Westinghouse

ENGINEER

SEPTEMBER 1964

A mere 3000-mile journey—by train at that—is a long way from nuclear-powered exploration of deep space. Yet the recent shipment of the NRX-A2 reactor from Pittsburgh to the Nuclear Rocket Development Station was one of the steps that program engineers expect to stretch into giant strides through space when nuclear-powered rockets have come of age. The chief theoretical advantage of a nuclear rocket over a chemically fueled rocket is the higher specific impulse that can be obtained from a relatively light nuclear reactor using hydrogen propellant.

The NRX-A2 is a test reactor for the NERVA program. (NERVA stands for Nuclear Engine for Rocket Vehicle Application.) In the program, Westinghouse Electric Corporation is responsible for design and development of a nuclear reactor for flight, and the Aerojet-General Corporation has overall responsibility for engine development. The program is being conducted for the Space Nuclear Propulsion Office, a joint office of the National Aeronautics and Space Administration and the Atomic Energy Commission.

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Autovalve, MSP, Prodac, Seri-Actrol

Cover Design: Glass, once a rare and expensive art medium, has become one of the mainstays of industry, science, and everyday life as ways have been developed to make and shape it economically. One of the newest processes is electric melting, described in this issue. The cover design is by Thomas Ruddy of Town Studios, Pittsburgh.

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Electric Glass Furnaces

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Electric melting of glass offers significant cost, quality, and control improvements over the fuel-fired melting process.

Electric glass melting has been common in Europe for many years because of shortage of fuels there. It is now beginning to be a significant factor in the American glass industry for other reasons: primarily, savings in capital investment (especially in new construction) and savings in many operating costs (such as furnace maintenance and raw materials). Another tangible advantage is improvement of glass quality, which has a marked effect on the quantity of rejects and on customer acceptance.

Furnace Types

In the conventional fuel-fired furnace, gas or oil is burned over the glass, and heat is radiated into the glass from the flame, from the furnace crown (roof), and from the furnace sidewalls. In electric melting, heat is generated within the glass itself by passing electric current through the glass.

Electric melting of glass is possible because molten glass is a fairly good conductor and becomes its own heating element when current is passed through it from electrodes. The electrodes are not used as heating elements, but only as means for getting current into the mass of molten glass. Melting is begun by burning fuel over the furnace charge, but, once the passage of current is initiated, it continues the melting and the burners are shut off.

Both electric and gas-fired glass furnaces usually are continuous units (Fig. 1). Batch (raw material) is fed in as required to maintain the desired level of molten glass. The molten glass flows from the bottom of the melting zone through a throat into the refiner. The principal function of the refiner is to distribute glass to the forehearths; little if any actual refining (i.e., elimination of bubbles and unmelted batch) takes place here. The forehearths, typically from one to six in number, are shallow troughs where the glass is cooled to the proper uniform working temperature. The refiner and forehearths are usually fuel-fired to keep the glass surface as hot as the body of the glass and to control the cooling rate. At the end of each forehearth is a shearing mechanism that cuts gobs (unformed pieces of soft glass) and delivers them to a forming machine to be made into containers, such as jars and bottles, and pressed ware, such as ashtrays.

Much smaller electric furnaces also are used, such as the day tanks employed for hand blowing operations. The term "day tank" is derived from the method of operation: batch is melted and refined at night to be ready for use the next day. The day tank is a simple one-compartment furnace that may hold from a few hundred pounds to a few tons of glass.

Comparison

Because heat is generated within the glass itself in electric melting, much less heat is lost; consequently, this type of furnace is more efficient than the fuel-fired type. Also, the entire melting area of the electric furnace can be covered with batch, which serves as an insulating blanket to conserve heat. An area clear of batch is necessary in a fuel-fired furnace so heat can be radiated into the body of the molten glass and to permit the escape of seeds and blisters (small and large gas bubbles, respectively).

Electric furnaces are much smaller than fuel-fired furnaces, which leads to many related advantages—less floor and building space, lower furnace construction costs, and lower furnace rebuilding and maintenance costs. One reason for the size difference is that the melting area of a fuel-fired furnace usually has to be about twice that of the more efficient electric furnace of the same capacity (Fig. 1). For some glasses that are difficult to melt, the ratio may be as high as four or five to one. An example is the electric furnace at Jeannette Glass Company, Jeannette, Pennsylvania, which produces an opal borosilicate glass that is hard to melt. The furnace area for this type of glass would be 10 to 15 square feet per ton-per-day capacity with fuel firing; in the electric furnace, the melting area required is less than three square feet per ton-per-day capacity. Another example is an electric container-glass furnace melting 25 tons per day of a dark blue glass with two to three square feet of melting area per ton-per-day production.

The other main reason for the smaller size of electric furnaces is that they do not require such accessories as regenerators for air preheating, flues and reversing valves, blowers for combustion air, and stacks for venting flue gases (Fig. 1).

