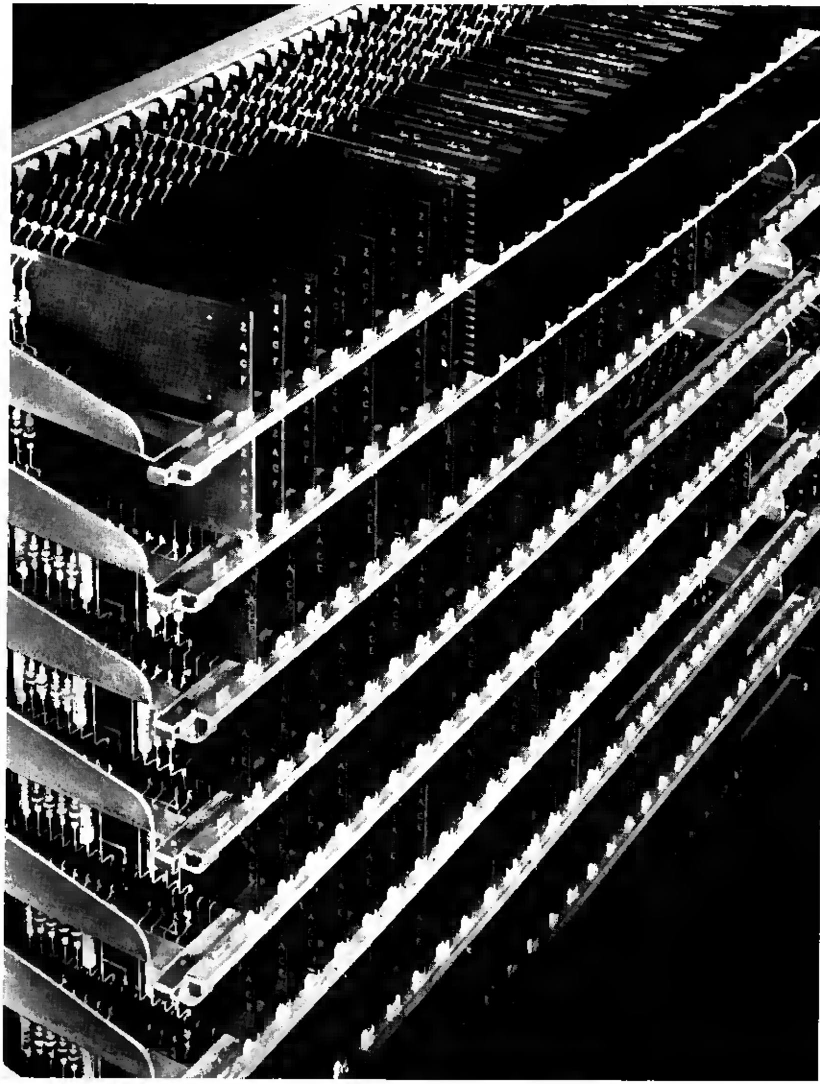


Fig. 25 — Equipment unit (front).

connector. This will prevent improper interchange of packages during assembly or maintenance.

As shown in Fig. 21, each component is fastened over the edge of the card by a clinch which mechanically holds it in position. A subsequent solder dip secures the mechanical clinch and at the same time makes the electrical connection to the printed wire circuit. The component lead makes an edge connection from one side of the board to the other if this is required. This method of mounting the components lends itself to low-cost automatic manufacturing techniques.

The connector shown in Fig. 22 is a reliable contact connector that can be produced and assembled by automatic or semi-automatic machinery. It is held together by spring clips, which also hold the connector in the mounting plate. Fig. 23 shows the connector and its component parts in more detail.

Fig. 24 is a portion of a circuit schematic showing the use of AND and OR gates, amplifiers and flip-flops. Circuitry of this type is developed into equipment as shown in Fig. 25. The gates, amplifiers and flip-flops are arranged for ease of maintenance, ease of manufacture and for proper circuit operation. In this equipment connectors are placed in the mounting plates and the mounting plates are arranged in groups and wired. The printed wire board assemblies are plugged into the connectors. The fronts of the boards are supported and aligned by means of a bar ar-

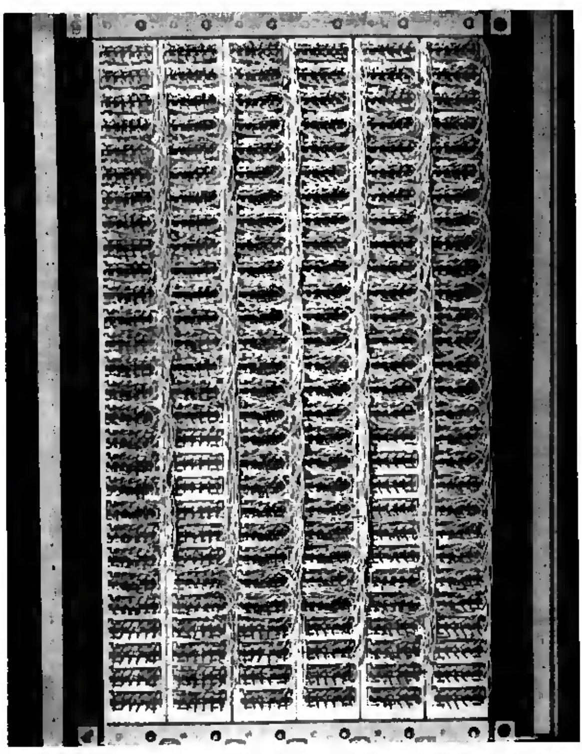


Fig. 26 — Equipment unit (rear).

ranged across the front of the framework, with the front edge of this bar also serving as a designation strip. Fig. 26 shows the rear of the equipment. Solderless wrapped connections are used exclusively, and the equipment can be automatically wired by programmed solderless wiring machines.

## VIII. CONCLUSION

The philosophy of semiconductor circuit design for the central control of an electronic switching system has been described. The primary emphasis in the design has been on low cost consistent with good margins, reliability and the meeting of all systems requirements.

The general-purpose nature of the building blocks affords great flexibility to the logic designer in implementing complex system functions. The ability of the circuits to handle widely varying loads allows the system to be engineered quite simply. An effort has been made to minimize the number of package types consistent with the many different functions to be performed. This results in economy in manufacture and maintenance.

The use of antisaturating low-gain circuits, clamping techniques and careful equipment design has resulted in operating speeds sufficient for the desired rate of information processing. Throughout the designs, generous allowances have been made for aging and variations in device and component parameters to assure high reliability consistent with economy. It is too early to give an accurate evaluation of the true reliability of the circuits, since this can only be obtained by relatively long field experience. However, present indications are that these circuits will result in a system furnishing service at least comparable with present-day electromechanical systems.

## IX. ACKNOWLEDGMENTS

The authors wish to express their appreciation for the contributions to this project made by their many co-workers. In particular, we wish to thank S. H. Washburn for his guidance and helpful comments.

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# Fundamental Concepts in the Design of the Flying Spot Store

By C. W. HOOVER, JR., R. E. STAEHLER and R. W. KETCHLEDGE

(Manuscript received May 15, 1958)

*The flying spot store is a semipermanent information storage system developed for use in an electronic switching system which utilizes cathode ray tube access to information stored on photographic emulsion. Parallel optical channels are used to provide high capacity and parallel readout. A feedback system provides rapid and precise beam positioning for writing and readout. This paper describes the fundamental design considerations for memory systems of this type, stressing physical realizability, speed, capacity and other system features. The characteristics of a laboratory model are presented.*

## I. INTRODUCTION

### 1.1 *Application in Electronic Switching Systems*

The flying spot store has been developed to meet the needs in electronic telephone switching systems for high capacity semipermanent memory arranged for rapid random access to stored words. In its present form the flying spot store meets all the requirements of the system described elsewhere in this issue.[1] The original suggestion for the use of a flying spot store was made by C. E. Brooks, leading to the development described in this paper.

Electronic switching systems of the future will also require memory of this type. Directory-to-equipment number translations alone will require millions of bits. In the form now under development, the flying spot store appears capable of meeting the memory requirements of these large systems.

### 1.2 *Form of the Flying Spot Store*

In the flying spot store information is stored as a pattern of transparent and opaque spots on developed photographic emulsion. Access for writing and reading is gained by means of a number of light beams generated by a single cathode ray tube and an array of objective lenses operated in parallel, each imaging the working area of the cathode ray tube face on a separate photographic area. A photomultiplier detector is located behind each area. One bit of each word is placed at the same address position, on each of the storage areas. In the laboratory model described each of these areas is on a separate photographic plate. A single access operation, i.e., beam positioning, therefore results in readout of a number of bits in parallel. In the usual instance, this number can be made equal to the word length.

Beam position is controlled by a closed-loop feedback system developed especially for this purpose. A number of the parallel optical channels contain code plates. The bar patterns on these code plates are chosen so that readout from these channels gives the cartesian coordinates of the beam position at all times in parallel binary form.

An error signal is derived by the comparison of this position readout with the required address position in a parallel digital comparator. The signal is applied to integrating and deflection amplifiers in tandem to drive the beam to within one spot diameter of the required position. Once within this area, the beam is locked by linear servo action to the single code-plate edge passing through the center of the cell at the address selected.

### 1.3 *Early Work on Photographic Storage Systems*

The basic idea of light-beam access to information stored on photographic emulsion is not new. It has been known for a long time that storage densities as high as $10^6$ bits per square millimeter can be obtained on emulsions.[2] With the incentive of low-cost high-capacity memory, many proposals were advanced in the literature for light beam access systems. King, Brown and Ridenour describe many of these proposals in an early survey article.[3] These early systems used only one source, objective lens and photodetector, and differed principally in means of access. Access was provided either by positioning a cathode ray tube beam or by mechanically addressing the point on the storage plane between fixed light source and detector.

Systems using the cathode ray tube or flying spot scanner type of access offered the possibility of rapid random access. However, in proposed systems of this type, the cathode ray tube and lens were operated at extremely high resolution in order to provide enough total capacity to make the system useful. This imposed requirements upon the devices, upon mechanical positioning tolerance and particularly upon the beam-positioning system which were extremely difficult to meet.

Systems using mechanical selection in whole or in part offered the possibility of high capacity but suffered from the slow sequential access dictated by the mechanical addressing arrangement.

For these reasons, neither type of system could have been used to satisfy the memory requirements of the electronic switching systems of the type now being developed.

### 1.4 *The Analog Accuracy Problem*

All of the early proposals for the flying spot scanner type of access recognized the severe difficulty in obtaining the required analog accuracy in beam positioning. In addition, our studies have shown that it is desirable to position the reading beam within $\pm 0.1$ spot diameter of the center of the written spot. Less precise positioning would degrade the discrimination ratio or force a reduction in capacity. In an analog beam positioning system used to establish an array of $500 \times 500$ spots at the cathode ray tube, the accelerating voltage supply for the cathode ray tube and the deflection amplifier power supply would have to be regulated to within $\pm 0.02$ per cent to maintain this position accuracy. Similar close requirements would appear at many other points in the system.

For these reasons, it has been a fundamental premise that beam positioning would be controlled by a closed-loop system in which the final positions for the cathode ray tube beam would be a set of mechanical reference edges fixed in space in the same plane as the stored information. This philosophy was suggested by R. W. Ketchledge, and implemented by multiple channel techniques in the first complete storage systems developed by R. E. Staehler and R. C. Davis.[4] A system of this type overcomes many of the difficulties in open-loop analog systems and makes the flying spot store type of system feasible.

### 1.5 *Purpose and Organization of the Paper*

This paper describes the fundamental design concepts and mode of operation of the flying spot store type of semipermanent memory system. The first section deals with physical realizability and shows that a wide variety of stores can be built. The succeeding sections discuss speed, capacity and other features and advantages resulting from parallel channel organization and the use of closed-loop beam-positioning systems. The characteristics of a laboratory model of the flying spot store are given.

We have attempted to include enough detail so that the interested reader can proceed directly from this to other future papers which will describe in detail the advances in systems, circuits, devices and measurement techniques which have made the flying spot store possible.

## II. FUNDAMENTAL DESIGN CONCEPTS

### 2.1 *Physical Realizability*

We turn first to a discussion of the physical realizability of a single-channel system like that shown in Fig. 1. This discussion is pertinent since the flying spot store is made up of many parallel single-channel systems. The major advances and improvements of the flying spot store depend upon our ability to operate many of these channels in parallel from a single cathode ray tube.[4]

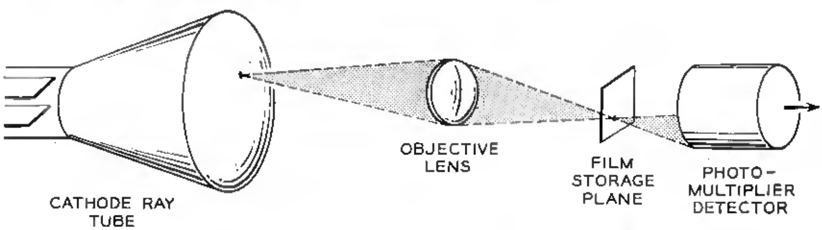


**Fig. 1 — Single-channel photographic storage system.**

At the start of the flying spot store project, work was begun on physical measurements of the properties of the devices used in the single-channel system: cathode ray tube, phosphor, optical elements, photographic emulsion, photocathode and electron multiplier. Very little of the data ordinarily available on these devices was in a form useful in designing the flying spot store. This measurement work has provided the necessary data on device characteristics for designing the laboratory model of the flying spot store described in Section III as well as the later systems now under construction.

#### 2.1.1 *Signal-to-Noise Considerations in Readout*

Fluctuations in the amplitudes of 1's and 0's read out from the store result from physical imperfections in various elements of the system and electrical noise generated in the photomultiplier. Fluctuations in the first category arise from beam malpositioning, which leads to partial superposition of information written on the plates; from halo and scattering effects in the phosphor screen; from limited spot spacing on centers set in order to get maximum capacity per channel; and from defocusing of the cathode ray tube beam. There are local contributions to this noise due to variations in phosphor light output as a function of position, film blemishes, variations in film density, optical aberrations and lens vignetting. In the model of the store described in this paper, the cathode ray tube beam-positioning system, the spot spacing on centers, the cathode ray tube itself and the objective and condenser lenses have been chosen so that fluctuations of this type do not reduce the discrimination ratio in readout below 10:1. Discrimination ratio is defined as:

$$\frac{S_{1,\min} - S_{0,\max}}{S_{0,\max}},$$

where $S_{1,\min}$ is the minimum amplitude of a 1 signal read out from the store and where $S_{0,\max}$ is the maximum amplitude of a 0 signal. Doubtless some improvement in this ratio can be obtained in later systems by more careful design. Redundancy is introduced to correct a major portion of the errors caused by film blemishes and the errors introduced on a per-channel basis by electrical fluctuations.

Electrical noise is superposed on the fluctuations due to physical imperfections. This noise arises from shot noise generated at the photocathode and from regenerative effects in the photomultiplier. Both components are "noise-in-signal" and affect primarily the amplitude of the 1 signal. The noise current from these sources is the dominant electrical noise. This is the case since the shot and regenerative components of the output current are much greater than the component that is due to thermionic emission from the photocathode in the wideband output circuits and at the output current level used in high-speed systems. The signal-to-noise ratio could be improved by increasing the light level at the photocathode. However, it is desirable to set the light level as low as possible, since this permits the widest possible choice in the selection of components and operating conditions for the other devices in the system. These considerations are made clear by the following example which gives the operating conditions in the information storage channels of the laboratory model of the flying spot store:

If sampling time is 0.1 microsecond, output circuit bandwidth is 3.5 megacycles and $\bar{\imath}_{pc}$, signal component of photocathode current is $400 \times 10^{-5}$ microamperes, then the shot noise component of photocathode current is given by:

$$i_{\text{shot, rms}} = (2e\Delta f \bar{\imath}_{pc})^{1/2} = 6.7 \times 10^{-5} \text{ microamperes.} \quad (1)$$

Under these conditions, the equivalent photocathode current required to account for the observed regenerative noise component of anode current would be

$$i_{\text{regen, rms}} = 35.0 \times 10^{-5} \text{ microamperes,}$$

and the thermal emission component of photocathode current at 25°C is

$$i_{\text{thermionic, rms}} = 0.05 \times 10^{-5} \text{ microamperes.}$$

Thus, the thermionic component is negligible compared to the shot and regenerative noise components.

Upon combining the noise components quadratically, the total electrical signal-to-noise ratio is found to be 11:1. If the only noise were that due to shot noise the signal-to-noise ratio would be 400/6.7 = 60:1 and the electrical fluctuations would be small compared to those due to physical imperfections.

A note about the nature of regeneration noise is appropriate here. Regeneration noise depends on anode current level and total tube gain. Recent experiments have shown that regeneration noise is associated with "after-pulsing" in the photomultiplier. Tubes of a new design have been tested which do not show after-pulsing or regeneration under the operating conditions required in use in the flying spot store. For this reason, in the discussion which follows, we consider shot noise as the limiting factor in setting the photocathode current.

### 2.1.2 *Flux Levels in the System*

In the preceding section we showed how the electrical signal-to-noise ratio depends upon the signal component of photocathode current, $\bar{i}_{pc}$. We now determine the radiant flux required to produce this photocathode current, and determine the flux levels throughout the system for a typical case.

2.1.2.1 *Choice of Phosphor and Photocathode.* The radiant flux required at the photocathode depends on the photocathode efficiency and on the match between the spectral sensitivity curve of the photocathode and the emission spectrum of the phosphor. All of our applications to date have used the type S-11 Cs-Sb photocathode. The region of high sensitivity of this photocathode is wide compared to the emission spectrum of most of the fast-decay phosphors.

The design objective for all systems studied has been maximum speed in access and readout. A fast phosphor is required for this purpose and type P-16 has been used in all applications. It exhibits an exponential decay, with a time constant after aging of 0.05 to 0.06 microsecond. Its spectrum is very well matched to the type S-11 photocathode and its efficiency is the highest of any of the fast-decay phosphors we have tested. Its spectrum does not change significantly with bombarding current density or life. The emission spectrum of type P-16 phosphor and the relative spectral sensitivity curve of the S-11 photocathode are shown in Fig. 2.

2.1.2.2 *Choice of Cathode Ray Tube.* The preceding paragraph shows that the phosphor and photocathode are chosen on the basis of compatibility and on the basis of speed of operation. Similar restrictions are met in the choice of cathode ray tube and optics. For the tube the limits are well defined: Here we want the brightest possible spot (i.e., greatest flux per unit area) of a specified size. The spot size may be fixed by system

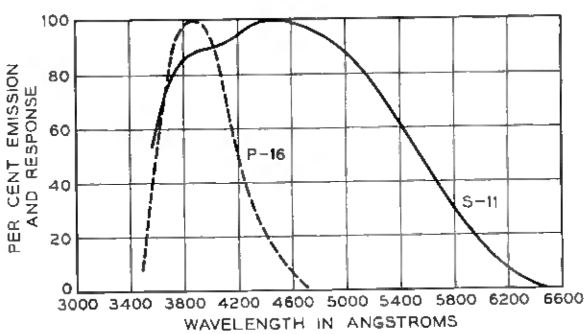


**Fig. 2** — Spectral emission characteristic of type P-16 phosphor and relative spectral sensitivity of type S-11 photocathode.

requirements or device limitations. For example, system speed may dictate that there be one spot on the cathode ray tube face for each word to be stored in the memory, in order that readout can be obtained in a single access operation. This requirement, coupled with limitations on the length and diameter of the cathode ray tube which may be used, then fixes spot size. In other cases, the spot size may be chosen, along with other parameters of the system, to provide an optimum system design for a specific application. In the laboratory model of the flying spot store spot size was set by device limitations.

The maximum current density in the focused spot depends directly on cathode loading and accelerating voltage.[5] The cathode loading must be limited to a value such that good tube life is obtained. In general, we make the accelerating voltage as high as possible, since light output increases with both the total current in the beam and the bombarding energy. However, limits are set on maximum accelerating voltage by the maximum deflection voltage which we wish to provide (deflection-amplifier power consumption sets this limit) and by the allowable phosphor loading, (watts/$cm^2$ to screen). In many cases the phosphor loading is the more severely limiting factor. This is easily understood because the power density at the phosphor screen may easily reach a kilowatt per square centimeter, screen weights are typically a few milligrams per square centimeter and the temperature rise of the screen with this power input is very rapid. Under such conditions, the exposure of any one point of the phosphor screen to the beam must be limited or the screen will be quickly destroyed. These factors determine the maximum total flux from a spot. If sufficient flux cannot be obtained for an intended application, either speed of operation or channel capacity must be reduced.

2.1.2.3 *Choice of Optics.* With the total flux determined by these factors, the objective lens must be chosen so that the fraction of the total light collected which reaches the photocathode produces the required photocathode current level.

Primary among the other factors that enter into the choice of optics is the effect of the lens on discrimination ratio. Since many parallel optical channels are combined in the store, the performance of the furthest off-axis lens is usually limiting. In this or any other channel the same beam initially writes, and later reads, the spots on the photographic emulsion. If the lens scatters light from the focused beam into adjacent areas of the emulsion, the spot spacing at the cathode ray tube must be made greater than that required to prevent overlap due to distribution in intensity in the source alone. Wider spacing reduces the number of spots in both coordinates of the array and reduces the total system capacity.

2.1.2.4 *Low Flux Level at Photocathode.* The fraction of the total flux which must be collected by each objective lens to provide the required photocathode current is very small, even in the highest-speed systems. For example, total flux of the order of 1500 microwatts can be obtained from each spot in a 256 $\times$ 256 array on a type P-16 screen operated at the phosphor loading limit. The flux required at the photocathode to produce a shot-noise-limited signal-to-rms. noise ratio of 20 to 1, with an output circuit bandwidth of 10 megacycles, is about 0.03 microwatts. Thus, the lens aperture and focal length and the system magnification ratio must be chosen so that a fraction $0.03/1500 = 1/45{,}000$ of the total flux emitted reaches the photocathode. The fraction is smaller in slower systems, with output circuits of narrower bandwidth.

The important fact that this fraction is small even for high-speed systems cannot be overemphasized. Thus, the requirement on the fraction of the total flux which must be intercepted by each objective lens can be met in a wide range of optical systems. For example, D. R. Herriott has shown several designs which utilize from 36 to 300 lenses. All of these are scaled from the operating conditions in the laboratory model of the flying spot store. All keep the flux at the photocathode constant at the level found in the laboratory model. It is this favorable situation which allows us to design stores with very diverse speeds and capacities. Many of the other desirable features of the system result from the use of lenses of small aperture operated at relatively large object distances. For example, increasing the $f/$ number of a lens permits a greater depth of focus to be achieved. This results in much less severe axial positioning tolerances on optical elements, cathode ray tube and film plane.

2.1.2.5 *Energy Required for Exposure.* One additional condition must be met. The flux density in the focused beam at the film plane must be sufficiently high to get a dense spot after development with a very short exposure time. This requirement arises directly from the mode of operation, in which the entire memory plane is prepared by exposure of each spot in turn. It has been found that the flux level which produces a satisfactory signal-to-noise ratio at the photocathode, even in the highest-speed system, produces, in quite ordinary photographic emulsions, a dense written spot with exposure times ranging from a few tens to a few hundred microseconds. Thus, the exposure of several million spots requires a few minutes at most and the process is reasonable. This is another of the large number of points of compatibility enjoyed by this particular form of system.

### 2.1.3 *Example of Single-Channel Design*

To illustrate the preceding discussion we present a design for a typical single channel suitable for use in a flying spot store. The components and operating conditions listed in Table I have been chosen with the idea of providing a large number of parallel channels. These parameters are intended to be a typical example rather than be descriptive of a working flying spot store. However, the flux levels throughout the system and other conditions listed in Table II have been verified in laboratory experiments.

## 2.2 *Multiple Optical Channels*

It has been shown that one can build multiple optical channel systems which satisfy flux level, mechanical and optical requirements. This section discusses some additional aspects of multiple channel organization.

TABLE I — CHARACTERISTICS OF DEVICES FOR INFORMATION STORAGE CHANNEL

| | |
|---|---|
| **Cathode Ray Tube** | |
| Array size: | 256 × 256 |
| Phosphor screen: | type P-16 |
| Operating conditions: | 10 kv, 10 microamperes in focused beam |
| Spot size: | 0.025 inch |
| **Photomultiplier Tube** | |
| Photocathode: | type S-11 |
| Photocathode sensitivity | ≈0.04 microampere/microwatt (including effect of spectral sensitivity and phosphor emission spectrum) |
| Gain: | $1.5 \times 10^5$ |
| Output current: | 350 microamperes (limited by dynode gain fatigue) |
| Output circuit bandwidth: | 3.5 mc (0.1 microsecond sampling time) |
| **Optics** | |
| Objective lens: | 6-inch, f/8.0 lens operated at 4/1 reduction |
| Condenser lens: | 4-inch f/3.5 |
| Absorption and scattering loss: | 1/4.2 (losses in objective and condenser lens, cathode ray tube face plate and emulsion — extreme off-axis channel) |

TABLE II — POWER LEVELS AND OPERATING CONDITIONS IN SINGLE-CHANNEL SYSTEM

| | |
|---|---|
| Power in cathode ray tube beam: | $10^5$ microwatts |
| Peak power in focused beam: | 250 watts/cm² max. (limiting factor) |
| Phosphor screen efficiency: | ≈1.5% |
| Flux from cathode ray tube: | 1,500 microwatts |
| Fraction of light collected by objective lens: | 1/6400 |
| Fraction of light lost by scattering and absorptions: | 1/4.2 |
| Flux at photomultiplier tube photocathode: | 0.055 microwatt |
| Peak flux density in film plane: | 885 microwatts/cm² min. |
| Exposure time: | ≈100 microseconds |
| Photocathode sensitivity to emission from P-16 phosphor: | 0.04 microampere/microwatt |
| Photocathode current: | 0.0023 microampere |
| Shot noise limited $S/N$; $S_{peak}/N_{rms}$: | 45/1 |
| Photomultiplier tube current gain: | $1.5 \times 10^5$ |
| Output current: | 350 microamperes |

2.2.1 *Information Assembly and Exposure*

In the multiple-channel store one bit of each word is stored on each information storage area of the photographic plate. The information to be stored is assembled so that all the bits to be placed on any one plate appear in sequence. The addresses of the binary 1's are then recorded on magnetic tape. Means are provided which allow the storage areas to be exposed one at a time. In writing the plates, one particular channel is selected and the position of the cathode ray tube beam is placed under the control of the information recorded on the magnetic tape. The beam is addressed to each spot of the array in turn. Opaque spots are generated by allowing the beam to dwell at a spot position for an appropriate exposure time — in the laboratory model this is approximately 600 microseconds for ordinary negative plates in which opaque spots result from exposure and 2000 microseconds where the plates' photographic image is reversed in development. The beam is swept quickly past those areas which must remain transparent. When the storage areas have been exposed, the plates are removed from the store, developed and reinserted.

### 2.2.2 *Parallel Readout*

In general, a number of channels equal to or greater in number than the word length can be supplied. With this organization, a single access operation (i.e., beam positioning) results in readout of the entire word in parallel. Because the access operation normally occupies much more than half of the cycle time, the time for readout of a given volume of information is minimized. This organization also eliminates the need for shift registers to assemble the parts of the word, which results in a minimum amount of output circuitry. It also affords advantages when redundancy is introduced for error detecting and correcting.

### 2.2.3 *Increased Total Capacity*

Multichannel stores can be built with capacities exceeding that of any single-channel store and without sacrificing speed of operation.

2.2.3.1 *Conditions on Maximum Number of Channels.* The maximum number of channels depends on the maximum off-axis angle at which the lens chosen can be used without degrading the discrimination ratio below a specified value, upon the lens aperture and object distance (distance from lens to cathode ray tube screen), and upon the maximum flux available from the spot on the cathode ray tube screen. The off-axis angle and object distance determine the area of the lens plane which may be used. The number of lenses which may be placed in this area depends upon the lens aperture and, because of the restriction that the image areas must not overlap, on the magnification ratio. This means that the magnification ratio is usually less than one and decreases as the number of channels is increased. However, the effect of film blemishes might become serious at very high storage densities, and this may become an additional limit on magnification ratio. For a given limiting angle, the usable area in the lens plane increases with the reduction in magnification ratio, so this additional requirement does not conflict with the growth in number of channels.

If we assume that the magnification ratio is fixed by one of the preceding requirements and focal length is fixed by the over-all length of the system, then the solid angle in which lenses may be placed increases and the solid angle subtended per lens decreases, with increasing $f/$ number. The maximum number of channels is proportional to this ratio and grows strongly with increasing $f/$ number.

2.2.3.2 *Effect on Array Size and Resolution.* All the changes which increase the number of channels do so at the expense of a reduction in resolution at the cathode ray tube. This reduction is made necessary by two factors. First, it is usually not possible to obtain the same resolution in wide off-axis positions as is obtained on axis. Second, the changes in the optical system which permit us to realize a large number of channels reduce the fraction of the light collected by each objective lens. If the light level at the photocathode and, as a consequence, the electrical signal-to-noise ratio and speed of operation are to remain constant as $N$ is increased, the flux at the cathode ray tube must be increased. Since the cathode ray tube is usually operated with phosphor loading as a limitation, this increase can only be obtained by increasing the spot size. Since the over-all size of the cathode ray tube remains fixed, this forces a reduction in the number of spots in the array.

2.2.3.3 *System Capacity.* System capacity is given by $C = NR^2$, where $N$ is the number of channels and $R$ is the number of spot positions in one row of the cathode ray tube array. As we change from a single on-axis channel to a multiple channel system, $N$ increases and $R$ decreases. However, $R$ falls much less rapidly than $1/\sqrt{N}$, so that the total system capacity increases strongly with the number of channels.

As the number of channels is increased the magnification ratio must eventually be reduced to avoid overlapping images. Likewise, off-axis operation tends to reduce the resolving capability of the lens. Finally, increasing the number of channels reduces the flux available to each lens, both because of the reduced effective aperture in the off-axis positions and because of the reduced solid angle subtended by each lens. These factors tend to require either larger spot size at the cathode ray tube to increase the available flux or wider spot spacing to hold the discrimination ratio constant. Thus, increasing the number of channels tends to force a reduction in the number of spots used on the cathode ray tube screen. However, this reduction is small enough to permit the total system capacity to continue to grow strongly with increasing number of channels.

The system may be scaled according to number of channels, system capacity and speed remaining constant. If, for example, the beam diameter is doubled the flux available is increased by a factor of four, and the number of spot positions in the cathode ray tube array is reduced by a factor of four. This increase in flux permits us to use a lens of one-half the aperture of that in our original system, so that four times as many lenses can be accommodated within the same area of the lens plane.

#### 2.2.4 *Independence of Bits of Stored Word*

The parallel-channel arrangement is well suited to the introduction of redundancy for error checking and detecting. If only one bit of each word is placed on each storage area in the film plane, a film blemish, regardless of size, can affect at most one bit of the stored word. Use of the Hamming code[6] then makes it possible to detect and correct one or more independent errors of this type. In this case, the information to be stored would be encoded in Hamming single-error-correcting, double-error-detecting code, with the check and correction features extending over the Hamming bits. These bits would be read out as part of the word and are available at once when needed.

There are virtually no interactions between the remote storage elements of one storage area or between the various channels.

#### 2.2.5 *Optical Beam Encoding*

The availability of multiple optical channels makes it possible to use some of these channels in the optical beam encoder, which is the sensor element of the closed-loop beam-positioning system. It will be shown that very considerable advantages accrue to the system as a whole through the use of this type of sensor element.

### 2.3 *Servo Control of Cathode Ray Tube Beam Position*

#### 2.3.1 *Description of Beam-Positioning System*

The feedback beam-positioning system is shown in block diagram form in Fig. 3(a). Since the positioning systems for the two deflection directions are identical and independent, only one is shown.

Beam positioning involves both the digital selection of a single storage cell and the precise positioning of the beam at the center of this cell.

These precise positions, as well as the binary coordinates of the cell, are established by the pattern on the code plates located in the same plane as the stored information. Each code plate occupies one channel of the system. Its associated lens forms a line image on the code plate of the spot on the cathode ray tube face. Patterns on the code plate are chosen so that the readout from the photomultipliers in these channels gives the beam coordinate in parallel binary code. This section of the system is termed the optical beam encoder.

In the first step in beam positioning, the position readout from these code plates is compared with the required position (address) in the digital comparator. This comparison yields an error signal which is applied to the integrating and deflection amplifiers in series to drive the beam to the desired position.

The simplest form of the comparator generates an error signal of the

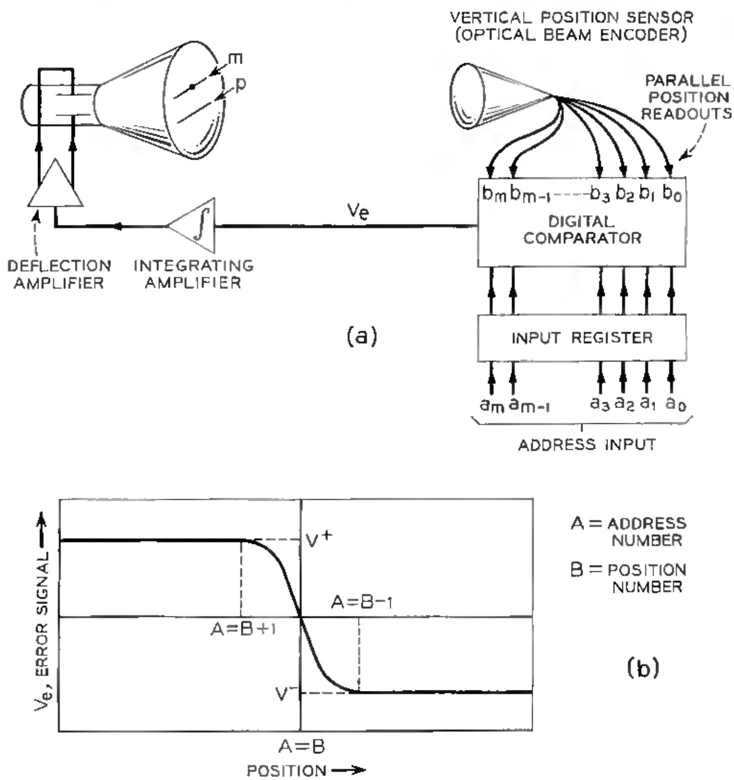


Fig. 3 — (a) Feedback beam-positioning system; (b) error signal as a function of position.

form shown in Fig. 3(b). The output has a constant positive value, $V^+$, when the beam is more than one full cell above the desired address and an equal negative value, $V^-$, when the beam is more than one full cell below the desired address. This particular type of comparator is known as the "sign-only" comparator. Comparators which yield sign and approximate magnitude of error have also been built and are being studied. This discussion is confined to the sign-only type, since this illustrates the important system features.

Once the beam is within the correct cell, the error signal becomes a continuous monotonic function of position, passing through zero precisely at the center of the cell. This error signal is generated by the passage of image of the beam across the transparent-to-opaque boundary in the one code plate column which has an edge at the required position. The comparator transmits the error signal without modification to the integrating and deflection amplifiers and the beam is locked by linear servo action to the code plate edge at the center of the cell.

### 2.3.2 *Operation of the Feedback Loop*

The action of the beam-positioning system using a sign-only comparator can be understood by considering its response to a change in address. The beam is assumed to be at rest at vertical level $m$. When a new address corresponding to level $p$ is suddenly applied to the $a$ or address terminals of the digital comparator, the previously existing match disappears. Since $p$ is below $m$ on the cathode ray tube screen, the error signal jumps to the full positive value $V^+$. This step function applied to the integrating amplifier produces an output which increases at a constant rate with time. This output is applied to the deflection amplifier, which drives the beam downward at a constant speed towards position $p$. When the beam reaches the position $p + 1$, the output from the comparator and the speed of drive start to decrease. This is the beginning of the region of linear servo action.

In order to understand the action of the loop as the beam settles to the equilibrium position within this linear region, we must consider the open-loop frequency response curve of the system. This is an integrating type of response with high constant gain from dc to a low-frequency break point. Above the break point the gain falls at 6 db per octave. The unity gain point occurs at a high frequency, typically 500 kc. To insure stability and a well-damped response in settling, the loop gain frequency response is shaped for 3 to 4 octaves beyond the frequency of unity loop gain. Because of the finite dc and low-frequency gain, the loop exhibits integrating behavior only for transient changes. After sufficient time, a step-function input results in a constant displacement by an amount corresponding to the dc loop gain, rather than a constant speed.

When the beam reaches the position $p + 1$ the magnitude of the signal from the comparator starts to decrease towards zero and the speed of drive diminishes. The beam actually comes to rest at the point of zero output voltage from the comparator only in the center of the raster. For all other positions the beam is off-centered by a small amount. The off-centering is just sufficient for the comparator output voltage, which is amplified by the full low-frequency gain of the integrating and deflection amplifiers, to maintain the beam deflection to the edge selected. Typically, the open-loop dc gain is very high and signals of $0.1V^-$ or $0.1V^+$ are sufficient to deflect the beam to steady-state positions at the raster edge. The dc gain is made high so that all transfers will be completed within the initial 10 per cent of the ultimate response of the integrating amplifier to the step-function input from the logic. Within this region the voltage output increases linearly with time and the beam moves at constant speed.

Although the full low-frequency loop gain is not needed to establish the readout position (readout is done before the beam settles to the accuracy guaranteed by the dc loop gain), the gain is available to oppose any perturbation imposed on the system. Thus, in a system already operating, 60 db of feedback opposes all changes occurring with frequencies up to 300 cycles per second and 20 db of cancellation is available at 50 kc. Therefore, the system is much less sensitive to electrical pickup and mechanical vibration. This is one of the important advantages gained for the system by the use of a closed-loop beam-positioning system.

### 2.3.3 *Speed Considerations*

The speed of positioning is of major importance. In this section we shall show the factors which limit the maximum speed of operation in systems using the sign-only digital comparator mentioned previously. We shall also discuss the conditions which determine the time to settle to the readout position within the selected cell. It will be seen that these two factors are related.

There is a well-defined maximum speed with which the beam may enter the linear region and settle without overshoot or with small overshoot. If this is exceeded by a small factor, oscillatory behavior results. In systems using the sign-only comparator the maximum slewing speed in the region more than one cell away from the match location is set by this requirement.

From the preceding discussion, we can see that both the slewing speed and the time to settle from the edge of the cell depend on the loop frequency response. The maximum crossover frequency (frequency of unity loop gain) which can be realized depends on the asymptotic gain characteristic of the loop. The asymptotic characteristic is steep because of the relatively large number of stages in the loop. In a typical loop design, amplifier and logic stages contribute 6 db per octave to the asymptotic slope above 8 or 10 mc, the effect of the finite persistence characteristic of the phosphor is seen as a 3-mc frequency cutoff, and the photomultiplier tube output circuit contributes one cutoff above 800 kc. Thus, in a typical loop, the asymptotic slope may reach 48 to 60 db per octave above 8 megacycles. The situation is further complicated in that the transit time around the loop contributes excess phase shift which cannot be equalized. This delay amounts to approximately 0.1 microsecond in the systems built, and seriously reduces the phase margin at high frequencies. Because of the steep asymptotic characteristic and excess delay phase, it is helpful to introduce a flat gain step in the frequency response below unity gain in order to preserve stability and insure a well-damped settling response.

One loop now in use has a loop crossover frequency of 500 kc. The maximum slewing speed is three spots per microsecond. The settling time to within 0.1 spot diameter of the final position is 0.8 microsecond.

#### 2.3.4 *The Optical Beam Encoder*

The concept of the use of the optical beam encoder in the beam-position servo system was developed by C. W. Hoover, Jr. One version is shown in Fig. 4. The pattern on the code plates can be chosen so that the readout is in any desired binary code. Gray code (reflected binary) is normally used in the flying spot store. The code plate edges which occur at the transitions from each number to the next are used as the reference edges to which the beam is locked by linear servo action.

2.3.4.1 *Coupling Between Cathode Ray Tube and Film Plane.* In an earlier section it was noted that high loop gain is available to oppose the effects of external perturbations. Since the code plates are located in the same plane as the stored information, a valuable coupling is introduced between the reading spot of light on the cathode ray tube face and the written information on the code plate. Thus, in the case of vibration, the cathode ray tube might move but the reading spot of light would remain fixed in space with its projected image on the written spot, as long as the position of the code-plate edges remains fixed with respect to the written spot. Likewise, movement of the image plane as a whole would not cause beam malpositioning.

2.3.4.2 *Frequency Cutoffs and Time Delay in Encoder.* Since the encoder forms part of the feedback loop, its gain and delay characteristics are important. Frequency cutoffs encountered in the encoder are the transit time cutoff of the cathode ray tube deflection plates, the cutoff due to the phosphor persistence characteristic and the dispersion cutoff of the photomultiplier and the photomultiplier tube output circuit. The transit time cutoff of the deflection plates and the effective dispersion cutoff of the multiplier occur above 100 mc. Thus, they contribute virtually no phase at or near the frequency of unity loop gain and can be excluded from the asymptotic characteristic in this region. The effect of the phosphor persistence can be represented by a 3-mc frequency cutoff. The other important cutoff is found in the photomultiplier tube output circuit. The output resistor is chosen to give a voltage swing of about 3 volts to operate the digital comparator with fatigue-limited maximum output current of the photomultiplier. Typically, this resistor, with the output and circuit capacitances, results in a frequency cutoff

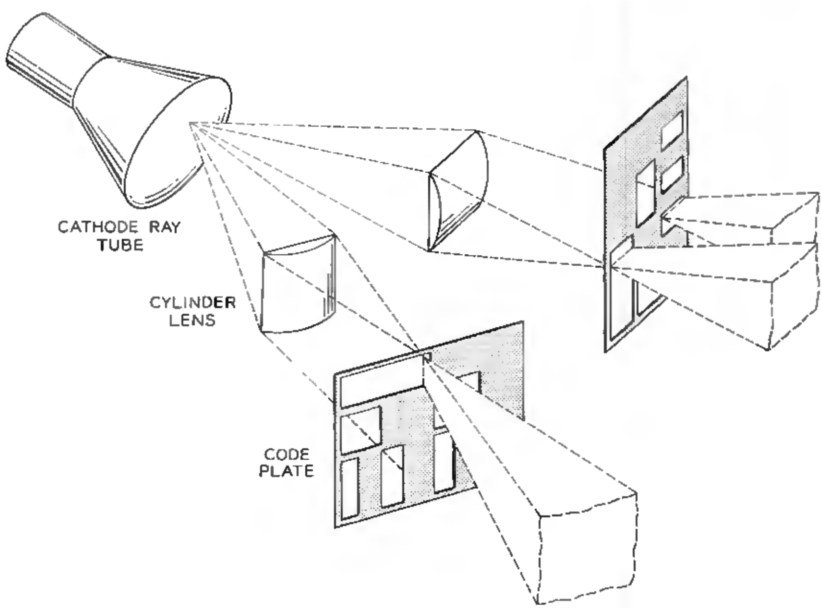


Fig. 4 — Two-axis optical beam encoder.

of about 800 kc. Thus, the optical beam encoder contributes only two important cutoffs to the asymptotic loop characteristic.

Most of the signal delay found in the loop is due to the encoder. Here are found the transit time delays because of the electron transit time from the deflection plates to the cathode ray tube screen (approximately 7 millimicroseconds), the time of flight of light in the optical path (approximately 4 millimicroseconds) and the electron transit time in the photomultiplier (50 millimicroseconds). As mentioned earlier, these delays play a part equally important to that of the open loop gain characteristic in determining the speed of slewing and settling.

2.3.4.3 *Encoding Transconductance.* One measure of the utility of an encoder is the encoding transconductance. For the optical beam encoder, considered as a unit from the cathode ray tube deflection plate terminals to the photomultiplier tube output, the encoding transconductance is defined as

$$g_{\text{enc}} = \left.\frac{\Delta i}{\Delta V}\right|_{\substack{1 \to 0 \\ 0 \to 1}}, \tag{2}$$

where $\Delta i$ is the current change at the output of the photomultiplier as the beam moves from an opaque area on the code plate to a clear area and $\Delta V$ is the voltage change at the deflection plates required to move the beam from the center of one spot position to the center of the adjacent spot position (i.e., to change a "1" to a "0" or vice versa on the least significant digit code plate). The maximum change in current, $\Delta i$, is limited by the maximum current which can be drawn from the photomultiplier tube without serious fatigue. J. R. Pierce (Ref. 5, p. 142) has shown that there is a theoretical limit on the minimum possible deflection sensibility [defined as volts to move the beam by an amount equal to the spot diameter — note that this is precisely $\Delta V$ in (2) above]. This dependence is given in

$$\Delta V \geqq K \left(\frac{j_{\max}}{j_0}\right)^{1/2} V_a , \tag{3}$$

where $j_{\max}$ is the maximum current density in the beam, $j_0$ is the cathode current density, $V_a$ is the potential in the deflection region and $K$ is a constant depending on the geometry of deflection plates and focusing system. In the cathode ray tube designs for the flying spot store, the ratio of current density in the spot to the cathode current density is very nearly constant. This is because current efficiency is sacrificed for intensity efficiency ($j_{max}/j_0$) in these tubes. At low current efficiency, the ratio ($j_{max}/j_0$) is nearly constant (Ref. 5, p. 125). Thus, we see that minimum deflection sensibility increases directly with the accelerating voltage. Therefore, to obtain maximum encoding transconductance in the encoder it is desirable to choose the minimum value of accelerating voltage at which the desired light output and resolution can be obtained. The transconductance does not increase with increasing accelerating voltage simply because the fatigue limit on photomultiplier output current prevents us from taking advantage of higher output currents due to greater light inputs to the multiplier.

A typical encoder has a deflection sensibility of 2 volts per spot and an output current of 2 ma. This yields an encoding transconductance of 1000 micromhos and should be compared with the encoding transconductances of 100 micromhos typically obtained in electron beam encoding tubes.

2.3.4.4 *Effect of Encoder on Cathode Ray Tube Requirements.* Finally, we note the effects of the use of the optical beam encoder on the cathode ray tube. Since the complete array of positions is specified by the fixed code-plate masks, it should be possible to change cathode ray tubes, after a failure, without rewriting the information on the storage plates. In addition, the requirements on cathode ray tube deflection direction orthogonality and uniformity of deflection sensibility with position are virtually eliminated. Consider, for example, the effect of a 10 per cent change in deflection sensibility. In analog beam-positioning systems this would result in local crowding or expansion of the pattern. In the closed-loop system this merely results in a very small change in the time required for the beam to settle to its final position.

### 2.3.5 *The Digital Comparator*

Several types of comparator have been developed by R. W. Ketchledge. However, since our purpose is to show general principles and mode of operation we confine our discussion to the sign-only form of the comparator. This form has been termed the sign-only servo.

As a first step in the description of the comparator we note all of the special conditions placed upon it by the application. These are as follows:

1. Parallel operation. Speed dictates that the comparator operate on parallel inputs from the input register and optical beam encoder.

