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The AEROVOX Research Worker

The Aerovox Research Worker is a monthly house organ of the Aerovox Corporation. It is published to bring to the Radio Experimenter and Engineer authoritative, first hand information on condensers and resistances for radio work.

VOL. 7, NO. 12 — DECEMBER, 1935 — 50c per year in U.S.A. 60c per year in Canada

Laws of Alternating Currents

PART 2

By the Engineering Department, Aerovox Corporation

Last month's problem and its solution have been found to be rather complicated. It is possible to simplify the work greatly by means of another algebraic method which avoids the continuous change from series circuit to equivalent parallel circuit and vice versa. Moreover this method employs laws which are similar to the laws employed for d.c. circuits. In this installment it is proposed to clarify this method and to solve the same problem as last month for purposes of comparison.

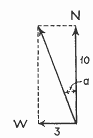

Fig. 1

Readers may have observed that for each step in last month's calculation vector diagrams could be drawn and that the whole problem could be solved graphically by drawing the vectors to scale. The one drawback to such a procedure would be that in some instances the paper would be too small and the construction would be inaccurate if one vector were several hundred or thousand times as long as another. We can however obtain the same result algebraically by a system which might be compared with bookkeeping. As an example let us return to the problem of the ship.

Figure 1 is a reproduction of last month's figure 1. A ship sails North at a speed of 10 knots; there is a current which makes the ship move westward at the rate of three knots. The resultant speed and direction is found as before. The two components of the resultant happen to be at right angles to each other and in the new method it is a rule to express each vector as the sum (vectorial sum) of two vector components which lie along two perpendicular axes. The position of these axes has been chosen for greatest convenience. In such a notation the resultant velocity of our ship would be written

$$Vs = N.\,10\ \text{knots} + W.\,3\ \text{knots}$$

It is true that this notation does not show immediately the absolute magnitude of the speed nor the direction, but these two quantities can be found immediately from the above expression whereas the form of the equation lends itself to the addition of vectors. Suppose a man walks on the ship from stern to stem (going North) at a rate of two nautical miles per hour, or, two knots, what is the direction and speed of the man with respect to the ocean? In our system of notation, the speed of the man with respect to the ship would be written as "No. 2 knots" because it happens to be along one of the axes and does not have to be resolved into components. The speed of the man with respect to the water is now found by adding the expression for the speed of the man and that for the speed of the ship, which gives the resultant

$$V = N.\,10 + 2\ \text{knots} + W.\,3\ \text{knots} = N.\,12\ \text{knots} + W.\,3\ \text{knots}.$$

The vector diagram of Figure 2 shows that this is true.

When two vectors are to be added, when both make an oblique angle with each one of the axes it is necessary to resolve both of them into components and to add components along the same axis. For example, consider an air-

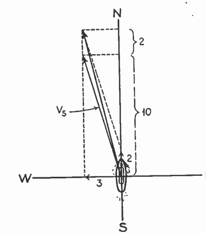

Fig. 2

plane flying at 100 miles an hour in a Northwesterly direction with respect to the surrounding air. This air, however, moves at a velocity of 40 m.p.h. in a Southwesterly direction and the airplane takes part in both motions. What is the direction and the velocity of the airplane?

Although it is not essential, suppose the axes in the direction N-S and E-W are retained. The speed of the airplane must then be resolved into two components, one towards the North and one towards the West and this would be written

Va = N. 71 m.p.h. + W. 71 m.p.h.
and similarly

Vw = S. 28 m.p.h. + W. 28 m.p.h.

The vector sum of the two is found by adding the components along the N-S axis and those along the E-W axis separately:

V = N. 43 m.p.h. + W. 99 m.p.h.