Batch costs are lower with electric furnaces because the

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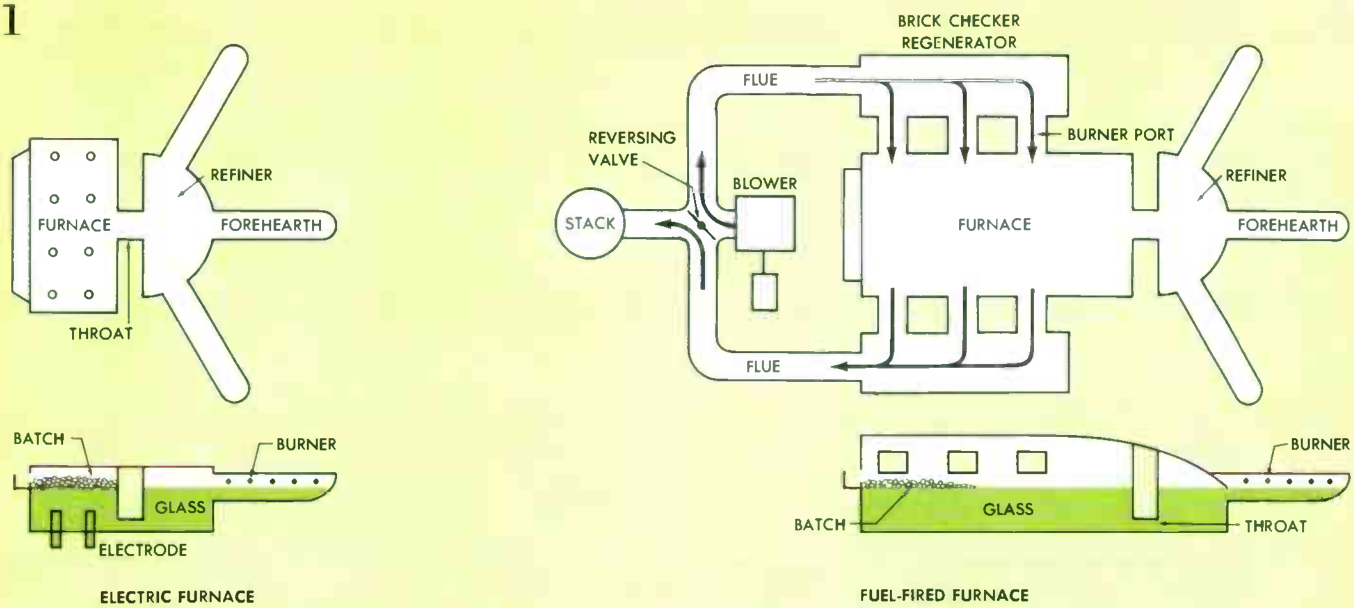
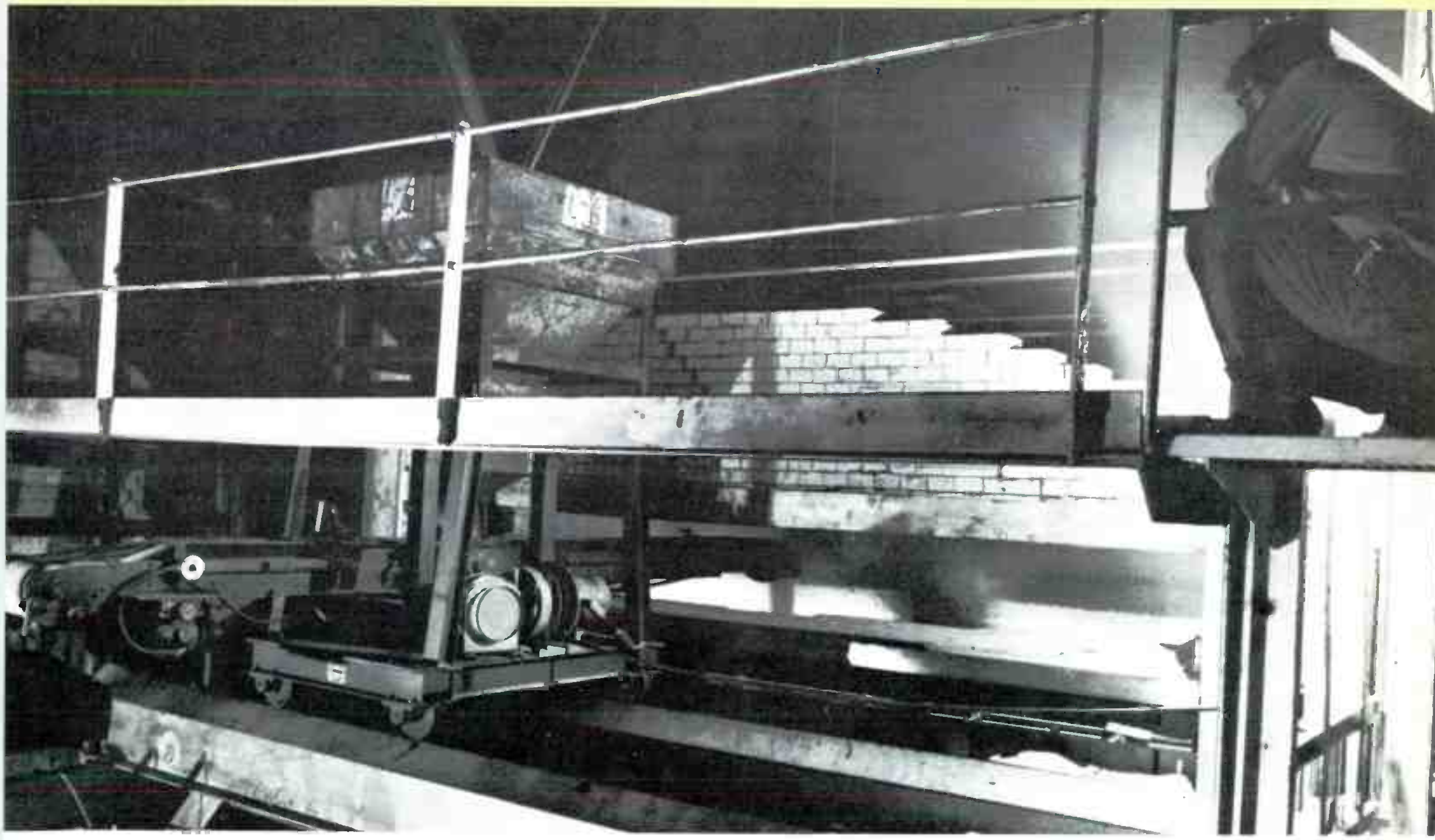


CONTINUOUS ELECTRIC GLASS FURNACE occupies much less space and is simpler than a fuel-fired furnace of the same capacity. The latter requires flues, regenerators to heat the combustion air, a stack, and a blower. In the electric furnace, heat is generated within the glass by passing current through it from electrodes. In both types, batch (raw material) is added as required to maintain the desired level of molten glass. Glass is taken out at the ends of the forehearths.



BATCH CHARGER for the continuous electric furnace at the Jeannette Glass Company travels along the side of the furnace. Its belt conveyor shuttles in and out to distribute batch evenly. The batch is held in a hopper that moves with the charger.

volatile materials such as opacifiers and coloring materials are not carried away in flue gases. For example, less than half as much cadmium-sulfide orange coloring agent is required with the electric day tank being operated at the World's Fair by The Pilgrim Glass Corporation, Ceredo, West Virginia, as would be needed with a comparable fuel-fired day tank. This difference stems from a basic difference in the melting operation. The batch floats on top of the molten glass in both electric and fuel-fired furnaces, but an electric furnace melts from the bottom of the batch layer while a fuel-fired furnace melts primarily from the top of the batch layer. The volatiles are condensed by the colder batch above in an electric furnace; they are released at the surface in a fuel-fired furnace and carried out the flue. In addition to material waste, the resulting dust pollutes the air.

The basic difference in melting also affects glass quality. When batch melts from the top, as it does in a fuel-fired furnace, it traps air and gas bubbles and carries them down into the molten glass to form seeds and blisters. Electric melting releases the gases through the dry unmelted batch. Quality is also improved in electric melting by the stirring action inherent with internally generated heat; since heat is generated within the body of the glass rather than at the surface, thermal currents mix the glass and improve its homogeneity.