2. Carry from most significant to least significant digit. When the position is changing rapidly the values of the encoder digits of low significance become indeterminate. Use of a carry in the normal direction would therefore seriously limit speed.

3. Linear transmission characteristic within selected cell. For signals of magnitude less than $V^+$ or $V^-$ the comparator merely transmits the error signal generated by one edge in the optical beam encoder.

4. Gray-binary comparison. The input addresses to the flying spot store are in parallel binary form. The code plates of the optical beam encoder read out the beam position in the Gray or reflected binary code. This code has the very desirable feature that one and only one digit changes in going from any one level to either adjacent one.

5. Compatibility with servo loop. The transmission and delay characteristics of the digital comparator must be such that it can be included in the servo loop without serious penalty in speed and stability. Its frequency response should be flat to well above the frequency of unity loop gain and it should contribute as few cutoffs as possible to the asymptotic characteristic. The delay should be such that the equivalent phase at gain crossover is tolerable.

Sign-only logic meeting these requirements has been built. We now turn to a discussion of the actual mechanism of number comparison.

The starting point is the following set of rules for obtaining the sign of the error between two parallel binary numbers. These are developed first for the case of binary-binary comparison and extended later for the case of binary-Gray comparison.

Consider two binary numbers, $A$ and $B$, with $A$ being the address number which remains fixed throughout the cycle of comparison and $B$ being the position number which takes on all successive digital values in approaching $A$. Thus we have two numbers:

$$\begin{aligned} &\text{Address, } A\colon a_m\,,\ a_{m-1}\,,\ a_{m-2}\,,\ \cdots\ a_j\,,\ \cdots\ a_0\,, \\ &\text{Position, } B\colon b_m\,,\ b_{m-1}\,,\ b_{m-2}\,,\ \cdots\ b_j\,,\ \cdots\ b_0\,, \end{aligned}$$

where $a_m$ and $b_m$ are the most significant digits, and where the first mismatch is found at the $j$th digit. The first mismatch determines the sign of the error. The rules are as follows:

1. Consider the pairs of digits $a_i$, $b_i$.
2. Write a third number $c_i$ resulting from each comparison.
3. Starting at the most significant digit, write $c_i = 1$ for all matches up to the first mismatch.
4. At the first mismatch, $a_j$, $b_j$, choose $c_j$ as follows:

$$\begin{aligned} c_j &= 1, \qquad a_j > b_j \text{ (i.e., } a_j = 1, \qquad b_j = 0), \\ c_j &= 0, \qquad a_j < b_j \text{ (i.e., } a_j = 0, \qquad b_j = 1). \end{aligned}$$

5. Following $c_j$, write $c_i = 1$ for all remaining digits, if $a_j \geqq b_j$.
6. Combine the $m$ digits, $c_m \cdots c_0$, resulting from these operations

in an $m$-input coincidence circuit. The output of this coincidence circuit will be 1, i.e., a coincidence resulting from an input on each of the $m$ leads whenever $B \leqq A$. As $B$ changes from equal to $A$ to greater than $A$ the coincidence disappears, and for all cases $B > A$ the output of the coincidence circuit is 0. Examples will make this clear. The $m$, $c_i$ outputs go to individual leads of an $m$-terminal coincidence circuit.

*Example 1:* $B < A$

| | | | | | | $i = j$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a_i$: | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| $b_i$: | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| $c_i$: | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

In this case the $c_i$ values are all 1. Therefore the coincidence circuit output is also 1. This output drives the beam in the direction required to reduce the value of $B$. This is defined as a down drive.

*Example 2:* $B = A$

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a_i$ : | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| $b_i$ : | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| $c_i$ : | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Here again the coincidence circuit output is 1 and the direction of drive is down.

*Example 3:* $B > A$

| | | | | | $i = j$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a_i$: | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| $b_i$: | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| $c_i$: | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

In this case comparison of the $c_i$ inputs in the coincidence circuit results in 0 and the drive direction is up.

From these examples we see that the beam continues to move after the point of numerical equality is reached. The rest position is one-half cell distant from the point of numerical equality in the direction in which $B$ increases. Thus it is seen that the storage cell boundaries are displaced by one-half cell from the code plate edges. When the Gray code is introduced, it will be seen that the significant advantage of this code is that, at any level, all digits except the one changing remain at full-amplitude readout as the beam is moved beyond match by one-half cell position.

Now we shall develop a generating function for the $m$ inputs, $c_m$, $c_{m-1}$, $\cdots c_0$, to the final coincidence circuit. Once we have this generating function in logical notation we can at once set down the block diagram of the logic circuit. We start by observing that, under our rules, the value of any $c$, say $c_k$, is determined either by the $k$th digit pair or by a mismatch in a more significant digit pair. We note that the function $a_k + b_k' = 1$ for $a_k \geqq b_k$, but that $a_k + b_k' = 0$ for $a_k < b_k$.

We can use this function to provide the correct output if the first mismatch occurs at the $k$th digit. However, our rules say that the most significant mismatch rules. Hence a mismatch occurring at the $k$th digit of the form $a_k < b_k$ ; $a_k + b_k' = 0$ must be overpowered by a more significant mismatch of the type $a_m > b_m$, so that the resultant $c_k$ is a 1. To take care of this possibility, we consider all more significant pairs. The function $a_x b_x'$ has the characteristic that it is 1 only for a mismatch condition in which $a > b$. Hence, if we combine all digit pairs for digits more significant than $k$ with a parallel OR circuit, $c_k$ will always be 1 when it is preceded by a mismatch of the type $a_j > b_j$. Finally, we note that $a_j + b_j' = 0$ at the first mismatch, for $a_j < b_j$, and that $a_k b_k' = 0$ for all $k > j$. Thus the correct down drive is attained. Our generating function is therefore

$$c_k = (a_k + b_k') + a_m b_m' + a_{m-1} b_{m-1}' + \cdots a_{k+1} b_{k+1}'.$$

The logical circuit realization is shown in Fig. 5. Note that the carry is formed by the long string of OR functions proceeding from the most significant digit towards the least significant digit. For reasons noted earlier, it is necessary to get a version of the comparator in which the position inputs, $b_m \cdots b_0$, in binary code are replaced by position inputs in Gray code, $g_m \cdots g_0$. We first note that any translation operation must translate the Gray-code inputs to the equivalent binary-code inputs, starting with the most significant digit and working down to the first mismatch. However, the translation must not destroy the property that only one digit changes at a time. If the translation has the proper

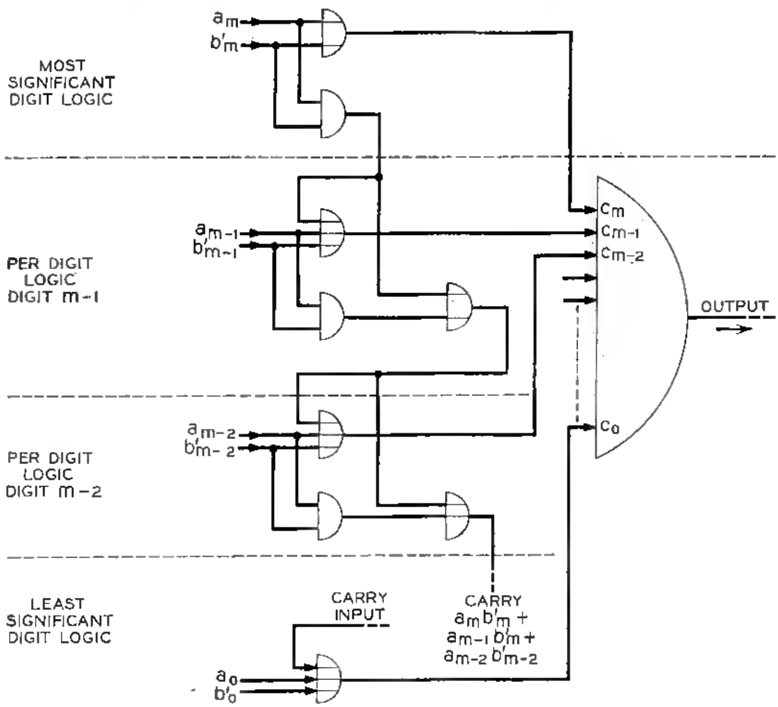


Fig. 5 — Sign-only comparator, binary-binary type.

behavior for match conditions and for the first mismatch, this function may be added to the circuit of Fig. 5, since all $c_k$ for digits beyond the first mismatch are disposed of by the carry which proceeds within the circuit already developed.

To start with, we note that the Gray-code number equivalent to any binary-code number can be generated by applying the following rule in sequence to the digits of the binary number:

$$g_k = b_k \oplus b_{k+1} .$$

The symbol $\oplus$ denotes the "ring sum" or "exclusive or". Its properties are shown by the following table:

| $b_k$ | $b_{k+1}$ | $b_k \oplus b_{k+1}$ |
|---|---|---|
| 1 | 1 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

Now consider the $n$th digit which is preceded by a match condition. In this case we may write

$$b_n = b_n \oplus b_{n+1} \oplus a_{n+1}$$

since $b_{n+1} \oplus a_{n+1} = 0$, all digits with $i > n$ are matched. In this regrouping we have the term $b_n \oplus b_{n+1}$, which is simply $g_n$. Therefore we may write

$$b_n = g_n \oplus a_{n+1} .$$

The number formed using this rule provides the correct translation down to and including the first mismatch. This translation also retains the Gray-code property of only one digit change at a time.

As we have noted, this is sufficient. R. W. Ketchledge, who discovered this translation property of the function $g_n \oplus a_{n+1}$, has termed the number formed using this rule a pseudo-binary number. It is the simplest form of translation which allows us to translate a Gray-code number to a form suitable for use in operating the comparator. Its utility can be easily understood if we compare this form to the usual rule for translating from Gray to binary, which involves counting down modulo 2 from the most significant digit to the digit to be translated, a process which clearly involves all of the more significant digits.

Transforming the preceding expression we have

$$b_n' = (g_n \oplus a_{n+1})' = g_n \oplus a_{n+1} \oplus 1 = g_n \oplus a_{n+1}'.$$

This gives us a form eminently suitable for direct replacement in our function $c_k$. Note that the new $c_k$ has the proper behavior at the first mismatch. Substituting, we then get our new form for $c_k$:

$$c_k = [a_k + (g_k \oplus a_{k+1}')] + a_m(g_m \oplus a_{m+1}') + \cdots a_{k+1}(g_{k+1} \oplus a_{k+2}').$$

Here we note that the term $a_{m+1}'$ is 1, since all digits in positions of greater significance than the most significant digit are necessarily zero. There we may write:

$$a_m(g_m \oplus a_{m+1}') = a_m g_m .$$

Substituting this into the expression for $c_k$ above results in a form for $c_k$ similar to that for the binary-binary comparator. The logic circuit can be drawn at once from the expression and is shown in Fig. 6.

## III. LABORATORY MODEL OF THE FLYING SPOT STORE

### 3.1 *Application*

The flying spot store discussed in this section was developed to meet the requirements of the electronic telephone switching system described elsewhere in this issue.[1] The objective was to provide the speed and capacity needed for a realistic test of the laboratory model of the switching system at as early a date as possible, rather than to obtain maximum capacity. As a consequence, many of the desirable features and refinements which can be incorporated in later stores are not included in this model. Nonetheless, it does provide all essential features required for the test.

### 3.2 *Requirements*

#### 3.2.1 *Capacity*

Because the laboratory model of the switching system serves only a few lines and because feasibility can be demonstrated by setting up a call without providing all of the ancillary routines of a full-sized switching system, the capacity required of the laboratory model of the flying spot store is small, 35,721 bits. Word lengths of 18 and 36 bits are uti-

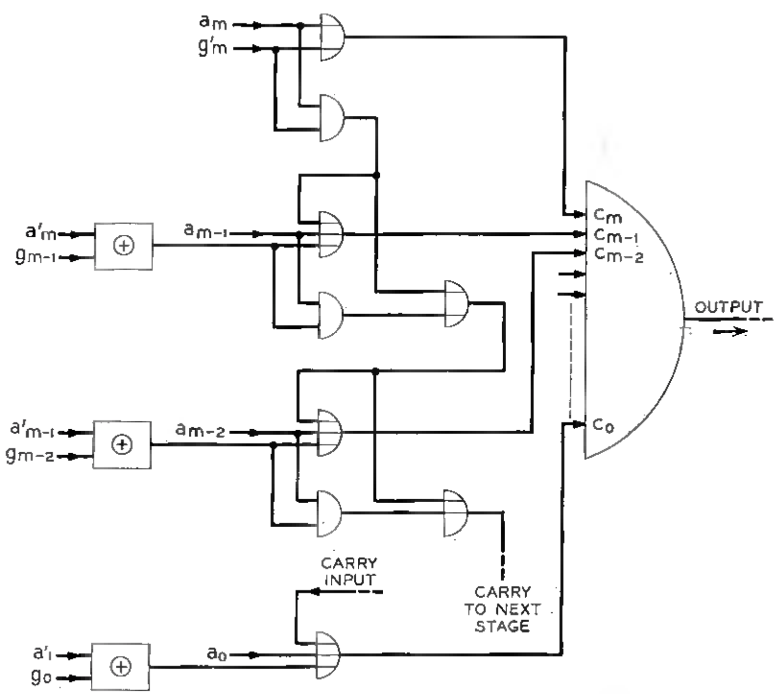


Fig. 6 — Sign-only comparator, binary-Gray type.

lized for this application and the output word is assembled from two or four readouts from the store. The X and Y addresses are each six binary bits. The array size is 64 × 64.

### 3.2.2 *Mode of Operation*

The laboratory model of the flying spot store has been in operation in the experimental electronic switching system for some time. In this application the store operates in response to input addresses and control signals (orders) supplied by the central control. A block diagram of the store showing input, output and control leads is given in Fig. 7. The output word is assembled in the output shift register from two readouts of nine bits each from the nine parallel storage channels. The central control of the system then takes this word from the output register.

Two main orders are used. These are "advance" and "transfer". In an advance order the word to be read out is located adjacent to the last word read out. This is the order most frequently used in the system. In

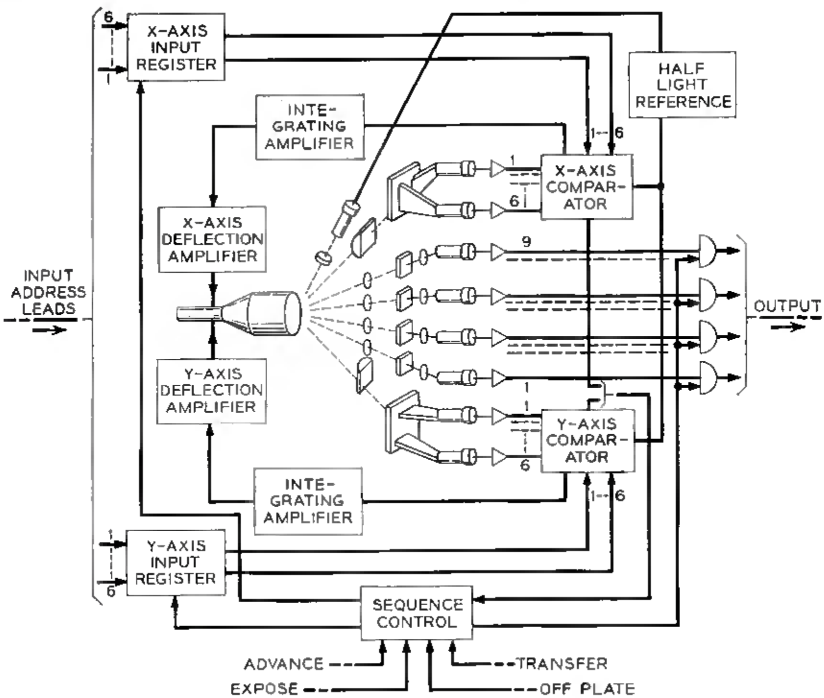


Fig. 7 — Block diagram of flying spot store.

this mode of operation a single signal is supplied from the central control to the advance lead. This signal brings about the following events in the internal program section of the store: The existing address in the x-axis input register is incremented by one. In response to this, the beam is driven to the adjacent location. Once it is there, a sampling signal is generated and applied to the sampling amplifier in each information storage channel. The resulting samples set the flip-flops in the output register. Following this, the internal program shifts the information to the second section of the shift register, again increments the input address register and, after a delay for beam positioning, generates another sampling pulse. The second sampling operation provides the second half of the word.

The transfer order differs in that the word to be read out is located remote from the preceding word read out. In this case the central control furnishes a signal on the transfer lead as well as a complete new address and a greater time is provided before sampling to permit the beam to reach the remote location. Cycle time in this case consists of beam transit time plus the normal sampling cycle time. At the completion of both transfer and advance orders a "cycle complete" signal is generated by the internal program, and sent to central control. In the few instances where the word length is 36 bits, the word is assembled in the central control after two successive readout operations from the store.

In addition to these functions the internal program of the flying spot store also provides special control signals used during plate preparation, line-up and trouble shooting.

#### 3.2.3 *Speed of Operation*

In the laboratory model of the electronic switching system the flying spot store operates at a cycle time of 12 microseconds in the advance mode. This includes the time for incrementing the address register twice, two beam positioning operations and two half-word readouts, plus the assembly of the word in the output register. The maximum time to complete a transfer order is 25 microseconds.

Operating by itself, this flying spot store has performed satisfactorily at a cycle time of 3 microseconds for the advance order and 25 microseconds for the transfer order. In this case, the one-spot beam position change required has been completed in 0.8 microsecond.

### 3.3 *Components*

Fig. 8 is a rear view of the flying spot store, showing the placement of the various devices.

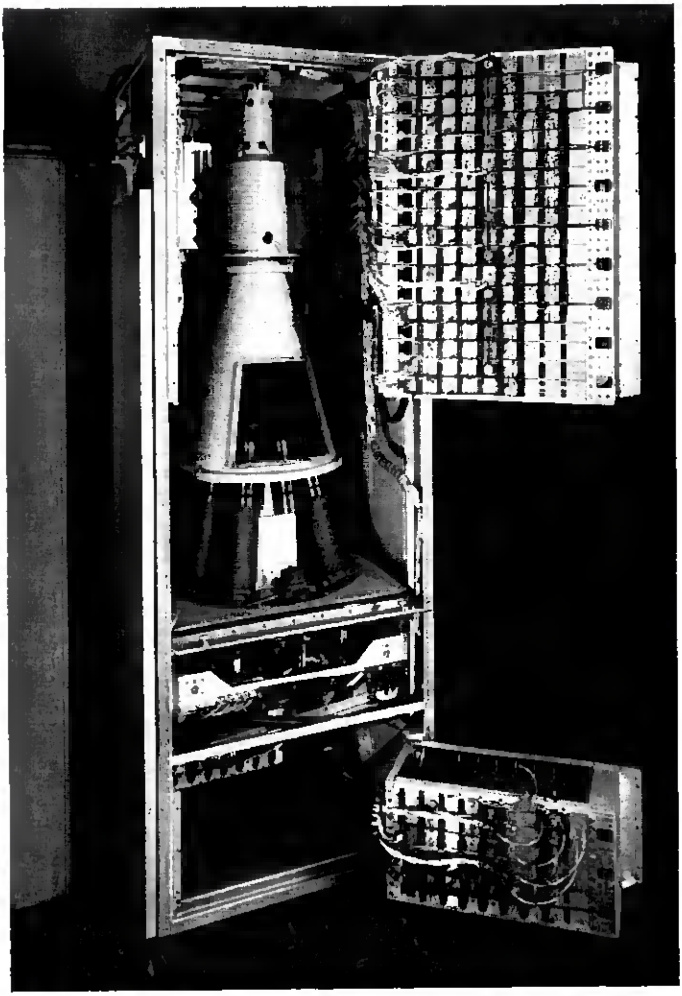

Fig. 8 — View of flying spot store with gates open.

### 3.3.1 *Cathode Ray Tube*

The cathode ray tube is shown at the top of the cabinet. It is supported above the main casting which holds the lenses, information storage and servo code plates, and is surrounded by a magnetic shield. The tube is 7 inches in diameter and is electrostatically focused and deflected. It is operated at a total accelerating voltage of 10 kv, with a beam current of 10 microamperes to the phosphor screen. The working area on the tube face is 3 $\times$ 3 inches. The phosphor used is type P-16. The screen is aluminized and has a weight of approximately 6 mg per square centimeter.

### 3.3.2 *Optical System*

The system uses nine Petzval lenses arranged in radial on-axis positions. Four of these can be seen protruding from the body of the support casting in Fig. 8. The storage plates are located within special mounts which are bolted to the main casting. A section of rubber tubing between the end of the lens and the storage-plate mount keeps the system light-tight and dust-free.

The lenses were originally intended for use in a much higher resolution system in which the available flux from each spot on the phosphor screen would have been lower by a factor of 16 or more. In the present application they have been stopped down to $f/13$. This provides greater depth of field and ease of adjustment, as well as reasonable exposure times for the emulsion chosen. The focal length of the lenses is 10 inches, and they are operated at a magnification ratio of 1:2. Thus, the storage area on each plate is $1.5 \times 1.5$ inches.

The storage plates are positioned in their mounts in the image plane of the system by three steel pins which bear against two ground edges of the glass photographic slides. A special magazine has been developed which allows a plate to be loaded into the channel for exposure, removed after exposure for development and reinserted after development.

A condenser lens is located beneath the image plane in each channel. This lens images the stop in the objective lens into a circle one-half inch in diameter on the face of the photomultiplier. This lens is required by the point-to-point nonuniformity of the photocathode response, which would introduce additional signal fluctuations into the channel if the narrow beam from the objective lens were to fall directly onto the photocathode.

### 3.3.3 *Photomultiplier Detector*

A photomultiplier tube is located in each storage channel. The tube used is a 10-stage type, operated with an output current of 350 microamperes in the information storage channels. The total gain required is about 200,000. At the input light level obtained in each channel of this system, the signal-to-shot-noise ratio is high. The discrimination ratio under these conditions is of the order of 10:1, where the fluctuations in the amplitude of the pulses read out from each channel are principally due to phosphor and film nonuniformity and to some regeneration noise in the photomultiplier tube. Satisfactory operation has been obtained with this signal-to-noise ratio.

### 3.3.4 *Sampling Amplifier and Output Register*

The output from each information channel photomultiplier detector is connected to a sampling amplifier. This section consists of an equalization and gain stage followed by a sampling stage and output amplifier. Gain is used following the photomultiplier tube output to make the detection of a 1 readout less dependent on power supply voltages.

The sampler outputs are fed to a shift register which is used to assemble the two or four readouts from the store into the word required by the system. The sampling amplifiers are arranged in a ring beneath the information channels. The output register packages are contained in a gate shown in Fig. 8 open at the upper right-hand side of the enclosure.

### 3.3.5 *Optical Beam Encoder*

The store uses an optical beam encoder of the type shown in Fig. 4. Two cylinder lenses are used, which are not visible in Fig. 8 since they are enclosed within the central part of the casting. They are arranged to provide two orthogonal line images, which fall on a pair of code plates with patterns arranged to give a Gray-code readout of position. A light pipe is provided beneath each column of each code plate to conduct a fraction of the light passing through that column to an individual photomultiplier detector. The light pipes are required because of space limitations on the code plates and the cylinder lens system.

In this type of encoder the length of the line image is made twice the width of the code plate to allow movement of the spot over the entire storage area without loss of position readout in any digit. Consequently, each digit column receives less than $\frac{1}{14}$ of the total light collected by each cylinder lens in the seven-digit encoder provided in this store (only six digits are presently in use). In addition, there are losses in the light pipes. For these reasons, the fraction of the light which must be collected by each cylinder lens is much larger than that which must be collected by each information channel lens. The cylinder lenses used in this system operate at $f/2.8$, with a focal length of 10.0 inches. They are used at a magnification ratio of 1:2.

### 3.3.6 *Beam Position Servo System*

The readouts from the optical beam encoder photomultipliers are transmitted directly to equalizing amplifiers located directly beneath them and connected to the digital comparator. The comparator used is the sign-only type discussed earlier. The $x$- and $y$-axis comparators are located at the bottom of the enclosure on gates (one is shown at the bottom right in Fig. 8). The error signal resulting from the comparison and code-plate readout drives the integrating and deflection amplifiers which are mounted at the top of the enclosure near the cathode ray tube.

## IV. SUMMARY AND CONCLUSIONS

### 4.1 *System Capacity and Number of Channels*

It has been shown that flux and signal-to-noise conditions must be met in flying spot stores with very high capacity and large number of channels. One important limitation is the maximum flux per unit area which can be obtained from the phosphor screen on the cathode ray tube. With specified maximum tube diameter and flux from the spot, the minimum spot diameter and maximum number of spots in the array on the tube face are fixed. As long as this limitation on the maximum number of spots in the array is not exceeded, maximum system capacity at a constant speed of sampling is not critically dependent upon the number of channels. Therefore, in most applications the number of channels may be chosen to provide word readout, together with error detection and correction bits, in a single-access operation.

System capacity increases with increase in sampling time but, due to resolution limitations on the optical elements and the cathode ray tube, this increase can usually be obtained only by increasing the number of channels.

The flying spot store type of memory is essentially a large-capacity device, since most of the elements must always be supplied and very little equipment is chargeable on a per-channel basis. This nearly fixed bulk of equipment in high-capacity stores means that the number of bits stored per active element in the control and sampling systems can be very large. This is one of the important advantages of this type of memory.

### 4.2 *Compatibility of Devices*

A very wide range of compatibility exists among the devices used in the store. Thus it has been shown that the emission spectrum of the most efficient fast-decay phosphor available is well matched to the spectral sensitivity curve of the efficient type S-11 photocathode. The efficiency and burn characteristics of this phosphor are such that the light level required at the photocathode in very high-speed systems can be obtained, even with lenses of high *f*/ number operated at magnification ratios considerably less than unity. The last two conditions permit very wide latitude in the choice of number of channels and result in reasonable tolerances on axial positioning of cathode ray tube, lens and storage plane. Finally, it has been found that the flux level at the photocathode which provides the required signal-to-noise level in readout is sufficient to create a dense image in the film after development, with short exposure times. This permits writing and reading with the same beam.

### 4.3 *Multiple Channel Organization*

The multiple channel organization permits large-capacity stores to be built without causing any of the devices to operate at the limit of resolution. This reduces the problems in the procurement of devices and tends to increase reliability in the system as a whole. It also provides for maximum possible speed of operation, since the total word is obtained in a single access operation. Some of the multiple channels are used in the optical beam encoder which is the sensor element of the servo. This type of sensor has been shown to be well suited for use in the servo loop. In addition, it introduces a valuable coupling between cathode ray tube and film plane which renders the system less sensitive to mechanical vibration and electrical pickup. Finally, the true independence of stored bits arranged so that only one bit of each word appears in each channel makes the flying spot store ideal for the use of error detection and correction codes.

### 4.4 *Closed-Loop Beam Positioning Systems*

Closed-loop beam positioning systems have been developed to overcome the analog accuracy problems encountered in open-loop systems. These systems provide digital beam positioning to within one spot diameter of the position for readout, followed by linear servo action in which the beam is locked to one of the code-plate edges of the optical beam encoder. A special digital comparator generates the error signal when the beam is more than one spot diameter distant from the required position for readout; it passes the proportional error signal without modification when the beam is within one spot diameter of the readout location. This comparator meets the special requirements imposed by its being included in the servo loop plus the requirements of the digital number comparison.

Complete systems based on this comparator plus the optical beam encoder are in operation in the laboratory and in the laboratory model of the flying spot store. They provide reproducible beam positioning with an accuracy of ±0.1 spot diameter. Typically, the time to position the beam to this accuracy from a one-spot distance is 0.8 microsecond, and the maximum slewing rate in the region more than one spot distant has been found to be about three spots per microsecond.

## V. ACKNOWLEDGMENTS

Many persons throughout the Bell Telephone Laboratories have made contributions to the development of the flying spot store. In particular, we want to acknowledge circuit contributions made by L. E. Gallaher, G. Haugk and K. K. Kennedy; mechanical and apparatus design contributions by J. J. Madden and D. C. Koehler; and optical and photographic studies by M. B. Purvis, G. V. Deverall, F. W. Clayden and D. R. Herriott. The encouragement and counsel of C. A. Lovell have provided the sound foundation for the project from the start.

We are also indebted to the Bausch and Lomb Optical Company for optical designs plus valued advice and to many persons at Eastman Kodak Company for advice on the photographic problems of this system.

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# A High-Speed Barrier Grid Store

By T. S. GREENWOOD and R. E. STAEHLER

(Manuscript received November 22, 1957)

*This paper describes a high-speed random access memory developed to serve an experimental electronic telephone switching system. The memory uses a barrier grid type electrostatic storage tube which is incorporated in a complete general-purpose store with a capacity of 16,384 bits. Random access to any bit together with a full storage cycle of reading and writing is complete in 2.5 microseconds, permitting a 400-kc repetition rate.*

*Experience with this store indicates that barrier grid storage can provide stable, compact, economical memory for data handling systems.*

## I. INTRODUCTION

The last two decades have seen a phenomenal growth in the field of digital data handling systems of large capacity. Prior to 1940 the telephone central office contained the only large-scale systems to be found. Since that time the number of special and general-purpose computers has mushroomed and, at the same time, the demands on special data handling devices for such systems have become more severe. One of the more thorny problems has been that of providing adequate storage for the large amounts of information received and generated by the systems. Early systems effectively used electromechanical relays for information storage, but the demands for speed and economy have forced the system designer to go far afield in his search for satisfactory memory devices.

While many devices have been developed for memory purposes, each particular system has requirements which limit the choice to relatively few of them. Those systems having most severe choice limitations required a random access memory with very high-speed reading and writing. Prior to 1950 the choice was limited to some form of electrostatic storage tube. With the introduction of magnetic core storage the field was widened. Because of the discrete nature of its storage unit and the convenience for individual experimentation, the core became the focus of considerably more attention in recent years. In contrast, the bulk nature of storage in tubes and the complex technology of such tubes required more extensive group efforts.

While widespread use of storage tubes has declined, the development of storage tubes and related techniques has continued and these tubes have retained a competitive position with cores. Storage tubes appear to have some advantage in speed and economy, while cores have a slight advantage in size and power consumption. Because these differences are small, the specialized requirements of a contemplated system may exert a profound influence on the choice of memory medium.

The form of electrostatic storage tube now most widely used is the barrier grid tube. Such tubes have been described by Jensen,[1] and Hines, Chruney and McCarthy.[2] They are fast, reliable and economical. This paper describes an unusually high-speed memory utilizing such a tube and is indicative of the barrier grid tube's present speed and capacity.

## II. SYSTEM REQUIREMENTS

This store was developed to provide the erasable memory of the experimental electronic telephone switching system described in detail in a companion paper.[3] To serve this system, the memory was required to perform a complete memory operation (a random access plus a reading and writing operation) in 2.5 microseconds. Because of the nature of the system, large numbers of single-bit words were used and a serial memory was adequate for longer words. For this reason, the memory was designed to read or write a single bit at a time.

The limited nature of the system experiment and the use of large amounts of nonerasable memory reduced the erasable storage requirement to a few thousand bits. However, since a working telephone system would require considerably more memory with no essential change in other requirements, the store was designed to have as large a capacity as possible consistent with other requirements.

Because an inherent feature of barrier grid storage is the need for periodic regeneration of information, the store imposed some restriction on the system. A requirement of the barrier grid tubes used was that every bit in the memory be regenerated at least once each second. Because of interaction between adjacent storage locations, a somewhat higher rate of regeneration was required in most areas of the memory. This rate depends upon the performance level of the storage tube and the specific pattern of memory consultation. For the particular system using the barrier grid tubes available, a study indicated that no more than 10 per cent of the working time of the system was needed for this regeneration. Since the system is a real-time data processer in which the full system time is used only during infrequent periods of maximum input data rate, the effect of regeneration time is very small.

Since the switching system did not provide common power supplies for its various units, the store was designed as a self-contained unit including power. Thus, the resulting store was a general-purpose memory which could be applicable to other data processing systems of a similar type.

## III. DESIGN OBJECTIVES

The storage capacity of a barrier grid tube depends upon the combined analog resolving power of the tube and the deflection circuitry. With high-speed random access, the settling time of the deflection system usually sets the limit on resolving power. Since the desired memory required the deflecting of only one barrier grid tube, it was decided to devote sufficient effort to the deflection system design to achieve nearly the ultimate resolution of the tube. At the performance level needed, the capacity of the tubes was around 16,000 bits; accordingly, the capacity objective was a square array of 16,384 ($128 \times 128$) bits.

Within the 2.5-microsecond cycle time imposed by the system, three separate store operations were required: beam deflection, reading and writing. The reading and writing durations were set by the tube design and at the capacity desired were 0.8 microsecond. With a 10 per cent design margin, the time remaining for deflection was 0.7 microsecond. Because of other requirements, it was necessary to deflect in somewhat less time and the design objective was from 0.4 to 0.5 microsecond. It is clear that any significant decrease in cycle time for the system requires faster barrier grid tubes; a 2.5-microsecond cycle is near the present practical upper limit.

To avoid excessive deviations in the duration of store operations, precise timing pulses were required within the store. The objective for these pulses was a single cycle of a squared 5-mc sine wave, which would give a pulse rise time of 0.05 microsecond. Since the inputs from the system were expected to be slower, input buffer circuits were provided. The pulse amplitude was set at a nominal 15 volts, with satisfactory operation at a 10-volt minimum.

At any designated location it was desired to have four alternative modes of operation. Each operation required the store to read and subsequently either (i) write a zero (WRITE 0); (ii) write a one (WRITE

1); (iii) write the same binary state as was read (REGENERATE); or (iv) write the binary state opposite to that read (REVERSE). Operations (i) and (ii) are the minimum necessary to provide storage. Because of the frequent occurrence of operations (iii) and (iv), it was observed that much system time could be saved by including the necessary logic for these operations within the store.

The functional block diagram of the store is shown in Fig. 1. The memory section contains the barrier grid tube and its associated dc control circuits. The deflection system contains all circuits necessary to direct the beam to the selected location and assure its proper focus. The control section contains the circuits which issue the proper sequence of pulses to execute the desired storage operation. The readout section provides gain and quantization of the readout pulses.

The implementation of the store operations requires pulses from the system on a selected set of 15 leads. The pulses on 14 of these leads define the binary state of a 14-digit binary number specifying the location of the bit. The remaining pulse, on one of four leads, specifies the storage operation to be carried out at the selected location. In normal operation, all 15 pulses arrive at the store simultaneously. However, proper operation of the store is achieved as long as no address pulses arrive in the 2.5-microsecond period following an operation order pulse. Each address digit is retained in the store until changed, so that the operation order will be carried out at the location specified by the complete set of most recent address pulses. Approximately 1.5 microseconds after the operation order, a readout 1 or 0 lead is pulsed by the store.

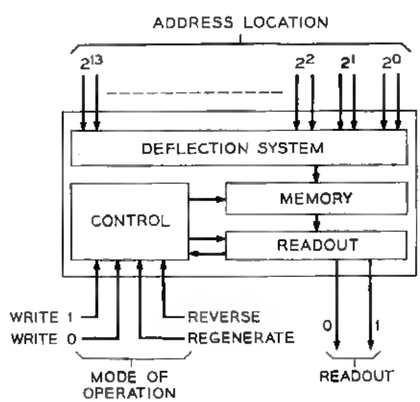


Fig. 1 — Barrier grid store system block diagram.

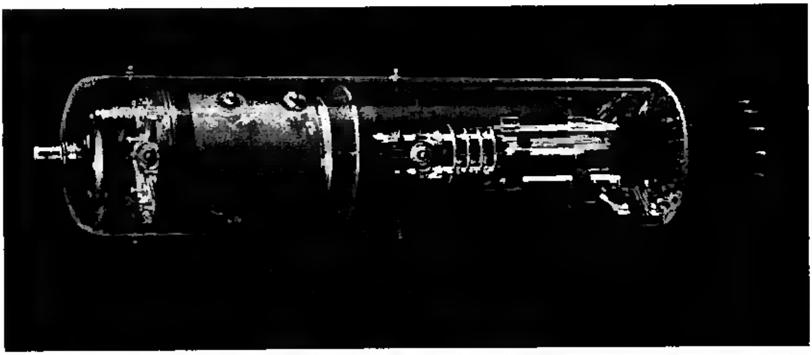

Fig. 2 — Barrier grid tube.

## IV. SECTIONS OF THE BARRIER GRID STORE

### 4.1 *The Memory*

The storage tube used in the memory is similar to that described in detail in Ref. 2. It uses electrostatic deflection and focusing and operates at an acceleration potential of 1000 volts, but it differs from that tube in having a double focusing lens. The two control apertures of the lens have elliptical instead of circular shapes and are placed with their major axes at right angles. The lens is aligned with the deflection plates so that each aperture corrects, more or less independently, for the defocusing caused by a particular set of deflection plates. This type of structure facilitates the application of dynamic correction for deflection defocusing. A photograph of the tube is shown in Fig. 2.

The target structure of the tube consists of a mica sheet approximately 0.001 inch thick placed between a 500-mesh barrier grid on the gun side and a conductive back plate on the opposite side. The space between deflection plates and target is occupied by a conical electron collector assembly.

When the beam is directed at a point on the mica, the area under the beam charges to an equilibrium potential approximately equal to the fixed barrier grid potential. The potential thus obtained represents the storage of a binary 0. Writing a binary 1 is a similar process, except that a positive potential is applied to the back plate during the mica charging process. When the back plate potential is restored to zero after the charging, the capacitive coupling to the mica causes it to finally assume a potential more negative than the barrier grid, representing a 1.

Reading is accomplished by the same operation as writing 0. During this process, the current flow away from the mica will depend upon whether the spot is already at equilibrium. Thus, there will be a different current flow for a 1 or a 0. This flow may be observed either at the collector or at the target (barrier grid plus back plate). The first method is called collector reading, and is used in this store. The second method is called target reading, and offers distinct advantages in signal uniformity and detection. Unfortunately, the current required to charge the back plate to barrier grid capacity for the write 1 process also flows in the target leads. This current is about one-half ampere, while the signal current is about one microampere. To maintain the desired cycle time of the store it is necessary for the readout amplifier to completely recover from the large writing pulse on the previous spot in about one-half microsecond. Ref. 2 describes a method of driving the back plate through a transformer wound with coaxial wire, which would reduce the interference from 500,000 to about 20 times the readout. Although this method holds considerable promise for the future, the residual recovery time problem was still a deterrent to its use in the present store.

With collector reading, both the 1 and 0 signal are of the same polarity and the 1 is 30 to 50 per cent larger than the 0. To decide whether a given readout is a 1 or a 0 requires an amplitude discriminator. With a fixed discrimination level, variation in the 0 amplitude over the surface represents a noise component. This is discussed by Hines under the subject "shading". In the tubes used, the construction was carefully controlled to minimize shading.

A figure of merit used for these storage tubes is the read-around-number (RAN). This refers to the number of complete storage operations* that can be carried out at a storage location without causing erroneous readout from an adjacent location. For any particular tube, the RAN figure of merit is the smallest such number obtained by testing all locations and all combinations of interfering and adjacent spots. Such a figure of merit gives an indication of how many times a spot may be used before its neighbors must be regenerated. In the tubes used the RAN was greater than 50 at full storage density.

The interaction effect of spots more distant than one unit may be expressed by a similar RAN, but it is found that this interference very rapidly becomes independent of distance. Thus, in the tubes used it was found that approximately one million operations in spots more than

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* A complete storage operation is any of those defined in Section III. This definition gives RAN's which are approximately one-half of those obtained using the definition given by the IRE Standards Committee on Electron Tubes.

three or four units removed from a given spot could be tolerated before errors occurred. These errors are caused by very small scale effects, such as reflected high-speed secondary electrons and positive ions.

To prevent errors from this cause, a requirement was imposed on the system to insure that each spot was used or regenerated at least once each second. At a 2.5-microsecond cycle time, this limited the maximum number of distant interferences to 400,000. In addition, the system was required to regenerate more frequently any spots that had high adjacent-spot interference.

### 4.2 *Deflection System*

The deflection system receives at its input a 14-digit binary number. It must convert this number into two analog voltages whose values are proportional to the binary numbers obtained by dividing the input number into two 7-digit numbers. These two voltages, applied to the deflecting plates of the tube, provide 128 discrete levels of beam position in each orthogonal axis. Since the adjacent spot interference depends upon accurate beam positioning, a high order of accuracy must be maintained in the conversion process. An error greater than 0.1 spot is deemed inadmissible for maintaining high RAN in the store. This means that in a jump from the lowest level to the highest level (128 spots) a positioning accuracy of better than 0.1 per cent is required.

The maximum regeneration interval of one second eases the accuracy requirement on slow drifts, since the regenerated array can move on the surface of the target. However, to avoid the necessity of extra target area, long-term drift must be minimized.

The requirement of 0.1 per cent accuracy has an important bearing on the attainable deflection speed, since the beam cannot be turned on until the deflection voltage has settled to within the 0.1-spot limit. To meet the design objectives, this must occur within 0.5 microsecond. If the voltage approaches its final value exponentially, this is equivalent to a 0.2-microsecond rise time in conventional terminology. Because of unavoidable overshoots, the amplifier must, in practice, be at least twice as fast. At the opposite end of the frequency spectrum, the response must extend to dc because, with random access, the store may operate almost continuously at any place on the storage surface.

Since accurate focusing of the beam is required for high RAN, the deflection system must have a minimum effect on focus. This requires, first, that the deflection plate voltage be applied in balance across the deflection plates to assure constant average potential between them.

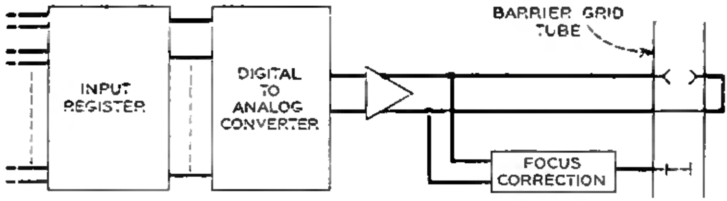


Fig. 3 — Deflection system functional diagram (one axis).

This alone is insufficient, and correction must be made for the remaining defocusing caused by the lens action of the deflecting plates, using circuitry which develops a voltage proportional to the square of the off-axis deflection. This voltage is then applied to the appropriate focusing electrodes. With the independent control in each axis offered by the double-focus lens, spot size can be held to a 30 per cent increase in the corners.

The circuit of each axis of the deflection system consists of four main parts, as shown in Fig. 3. The input register consists of flip-flops which convert the incoming pulse into binary dc levels. These levels actuate the digital-to-analog converter to produce a low-level analog signal proportional to the input binary number. These analog signals are amplified and applied to the deflecting plates and the focus correction circuit.

The register flip-flops are identical to those shown in a later section

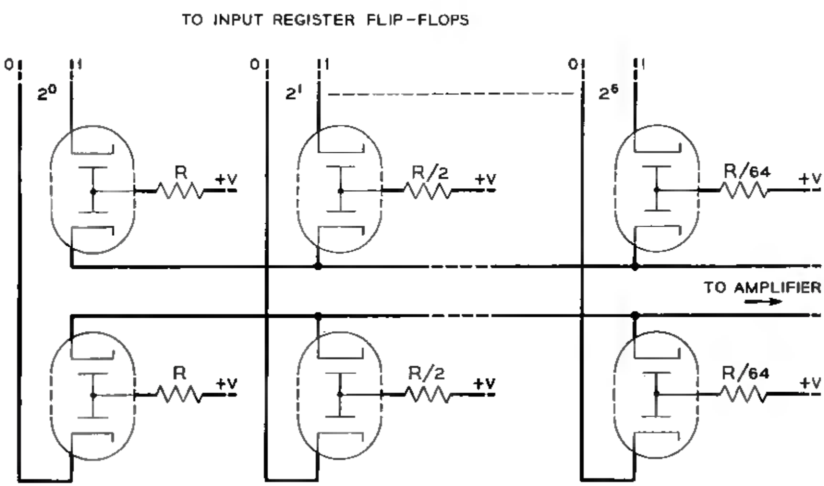


Fig. 4 — Digital-to-analog converter schematic diagram.

and are conventional Eccles-Jordan type with cathode follower outputs. They each drive an input of the digital-to-analog converter whose circuit is shown in Fig. 4. This is a well-known circuit in which there is a current established by each resistance from a common supply. The current may flow into the low-impedance lead to the amplifier or back into its respective flip-flop. The potential at the junction of each diode pair is held to within a volt or so of ground, while the supply voltage, $+V$, is approximately 100 volts. This means that there is essentially a constant current in each resistance and residual inductance effects are minimized. The action of the converter is to switch a number of these binary-weighted currents into the input leads to the amplifier, where they add to produce the desired analog voltage across the input resistance of the amplifier. To meet the required balanced output voltage, both sides of the flip-flops are used to create complementary currents in the two output leads.

The accuracy of the converter depends on the accuracy of the resistors and the power supply. The latter must be well regulated, to at least 0.05 per cent over a one-second period and better than 1 per cent over longer periods. The resistors, chosen for good high-frequency performance, are carbon film types and are not a factor in short-term drift. However, their long-term drift is important, particularly if they do not all drift equally. This could cause crowding of lines and columns of the storage array and serious degradation of RAN.

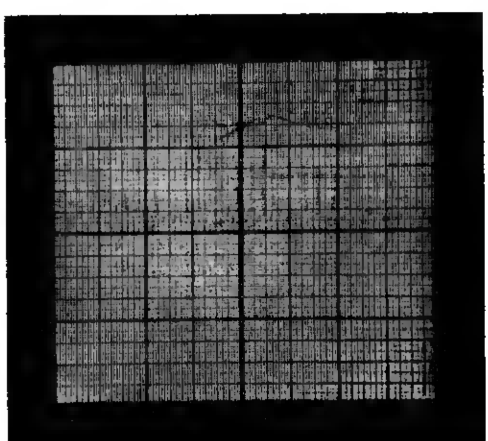

Fig. 5 — Barrier grid tube storage raster showing "plaid" pattern for drift protection.