This problem has been illustrated in Figure 3. When the absolute speed is required, it is found from the well known Pythagorean theorem

$$V_S = \sqrt{43^2+99^2} = \sqrt{11,650} = 108 \text{ m.p.h.}$$

and the direction is given by the angle a

$$a = \tan^{-1} \frac{99}{43} = 66°36'$$

Vectors representing electrical quantities are treated the same way and since there is no North or South here, the directions are chosen for greatest convenience. Resistive components are measured along the X axis reactive components along the Y axis. It is necessary, when writing expressions for vectors, to mark the ones along X and Y axes so that the proper terms will be added and there be no possibility of mixing them. For reasons to be explained later it is customary to write a j in front of the reactive component. As long as the problem contains nothing but additions this j need not be considered anything but a marker, to prevent one from adding terms with a j to terms without a j. For example, in the coupling circuit between the stages of an amplifier there might be a condenser in series with a choke (see Figure 4). Supposing the condenser to have a capacity of .1 mfd and a resistance of 2 ohms, the choke having an inductance of 400 henries and a resistance of 5000 ohms. what is the total impedance at 400 cycles?

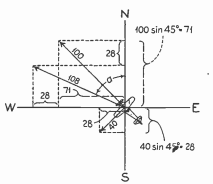

Fig. 3

It has become a convention to call a reactive component positive when it represents inductive reactance or the voltage across an inductance. Similarly, capacitive reactance or the voltage across a condenser will be marked by —j in our notation. Vectors representing current are just the opposite. A current through a condenser would be considered positive and through an inductance negative. That this must be so is easily understood when one remembers that vectors are rotating in an anti-clockwise direction. A voltage across an inductance is 90 degrees ahead of the current through it. When vector diagrams of voltages are made, the current vector is generally the reference vector so that the voltage vector must be made positive to be 90 degrees in the lead. Vector diagrams of currents have the impressed voltage as a reference vector so, in an inductive circuit when the current lags behind, the vector must be drawn the other way and in the notation it is negative.

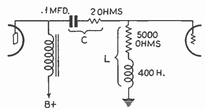

Fig. 4

Returning to the problem

$$Z_L = 5000 + j\, 6.28 \times 400 \times 400 \text{ OHMS}$$
$$= 5000 + j\, 10,060,000 \text{ OHMS}$$

$$Z_C = 2 - j \frac{10^6}{6.28 \times 400 \times .1} = 2 - j400 \text{ OHMS}$$

The impedance of the two in series is then

$$Z = 5002 + j\, 10,056,000 \text{ OHMS}$$

This is still in its vector notation. When writing the scalar value for the same impedance it is necessary to distinguish the symbol from the one representing the vector value. In many engineering texts, the scalar value is indicated by the same letter but placed between two vertical lines:

$$|Z| = \sqrt{5002^2 + 10,056,000^2} = 10,056,000 \text{ approx.}$$

This is the same impedance as the one in the previous equation but here we are concerned with its absolute value only without regard to its direction. From the above it should be clear that the addition of vectors does not offer any difficulties.

The next problem is to solve networks which involve multiplication. When resistances are in parallel we can find the resultant resistance easily by the equation

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

The idea is now to give the letter j such a value that it will enable us to use the same formulas for a.c. circuits as were used for d.c. circuits. This applies to formulas for parallel circuits as well as Ohm's Law. It is beyond the scope of this article to show all the reasons for giving to j the value it has but we quote only one. A vector along the X-axis, that is, a resistive impedance would be indicated by the notation x. Rotating this vector 90 degrees ahead can be accomplished by multiplying it with j, because jx would be a reactive impedance, 90 degrees ahead of x. Then multiplying the vector again by j should again result in rotating it 90 degrees ahead and it becomes resistive again; it is now negative and would be written — x. Thus, twice multiplying by j is equivalent to multiplying by —1 therefore

$$j^2 = -1 \qquad j = \sqrt{-1}$$

It so happens that when j is given this value, the laws for parallel circuits are the same as those used in d.c. circuits and Ohm's Law can be applied in the same way. Keeping in mind only that j^2 can be replaced by —1 the rest is just ordinary algebra. In algebraic textbooks, terms involving the imaginary value are using the letter i and this is also employed in mechanical engineering. Electrical engineers have to use j because the letter i is already widely used to denote the current.