Electric Furnaces

Electrodes—Electric current is introduced into the furnace by electrodes immersed in the glass. These may be inserted through the bottom or through the sidewalls. No effective method for inserting the electrodes down through the batch has been found; the major problem is oxidation at the glass surface.

Many different electrode materials have been used, including graphite, molybdenum, iron, and platinum. Graphite and especially iron discolor glass, and platinum is too expensive for most applications. Molybdenum is usually the best material. It has excellent high-temperature strength, makes good electrical contact with glass, and is not attacked by most glass constituents. Moreover, molybdenum can be fabricated by rolling, machining, and other common metal-working techniques into any desired shape and size.

The Jeannette Glass Company furnace has eight cylindrical vertical electrodes extending through the bottom up into the glass (Fig. 2). The power connections are two-phase four-wire for each set of four electrodes. This arrangement fits nicely with conventional rectangular furnace construction because it distributes power quite evenly over the furnace area for uniform heating. Smaller continuous furnaces require only a single set of four electrodes in a square arrangement.

Electrodes extend through a wall in the World's Fair day tank (Fig. 3). This arrangement keeps the electrodes fully immersed as the workers take glass out of the tank. The operation of this type of furnace is different from that of continuous furnaces because the level changes over a wide range and glass temperature is considerably higher for melting than for working. These two factors require a fairly wide range in power input and consequently in voltage.

Batch Charging—Batch charging into a continuous electric furnace can be considerably simpler than the charging of a fuel-fired furnace. The temperature above the batch blanket is so low in an electric furnace that a simple conveyor, even with a rubber belt, can be used to distribute batch over the melting area. (See photograph, page 131.)

Electrical Design—Both temperature and composition determine the electrical resistivity of glass (Fig. 4). Consequently, both affect power-supply design. As temperature increases, resistivity decreases. This negative temperature characteristic requires special consideration in

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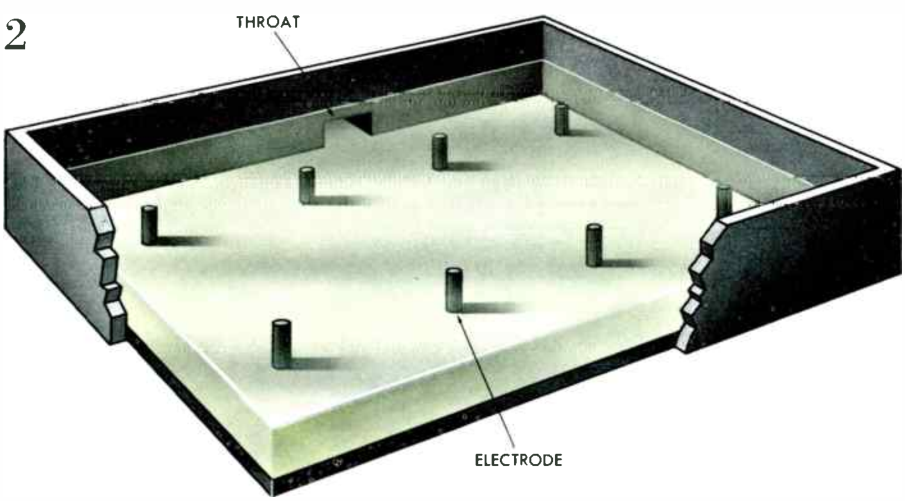

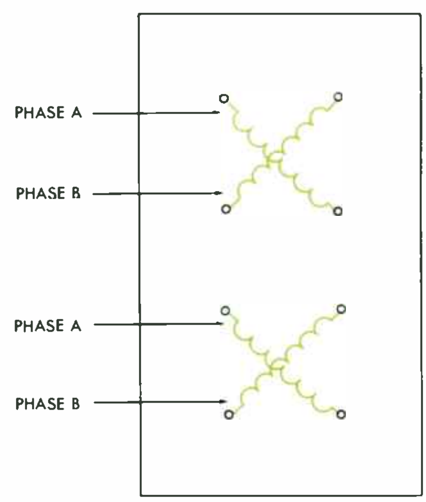

TYPICAL ELECTRIC FURNACE has eight vertical cylindrical electrodes (left). The electrodes are connected in a two-phase four-wire arrangement (right), an inherently square configuration. Using two sets of four electrodes gives even heat distribution over a conventional rectangular furnace; a smaller square furnace would need only one set.

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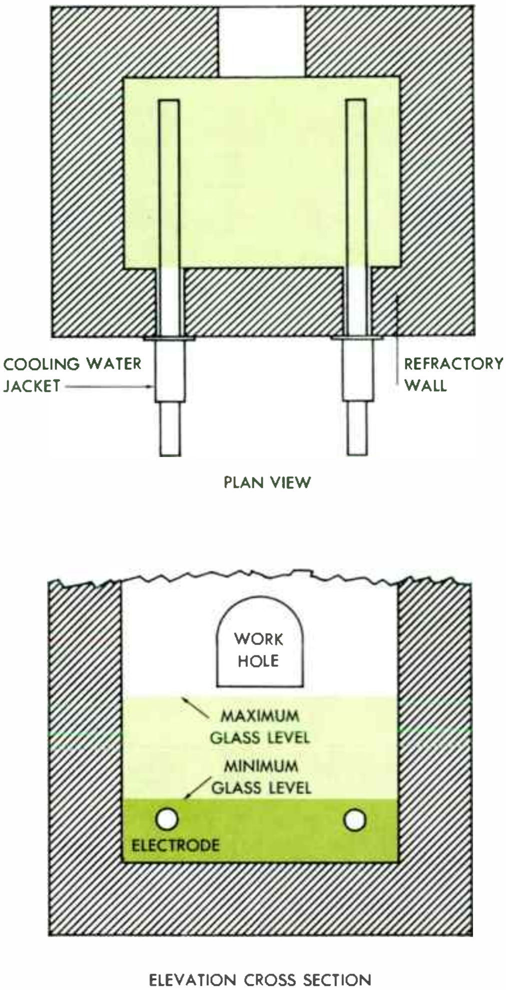

ELECTRIC DAY TANK, a small furnace, has two horizontal electrodes extending through a wall. This arrangement keeps electrodes fully immersed as workers take glass out of the tank.