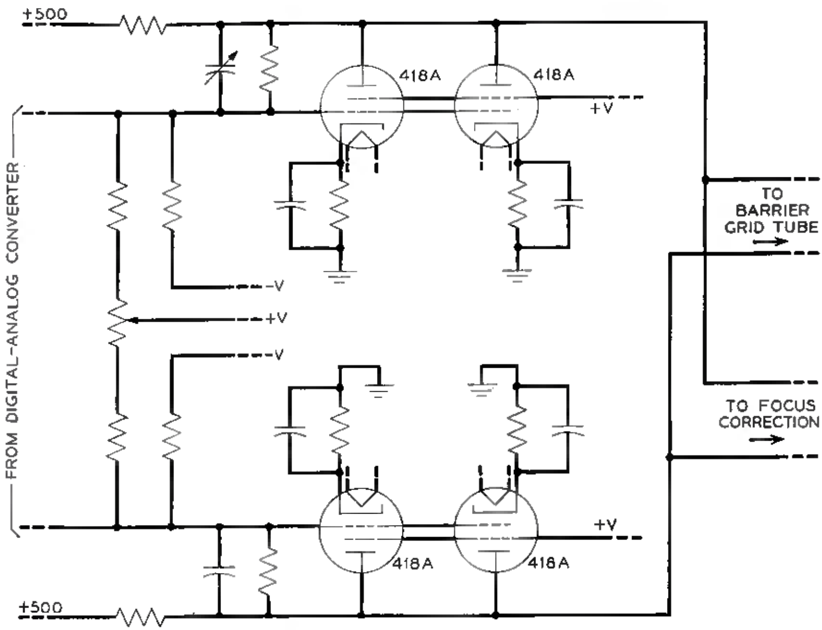


Fig. 6 — Deflection amplifier schematic diagram.

In order to permit some drift, the resistors were chosen with a weighting ratio of 2.02 between digits instead of 2.0. These result in a plaid pattern, as shown in Fig. 5, which occupies a square approximately 8 per cent larger than a linear-spaced array with the same minimum spacing. However, a $\pm 1$ per cent drift in the resistors is allowed before the minimum spacing is exceeded. Generally, this much protection is more than adequate, since the resistors tend to drift in the same direction.

The output of the converter drives an amplifier whose simplified schematic is shown in Fig. 6. To stabilize the gain of this amplifier, feedback from grid to plate is used. This also provides the necessary low-input impedance required by the converter circuit. The output of the amplifier drives the deflection plates of the barrier grid tube directly. To allow for differing deflection sensitivities, the effective converter supply voltage is variable. Centering is accomplished by differential current injection at the amplifier grid. The total swing at each deflection plate of the tube is about 75 volts, centered at 145 volts. To achieve optimum focus, the average value of the plates is adjustable by means of the bias supply voltage ($-V$). This supply, like the converter supply, must be well regulated.

To achieve an over-all deflection rise time of 0.1 microsecond with negligible overshoot requires careful layout of the output circuit to minimize stray capacity. It was for this reason that size and centering were placed in the grid circuit. Careful adjustment of the feedback capacitors permits the overall deflection system to achieve rise times of 0.08 microsecond and to settle to within 0.1 per cent of final value in 0.5 microsecond.

The defocusing caused by the full beam deflection requires correction at the focus electrodes. Ideally, the correction should vary as the square of deflection deviation; in practice, a power term whose exponent lies between 1.5 and 2.5 is equally satisfactory. For this reason, the correction is developed by using the approximate multiplicative action of grids on beam current in multigrid tubes. The circuit of Fig. 7 has input pentodes whose control and suppressor grids are driven by the same signal. Each deflection plate drives such a stage and outputs from corresponding plates are added. The result is very closely parabolic and

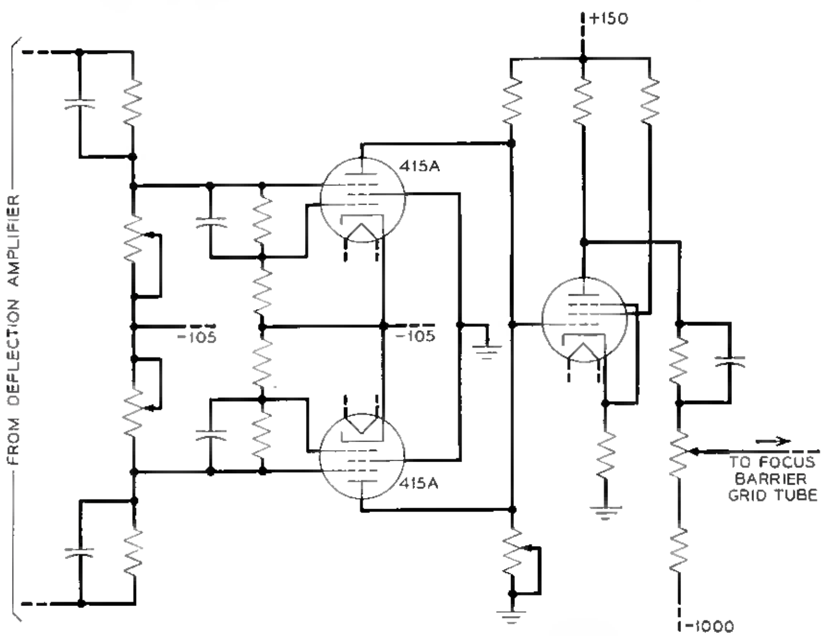


Fig. 7 — Focus correction schematic diagram.

symmetric about the center of deflection. Further amplification is necessary to provide the required correction amplitude.

### 4.3 *Control System*

During a storage cycle the change of deflection plate potential and focus correction must be completed before the beam is turned on. As previously indicated, a 0.7-microsecond period is allowed for this action. When this interval has elapsed the beam is turned on, with the back plate at its lower potential (the backplate is normally held at this potential). This action (see Fig. 8) constitutes both a reading and a writing of a 0 and occupies 0.8 microsecond. During this period, the output current from the collector is sampled to determine the value of the spot stored and, at the end of the period, the spot has been written to a 0. Up to this point, the action is identical for all operational modes. During the succeeding 0.8 microsecond the action depends on the particular polarity required in the write operation. The circuit may either remain idle and leave the 0 just written, or the beam may be turned on and the backplate potential raised, writing a 1.

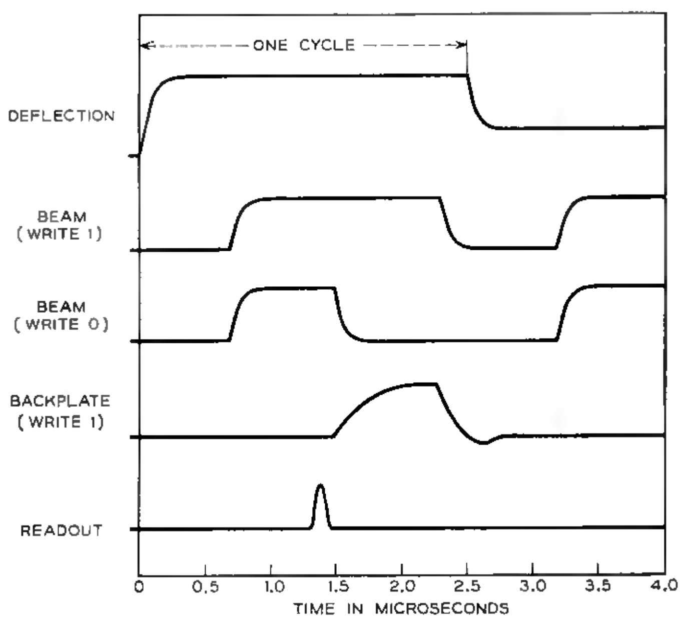


Fig. 8 — Timing sequence.

It is the function of the control circuit to time and initiate these actions, using the circuit whose simplified schematic is shown in Fig. 9. The circuit is constructed of logical gates, passive delay lines and flip-flops. Delay-line losses are made up by gate circuits (which have gain stages) either in conjunction with their normal logical function or, in some cases, specifically for gain purposes. Gates in the latter class have been omitted from the schematic.

There are two basic gate units providing the logical "or" and logical "and" functions; their circuits are shown in Fig. 10. They consist of a basic-pulse gain stage preceded by a logic stage. The basic output pulse is of 0.1-microsecond duration and uses 100-ohm terminated pulse cables with low-impedance delay lines. The output transformer of each unit acts as a reshaper and provides a uniform pulse shape throughout the system.

The flip-flop circuit is shown in Fig. 11. This same circuit is used in both the deflection registers and the control circuit.

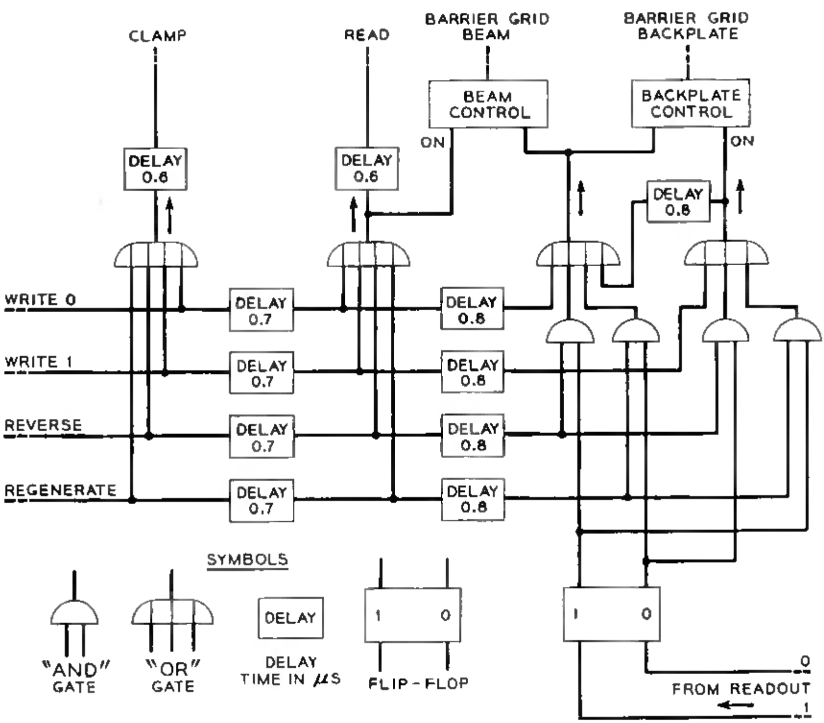


Fig. 9 — Logic of control section.

There are two special circuits in the control circuit which are used functionally as flip-flops but whose special requirements dictate a circuit different from that just given. These are called drive circuits. A common simplified schematic is shown in Fig. 12.

These circuits are required to drive an essentially capacitive load from one voltage level to another. The change must be rapid and operate on the receipt of a "set" or "reset" pulse. Normally the circuit is in its lower voltage state and is driven to its upper level for only short periods of time. Due to the essentially nondissipative character of the load, current is required from these circuits only during the transition from one level to another. However, to achieve the required rise and fall times (typically, 0.3 microsecond for a 50-volt transition across 2500 micromicrofarads at the back plate) the peak currents required are of the order of one-half ampere.

Tube $V_3$ acts as a cathode follower and in response to a "set" pulse provides a large pulse of current to cause a positive-going transition. Because the load is capacitive, it remains at its upper potential after

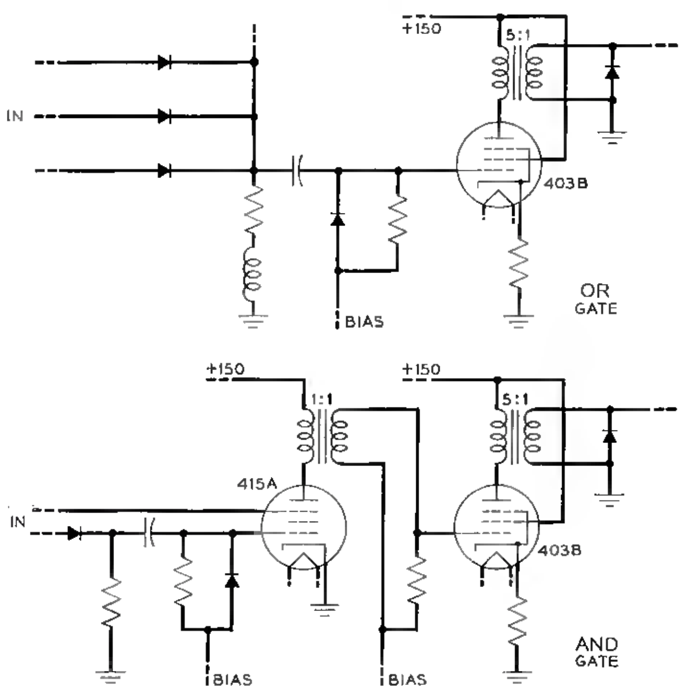


Fig. 10 — Logic circuit schematics.

$V_3$ is biased off. When a return to the normal lower potential is desired, $V_4$ is pulsed to discharge the load. This tube must discharge the load sufficiently to allow $V_3$ to become biased on and assume control of the output voltage.

To assure that the output current does not drift upward during periods of idleness, a constant drain current is provided by $V_5$. The width of the system pulses are insufficient to insure current flow during the entire transitions, so these pulses are stretched by the input diodes and grid capacity and are clipped and amplified by $V_1$ and $V_2$.

In the specific circuit for driving the back plate, the load is sufficiently large to require doubling $V_3$ and $V_4$. In the case of the grid-drive circuit, single tubes are adequate, but the lesser capacitance results in considerably more droop along the top of the pulse. Since proper operation of the barrier grid tube requires this top to have very little droop, the actual output is severely clipped by the circuit of Fig. 13 before it reaches the

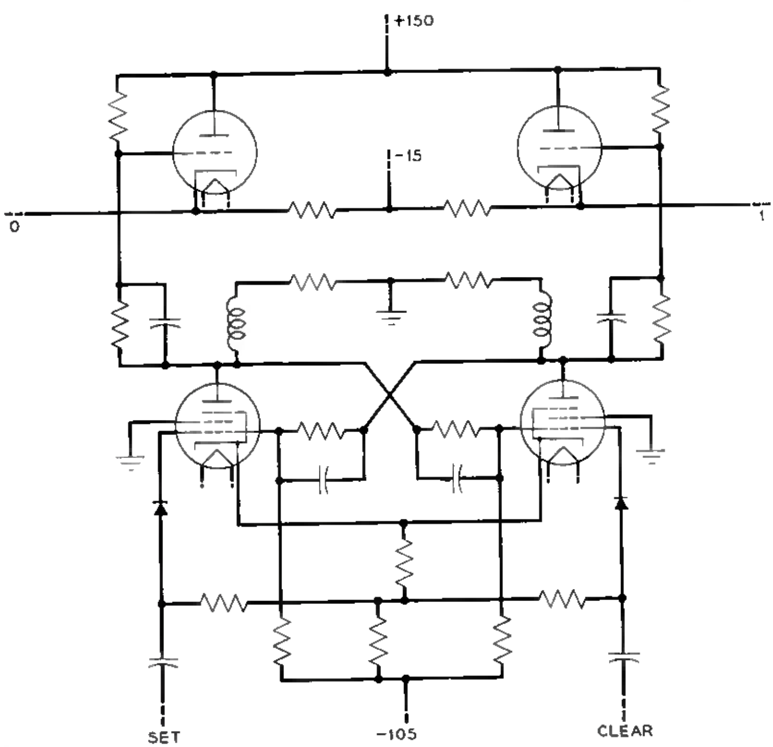


Fig. 11 — Flip-flop circuit schematic.

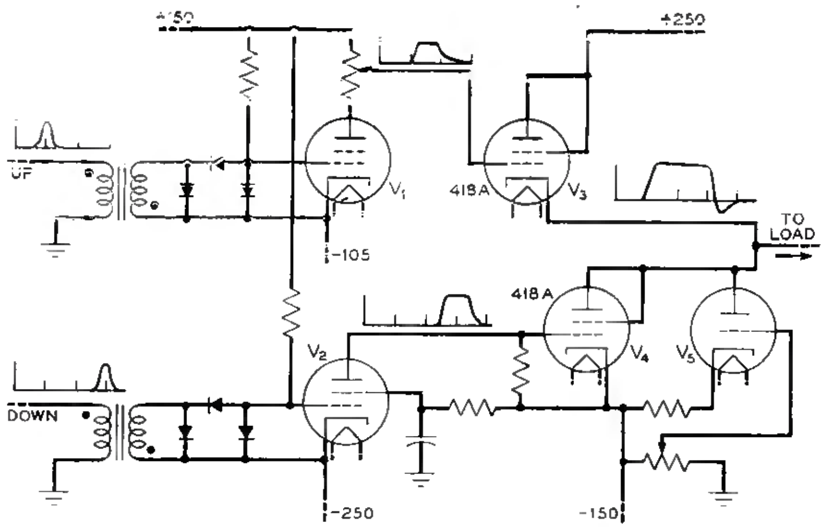


Fig. 12 — Driver circuit simplified schematic.

grid. This circuit clamps the grid to either −1000 volts or to −$V$ volts. The cathode bias is adjusted so that when the grid is at −1000 volts the beam current is at the desired value. The supply, −$V$, is adjusted to insure the tube is cutoff when the grid is at −$V$ volts. This method of clipping provides a fairly simple circuit, but a change in operating beam current requires a change in cathode voltage, with a resulting change in accelerating voltage and deflection sensitivity. In practice, the usable range of beam currents can be achieved by a change in accelerating potential of less than 2 per cent. For most adjustments of current this does not require readjustment of deflection.

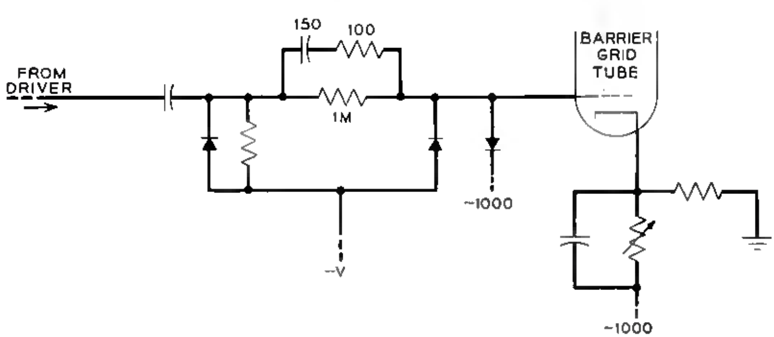


Fig. 13 — Beam drive clamp circuit.

### 4.4 *Readout Section*

The output current of the collector is of the order of 1 microampere. Since the required rise time of the output voltage and the collector and wiring capacity limit the load resistor to 2 kilohms, a voltage of 2 millivolts represents the available output signal. Of this, only about one-third represents the difference between 1's and 0's. Amplification must be provided to achieve satisfactory discrimination between these two states.

Unfortunately, both 0 and 1 signals have a dc component which depends upon duty factor. To make the proper discrimination, this dc component must be preserved in the amplification process. At these input levels and bandwidths (the amplifier rise time is 0.3 microsecond) stable dc amplification is not achieved simply. Fortunately, the waveform is such that an ac amplifier followed by dc restoration may be used. The schematic of the amplifier is shown in Fig. 14. The signal output of the amplifier for several conditions is shown in Fig. 15.

Each figure shows the superimposed readouts from all spots in the store. Since the pattern of 1's and 0's in the store formed a checkerboard, half of the readouts represent 1's and half represent 0's. The central 2.5 microseconds of each sweep represents a complete storage cycle

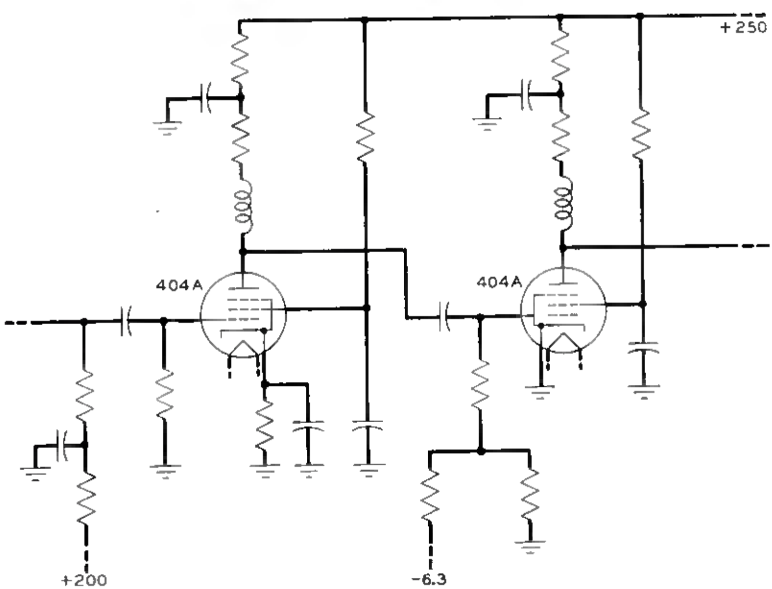


Fig. 14 — Readout circuit input section.

consisting of deflection (D), reading (R), and writing (W). This is most clearly seen in Fig. 15(a), where the store is operated at a low repetition rate. In Figs. 15(b) and 15(c) the repetition rate is 400 kc and the noise from the writing in one cycle extends into the deflection period of the next cycle.

During the period R the differing traces represent the 1 and 0 readouts and the amplitude must be sampled during this period. The store delivers a 1 output pulse only if the signal is more negative than approximately −3 volts. In Fig. 15(b), where no dc restoration is provided, the effect of the higher repetition rate can clearly be seen to be essentially a shift in the dc level of the readouts, and the preceding discriminator would not indicate the presence of 1's. Since all traces in Fig. 15(b) coincide just prior to readout, the voltage at this point may be clamped to the same value (0 volts) it has in Fig. 15(a), and the result is shown in Fig. 15(c). Here it can be seen that the discriminator will give the correct

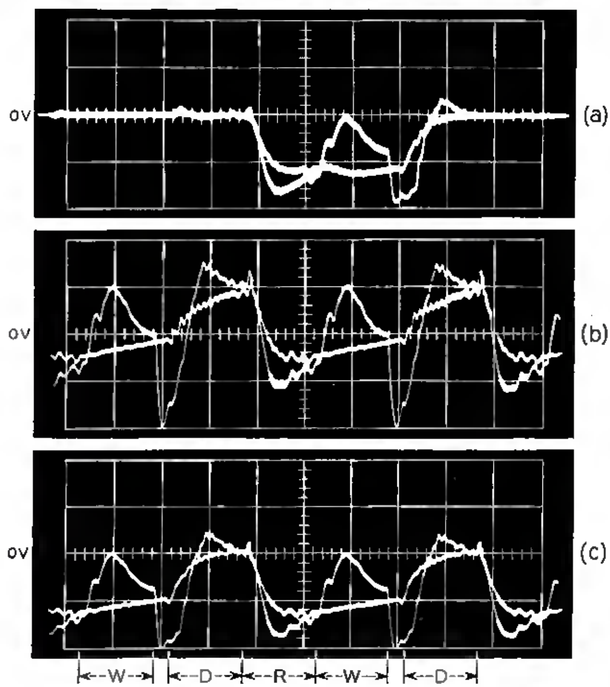


Fig. 15 — Signal outputs from readout amplifier: (a) low repetition rate; (b) high repetition rate without clamp; (c) high repetition rate with clamp. Horizontal scale, 0.5 microsecond per division; vertical scale, 2 volts per division. D = deflection; R = read; W = write.

readout. The actual sample of the readout is taken at the center of the trace and is about 0.1 microsecond in duration. The apparent improvement in the separation at this point in Fig. 15(c) relative to Fig. 15(a) is due to a favorable interference pattern and, in practice, the levels are essentially identical between low-frequency and high-frequency operation.

These views of the readout indicate that a more favorable sample might be taken earlier than at the time indicated. Unfortunately, it is difficult to reproduce marginal 1 signals photographically, and these do not show the large separation from the 0's early in the readout period. The optimum time in an actual store is determined by adjusting for best RAN, and it is close to the indicated time in all tubes.

The clamping circuit and amplitude discriminator are shown in Fig. 16. Vacuum tube diodes were used to achieve fast clamping with minimum loading. The amplitude discriminator consists of a cathode follower driving a grounded grid amplifier through a series diode. This configuration has proved to be very accurate, stable and fast, and it has a large overload capacity. However, since it is not regenerative it must be

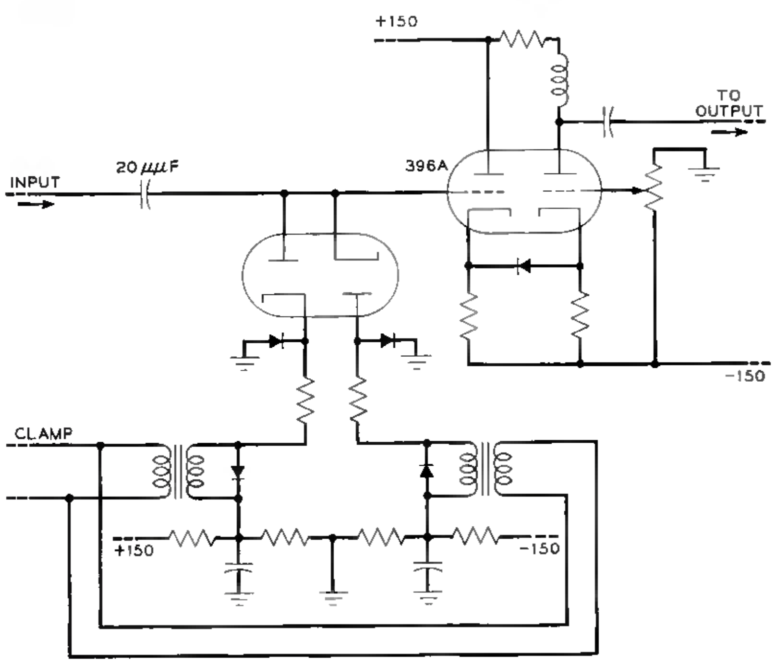


Fig. 16 — Readout circuit clamp and discriminator section.

followed by considerable gain. This is shown in Fig. 17. The first stage is straightforward voltage gain, followed by a regenerative phase splitter. This circuit has characteristics similar to a Schmidt trigger circuit,[4] but, since the feedback is ac coupled, there tends to be a small amount of turn-off delay. This is minimized by limiting the amplitude of the feedback. This stage drives a pair of diode AND gates which are sampled at the appropriate time by the control circuit to generate a 1 or 0 output.

### 4.5 *Equipment*

In packaging the various circuits which constitute the store several objectives were sought. It was desired to evaluate the problems existing in an actual telephone system where continuous operation is mandatory. Clearly, continuous operation requires the system to have standby

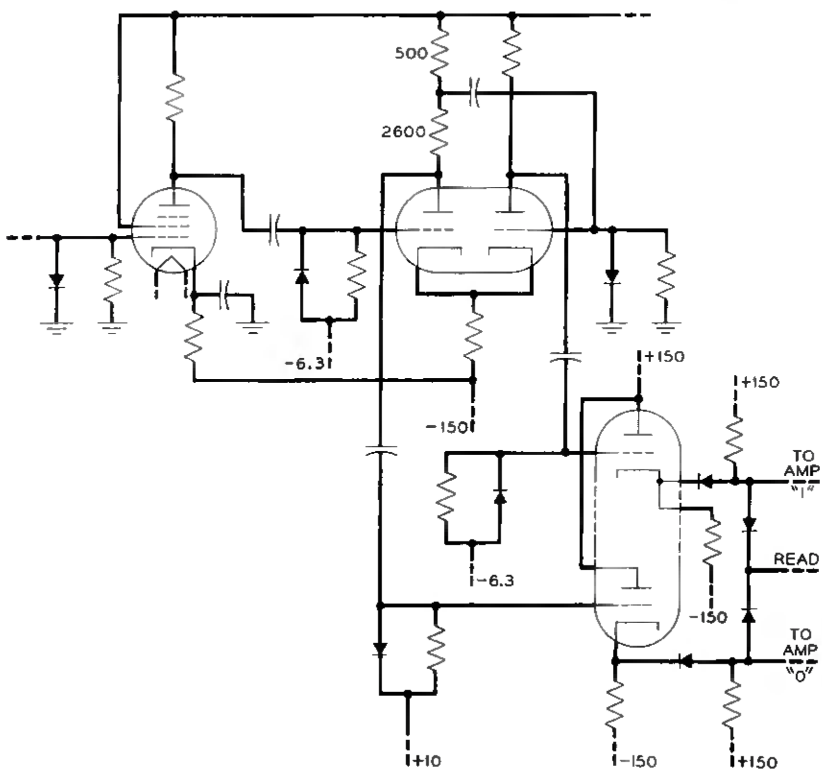


Fig. 17 — Readout circuit output signal generation.

storage facilities. Since the amount of such storage depends upon the expected out-of-service time of faulty stores, the stores must be arranged for simple and rapid maintenance and repair. To assure this, practically all components were included in plug-in packages. Exceptions to this rule were made in the case of highly reliable elements such as delay lines, but in general the only fixed elements in the store cabinet were wiring and connectors.

The control section and part of the deflection system readily divide into small functional units which may be packaged conveniently, but the other sections of the store cannot be so readily separated. If their breakdown is into rather small units, stray capacitance due to extra inter-unit wiring presents a problem in keeping within the cycle time. Too large a unit imposes a mechanical problem and tends to require excessive spare equipment. However, what proves to be the governing consideration is the need for grouping all alignment controls and their circuits on a single plug-in unit so that store realignment is not necessary when a package is replaced. The alignment controls in the store include adjustment for deflection size and centering, focusing correction and level, beam current, output gain and discrimination level and a few dc parameters of the storage tube. The number of these adjustments requires this one package to be relatively large.

This package, called the barrier grid tube unit, is shown in Fig. 18. It includes the barrier grid tube and its dc circuitry, the deflection amplifiers, the focus correction unit and the readout section. Fig. 18(a) shows the partially assembled unit with the barrier grid tube centrally mounted in a Mu-metal box for magnetic shielding. Electrostatic shielding is obtained from the copper mesh placed directly around the tube. The deflection amplifiers are mounted on either side of the tube outside of the shields, and they are wired directly to the deflection plates. In the forward left-hand corner is the focus correction unit. In Fig. 18(b) is shown the unit with the Mu-metal cover and complete readout section in place. Fig. 18(c) shows the fully assembled unit. For installation, the unit rests on drawer-type slides in the center of the store.

A view of the full store is shown on the left side of Fig. 19. Below the barrier grid unit is the remainder of the deflection section. The individual flip-flop units are on the sides, while the digital-to-analog converter is in the center. The latter is a single unit that plugs into the rear of the store. Above the barrier grid unit is the control section. The uppermost unit is the combined grid and back plate drivers.

The small functional packages in the control circuit and deflection

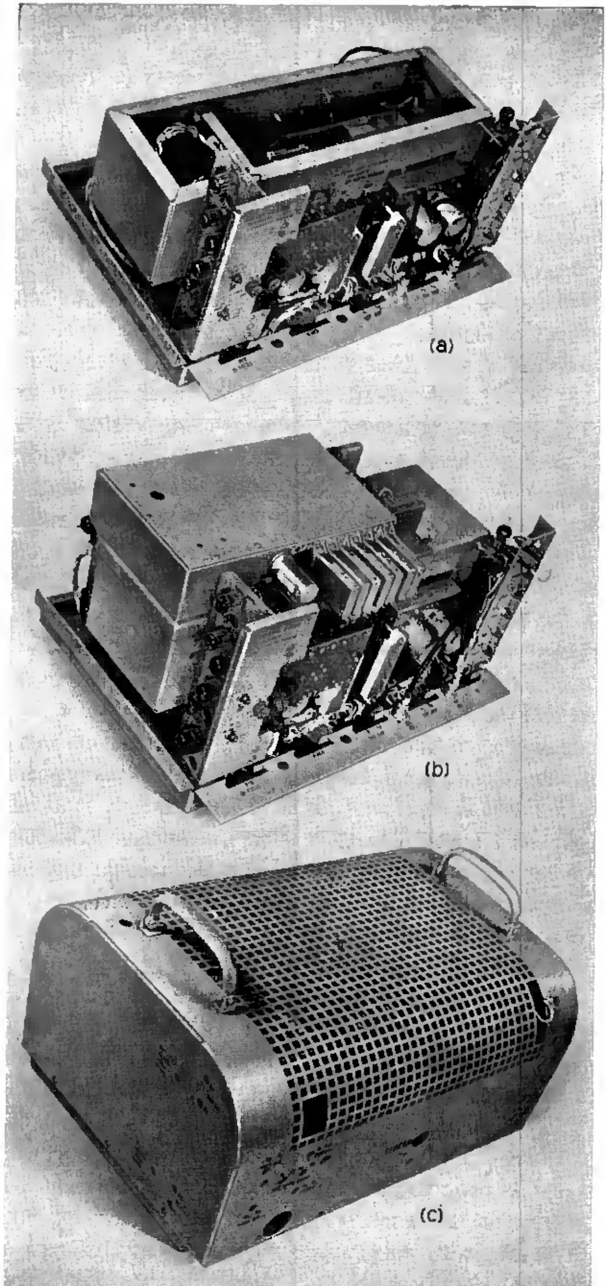


Fig. 18 — Barrier grid tube unit: (a) mounting of barrier grid tube; (b) unit without cover; (c) fully assembled unit.

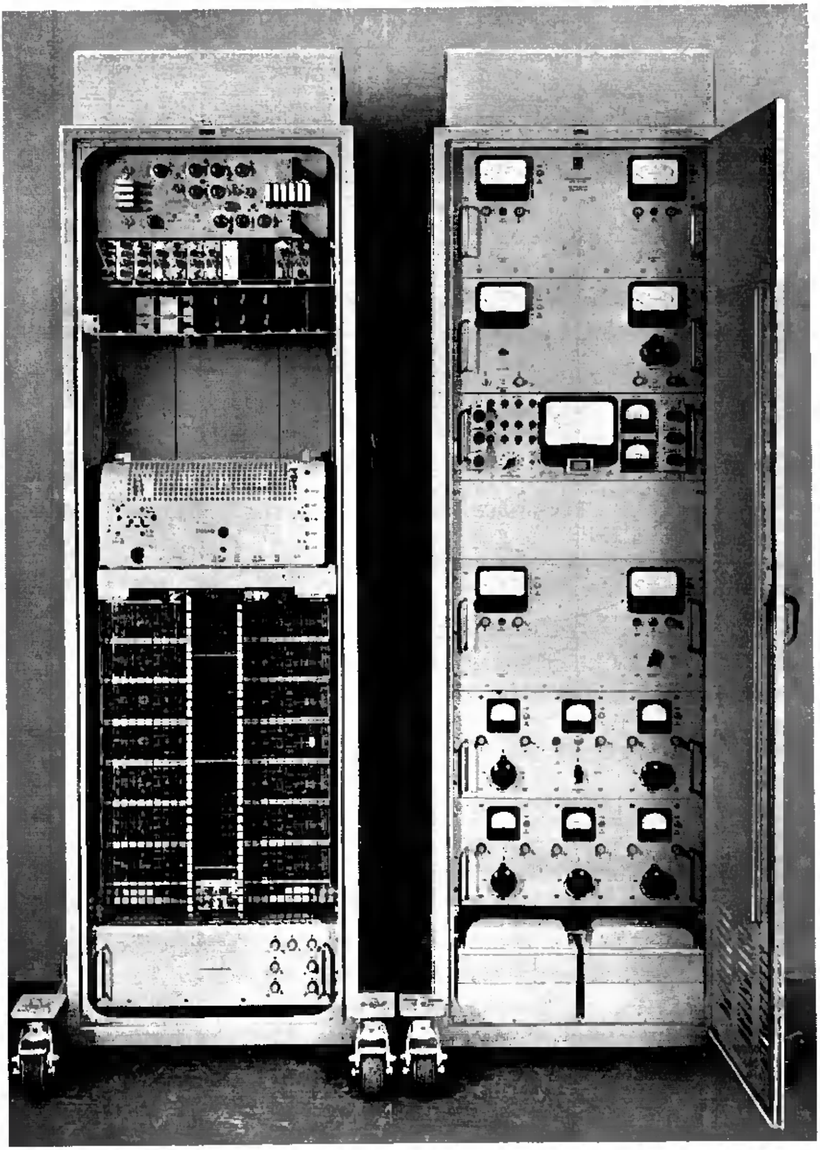

Fig. 19 — The barrier grid store, left, and its power supply, right.

input register are shown in Fig. 20. These are the gates and flip-flop circuits. Three types of mechanical designs were used for evaluation purposes. All used printed wiring, but various amounts of mechanical

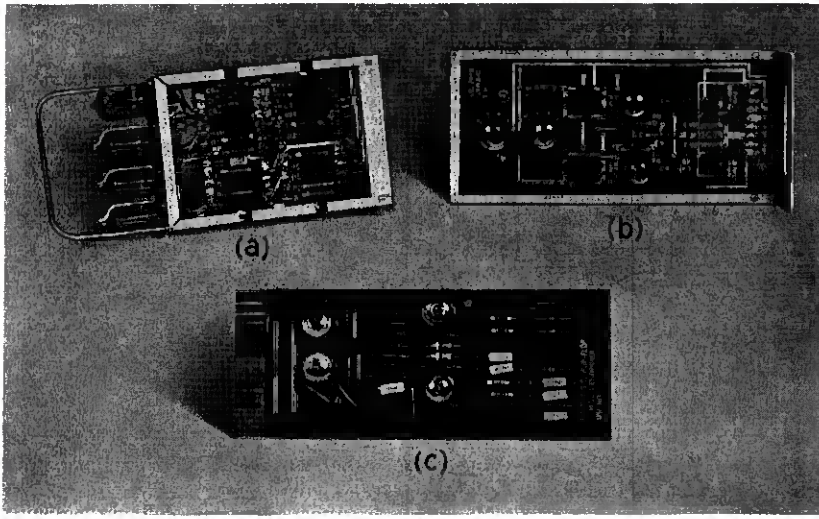


Fig. 20 — Construction of small packages.

support were used. Experience has indicated that in the relatively non-severe type of usage encountered in this type of system a self-supporting card of type (c) is entirely satisfactory.

### 4.6 *Power Supplies*

The power supply for the store is shown on the right side of Fig. 19. Twelve voltage levels are used. Eight of these, which supply 75 per cent of the dc power, are simple rectifiers achieving ±2 per cent regulation through line voltage regulation of ±1 per cent in resonant regulators. The remaining power is supplied by electronically regulated supplies meeting ±0.5 per cent regulation for periods longer than 1 second and ±0.05 per cent for shorter periods. The total dc consumption of the store is 500 watts.

In addition to the dc power, a filament supply of 220 watts is required. Although these power requirements are quite modest, a largely transistorized version of the store would offer significant power economy. Such a version appears to be feasible with transistors now available.

## V. OPERATING EXPERIENCE

The laboratory operating experience with two stores over a combined time of 10,000 hours has been very favorable. The most difficult operating aspect has been that of initial alignment. The operator has under his control about a dozen parameters which he must initially optimize for system operation. The general alignment procedure which has evolved is first to adjust size, shape and centering of the deflection raster. This presents some problems because the raster is not directly viewable and the effective edge of the storage area is not exactly definable. The usual procedure is to define the edge by, say, a 20 per cent reduction in a 1 readout. This permits the deflection sensitivity to be measured and the shape and centering and approximate size to be set. The "correct" size may be set later during more sensitive tests.

For setting focus and discrimination it is necessary to have test equipment available which will evaluate RAN (or an equivalent figure of merit) both at a point (or over a small area) and over the entire storage surface. The focus correction can then be determined by optimizing small-area RAN at the center and four sides. For setting the actual dc level of focus, it is preferable to use an over-all RAN, since focus should be optimized at the point of lowest RAN. Likewise, the discrimination level should be set for optimization of over-all RAN. The actual alignment procedure can be mechanized. However, the alignment procedure was an important consideration in the original store design since an unwanted interdependency of adjustments might have seriously complicated the procedure.

Once aligned, the store requires little further adjustment. The stability of alignment has proven very good with respect to both continuous operation and package replacement. Prealignment of a spare barrier grid unit is entirely practical, although a slight amount of touch-up alignment is usually done after replacing this package. The only drift effects that have proved noticeable have been from changes in storage tube beam current. These, however, are easily corrected by periodic observation and minor adjustment. No other drifts of important magnitude have been apparent. Observations on one store over 2500 hours showed a drift in raster position of only one spot.

The RAN's that have been obtained for the store are in the range of 50 to 60 at the worst areas of the surface. These are generally the edges and corners, and RAN's of 150 are more common for most spots. No changes in storage characteristics have been noted over periods of several thousand hours. While cathode life would thus seem to be the main factor in limiting the life of the tubes, the small numbers of tubes used and the short periods involved have not permitted a complete evaluation of expected life.

For the store as an entity, a measure of reliability would be desirable. Such measures are difficult to obtain and are often of nebulous meaning, but, to the degree that they establish bounds, they are nonetheless useful. For the store described, tests were carried out in which known storage patterns were regenerated for long periods of time. From such tests, the store, its power supplies and its test equipment appear to have an error rate of approximately 1 to 2 errors every 24 hours, or about 1 error in $10^{11}$ operations. At this low rate, protection of important information which must be held for long periods can be assured by single error-correcting codes.

## VI. SUMMARY AND CONCLUSIONS

Using a single barrier grid tube, a complete self-contained data storage unit has been achieved. The unit has a 16,384-bit capacity, and an access to any bit, including reading and writing, may be made every 2.5 microseconds. While the specific unit was designed for serial operation, its high input impedance makes paralleling of stores straightforward. Regeneration is necessary, but the interaction between spots is low and not more than 10 per cent extra system time is required.

The store has a volumetric efficiency of 4 cubic inches per bit (inclusive of power) and a power efficiency of 45 milliwatts per bit. As a measure of economy, the component count (tubes, resistors, condensers, diodes, transformers, switches, sockets) indicates a usage of approximately 0.1 unit per bit, of which only 10 per cent are active elements. Actual operation has indicated good stability and reliability.

In the field of random access memory for data handling systems, barrier grid tubes can provide high-speed memory with moderately large capacity per tube. The resulting stores are economical and offer considerable promise for future electronic telephone switching systems.

## VII. ACKNOWLEDGMENTS

As with any project of this nature, many persons have been involved. We especially acknowledge the efforts of C. F. Ault, S. J. Dudek, R. M. Genke and J. A. Ruff for circuit contributions and E. Ley for mechanical design. The encouragement of R. W. Ketchledge has been invaluable.

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# Linear Least-Squares Smoothing and Prediction, with Applications

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*This paper describes the calculation of smoothing and prediction operators of the linear least-squares sort using techniques derived from a circuit theory point of view. The techniques are developed explicitly for time series which are continuous and statistically stationary. Other situations are explored more briefly, however, in which the time series are either discrete or statistically nonstationary.*

*For the most part, functions of time are replaced by functions of frequency, representing their transforms. Mathematical complications are avoided by restricting statistical ensembles to those which have rational power spectra. In practice, actual spectra can be approximated sufficiently well by rational spectra, and the simplified methods are sufficiently general for engineering applications of many different sorts. Both finite and semi-infinite smoothing intervals are permitted, with or without constraints of various sorts. The assumption of rational spectra does not apply directly to nonstationary time series, but it may be replaced by a closely analogous restriction which does apply. Then there are nonstationary operations which are closely analogous to the stationary operations, developed for stationary systems. A brief examination of these analogies is of interest, even though the nonstationary operations are usually too complicated for engineering purposes.*

*The general techniques are developed in terms of specific problems, chosen for purposes of exposition and because of their engineering interest.*

## I. INTRODUCTION

For a number of years, beginning roughly at the end of World War II, there has been a growing interest in theories of optimum smoothing and prediction. Much of the work has been concerned with optimum smoothing and prediction of the linear least-squares sort applied to statistically stationary time series — a subject which is both attractive to mathematicians and important in various engineering problems.

This paper describes techniques for solving smoothing and prediction problems of the stationary, linear, least-squares sort using a circuit theory point of view. It avoids the more difficult mathematics of the very general, completely rigorous treatments, but maintains sufficient generality for many engineering applications. It develops general techniques in terms of specific engineering problems, which are of real interest in themselves and may also serve as patterns to be followed in solving other problems. Among the problems considered explicitly are the following: classical smoothing and prediction problems solved by Wiener,[1] Kolmogoroff,[2] Zadeh and Ragazzini,[3] etc.; the simultaneous use of different instruments, with different error spectra, for the observation of single physical variables; applications of the *mathematics* of data smoothing to circuit design problems which do not actually involve data smoothing as such.

The general techniques described here have been developed over a period of years. Some of the results have already been stated, in specialized reports describing specific applications of one sort or another.[4, 5, 6]

### 1.1 *Further Background*

Present-day theories of smoothing and prediction may be said to have started with the classic papers of Wiener[1] and Kolmogoroff,[2] which were written during World War II. They assumed linear least-squares operations, stationary statistics and observations available for all past times. Zadeh and Ragazzini[3] modified the theory for observations which are available only over a past interval of finite duration. By now, many other papers have been published, aimed at generalizing, modifying, interpreting or applying the original theories. A complete bibliography would be very extensive, and will not be attempted in this paper.*

Differences in points of view can result in quite different formulations of smoothing and prediction theory, even though the formulations must reflect the same mathematical fundamentals. This is important, because the classic papers of Wiener, Kolmogoroff, Zadeh and Ragazzini, etc. contain quite formidable mathematics, which is not generally accessible to engineers. Much of this mathematics can be avoided by imposing certain additional restrictions, which are generally minor in terms of resulting restrictions on engineering applications. Further complications can be avoided by not requiring perfect rigor in regard to all singular, or mathematically "pathological" situations. This point of view is quite different from that adopted, for example, by Doob[8] in his very general treatment of smoothing and prediction in terms of the general theory of stochastic processes.

Bode and Shannon[9] simplified the derivation of Wiener and Kolmogoroff's most important result, using circuit theory concepts to interpret the mathematical operations in physical terms. Their physical interpretations are very powerful tools for engineers who must solve the mathematical problems and, in fact, their paper is our principal reference in what follows. Their method of solving the Wiener-Kolmogoroff problem, however, does not apply to the Zadeh-Ragazzini problem or to various other generalizations of engineering interest (unless it is complicated in ways which destroy most of its advantages). Furthermore, their solution of the Wiener-Kolmogoroff problem itself is not simple in numerical applications.

This paper uses a circuit theory point of view in a somewhat different way, which leads to more general applications and to simpler computations. The advantages are obtained at the price of an additional mathematical restriction. Functions of frequency representing statistical "power spectra" are required to be *rational*, whereas more general theories allow more general functional forms.† This is a minor restriction in most engineering applications, where nonrational functions can be replaced by rational approximations.

* For an extensive bibliography (as of 1955) see Stumper.[7]

† Bode and Shannon mention rational spectra as simplifying one part of their method — the evaluation of a loss-phase integral — but they do not seek other simplifications, which may be realized by a rather different method of solution.

Under the assumption of rationality, most of the analysis can be carried out in the frequency domain, in terms of the more usual operations of circuit theory. The concepts of generalized Borel fields, measurable spaces and even Hilbert spaces need not be used at all. Usually, Wiener-Hopf equations can be replaced by contour integrals in the complex plane. When Wiener-Hopf equations do appear, they may be replaced quickly by conditions applied to the analytic properties of functions of frequency. This avoids the usual difficulties with "$\delta$ functions" and their derivatives, and states conditions in forms more familiar to circuit theorists. Frequently, end results may be expressed as conditions which determine network zeros and poles more or less directly. These circumstances all depend, however, on the basic assumptions of linear least-squares smoothing. For the simple methods, the assumption of stationary statistics also is essential; more complicated analogous methods apply to time-variable situations, at least in principle. Nonstationary systems are discussed in this paper only briefly, in Section 3.6.