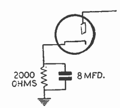

Fig. 5

Here is an example of the solution of parallel circuits and the use of Ohm's Law. Figure 5 shows the diagram of a tube having a bias resistor of 2000 ohms bypassed by a condenser of 8 mfd. Neglecting the resistance of the condenser, what is the impedance of the parallel circuit and what is the phase angle between current and voltage?

Applying the formula for parallel circuits:

$$Z_C = -j\, 200 \text{ OHMS} \qquad Z_R = 2000 \text{ OHMS}$$

$$Z = \frac{-j\, 200 \times 2000}{2000 - j\, 200} = \frac{-j\, 4000}{20 - j\, 2} \text{ OHMS}$$

This form has to be simplified to the general form a + jb and that is always possible. In order to remove the term containing j from the denominator, multiply both the denominator and numerator by 20 + j 2, which is called the conjugate impedance of the one shown in the denominator.

$$Z = \frac{-j\, 4000\,(20 + j\, 2)}{(20 - j\, 2)(20 + j\, 2)}$$
$$= \frac{-80,000j - 8000j^2}{20^2 - j^2 2^2}$$

$$Z = \frac{8000 - j\, 80,000}{400 + 4} = \frac{8000}{404} - j \frac{80,000}{404}$$
$$= 19.8 - j\, 198 \text{ OHMS}$$

$$|Z| = \sqrt{19.8^2 + 198^2} = 199 \text{ OHMS}$$

and the phase angle between current and voltage is given by

$$a = \tan^{-1} \frac{-198}{19.8} = \tan^{-1} -10 = -84°18'$$

Note that the angle is negative which means that the voltage lags behind the current. If the angle was found from the vectors representing the current, the angle would be found to be positive because the current is ahead of the voltage. This can be shown by applying Ohm's Law to the result and figuring out the current. The phase angle with respect to the reference vector (which is the voltage across the combination) will then be found positive and, of course, it will be the same.

$$I = \frac{E}{Z}$$

$$I = \frac{E}{19.8 - j\, 198} = \frac{E\,(19.8 + j\, 198)}{19.8^2 + 198^2}$$
$$= .000505\, E + j\, .00505\, E \text{ AMPS.}$$

and the phase angle

$$b = \tan^{-1} \frac{.00505\, E}{.000505\, E} = \tan^{-1} 10 = +84°18'$$

These examples demonstrate that the chosen value of j makes it possible to employ the same laws for a.c. circuits which are in use for d.c. circuits. When the correct signs are given to vectors at the beginning of the problem the results will automatically show the magnitude and direction of the desired vector.

Returning now to last month's filter problem Figure 6 again shows the same filter with the same constants. Note that the calculation which follows is now much shorter and gives the same answer. Due to the fact that some of the numbers have been approximated, rounded off, a small discrepancy was expected. Incidentally, multiplication of the large numbers has been done with logarithms which meant approximating the numbers to four or five significant figures. This has been done with the problem of last month and this month and this work is not shown here.

The first operation is to combine the branches A, B, and C, and add them to impedance D. It is done easiest by first combining branches A and C because they are both resistive.

Note that the parallel branch, representing the leakage of the condenser has the effect of adding only slightly to the series resistance of the condenser. In reality, the reactance has become slightly smaller, but so slightly that it cannot be seen in the three significant figures above. The actual value is now 332 ohms multiplied by 1,000,000/1,000,040.101124. This would make the reactive component 4 parts in 100,000 smaller and the capacity that much larger.