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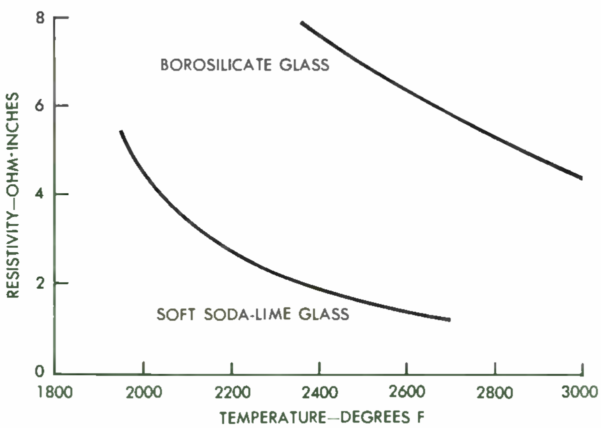

ELECTRICAL RESISTIVITY OF GLASS varies inversely with temperature, and it also depends on the chemical composition of the glass. This wide variation in resistivity encountered among various glasses and at various temperatures necessitates careful design of electric furnaces and their power supplies. The characteristics illustrated here are for a borosilicate glass (for ovenware) and a soda-lime glass (common container glass).

power-supply design for stability. Power adjustment by transformer taps alone is inherently unstable; either the voltage is too high and power input (heat) increases as the temperature rises, or it is too low and power input decreases as the temperature drops.

The required heat input (kilowatts) is determined by the glass production required, but the ratio of volts to amperes to produce these kilowatts depends on glass chemical composition. The primary current carriers are alkalies—sodium and potassium oxides. Since potassium is twice as resistive as sodium, the glass resistivity is roughly inversely proportional to the percentage of sodium plus half the percentage of potassium.

Single-phase power is ideal for small furnaces such as day tanks because the equipment is simple and inexpensive. For larger continuous furnaces, single-phase is objectionable to the power companies because they prefer balanced three-phase loads. Two-phase power is convenient for furnace use, as illustrated in Fig. 2, since it fits conveniently with rectangular or square furnace construction. Two-phase power was used in the continuous-furnace power supplies mentioned earlier. A Scott transformer connection converts from a balanced two-phase furnace load to a balanced three-phase load for the power system.

The main melting-area power supplies are a combination tapped-transformer and saturable-reactor type known as Seri-Actrol (Fig. 5). Tap adjustments change power for relatively large changes in furnace output. Tap changes are also necessary during startup, when a relatively high voltage is needed because of high glass resistance. The tap changer operates at no load: the main incoming line switch must be opened to make a change. (Because tap changes are infrequent, there is no necessity to provide for tap changing under load.) Electrical interlocking between the tap changer and the switch insures safe operation.

For fine power-level control, the operator adjusts the saturable-reactor control system that is an integral part of the melter power supply. This fine adjustment is limited to approximately 15 percent of the top voltage tap. As the tap setting (voltage) is reduced, the percentage of control available by means of the saturable reactors increases to approximately 25 percent at the lowest tap setting. Since power input to the furnace is a function of voltage squared, 15 percent voltage is equivalent to about 30 percent power or heat. This is adequate for normal furnace operating adjustments. The considerable overlap of voltages obtainable on each tap reduces the number of tap changes required (Fig. 6).

The voltage-current characteristics of the Seri-Actrol power supply are inherently suited to electric glass melting because the curve slopes steeply over the reactor-control range of adjustment (Fig. 7). The power supply works almost as a current regulator. Power (heat) into the furnace is represented by the expression I^2R, where I is current and R is the furnace resistance. As the glass temperature increases, the power input to the furnace decreases because resistance drops. Conversely, as the temperature decreases, the heat input to the furnace increases. This control has proved to be stable, and operators have had little difficulty maintaining the desired melting conditions with it.

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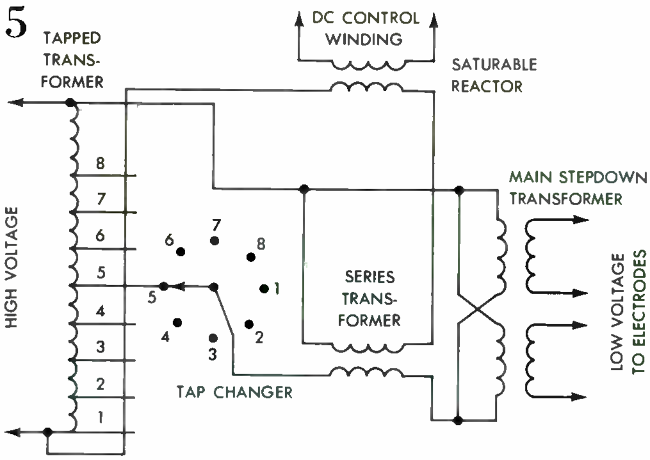

SERI-ACTROL FURNACE POWER SUPPLY includes a transformer tap changer for coarse power adjustments and saturable reactors for fine adjustments. For simplicity, a single-phase unit is illustrated rather than the two- or three-phase type actually used for continuous-furnace melting.

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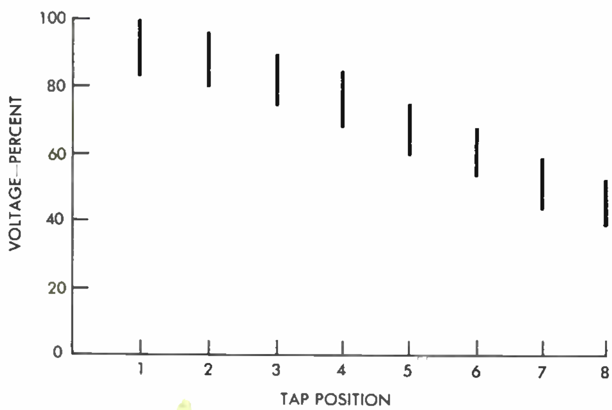

VOLTAGE CONTROL available at each transformer tap position by reactor adjustment overlaps that available at the adjacent positions. This reduces the amount of tap changing required.

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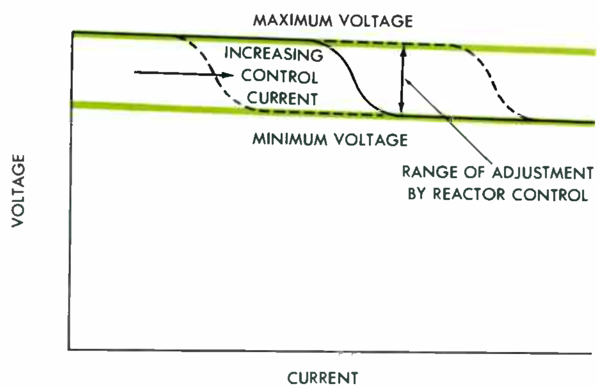

VOLTAGE-CURRENT CHARACTERISTICS of the furnace power supply for one transformer tap position. The two colored lines indicate maximum and minimum voltages available at this tap position. The solid line shows the characteristic for one specific setting of control current in the saturable reactor; other control-current settings would give the characteristics indicated by the dashed lines.