For the most part, continuous-data systems are assumed. However, the techniques developed for continuous data can readily be adapted to sampled-data problems, by methods which are outlined in Section 3.5. These methods have not yet been compared in detail with more direct methods of handling sampled-data problems such as, for example, those of Levinson (Ref. 1, Appendix B), and Lloyd and McMillan.[10]

Chang[11] has described a frequency-domain equivalent of Wiener and Kolmogoroff's central result. He starts with contour integration, but does not simplify the solution by assuming rational spectra. He does not extend the method to the finite memory problem solved by Zadeh and Ragazzini. Zadeh and Ragazzini themselves describe a solution which assumes rational spectra, but they use a time-domain analysis which is less simple than an analysis in the frequency domain. Laning and Batten[12] also describe smoothing and prediction in time-domain terms, subject to the assumption of rational spectra.

In Ref. 11 Chang also points out that the *mathematics* of smoothing and prediction may be applied to network synthesis problems which do not actually involve data smoothing or prediction. Basically, Chang proposes designing to a least-squares error criterion, where the error is the magnitude of the complex difference between a physical transfer function and a nonphysical ideal function which it is to approximate. This appears to be a quite rewarding approach to various nonstochastic problems in network synthesis, particularly after the frequency-domain method has been extended to more general smoothing and prediction problems.

Within the general field of data smoothing, an important variation of the classical problem is as follows: In the classical problem, one is concerned with the optimum smoothing and prediction of a statistical signal, contaminated with a statistical noise, when the statistics of the signal and noise are known. In the variation, one is concerned with the simultaneous use of *two different* instruments to measure a single physical variable or signal. In the simplest form, the readings of the two instruments are combined through optimum linear operations, subject to the condition that the net error is to depend only on the instrumental error. Then the signal statistics do not enter at all, but two different statistics must still be considered, corresponding to the two different instrumental errors. If the errors of the two instruments have quite different frequency characteristics, the two-instrument combination can give much greater accuracy than either instrument alone. This may be compared with the use of "woofer" and "tweeter" speakers in high fidelity sound systems.

The techniques described here were developed, to a considerable extent, in connection with specific applications of the two-instrument problem described in Refs. 4, 5 and 6. The two-instrument optimization problem was suggested by previous uses of two kinds of instruments, combined through arbitrary, non-optimum linear operations.* More recently, two-instrument optimization principles have been described in papers by Bendat,[14] and Stewart and Parks.[15]

### 1.2 *Organization of the Paper*

The remainder of this paper is organized as follows: Section II formulates a fairly general smoothing and prediction problem in mathematical and physical terms. At the same time it reviews certain mathematical relations which will be needed in the sequel, including some elementary Fourier transforms, some properties of "physical" networks and some properties of stationary Gaussian noise.

Section III develops techniques for solving the general problem. The techniques are explained in terms of specific problems, which are special cases of the general problem (or reasonable variations of it) and are especially suitable for purposes of explanation. Section IV describes other specific problems and engineering applications which have been chosen primarily for their practical interest.

Some of the specific problems illustrate existing engineering applications. Others are merely potentially useful or of interest for largely the-

* Examples are: an instrument made by North American Aviation for the Sandia Corporation (which furnished a starting point for Ref. 4) and a proposal of Crooks.[13]

oretical reasons. Some of the problems may not have been solved before. Others have well-known solutions, in one form or another, and are included purely to illustrate the generality and efficiency of the techniques under discussion.

A more detailed outline of the paper may be seen in the table of contents at the beginning of this paper.

## II. FORMULATION OF GENERAL RELATIONS

In this section we formulate a central problem, in about the same way as Bode and Shannon, and review some mathematics which will be needed in the sequel. In later sections, we shall modify some of the details, but within a set of fundamental restrictions which are included in the formulation described below.

### 2.1 *The Central Problem*

The central problem is as follows: We are given a time function $f(t)$, representing a signal $s(t)$ contaminated by noise $n(t)$:

$$f(t) = s(t) + n(t). \tag{1}$$

The time functions $s(t)$ and $n(t)$ are drawn from statistical ensembles of such functions, and we assume that the pertinent statistical characteristics of the ensembles are known.

We are to derive from $f(t)$ an estimate $g(t)$ of $s(t + \alpha)$. When $\alpha$ is positive, $g(t)$ is a prediction of what the true signal $s$ will be $\alpha$ seconds from present time $t$. When $\alpha$ is negative, $\alpha = -\beta$ and $g(t)$ is an estimate of what the true signal was $\beta$ seconds before present time $t$. The operations to be used in deriving $g(t)$ from $f(t)$ are restricted in various ways. Then $g(t)$ generally will not match $s(t + \alpha)$ exactly, but will be in error, by an amount $\epsilon(t)$:

$$g(t) = s(t + \alpha) + \epsilon(t). \tag{2}$$

The permitted operations are to be used in such a way that $g(t)$ is an *optimum* estimate of $s(t + \alpha)$, as judged by a specific criterion applied to the statistics of the error $\epsilon(t)$. The problem is to find the specific combination of operations within the permitted operations which will, in fact, yield the optimum $g(t)$.

An engineering representation of the problem is illustrated in Fig. 1. The "observed" signal $f(t)$ differs from the "true" signal $s(t)$ by the noise $n(t)$. The observed signal is to be modified by passing it through some sort of device, such as an electrical network, to obtain the output signal $g(t)$, which is to represent an estimate of $s(t + \alpha)$. The action of the de-


Fig. 1 — A physical representation of the smoothing and prediction problem.

vice is described by a mathematical "operator", which is simply a symbolic representation of the corresponding mathematical process relating $g(t)$ to $f(t)$. If the device must be chosen from some class of permitted devices, this class of permitted devices will determine a class of permitted operators. The problem is to determine the optimum operator within the permitted class, as a first step in designing an optimum device.

### 2.2 *General Assumptions*

The specific method of analysis depends on specific assumptions regarding the statistics of the signal and noise, the criterion used to define optimum estimates and the class of permitted operators. The six conditions stated below will be assumed throughout this paper, except in a few instances where certain specific departures will be noted. Other conditions will vary with different circumstances, considered in different sections, and will be noted when needed.

i.* The signal and noise statistics are assumed to be *stationary*. Thus, statistical characteristics which refer to a single time are the same for all times, correlations involving more than one time depend only on time differences, etc.

ii. The criterion used to define an optimum estimate is to be the *average square error*, or variance $\sigma^2 = \text{ave } \epsilon^2$. In other words, the permitted operations are to be used in such a way that $\sigma^2$ is a minimum.

iii. The permitted operations are to be *linear*. A linear operation, applied to $f(t)$, may yield a sum of terms of the following sorts: values of $f(t)$ at specific times, derivatives of $f(t)$ of any order, weighted integrals of $f(t)$.

iv.† The "power spectra" of the signal and noise are to be *rational* functions of frequency $\omega$ — real when $\omega$ is real. (Power spectra are Fourier transforms of covariance functions.)

v. The statistics and the permitted operations must be such that estimates with *bounded average square errors* are possible.

vi. In general, the permitted operations will be an *assigned subset* of the class of all linear operations (for example, the class of all "physical"

* Except in Section 3.6, which shows how the methods used for stationary statistics may be applied to time-variable systems of certain very special kinds.

† Except for possible departures in Sections 3.2.3 and 4.3.

linear operations). Sums and differences of permitted operations will always be linear operations, but they will not always be *permitted linear* operations. However, we will assume that the class $C_Y$, of permitted operations $Y$, always has the following property: If $Y_1$ and $Y_2$ are permitted operations and $k$ is an arbitrary positive or negative real constant, there must always exist a permitted operation $Y_3$ such that*

$$Y_3 - Y_1 = k(Y_2 - Y_1). \tag{3}$$

Conditions i, ii and iii are fundamental to the theories of Wiener,[1] Kolmogoroff,[2] Zadeh and Ragazzini,[3] and Bode and Shannon.[9] It is these conditions which make the mathematics tractable. Conditions i and iii are clearly appropriate for a treatment using conventional theories of fixed linear circuits. Under condition ii, the optimization depends only on linear correlations, as will be confirmed in Section 2.6. Then the actual statistics may be replaced by any more convenient statistical models which have the same linear correlations. Bode and Shannon discuss ways in which these three assumptions do and do not limit engineering applications. The limits should be clearly understood before practical applications are attempted.

Under conditions i, ii and iii, with no further restrictions, mathematically "pathological" situations must be accounted for, and these lead to quite formidable (although tractable) mathematics. Condition iv, assumed here, excludes the more pathological situations. The resulting simplifications in the necessary mathematics are very substantial. While the requirement of rational spectra is an arbitrary restriction, it does not restrict engineering applications to a serious extent. The nonrational spectra usually encountered can be approximated sufficiently well with rational functions.

Condition v is not a significant restriction on the usefulness of a design method. The convergence of certain integrals in which we will be interested depends on this condition, and it is stated here for ready reference. Note that v does *not* require convergence of the integrals of the signal and noise spectra alone, provided the permitted operations can lead to estimates with bounded average square errors. (See, for example, Section 4.1.1.)

Condition vi guarantees that the optimum permitted operator will correspond to a "stationary point" in the usual calculus of variations

* L. A. MacColl has shown that condition vi is equivalent to the following: Let $Y_0$ be any one permitted operation, and let $V$ be the class of operations $Y - Y_0$. Then $V$ is a "linear subspace" and $C_Y$ a "flat subspace" in the linear vector space of all linear operations.

sense. It can frequently be simplified to the following: If $Y_1$ is a permitted operator, then $kY_1$ is a permitted operator. We are going to examine certain constraints, however, which exclude the simple form. The typical constraint of this sort permits only linear operations $Y$ which make no change in in some particular (specified) time function (for example, a constant, or dc, signal). If $f_0(t)$ is the particular time function, and $Y \cdot f_0(t)$ is the result of applying operator $Y$ to $f_0(t)$, the constraint requires $Y \cdot f_0(t) = f_0(t)$. But then $(kY) \cdot f_0(t)$ becomes $kf_0(t)$, and is not permitted. On the other hand, $(Y_2 - Y_1) \cdot f_0(t)$ becomes $[f_0(t) - f_0(t)] = 0$, and the same is true of $k(Y_2 - Y_1)$.

### 2.3 *Substitution of Gaussian Ensembles*

We now replace the actual signal and noise ensembles by Gaussian ensembles with the same linear correlations (as permitted under assumption ii). For a more specific physical representation, we may think of the new $f(t)$ and $g(t)$ as electrical signals (voltages or currents), provided the pertinent statistical characteristics are retained. Stationary Gaussian ensembles may be generated by passing white noise through (idealized) linear networks. Under assumption iii, the operations used to derive $g(t)$ from $f(t)$ also correspond to some *linear* network. Then, Fig. 1 may be replaced by Fig. 2. The two white noise ensembles are uncorrelated (assuming that signal and noise are uncorrelated). Their spectral densities are unity (scale factors appear as gains or losses associated with the networks, which are permitted to include amplifiers). The linear operations performed by the networks may be represented by frequency functions $Y_S(p)$, $Y_N(p)$, $Y_G(p)$, where $p = i\omega$. Responses to any time functions may be found from the frequency functions by means of Fourier transforms. We shall say more about these later on, and also about the properties of white noise.

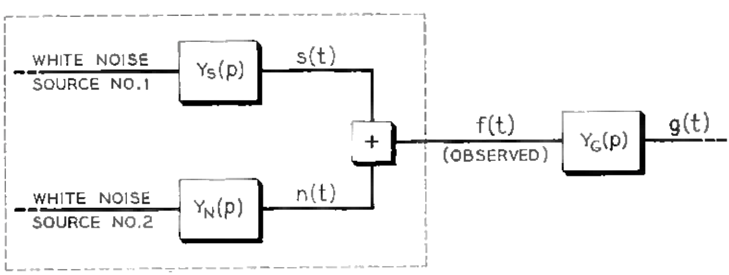



Fig. 2 — A Gaussian physical model.

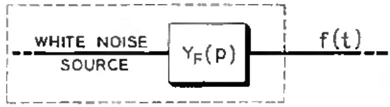



Fig. 3 — An alternative physical model for $f(t)$.

The noise sources themselves are not available to the observer, who sees only $f(t)$. The noise sources and associated networks corresponding to $Y_S(p)$ and $Y_N(p)$ are merely imagined devices which permit the pertinent signal and noise statistics to be described in physical terms. Our problem is to find that particular permitted $Y_G(p)$ (regarded as a linear operator) which converts $f(t)$ into the optimum $g(t)$.

In Fig. 2, $f(t)$ is the sum of two Gaussian ensembles. Since the sum of two Gaussian ensembles is a Gaussian ensemble, $f(t)$ may be represented more simply, as in Fig. 3. This representation does not show the correlation between $f(t)$ and $s(t)$ or $n(t)$, but it will be useful in some of our analysis. The power spectrum of $f(t)$, viewed as a single ensemble, is the sum of the power spectra of the (uncorrelated) signal and noise.

The auto-covariance of any one of the ensembles (signal, noise or signal plus noise) may be specified in any one of three ways: directly as a function of time difference [average of $s(t)\,s(t + \tau)$, etc.]; by means of the power spectrum (which is the Fourier transform of the auto-covariance function); or by means of a network function, $Y_N$, $Y_S$ or $Y_F$ of Figs. 2 or 3 (from which the power spectrum can easily be computed). We will use the following notation:

| Ensemble | Auto-covariance Function | Power Spectrum | Network Function |
|---|---|---|---|
| $s(t)$ | $\Phi_S(\tau)$ | $S$ | $Y_S(p)$ |
| $n(t)$ | $\Phi_N(\tau)$ | $N$ | $Y_N(p)$ |
| $f(t) = s(t) + n(t)$ | $\Phi_F(\tau)$ | $F = S + N$ | $Y_F(p)$ |

Let $E$ and $Y_E$ be any of the three spectra and its corresponding network function. Let $\tilde{Y}(p)$ designate $Y(-p)$:

$$\tilde{Y}(p) = Y(-p). \tag{4}$$

Then an elementary property of power spectra requires:*

$$E = Y_E(p)\,\tilde{Y}_E(p). \tag{5}$$

We need to know how to find $Y_E$ when $E$ is given. In general, the relation of $Y_E$ to $E$ involves the general loss-phase integrals, as described by

* Related to equations (T-9) and (T-14) of Table I, Section 2.5, and the properties of white noise.

Bode.[16] Under our assumption of rationality, however, the loss-phase integrals can be replaced by simple relations between zeros and poles (also in accordance with Bode[16]).

Equation (5) makes $E$ an even function of $p$, and also an even function of $\omega$ (since $p^2 = -\omega^2$). At real *frequencies*, $E = | Y_E(i\omega) |^2$ and is nonnegative. The zeros of $E$ occur in positive and negative pairs, like $+p_\sigma$ and $-p_\sigma$, and so do the poles. Of each pair, one is a zero or pole of $Y_E$ and the other a zero or pole of $\tilde{Y}_E$. When $E$ is given, the zeros and poles can be arranged in $+$ and $-$ pairs, and one of each pair can be assigned to $Y_E$. The possible assignments are not unique, however, unless some further restriction is imposed. For our purposes, we will need the specific assignment described below.

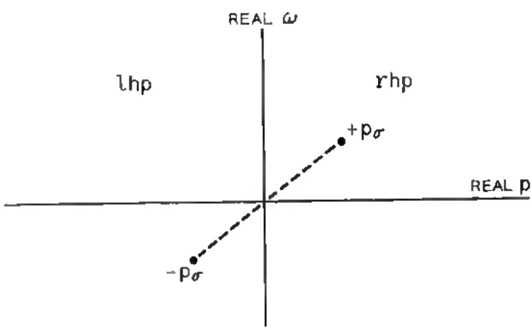



**Fig. 4 — The complex plane for $p = i\omega$.**

Referring to the complex plane for $p$ oriented as in Fig. 4, if $p_\sigma$ lies on one side of the real $\omega$ axis, $-p_\sigma$ lies on the other side. Using "lhp" to designate "left-half plane", we require:

$$\text{The zeros and poles of } Y_E \text{ are the lhp zeros and poles of } E, \quad \text{where } E = S, N, F. \tag{6}$$

There may also be zeros of $E$ *on* the real $\omega$ axis, but these always occur in *identical* pairs, one of which goes to $Y_E$.*

## 2.4 *Properties of Physical Frequency Functions*

Let "rrhp" designate "regular in (the finite part of) the right-half plane." Then an immediate consequence of (6) is

$$Y_E \text{ and } 1/Y_E \text{ are rrhp where } E = S, N, F. \tag{7}$$

* As an immediate consequence of the nonnegative character of $E = | Y(i\omega) |^2$ at real frequencies.

Then both $Y_E$ and $1/Y_E$ are "physically realizable" in the general sense of Bode.[16]* Under assumption iv they are realizable with finite networks of lumped elements, provided these are idealized to include multiple poles at $p = \infty$ (which can be approximated within reasonable limits by active circuits).

The function $Y_G(p)$ in Fig. 2, which converts the observed $f(t)$ into the estimate $g(t)$, may not be rational even though $Y_S$ and $Y_N$ are rational. When a nonrational $Y_G$ is required to be "physical", it must still be regular in the finite part of the rhp. Now, however, it may have an essential singularity at $p = \infty$, and this must meet an additional restriction.

A general definition of a "physical" frequency function, $Y(p)$, merely requires that it be "causal". A causal $Y(p)$, applied as an operator to any time function $f(t)$, produces a response $g(t)$ which depends only on present and past values of $f(t)$. Causal frequency functions have been studied in very general terms, by Wiener,[1] Beurling,[17] Nyman[18] and Youla, Castriota, and Carlin,[19] but the general mathematics is relatively complicated. A less comprehensive description of the conditions for physicalness will be sufficient for our purposes. Let the definition of rrhp be extended, arbitrarily, to

rrhp means:

*a.* Regular in the finite part of the rhp,

*b.* Approaches $\sum_{m,\gamma} (C_{m,\gamma} p^m e^{-\gamma p})$ as $p \to \infty$, (8)

$$m = \pm\text{integer},$$

$$\gamma = \text{real and } \geqq 0,$$

$$C_{m,\gamma} = \text{real}.$$

Then, for purposes of this paper, it is sufficient to define "physical" by:

*A physical $Y(p)$ is any real function of $p$ which is rrhp.* (9)

(A real function of $p$ is one which is real when $p$ is real.)

In (8), the second condition permits $Y(p)$ to behave like $p^m e^{-\gamma t}$ at $p \to \infty$, provided $\gamma$ is *positive*. Behaviors of this kind can be obtained with networks of lumped elements and ideal delay lines.

We will need corresponding relations describing $\tilde{Y}(p) = Y(-p)$. Let rlhp be defined by

* As used here "physically realizable" includes the requirement of stability. This is the usual interpretation of the conventional theory of linear networks. An unstable linear device can in fact exist and it can be driven by an input which is a general function of time, but only over a finite time interval.

rlhp means:

$$\begin{aligned} &a.\ \text{Regular in the finite part of the lhp,} \\ &b.\ \text{Approaches } \textstyle\sum_{m,\gamma} (C_{m,\gamma} p^m e^{+\gamma p}) \text{ as } p \to \infty, \\ &\qquad m = \pm\text{integer}, \\ &\qquad \gamma = \text{real and } \geqq 0, \\ &\qquad C_{m,\gamma} = \text{real}. \end{aligned} \tag{10}$$

Then, it follows from (8) and (9) that:

*A physical $\tilde{Y}(p)$ is any real function of p which is rlhp.* (11)

2.4.1 *An Essential Integral Theorem*

Bode[16] has derived a number of special properties of "physical" functions, in terms of integrals in the complex $p$ plane. These include the loss-phase relations, the integral in the definition of resistance efficiency and other similar integrals. One particularly simple theorem of this sort is essential to our method of solving smoothing and prediction problems.

For our purposes, the theorem may be stated as follows:

$$\begin{aligned} &\text{If: } a.\ H(p) \text{ is either rrhp or rlph and} \\ &\qquad b.\ |H(i\omega)| = O\omega^{-2} \text{ when } \omega \text{ is real and} \to \infty^*, \\ &\text{then: } \int_{-\infty}^{+\infty} H(i\omega)\, d\omega = 0. \end{aligned} \tag{12}$$

The function $H(p)$ is not necessarily one of our network functions, $Y_E(p)$ or $Y_O(p)$, provided it is rrhp or rlhp in the sense of (8) or (10) and also meets the convergence condition. Generally, it will be a combination of our network and spectral functions.

The theorem is easily proved by closing the path of integration with an arc at $\infty$. The arc encloses either the rhp or the lhp, as in Fig. 5(a) or 5(b), depending upon whether $H(p)$ is rrhp or rlhp. Because of (12*a*) the integration around the closed contour is 0. Because of (12*b*), the integral over the arc at $\infty$ is 0 [provided $\gamma \geqq 0$ in (8) or (10), as required].

2.5 *Fourier Transforms*

Equations (1) and (2) describe our problem in terms of time functions, while Fig. 2 describes it in terms of frequency functions (and the proper-

* The symbol $O$ has the following meaning: if $q(\omega) = Or(\omega)$, then $q(\omega)/r(\omega)$ is bounded; thus, $H = O\omega^{-2}$ means $\omega^2 H$ is bounded.

ties of white noise). In general, a facility at transforming quickly between time-domain and frequency-domain formulations is an important tool in smoothing and prediction problems. Time-domain and frequency-domain formulations are, of course, mathematically equivalent, and are related by Fourier transformations. A few elementary theorems regarding Fourier transforms which will be referred to in later sections are reviewed in Table I.

In order for the Fourier transforms to exist, certain mild conditions must be met. We will assume, a priori, that the conditions are satisfied wherever we use the transforms. This is a departure from strict rigor, but our use of the transforms will be entirely reasonable, under our assumptions iv and v.

When $Y(p)$ is the frequency function of a network or equivalent device, $Y(i\omega)$ indicates the steady state response to a sinusoidal input. The inverse transform, $K(t)$, is the response to an ideal unit impulse, or $\delta$ function, applied at time $t = 0$. A general input time function, $f(t)$, may be thought of as a series of impulses. The effect of any one impulse on the response $g(t)$ at a given time $t$ depends on the amplitude of the impulse and on the length of time which has elapsed since the impulse was applied. Then

$$g(t) = \int_{-\infty}^{+\infty} f(t - \tau)K(\tau)\, d\tau = f(t) * K(t). \qquad (13)$$

Thus, $g(t)$ is a weighted integral of $f(t)$, in which the weight factor is $K(\tau)$ and $\tau$ represents the "age of data".

The response of a physical device cannot depend on future inputs.

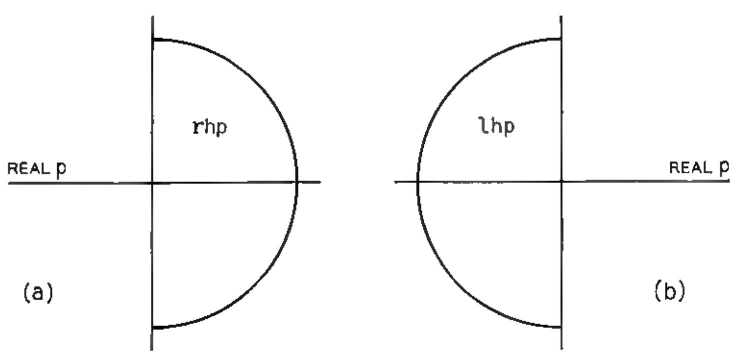



Fig. 5 — Arcs at infinity, enclosing half-planes.

TABLE I — FOURIER TRANSFORMS

Definitions

Fourier Transform:

$$Y(i\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} K(t)\, e^{-i\omega t}\, dt \qquad \text{(T-1)}$$

Inverse Transform:

$$K(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} Y(i\omega)\, e^{+i\omega t}\, d\omega \qquad \text{(T-2)}$$

Convolution:

$$K_1(t) * K_2(t) = \int_{-\infty}^{+\infty} K_1(t-\tau)\, K_2(\tau)\, d\tau \qquad \text{(T-3)}$$

Delta Function:

$$\delta(t) = 0 \quad \text{when} \quad t \neq 0, \int_{-\epsilon}^{+\epsilon} \delta(t)\, dt = 1 \qquad \text{(T-4)}$$

Some Fourier Transform Pairs

| | | |
|---|---|---|
| $K(-t)$ | $Y(-p) = \tilde{Y}(p)$ | (T-5) |
| $kK(t)$ | $kY(p)$ | (T-6) |
| $K_1(t) + K_2(t)$ | $Y_1(p) + Y_2(p)$ | (T-7) |
| $\delta(t)$ | $1/\sqrt{2\pi}$ | (T-8) |
| $K(t-T)$ | $Y(p)e^{-Tp}$ | (T-9) |
| $K_1(t) * K_2(t)$ | $Y_1(p)Y_2(p)$ | (T-10) |
| $K(t) * K(-t)$ | $Y(p)\tilde{Y}(p)$ | (T-11) |
| $K(t) = 0, \quad t < 0$ | $Y(p)$ rrhp* | (T-12) |
| $K(t) = 0, \quad t > 0$ | $Y(p)$ rlhp* | (T-13) |

Some Related Formulas

Parseval's Equation:

$$\int_{-\infty}^{+\infty} K_1(t)\, K_2(t)\, dt = \int_{-\infty}^{+\infty} Y_1(i\omega)\, \tilde{Y}_2(i\omega)\, d\omega \qquad \text{(T-14)}$$

If $Y(p)$ = real when $p$ is real:

$$\int_{-\infty}^{+\infty} [K(t)]^2\, d\omega = \int_{-\infty}^{+\infty} |\, Y(i\omega)\, |^2\, d\omega \qquad \text{(T-15)}$$

Power Spectrum = Fourier Transform of Covariance. (T-16)

If $g(t)$ is the response of a linear device to an input $f(t)$ and if $g(t) = Y(p)e^{pt}$ when $f(t) = e^{pt}$ and $g(t) = K(t)$ when $f(t) = \delta(t)$, then $Y(p)$ and $K(t)$ are a Fourier transform pair and, for a general $f(t)$,

$$g(t) = f(t) * K(t) = K(t) * f(t) \qquad \text{(T-17)}$$

* Provided certain pathological $Y(p)$ and $K(t)$ are excluded, in a way consistent with our definitions of "rrhp," "rlhp," and "physical."

Thus, (13) requires:

$$\textit{If } K(t) \textit{ is physical, } K(t) = 0 \textit{ when } t < 0. \tag{14}$$

When $t \geqq 0$, $K(t)$ need not be particularly well behaved, for it can include $\delta$ functions and their derivatives. If it contains nothing worse than derivatives of $\delta$ functions it can be approximated with combinations of transversal filters and differentiators.

### 2.6 *A Variational Condition, Equivalent to the Optimization Requirement.*

In Figs. 2 and 3 our inputs are drawn from unit-level white noise ensembles. White noise may be described in either frequency-domain or time-domain terms, in accordance with Rice.[20] In the frequency domain it is a sum of sinusoids of all frequencies, with phases that are completely random. The "spectral density" is constant over all frequencies. If the white noise is applied to a network described by $Y(p)$ the corresponding output has similar properties, except that the amplitudes of the sinusoids of different frequencies, $\omega$, are changed by a factor $| Y(i\omega) |$. Then the power spectrum of the ensemble has density $E$ given by the following (at real frequencies):

$$E = | Y(i\omega) |^2 = Y(p)\tilde{Y}(p). \tag{15}$$

Since phases are initially entirely random, phases added by the network do not change the character of the ensemble. Changing the phase of $Y(i\omega)$ without changing $| Y(i\omega) |$ merely maps individual time functions into other, equally probable time functions of the same ensemble.

The average square, $\sigma^2$, of the response to the white noise, is simply the sum of the average squares for the individual frequencies. Thus,

$$\sigma^2 = \int_{-\infty}^{+\infty} | Y(i\omega) |^2 \, d\omega = 2 \int_0^{+\infty} | Y(i\omega) |^2 \, d\omega. \tag{16}$$

Here, $\sigma^2$ represents both the "time average" and the "ensemble average", since the two are identical when they refer to a stationary Gaussian ensemble. The two ranges of integration are equally permissible, as shown, because $| Y(i\omega) |^2$ is an even function of $\omega$.*

In the time domain, the white noise may be described as a sequence of impulses, with infinitesimal spacing along the time scale. The amplitudes of the impulses are uncorrelated Gaussian random variables. An

* It is assumed here that a spectrum $E$ is so scaled that integrating $E$ from $\omega = -\infty$ to $+\infty$ gives the variance, $\sigma^2$. "Unit level" white noise is to be consistent with this assumption and (15). Sometimes the scale of $E$ is doubled, so that integrating $E$ from $\omega = 0$ to $\infty$ gives $\sigma^2$.

impulse at time $t - \tau$ contributes to the response of a network at time $t$ in proportion to $K(\tau)$ and $\sigma^2$ may now be expressed as the sum of the average squares contributed by the individual (uncorrelated) impulses. When limits are taken properly, the result is:

$$\sigma^2 = \int_{-\infty}^{+\infty} [K(t)]^2 \, dt. \tag{17}$$

Equations (16) and (17) are, of course, consistent with (T-15), in Table I, which is a special form of Parseval's equation.

Referring again to Fig. 2, if the output $g(t)$ is interpreted as an estimate of the true future signal $s(t + \alpha)$, the error $\epsilon(t)$ must be

$$\epsilon(t) = g(t) - s(t + \alpha). \tag{18}$$

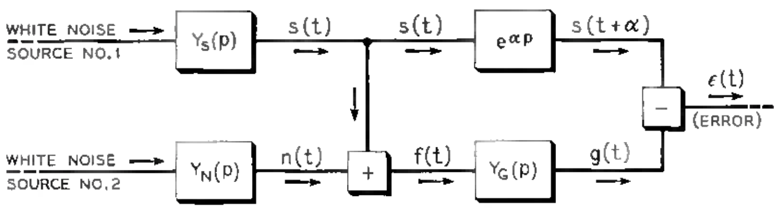



Fig. 6 — A physical model for the calculation of the error $\epsilon(t)$.

We may think of $\epsilon(t)$ as the output of the (unattainable) circuit shown in Fig. 6, in which the responses of the different parts are again described in frequency domain terms.* The average squared error $\sigma^2$ is now the sum of the contributions from the two uncorrelated white noise sources. Evaluating these in terms of (16) gives

$$\sigma^2 = \int_{-\infty}^{+\infty} (| Y_G |^2 N + | Y_G - e^{\alpha i\omega} |^2 S) \, d\omega. \tag{19}$$

The optimization problem may now be stated as follows: Given the class of permitted functions $Y_G$ corresponding to all networks of the permitted sort, find the particular $Y_G$, say $Y_M$, such that $\sigma^2$ is a minimum. Assume tentatively that $Y_M$ exists and let $\Delta_Y(p)$ be defined by

$$Y_G(p) = Y_M(p) + \Delta_Y(p). \tag{20}$$

* When $\alpha > 0$, the box marked $e^{\alpha p}$ is nonphysical, since $e^{\alpha p}$ corresponds to a negative delay, or ideal prediction. So long as the noise is present, the box is connected to an unavailable signal source, $s(t)$ without $n(t)$, whatever the value of $\alpha$. These circumstances are what makes $\epsilon(t)$ the error, instead of an observable correction which could be used to determine $s(t + \alpha)$ exactly.

Under assumption vi, if $Y_G(p)$ in (20) is a permitted $Y_G(p)$, then

$$Y_M(p) + k\Delta_Y(p)$$

is a permitted $Y_G(p)$, where $k$ is any positive or negative real constant.

Substituting $Y_M + k\Delta_Y$ for $Y_G$ in (19) gives:

$$\begin{aligned}\sigma^2 = &\int_{-\infty}^{+\infty} (|Y_M|^2 N + |Y_M - e^{\alpha i\omega}|^2 S)\, d\omega \\ &+ k^2 \int_{-\infty}^{+\infty} (N + S) |\Delta_Y|^2\, d\omega \\ &+ k \int_{-\infty}^{+\infty} [Y_M(N + S) - e^{\alpha i\omega} S]\tilde{\Delta}_Y\, d\omega \\ &+ k \int_{-\infty}^{+\infty} [\tilde{Y}_M(N + S) - e^{-\alpha i\omega} S]\Delta_Y\, d\omega.\end{aligned} \tag{21}$$

In this expression the last two integrals are equal, for the following reasons: First, each of the two integrals is *real*, for the imaginary part of the integrand is always an *odd* function of $\omega$. Second, the two integrals can at most be conjugates of each other, since their integrands are conjugates at real $\omega$. Replacing the two integrals by twice the first leaves

$$\begin{aligned}\sigma^2 = &\int_{-\infty}^{+\infty} (|Y_M|^2 N + |Y_M - e^{\alpha i\omega}|^2 S)\, d\omega \\ &+ k^2 \int_{-\infty}^{+\infty} (N + S) |\Delta_Y|^2\, d\omega \\ &+ 2k \int_{-\infty}^{+\infty} [Y_M(N + S) - e^{\alpha i\omega} S]\tilde{\Delta}_Y\, d\omega.\end{aligned} \tag{22}$$

When $k = 0$, $Y_M + k\Delta_Y = Y_M$, and $\sigma^2$ must be a minimum. This will be true only if the coefficient of $k$ in (22) is zero; hence the third integral must be zero. Furthermore, for $Y_M$ to be a true optimum, the integral must be zero whatever permitted $Y_G$ is used in (20) in determining $\Delta_Y$. If the variable of integration, $\omega$, is replaced by $p = i\omega$ (for convenience in what follows), this requires:

For every permitted $\Delta_Y$:

$$\int_{-i\infty}^{+i\infty} [Y_M(N + S) - e^{\alpha p} S]\tilde{\Delta}_Y\, dp = 0. \tag{23}$$

Our principal concern, in Sections III and IV, will be the solution of (23) for $Y_M$, for different classes of permitted functions $Y_G$. Generally, the class of permitted functions will exclude the obvious solution which makes the bracket, [ ], in (23) identically zero. Then the integrand will not be identically zero, but will have to be such that the integral will be zero for every $\tilde{\Delta}_Y$ which may be derived by using a permitted $Y_G$ in (20).

When (23) is true, (22) may be written as follows, for every permitted $Y_G$:

$$\sigma^2 = \sigma_M^2 + k^2 \int_{-\infty}^{+\infty} (N + S) \, | \Delta_Y |^2 \, d\omega$$
$$\sigma_M^2 = \int_{-\infty}^{+\infty} ( | Y_M |^2 N + | Y_M - e^{\alpha i \omega} |^2 S) \, d\omega. \tag{24}$$

Here $\sigma_M^2$ is the $\sigma^2$ achieved with $Y_G = Y_M$, and the second term in $\sigma^2$ is clearly nonnegative.

Equation (24) implies the following situation: Under assumptions v and vi, a true minimum variance $\sigma_M^2$ exists (at least as a limit of a sequence of $\sigma^2$'s corresponding to a sequence of permitted functions $Y_G$). Any solution of (23) for $Y_M$ within the class of permitted functions $Y_G$ must yield the true $\sigma_M^2$. Since $N + S$ is nonnegative in (23), no other permitted $Y_G$ can yield a smaller $\sigma^2$. When $N + S$ is also nonzero at real frequencies, no other $Y_G$ can yield as small a $\sigma^2$, and the solution for $Y_M$ is at once unique.

When $N + S$ is zero at one or more real frequencies, the situation regarding uniqueness is not so clear, for any $\Delta_Y$ which is nonzero at those frequencies but zero at all other real frequencies will lead to a new $Y_G$, yielding the same $\sigma^2 = \sigma_M^2$. Under assumption iv, $N + S$ can be zero only at discrete frequencies. As a result, if there are two solutions for $Y_M$ in (23), at least one must include transfer functions of filters with infinitesimal bandwidths. It is questionable whether zero-bandwidth filters may be called "physical"; in any event, they cannot be built and alternative solutions do not reduce $\sigma_M^2$. Accordingly, any solution of (23) for $Y_M$ which does not include transfer functions of zero-bandwidth filters will be called unique, whether or not $N + S$ has zeros at real frequencies.

Certain statements about convergence will be useful later on. Under assumption v, $Y_M$ will make $\sigma^2$ finite. In seeking $Y_M$, then, we may start by excluding all $Y_G$ for which $\sigma^2 = \infty$. Since the two integrals in (24) are nonnegative, $\sigma^2$ will be bounded only if both integrals converge. Each of the two integrands is a sum of two nonnegative terms. It follows that each term must meet convergence conditions. Combining these yields an additional useful condition, which will be satisfied by the integrand in

(23). The five conditions may be written collectively as follows:

When $\omega$ is real and $\rightarrow \infty$ the following five functions $= O\omega^{-2}$:†

$$
\begin{aligned}
&| Y_M |^2 N; \qquad | Y_M - e^{\alpha p} |^2 S \\
&| \Delta_Y |^2 N; \qquad | \Delta_Y |^2 S \\
&[Y_M(N + S) - e^{\alpha p} S]\tilde{\Delta}_Y
\end{aligned}
\tag{25}
$$

#### 2.6.1 *An Equivalent Formulation in the Time Domain*

Time-domain equivalents of (23) and (24) can easily be derived. The above analysis can be paralleled in time-domain terms or, alternatively, (T-14) and (T-15) of Table I can be applied directly to (23) and (24). Let $K_G(t)$, $K_M(t)$, $\Delta_K(t)$ be the inverse transforms of $Y_G(p)$, $Y_M(p)$, $\Delta_Y(p)$. Then (23) becomes:

For every permitted $\Delta_K$,

$$\int_{-\infty}^{+\infty} [K_M(\tau) * \Phi_F(\tau) - \Phi_S(\tau + \alpha)]\Delta_K(\tau)\, d\tau = 0. \tag{26}$$

Equation (24) can be transformed into various time-domain equivalents, of which the following is perhaps the most interesting:

$$
\begin{aligned}
\sigma^2 &= \sigma_M^2 + \int_{-\infty}^{+\infty} [K_F * \Delta_K]^2\, d\tau, \\
\sigma_M^2 &= \int_{-\infty}^{-\infty} \{(K_M * K_N)^2 + [(K_M - \delta(\tau + \alpha)) * K_S]^2\}\, d\tau.
\end{aligned}
\tag{27}
$$

### 2.7 *Breadth of the Optimum*

The $Y_M$ determined by (23) may or may not be realizable with a finite network of lumped elements, even though assumption iv insures that $Y_N$ and $Y_S$ of Fig. 2 could be so realized. When $Y_M$ cannot be realized with a finite network of lumped elements, it may be necessary to replace $Y_M$ by a reasonable approximation to it, which can be so realized (and similarly for equivalent nonelectrical devices). A reasonable approximation will be one which can be realized in a reasonable way, and which makes $\sigma^2$ only a little greater than the minimum, $\sigma_M^2$.

The approximation of general "physical" network functions with "finite network approximations" is a familiar problem in general network

† Recall the meaning of $O$ noted in Section 2.4.1.

theory, which need not be developed here. In this connection, however, it is important to note the situation described briefly below.

In engineering problems, the minimum exhibited by $\sigma^2$ as a function of $Y_G$ is usually quite broad. In other words, $Y_G$ may be made quite a little different from $Y_M$ without increasing $\sigma^2$ very much. This has not been proved but is simply a matter of experience in problems of the engineering sort (and, in fact, it cannot even be stated exactly without assigning a more quantitative meaning to the expression "broad minimum").

A broad minimum does not mean that *all* small departures from $Y_M$ have small effects on $\sigma^2$. For example, a change from order $c/p^m$ at $p = \infty$, to order of $c/p^{m-1}$ may yield only small departures from $Y_M(p)$ at all real frequencies, but it is likely to change $\sigma^2 = {\sigma_M}^2$ into $\sigma^2 = \infty$. Generally, reasonable *percentage* changes, relative to the magnitude of $Y_M$ at corresponding frequencies, may be tolerated. The percentage changes may be frequency-dependent, and may be real or complex. The specific sensitivities to changes, however, will depend on the specific values of $N$ and $S$.

The effect of specific departures from $Y_M$, in specific problems, may be calculated by means of (24) or (27).

## III. GENERAL TECHNIQUES, IN TERMS OF SPECIFIC PROBLEMS

The remainder of the paper describes the calculation of $Y_M$ from (23), and modifications thereof. The specific $Y_M$ determined by (23) depends on the class of permitted functions $Y_G$, within which $Y_M$ is to be the optimum choice. A number of different classes are of interest, on both theoretical and practical grounds. Furthermore, the appropriate techniques for calculating $Y_M$ vary with the permitted class, in ways which are likely to be nontrivial.

In this section we consider some fairly general classes of permitted functions, which will illustrate general techniques. In Section IV we shall examine variations and special cases, chosen primarily for their engineering interest.

In describing the properties of the different classes, it will be convenient to use the following general notation:

*a.* $C_Y$ = the class of permitted functions $Y_G$, within which $Y_M$ is to be the optimum choice;
*b.* $C_\Delta$ = the class of functions $Y_{G1} - Y_{G2}$, where $Y_{G1}$ and $Y_{G2}$ are any two $Y_G$ in $C_Y$. (28)

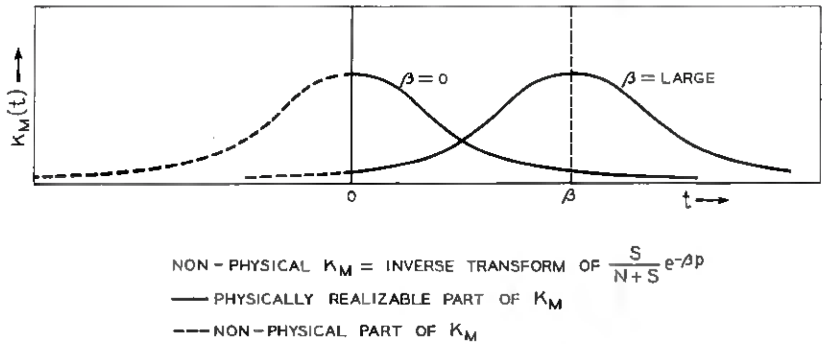



**Fig. 7 — The optimum nonphysical impulse response when $\beta$ is large.**

The class $C_\Delta$ is completely determined by the class $C_Y$. In terms of $C_\Delta$, assumption vi of Section 2.2 may be written as follows:

$$\text{If } \Delta_Y \text{ is in } C_\Delta, \qquad k\Delta_Y \text{ is in } C_\Delta. \tag{29}$$

### 3.1 *Optimum Nonphysical Network*

The integral in (23) will surely vanish if the integrand is zero identically for every permitted $\Delta_Y$. The integrand will be zero if

$$Y_M = \frac{S}{N + S} e^{\alpha p}. \tag{30}$$

This $Y_M$, however, is generally nonphysical. First, it is not generally regular in the finite part of rhp. Second, when $\alpha > 0$ it behaves improperly at $p \to \infty$.

When $\alpha$ is negative, $Y_M$ behaves properly at $p \to \infty$, but there are still singularities in the rhp [except when $S/(N + S)$ is a constant]. Let $\alpha$ equal $-\beta$. Then $g(t)$ is an estimate of $s(t)$ for a time $\beta$ seconds in the past. If $\beta$ is sufficiently large, $Y_M$ can be approximated closely with a physical $Y$.* This is illustrated in Fig. 7, in time-domain terms. Note that the inverse transform of $Y_M$ is symmetrical about the time $t = \beta$, which represents the length of the time interval from the time for which the signal $s(t - \beta)$ is estimated up to present time $t$.

A large $\beta$ is appropriate for reducing data long after they are collected, such as reconstructing the flight of an experimental missile from recorded positions or velocities. Then $\beta$ is the length of time by which the interval of observation extends beyond the time for which $S$ is to be estimated.

* See Ref. 9, Section V.

When the $Y_M$ of (30) is used in (24), the minimum average square error is (after some simple manipulation)

$$\sigma_M{}^2 = \int_{-\infty}^{+\infty} \frac{NS}{N + S}\, d\omega. \tag{31}$$

Then assumption v of Section 2.2 requires

$$\frac{NS}{N + S} = O\omega^{-2} \quad \text{when } \omega \to \infty. \tag{32}$$

The same restriction on $N$ and $S$ will surely still be necessary (although perhaps not sufficient) when the choice of $Y_M$ is restricted to any subset of the present $C_Y$.

3.2 *Optimum Physical Network*

In this section, we restrict the function class of $C_Y$, from which $Y_M$ is to be chosen, to the "physical" subset of the class permitted in the previous section.

$$C_Y = \text{the class of all physical frequency functions.} \tag{33}$$

by "physical", we again mean rrhp as in (8) and (9). Since the difference of two rrhp functions is also rrhp, (33) implies

$$C_\Delta = \text{the class of all physical frequency functions.} \tag{34}$$

The integrand in (23) can no longer be identically zero for all the $\tilde{\Delta}_Y$ permitted by $C_\Delta$, but the restrictions on $\tilde{\Delta}_Y$ are such that they may be taken advantage of, in one way or other. Note that the $C_\Delta$ of (34) obeys (29), and hence also our assumption vi.

The optimum $Y_M$ may be determined in the following way: First, (33) is applied to (23), to obtain tentative conditions on $Y_M$ which are consistent with (33) and in which $\Delta_Y$ does not appear. As derived, these are sufficient conditions. Any corresponding $Y_M$, if one exists, must be the correct $Y_M$, but it is not at once apparent that one does exist. The necessity of the conditions is then established by demonstrating that a corresponding $Y_M$ does, in fact, exist. (Recall that the correct $Y_M$ is unique.) This is a procedure which is useful in many variations of the smoothing and prediction problem.

The integral in (23) will be zero if its integrand behaves like $H(p)$ of (12). By (25), the integrand behaves properly when $\omega$ is real and $\to \infty$. By (33), the factor $\tilde{\Delta}_Y$ in the integrand in (23) is rlhp. Therefore, if the remaining factor in the integrand in (23) is made rlhp both (12*a*) and

(12*b*) will be true, and (23) will be satisfied. Assembling this condition with (25) and (33) gives the following set of *sufficient* conditions on $Y_M$ :

$$\begin{aligned} &a.\ Y_M \quad \text{is rrhp,} \\ &b.\ Y_M (N + S) - e^{\alpha p}S \text{ is rlhp,} \\ &c.\ \text{When } \omega \text{ is real and} \to \infty, \\ &\quad | Y_M |^2 N \quad \text{and} \quad | Y_M - e^{\alpha p} |^2 S = O\omega^{-2}. \end{aligned} \tag{35}$$

It remains to be shown that a $Y_M$ meeting (35) does, in fact, exist. The demonstration is somewhat different for positive and negative values of $\alpha$.