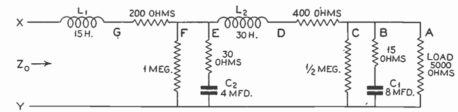

Fig. 6

The impedance of Zabc is found in accordance with the principles explained above:

$$Z_{AC} = \frac{Z_A Z_C}{Z_A + Z_C} = \frac{5000 \times 500,000}{505,000}$$
$$= 4950.5 \text{ OHMS}$$

$$Z_B = 15 - j\, 166 \text{ OHMS}$$

$$Z_{ABC} = \frac{Z_{AC} Z_B}{Z_{AC} + Z_B} = \frac{4950.5(15 - j\, 166)}{4965.5 - j\, 166} \text{ OHMS}$$

multiplying numerator and denominator by (4965.5+j 166)

$$Z_{ABC} = \frac{4950.5\,(15 - j166)(4965.5 + j\, 166)}{4965.5^2 + 166^2}$$
$$= \frac{4950.5\,(15 \times 4965.5 + 166^2 - j4950.5 \times 166)}{2468 \times 10^4}$$
$$= 20.5 - j\, 164.8 \text{ OHMS}$$

$$|Z_{ABC}| = \sqrt{20.5^2 + 164.8^2} = 166.1 \text{ OHMS}$$

This result must now be added vectorially to the impedance D. The absolute value of Zabc is not needed for that but it will be required later.

$$Z_{A-D} = Z_{ABC} + Z_D$$
$$= 20.5 + 400 + j\, 22,608 - j\, 164.8$$
$$= 420.5 + j\, 22,443.2 \text{ OHMS}$$

$$Z_{A-D} = \sqrt{420.5^2 + 22,443.2^2} = 22,447 \text{ OHMS}$$

$$Z_{EF} = \frac{Z_E Z_F}{Z_E + Z_F} = \frac{10^6\,(30 - j\, 332)}{1,000,030 - j\, 332}$$
$$= \frac{10^6\,(30 - j\, 332)(1,000,030 + j\, 332)}{1,000,030^2 + 332^2}$$
$$= \frac{10^6\,(30 \times 1,000,030 + 332^2 - j\, 10^6 \times 332)}{100,004 \times 10^7}$$
$$= 30.1 - j\, 332 \text{ OHMS}$$

Zef is in parallel with Za-d and to find the resultant impedance the same rule is used again

$$Z_{A-F} = \frac{Z_{EF} Z_{A-D}}{Z_{EF} + Z_{A-D}}$$
$$= \frac{(30.1 - j\, 332)(420.5 + j\, 22,443.2)}{450.6 + j\, 22,111.2}$$
$$\frac{420.5 \times 30 + 22,443.2 \times 332}{450.6 + j\, 22,111.2} +$$
$$+ \frac{j\, 30.1 \times 22,443.2 - j\, 420.5 \times 332}{450.6 + j\, 22,111.2}$$
$$= \frac{7,463,800 + j\, 535,930}{450.6 + j\, 22,111.2}$$
$$= \frac{(7,463,800 + j\, 535,930)(450.6 - j\, 22,111.2)}{450.6^2 + 22,111.2^2}$$
$$= \frac{7,463,800 \times 450.6 + 535,930 \times 22,111.2}{203,000 + 488,900,000} +$$
$$+ \frac{j\, 450.6 \times 535,390 - j\, 7,463,800 \times 22,111.2}{203,000 + 488,900,000}$$
$$= 31.2 - j\, 337.9 \text{ OHMS}$$

and

$$|Z_{A-F}| = \sqrt{31.2^2 + 337.9^2} = 339 \text{ OHMS}$$

while

$$Z_0 = Z_G + Z_{A-F} = 200 + 31.2 + j\, 11,304 -$$
$$- j\, 337.9 = 231.2 + j\, 10,966 \text{ OHMS}$$

$$|Z_0| = \sqrt{231.2^2 + 10,966^2} = 10,970 \text{ OHMS}$$

This differs in value from the result found last month for Zo due to an error in that very line in last month's calculation. The result should have read 10970 ohms approximately. The hum voltages across the condensers are found in the usual way and need no further comment.