The small day tank at the World's Fair has a tapped transformer and a full-range saturable reactor with a current regulator. The glass blowers adjust glass viscosity to the desired working condition by simply changing the current value set-point of the regulator. This type of power supply is also useful for controlling heat in the refiners of larger furnaces.

Control—Highly automated and sophisticated control is not necessary for electric glass melting. Electrical instruments supply the operator with information on voltage, current, power, power factor, and energy consumption (kwhr). Temperature recorders monitor refractory and glass temperatures at critical spots throughout the furnace. Glass-level and batch-charging controls are provided on the furnace control panel, and switches and indicating lights there give the operator complete control and supervision of the tap changer, reactor control, and switchgear. An alarm annunciator warns the operator in the event of such conditions as dangerous furnace temperatures, loss of electrode cooling water, and malfunction of electrical equipment.

Electric furnaces are easy to operate and control. Combustion control is much more complex and somewhat more of an art than control of electric power—fuel-to-air ratio, flame appearance, furnace pressure, protection of the crown refractories, and control of air reversal through the regenerators all contribute to the greater complexity of fuel firing.

Operating Characteristics—Electric melting requires between 500 and 600 kwhr per ton of glass produced, depending on the type of glass. Heat losses from the furnace must be added to these figures to find total energy consumption. Losses for a continuous furnace producing 25 tons per day would be approximately 200 kw, or a total of just over 800 kwhr per ton (based on a theoretical base of 600 kwhr per ton). Larger furnaces operate at a lower figure. A good average for continuous furnaces is 700 to 800 kwhr per ton. Figures for small day tanks are more difficult to express; most of the power is used to maintain working temperature and therefore is all loss. Average power consumption for a day tank such as The Pilgrim Glass Corporation's tank at the World's Fair is approximately 45 kw over the complete 24-hour melt-down and working cycle.

Continuous electric furnaces can operate consistently at more than 90-percent power factor. Since many glass plants have low power factor, an electric furnace can improve overall plant power factor and reduce electric power rates for the entire plant. Frequently, the added load also justifies a lower rate schedule that applies to the entire plant.

Conclusion

Electric glass melting is a practical process, especially suited for glasses that are hard to melt and for special glasses in which quality or color are important. The glass industry in the United States has shown much interest in the process since the successful installations of the electric furnaces mentioned in this article; this interest indicates that electric melting is likely to be applied to an ever-widening range of furnace sizes and glass types.

Westinghouse ENGINEER Sept. 1964

Arc-Welding Power Supplies

Emil F. Steinert, Fellow Engineer, Westing-Arc Division, Westinghouse Electric Corporation, Buffalo, New York.

Arc welding has become a major industrial process, and a wide variety of power supplies has been developed to fit the many different needs.

The arc-welding process has grown in applications and in complexity in the past 20 years or more until there is now a rather large family of welding power supplies (commonly known as welders). Welders can be classified in several ways, however, and such classification helps in determining which is the right one for a given application.

One main division is that of *rotating* and *static* types, both of which have been developed for a long time and are equally reliable. The rotating type includes motor- and engine-driven generators. Engine-driven welders are still the main source of welding power in areas where electric utility power is not available.

The static type includes the transformer ac and transformer-rectifier dc types. The transformer-rectifier dc welder is one of the later developments stemming from technological advances in rectifiers. These rectifier developments have brought about a general trend toward replacement of rotating apparatus throughout the electrical industry, including arc welding. Static apparatus has the advantages of higher electrical efficiency, simpler construction, less volume, less weight, and long life with minimum maintenance.

The second main division of arc welders is that of *ac* and *dc output*. Because the type of current affects the characteristics of the arc, it is an important factor. Direct current is continuous in wave form, and once it is established by striking an arc it is only necessary thereafter to control the magnitude of the current. With alternating current, the arc is extinguished each half cycle as the current passes through zero. This condition requires that certain ingredients be included in the electrode coating to provide easy ignition without time lapse at each current reversal. The open-circuit voltage of the welder must be adequate to suit the electrode characteristics, or vice versa.

The choice between alternating and direct current is still controversial, but certain basic factors serve as guides:

1) Direct current is the most universal in application; it can be used in practically all welding operations. An exception is tungsten inert-gas (Tig) welding of aluminum and magnesium, usually done with alternating current.

2) Except for some combination ac-dc welders, dc welders use balanced three-phase input power. Transformer ac welders are inherently single phase and can present some load-unbalance and voltage-regulation problems where the power-supply capacity is limited.

3) Transformer ac welders are simplest in construction and lowest in cost because they need include only the essentials for supplying a welding arc—a transformer and reactor that are usually an integral unit. Electrical efficiency is high because only a single power conversion is made, instead of several conversions as in welders employing motor and generator or transformer and rectifier.

4) Arc blow, a deflection of the arc caused by reaction between the arc current and the magnetic field surrounding the arc, is a problem with direct current because it makes it difficult or impossible for the operator to position the weld puddle. Arc blow is practically nonexistent with alternating current, an important advantage at higher welding currents (above 200 amperes).

5) Practically all commercial electrodes can be used with direct current, while only those so designated are usable with alternating current.

6) The peak value of the open-circuit voltage from an ac welder is 41 percent higher than the RMS value of the open-circuit voltage at which it is rated. This higher peak voltage and the physiological effect of alternating current make the shock hazard of ac welders greater than that of dc welders. This is especially significant where the operator is subject to wet or otherwise hazardous working conditions.

The advent of reliable silicon rectifiers has reduced the electrical-efficiency advantage of ac welding over dc. In fact, the lower forward drop in silicon rectifiers, as compared with selenium rectifiers, has improved overall efficiency of transformer-rectifier welders to a point approaching that of transformer welders. The one remaining principal advantage of alternating current is its relative freedom from arc blow.

Finally, both ac and dc welders are further classified by manufacturers' standards as *limited input*, *limited service*, and *industrial*.

AC Welders

Limited Input—This class was developed to provide a welder suitable for operation on rural and other limited-capacity single-phase power lines without causing undue

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voltage fluctuation. Output rating is 180 amperes (on a 25-volt arc load); this is also the maximum output, there being no overrange as is usual on larger welders. The limitations necessitate a lower open-circuit voltage, especially on the upper current settings, and this limits the welder to certain highly stabilized low-voltage ac electrodes. Most welders of this type automatically provide higher open-circuit voltage on the lower portion of the current adjustment range, which provides better arc striking and stability with a broader selection of electrodes. The rated duty cycle is 20 percent at maximum output rating, but higher duty-cycle operation usually is possible at lower current settings.