### 3.2.1 *Prediction for a Present or Future Time* ($\alpha \geqq 0$)

When $\alpha$ is positive in (35*b*) the behavior of $e^{\alpha p}S$ as $p \to \infty$ is consistent with rlhp. Then $Y_M(p)$ may reasonably be rational (under our assumption of rational $N$ and $S$). When $Y_M$ must be physical and $\alpha$ is nonzero and positive, $| Y_M - e^{\alpha p} |^2$ cannot $\to 0$, when $\omega$ is real and $\to \infty$. Then assumption v, as reflected in condition (25), requires $S = O\omega^{-2}$ as $\omega \to \infty$. (The limiting case of $\alpha = 0$ will be examined later.) When $S = O\omega^{-2}$ as $\omega \to \infty$, conditions (35) can easily be translated into the following set, which refers to poles and zeros in the finite part of the $p$ plane:

*a*. The poles of $Y_M$ are the lhp zeros of $N + S$,

*b*. At the lhp poles of $N$ and $S$

$$Y_M = \frac{S}{N + S} e^{\alpha p}, \tag{36}$$

*c*. The degree of the numerator of $Y_M$ is a minimum,within conditions *a* and *b*.

These conditions determine a $Y_M$ uniquely. The $Y_M$ so determined will also satisfy (35), provided the degree determined by (36*c*) is such that (35*c*) is satisfied. It is shown below that (36*c*) is, in fact, just consistent with (35*c*).

Recall that $N$ and $S$ are even functions, with half of their zeros and poles in each half plane. Also, $Y_M\tilde{Y}_M$, which is $| Y_M |^2$ at real $\omega$, is an even function, in which exactly half of the zeros and poles are zeros and poles of $Y_M$. Then, under (36*a*), the number of (finite) poles of $Y_M\tilde{Y}_M$ is exactly the number of (finite) zeros of $(N + S)$. On the other hand, the number of zeros of $Y_M$ is *one less* than the number of lhp poles of $N + S$, for the scale of $Y_M$ can be adjusted, as well as the zeros, in meeting (35*b*). Then the number of zeros of $Y_M \tilde{Y}_M$ is exactly *two less* than the number of poles of $N + S$. As a result,

$$| Y_M |^2 (N + S) \rightarrow c/\omega^2 \text{ as } \omega \rightarrow \infty. \tag{37}$$

Finally, since $N$ and $S$ are nonnegative at real $\omega$ (and hence cannot cancel each other at $\omega \rightarrow \infty$), both

$$| Y_M |^2 N \quad \text{and} \quad | Y_M |^2 S = O\omega^{-2} \text{ as } \omega \rightarrow \infty. \tag{38}$$

Since we already know that $S = O\omega^{-2}$, this is sufficient to establish (35).

Conditions (36) need further interpretations for special cases. When zeros or poles of $N + S$ occur *on* the axis of real frequencies, they occur in identical pairs.* One of each pair is to be interpreted as in the lhp. Certain zeros of $N + S$ may coincide with poles common to $N$ and $S$, when they are computed from the numerators and denominators of the (rational) $N$ and $S$. Lhp zeros of this sort are to be retained as poles of $Y_M$.†

When $\alpha = 0$, the second condition in (35*c*) becomes $| Y_M - 1 |^2 S = O\omega^{-2}$ as $\omega \rightarrow \infty$. So long as $S = O\omega^{-2}$ as $\omega \rightarrow \infty$, conditions (36) are still appropriate. Now, however, $S$ may be nonzero, or even unbounded at $\omega = \infty$. When $S$ is nonzero but bounded, $Y_M \rightarrow 1$ as $\omega \rightarrow \infty$. When $S$ has poles at $\infty$, $(Y_M - 1) \rightarrow c/p^m$. When (36) is properly modified to include the new conditions, a unique $Y_M$ is again determined, which again satisfies (35), provided $N$ is such that (32) is still satisfied. When $S \neq 0$ at $\infty$, however, (32) will not be satisfied unless $N = O\omega^{-2}$ as $\omega \rightarrow \infty$.

### 3.2.2 *Estimation for a Past Time* ($\alpha < 0$)

If $\alpha = -\beta$, (35) becomes:

$$\begin{aligned} &a.\ Y_M \text{ is rrhp,} \\ &b.\ Y_M(N + S) - e^{-\beta p} S \text{ is rlhp,} \\ &c.\ \text{When } \omega \text{ is real and} \rightarrow \infty, \\ &\qquad | Y_M |^2 N \quad \text{and} \quad | Y_M - e^{-\beta p} |^2 S = O\omega^{-2}. \end{aligned} \tag{39}$$

When $\beta$ is positive, $e^{-\beta p} S$ does not behave at infinity in an rlhp manner,

* A necessary consequence of the *positiveness* of spectra $N$ and $S$.
† This is confirmed by (36*b*), which makes $Y_M$ infinite at a lhp pole of $S$ which is cancelled by a like pole of $N$ in $N + S$.

as defined by (10). Hence $Y_M(p)$ can no longer be rational, but must contain a term which annuls the forbidden behavior at infinity.

Let $Y_M$ be represented as

$$Y_M = A + \frac{S}{N + S} e^{-\beta p}. \tag{40}$$

Then, if (39$c$) is rearranged with due regard for (32) and the positiveness of $N$ and $S$, (39) now becomes

$$\begin{aligned} &a.\ \ A + \frac{S}{N+S} e^{-\beta p} \text{ is rrhp,} \\ &b.\ \ A(N + S) \qquad \text{is rlhp,} \\ &c.^*\ \text{When } \omega \text{ is real and} \to \infty, \\ &\qquad |A|^2(N + S) = O\omega^{-2}. \end{aligned} \tag{41}$$

The function $A$ can now be factored, recalling $Y_F$ of Section 2.3 and Fig. 3:

$$\begin{aligned} Y_F\tilde{Y}_F &= N + S, \\ Y_F,\ 1/Y_F &\text{ are rrhp.} \end{aligned} \tag{42}$$

We can multiply the function in (41$a$) by $Y_F$, without changing its rrhp character. (If rrhp, it will remain rrhp; if not rrhp, it will remain not rrhp.) Similarly, we can divide the function in (41$b$) (arbitrarily) by $\tilde{Y}_F$. Then (40) and (41) may be written as follows [if (42) is again used]:

$$\begin{aligned} &Y_M = \frac{1}{Y_F}\left(B + \frac{S}{\tilde{Y}_F} e^{-\beta p}\right), \\ &a.\ B + \frac{S}{\tilde{Y}_F} e^{-\beta p} \text{ is rrhp,} \\ &b.\ B \qquad\qquad \text{is rlhp,} \\ &c.\ \text{When } \omega \text{ is real and} \to \infty,\ |B|^2 = O\omega^{-2}. \end{aligned} \tag{43}$$

The conditions (43) can only be realized with a unique rational $B$. The poles of $B$ are in the rhp but they are cancelled in $Y_M$ by poles of

* This may be derived in the following way: Use the $Y_M$ of (40) in the two functions in (39$c$). Add the two functions, rearrange, and separate out the function in (41$c$), making use of the following theorem: Let $U_1$ and $U_2$ be two functions of $\omega$; if $U_1 = O\omega^{-2}$ and $U_2 = O\omega^{-2}$, then $U_1 + U_2 = O\omega^{-2}$; conversely, if $U_1 + U_2 = O\omega^{-2}$, and both $U_1$ and $U_2$ are nonnegative, then $U_1 = O\omega^{-2}$ and $U_2 = O\omega^{-2}$.

$S/\tilde{Y}_F$. Because the poles of $B$ are in the rhp, $Y_M$ cannot be realized exactly by a finite network of lumped elements and delay lines. It can be approximated arbitrarily closely, however, by transversal filter techniques.

### 3.2.3 *A Time-Domain Interpretation*

A time-domain interpretation of $Y_M$ may be derived from (35) which coincides with, for example, Bode and Shannon's interpretation.[9] This may be accomplished by using the functions $Y_F$ and $\tilde{Y}_F$ much as in the previous section.

Since $\tilde{Y}_F$ and $1/\tilde{Y}_F$ are both rlhp, the function in (35*b*) may be multiplied by $1/\tilde{Y}_F$ without altering its rlhp character. (If rlhp, it will remain rlhp; if not rlhp, it will remain not rlhp.) Then (35*b*) becomes

$$Y_M Y_F - \frac{S}{\tilde{Y}_F} e^{\alpha p} \text{ is rlhp.} \tag{44}$$

By (T-13), the inverse transform must be 0 when $t > 0$. Hence, the two terms in the difference must be equal when $t > 0$. Since $Y_M$ and $Y_F$ are both physical, $Y_M Y_F$ is physical and its inverse transform must be zero when $t < 0$. Then (41) becomes, by (T-10),

$$\begin{aligned} K_M * K_F &= \text{inverse transform of } \frac{S}{\tilde{Y}_F} e^{\alpha p} \text{ when } t > 0 \\ &= 0 \text{ when } t < 0. \end{aligned} \tag{45}$$

Equation (45) determines $K_M * K_F$ uniquely. The transform is $Y_M Y_F$, and $Y_M$ can be found by dividing out the (minimum phase, physical) factor $Y_F$. Note that (45) can be solved for $K_M$ even though the spectra are nonrational: $F = N + S$ must be such that $Y_F$ can be found from $F = Y_F \tilde{Y}_F$ by means of the loss-phase integral; and various transforms and convolutions must be evaluated.

### 3.2.4 *A Further Interpretation*

A further interpretation of (45) is useful when one seeks certain generalizations of the present problem. The time series which are fundamental to the problem are the true signal $s(t)$ and the observed signal $f(t)$. When $s(t)$ is to be predicted for a specific time, $t + \alpha$, from values of $f(t)$ observed over a specific interval, $(-\infty < \tau < t)$, only certain of the linear covariances, describing statistical characteristics of $s$ and

$f$, are actually pertinent to the problem. The pertinent linear covariances are:

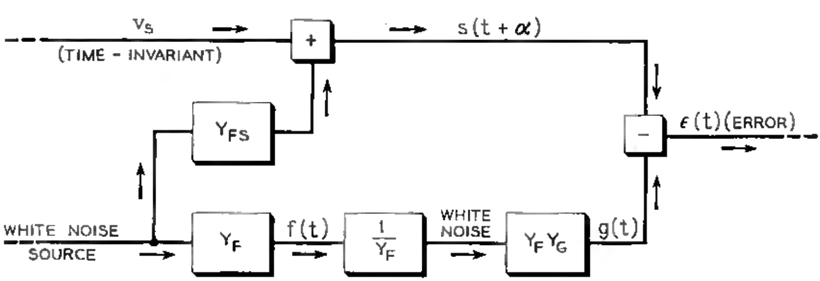
1. the auto-covariance of the observed signal $f$ at any two times, $t_1$, $t_2$, in the observation interval;

2. the cross-covariance between $f$ at any time, $t_1$, in the observation interval and $s$ at the special time $t + \alpha$;

3. the expected value of $s^2$, which is the zero lag covariance of $s$, at the special time $t + \alpha$.

The auto-covariance function $\Phi_s$, or the corresponding spectrum $S$, is pertinent to our present problem only as it affects the covariances of $f$ through the relation $f(t) = s(t) + m(t)$.

It follows from the above that the original Gaussian model may be replaced by any other Gaussian model which retains the same pertinent linear covariances, without affecting either $Y_M$ or $\sigma_M^2$. For example, the model represented by Fig. 6 may be replaced by that shown in Fig. 8. The network with frequency function $Y_F$ generates an $f(t)$ with the correct auto-covariance (in accordance with Section 2.3 and Fig. 3). The network with frequency function $Y_{FS}$, in the presence of the other network, supplies the correct cross-covariance between $f(t_1)$ and $s(t + \alpha)$. (The cross-covariance arises from the sharing of the same white noise, by $s$ and $f$.) Note that $Y_{FS}$ in Fig. 8 is exactly the function $(S/\tilde{Y}_F)e^{\alpha p}$ in (44). The single (time-invariant) Gaussian random variable $v_s$ contributes only to $s$, and gives the correct variance of $s$ when added to the contribution from the white noise. Note that the variance of $v_s$ is exactly the $\sigma_M^2$ of Section 3.1, corresponding to the optimum nonphysical $Y_M$.




$$Y_F \tilde{Y}_F = S + N \qquad Y_{FS} = \frac{S}{\tilde{Y}_F} e^{\alpha p} \qquad E[v_S^2] = \int_{-\infty}^{+\infty} \frac{NS}{N+S} d\omega$$

Fig. 8 — An alternate physical model, which retains the pertinent covariances.

Because $Y_F$ has been chosen in such a way that $1/Y_F$ is physical, the white noise of Fig. 8 can be recovered. If $Y_M Y_F = Y_{FS}$, the estimate $g(t)$, of $s(t + \alpha)$, is exactly the contribution of the white noise to $s(t + \alpha)$, and the error in the estimate is exactly $v_a$. If $Y_M$ must be physical, $Y_M Y_F$ can yield only the contributions to $s(t + \alpha)$ from present and past values of the white noise. This is roughly the description of the solution used by Bode and Shannon (in Ref. 9, Section VII).

### 3.3 *Optimum Network with a Finite Memory*

In Section 3.2 we assumed that all past values of $f(t)$ (the signal plus noise) were available, back to $t = -\infty$. Furthermore, the $Y_M(p)$ of (35) does generally lead to the utilization of all past values. In practical applications, the corresponding $K_M(\tau)$ is usually very small when the age of data $\tau$ is sufficiently large, say $\tau > \tau_m$; and data older than $\tau_m$ may reasonably be neglected. Frequently, however, $f(t)$ is available only for a smaller interval, say $0 < \tau < T$; and then the procedure must be revised. This is the central problem considered by Zadeh and Ragazzini.[3] It may be referred to as the "finite memory" problem, as opposed to the "infinite memory" considered in Section 3.2.

We can restrict ourselves to values of $f(t)$ no older than $T$ by making our function class $C_Y$ correspond to physical networks with memories which extend only $T$ seconds into the past. With fixed networks of this sort, $T$ is constant, and the used data begins at a variable time $t - T$. When $f(t)$ is available from a fixed starting time, $t_0$, to present time $t$, $T$ is a function of $t$. Then a variable network must be designed with a response which equals or approximates the response of a different fixed network at each different $t$.

Our smoothing and prediction device will remember only $T$ seconds into the past if the impulse response $K_G$ is restricted by

$$K_G(t) = 0, \quad \text{except when} \quad 0 < t < T. \tag{46}$$

An equivalent frequency domain restriction may be derived as follows: Referring to Fig. 9, if $K_G(t) = 0$ except when $0 < t < T$, it follows that $K_G(-t) = 0$ except when $-T < t < 0$. But then $K_G[-(t - T)] = 0$ except when $0 < t < T$, which meets the conditions for physicalness. If $Y_G$ is the transform of $K_G$, then $\tilde{Y}_G e^{-Tp}$ is the transform of $K_G[-(t - T)]$, by (T-5) and (T-9). Thus (46) corresponds to

$C_Y$ is the class of functions $Y_G(p)$ such that:

$$\begin{aligned} &a.\ Y_G \quad \text{is rrhp,} \\ &b.\ \tilde{Y}_G e^{-Tp} \text{ is rrhp.} \end{aligned} \tag{47}$$

Since differences of $K_G$'s of the form (46) also obey (47), it follows that

$$C_\Delta \text{ is the class of functions } \Delta_Y \text{ such that:}$$

$$\begin{aligned} &a.\ \Delta_Y \quad \text{is rrhp,} \\ &b.\ \tilde{\Delta}_Y e^{-Tp} \text{ is rrhp.} \end{aligned} \tag{48}$$

Note that (48) is consistent with (29) and hence also with assumption vi of Section 2.2.

The following is an important property of functions in the class (47): Replacing $Y_G$ by $\tilde{Y}_G$ maps singularities from either half plane into the other. Then (47) excludes singularities from all finite parts of the $p$ plane. Let rfpp mean "regular in the finite part of the plane". Then, (47) implies that

$$Y_G \text{ is rfpp.} \tag{49}$$

There will be no $Y_M$ of the class (47) which satisfies (35). Hence, a new relation must be derived from (23). To take advantage of (47), $Y_M$ may be expressed in the following arbitrary way:

$$Y_M = A + Be^{-Tp}. \tag{50}$$

If this expression is used in (23), the integral in (23) may be divided into two parts, as follows:

$$\int_{-i\infty}^{+i\infty} [A(N + S) - e^{\alpha p}S]\tilde{\Delta}_Y\,dp + \int_{-i\infty}^{+i\infty} [B(N + S)]e^{-Tp}\tilde{\Delta}_Y\,dp = 0. \tag{51}$$

The two integrals in (51) will be zero, individually, if the two integrands behave like $H(p)$ of (12). This will be true under conditions noted below.

In the first integrand, $\tilde{\Delta}_Y$ is rlhp. In the second, $\tilde{\Delta}_Y e^{-Tp}$ is rrhp by

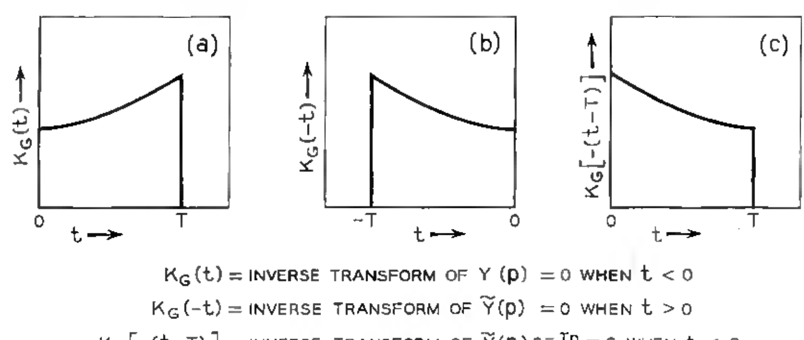



Fig. 9 — Impulse responses of finite duration.

(48). For (12$a$) to be true, the other factor in each integrand must be regular in the same half plane. The convergence condition (12$b$) must also be applied, and it can be applied if (25) is properly taken into account. When (47) also is transcribed the following conditions may be assembled as sufficient conditions on $A$ and $B$:

$$\begin{aligned} &a.\ A + Be^{-Tp} && \text{is rrhp,} \\ &b.\ \tilde{B} + \tilde{A}e^{-Tp} && \text{is rrhp,} \\ &c.\ A(N + S) - e^{\alpha p}S && \text{is rlhp,} \\ &d.\ B(N + S) && \text{is rrhp,} \end{aligned} \tag{52}$$

$e$. When $\omega$ is real and $\to \infty$, the following functions $= O\omega^{-2}$:

$$| A |^2N, \qquad | A - e^{\alpha p} |^2S$$
$$| B |^2N, \qquad | B |^2S.$$

Functions $A$ and $B$ which satisfy (52) do, in fact, exist; hence the conditions are necessary as well as sufficient. As in Section 3.2, the solution takes different forms for positive and negative values of $\alpha$.

#### 3.3.1 *Prediction for a Present or Future Time* ($\alpha \geqq 0$).

When $\alpha$ is positive in (52$c$) $A$ and $B$ are rational. By the definition of rrhp in (8), conditions $a$ and $b$ will be satisfied if the rational $A$ and $B$ satisfy (49). When $S = O\omega^{-2}$ as $\omega \to \infty$ (as it must if $\alpha$ is nonzero and positive), (52) may be translated into the following set of conditions, which refer to poles and zeros in the finite part of the $p$ plane:

$a$. The poles of $A$ and $B$ are the zeros of $N + S$, in both half planes,

$b$. At the lhp poles of $N$ and $S$

$$A = \frac{S}{N + S} e^{\alpha p},$$

$c$. The rhp poles of $N$ and $S$ are zeros of $B$, (53)

$d$. At the poles of $A$ and $B$

$$-\frac{A}{B} = e^{-Tp},$$

$e$. The combined degrees of the numerators of $A$ and $B$ are to be as small as possible and the relative degrees are to be adjusted as required by (52$e$).

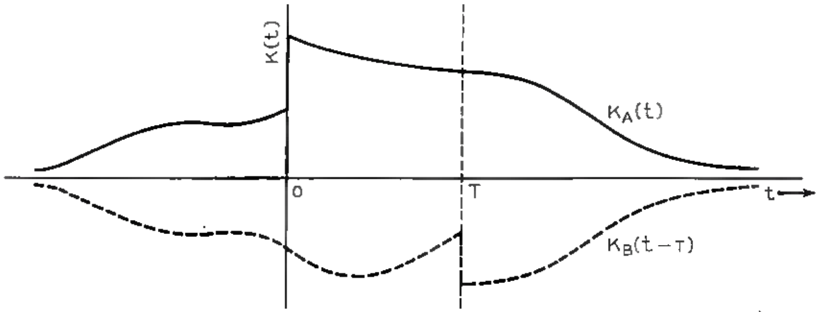



**Fig. 10 — Inverse transforms of $A$ and $Be^{-Tp}$.**

An analysis of degrees, as in Section 3.2.1, shows that (53) determines a unique $A$ and $B$, which satisfy (52) when $S = O\omega^{-2}$ as $\omega \rightarrow \infty$. When $\alpha = 0$, $S$ is not necessarily 0 at $\infty$. Then (52$c$) imposes additional conditions on $A$. When (53) is suitably modified to include these new conditions, a unique $A$ and $B$ are again determined.

It is interesting to examine the forms of the inverse transforms of $A$ and $Be^{-Tp}$, which together make up the inverse transform $K_M(t)$ of $Y_M(p)$. The inverse transforms of general rational functions are equal piecewise to sums of exponentials. The exponentials are different for $t > 0$ and $t < 0$, and there may be discontinuities (and also $\delta$ functions) at $t = 0$. In $Be^{-Tp}$, the discontinuities are displaced to $t = T$. Then the two time functions may take the form suggested in Fig. 10.

As in Section 3.2.2, $Y_M$ cannot generally be realized with a finite network of lumped elements and ideal delay lines. It can be approximated by a transversal filter; plus differentiations if $K_M(t)$ is not a bounded function. The transversal filter need not involve transmission times greater than $T$. If preferred, the rhp parts of $A$ and $B$ can be separated out and realized with a finite network of lumped elements plus a single delay line.

### 3.3.2 *Prediction for a Past Time* ($\alpha < 0$).

When $\alpha = -\beta$, (52$c$) excludes a rational $A$. Recalling (40), we let $A$ be

$$A = D + \frac{S}{N + S} e^{-\beta p}. \tag{54}$$

Then when $\alpha = -\beta$, (52) becomes the following:*

$$
\begin{array}{lll}
a. & D + \dfrac{S}{N+S} e^{-\beta p} + Be^{-Tp} & \text{is rrhp,} \\
b. & \tilde{B} + \tilde{D}e^{-Tp} + \dfrac{S}{N+S} e^{-(T-\beta)p} & \text{is rrhp,} \\
c. & D(N+S) & \text{is rlhp,} \\
d. & B(N+S) & \text{is rrhp,}
\end{array}
\qquad (55)
$$

*e.* When $\omega$ is real and $\rightarrow \infty$, the following functions $= O\omega^{-2}$:

$$
|D|^2N, \qquad |D|^2S
$$
$$
|B|^2N, \qquad |B|^2S.
$$

Consider the term $e^{-(t-\beta)p}$ in (55*b*). When $\beta < T$, the exponential behaves at $\infty$ in a manner suitable for an rrhp function, as defined in (8). Then $D$ and $B$ are both rational, and can be calculated from zero and pole conditions implied by (55).

When $\beta > T$, the exponential $e^{-(T-\beta)p}$ is not suitable for an rrhp function, and it must be annulled by an exponential term in $B$. Let

$$
B = J - \frac{S}{N+S} e^{-(\beta-T)p}. \tag{56}
$$

Using both (54) and (56) in (50) and (52) gives

$$
Y_M = D + Je^{-Tp}
$$

$$
\begin{array}{lll}
a. & D + Je^{-Tp} & \text{is rrhp,} \\
b. & \tilde{J} + \tilde{D}e^{-Tp} & \text{is rrhp,} \\
c. & D(N+S) & \text{is rlhp,} \\
d. & J(N+S) - Se^{-(\beta-T)p} & \text{is rrhp,}
\end{array}
\qquad (57)
$$

*e.* When $\omega$ is real and $\rightarrow \infty$, the following functions $= O\omega^{-2}$:

$$
|D|^2N, \qquad |D|^2S
$$
$$
|J|^2N, \qquad |J - e^{-(\beta-T)p}|^2S.
$$

When $\beta > T$, $D$ and $J$ are rational, and can be found from (57).

A comparison of (57) and (52) indicates the following relationship:†

---

* Because $N$ and $S$ are even, $\tilde{N} = N$ and $\tilde{S} = S$.

† In making the comparison, recall that $Y$ is rrhp when $\tilde{Y}$ is rlhp, and vice versa.

*If $Y_M$ is the optimum $Y_G$ for estimating $s(t + \alpha)$, then $\tilde{Y}_M e^{-Tp}$ is the optimum $Y_G$ for estimating $s(t - T - \alpha)$.* (58)

The two times, $t + \alpha$ and $t - T - \alpha$, are symmetrically located relative to the interval $t - T < \tau < t$, for which values of $f(t)$ are available. The relation (58) may be viewed as a direct consequence of the symmetry of auto-covariance functions.

### 3.3.3 *A Time-Domain Interpretation*

In Section 3.2.3 we derived Bode and Shannon's explicit time-domain solution of the infinite memory problem. When memories must be finite there are difficulties which exclude an explicit time-domain solution of the same sort. The difficulties themselves are informative, however, and a simple time-domain analysis also furnishes an alternative derivation of (52).

The starting point is now (26) in place of (23). Like $K_G$, $\Delta_K$ is now restricted by (46). Then the limits of integration may just as well be reduced to $0 < \tau < T$ and (26) becomes

$$\int_0^T [K_M(\tau) * \Phi_F(\tau) - \Phi_S(\tau + \alpha)]\Delta_K(\tau)\, d\tau = 0. \tag{59}$$

The variation function $\Delta_K$ can have any arbitrary values at times within the interval of integration. Therefore, the other factor in the integrand must be zero, over the same interval $0 < \tau < T$. Referring to Fig. 11, we may divide it into two parts, one of which is zero when $\tau > 0$, and the other when $\tau < T$. These may be written as follows:

$$K_M(\tau) * \Phi_F(\tau) - \Phi_S(\tau + \alpha) = K_U(\tau) + K_V(\tau - T), \tag{60}$$

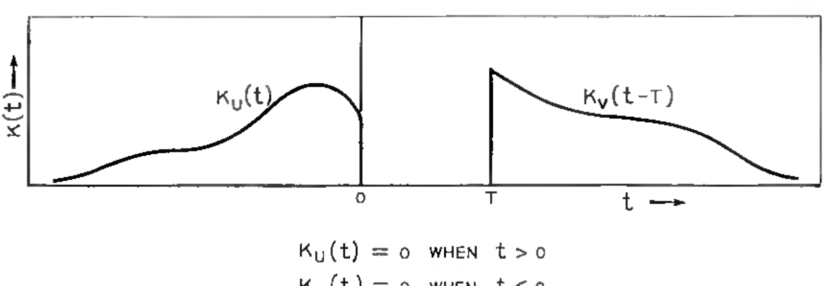



Fig. 11 — The functions $K_u$ and $K_v$.

in which

$$K_U(\tau) = 0 \quad \text{when} \quad \tau > 0,$$

$$K_V(\tau) = 0 \quad \text{when} \quad \tau < 0.$$

Take the transform of both sides of (60), and recall that the transforms of $K_M$, $\Phi_F$, $\Phi_S$ are $Y_M$, $N + S$, $S$. Then

$$\begin{gathered} Y_M(N + S) - e^{\alpha p}S = U + Ve^{-Tp}, \\ U \text{ is rlhp}, \\ V \text{ is rrhp}. \end{gathered} \tag{61}$$

Compare this with (35*b*) of Section 3.2.

In Section 3.2.3, we divided (35b) by $\tilde{Y}_F$ to obtain (44) and a simple physical interpretation. What happens if (61) is divided by $\tilde{Y}_F$? The result is as follows:

$$\begin{gathered} Y_M Y_F - \frac{S}{\tilde{Y}_F} e^{\alpha p} = \frac{U}{\tilde{Y}_F} + \frac{V}{\tilde{Y}_F} e^{-Tp}, \\ U/\tilde{Y}_F \text{ is rlhp}, \\ V/\tilde{Y}_F \text{ is not rrhp}. \end{gathered} \tag{62}$$

Because $V/\tilde{Y}_F$ is not rrhp, the inverse transform of the left-hand side is *not* zero in the interval $0 < \tau < T$ (nor in any other significant interval). Furthermore, while a $Y_M Y_F$ which is rrhp still implies that $Y_M$ is rrhp, it is not simple to solve for $Y_M Y_F$ in such a way that $\tilde{Y}_M e^{-Tp}$ is also rrhp. Thus division by $\tilde{Y}_F$ is no longer useful.

To obtain an alternative derivation of (52), use the $Y_M$ of (50) in (61). Then

$$\begin{gathered} (A + Be^{-Tp})(N + S) - e^{\alpha p}S = U + Ve^{-Tp}, \\ U \text{ is rlhp}, \\ V \text{ is rrhp}. \end{gathered} \tag{63}$$

Equate, separately, the terms which do and do not involve $e^{-Tp}$, and cancel the rrhp factor $e^{-Tp}$ from those that do. Then

$$\begin{aligned} A(N + S) - e^{\alpha p}S &= U, \text{ which is rlhp}, \\ B(N + S) &= V, \text{ which is rrhp}. \end{aligned} \tag{64}$$

These equations correspond exactly to conditions *c* and *d* of (52).

3.3.4 *A Somewhat More General Method*

In some problems (59) must be replaced by a slightly more general expression. In particular, $\Phi_S(\tau + \alpha)$, the last term in the square brackets, sometimes must be replaced by a more general function of $\tau$, say $K_c(\tau)$. This is characteristic of problems involving linear constraints, which will be discussed further in Section IV. The methods described above may be modified in such a way that the transform of $K_C(\tau)$ need not be rational, provided $\Phi_F$ still corresponds to a rational spectrum.

Suppose $Y_M$ is to be determined from conditions of the following sort (in which $Y_M$, $\Delta_Y$, $F$, $Y_C$ are of course the transforms of $K_M$, $\Delta_K$, $\Phi_F$, $K_C$):

$$\begin{aligned}
&a.\ \int_0^T [K_M(\tau) * \Phi_F(\tau) - K_C(\tau)]\Delta_K(\tau)\, d\tau = 0; \\
&b.\ K_M(t),\ \Delta_K(t) = 0 \text{ except when } 0 < t < T \\
&\quad [\text{then } Y_M,\ \Delta_Y \text{ belong to the classes (47), (48)}]; \\
&c.\ |\, Y_M\,|^2 F = O\omega^{-2}, \text{ when } \omega \text{ is real and} \to \infty; \qquad (65) \\
&d.\ \Phi_F(t) = \text{an auto-covariance function}, \\
&\quad F = Y_F(p)\tilde{Y}_F(p) = \text{a rational function}; \\
&e.\ K_C(t) = \text{a given time function, } Y_C(p) \text{ need not be rational.}
\end{aligned}$$

Let a new time function $\hat{K}_C(t)$ with transform $\hat{Y}_C(p)$ be defined as follows:

$$\begin{aligned}
\hat{K}_C &= K_C(t) \quad \text{when} \quad 0 < t < T, \\
&= 0 \quad \text{when} \quad t < 0 \text{ or } > T.
\end{aligned} \qquad (66)$$

Then, exactly as with $Y_M$, we must have

$$\hat{Y}_C(p) \quad \text{and} \quad \tilde{\hat{Y}}_C e^{-Tp} \text{ are rrhp and rfpp.} \qquad (67)$$

In these terms (65) implies the following, replacing our previous (60):

$$\begin{aligned}
K_M(\tau) * \Phi_F(\tau) - \hat{K}_C(\tau) &= \hat{K}_U(\tau) + \hat{K}_V(t - \tau), \\
\hat{K}_U(\tau) &= 0 \quad \text{when} \quad \tau > 0, \qquad (68) \\
\hat{K}_V(\tau) &= 0 \quad \text{when} \quad \tau < 0.
\end{aligned}$$

The functions $\hat{K}_U$ and $\hat{K}_V$ are like $K_U$ and $K_V$ of (60), except that they include the "tails" of $K_C$ lying outside the interval $0 < \tau < T$.

Now take the transforms of all functions in (68), and solve for $Y_M(p)$. If $\hat{U}$ and $\hat{V}$ are the transforms of $\hat{K}_U$ and $\hat{K}_V$, the result is

$$Y_M = \frac{\hat{Y}_C + \hat{U} + \hat{V}e^{-Tp}}{F},$$

$$\begin{aligned} &a. \ \hat{U} \text{ is rlhp}, \\ &b. \ \hat{V} \text{ is rrhp}, \\ &c. \ |Y_M|^2F = O\omega^{-2} \text{ when } \omega \text{ is real and} \to \infty. \end{aligned} \tag{69}$$

The functions $\hat{U}$ and $\hat{V}$ must be such that $Y_M$ is rfpp. But $\hat{Y}_C$ itself is rfpp. As a result,

$$\begin{aligned} &a. \text{ The finite poles of } \hat{U} = \text{the rhp poles of } F; \\ &b. \text{ The finite poles of } \hat{V} = \text{the lhp poles of } F; \\ &c. \text{ If } p = p_k \text{ at a finite zero of } F, \\ &\qquad \hat{Y}_C(p_k) + \hat{U}(p_k) + \hat{V}(p_k)e^{-Tp_k} = 0; \\ &d. \text{ When } \omega \text{ is real and} \to \infty, \hat{U} \text{ and } \hat{V} \text{ must behave} \\ &\qquad \text{in such a way that } |Y_M|^2F = O\omega^{-2}. \end{aligned} \tag{70}$$

These conditions determine a unique rational $\hat{U}$ and $\hat{V}$ when $F$ is rational, provided $K_C(t)$ satisfies certain requirements relating to continuity. If $F \to C\omega^{-2m}$ as $\omega \to \infty$, then $K_C(t)$ and its first $m - 1$ derivatives must be continuous in the interval $0 < t < T$. When the continuity condition is violated, there will be no $Y_M$ which meets all the conditions (65), unless (65*c*) is modified. (See Section 4.3 for an interpretation in connection with a particular application.)

The transform $Y_C(p)$ of $K_C(t)$ need not be rational. Furthermore, when $K_C(t)$ is given the time function can be calculated without actually evaluating $Y_C(p)$, except at the special points $p = p_k$. In particular, the following two relations may be used in evaluating the inverse transform of the right hand side of (69), with help of (70):

$$a. \ \hat{Y}_C(p_k) = \frac{1}{\sqrt{2\pi}} \int_0^T K_C(\tau)e^{-p_k\tau}\, d\tau;$$

$$b. \ \text{Inverse transform of } \frac{\hat{Y}_C}{F} = \frac{1}{\sqrt{2\pi}} \int_0^T K_C(\tau)K_{(1/F)}(t-\tau)\, d\tau, \tag{71}$$

where $K_{(1/F)}(t)$ = inverse transform of $1/F$.

### 3.4 *Simultaneous Optimization of Two Network Functions*

In each of our problems so far we have been required to find a single frequency function, $Y_M(p)$, which is to represent the optimum choice of a single linear operator, $Y_G(p)$. There are other problems, however, in which two or more frequency functions are to be found, corresponding to the simultaneously optimum choices of two or more different but related linear operators. This section develops general methods in terms of one such problem.

Suppose the same signal can be observed in two different ways, or at two different places, involving contamination with noise from two different (uncorrelated) sources. An important example which will be discussed later (but not the only example) is the observation of a single physical variable with instruments of two different kinds. There are now two observed time functions, $f_1(t)$ and $f_2(t)$, related to $s(t)$ by

$$\begin{aligned} f_1(t) &= s(t) + n_1(t), \\ f_2(t) &= s(t) + n_2(t). \end{aligned} \tag{72}$$

The two different time functions are to be modified by (different) linear operations, and the results are to be added, to obtain an optimum estimate of $s(t + \alpha)$. All the assumptions of Section 2.2 are to be retained. Thus, Gaussian ensembles may be substituted for $s(t)$, $n_1(t)$ and $n_2(t)$, and Fig. 2 may be replaced by Fig. 12.

From Fig. 12, the following integral is easily obtained, in place of (19):

$$\sigma^2 = \int_{-\infty}^{+\infty} (|Y_{G1}|^2 N_1 + |Y_{G2}|^2 N_2 + |Y_{G1} + Y_{G2} - e^{\alpha i\omega}|^2 S)\, d\omega. \tag{73}$$

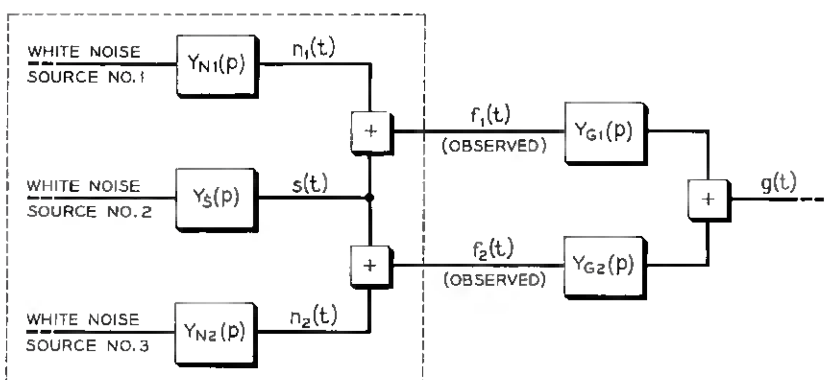



Fig. 12 — A physical model for a signal which is observed in two different ways.

Here $N_1$ and $N_2$ are, of course, the power spectra of the two noise ensembles (assumed to be uncorrelated). Let $Y_{M1}$ and $Y_{M2}$ be the optimum choices of $Y_{G1}$ and $Y_{G2}$, and define $\Delta_{Y1}$ and $\Delta_{Y2}$ by

$$\begin{aligned} Y_{G1} &= Y_{M1} + \Delta_{Y1}, \\ Y_{G2} &= Y_{M2} + \Delta_{Y2}. \end{aligned} \tag{74}$$

Then (73) may be replaced by

$$\begin{aligned} \sigma^2 = &\int_{-\infty}^{+\infty} (| Y_{M1} |^2 N_1 + | Y_{M2} |^2 N_2 + | Y_{M1} + Y_{M2} - e^{\alpha p} |^2 S) \, d\omega \\ &+ \int_{-\infty}^{+\infty} (| \Delta_{Y1} |^2 N_1 + | \Delta_{Y2} |^2 N_2 + | \Delta_{Y1} + \Delta_{Y2} |^2 S) \, d\omega \\ &+ 2 \int_{-\infty}^{+\infty} \{[Y_{M1}(N_1 + S) + Y_{M2}S - e^{\alpha p}S]\tilde{\Delta}_{Y1} \\ &\qquad + [Y_{M1}S + Y_{M2}(N_2 + S) - e^{\alpha p}S]\tilde{\Delta}_{Y2}\} \, d\omega. \end{aligned} \tag{75}$$

The first integral in (75) is $\sigma_M^2$, corresponding to a $\Delta_{Y1}$ and $\Delta_{Y2}$ which are zero identically. The second integral is positive for any $\Delta_{Y1}$ and $\Delta_{Y2}$ which are not zero identically. Then, if $\Delta_{Y1}$ and $\Delta_{Y2}$ each obeys (29), the last integral must be zero. This replaces (23) by the following:

For every permitted $\Delta_{Y1}$ and $\Delta_{Y2}$:

$$\begin{aligned} \int_{-i\infty}^{+i\infty} &\{[Y_{M1}(N_1 + S) + Y_{M2}S - e^{\alpha p}S]\tilde{\Delta}_{Y1} \\ &+ [Y_{M1}S + Y_{M2}(N_2 + S) - e^{\alpha p}S]\tilde{\Delta}_{Y2}\} \, dp = 0. \end{aligned} \tag{76}$$

The one integral can often be split into two, each involving one $\tilde{\Delta}_Y$, and each equal to zero. The separation may not be permissible however, when $N_1/S = O\omega^{-2}$ at $\omega = \infty$. An examination of convergence rules, noted below, will clarify this statement. Convergence conditions may again be obtained, by excluding at once, all $Y_M$'s and $\Delta_Y$'s for which $\sigma^2 = \infty$. Conditions are easily derived from the first two integrals in (75). These may be combined to obtain a condition on the integrand in (76). The result is

When $\omega$ is real and $\to \infty$, the following seven functions $= O\omega^{-2}$:

$$\begin{array}{lll} | Y_{M1} |^2 N_1, & | Y_{M2} |^2 N_2, & | Y_{M1} + Y_{M2} - e^{\alpha p} |^2 S, \\ | \Delta_{Y1} |^2 N_1, & | \Delta_{Y2} |^2 N_2, & | \Delta_{Y1} + \Delta_{Y2} |^2 S, \end{array} \tag{77}$$

and the integrand in (76).

Note that $|\Delta_{Y1}|^2 S$ and $|\Delta_{Y2}|^2 S$ do *not* necessarily $= O\omega^{-2}$, provided they are restricted by a linear relation at $\omega \to \infty$.*

3.4.1 *Optimum Physical Networks When $\alpha \geqq 0$*

Suppose $Y_{M1}$ and $Y_{M2}$ are to be selected from the class of all "physical" frequency functions, that is, from the $C_Y$ of (33). Then $\Delta_{Y1}$ and $\Delta_{Y2}$ belong to the $C_\Delta$ of (34). Following our usual procedure, we fit the integrand of (76) to $H(p)$ of (12). Behavior at $\infty$ is already proper, and $\tilde{\Delta}_{Y1}$ and $\tilde{\Delta}_{Y2}$ are both rlhp. Then we have only to make the two associated factors also rlhp. Assembling these requirements with conditions of physicalness and convergence gives:

$$\begin{aligned} &a.\ Y_{M1} && \text{is rrhp,} \\ &b.\ Y_{M2} && \text{is rrhp,} \\ &c.\ Y_{M1}(N_1 + S) + Y_{M2}S - e^{\alpha p}S && \text{is rlhp,} \\ &d.\ Y_{M1}S + Y_{M2}(N_2 + S) - e^{\alpha p}S && \text{is rlhp,} \end{aligned} \tag{78}$$

*e.* When $\omega$ is real and $\to \infty$, the following three functions $= O\omega^{-2}$:

$$|Y_{M1}|^2 N_1, \qquad |Y_{M2}|^2 N_2, \qquad |Y_{M1} + Y_{M2} - e^{\alpha p}|^2 S.$$

When $\alpha \geqq 0$, $Y_{M1}$ and $Y_{M2}$ are rational (assuming rational $N_1$, $N_2$, $S$). It is not at once apparent, however, what the poles of $Y_{M1}$ and $Y_{M2}$ will be. From (78c) and (78d), we can define two rlhp functions, say $\tilde{U}_1$ and $\tilde{U}_2$, by:

$$\begin{aligned} Y_{M1}(N_1 + S) + Y_{M2}S - e^{\alpha p}S &= \tilde{U}_1, \\ Y_{M1}S + Y_{M2}(N_2 + S) - e^{\alpha p}S &= \tilde{U}_2. \end{aligned} \tag{79}$$

Solving for $Y_{M1}$ and $Y_{M2}$ gives:

$$\begin{aligned} Y_{M1} &= \frac{\tilde{U}_1 N_2 + (\tilde{U}_1 - \tilde{U}_2)S + e^{\alpha p}SN_2}{N_1N_2 + (N_1 + N_2)S}, \\ Y_{M2} &= \frac{\tilde{U}_2 N_1 - (\tilde{U}_1 - \tilde{U}_2)S + e^{\alpha p}SN_1}{N_1N_2 + (N_1 + N_2)S}. \end{aligned} \tag{80}$$

Since $Y_{M1}$ and $Y_{M2}$ are to be rrhp, while $\tilde{U}_1$ and $\tilde{U}_2$ are to be rlhp, the

* For example, suppose $N_1$, $N_2 = O\omega^{-4}$ as $\omega \to \infty$, and $S = O\omega^{-2}$. Then $|\Delta_{Y1}|^2$ and $|\Delta_{Y2}|^2$ may $= O\omega^{+2}$, provided $\Delta_{Y1}$ and $\Delta_{Y2}$ are so related that $|\Delta_{Y1} + \Delta_{Y2}|^2 = OC$. But this $\Delta_{Y1}$ and $\Delta_{Y2}$ cannot be chosen entirely independently, and then the integral in (76) must not be split in two.

two pairs of functions can have no finite poles in common. Then every finite pole of $Y_{M1}$ or $Y_{M2}$ in (78) must belong to one or the other of the following classes:

*a*. The lhp zeros of $N_1N_2 + (N_1 + N_2)S$;

*b*. For $Y_{M1}$, any lhp poles of $N_2$ or $S$ which are not poles of $N_1N_2 + (N_1 + N_2)S$; for $Y_{M2}$, any lhp poles of $N_1$ or $S$ which are not poles of $N_1N_2 + (N_1 + N_2)S$. (81)

The second class occurs only in special or degenerate cases, such as when zeros of $N_1 + N_2$ happen to coincide with poles of $S$.

After the permissible poles have been determined by (81), (78) may be used to find the numerators of $Y_{M1}$ and $Y_{M2}$. Simultaneous linear equations in the numerator coefficients may be derived from required behavior at lhp poles of $S$, $N_1$, $N_2$ and also at poles of $Y_{M1}$ and $Y_{M2}$ themselves. For the latter, take the difference of (78c) and (78d) to get:

$$(Y_{M1}N_1 - Y_{M2}N_2) \text{ is rlhp.} \tag{82}$$

Numerators of the minimum degree are determined uniquely, and also turn out to be just compatible with (78*e*). In special or degenerate cases, certain of the poles, permitted by (81), may coincide with zeros of the corresponding numerators of $Y_{M1}$ and $Y_{M2}$ and can be cancelled out.

### 3.4.2 *Optimum Physical Networks When $\alpha < 0$*

When $\alpha = -\beta$, with $\beta > 0$, Sections 3.2.2 and 3.4.1 may be combined to get

$$\begin{aligned} Y_{M1} &= A_1 + \frac{SN_2e^{-\beta p}}{N_1N_2 + (N_1 + N_2)S}, \\ Y_{M2} &= A_2 + \frac{SN_1e^{-\beta p}}{N_1N_2 + (N_1 + N_2)S}. \end{aligned} \tag{83}$$

Here, $A_1$ and $A_2$ are two different rational functions, not necessarily rrhp (even though adding the exponential terms must make $Y_{M1}$, $Y_{M2}$ rrhp). Substituting in (78) and using a generalization of (32) gives*

* Paralleling Section 3.1 gives, in place of (32):

$$\frac{N_1N_2S}{N_1N_2 + (N_1 + N_2)S} = O\omega^{-2} \text{ as } \omega \to \infty$$

$$
\begin{aligned}
&a.\ A_1 + \frac{SN_2e^{-\beta p}}{N_1N_2 + (N_1 + N_2)S} \text{ is rrhp,} \\
&b.\ A_2 + \frac{SN_1e^{-\beta p}}{N_1N_2 + (N_1 + N_2)S} \text{ is rrhp,} \\
&c.\ A_1(N_1 + S) + A_2S \qquad \text{is rlhp,} \\
&d.\ A_1S + A_2(N_2 + S) \qquad \text{is rlhp,}
\end{aligned} \tag{84}
$$

*e*. When $\omega$ is real and $\rightarrow \infty$, the following three functions $= O\omega^{-2}$:

$$
|A_1|^2N_1, \qquad |A_2|^2N_2, \qquad |A_1 + A_2|^2S.
$$

In (84), conditions *a* and *b* put the rhp poles of $A_1$ and $A_2$ at the rhp poles of the functions which multiply $e^{-\beta p}$. Conditions *c* and *d* permit the same lhp poles as the poles of $Y_{M1}$ and $Y_{M2}$ for $\alpha > 0$, which are determined by (81). Combining the lhp and rhp conditions shows that every finite pole of $A_1$ or $A_2$ must belong to one or the other of the following classes:

*a*. The zeros of $N_1N_2 + (N_1 + N_2)S$;

*b*. For $Y_{M1}$, any poles of $N_2$ or $S$ which are not poles of $N_1N_2 + (N_1 + N_2)S$; for $Y_{M2}$, any poles of $N_1$ or $S$ which are not poles of $N_1N_2 + (N_1 + N_2)S$. (85)

These are merely the extensions of the classes (81), to include poles in both half planes.

The poles of $A_1$ and $A_2$ are twice as numerous as the poles of $Y_{M1}$ and $Y_{M2}$ for $\alpha > 0$. As a result there are more numerator coefficients to be determined, but they are still determined by simultaneous *linear* equations.

### 3.4.3 *A Time-Domain Interpretation*

Paralleling Section 3.2.3 gives a time-domain interpretation. Although it does not represent a significant simplification of the present problem, it does have theoretical interest, and also a potential usefulness in variations of the present problem.

In Section 3.2.3, we multiplied the rlhp function in (35*b*) by the rlhp function $1/\tilde{Y}_s$, so as to obtain the rlhp function (44), in which the rrhp term $Y_MY_F$ may be regarded as the only unknown. We now multiply the two rlhp functions in (78) by two different rlhp functions and add the result. The object is to obtain an rlhp function with a single unknown term which is itself rrhp.

Let $Q_1$ and $Q_2$ be two (as yet unknown) functions of $p$, both required to be rrhp. Then $\tilde{Q}_1$ and $\tilde{Q}_2$ are both rlhp. If we multiply the two rlhp functions in (78$c$) and (78$d$) by $\tilde{Q}_1$ and $\tilde{Q}_2$ and add the result the sum will have to be rlhp. The sum can be written as follows:

$$\begin{aligned} (Y_{M1}Y_a + Y_{M2}Y_b) - e^{\alpha p}S(\tilde{Q}_1 + \tilde{Q}_2) &\text{ is rlhp,} \\ Y_a = \tilde{Q}_1(N_1 + S) + \tilde{Q}_2 S&, \\ Y_b = \tilde{Q}_1 S + \tilde{Q}_2(N_2 + S)&. \end{aligned} \tag{86}$$

If $Y_a$ and $Y_b$ are both rrhp, as well as $Y_{M1}$ and $Y_{M2}$, the function $(Y_{M1}Y_a + Y_{M2}Y_b)$ will be rrhp and can be found from (86) by the Bode-Shannon method. This function can be used to express (82) in terms of $Y_{M1}$ or $Y_{M2}$ alone. Thus, $Y_{M1}$ and $Y_{M2}$ may be found by a sequence of straightforward calculations, as soon as $\tilde{Q}_1$ and $\tilde{Q}_2$ are known. The problem is to find a $\tilde{Q}_1$ and a $\tilde{Q}_2$ such that $Y_a$ and $Y_b$ are, in fact, rrhp. It may be more informative to use the equivalent requirement that $\tilde{Y}_a$ and $\tilde{Y}_b$ must be rlhp. Then (since $N_1$, $N_2$, $S$ are even), (86) requires

$$\begin{aligned} &a.\ \tilde{Y}_a = Q_1(N_1 + S) + Q_2 S \text{ is rlhp,} \\ &b.\ \tilde{Y}_b = Q_1 S + Q_2(N_2 + S) \text{ is rlhp,} \\ &c.\ Q_1, Q_2 \qquad\qquad\qquad\qquad \text{are rrhp.} \end{aligned} \tag{87}$$

Equations (87) are about as hard to solve for $Q_1$ and $Q_2$ as (78) is for $Y_{M1}$ and $Y_{M2}$. The poles of $Q_1$ and $Q_2$ are exactly the poles of $Y_{M1}$ and $Y_{M2}$. The numerators are different and are not uniquely determined.

Note that the calculation of $Q_1$ and $Q_2$ does not involve the terms $e^{\alpha p}S$ of (78). These enter only in the Bode-Shannon type of analysis. The method has potential usefulness in variations of the present problem, in which the terms $e^{\alpha p}S$ are replaced by more complicated functions. (Compare this section with Section 3.3.4).

### 3.5 *Sampled-Data Systems*

This section shows how the methods which we applied in the previous sections to continuous-data systems may be modified for application to discrete, or sampled-data systems. Methods appropriate for discrete systems can, of course, be derived without reference to continuous-data systems; and they have been so derived by, for example, Levinson (Ref.

1, Appendix B), and Lloyd and McMillan.[10] It is interesting to observe, however, how simply the transformation may be accomplished, from techniques for continuous-data systems to techniques for discrete-data systems. For this purpose, it will be sufficient to outline an appropriate procedure, without filling in details.

Two different kinds of discrete-data systems may be considered. The signal and noise may be discrete time series, described by statistics appropriate for such series. On the other hand, the signal, $s(t)$, and the noise, $n(t)$, may themselves be continuous time series, with the observations of signal-plus-noise, $f(t)$, limited to discrete "sampling" times. Methods appropriate for both kinds of discrete-data systems may be derived from a study of continuous-data systems of a special kind.

### 3.5.1 *A Special Kind of Continuous-Data System*

Following the usual methods of "$z$ transform" theory, let $z = e^{-Tp}$, in which $T$ is to be the sampling interval. Rational functions of $z$, when treated as functions $p$, now have inverse transforms which are zero except at the discrete times $\sigma T$. The methods which we have applied to continuous-data systems do not apply directly to spectra, $S$ and $N$, which are simply rational functions of $z$. The behavior at $p = \infty$ leads to divergent integrals such as, for example, the integral in (31). On the other hand, spectra of the sort described below meet the convergence conditions and also lead to $Y_M(p)$ which are exactly rational functions of $z$. Then the only data which are actually utilized are the data observed at the discrete times $t - \sigma T$.

Note that $\tilde{z} = 1/z$, and that $|z| > 1$ in the left half of the $p$ plane. Use the notation $Y(p) = Y_z(z)$ and $\tilde{Y}_z(z) = Y_z(1/z)$. In these terms, $N$ and $S$ are to be

$$a.\ S = \frac{q}{1 + \epsilon^2\omega^2} Y_S \tilde{Y}_S ,$$

$$b.\ N = \frac{q}{1 + \epsilon^2\omega^2} Y_N \tilde{Y}_N , \tag{88}$$

$c.$ $Y_{Sz}(z)$, $Y_{Nz}(z)$ are rational in $z$,

$d.$ $Y_{Sz}$, $Y_{Nz}$, $1/Y_{Sz}$, $1/Y_{Nz}$ are regular at $|z| < 1$.

The constants $q$ and $\epsilon$ are to be real and positive.

Suppose $\alpha > 0$, in $s(t + \alpha)$, and $Y_M$ is to be the optimum "physical" $Y_G$, when $N$ and $S$ are described by (88). The procedure explained in Section 3.2 is easily adapted to the new problem. Corresponding to

(35), one gets:

$$\begin{aligned}
&a.\ Y_M(p) \qquad \text{is rrhp,}\\
&b.\ \frac{q}{1-\epsilon^2 p^2} Y_\Sigma(p) \text{ is rlhp}\\
&\quad Y_{\Sigma z} = Y_{Mz}(Y_{Sz}\tilde{Y}_{Sz} + Y_{Nz}\tilde{Y}_{Nz}) - \frac{Y_{Sz}\tilde{Y}_{Sz}}{z^{\alpha/T}},\\
&c.\ |Y_{\Sigma z}| < \infty \text{ at } z = \infty.
\end{aligned} \tag{89}$$

These conditions are satisfied by a $Y_M(p)$ such that $Y_{Mz}(z)$ is rational and meets the following conditions:

$$\begin{aligned}
&a.\ Y_{Mz}(z) \text{ is regular} && \text{when } |z| < 1,\\
&b.\ Y_{\Sigma z}(z) \text{ is regular} && \text{when } |z| > 1,\\
&c.\ Y_{\Sigma z}(z) = 0 && \text{when } 1 + \epsilon p = 0,\\
&d.\ 0 < |Y_{\Sigma z}(z)| < \infty && \text{at } z = \infty.
\end{aligned} \tag{90}$$

The factor $q/1 + \epsilon^2\omega^2$ was applied to $N$ and $S$ in (88) to obtain convergence of integrals in the derivation of (89) and (90). However, we can make $\epsilon$ as small as we wish, and $Y_M$ approaches a reasonable limit as $\epsilon \to 0$. The poles of $Y_{Mz}$ are completely independent of $\epsilon$. The numerator approaches a limit such that condition (90*c*) disappears, and (90*d*) changes to

$$Y_{\Sigma z}(z) = 0 \quad \text{at} \quad z = \infty. \tag{91}$$

This follows from the fact that (90*c*) requires $Y_{\Sigma z}$ to have a numerator factor in $z$ which is zero at the zero of $1 + \epsilon p$ — namely, a factor

$$1 - e^{-(T/\epsilon)}z = 1 - e^{-(T/\epsilon)(1+\epsilon p)}. \tag{92}$$

When $\epsilon$ is small, but not zero, the autocovariances corresponding to the $N$ and $S$ of (88) take the form shown in Fig. 13. The "spikes" become sharper as $\epsilon$ becomes smaller. As $\epsilon \to 0$, the integrals of the $N$ and $S$ of (88) become infinite unless $q$ also $\to 0$ to order of $\epsilon$. The function $Y_M$, however, is entirely independent of $q$, which need be considered quantitatively only in calculating the corresponding variance, $\sigma_M^2$.

### 3.5.2 *Discrete Time Series*

Suppose $S$ and $N$ are the spectra of continuous time series $s(t)$ and $n(t)$, such that the covariances are exactly zero except when the lag

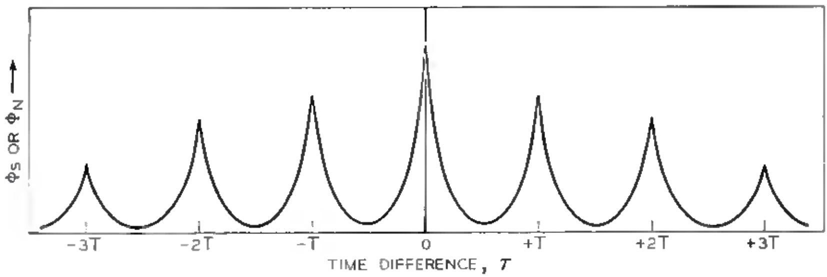



**Fig. 13 — A special form of auto-covariance function.**

time is $\sigma T$, with $\sigma$ an integer. Let the prediction time $\alpha$ also be an integral multiple of $T$. The corresponding $Y_M$ will be such that the estimate $g(t)$, of $s(t + \alpha)$, will depend only on the values assumed by $s(t)$, and $n(t)$ at the discrete series of times $t - \sigma T$. Since the values of $s(t)$ and $n(t)$ which occur at intermediate times are not utilized, and are not correlated to the values which are utilized, they can have no significant relation to the problem; they may be regarded as undefined without altering the solution. Thus the special continuous-data system yields the same $Y_M$ as though $s(t)$ and $n(t)$ were discrete, ordered sequences of random variables matching the values which are assumed by the continuous time series at the discrete times $\sigma T$.

Conversely, when $s$ and $n$ are initially discrete series, corresponding continuous ensembles may be constructed for calculating the optimum smoothing and prediction. When the covariances of the discrete series are sums of exponentials in $m_2 - m_1$, where $m_1$ and $m_2$ are the order numbers of the samples involved, the corresponding $S$ and $N$ will be rational functions of $z = e^{-Tp}$, and $Y_M$ and $K_M$ may be found by applying Section 3.5.1. Many details, of course, remain to be filled in.

### 3.5.3 *Discrete Samples of Continuous Time Series*

Now suppose that $s(t)$ and $n(t)$ are continuous time series with rational spectra $S$ and $N$, but that $f(t) = s(t) + n(t)$ may be observed only at discrete "sampling times", $t = \sigma T$. Let "present time" $t$ coincide with one of the sampling times and consider the calculation of the optimum $Y_M$ for estimating $s(t + \alpha)$, with $\alpha > 0$.

In Section 3.2.4, we replaced the Gaussian statistical model shown in Fig. 6 by that shown in Fig. 8, which has the same pertinent statistical characteristics. Now, the pertinent statistics are even more restricted, and a further change in the Gaussian model is useful. In particular, the frequency functions $Y_F$, $Y_{FS}$ and the single random quantity $v_s$ may be modified in any way which leaves the following covariances unchanged (in which $\sigma$ is a nonnegative integer): the auto-covariance of $f(t)$ at the discrete lag times $\sigma T$, the cross-covariance of $f(t)$ and $s(t)$ at the discrete lag times $(\alpha + \sigma T)$ and the auto-covariance of $s(t)$ at the lag time zero.

Our present problem may be solved by changing $Y_F$, $Y_{FS}$, $N_S$ (without changing the pertinent statistics), in such a way that the optimum $Y_M(p)$, computed on a continuous-data basis, will be the transform of a $K_M(t)$ which is zero except when $t = \sigma T$. The output of a corresponding device, at any sampling time, will depend only on the values of the input at sampling times. A suitable mechanization for the sampled-data system, derived from this $Y_M$ or $K_M$, may be either a fixed network (or equivalent device) with sampling at input or output, or digital computations carried out once each computing cycle.

When $N$ and $S$ are rational functions of frequency, $Y_F$ and $Y_{FS}$ may be changed to rational functions of $z$, and Section 3.5.1 may be applied to find $Y_M$ as a rational function of $z$. When $\alpha = mT$, with $m$ an integer, it is sufficient to change $N$ and $S$ to suitable rational functions of $z$, preserving auto-covariances of $s(t)$ and $n(t)$ at lag times $\sigma T$, and then to apply Section 3.5.1. When $\alpha \neq mT$, however, $Y_{FS}$ must be modified further (or the related function $Se^{\alpha p}$). As in the previous section, many details remain to be filled in.

### 3.6 *Nonstationary Systems*

In this section, we consider signal and noise ensembles which are statistically nonstationary. Some years ago, Booton[21] described integral equations which determine the corresponding linear least-squares smoothing and prediction operators. The general integral equations, however, are very much more difficult to solve than are the equations corresponding to stationary systems. On the other hand, the techniques described above, for stationary systems, may be paralleled, in principle at least, for analogous nonstationary systems. More specifically, a one-to-one correspondence may be established between the individual relationships and operations used in the techniques and a new set of relationships and operations which are appropriate for at least a large class of nonstationary systems.

While it is clear that the new relationships and operations are appropriate for many nonstationary systems, the exact range of their applicability has not been fully established. In addition, the manipulations required in numerical problems are very much more complicated than the stationary counterparts and they may be feasible only for quite simple systems, even when electronic computers are available. Accordingly, a very brief outline may be sufficient here, leaving a fuller exposition for a later paper.

#### 3.6.1 *A Class of Nonstationary Systems*

In previous sections, we considered finite networks of lumped elements, driven by white noise, as physical models which generate stationary Gaussian ensembles with rational spectra (Figs. 2, 3, 6 of Section II and Fig. 8 of Section 3.2.4). We now change the picture only by permitting the network components to be time variable; we thereby define a class of nonstationary Gaussian ensembles, analogous to the stationary ensembles which have rational spectra. Miller and Zadeh[22] have studied essentially the same class of nonstationary ensembles, in somewhat different terms.

Let $V$ be the input and $E$ the output of a finite network of lumped, linear, time-variable elements. Then

$$PV = QE,$$

$$P = a_0 + a_1 \frac{d}{dt} + \cdots + \frac{d^n}{dt^n}, \tag{93}$$

$$Q = H\left(b_0 + b_1 \frac{d}{dt} + \cdots + \frac{d^m}{dt^m}\right).$$

The coefficients $a_\sigma$, $b_\sigma$, $H$ are now generally time-variable.

Consider the functions $U_\sigma$, defined by $PU_\sigma = 0$, and the functions $L_\sigma$, defined by $QL_\sigma = 0$. There will always exist $n$ linearly independent $U_\sigma$'s and $m$ linearly independent $L_\sigma$'s. The $U_\sigma$'s may be referred to as the basis functions, or bf's, and the $L_\sigma$'s as the zero-response functions, or zrf's. In the theory of nonstationary systems, the bf's correspond to the poles of the admittances of stationary systems, and the zrf's correspond to the zeros. In general, the bf's and the zrf's may be chosen in any of various ways, since linear transformations of legitimate choices are also legitimate choices. Frequently, however, a specific choice is particularly useful, as in the situation described below.

Existence theorems are more easily established if the coefficients $a_\sigma$, $b_\sigma$, $H$ in $P$ and $Q$, when viewed as functions of time $t$, are required to be analytic at $t = -\infty$. Then, except in degenerate cases which need not be considered here, the bf's, $U_\sigma$, and the zrf's, $L_\sigma$, may be so chosen that they behave as follows, in accordance with Bellman:[23]

$$\begin{aligned} &a.\ \text{When } a_\sigma,\ b_\sigma,\ H \text{ are analytic at } t \to -\infty, \\ &b.\ \text{Then } U_\sigma \to t^{\gamma_\sigma'} e^{p_\sigma' t} \text{ and } L_\sigma \to t^{\gamma_\sigma''} e^{p_\sigma'' t}. \end{aligned} \tag{94}$$

The coefficients $\gamma_\sigma$ and $p_\sigma$ are *constants*. The $p_\sigma$'s may be described further in terms of $P$ and $Q$ of (93). Let $P_\infty(p)$ and $Q_\infty(p)$ be the polynomials derived from $P$ and $Q$ in the following way: Replace $d/dt$ by $p$. For coefficients, use the values assumed by the $a_\sigma$'s and $b_\sigma$'s at $t = -\infty$. Then the $p_\sigma'$'s are the zeros of $P_\infty(p)$ and the $p_\sigma''$'s are the zeros of $Q_\infty(p)$. The same sort of conditions for "physical" networks may now be applied to the $p_\sigma'$'s and $p_\sigma''$'s as to the poles and zeros of stationary admittances. (The coefficients must also behave "reasonably" in some sense, although not necessarily analytically, at times other than $t \to -\infty$.)

When the $p_\sigma'$'s have negative real parts, as required for "physical" networks, the integration of (93) gives (for $m < n$):

$$\begin{aligned} &a.\ V(t) = \int_{-\infty}^{+\infty} K(t, \tau) E(\tau)\, d\tau, \\ &b.\ K(t, \tau) = \sum_{\sigma=1}^{n} U_\sigma(t) X_\sigma(\tau) \quad \text{when} \quad \tau < t, \\ &\qquad\qquad = 0 \qquad\qquad\qquad\quad \text{when } \tau > t. \end{aligned} \tag{95}$$

Here, $K$ is the impulse response, as in previous sections, but it is no longer a function of the single variable $t - \tau$. The $U_\sigma$'s are again the bf's of the differential equation (93) and the $X_\sigma$'s are new functions. Their calculation is at least straightforward, provided the $U_\sigma$'s are known, as well as the coefficients of $Q$ in the differential equation. The $X_\sigma$'s are roughly analogous to the residues at the poles of the transfer admittance of a stationary network. (The functions $X_\sigma/U_\sigma$ are a closer analog.)

### 3.6.2 *Manipulations of Differential Equations*

When a time-variable network is made up of networks in tandem or networks in parallel the differential equation for the complete network may be found from the differential equations which correspond to the different parts. The processes are analogous to, but not the same as, the algebraic products and sums which are used to combine the rational admittance functions of stationary partial networks. The differences may be said to stem from the noncommutability of time-variable coefficients and the derivative operator, $d/dt$.

$$\left( C \frac{d}{dt} V \neq \frac{d}{dt} CV, \text{ when } C \text{ is time-variable.} \right)$$

Conversely, differential equations such as (93) may be decomposed into sets of simpler differential equations, corresponding to partial networks connected either in tandem or parallel fashion. For this, however, one needs to know the bf's of the differential equation (and also the zrf's, for tandem circuit decompositions), just as one needs to know the poles (and also the zeros, for tandem circuit decompositions) when decomposing the admittances of stationary circuits. The operations are of course analogous to, but not the same as, the factoring and partial fraction expansion of admittance functions.

Manipulations of the sort noted above are described in more detail in a previous paper by the author.[24]

### 3.6.3 *Auto-Covariances*

When the input signal $V$ is (unit level) white noise, the auto-covariance, $\Phi$, of the output signal, $E$, may be calculated from the impulse response, $K$. When $K$ is as in (95*b*), $\Phi$ may be expressed as a somewhat similar finite sum. More specifically,

$$\begin{aligned} a.\ \Phi(t_2, t_1) &= \int_{-\infty}^{+\infty} K(t_2, \tau)K(t_1, \tau)\, d\tau, \\ b.\ \Phi(t_2, t_1) &= \sum_{\sigma=1}^{n} U_\sigma(t_2)W_\sigma(t_1) \quad \text{when} \quad t_1 < t_2 \qquad (96) \\ &= \sum_{\sigma=1}^{n} W_\sigma(t_2)U_\sigma(t_1) \quad \text{when} \quad t_1 > t_2 . \end{aligned}$$

The $U_\sigma$'s are again the basis functions or bf's of the differential equation (93). The $W_\sigma$'s are new functions, which may be calculated in any of various ways.

A differential equation, corresponding to $\Phi$, may be defined as the equation connecting two time functions, say $G_1$ and $G_2$, in such a way that

$$G_2(t_2) = \int_{-\infty}^{+\infty} \Phi(t_2, t_1)\, G_1(t_1)\, dt_1 .$$

The differential equation is of order $2n$. Its bf's comprise both the $U_\sigma$'s and $W_\sigma$'s of (96), but its zrf's can be found only by solving a homogeneous differential equation with time-variable coefficients.

In (96*a*), $K(t_1, \tau)$ may be replaced by $K^a(\tau, t_1)$, where $K^a$ is the "adjoint" of $K$, defined as the function obtained from $K(t, \tau)$ by interchanging, within the function, the input time, $\tau$, and the output time, $t$. The integral of the product $K(t_2, \tau)K^a(\tau, t_1)$ is a convolution, representing the impulse response of the tandem combination of two networks. The two networks are the original network preceded by a (nonphysical) network with the adjoint response, as in Fig. 14.

We may now relate $G_1$ and $G_2$ through an intermediate variable, say $G_m$, by means of

$$\begin{aligned} a.\ & P^a G_m = Q^a G_1, \\ b.\ & P G_2 \;\; = Q G_m . \end{aligned} \tag{97}$$

Here, $P$ and $Q$ are as in (93), and $P^a$ and $Q^a$ are similar operators, which lead to the adjoint $K^a(t, \tau)$ of the impulse response $K(t, \tau)$.

When systems are stationary, the transform of $\Phi$ is the product $Y(p)\tilde{Y}(p)$. When systems are nonstationary, the bf's and zrf's of (97*a*) are analogous to the poles and zeros of $\tilde{Y}(p)$, just as the bf's and zrf's of (97*b*) are analogous to the poles and zeros of $Y(p)$. The simple product of $Y$ and $\tilde{Y}$ is replaced by the construction of a single differential equation, from the two equations of (97), through the elimination of the intermediate variable $G_m$. One half of the bf's of $\Phi$ are exactly the bf's of $K$, and one half of the zrf's of $\Phi$ are the zrf's of $K^a$. The other halves of both the bf's and zrf's of $\Phi$ are *not* exactly the bf's of $K^a$ and the zrf's of $K$, except when systems are stationary.

The function $\Phi(t_2, t_1)$ is symmetrical in $t_2$ and $t_1$. As a result, $\Phi$ is its own adjoint. The self-adjoint property of $\Phi(t_2, t_1)$ corresponds to the evenness of the frequency function $Y(p)\tilde{Y}(p)$, which was noted in the analysis of stationary systems.

When (uncorrelated) signal and noise ensembles each have auto-covariances of the general form (96*b*), the auto-covariance of the signal-plus-noise is $\Phi_S + \Phi_N$, and it has the same general form. The $U_\sigma$'s and $W_\sigma$'s of $\Phi_F$ simply comprise all the $U_\sigma$'s and $W_\sigma$'s of $\Phi_S$ and $\Phi_N$.

### 3.6.4. *An Analog of the Bode-Shannon Method*

Now consider the calculation of linear least-squares smoothing and prediction operators when auto-covariances, $\Phi_S$ and $\Phi_N$, of signal and

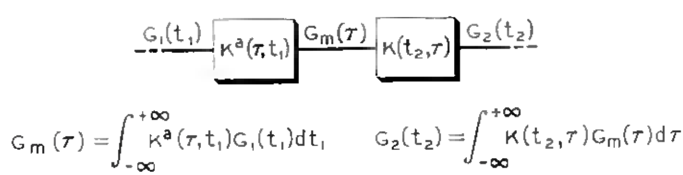



Fig. 14 — A (nonphysical) network whose impulse response is $\Phi(t_2, t_1)$.

noise are known, have the form (96*b*) and satisfy condition (94a). The method described below is analogous to the method of Bode and Shannon, as interpreted in Section 3.2.4.

We begin with the physical model illustrated in Fig. 15, which is exactly like Fig. 8, of Section 3.2.4, except that the (now nonstationary) impulse responses of the networks have the form $K(t, \tau)$, instead of $K(t - \tau)$ [the inverse transform of $Y(p)$]. The method described in Section 3.2.4 can be adapted to the model of Fig. 15, in principle at least, provided an impulse response $K_F(t, \tau)$ of the form (96*b*) does, in fact, exist which has the following properties: it must turn white noise into an $f(t) = s(t) + n(t)$ with the required auto-covariance, $\Phi_F(t_2, t_1)$; it must be "physical"; there must exist a "physical" impulse response, $K_F^{-1}(t, \tau)$, which turns $f(t)$ back into the white noise.

The manipulations described in Section 3.6.2 may be used to decompose the differential equation corresponding to $\Phi_F$ into a pair of differential equations which are at least superficially like (97*a*) and (97*b*). Under condition (94*a*), a particular decomposition will always have the following properties: the orders of the two equations are the same; the two equations have identical coefficients $H$ of (93); the bf's and zrf's of (97*a*) are all "nonphysical", while those of (97*b*) are all "physical" [as determined by the real parts of the $p_\sigma$'s of (94*b*)]; the corresponding impulses will be respectively $K_F^a$ and $K_F$ provided they are, in fact, adjoints, each of the other.

While a rigorous proof has not been completed, there is strong evidence that the two impulse responses will, in fact, be adjoints when derived in the manner described from auto-covariances of the form (96*b*) subject to the condition (94*a*). The same probably holds true in many situations where (94*a*) is not satisfied. When (97*a*) and (97*b*) have been determined, $K_F^{-1}$ may also be found, by merely interchanging $P$ and $Q$ in (97*b*) and

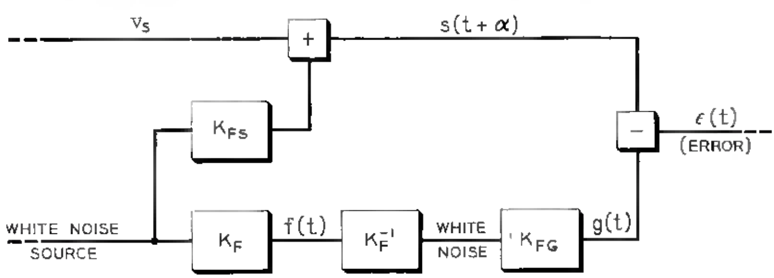



Fig. 15 — A physical model of a nonstationary system.

then calculating the corresponding impulse response. It will be physical because the zrf's of $K$ have been made physical.

For the most part, the calculations described above are straightforward but laborious. The bf's of the differential equation for $\Phi_F$ are the basis functions $U_\sigma$ and $W_\sigma$ of $\Phi_S$ and $\Phi_N$, and are known when $\Phi_S$ and $\Phi_N$ are known in the form (96$b$). The zrf's of $\Phi_F$, however, must be calculated as solutions of a homogeneous linear differential equation with time-variable coefficients. This is analogous to finding the zeros of the spectrum $N + S$ when given rational spectra $N$ and $S$. The computational difficulties, however, are very much greater, and they are likely to limit applications to quite simple problems.

### 3.6.5 *An Analog of the Zero and Pole Method*

Some of the laborious calculations (but not the calculation of zrf's of $\Phi_F$) may be avoided by techniques analogous to the zero and pole techniques described in Section 3.2, etc. For definiteness, let the optimum impulse response, $K_M(t, \tau)$, be chosen from the class of all stationary and nonstationary physical impulse responses (infinite memories included) and let the prediction interval $\alpha$ be greater than 0. Methods similar to those used in Sections 2.6 and 3.2.3 yield a Wiener-Hopf equation of the following sort [a generalization of (45)]:

$$\int_{-\infty}^{t} K_M(t, t_1)\Phi_F(t_2, t_1)\, dt_1 = \Phi_S(t + \alpha, t_2), \quad t_2 < t. \tag{98}$$

When $\Phi_F = \Phi_S + \Phi_N$, and $\Phi_S$, $\Phi_N$ have the form (96$b$), (98) becomes:

$$\left.\begin{aligned} &\sum_\sigma U_\sigma(t_2) \int_{-\infty}^{t_2} K_M(t, t_1) W_\sigma(t_1)\, dt_1 \\ &\quad + \sum_\sigma W_\sigma(t_2) \int_{t_2}^{t} K_M(t, t_1) U_\sigma(t_1)\, dt_1 \end{aligned}\right\} = \sum_\rho W_\rho(t_2) U_\rho(t + \alpha),\ t_2 < t. \tag{99}$$

The sum $\sum_\rho$ includes those terms in $\sum_\sigma$ which are contributed by $\Phi_S$, but not those from $\Phi_N$.

A differential equation may be derived from (99) which is exactly the differential equation corresponding to $\Phi_F$, with $K_M$ taking the role of the "input" function, $G_1(t_1)$, and the right-hand side of (99) taking the role of the "output" function $G_2(t_2)$. Time $t$ is a constant in the derivation and analysis of this differential equation. Then $G_2(t_2)$ is a sum of some of the bf's of the differential equation. As a result, $K_M$ must be a linear combination of zrf's of $\Phi_F$, except that $\delta$ functions may be permitted at the limit of the integration, $t_1 = t$. More exactly, $K_M$ is a linear combination of the *physical* zrf's of $\Phi_F$, plus possible $\delta$ functions. This is analogous to condition (36*a*) of Section 3.2.1, which makes the poles of $Y_M$ the lhp zeros of $N + S$.

The coefficients of the various terms in the linear combination may be determined by using the linear combination with general coefficients, in place of $K_M$ in (99). Evaluating the integrals yields a linear combination of the $U_\sigma$'s, which are the physical bf's of $\Phi_F$. Since the combination must be identically zero [when the right-hand side of (99) is included], the net coefficient of each $U_\sigma$ must be zero. The result is a set of linear equations which determine the coefficients of the various terms in $K_M$. The equations are analogous to the conditions (36*b*) on the behavior of $Y_M$ at lhp poles of $N$ and $S$.

## IV. FURTHER SPECIFIC PROBLEMS AND APPLICATIONS

The central problems described in Section III may be adjusted to fit various engineering problems. The adjustments, however, may require changes in various details. The examples described in this section illustrate both engineering usefulness and ways in which details may be changed. Some of the examples represent existing engineering applications. Some are merely potentially useful. Others are of interest primarily for theoretical reasons. The specific changes in the central problems are reviewed in more general terms in Section 4.6.

### 4.1 *Problems Related to Anti-Aircraft Fire Control*

The correct aiming of anti-aircraft artillery depends, fundamentally, on data smoothing and prediction. It also illustrates several of the ways in which the central problems may be modified to meet practical requirements. The anti-aircraft problem, as such, will not be developed in more detail than is needed for purposes of illustration.

An anti-aircraft projectile must be aimed at a predicted future position of the target, for which the prediction is based on positions of the target observed at present and past times. The observations (optical or radar) are contaminated by fluctuating observational errors, corresponding to some statistical ensemble. The true motion of the target also corresponds to a statistical ensemble, for the direction of flight and the speed will change with time, in ways which are not entirely predictable. Then the observational errors correspond to our noise, $n(t)$, and the true position of the target corresponds to our true signal, $s(t)$. While the problem is really three-dimensional, it will be sufficient for our purposes to consider a single one-dimensional component.

It is generally reasonable to use a Gaussian model for the observational errors. The observations, however, are likely to be available only for a limited interval. Thus the finite-memory form of the general problem should be assumed.

The true-signal (target) statistics are *not* well represented by a Gaussian model. Furthermore, the average square error is not a reasonable criterion of accuracy, without a careful interpretation. (The average square error gives most weight to large errors, while the "kill probability" depends on the frequency of small errors.) A somewhat nonoptimal solution is generally accepted, as described below.

Most anti-aircraft fire control systems are designed around the following assumption:

*In the absence of observational errors, the future position of a nonaccelerating target is to be predicted perfectly.* (100)

A nonaccelerating target flies a straight-line, constant speed course. Under (100), the actual errors in prediction will depend on the actual observational errors and on the actual accelerations of the target during the combined observation and prediction intervals ($t - T$ to $t + \alpha$).

The future position of a nonaccelerating target may be determined from its present position and present velocity. Hence, physical linear operators *can* satisfy (100). Generally, the truly optimum linear operator cannot be very different. Anti-aircraft systems must have a high *percentage* accuracy. That is, position errors must be small compared with typical distances from tracker to target or from present target position to predicted future position. Furthermore, the signal ensemble may generally be regarded as centered about the nonaccelerating target courses. Therefore, the truly optimum linear operator must sum the observations in a way which comes very close to giving perfect prediction under the special condition stated in (100). Then small changes will make these particular predictions perfect.

Let $x$ be any one component of position. Then

$$x(t + \alpha) = x(t) + \alpha\bar{\dot{x}} . \tag{101}$$

Here, $\bar{\dot{x}}$ is the average rate of change of $x$, averaged over the prediction interval, $t$ to $t + \alpha$. Under (100), separately optimum estimates of $x(t)$ and $\bar{\dot{x}}$, used in (101), give the optimum estimate of $x(t + \alpha)$. (A proof is not needed for our present purposes.) In anti-aircraft systems, errors in $x(t)$ are usually less significant than errors in $\alpha\bar{\dot{x}}$, and they are generally less subject to reduction by data smoothing. Then attention is centered on the optimum estimation of $\bar{\dot{x}}$.

The optimum estimation of $\bar{\dot{x}}$ is described in Sections 4.1.1 through 4.1.4. In these sections, our signal, $s(t)$, becomes the true volocity $\dot{x}(t)$. Instead of predicting $\dot{x}(t + \alpha)$, we are to predict $\bar{\dot{x}}$, which is the average of $\dot{x}(t)$ over the interval $t$ to $t + \alpha$ ($\bar{\dot{x}}$ is a functional of $\dot{x}$). The prediction is to be obtained by linear operations applied to observed positions $x(t)$. Since a factor $p$ in a frequency function corresponds to differentiation, we may write

$$g(t) = \bar{\dot{x}} + \epsilon(t) = x(t) \text{ modified by a linear operator } Y_G(p)p. \quad (102)$$

Here, $Y_G(p)$ represents the data smoothing applied to the apparent rate of change, $f(t)$ = observed $\dot{x}(t)$. The error part of $f(t)$ is described by the spectrum

$$N = Y_N \tilde{Y}_N = \omega^2 N_x , \quad (103)$$

where $N_x$ is the error spectrum for $x(t)$ itself.

4.1.1 *Optimum Measurement of a Constant Velocity**

In this section we assume the following conditions:

$a$. Positions $x$ are observed from $t - T$ to $t$,

$b$. Conditions (100) are to be satisfied, (104)

$c$. The true $\dot{x}(t)$ is constant from $t - T$ to $t + \alpha$.

When condition (100) is satisfied and the actual $\dot{x}(t)$ is constant, the entire error $\epsilon(t)$ must be due to errors in observation. Then, by (103) and the noise part of (19) or of Fig. 2,

$$\sigma^2 = \int_{-\infty}^{+\infty} | Y_G |^2 N_x \omega^2 \, d\omega. \quad (105)$$

When $\dot{x}(t)$ is constant, present $\dot{x}(t) = \bar{\dot{x}}$, and prediction time $\alpha$ need not appear at all. Then (104*b*) requires that $Y_G(p)$ applied to a constant must yield the same constant. Since the response to a constant signal $C$ is $CY_G(0)$, we now have

$C_Y$ is the subclass of the class (47) such that $Y_G(0) = 1$. (106)

If $Y_{Gj}(0) = Y_{Gk}(0) = 1$, the difference $\Delta_Y(0) = 0$. Then

$C_\Delta$ is the subclass of the class (48) such that $\Delta_Y(0) = 0$. (107)

Note that (107) is consistent with (29).

* An early treatment of this problem, yielding solutions of special cases is included in a wartime report by Phillips and Weiss.[25]

The optimum operator $Y_M$ is now the $Y_G$ of the class (106) which minimizes $\sigma^2$ in (105). As in Section 3.3, use

$$\begin{aligned} &Y_M = A + Be^{-Tp}, \\ &Y_M\,,\ \tilde{Y}_M e^{-Tp} \text{ are rrhp.} \end{aligned} \tag{108}$$

Equation (51) may now be adapted to the present problem as follows: Omit terms proportional to $S$. Replace $N$ by $N_x\omega^2$, or, in terms of $p^2 = -\omega^2$, by $-N_x p^2$. To take account of (107), rearrange the integrands, so that $\tilde{\Delta}_Y/p$ becomes the variation factor rather than $\tilde{\Delta}_Y$ [(107) excludes poles of $\tilde{\Delta}_Y/p$ at $p = 0$]. This changes (51) into

$$\int_{-i\infty}^{+i\infty} (AN_x p^2 p)\,\frac{\tilde{\Delta}Y}{p}\,dp + \int_{-\infty}^{+\infty} (BN_x p^2 p)e^{-Tp}\,\frac{\tilde{\Delta}Y}{p}\,dp = 0. \tag{109}$$

Section 3.3 may now be paralleled further, to assemble conditions corresponding to (52). Since there are no terms in $e^{\alpha p}$ in (109), a rational $N_x$ leads to rational $A$ and $B$. Then (52*a*) and (52*b*) may be replaced by (49). The resulting list of conditions is as follows:

$$\begin{aligned} &a.\ A + Be^{-Tp} = 1 \text{ at } p = 0, \\ &b.\ A + Be^{-Tp} \text{ is rfpp}, \\ &c.\ AN_x p^3 \text{ is rlhp}, \\ &d.\ BN_x p^3 \text{ is rrhp}, \\ &e.\ \text{When } \omega \text{ is real and} \to \infty, \\ &\quad |A|^2 N_x\omega^2 \text{ and } |B|^2 N_x\omega^2 = O\omega^{-2}. \end{aligned} \tag{110}$$

When $N_x$ is rational, these conditions are just sufficient to determine a rational $A$ and $B$.

As an example, suppose $N_x$ is*

$$N_x = \frac{1}{\pi}\,\frac{\sigma_x^2\omega_0}{\omega_0^2 + \omega^2} = \frac{1}{\pi}\,\frac{\sigma_x^2\omega_0}{\omega_0^2 - p^2}. \tag{111}$$

Then (110*c*), (110*d*) and (110*e*) restrict $A$ and $B$ to

$$A = \frac{a_1 + a_2 p + a_3 p^2}{p^3}, \qquad B = \frac{b_1 + b_2 p + b_3 p^2}{p^3}. \tag{112}$$

* The scale factor has been designated in such a way that $\sigma_x^2$ is in fact the average squared position error which is related to $N_x$ by

$$\sigma_x^2 = \int_{-\infty}^{+\infty} N_x\,d\omega$$

Given (112), one can use (110a) through (110d) to find $a_1$, $a_2$, $a_3$, $b_1$, $b_2$, $b_3$. The resulting $Y_M$ may be arranged as follows:

$$Y_M = -12J\left[\frac{\left(1+\frac{p}{\omega_0}\right)(1-cp)}{T^3p^3} - \frac{\left(1-\frac{p}{\omega_0}\right)(1+cp)}{T^3p^3}e^{-Tp}\right], \quad (113)$$

in which the constants $J$, $c$ are related to $T$, $\omega_0$, by

$$c = \frac{T}{2}\left(1+\frac{2}{T\omega_0}\right),$$
$$J = \frac{1}{1+\frac{6}{T\omega_0}+\frac{12}{T^2\omega_0^2}}. \quad (114)$$

The corresponding impulse response $K_M(t)$ may be arranged as follows:

$$K_M(t) = JK_1(t) + (1-J)K_2(t),$$
$$K_1(t) = \frac{6}{T^3}t(T-t) \quad \text{when} \quad 0 < t < T,$$
$$K_2(t) = \frac{1}{T} \quad \text{when} \quad 0 < t < T, \quad (115)$$
$$K_1(t) = 0, K_2(t) = 0 \quad \text{when} \quad t < 0 \text{ or } > T.$$

The two functions $K_1$ and $K_2$ are the unit-area parabola and unit-area step shown in Figs. 16(a) and 16(b). Then $K_M$ is the combination shown in Fig. 16(c).

The minimum $\sigma^2$ determined by (105) may be found by evaluating

$$\sigma^2 = \int_{-\infty}^{+\infty}(A + Be^{-Ti\omega})(\tilde{A} + \tilde{B}e^{Ti\omega})N_x\omega^2\,d\omega. \quad (116)$$

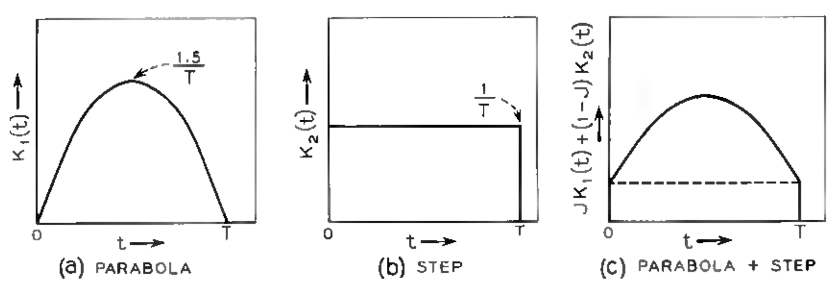



Fig. 16 — Parabolic smoothing.

The evaluation is complicated by the presence of the two exponentials in the integrand, and the multiple poles of $A$ and $B$ at $p = 0$, as determined by (112). The complications may be resolved by splitting the integral into the following two integrals:

$$\sigma^2 = -\int_{-i\infty}^{+i\infty} \left(A + Be^{-Tp} - \frac{1}{1 + cp}\right) \tilde{A} N_x p^2 \, dp - \int_{-i\infty}^{+i\infty} \left[\frac{\tilde{A} N_x p^2}{1 + cp} + (Ae^{Tp} + B)\tilde{B} N_x p^2\right] dp. \tag{117}$$

Each integrand $= O\omega^{-2}$ when $\omega$ is real and $\to \infty$; it is regular at $p = 0$ [by (110$a$) and (110$b$)]; and it involves only a single exponential. The first integrand is rrhp; the second is rlhp except for a single pole of $\tilde{A} N_x p^2/(1 + cp)$ at $p = -\omega_0$. Closing the contours of integration by suitable arcs at $\infty$, and applying the residue theorem gives

$$\sigma^2 = \frac{24J}{T^3} \frac{\sigma_x^2}{\omega_0}. \tag{118}$$

4.1.2 *A White Noise Approximation*

The constant $J$ in (113) and (114) is a function of $T\omega_0$. Generally, $T\omega_0$ is quite large. In the limit, as $T\omega_0 \to \infty$, $J \to 1$ and (113), (115) and (118) become

$$\begin{aligned} Y_M &= 12 \frac{-\left(1 - \frac{Tp}{2}\right) + \left(1 + \frac{Tp}{2}\right) e^{-Tp}}{T^3 p^3}, \\ K_M &= \frac{6}{T^3} t(T - t), \\ \sigma^2 &= \frac{24}{T^3} \frac{\sigma_x^2}{\omega_0} = \frac{12\pi}{T^3} N_x(0). \end{aligned} \tag{119}$$

The parabolic smoothing represented by (119) was derived by Bode[26] in a quite different way.

Note that $N_x(0)$ is the spectral density of the position errors at low frequencies. The result, (119), may be obtained more simply by substituting the constant $N_x(0)$ for $N_x(p)$ in (110).

The nature of the approximation is explained further by Fig. 17. Curve ($a$) represents a more general $N_x$, including a "spike" at $\omega = 0$ to allow for drift errors, etc. Curve ($b$) indicates the (qualitative) nature of the corresponding function $|Y_M|^2\omega^2$. The variance, $\sigma^2$, is the integral of the product of these two functions. Therefore, the value of $N_x$ is unim-

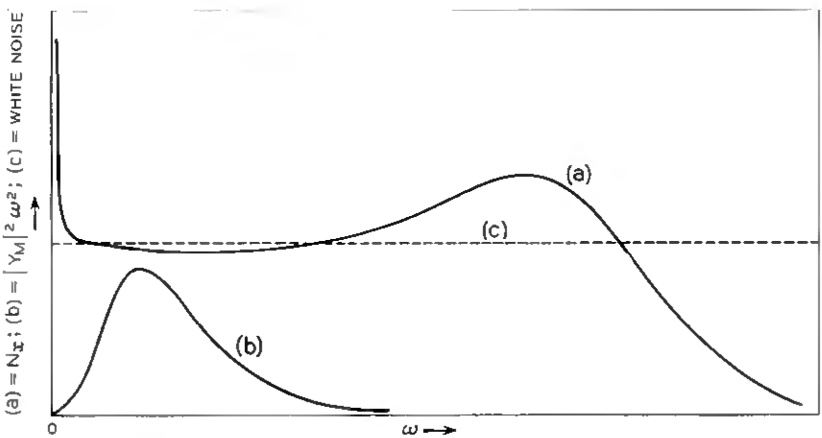



Fig. 17 — Explanation of a white noise approximation.

portant in regions where $|Y_M|^2\omega^2$ is very small. Thus, the white noise spectrum c gives about the same result as the actual spectrum $N_x$.