Limited Service—This class is intended to cover the field of application between the limited-input and the much heavier industrial ratings. The rating basis is the same as for the limited-input class; that is, on maximum output current at 20-percent duty cycle. Also, like the limited-input class, most permit higher duty-cycle operation at reduced current settings. This class of welder is economically advantageous where the required welding operations are highly intermittent and within the duty-cycle capacity of the welder.

Because there is no limitation on input, these welders can be designed to have open-circuit voltage suitable for a wide selection of ac electrodes, including low-hydrogen types. The largest rating recognized by manufacturer's standards is 295 amperes maximum (at 30 volts load). This odd figure was chosen to distinguish the rating from an industrial 300-ampere rating.

Industrial—Industrial ac welders are designed and rated to withstand production service at high duty cycles under conditions normally encountered in industry. The class is divided arbitrarily into manual welders, with ratings below 750 amperes, and automatic welders, with ratings of 750 amperes and higher.

Manual industrial welders are normally rated at 60-percent duty cycle. Their application is not limited to manual operator welding; they can be applied on automatic or semiautomatic service within their equivalent continuous duty ratings. Because industrial welders must perform satisfactorily on all commercial types of ac electrodes, they have open-circuit voltages of 70 to 80 volts; 80 volts is generally recognized as the maximum allowable for manual welding.

As with all transformer welders, the power drawn from the supply line is inherently single-phase. Power factor is about 55 to 65 percent at full-load rating. Both the single-phase load and the low power factor contribute to unbalanced loading on a three-phase supply, with resultant line voltage regulation that could be excessive on lines of limited capacity. Consequently, industrial welders are usually equipped with capacitors for power-factor correction to at least 75 percent. This feature also reduces the line current and kva drawn by the welder. Because the capacitors draw the same corrective (leading) kva under all load conditions, the power factor increases with decrease in current output of the welder. At lower current settings, the power factor may be leading. When idling, the capacitors are drawing their full corrective kva, thus contributing power-factor correction to the remainder of the load on the system (Fig. 1). For this reason, too much power-factor correction can be undesirable. If a number of welders are connected to a supply line, the possibility should be considered that the combined corrective (leading) kva will upset the voltage stability of the line (when, for example, all units are idling). Leading-power-factor kva will cause a rise in line voltage that could be harmful.

Where a number of single-phase welders are connected to a three-phase power supply, a partial balance in loading can be obtained by distributing the units on the three phases. However, this cannot be done when welder secondaries are connected in parallel to obtain arc current higher than the rating of a single welder.

Automatic, or high-current, ac industrial welders are practically limited to submerged-melt ac welding. Arc blow, electrical efficiency, and cost are the deciding factors in application of ac at these high welding currents. The units usually have connections to give open-circuit voltages of 85 and 100 volts. The higher voltage is usually necessary to compensate for the higher reactance voltage drop that is encountered because of the high current and large welding-circuit loop.

DC Welders

The dc welders are classified as constant-current and constant-voltage types. Actually, these characteristics are only relatively constant.

Constant Current—This type's characteristics are represented by a family of drooping volt-ampere curves (Fig. 2). Because the slope of the curves is relatively steep in the working voltage range, at a given machine setting a change in load voltage produces only a small change in current. The constant-current type is the more conventional of the two; it is used for manual-electrode, submerged-melt, and Tig welding, where the length of the arc is controlled manually by the operator or automatically by a control that senses the arc voltage.

These welders have open-circuit voltages of about 70 to 80 volts. Welding-current output is adjusted to the desired value by varying series impedance (usually ac inductive reactance).

Although manufacturers' standards recognize a *limited-input* dc welder, its commercial availability and popularity are insignificant. The ac welder predominates in this field for economic reasons.

Limited-service dc welders are similar to ac limited-service welders. For economic reasons, they are designed for single-phase input, with a full-wave rectifier. Most are made for combination ac-dc output, a design that permits the use of standard ac welder components with the addition of rectifier, inductor, and switching devices to select ac or dc welding current.

Single-phase full-wave rectification produces a non-continuous direct current in the form of a series of half wave loops, with a current zero at each half cycle. Only the ac type of electrode would perform satisfactorily with such current. To overcome this deficiency, ac-dc welders have a built-in dc inductor. It provides current continuity, thereby eliminating arc extinction at each half cycle and making the welding current output suitable for use with dc electrodes.

Manufacturers' standards for *industrial* dc welders include ratings of 200 amperes and above, at 60-percent duty

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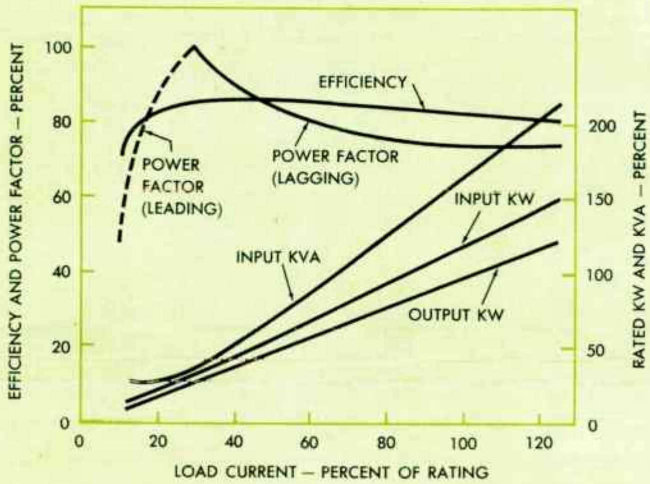

AC ARC WELDERS of the transformer type usually have capacitors for power-factor correction, giving electrical performance illustrated by these typical curves. Because the capacitors draw the same corrective kva under all load conditions, power factor increases with decreasing current output. Idling input is approximately 1.5 kva per 100 amperes of rating, and power factor is 100 percent leading.