4.1.3 *Constant Target Accelerations*

According to (118) and (119) increasing $T$ always decreases $\sigma^2$. The $\sigma^2$ of (118) and (119), however, is not the total variance $\sigma_{\bar{x}}^2$ of the predicted $\bar{x}$, except when the target acceleration may be neglected. The effect of a given target acceleration increases as $T$ increases, and this limits the values of $T$ for which (118) and (119) are good approximations to $\sigma_{\bar{x}}^2$.

A higher order of approximation takes account of target accelerations, but assumes that they are constant over the interval $t - T$ to $t + \alpha$. If the $Y_M$ of (119) is retained unchanged, $\sigma_{\bar{x}}^2$ becomes

$$\sigma_{\bar{x}}^2 = \frac{12\pi}{T^3} N_x(0) + \frac{(T + \alpha)^2}{4} \sigma_{\ddot{x}}^2. \tag{120}$$

Here, $\sigma_{\ddot{x}}^2$ is the (ensemble) average squared acceleration of the target.

A smaller $\sigma_{\bar{x}}^2$ may be obtained by modifying the conditions which determine $Y_M$ in a way which takes account of target accelerations. When the expected accelerations are large, it may be reasonable to strengthen (100) to the following:

*In the absence of observational errors, the future position of a target with constant acceleration is to be predicted perfectly.* (121)

The stronger condition reduces the function classes $C_Y$ and $C_\Delta$ to the subclasses of (47) and (48), such that

$$\begin{aligned} a.\ & Y_G(p) \to 1 + \tfrac{1}{2}\alpha p \quad \text{as} \quad p \to 0, \\ b.\ & \Delta_Y(p) \lesssim cp^2 \quad \text{as} \quad p \to 0. \end{aligned} \tag{122}$$

These changes in $C_Y$ and $C_\Delta$ lead to the following changes in (110): condition (110*a*) is replaced by (122*a*); the factor $p^3$ in (110*c*) and (110*d*) is replaced by $p^4$. The white noise assumption leads to the following, in place of (119):

$$\begin{aligned} K_M(t) &= \frac{6}{T^3}\, t(T - t)\left[1 + \frac{5(T+\alpha)}{T^2}(T - 2t)\right], \\ \sigma^2 &= \frac{12\pi}{T^3}\left[1 + 15\left(1 + \frac{\alpha}{T}\right)^2\right] N_x(0). \end{aligned} \tag{123}$$

If the expected acceleration does not have a large effect in (120) with the largest $T$ for which observations are available, full compensation may not be justified. (The compensation increases the sensitivity to observational errors unless $T$ can be increased.) A possible, though more complicated procedure is then as follows: Assume that the target acceleration is nonzero but time-invariant, and that the ensemble average of the squared acceleration is $\sigma_{\ddot{x}}^2$. Then minimize the combined effects of the target acceleration and the observational errors.

The combined effects now give

$$\sigma_{\bar{x}}^2 = \int_{-\infty}^{+\infty} |\, Y_G\,|^2 N_x \omega^2\, d\omega + \sigma_{\ddot{x}}^2 \left[\frac{Y_G - 1}{p} - \frac{\alpha}{2}\right]_{p=0}^2. \tag{124}$$

To keep the second term finite, (106) and (107) are again in order. For a minimum $\sigma_{\bar{x}}^2$,

$$\int_{-i\infty}^{+i\infty} [\quad]\, dp + \int_{-i\infty}^{+i\infty} [\quad]\, dp + 2\left[\frac{\tilde{\Delta} Y}{p}\left(\frac{Y_M - 1}{p} - \frac{\alpha}{2}\right)\right]_{p=0} \sigma_{\ddot{x}}^2 = 0, \tag{125}$$

in which the integrals are the same as in (109). The conditions (110) are now modified to permit simple poles of the two integrands at $p = 0$, which cancel out in their sum. The two contours of integration may be indented at $p = 0$, provided they are indented in the same way. Conditions (110*c*) and (110*d*) permit one contour to be closed at infinity around the lhp, and the other around the rhp. Then the residue theorem makes one integral proportional to $\tilde{\Delta}_Y/p$ at $p = 0$, and parameters can be adjusted to cancel the last term in (125).* The result is

* The contour may be indented to pass to either side of $p = 0$, provided the indentation is the same for both integrals. The final results are the same because the poles of the integrands at $p = 0$ cancel out in their sum.

$$K_M = \frac{6}{T^3} t(T - t) \left\{1 + Q\left[\frac{5(T + \alpha)}{T^2} (T - 2t)\right]\right\},$$

$$\sigma^2 = \frac{12\pi}{T^3} \left\{1 + Q\left[15\left(1 + \frac{\alpha}{T}\right)^2\right] N_x(0)\right\}, \tag{126}$$

$$Q = \frac{\sigma_{\ddot{x}}^2}{\sigma_{\ddot{x}}^2 + \frac{720}{T^5} N_x(0)}.$$

#### 4.1.4 *More General Target Accelerations*

Comparing (123) with (120) shows that compensation for constant target acceleration increases the sensitivity to observational errors, unless $T$ is increased. (This is because the observation of acceleration, as well as velocity, is included implicitly in the new $Y_M$.) Sensitivity to fluctuations in the target acceleration is also increased, and becomes greater as $T$ increases. In principle, $Y_M$ can be modified further, to give perfect prediction in the absence of observational errors, whenever the acceleration is matched perfectly by, say, an $n$th degree polynomial with random coefficients.* The sensitivity to observational errors will be further increased, unless $T$ is increased. However, any reasonable acceleration ensemble which involves only a finite number of random parameters will lead to a $Y_M$ such that $\sigma^2 \to 0$ as $T \to \infty$.

The infinite memory problem may be handled more reasonably by assigning a *spectrum*, $N_{\ddot{x}}$, to the target accelerations. Then (105) may be replaced by

$$\sigma_{\ddot{x}}^2 = \int_{-\infty}^{+\infty} \left[ | Y_G |^2 N_x \omega^2 + \left| Y_G - \frac{e^{\alpha i \omega} - 1}{\alpha i \omega} \right|^2 \left(\frac{N_{\ddot{x}}}{\omega^2}\right)\right] d\omega. \tag{127}$$

If the optimum impulse response $K_M(t)$ corresponding to $Y_M(p)$ is negligible when $t > t_m$, there may be no advantage in extending the interval of observation to times older than $t - t_m$. The practical significance of a quantitative limit may be weakened, however, by the non-Gaussian character of actual target statistics.

### 4.2 *Measurements with Multiple Instruments*†

This section describes further the two-instrument problem noted in Section 3.4. It will be assumed that $s(t)$ is to be estimated for present

* Blackman[26] has related the corresponding $K_M(t)$ to Legendre polynomials.

† The use of multiple instruments is described in more detail in reports[4, 5, 6] relating to specific applications. Principles are described in papers by Bendat[14] and Stewart and Parks.[15]

time, $t$, and need not be predicted for a future time, $t + \alpha$. When $\alpha = 0$, (73) becomes

$$\sigma^2 = \int_{-\infty}^{+\infty} (|Y_{G1}|^2 N_1 + |Y_{G2}|^2 N_2 + |Y_{G1} + Y_{G2} - 1|^2 S)\, d\omega. \quad (128)$$

In (72) $f_1(t)$ and $f_2(t)$ are now the results of observing a physical variable, $s(t)$, with two different instruments. Then $N_1$ and $N_2$ are the spectra of the instrumental errors $n_1(t)$ and $n_2(t)$.

#### 4.2.1 *Elimination of Errors Proportional to True Signal*

When the two instruments have reasonably high percentage accuracies, $S \gg N_1$ and $N_2$. Then the $Y_{G1}$ and $Y_{G2}$ which minimize $\sigma^2$ in (128) will make $Y_{G1} + Y_{G2} - 1$ very small. Under these conditions, it is reasonable to approximate the true optimum by making the factor exactly zero. Then $Y_{G2}$ is related to $Y_{G1}$ by

$$Y_{G2} = 1 - Y_{G1}. \quad (129)$$

Under (129), the $S$ term in (128) drops out. Using $Y_G$ to represent $Y_{G1}$ and $1 - Y_G$ to represent $Y_{G2}$ gives

$$\sigma^2 = \int_{-\infty}^{+\infty} (|Y_G|^2 N_1 + |1 - Y_G|^2 N_2)\, d\omega. \quad (130)$$

Now, $\sigma^2$ is to be a minimum with respect to the single-frequency function $Y_G$.

Formally, (130) is exactly the same as (19) (with $\alpha = 0$). Rational spectra $N$ and $S$ have merely been replaced by rational spectra $N_1$ and $N_2$. Thus the whole of Sections 3.2 and 3.3 may be applied.

#### 4.2.2 *Determination of Position from Position and Velocity Measurements*

As an example, suppose an aircraft's position is measured with one instrument and its velocity with another, and that the two measurements are to be combined to determine the present position to high accuracy. Let $N_x$ and $N_v$ be the error spectra of the position and velocity observations. Then, if all errors are referred to positions, $N_1 = N_x$, $N_2 = N_v/\omega^2$, and (130) becomes

$$\sigma^2 = \int_{-\infty}^{+\infty} \left(|Y_G|^2 N_x + |1 - Y_G|^2 \frac{N_v}{\omega^2}\right) d\omega. \quad (131)$$

In (131), $\sigma^2$ will be bounded only if $Y_G(0) = 1$ [assuming that $N_v(0) \neq 0$]. When $Y_G(0) = 1$, applying $Y_G$ to a constant position leaves the position unchanged. This situation may be interpreted as follows: By (13), any $Y_G$ applied to the true position yields a weighted integral of the true positions existing during the "smoothing interval". When $Y_G(0) = 1$, the weighted integral is a weighted *average*, and the position measurements are used to determine a weighted average position, averaged over the smoothing interval. When the position is not constant, the present position will generally be different from the average, and the difference may be calculated from the velocities observed during the averaging or smoothing interval. The correction may be said to "update" the average. The first term in the integrand in (131) corresponds to errors in the weighted average position, determined from the position measurements, and the second term corresponds to errors in the updating, determined from the velocity measurements.

If the position measurements are used alone and if guessed velocities are, in fact, quite uncertain, an adequate smoothing interval may lead to large updating errors. On the other hand, if the velocity measurements are used alone, they must be integrated over the entire time of flight, and velocity errors may accumulate into large position errors. Thus, the two instruments together may give a very much higher accuracy than is possible with either instrument alone. This may be explained further by citing differences between the spectra $N_x$ and $N_v/\omega^2$, and comparing $Y_G$ and $1 - Y_G$, in (131), to the transfer functions of a pair of separating filters.

#### 4.2.3 *Precalculated Information*

Sometimes, the second measurements may be either replaced by, or augmented by previous nonstatistical information concerning the physical variable. The "biased statistics" may be taken account of as follows: Let $s_0(t)$ be a precalculated "nominal" $s(t)$, which may be regarded as the ensemble average of $s(t)$. Then let

$$\begin{aligned} s(t) &= s_r(t) + s_0(t), \\ f(t) &= f_r(t) + s_0(t), \\ g(t) &= g_r(t) + s_0(t), \\ S &= \text{spectrum of } s_r(t) \text{ ensemble}. \end{aligned} \tag{132}$$

The time series $s_r(t)$ with spectrum $S$ may be regarded as the error in the prediction of $s(t)$ without measurements, by precalculation alone. The time function $g_r(t)$ is to be obtained by applying operator $Y_G(p)$ to

$f_r(t) = f(t) - s_0(t)$. Then $g(t)$ may be found by adding $s_0(t)$ to $g_r(t)$. The error integral (128) is unchanged, provided $S$ is redefined as in (132).

If $Y_{G2} = 0$, in (128), the second measurements are replaced entirely by the precalculation of $s_0(t)$, and the error in the second measurements is replaced by the error in the precalculation. If $Y_{G2}$ is neither 0 nor $1 - Y_{G1}$, the estimate of $s(t)$ is based on three sources of information: the two kinds of measurements and the precalculation of $s_0(t)$. Whether the full generality is justified depends on the relative magnitudes and spectra of the three corresponding errors.

### 4.3 *A Signal Detection Problem.*

This section describes a simple problem related to signal detection. The time function $f(t) = s(t) + n(t)$ is again observed and $g(t)$ is again produced by applying, to $f(t)$, a *physical* linear operator $Y_G(p)$. Now, however, the true signal $s(t)$ has the following properties: it is a time function which has finite duration and a known shape, but which starts at an unknown time. It may be represented as follows:

$$\begin{aligned} s(t) &= r(t - t_1), \quad t_1 < t < t_1 + w \\ &= 0, \quad t < t_1 \quad \text{or} \quad > t_1 + w \qquad (133) \\ t_1 &= \text{a random variable.} \end{aligned}$$

In the absence of noise, the response $g(t)$ will have a maximum value at some value of $(t - t_1)$. The contribution to $g(t)$ from the noise $n(t)$ will have an rms value. We are to find a particular linear operator $Y_M$ which minimizes the ratio of rms noise response, to maximum response to true signal.

Since only ratios are of interest, the scale of $Y_M$ may be chosen to give a unit maximum true response. Then the problem is to minimize the rms noise response within this constraint. Given any valid solution for $Y_M$ producing a maximum true response at, say, $t_1 + t_m$, there will be equally valid solutions producing maximum responses at later times. The operator $Y_M$ can always be multiplied by $e^{-\beta p}$, representing an ideal delay $\beta$. Thus, if $t_1 + t_m$ is the time of maximum true response, $t_m$ may be treated as an arbitrary parameter, provided the final results are examined, to determine what values of $t_m$ are, in fact, valid.

When $t = t_1 + t_m$ (133) makes $s(t - \tau)$ become $r(t_m - \tau)$. Then $Y_M$ is the physical $Y_G$ which minimizes the following $\sigma^2$, subject to the following constraint:

$$\sigma^2 = \int_{-\infty}^{+\infty} |Y_G|^2 N \, d\omega,$$
$$\int_{-\infty}^{+\infty} r(t_m - \tau) K_G(\tau) \, d\tau = 1. \tag{134}$$

Let $Y_r(p)$ be the transform of $r(t_m - \tau)$, regarded as a function of $\tau$. Then, by Parseval's equation (T-14), the constraint may be written

$$\int_{-\infty}^{+\infty} Y_r \tilde{Y}_G \, d\omega = 1,$$
$$Y_r(p) = \text{transform of } r(t_m - \tau). \tag{135}$$

The isoperimetric method of the calculus of variations may now be applied in the following way: When $Y_G$ is replaced by $Y_M + \Delta_Y$, (134) and (135) each yield an integral which must vanish. The two integrals may be summed in arbitrary proportion to get

$$\int_{-i\infty}^{+i\infty} (Y_M N - k Y_r) \tilde{\Delta}_Y \, dp = 0, \tag{136}$$

in which $k$ is an initially undetermined constant. The methods of Section 3.2 then give†

$$Y_M Y_N - k \frac{Y_r}{\tilde{Y}_N} \text{ is rlhp.} \tag{137}$$

These can be solved for the physical $Y_M$, in either frequency-domain or time-domain terms. The resulting $Y_M$ is proportional to $k$, and $k$ may then be determined by (135).

After further manipulation, the corresponding noise ratio turns out to be

$$\sigma_m^2 = k = \frac{1}{\int_0^\infty [K_{rN}(\tau)]^2 \, d\tau},$$
$$K_{rN}(\tau) = \text{inverse transform of } \frac{Y_r(p)}{\tilde{Y}_N(p)},$$
$$= r(t_m - \tau) * \left( \text{inverse transform of } \frac{1}{\tilde{Y}_N(p)} \right). \tag{138}$$

In the formula for $\sigma^2$, the integration runs from 0 to $\infty$. In general, $K_{rN}(\tau) \neq 0$ when $\tau < 0$, but increasing $\tau_m$ shifts $K_{rN}(\tau)$ along the time

† Recall the definition of $Y_N$ by $N = Y_N \tilde{Y}_N$, in Section 2.3.

axis in the direction of $\tau > 0$. Then the corresponding $\sigma_m^2$ must decrease. It approaches a minimum value asymptotically, as $t_m$ becomes so large that the "tail" of $[K_{rN}(\tau)]^2$ at $\tau < 0$ becomes negligibly small.

When $N$ is either a constant or the reciprocal of a polynomial $1/\tilde{Y}_N$ is at most a polynomial in $p$ and $K_{rN}(\tau) = 0$ when $\tau > w$, which is the length of the signal $r(t - t_1)$ as defined in (133). Then $\sigma^2$ reaches its asymptotic value as soon as $t_m \geqq w$. When $N$ has zeros at finite values of $p$, $\sigma_m^2$ approaches its asymptotic value only when $t_m$ exceeds $w$ by the effective correlation time of the spectrum $1/N$.

When $N$ is a constant, corresponding to white noise, the optimum impulse response is $K_M(\tau) = r(t_m - \tau)$. Then $K_M$ may be described as a mirror image of the given signal form, $r(t - t_1)$, as illustrated in Fig. 18. This is an old principle described, for example, by North.[27] The more general solution, for $N \neq$ constant, has been described by Zadeh and Ragazzini.[28]

A variation of the present problem restricts the class $C_Y$, of permitted frequency functions, $Y_G$, to the finite memory class considered in Section 3.3. The problem may be solved by combining the methods of this section and of Section 3.3.4.

When $N \to 0$ as $\omega \to \infty$, it is easy to find pulse shapes such that the values of Section 2.2 and 2.3 regarding behavior at $\infty$ are violated unless $k = 0$ in (136). The corresponding $\sigma_m^2$ does, in fact, $= 0$. An explanation is as follows:

Suppose the $(m - 1)$th derivative of $r(t - t_1)$ is discontinuous and consider a $Y_G(p)$ which approaches $cp_m$ as $p \to \infty$. The corresponding response to $r(t - t_1)$ will include a $\delta$ function and its maximum value will be $\infty$. If, at the same time, the noise spectrum $N = O\omega^{-2(m+1)}$ as $\omega \to \infty$, the rms response to the noise will be bounded and the ratio of rms response to noise to maximum response to true signal will be 0. If $N \to c\omega^{-2m}$, the rms noise itself will be $\infty$, but it can be shown that the ratio of noise to maximum signal response will still be 0. Compare these

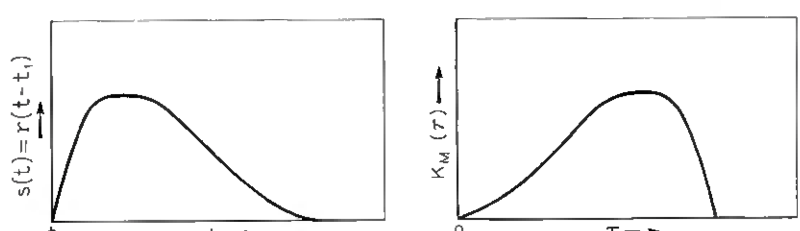



Fig. 18 — Time functions which are mirror images.

conditions with the continuity conditions described in Section 3.3.4. *Actual* pulse shapes and noise spectra are not likely to behave in this way, but what appear to be good approximations may. Care must be used to choose approximations which behave properly as $p \to \infty$.

### 4.4 *A Principle Relating to Diversity Systems*

This section adapts the method of Section 3.4 to the following problem: A single signal $s(t)$, with known spectrum $S(\omega)$, is observed by $m$ different devices. (These may be, for example, the receivers of a communication system using the diversity principle.) The signal actually observed by device $k$ is $f_k(t) = s(t) + n_k(t)$. The $m$ different $n_k(t)$ are uncorrelated and have known spectra $N_k(\omega)$. Finally, the various noise spectra all have the same shape, but they differ in amplitude. Thus,

$$N_k(\omega) = J_k N(\omega), \qquad k = 1, \cdots, m, \tag{139}$$

where the $J_k$ are known constants and $N(\omega)$ is a known function. The optimum physical filters are to be determined for estimating $s(t + \alpha)$ by summing linear operations on the various $f_k(t)$.

Paralleling (78) gives

$$\begin{aligned} &a.\ Y_{Mk} \text{ is rrhp,} \\ &b.\ \sum_j Y_{Mk} F_{kj} - e^{\alpha p} S \quad \text{is} \quad \text{rlhp,} \\ &\quad F_{kk} = S + J_k N, \\ &\quad F_{kj} = S, \qquad j \neq k, \\ &c.\ \text{When } \omega \text{ is real and} \to \infty, \\ &\quad |Y_{Mk}|^2 N \text{ and } \left| \sum_k Y_{Mk} - e^{\alpha p} \right|^2 S = O\omega^{-2}. \end{aligned} \tag{140}$$

The indices $k$ and $j$ run through the integers 1 to $m$.

A set of $m$ equations similar to the pair in (79) may now be solved for the $Y_{Mk}$, in terms of (unknown) rlhp functions $U_k$. Under the special conditions which we are now assuming, however, a solution is obtained more simply by examining linear combinations of the conditions (140$b$), which give

$$\begin{gathered} \left( \frac{N}{\sum_k \frac{1}{J_k}} + S \right) \sum_k Y_{Mk} - e^{\alpha p} S \text{ is rlhp,} \\ (J_k Y_{Mk} - J_j Y_{Mj}) N \text{ is rlhp.} \end{gathered} \tag{141}$$

The second of these conditions can be satisfied, within the convergence condition (140*c*), only if

$$J_k Y_{Mk} = J_j Y_{Mj} . \tag{142}$$

Because of (142), the various $f_k(t)$ may first be summed with weights proportional to $1/J_k$ and then a single frequency-dependent operator may be applied to the sum. In other words, only a single "filtering" device is needed.* This confirms a result which might reasonably be expected without formal analysis. The optimum filter characteristic is proportional to $\sum Y_{Mk}$, and may be found by applying the methods of Section 3.2 to the first condition of (141).

The use of a single filter for all the channels may actually be dictated by cost considerations. The above analysis confirms its use on a performance basis.

### 4.5 *Nonstatistical Network Synthesis Applications*

Nonstatistical problems in network synthesis sometimes may be formulated in terms of the mathematics of data smoothing, even though no data smoothing is involved. In particular, reasonable solutions sometimes may be found by minimizing integrals similar to those which represent our $\sigma^2$. This has been pointed out by Chang,[11] with illustrations in terms of a frequency-domain theory of optimum infinite-memory networks. The possibilities appear to be much greater when a frequency-domain form of the finite-memory problem is available. Possible uses, however, have not been explored in detail. The example described below will illustrate how problems may be formulated.

It will be simplest to develop the example in two stages. We will begin by seeking a physical network function $Y_G(p)$ with the following properties: The "step response" is to have a "rise time" $T$ and is to be exactly 1 thereafter, as in Fig. 19(a). At the same time, $|Y_G|^2$ is to be small at real frequencies above a cutoff frequency, $\omega = \omega_c$, as in Fig. 19(b). More exactly, $|Y_G|^2$ is to be as small as possible, within the rise-time restriction. In order to apply the mathematics of data-smoothing, we will use an average square criterion $\sigma^2$ to judge the effective smallness.

The impulse response $K_G(t)$ is the derivative of the step response. It will have to be zero when $t > T$, as in Fig. 19(a). Also, $Y(0)$ is equal to the step response at $t = \infty$, and it will have to be exactly 1:

$$\begin{aligned} K_G(t) &= 0 \quad \text{when} \quad t > T, \\ Y_G(0) &= 1. \end{aligned} \tag{143}$$

* When the $N_k$ are not related as in (139), the $Y_{Mk}$ generally share a common set of poles, but differ in regard to the residues at the poles.

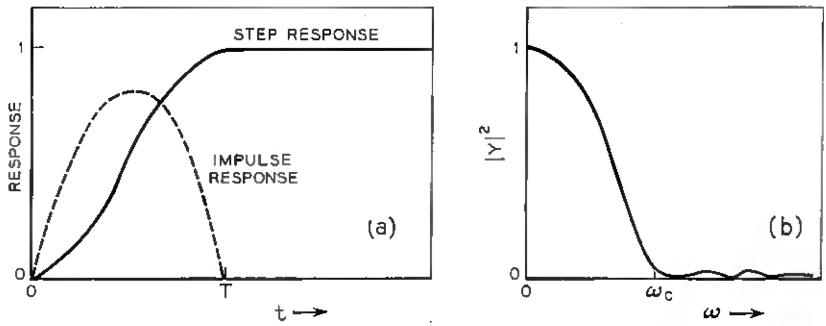



Fig. 19 — Requirements imposed on a network response.

If the average-square criterion $\sigma^2$, applied to $|Y_G|^2$, is unweighted,

$$\sigma^2 = \int_{\omega_c}^{\infty} |Y_G|^2 \, d\omega. \tag{144}$$

The limited range of integration, however, makes the minimization problem very difficult. A more tractable compromise is the following weighted average-square:

$$\sigma^2 = \int_{-\infty}^{+\infty} |Y_G|^2 R \, d\omega, \tag{145}$$

where

$$\begin{aligned} R &= \text{a rational function of } \omega, \\ &= \text{an even function} \geqq 0, \\ &= \text{small when } \omega \text{ is small.} \end{aligned}$$

The problem now is as follows: find the particular "physical" function $Y_M(p)$ which makes $\sigma^2$ of (145) a minimum, within the constraints (143). Mathematically, this is exactly the problem described in Section 4.1.1, except that the weight factor $R$ replaces spectrum $N_x\omega^2$.

Details of the weight factor $R$ are arbitrary. They influence the distribution of the loss, due to the network, in the high-loss region $\omega > \omega_c$. An efficient choice of $R$ may place zeros and poles on the axis of real $\omega$, with zeros $< \omega_c$ and poles $> \omega_c$. Real zeros and poles must be in identical pairs to preserve $R \geqq 0$. One of each pair is interpreted as in each half plane, and $|Y_M|^2$ will always have zeros at real $\omega$ poles of $R$. Further analysis indicates the following:

When all zeros and poles of $R$ are at real $\omega$,

$$R = W^2,$$
$$Y_M = \frac{1}{W}(u - \tilde{u}e^{-Tp}). \tag{146}$$

Poles of $1/W$ are canceled in $Y_M$ by zeros of the other factor. The function $u$ may be determined by adapting the method of Section 4.1.1.

The problem may now be modified in the following way: the step response need only approximate 1 when $t > T$. The approximation is to be judged on an average-square basis. There are now two average square criteria, referring respectively to the step response and the frequency suppression. The two criteria are to be added to obtain a measure of over-all performance. Since the step response is the integral of the impulse response

$$\sigma^2 = \int_{-\infty}^{+\infty} |Y_G|^2 R\, d\omega + \int_T^\infty \left[\int_0^\tau K_G(\tau_1)\, d\tau_1 - 1\right]^2 d\tau, \tag{147}$$

the relative importance of the frequency suppression and the step response may be adjusted by adjusting the scale of the weight factor $R$.

The problem may be solved by splitting $Y_G$ into the following two parts:

$$\begin{aligned} Y_G(p) &= Y_1(p) + pY_2(p), \\ K_1(t) &= 0 \quad \text{when} \quad t > T, \\ Y_1(0) &= 1, \\ K_2(t) &= 0 \quad \text{when} \quad t < T. \end{aligned} \tag{148}$$

The impulse response of $Y_2$ is the integral of the impulse response of $pY_2$ (and is therefore equal to the step response of $pY_2$). Then the constraints on $K_1(\tau)$ and $K_2(\tau)$ are such that

$$\int_T^\infty \left[\int_0^\tau K_G(\tau_1)\, d\tau_1 - 1\right]^2 d\tau = \int_{-\infty}^\infty [K_2(\tau)]^2\, d\tau. \tag{149}$$

Applying Parseval's equation (T-15) now gives:

$$\sigma^2 = \int_{-\infty}^{+\infty} [|Y_1 + pY_2|^2 R + |Y_2|^2]\, d\omega. \tag{150}$$

The problem now is as follows: Find the "physical" $Y_1$ and $Y_2$ which make $\sigma^2$ a minimum, within the constraints (148). The problem may be solved by combining the method of Sections 3.3, 3.4 and 4.1.1.

The optimum network function $Y_M(p)$ is not realizable with a finite network in either form of the problem, but it may be approximated arbitrarily closely. It should also furnish a reference for judging the performance of finite networks designed in other ways. The method has not yet been tested by detailed calculations.

### 4.6 *More General Modifications of the Central Problems*

The specific problems described in Sections 4.1 to 4.5 illustrate various more general ways in which the central data smoothing and prediction problems may be modified. These include the following:

i. The restriction of the function class $C_Y$, by constraints which specify the response to certain frequencies (Section 4.1) or to certain more general time functions (Section 4.3).

ii. The substitution of simplified spectra, which approximate true spectra only at frequencies which are actually utilized (Section 4.1.2).

iii. The addition of signal or noise functions which involve a finite number of random variables (Sections 4.1.3, 4.1.4).

iv. The estimation or prediction of a functional of the true signal, rather than the signal itself (Sections 4.1.3, 4.1.4).

v. The substitution of random variables other than signal and noise, and the treatment of "biased" statistics (Section 4.2).

vi. The use of more than two simultaneous observations (Section 4.4).

vii. The application of the mathematics of data smoothing to non-statistical problems (Section 4.5).

Other modifications are possible, which have not been illustrated. For example, correlations between signal and noise may be handled very simply, by modifying the physical model illustrated in Fig. 8. It is only necessary to change the associated frequency functions, so as to generate the correct pertinent covariances listed in Section 3.2.4. The methods of Section 3.3 and 3.4 may be combined, to find two optimum operators $Y_{G1}$, $Y_{G2}$ restricted to finite memories. The single operator problem may be solved for signal and noise situations which are different in different segments of past time. In a special case, $n(\tau)$ is observed when $-\infty < \tau < t - T$, and $s(\tau) + n(\tau)$ is observed when $t - T < \tau < t$. Added complications, however, are likely to increase very drastically the number of simultaneous linear equations which must be solved to find $Y_M(p)$.

A general solution of the following problem would be of interest to engineers, particularly in connection with preliminary system studies: Suppose the signal or noise spectrum is not known in detail but is known to lie within some sort of limits. What $Y_G$ will give the best protection against a large $\sigma^2$? If the most unfavorable permitted spectrum is associated with each $Y_G$, what $Y_G$ will make the corresponding $\sigma^2$ a minimum, and how large will the minimum be? It appears that no general solution of this problem has been achieved.

## V. ACKNOWLEDGMENTS

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# Automatic Number Identification

## and Its Application to No. 1 Crossbar, Panel and Step-by-Step Offices

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*Automatic number identification provides facilities for automatically obtaining calling customers' directory numbers in No. 1 crossbar, panel and step-by-step central offices and transmitting these numbers to a tandem or toll office for recording on centralized automatic message accounting equipment for billing purposes. Present centralized automatic message accounting systems require operators to request the calling customers' numbers and to key these numbers into recording equipment. The new facilities feature a simplified procedure for customers, improvement in speed of service and greater accuracy of billing records. Offices with local automatic message accounting facilities already have these features and are not candidates for automatic number identification.*

### I. INTRODUCTION

As the range of customer dialing has been extended, it has become increasingly important to provide arrangements for automatically recording and processing the data required for billing these calls. To accomplish this, a system using punched tape recording and known as automatic message accounting (AMA) has been developed. This system has been arranged for local office application in No. 1 crossbar, No. 5 crossbar and step-by-step offices. Here, automatic number identification is an integral part of the system. This application is economical only when calling rates for customer-dialed multiunit traffic and toll traffic are relatively high; in places where the calling rate is low, AMA features may be located more economically at a tandem or toll-switching location. This is known as centralized automatic message accounting (CAMA). In its earlier arrangement, an operator has been bridged on the connection momentarily to obtain the calling number and to key it into the system. Automatic means of performing this action have been developed and it is the purpose of this article to describe automatic number identification (ANI) as it applies to No. 1 crossbar, panel and step-by-step types of local offices. As it is to be applied to No. 5 crossbar offices without local AMA, automatic number identification is of a different type and is beyond the scope of this article.

Automatic number identification as presently developed is for use with one- and two-party lines. Multiparty lines are recognized as such and their calls will be referred to a CAMA operator for identification.

## II. METHOD OF OPERATION

Fig. 1 is a block diagram of the major equipment items required for automatic number identification, exclusive of maintenance facilities. These are:

1. an ANI outgoing trunk circuit
2. a link circuit to connect the trunk to an outpulser
3. an outpulser and identifier-connector circuit to seize and prime an identifier
4. an identifier circuit to determine the calling customer's number and forward it to the outpulser, which in turn transmits the number to the CAMA office
5. a number network and bus system to connect each customer's directory-number sleeve wire to a grid of bus panels and to connect the output of these panels via an identifier connector to an identifier.

Fundamentally, the operation is quite simple. A call proceeds in the normal fashion until the called number has been transmitted to the

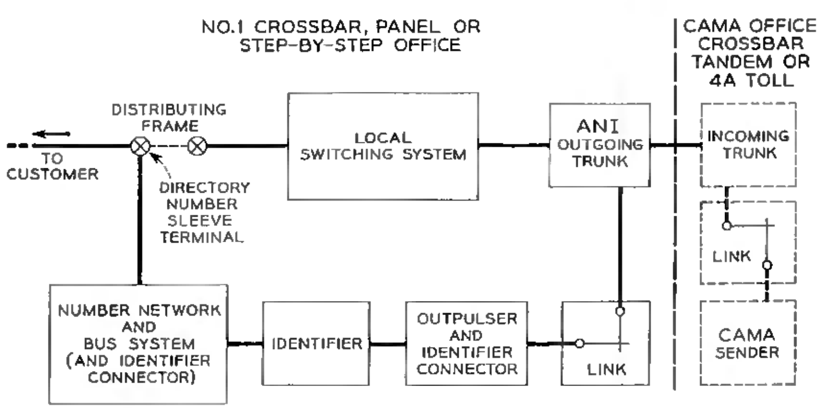


Fig. 1 — Block diagram of automatic number identification system.

CAMA sender, whereupon the identification equipment comes into play. This action is initiated by the outgoing trunk, which establishes a connection through the link to an outpulser. This circuit, by means of connecting facilities within itself, seizes an identifier. The identifier connects itself to the number network and bus system and signals the trunk to apply an identification signal to the sleeve wire toward the local switch train. This identification signal is a 5,800-cycle frequency at approximately two volts, and it finds its way over the sleeve of the switch train and back to the customer's line equipment. Here the path continues through the distributing-frame cross connections that attach directory-number significance to the line circuit, and the identification signal reaches the directory-number sleeve terminal. All of these sleeve terminals are cabled individually to networks connected to a bus system. These buses are arranged in a grid pattern in such a manner that the identifier can quickly scan the groups of output leads and identify the central office and the four digits of the calling number. This information is transferred to the outpulser, which forwards it to the CAMA point by multifrequency pulsing. Then the outpulser releases its connection through the link and the trunk connects the transmission circuit through for talking.

At the CAMA office the calling number is recorded on AMA tape along with the other information required for billing purposes. After switching of the call is completed, the called customer's line is rung, and when answer and disconnect take place, the corresponding timing entries are recorded under control of the supervisory equipment at the CAMA office.

## III. NUMBER NETWORK AND BUS SYSTEM

A unique and fundamental part of the identification arrangement is the network and bus system, for which the electrical equivalent is shown in Fig. 2. The customers' directory number sleeve wires are cabled from the distributing frame to terminals on the number networks at panels in the primary bus system. The sleeve terminations are arranged in a square pattern of 100 rows and 100 columns. Each sleeve wire is connected through a 0.05-microfarad capacitor and 510-ohm resistor to ground; the junction of these components is connected through 20,000-ohm resistors to one vertical and one horizontal bus in the grid. Thus, each sleeve is associated with one of the 10,000 coordinate points in the grid and may be identified in terms of the vertical and horizontal buses to which it is attached.

If it were practical for the identifier to examine 200 buses to determine the calling number, no additional bus system would be needed. This, however, would require an excessive number of detectors. Therefore, an arrangement is provided using two secondary systems, each arranged in a square pattern of ten rows and ten columns. The primary buses are concentrated in the secondaries as shown in Fig. 2, with the vertical buses connected to one secondary grid and the horizontal buses to the other secondary grid. With this arrangement, an identifier equipped with ten detectors may be switched from one group of ten secondary buses to another. With an input signal at one of the number networks, an output signal will appear on one bus in each of the four secondary groups of ten, and the buses so marked correspond to the numerical digits of the customer's number. Thus, the identifier makes four successive tests to identify completely a number in a 10,000-line unit.

In the primary bus arrangement, the vertical buses are designated thousands and units and the horizontal buses hundreds and tens, as shown in Fig. 2. This particular numbering arrangement of the buses is

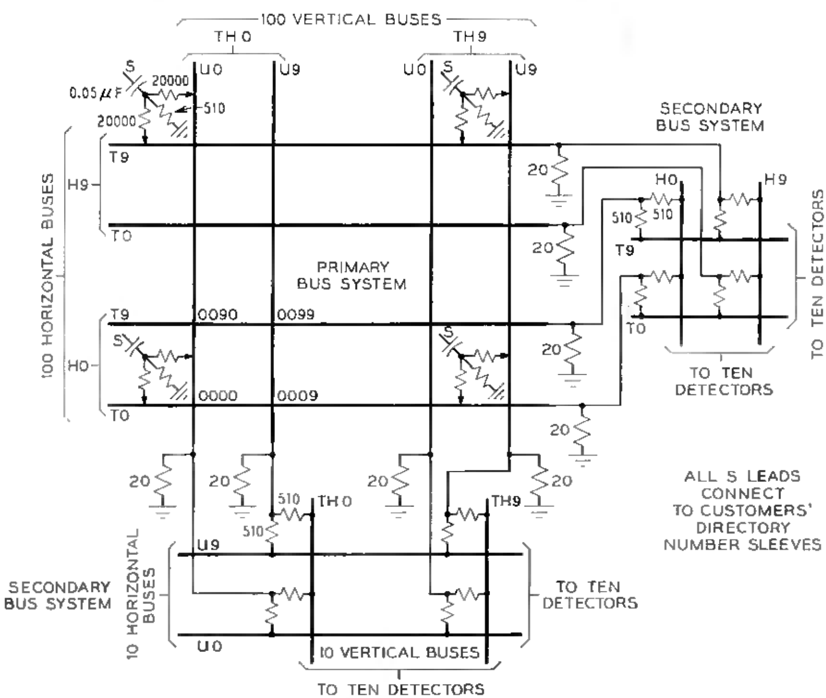


Fig. 2 — Number network and bus system.

used to provide the most convenient grouping of networks on the number network panels, as illustrated for the 00-hundred group in Fig. 2.

In multioffice buildings, a single group of ANI equipment may serve as many as six central offices, each having a maximum of 10,000 numbers. In buildings containing more than six central offices, a second group of identification equipment is required and simultaneous identifications may be made in the two groups. Successive groups of thousands buses are tested until a signal is found, then the hundreds, tens and units are examined to complete the identification. Office identification is accomplished by recognizing the particular thousands group in which the signal is found.

In offices with two-party lines, the tip parties are connected to a second set of primary buses. Before the identifier connects to the secondary buses, it is provided with information as to whether the calling customer is a tip party and, if so, it transfers the two secondary grids from the primary that contains the ring party numbers to the one that contains the tip parties. In this way, it differentiates between the two parties on a line in spite of the fact that the signal is present in the number networks for both parties.

Many primary buses will have less than 100 number networks connected to them. This will occur in offices with two-party lines where the networks are divided between tip and ring bus fields; it will also occur in offices with four-party, multiparty or PBX lines, for reasons that will be described later. Actually, the handling of PBX lines is such as to result in connecting more than 100 networks to some buses. This means that the number of networks connected to a particular bus may vary from none connected to a full complement of 100 or more. In order to minimize the resultant variation in primary bus voltage, a 20-ohm termination to ground is provided on each primary bus. This makes it easier for the detectors to discriminate between the wanted signal and the unwanted signals which result from backup paths through other networks.

A number network consists of the capacitor and three resistors associated with a single customer's directory number. These networks are mounted in groups of ten on a card approximately nine inches by three inches, photographs of which are shown in Fig. 3. The components are connected to brass details staked to the card in an unique pattern that facilitates joining them together and to terminals for external connections. The terminals for the capacitors appear at one edge of the card for cabling to the distributing frames. The terminals for the 20,000-

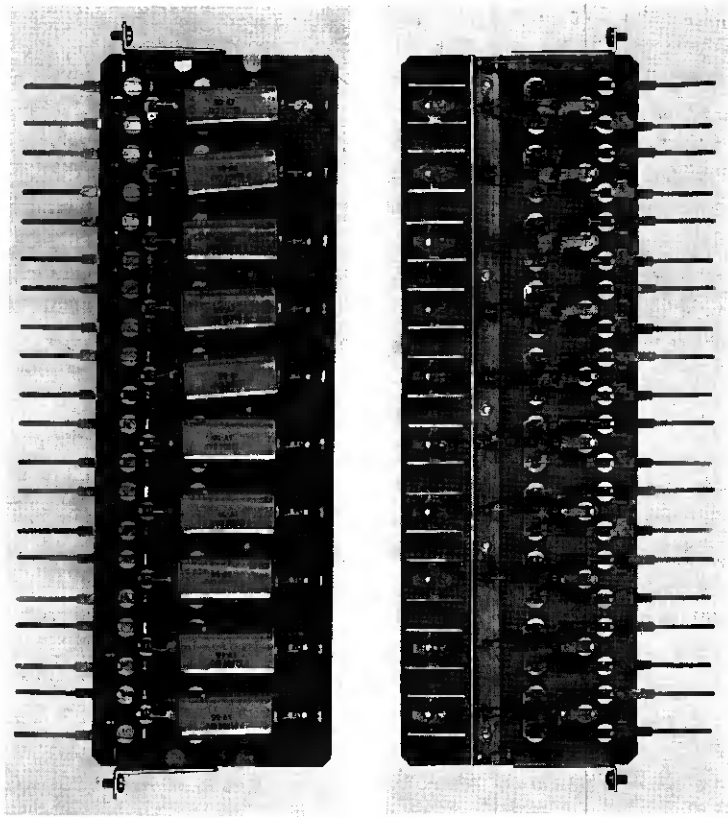

Fig. 3 — Opposing sides of card on which number networks are mounted in groups of ten.

ohm resistors project from the opposite edge of the card in positions that facilitate strapping to either tip or ring party buses.

The primary bus system is made up of panels, each of which accomodates twenty cards or two hundred number networks. A photograph of one of these bus units, partially equipped, is shown in Fig. 4. The panel itself is made of phenol fabric and, on the rear side, a grid of small buses is attached. These buses are arranged in vertical and horizontal groups. One bus of each group belongs to the tip party system and the other to the ring party system. At each position on the panel where buses cross, projections of these buses are carried through the panel to the front, where they appear as terminals for connection to the networks. Also at each position are two holes suitably positioned so that a card of ten-number networks may be mounted vertically on the rear of the panel and have its 20,000-ohm resistor terminals project through the panel to the front. Ten bus panels are mounted vertically on a 23-inch frame, which thus serves 2,000 numbers. Five such frames are needed for a full 10,000-number unit. Corresponding vertical buses on the ten panels of one frame are multipled vertically. Similar connections for the horizontal buses of the five frames of a central office are provided by interframe wiring. In crossbar offices with coded or "X" numbers, one or two additional frames are required for these networks (the method of connecting them will be described below). These numbers are outside of the 10,000-number series, and are provided for PBX use.

In addition to the two vertical buses that provide for tip and ring party identification, a third vertical bus (not indicated in Fig. 2) is provided. To this are connected all four-party or multiparty numbers; one of the 20,000-ohm legs is used for this purpose and the other is left unused. The four-party and multiparty buses are grouped together through suitable networks so they can be examined by a single "party"

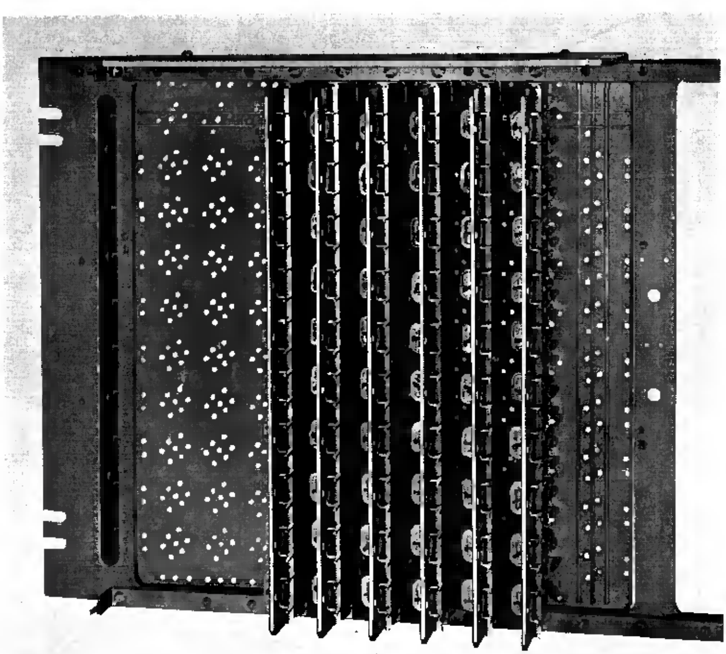

Fig. 4 — Primary bus system unit, partially equipped.