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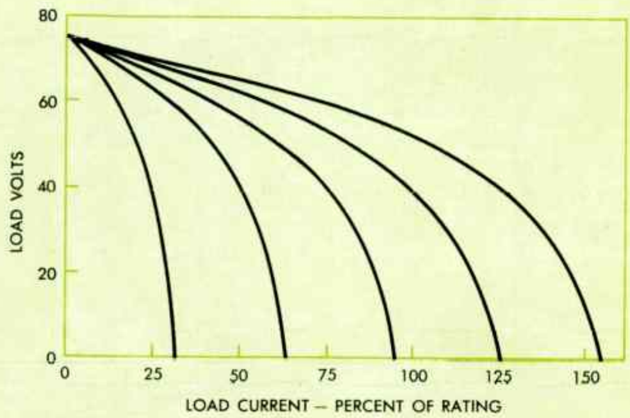


CONSTANT-CURRENT DC WELDERS have electrical characteristics as shown by these typical volt-ampere curves. Because the curves slope steeply in the working voltage range, a change in load voltage at a given machine setting produces only a relatively small current change.

WELDERS AND WELDING PROCESSES

Arc welding probably originated in 1885, when a Russian named Bernadoz used a carbon arc for welding. Another Russian, Slavianoff, originated the metallic arc process in 1891. The first recorded arc-welding plant in the United States was at the Baldwin Locomotive Works in 1902.

Early welding sets consisted of a rather crude assembly of motors, generators, resistors, and the other necessary components. Successive steps finally produced the motor-generator arc welder of unified functional design, which became the standard welder and remained so for many years.

Only dc welders were available until around 1920, when some experimental work was done on ac welding. The development was halted because the existing electrodes, while suitable for direct current, required too high an open-circuit voltage with alternating current; ac welding did not really get started until aroung 1933 when development of suitable electrodes made it feasible.

The *rotating* type of welder is basically a generator, usually dc, that is driven by a directly coupled motor or engine and has static volt ampere and dynamic characteristics suited to welding. The dc generator usually is differentially compound with series and shunt field control for output current adjustment. Cross-field, shifting-brush, and separately excited generators also are used.

The *static type* includes transformer and transformer-rectifier welders. The *transformer welder* is an ac unit consisting essentially of a step-down transformer and an adjustable series reactor. The reactor provides the necessary phase shift and adjustable impedance for arc ignition, stabilization, and current adjustment. The *transformer-rectifier* type is a transformer welder with the addition of a rectifier on the output side to convert the output to dc power.

Arc welders are rated in terms of permissible *duty cycle,* which is the ratio of arc time to total time. A unit with a 60-percent duty cycle, for example, could sustain welding load continuously for six minutes of a ten-minute cycle repeated indefinitely.

Manual welding is the most common arc-welding process. The operator usually employs a consumable coated electrode, which provides filler metal, clamped in a holder that he holds in his hand. He controls the length of the arc by raising or lowering the tip of the electrode. The term is also applied to a process in which the operator uses a nonconsumable tungsten electrode to weld by fusion of the parent metal or by addition of a filler rod fed into the arc.

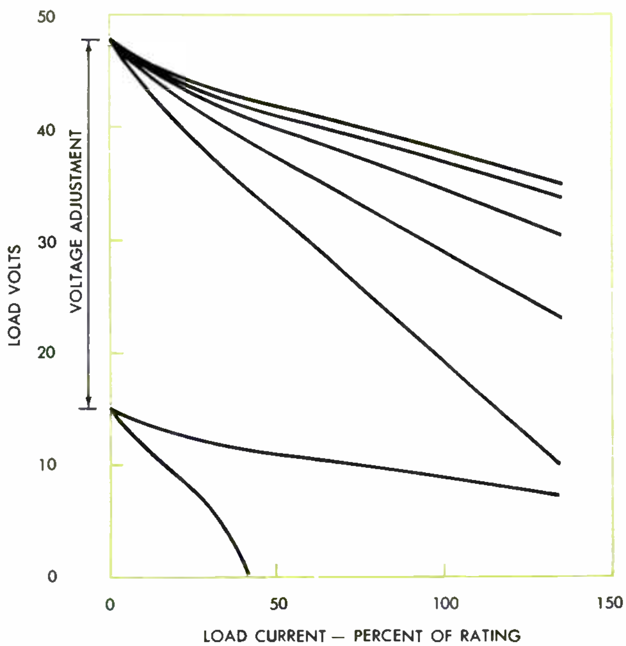
In *automatic welding,* the electrode is automatically positioned for arc length and advanced along the seam mechanically. The electrode, usually a continuous consumable wire, is fed into the arc automatically; a voltage-sensing control regulates the wire drive mechanism to maintain a constant preset arc voltage. The term is also applied to the nonconsumable-electrode process when the electrode is raised and lowered automatically to maintain constant arc voltage.

In *semiautomatic welding,* the consumable electrode is fed into the arc automatically, but the operator positions and advances it manually.

The welding zone is nearly always *shielded* to prevent oxidation or contamination by gases in the atmosphere. The covering on conventional coated electrode provides gas and slag for shielding. In *submerged-melt* ac welding, a blanket of fusible granular material known as flux is deposited on the welding zone. The submerged-arc process may be automatic or semiautomatic, and it requires relatively high arc current. *Gas-shielded processes* surround the welding zone with inert gas (argon or helium), with carbon dioxide, or with mixtures of gases. *Tungsten inert-gas* (Tig) welding employs a nonconsumable electrode and a shielding gas.

Stud welding is the process of welding studs or similar metal parts to a work piece. A stud-welding gun maintains an arc between stud and work piece until the parts are properly heated and then presses the stud against the work piece. Control equipment sequences the operations.

Carbon air cutting is a fast, economical method of cutting and gouging iron and steel. An arc is drawn between a carbon electrode and the work; an air jet is directed into the melted puddle to blow out the molten metal.



CONSTANT-VOLTAGE DC WELDERS have volt-ampere curves that are nearly flat. In some, the slope of the curves can be adjusted as shown. Voltage also is adjustable over a wide range.

cycle. These welders are designed and rated for heavy production welding, manual or automatic. For automatic-welding duty cycles higher than 60 percent, a welder of higher ampere rating can be used at derated current. For higher current capacity, two or more similar units can easily be paralleled. Practically all welders of this class are designed for balanced three-phase power input.

The first commercially successful selenium rectifier welders were introduced in 1950. These were followed in 1958 by silicon-diode rectifier welders. Since then, nearly all welder manufacturers have changed to silicon rectifiers. The advantages of silicon rectifiers include higher electrical efficiency because of lower forward and reverse power loss and almost insignificant leakage loss during idling. Silicon diodes do not age, a feature that assures continuing high efficiency for many years. Their reliability has been developed to such a high degree that Westinghouse silicon power diodes in Westinghouse welders are guaranteed for the life of the welder.* Moreover, silicon diodes are hermetically sealed against corrosive elements, and they are smaller in size and weight than selenium rectifiers.