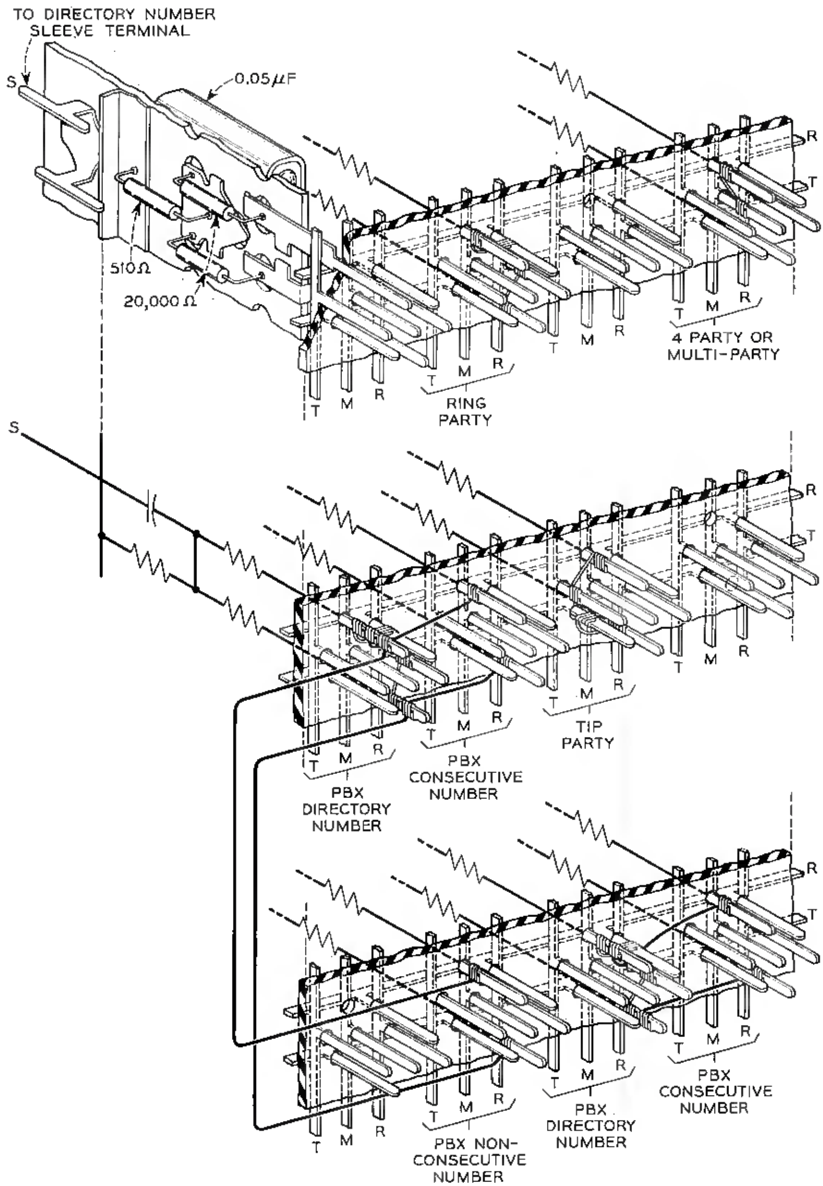


Fig. 5 — Section of primary bus panel, showing arrangement of buses and connections for typical networks.

detector in each identifier. The appearance of a signal results in the call being routed to an operator at the CAMA office, who will ask the calling customer for his number and key it into the CAMA equipment for recording on the AMA tape.

Fig. 5 is a sketch of a transparent section of a primary bus panel, facing the front. It shows the physical arrangement of the buses and typical network connections, including a ring party, a tip party and a four-party or multiparty number, as well as the connections for PBX

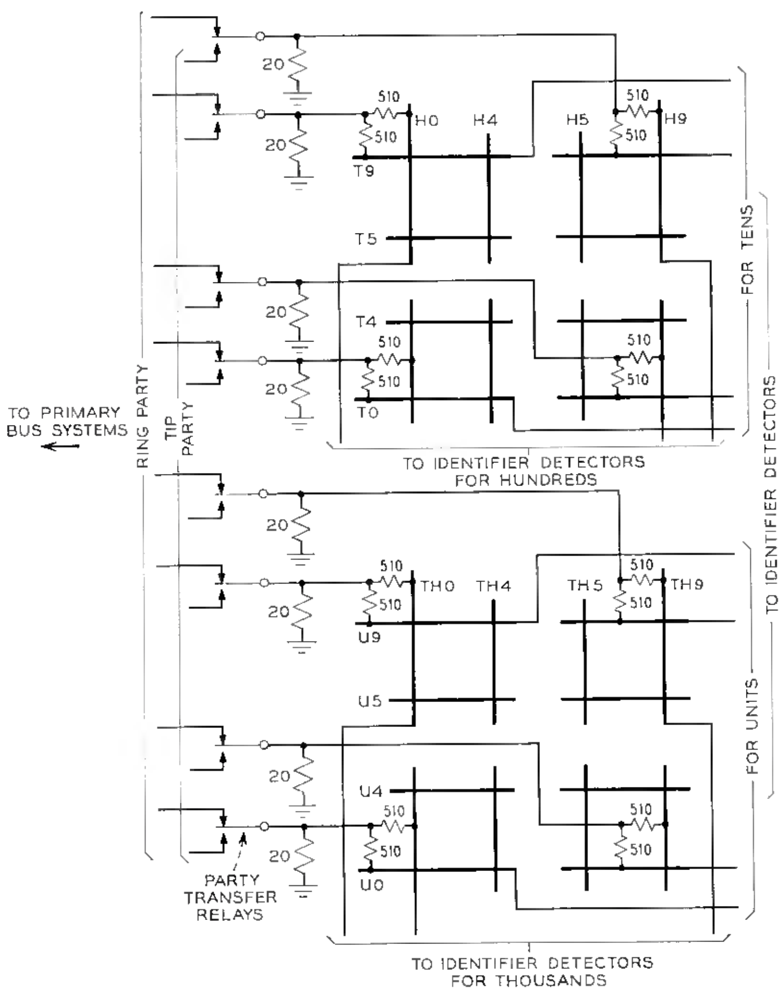


Fig. 6 — Secondary bus systems.

numbers which will be described later. The connections between network terminals and bus terminals are solderless-wrapped, with short bare-wire straps being used for adjacent terminals and insulated wire for the others. Changes in party line assignments are easily effected by unwrapping the two straps (one if multiparty) and wrapping two new ones.

The secondary bus system has been so named because the term is useful in understanding its circuit function in conjunction with the primary system. Actually, it does not use buses like those in the primary system. The resistors that make up the secondary grids are mounted on the identifier frames, together with the relays that serve to transfer these grids from the ring to the tip primary system when a tip calling party has been indicated. Fig. 6 shows the secondary bus systems in an array that is convenient for circuit understanding. Each bus is divided at the middle and its two halves are connected to separate primary windings of the input transformers in the amplifier detectors. These primary windings are connected opposing each other so that the noise and other unwanted signals on the secondary buses will be largely cancelled out.

## IV. IDENTIFICATION ON PARTY LINES

On calls from two-party lines, party information must be obtained and forwarded to the identifier so that the proper one of the two primary bus systems may be switched in before number identification. In the No. 1 crossbar system, tip and ring party information is registered in the originating marker in the standard manner while the call is being switched. This information is forwarded to the ANI trunk while the marker is setting up the switch linkages. From the trunk, the information is passed on through the outpulser link to the outpulser and then to the identifier. In the panel system, the district selector makes the party test and records party information but it is not feasible to pass the information forward to the trunk. Therefore, the ANI equipment must make a party test of its own. This is done by the outpulser, which recognizes the conventional ringer ground through the switchhook as indicating a tip party. In the step-by-step system, a party test is made by the ANI trunk during the interdigital time between the first two digits dialed after the trunk is reached, and the result is forwarded to the outpulser, as in the case of the No. 1 crossbar trunk.

As indicated previously, calls for four-party and multiparty rural lines will not be individually identified but will be recognized as calls requiring operator identification. This recognition is made by connecting the networks for these lines to the third vertical bus in the primary bus system and providing a special multiparty detector in the identifier to examine the output of the multiparty buses, after attenuation to simulate losses in the secondary bus system. In the event that an economical method for identifying the parties on a four-party line is obtained, automatic identification of these numbers can be made by providing an additional primary grid system for the third and fourth party numbers. By confining these numbers to selected hundreds groups, the additional grid would need to be only a portion of a full primary bus system, and this would serve to minimize the cost.

### V. PBX FEATURE

Since calls from all lines in a PBX are ordinarily billed to a single account number, it is desirable in any automatic message accounting system to be able to record all these calls by account number on the original tape. This avoids cumbersome translating procedures in the accounting center. With ANI, this feature is readily available. The network corresponding to the billing number, like any ring-party customer, is connected to the ring-field primary bus system. The networks representing the other lines in the PBX, however, are not connected to the bus system at the locations where they are mounted. These are multipled, as shown in Fig. 5, to the terminals of the directory or billing number network. Electrically, therefore, they act as if they were all connected to the primary bus grid at the same location as the directory number network. This arrangement can be used for nonconsecutively as well as consecutively numbered PBX lines, although the nonconsecutively numbered lines require longer jumper wires for multipling. Some PBX groups may extend beyond a single number network frame. To care for these cases, terminal blocks are provided at the top of all number network frames and interframe tie cables furnish the connections between frames.

As indicated earlier, in No. 1 crossbar there may exist a special category of numbers sometimes called "X numbers" and sometimes "coded numbers" which are outside the 10,000-number series comprising a central office unit and which serve as additional lines for PBX groups. It is desirable that calls from these lines, like those from other PBX lines, be billed to a common billing number in the 10,000 unit. This is accomplished electrically in the same manner as for the nonconsecutive PBX lines. Physically, since they have no position in the 10,000-number grid, they are connected to number networks mounted on a separate frame and cabled over to the base-number network.

## VI. OFFICE IDENTIFICATION AND OUTPULSING THE CALLING NUMBER

During its search for the calling number, the identifier scans the groups of secondary buses by connecting its detectors to the thousands buses of each office secondary grid, one after another. Meanwhile, the identifier keeps tract of its progress and, when it finds the signal, it grounds a corresponding lead to the outpulser, thus enabling that circuit to register the office of the calling customer. During this action, the particular thousands digit registered in the identifier is being transferred to the outpulser. Thereafter the identifier scans the hundreds, tens and units buses and registers these digits in the outpulser. Although identification is made on a one-out-of-ten basis, a translation is introduced so that registration in the outpulser is on a two-out-of-five basis.

With the registers in the outpulser full and checked against missing or extra information, and the digit representing the office translated to the corresponding three-digit office code, the outpulser releases the identifier and starts outpulsing the calling number to the CAMA office. The information is sent in the following order: KP signal, information digit, three-digit office code, four numerical digits and ST signal. The KP and ST signals use the conventional frequencies that serve to actuate a receiver at the beginning and end of a sequence of information. The information digit serves to indicate one of four conditions:

1. Calling customer identified automatically.
2. Calling customer on a four-party or multiparty line, and therefore requires identification by the CAMA operator. No office or numerical digits are sent for these calls.
3. Calling customer is under service observation, and therefore the AMA record for his call requires a service-observing mark in addition to the usual information.
4. Calling customer could not be identified because of trouble in the automatic equipment. This condition requires identification by the CAMA operator. No office or numerical digits sent for these calls.

When all digits have been outpulsed, the outpulser is released and the trunk is closed through for the talking condition.

## VII. AUTOMATIC NUMBER IDENTIFICATION IN COMBINATION WITH DIRECT DISTANCE DIALING

The introduction of direct distance dialing (DDD) eliminates the need for an operator to control the switching of a call, and automatic number identification will remove the need for an operator to obtain and record the calling number information. Hence it is natural that the two facilities would appear together in many areas. However, there are no design or operating conditions that make this imperative. Automatic number identification may be used in No. 1 crossbar and panel offices for seven- and ten-digit calls switched through tandem or toll CAMA offices. For the ten-digit calls, the called number is always outpulsed on a multifrequency (MF) pulsing basis and this requires an "MF-type" ANI outgoing trunk in the local offices. Seven-digit calls originating in these offices and switched through tandem may use the panel call indicator method of outpulsing the called number and for this a "PCI-type" ANI outgoing trunk is provided. Other seven-digit ANI calls use the "MF-type" pulsing.

Direct distance dialing calls in step-by-step offices with ANI may be switched through tandem, toll, No. 5 crossbar or step-by-step intertoll offices equipped with CAMA. These calls outpulse the called number on a dial pulse basis, and usually require a preliminary or directing code to switch the call through the originating step-by-step office. Offices with automatic ticketing, whether converted to AMA or not, are not candidates for ANI, since they already have their own arrangements for identifying the calling customer.

## VIII. CIRCUITS

### 8.1 *Trunk Circuits*

The fundamental requirements for the ANI trunk circuits are similar in the three switching systems. These trunks must be able to recognize the correct time to perform the ANI function and then seize an outpulser through the outpulser link. They must participate in several ways in the party-test function before number identification, and they must provide a path between outpulser link and outgoing cable over which the outpulser can forward the calling number after it has been identified. Then, after release of the outpulser, they must provide a transmission path with talking battery and supervision toward the calling customer and trunk supervision toward the tandem end. Also, they must provide the necessary sleeve ground to hold the originating switch train.

Due to inherent differences in the three switching systems, it has been found best to design separate trunks for each. Furthermore, variations within each of the systems in methods of pulsing and signaling have resulted in two types of trunks for each system. In No. 1 crossbar and panel, one type is provided when the called number is to be transmitted by MF pulsing and the other type when PCI pulsing is used. In step-by-step, one type of trunk is provided for loop signaling and the other for the so-called "E and M lead" signaling which is required for the longer distances and when voice repeaters or carrier circuits are used.

Crossbar and panel trunks must receive a signal from CAMA indicating readiness to receive the called number. For MF circuits, this signal is a momentary reversal of battery and ground and is relayed through the ANI trunk and back to the DDD sender as a go-ahead signal. For PCI circuits, the corresponding signal is the removal of battery and ground at the CAMA end of the trunk. At the time this occurs, the ANI trunk is cut through and the signal is transmitted directly to the subscriber sender (in this case, there is no DDD sender connected).

All the ANI trunks need a "start identification" signal from CAMA to indicate when that equipment is ready to receive the calling number. Crossbar and panel trunks must also recognize when the district junctor or selector has reached the cut-through position. Only then is the sleeve continuous, as required for transmission of the identification signal. Receipt of the start identification signal, together with detection of district junctor or selector "cut-through," when required, causes the ANI trunk to initiate the identification function.

### 8.2 *Outpulser Link Circuit*

This circuit uses a six-wire, 100-point crossbar switch as the linkage medium for connecting trunks to outpulsers. One switch is required for each group of ten trunks. The control features are very simple and require one preference chain relay for each trunk and a group of three control relays per outpulser for each group of ten trunks.

### 8.3 *Outpulser Circuit*

The outpulser has a number of functions to perform. On some calls, as described earlier, it will receive party information from the trunk and on others it will be told to make party test by a signal from the trunk. Then the outpulser seizes an identifier, which, as it finds the signal, registers in the outpulser the office identity and numerical digits of the calling customer. If the calling customer's line is one on which service observations are being made, the connected service-observing equipment will operate a service-observing detector in the identifier. An indication to this effect is registered in the outpulser. As the digits are registered in the outpulser, they are checked and the digit representing the office is translated to the three digits representing the calling office code. Outpulsing to CAMA then takes place, with the calling number preceded, of course, by the information digit. This outpulsing is done on a multifrequency basis at the conventional rate of approximately seven digits per second.

The outpulser is provided with checking and timing features so that it can detect a trouble promptly. In this event, the outpulser calls in a trouble-recording medium known as the "trouble ticketer" and provides it with the significant information preparatory to printing the trouble record. The trouble ticketer is a new facility introduced with the ANI equipment and will be described later. After making the trouble record, the outpulser makes a second trial identifier seizure.

### 8.4 *Identifier Circuit*

Consider a building with several central office units and a common group of identification equipment. For each ANI call, the identifier must test the thousands secondary buses for each office, one after another, and, when the signal is found, test successively the hundreds, tens and units buses in that office. Fig. 7 shows how the identifier performs the steering and testing functions and registers the results of its search in the outpulser.

During the search from one office to another, the thousands steering relay THS in the digit steering circuit is held operated. Relays OF0, OF1, etc., in the office-steering circuit operate successive identifier connector thousands relays TH in the secondary network and bus connector circuit. These relays close, in succession, the detector inputs to the thousands secondary buses of the different offices. During this time the register-steering circuit holds the detector outputs closed to the thousands translation circuit relays THR0-9. When a signal is found, the operated detector energizes its associated THR- relay representing the digit received. The THR- relay operates two-out-of-five thousands relays TH- in the thousands register of the outpulser circuit, thus registering the thousands digit of the calling customer's number. At the same time, the identifier office-check relay OFK operates and, in conjunction with the energized office steering relay OF-, operates an office relay OF- in the office register of the outpulser circuit. This latter OF- relay registers the office containing the calling customer's number. Further advance of the office steering circuit is blocked by relays not shown.

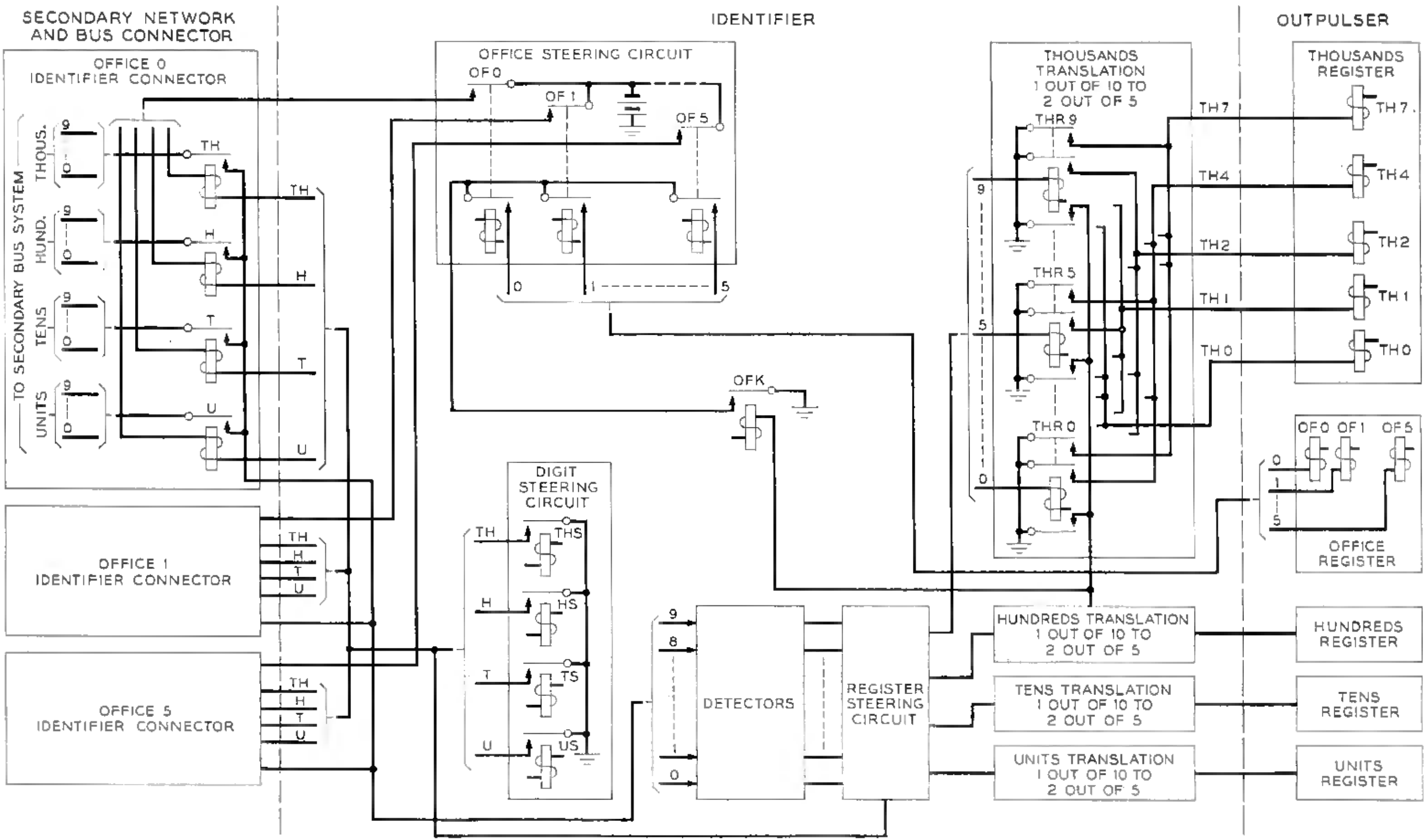


Fig. 7 — Identifier and outpulser steering and register functions.

Meanwhile, the operation of the OFK relay has brought the digit steering circuit into play, and it operates, successively, the hundreds, tens and units steering relays HS, TS, and US over paths not shown. These relays operate the corresponding secondary network and bus connector H, T and U relays, which serve to connect the detector inputs successively to the hundreds, tens and units secondary buses of the identified office. At the same time and in synchronism with this sequence, the detector outputs are connected to the hundreds, tens and units translation relays, which function to register the corresponding digits in the proper outpulser registers. During periods of light load, the identifier will not only scan successive office buses until the signal is found but will continue its search thoughout the remaining offices, in order to pick up troubles that cause wrong detector operation. The action of the circuit in case it encounters a trouble condition will be described later.

In one identifier group, which may serve as many as six central offices, only one identifier can be connected to the bus system at a time. Therefore, it is important that the search be completed quickly so as not to delay the traffic. The average identifier holding time is approximately 0.27 second for the largest groups and is less, of course, for others. With this speed of operation, the "one-at-a-time" identification method will care for traffic greatly exceeding the amount now foreseen for any group of six offices. In buildings having more than six offices, two or more independent identifier groups will be furnished and may operate simultaneously. Features in the detectors for discriminating between real and spurious signals require that the detectors be connected to the buses for 12 to 15 milliseconds for each test. Additional time for operating the register relays in the outpulser and for transferring detectors between tests, as well as for the initial circuit preparation, bring the over-all time up to the value quoted.

### 8.5 *Operation in Case of Identification Failure*

The identifying signal may be blocked temporarily in the panel system if the brushes of a hunting selector bridge a grounded terminal to a terminal carrying a signal. It may also be blocked temporarily in a step-by-step office as a selector wiper hunts across a terminal carrying a signal. These conditions can result in a failure to identify one or more digits of the calling number. To care for this situation, the identifier is arranged so that, if digits are missing, it will make a retest to fill in the missing digit or digits. If the retest is successful, no trouble record will be made.

If the identifier is unable to make an identification in two tests, a trouble is assumed and the outpulser is so notified. This circuit seizes the trouble ticketer (as previously described) and gives it information for a trouble record. The outpulser then releases the identifier and makes a second trial, using the other identifier, if available. In the event that the second trial should fail in two tests, the outpulser sends out the proper information digit to call in the CAMA operator for identification.

## IX. TRANSMISSION AND DETECTION OF IDENTIFICATION SIGNAL

Considerable study and testing, both in the laboratory and in the field, was done to determine a suitable voltage and frequency for the identification signal and to develop a reliable amplifier-detector. The upper limit on the signal voltage was found to be controlled principally by induction into adjacent circuits on panel office selector banks and consequent disturbance to transmission. Field tests indicated that the voltage of the signal on the switch train sleeve must not greatly exceed two volts, assuming operation to be within the frequency range imposed by limitations to be described later. The signal undergoes considerable attenuation, particularly as it is transmitted through the translating resistors of the number networks and through the coupling resistors between primary and secondary bus systems. Fig. 8 is a simplified circuit showing the path of the signal from the directory number sleeve to the amplifier. This path is, in effect, a series of voltage dividers. Typical voltages are given at successive junctions along the path. The nominal value of the final voltage at the input to the amplifier-detector is 90 microvolts. This signal must be amplified to approximately 90 volts to operate the registering device, representing a voltage amplification of a million times, or 120 db.

The determination of a suitable frequency for the identification signal was made after experimentation with several lower values in the field. At 1,900 cycles, the induction crosstalk into the panel selector banks was sufficient to be audible to customers using other circuits in the office. Frequencies in the nighborhood of 200 cycles are clear of this objection, partly because of the reduced sensitivity of the ear and partly because of lower coupling. However, this frequency would be more difficult to distinguish from surges and would require a longer operate time in the detector, as will be evident later. When the frequency was raised from 1,900 to 5,800 cycles, the noise on disturbed circuits was reduced to an insignificant level. This was due in part to the effect of lessened receiver sensitivity and in part to masking by noise from other sources in this frequency range. Regarding the upper limit on the frequency, a major consideration is the avoidance of interference into program circuits and carrier facilities by secondary induction. Although these circuits would not be in the same cable with the signal carrying sleeve, they may be in the same outside cable with the identified customer's pair, which does pick up some signal because of electromagnetic coupling with the associated sleeve lead through the switch train.

As for the transmission path of the identification signal, large variations must be met in the conditions contributing to the load on the sleeve lead and in the elements causing momentary interference. As previously described, the selectors in the panel and crossbar switch trains are in the "operator talking" condition, and this requires a very low-resistance dc ground on the sleeve from the ANI trunk during identification. To meet this requirement without placing too heavy a load on the oscillator, an inductor is used which has approximately 0.5 ohm dc resistance and 450 ohms impedance to the 5,800-cycle signal. Panel offices, unlike crossbar and step-by-step offices, present a 2-microfarad capacitor in series

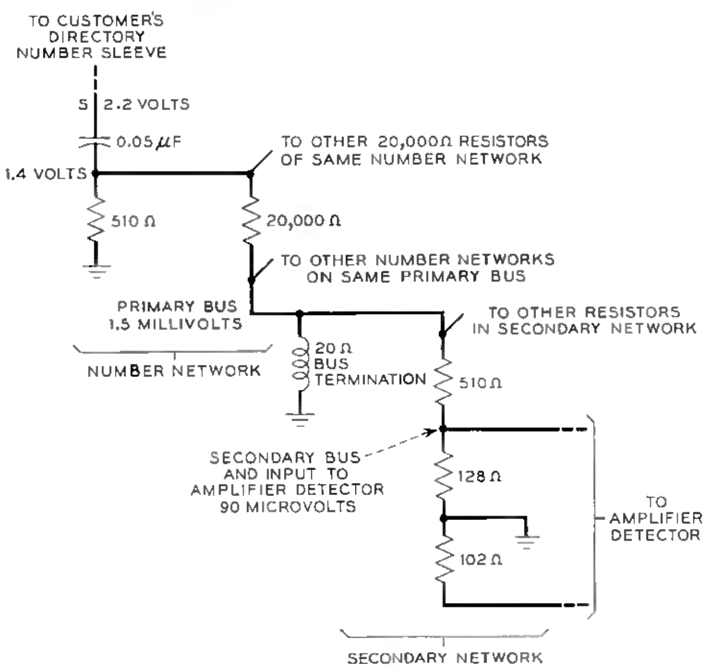


Fig. 8 — Path of identification signal through number network and bus systems. Voltages shown are nominal values.

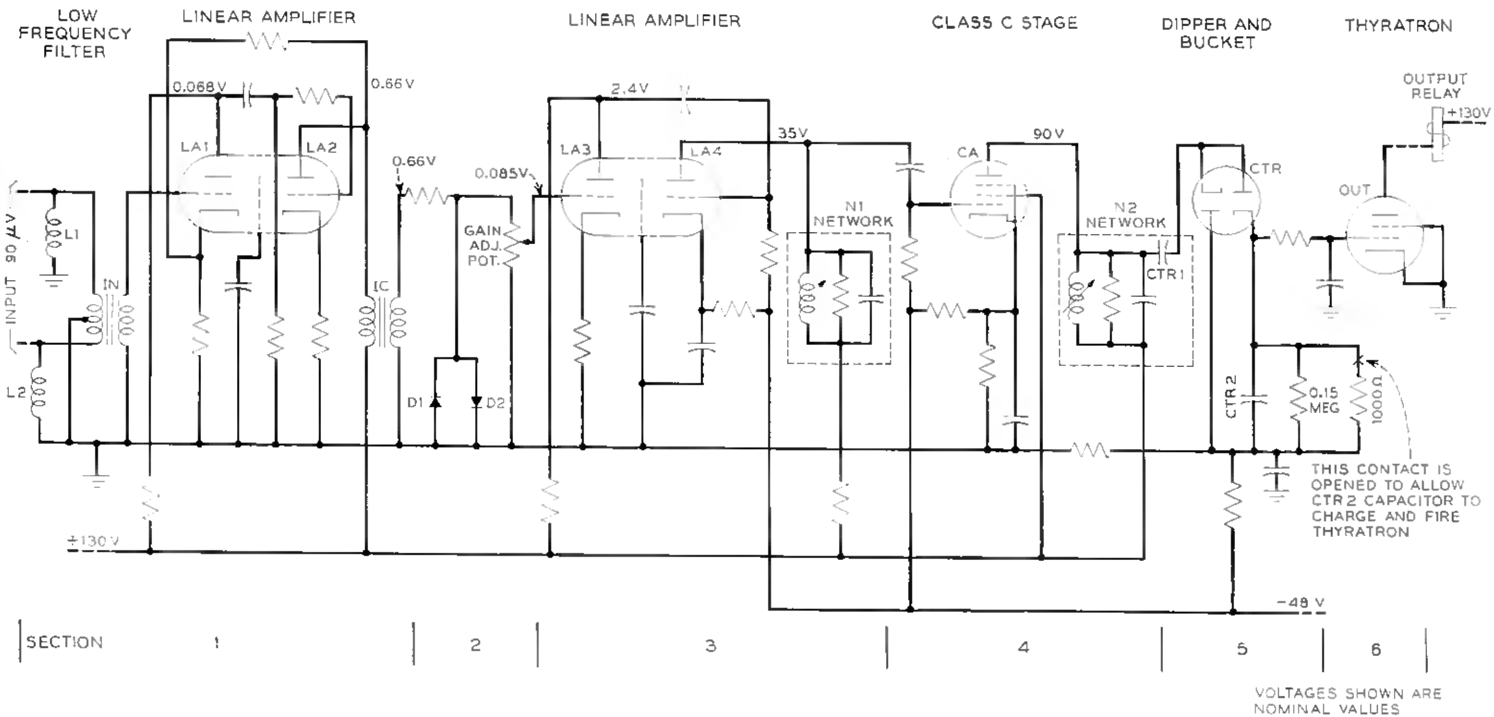


Fig. 9 — Amplifier-detector circuit. Voltages shown are nominal values.

with the switch train sleeve, and either 220 ohms to battery or 112 ohms to ground on the number network side of the capacitor. Additional resistance in the transmission path may be encountered at the sequence switch brushes and multiple bank brushes, or even at relay contacts. All systems have a great variety of relay or crossbar switch hold-magnet windings connected to the sleeve. On the identified sleeve, these constitute an additional load. On nonidentified sleeves, these magnets may be releasing and producing surges and voltage peaks which impose a nonoperate requirement on detectors that are examining the corresponding buses at the time. All of these conditions contribute to the operating requirements and margins of the amplifier-detectors and of the ANI system.

## X. FEATURES OF THE AMPLIFIER-DETECTOR

In much of conventional amplifier-detector design, the margin between operate and nonoperate conditions is largely determined by the signal-to-noise ratio. Where both signal and noise are in the same frequency band, discrimination is made by amplitude. In ANI, however, there are times when the unwanted signal approaches or even exceeds the wanted signal in amplitude and is indistinguishable by frequency. Fortunately, these times of high unwanted signal are of short duration, because they are caused by various transient conditions in the switching systems.

Some of the features of the amplifier-detector that provide the needed amplification and yet retain adequate discrimination against the summation of unwanted signals are of interest. Fig. 9 is a simplified sketch of the amplifier, divided into sections for descriptive purposes and with representative voltages indicated at significant points. As previously described, the two input leads of the amplifier are connected to the two halves of the secondary bus and the primary windings of IN transformer are connected opposing. With this arrangement, much of the noise on the secondary buses is cancelled out. The L1 and L2 inductors, in conjunction with the reflected capacitative impedance looking into the primary winding of IN transformer, constitute a bandpass filter. As low-frequency components of noise enter the amplifier, this filter removes practically all of each surge except the spikes associated with the steep wave fronts. This is necessary in order to prevent surges and 60-cycle pickup from blocking the legitimate signals by overloading the amplifiers. The spikes are of such short duration that, in spite of their considerable amplitude, they do not cause blocking. The first section includes a twin-triode linear amplifier LA1-LA2 with the two halves in cascade and with transformer coupling to input and output. The voltage gain in this section, which includes the IN and IC transformers, is some 7,300 times, or 77 db.

The second section uses two oppositely poled silicon diode clippers D1 and D2 which limit both peaks of the spikes before they can reach the tuned circuit in the next section. Since the tuned circuit would oscillate, it is necessary to limit the sudden voltages which it receives. This application of the silicon diodes utilizes a characteristic that makes them, in effect, reverse-biased up to 0.6 volt in the forward direction. The identification signal is too low in this stage to be affected by the clippers. The adjusting potentiometer GN is just beyond the clippers. With normal settings, a loss of about 18 db exists in the second section.

The third section is another twin-triode amplifier LA3-LA4 working into a 5,800-cycle tuned network designated N1. The resistance in this network is provided to reduce any tendency to oscillate. The voltage gain in this section is about 410 times, or 52 db.

The fourth section, using tube CA, is a class C stage designed to eliminate signals below the expected minimum operating signal. Since this stage is biased beyond cutoff, it ignores signals below the desired level. Otherwise, these signals would help build up the grid voltage at the output tube and impose a more severe nonoperate condition on the detector. This stage has a voltage amplification of about 2.6 times, or 9 db. It has a tuned plate circuit designated N2 in Fig. 9.

The fifth section is a "dipper and bucket counter" comprising a double-diode CTR and two capacitors. On each negative half-cycle of a signal the small capacitor (CTR1) is charged, and on each positive half-cycle it spills its charge into a larger bucket capacitor (CTR2) which therefore gradually increases its voltage in steps until it operates a thyratron tube in the sixth section. A resistance is bridged around the bucket capacitor to enable the detector to forget comparatively small numbers of pulses resulting from random surges. When the detector is not testing for a signal, a low-resistance 1,000-ohm circuit is maintained across the bucket capacitor. The testing time is controlled by the identifier, which opens the 1,000-ohm circuit for an interval of 12 to 15 milliseconds.

From this description, it can be seen that the amplifier-detector not not only has the amplification necessary to raise a 90-microvolt signal to a usable value but also discriminates by filtering, by blocking out signals known to be spurious because they are too big or too small, and by counting an appropriate number of cycles of legitimate signal before being satisfied.

## XI. MAINTENANCE FACILITIES

In the event of an identification failure or certain other irregularities, a trouble record is made. This shows the identity of the various ANI circuits involved and the stage to which each had progressed at the time the trouble appeared. This record will be recorded by a ticket printer of the type used in step-by-step automatic ticketing. After the trouble record, a second attempt is made using the other identifier when provided. If this attempt should also fail, due to the trouble being in the network or somewhere else outside the identifier, no second trial trouble record is normally made. This is because the trouble ticketer will be busy printing a record of the first failure. If it is desired to take a trouble record after second trial instead of first trial failures, a key must be operated that is provided for this purpose. Trouble records cannot be taken after both first and second trial failures because the functioning time of the trouble ticketer is about six seconds, and to wait for it to become free would seriously extend the second trial identifier-holding time. In the event of a second trial failure, the appropriate information digit is sent to cause the CAMA operator to make the identification. A special trouble record will be made at tandem and will include the calling customer's number as obtained by the operator; this record will be returned or reported to the originating office for maintenance use.

Testing equipment is provided for the ANI trunks, outpulsers, identifiers and networks, with means for selecting any combination of these circuits. Tests are made by selecting the trunks directly rather than through the switch train. Normally, the identification signal, instead of following the switch train path back to the number networks, is diverted to an artificial test network. This facilitates applying marginal tests to the detectors to anticipate approaching troubles. It also permits holding the test connection while service calls are allowed to use the regular number network and bus system. If it is desired to include the regular number network system in the test path, a patching connection must be made on one of the number network frames to a particular number network. In either case, the identified number, instead of being pulsed out to tandem, is steered back to the test equipment and displayed on a set of lamps.

When new customers are given service or when line numbers are changed, it is important that the corresponding service order work be checked to verify correct identification on calls from these lines. This identification is made by the ANI equipment. In No. 1 crossbar and step-by-step offices, existing test train facilities are used to direct a test call to the line to be verified. In panel offices, this connection is established by patching at a distributing frame. In all cases, the test connecting facilities will have access to the ANI outpulsers through the outpulser link. After the line to be verified is reached, the ANI equipment is actuated and the outpulser obtains an identification by the normal process. Instead of forwarding this number to a distant CAMA office, the outpulser will display it on lamps convenient to the observer.

A new facility will be available in No. 1 crossbar and panel offices for identifying lines that have been routed to permanent-signal holding trunks because of receiver off-hook or line trouble conditions. Here an auxiliary circuit will be provided with access to the outpulsers through the outpulser link. To determine the number of a line that has become connected to a permanent-signal trunk, the test man will patch the new auxiliary circuit to the jack of the permanent-signal trunk. Upon receipt of a start signal, the ANI equipment will function and the identified number will be displayed on a special ticket produced by the trouble ticketer. Since line troubles might or might not give a tip party indication if party test were made, the identifier will always test first in the ring field on permanent-signal calls. If no identification is made, it will then test in the tip field, since the trouble may be on a party line with only a tip station connected to the identification buses. It is not planned to provide this facility in step-by-step offices because, in the absence of any common control equipment, routing of these calls to permanent signal holding trunks is not feasible.

## XII. CONCLUSION

The development of automatic number identification is a major advance toward full automation of all customer-dialed calls in the Bell System. It provides a simplified procedure for the customers, faster service in completing calls and greater accuracy for billing records.

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# Contributors to This Issue

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# The First Ten Years of the Transistor

## A Guest Editorial

By W. O. BAKER

The technology in which it was to be used was already about 35 years old when Bardeen, Brattain and Shockley discovered the transistor ten years ago. This circumstance has led to an unusual course of development and application. The transistor has affected the technical and commercial growth of electronics and communications more quickly than might have been expected from the somewhat offhand newspaper comments of July 1, 1948. (The news story about the transistor was carried on page 46 in one famous case).

The quick growth of the transistor art after the original discovery came about because electronics based on an elaborate functionality of the vacuum tube as amplifier, modulator, oscillator and such in our technology was already preeminent. The transistor provided an alternative to the vacuum tube in these functions, so electrotechnology, one of the most elegant intellectual attainments of history, was ready for any and all transistors.

This was as though Faraday's capacitors and dielectric principles had been first reported to a meeting of the A. I. E. E., or antibiotics had been announced in finished form to a congress of bacteriologists and clinicians who had already established therapeutic procedures for dealing with all classes of germs. Actually, electrical capacitors, inductors, resistors and thermionics were extensively explored before any uses were known for them; bacterial inhibition by antibiotics was studied for decades before medical application was worked out; even with war-time haste, several years of intensive effort separated Hahn and Strassmann's discovery of nuclear fission from the production of significant energy by the process. Thus, in each of these cases there was time to ponder, to try, but most of all to imagine, beyond any known framework of use, what the discovery would come to.

Much of this was denied the transistor in its early years. To a degree, perspective was lost in the astoundingly rapid exploitation of transistor function. In late 1952 there were commercial transistorized hearing aids. Their performance now surpasses Dr. Bell's highest hopes for comfort for the deaf. About 100,000 transistors for defense use studies were made the same year. Three million were made annually for auto radios by 1957, in the United States alone. The cost of this being a child prodigy, of this growing old in infancy, was early made clear by crises in reliability. An article in *Time* for September 7, 1953, said, "Still neither . . . nor other scientists have lost all hope concerning the difficulties then widespread in production and performance of the device." Satisfactory yields from production have been erratic, and economics consequently often frenzied.

But other costs may have been far greater, costs incurred because the world of electronics foolishly thought it knew all about the possible uses and the possible consequences of the transistor through experience gained long before the transistor was created. Because of this, we must at the end of this first decade give thought to what the discovery of the transistor means in a larger frame of reference than that even all of the present wonders of electrotechnology—communications, automation, control—can give it.

The transistor discovery has in many ways altered our age-old ideas about the electrical behavior of matter. For example, we are accustomed to the idea that positive and negative charges of electricity accessible to each other will amost immediately neutralize one another, and that common fluid media, for example, contain charged particles of opposite sign only because of stabilization as ions and the continuing replenishment of the supply by thermal agitation. In the crystals of which transistors are made, however, positive and negative charges do coexist for appreciable times in both the so-called p and n regions. Thus, p-type crystals having a multitude of positive charges contained in their rigid atomic skeleton also have electrons "free" among them. Such long-life times of opposite charges for recombination suggest an abundance of new effects, in addition to transistor action across junctions. For instance, the controlled recombination can give light emission which may itself some day be useful, and which already has revealed much about electrical charge interactions. The conductivity modulation of semiconductors contrasts them with metals.

Further, and equally exciting, are the phenomena related to the mobility of the charged particles. We find that, although the conditions for this new control of electric charges are indeed built firmly into the crystal lattice by the substitution of foreign atoms, a remarkable mobility of charge remains. Thus, responses of these systems to electromagnetic waves are lively.

Significantly, this liveliness is different from the acceleration of electrons in the vacuum tube, which is proportional to an electric field, and it is also different from the behavior of the electron cloud conceived to be in metals. For instance, in the crystals preferred for transistors, electrons move at about 1/1000th the speed of light and hit some atom and bounce off in a way that corresponds to a new motion in each distance of about 1000 atom diameters. They can keep this spirited pace over such long distances because of the perfect periodic arrangement of atoms in the crystal. Through such an array of atoms, electrons can proceed like waves, as Davisson and Germer discovered at Bell Telephone Laboratories more than 25 years ago. These electron movements are such that the speed and not the acceleration is proportional to the applied electric field. A pressing challenge now is discovery of some principle which will give us other ways of controlling this speed and thus, in effect, raise the frequency response of transistors from a few hundred megacycles up to thousands and tens of thousands. Existing knowledge seems inadequate.

The second main aspect of the transistor discovery which is influencing nearly all modern science and technology of solids is the novel forms of matter it evoked. Pfann's zone refining produced germanium and silicon crystals of such purity and relative perfection that the ideal simplicity of solid state theory was at last approximated. It can, in fact, be argued that silicon and germanium are the best understood solids. At least they were understood perfectly enough so that, in them, imperfections (in crystal order) were first identified. Thus a mystery as old as science and as crystals was exposed. It is because of such imperfections that the strength of metals and solid matter is generally so much less for both yield and fracture than the forces between the atoms of the ideal crystals would allow. The explanation is, as long guessed but not before shown, that wires and girders and bridges and wheels have to be the extra size they are because the atoms are missing from some places they ought to be in; that is, dislocations and vacancies limit the properties of real solids.

Such meaningful findings as these coupled directly back to the transistor technology itself, since imperfections control directly certain performance of the devices. This all shows what a few new ideas can mean to the entirety of science and engineering.

Likewise, of course, these exquisite refinements in the knowledge of what electrons and their positive equivalents or "holes" could do in suitable crystals have engendered new families of devices, of deep import for the future. Thus, in the past decade, have been created junction devices yielding nearly perfect rectifiers, electronic switches, photo and solar cells and lightning protectors or nondestructive fuses. Indeed, such controlled breakdown has been made to give out visible light associated with microplasmas, with implications for visual communication which are just now being pondered.

These aspects of the transistor discovery have fired the imaginations of engineers and scientists throughout the world. While substitution for the vacuum tube has itself attracted many licensees of patents through the Western Electric Company, including agreements in England, France, Germany, Switzerland, Sweden, Italy, the Netherlands, Japan, as well as North America, the recognition also of additional features is seen in the accompanying active and historic exchange of technical information, which is growing rapidly. Thus, beginning with a couple of international symposia early held at Bell Telephone Laboratories, there has now arisen a vigorous worldwide system of conferences in this new field. This may be illustrated by the Solid State School held at Varenna on Lake Como, Italy, in 1957, the International Conference on Solid State Physics in Electronics and Telecommunications in Brussels in 1958, and the forthcoming International Conference at Rochester. One should also mention the formation of a new division of solid state physics of the American Physical Society which provided 18 of 24 sessions at the Chicago meeting of the Society in March of 1958. Publication has been equally strong evidence that the transistor has had a bearing on modern science far beyond that of a specific electronic device. Several thousand papers have now been published in journals all around the world, including the Indian *Journal of Physics*, the Czech *Slaboproudy Obzor* and the Russian *Uspekhi Fizicheskikh Nauk*.

The results of this world-wide interest and attention have in ten years, as in the case of transistor technology, spread transistor science far beyond its original scope. Thus the most perfect solids, the nearly ideal crystals, have enabled qualities of separate electrons rather than characteristics of electron swarms to be studied and eventually to be exploited. Paramagnetic systems arise from the unpaired electron spins in germanium and silicon crystals containing certain well-known donor atoms. These exhibit resonance of the thermal motion of the isolated magnetic electrons with radio frequency or microwave frequency fields, in the presence of an external magnetic bias. A great new realm of behavior of electric waves has been revealed. This is in their interaction with electron spins in which energy, and hence signals, can be exchanged among waves and crystals not by the movement of electric charges, but by the reorientation of the magnetic moments of electrons. These may be distributed in nonconductors easily accessible to the penetration of even microwaves and, of course, this realm extends far beyond semiconductors. Solid state MASERS, as some of the devices embodying these principles have come to be called, promise a new freedom from noise in electronic amplifiers, so that signals from outer space, and indeed the signals from the stars which radio astronomers study, will be exactly received. Further, the international scientific interest in both the concept and the compositions of transistors has recently succeeded in identifying, by the methods of cyclotron resonance, that electronic charge carriers, the essence of all communications and electrical engineering, can exhibit even in metal systems more than one effective mass. This means that knowledge of conductors is being sharply revised and extended, since details of the band structure concept are being revealed in formerly unimagined detail. This study has indeed finally illuminated the detailed structure of graphitic carbon—the patriarch of electronics—in filaments, microphones, resistors, electrodes and contacts.

But finally, what does all this replacement of the vacuum tube at one end and advancement of the frontiers of science at the other during the decade of the transistor mean for communications itself? Any comment, and especially any comment looking to the future, ought to be related to the shape and philosophy of electrical communications expected to come. Therewith a certain harmony in time appears, because this is also the tenth anniversary of Shannon's *Mathematical Theory of Communication* and of Wiener's *Cybernetics*, as well as the 30th anniversary of Hartley's forerunner of modern analysis of the communication of information. Shannon's monumental theory is particularly rich in its applications to situations involving discrete coding, as in the so-called "digitalization" of sight and sound. That is, instead of using electromagnetic waves whose amplitude or frequency are analogues to acoustic or visual signals, these same signals are represented by pulses, giving on-or-off, 1-or-0 conditions. Such pulse systems are particularly suited to modern wideband transmission media.

The transistor and its associated devices are perfectly proportioned for communications and data handling, for control and sensing, based on pulses. The pulse code modulation now developed for exchange trunk carrier in telephony seems destined also for widespread video transmission by sending 70 million pulses per second. This now seems possible over ordinary pairs of wires, providing that simple transistor amplifiers are frequently inserted in the lines. Important new uses of existing plant facilities may soon appear. But a still greater unity of communications systems based on binary signal processes is forthcoming. Switching and the related logical elements of computing comprise extensive digital operations. While most of the work here may be well done by improved diodes, which came along with transistor technology, the transistors themselves are, as in transmission, elegantly suited for the amplifiers, inverters, pulse stretchers, power regulators and tone generators necessary in the new schemes of communications, in which at last the technical essences of transmission and switching are joined.

Junction devices seem uniquely suitable for digital operations; on-or-off, plus-or-minus now look like the alphabets of communication—through neurons, minds, telegraphs. Will it come to pass, that, after all, the vacuum tube electronics will not be converted to transistor electronics, but rather, that it will just be forgotten? Perhaps transistors will next enjoy Shaw's kind of childhood—one too wonderful to have been wasted on the young.