Constant Voltage—Constant-voltage welders differ from the constant-current type in their mode of adjustment, open-circuit voltage, and volt-ampere characteristic.

The voltage drop in an electric arc does not increase proportionally with increase in current. Therefore, if a constant voltage were applied across the arc the current would be relatively unlimited. Some ballast in the form of series resistance or reactance is required for electrical stability of the arc. For this reason, constant-voltage welders are not generally applicable for manual operator welding.

The volt-ampere curves of constant-voltage welders are approximately straight lines having a slight droop, indicating nearly constant voltage with change of output current. The principal adjustment is in open-circuit voltage, which is adjustable over a range. In operation, the voltage setting is equal to the arc voltage plus the relatively small voltage drop in the circuit.

The constant-voltage welder is used for automatic and semiautomatic arc welding (usually gas shielded). Consumable electrode is fed into the arc at an adjustable but constant rate. Welding current is adjusted by increasing or decreasing the rate of electrode feed. For a given rate of feed, the current adjusts itself to melt the electrode at the rate required to maintain an arc length corresponding to the voltage setting of the welder.

From the power-supply viewpoint, the constant-voltage process has two main advantages over constant current. First, input kva is low and power factor high. Because the system operates at an open-circuit voltage only slightly above the arc voltage (40 volts or less), the kva, which is proportional to the product of open-circuit voltage and load amperes, is about 50 percent of that for a constant-current system. This reduction in kva drawn from the line also reduces the physical kva size of the welder and gives a higher operating primary power factor—approaching 100 percent. Second, the adjustable but constant-rate wire feed control is simpler and less costly than the control required for a constant-current system, which controls arc length by sensing arc voltage.

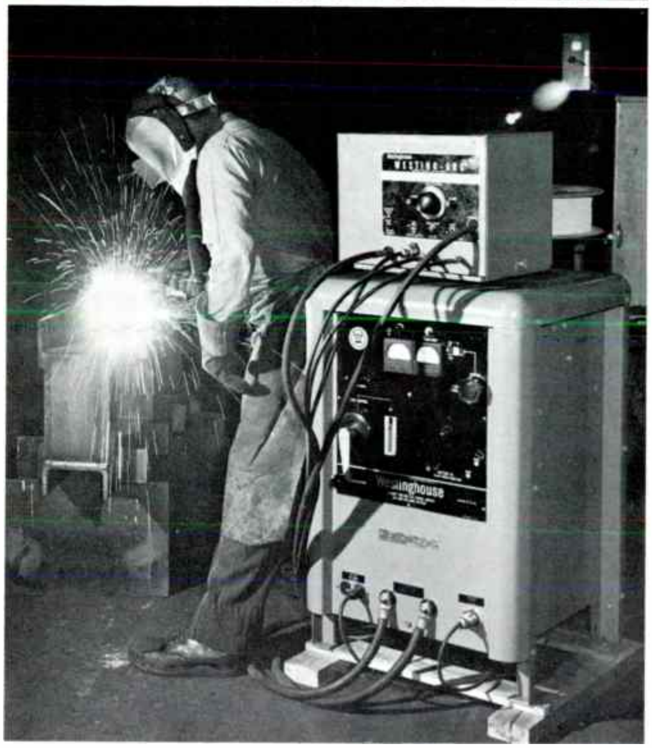
The constant-voltage unit, with its relatively flat volt-ampere characteristic, is best suited to the gas-shielded consumable-electrode process and the spray-transfer condition. This condition is metal transfer in the form of a fine spray of droplets, and it exists at the higher levels of electrode current density (for example, above 300 amperes with 1/16-inch electrode). The high short-circuit current that can be supplied by this type of unit results in instantaneous fusion of the electrode at contact with the work and instant arc ignition.

Constant-voltage welders with flat characteristic are applied in the higher current applications and are, therefore, available in the higher ampere ratings above 300 amperes. They are designed for balanced three-phase power input, and their range of open-circuit voltage adjustment is around 15 to 40 volts. A fine stepless adjustment that can be regulated under load is required.

At electrode current densities below the critical value, the metal transfer across the arc changes from a spray to a globular type of transfer. This condition causes repetitive momentary short circuits resulting from metal bridging the arc gap. The high momentary short-circuit current causes excessive metal spatter. Therefore, the power supply requires some taming for the globular transfer condition. The modifications currently in use on commercial constant-voltage power supplies are:

1) A dynamic reactor is included. This is simply a dc inductor connected in series with the welding circuit, and

**Westinghouse warrants to the original purchaser that it will correct any defect or defects in workmanship or material, by repair or replacement, f.o.b. factory, for any JEDEC type silicon power semiconductor during the life of the equipment in which it is originally installed, provided said device is used within manufacturer's published ratings and is applied in accordance with good engineering practice. This warranty is applicable to devices of the stated types shipped after April 1, 1964, until further notice. This warranty shall constitute a fulfillment of all Westinghouse liabilities in respect to said products. This warranty is in lieu of all other warranties expressed or implied. Westinghouse shall not be liable for any consequential damages.*



it reacts to a change in current in such a manner as to hold the current constant. Its effect is strictly dynamic, thus limiting only the rate of change of current and not the final steady-state value. A mechanical analogy of the dynamic reactor is the flywheel, which tends to level out what would otherwise be abrupt speed changes.

2) Adjustable slope in the volt-ampere characteristic is provided by adjustable ac inductive reactance in series with the ac circuit of the power unit (Fig. 3). In principle, this scheme is similar to that employed in constant-current welders except that the amount of reactance employed is much less, giving a smaller amount of droop. The ac reactance has the same effect of limiting the rate of change of current, but it differs in that the final or steady-state current is also limited. This limitation in steady-state current also limits the short-circuit current. While this is desirable in other respects, it could have an adverse effect on arc starting, where a high current is essential to rapid fusion of the electrode. Commercial welders of the adjustable-slope constant-voltage type have two adjustments—one for adjusting the open-circuit voltage and the other for adjusting the reactance, or degree of slope.

3) More sophisticated welders of this class have both dynamic-reactor and adjustable-slope control. On these designs, the dynamic reactor is also made adjustable. The Westinghouse type RCS, for example, has an inductor with a control winding regulated through a rheostat to vary the dynamic characteristics. The operator has three stepless adjustments, which can be set to give optimum welding performance under practically all conditions.

4) Selecting optimum conditions with three or even two independent adjustments may be difficult, even for an experienced operator. As a highly practical approach to this problem, Westinghouse introduced the type RS power supply with fixed slope characteristic. It has a fixed-slope ac reactor, reducing operator adjustment to one control—voltage. The degree of slope and other characteristics were selected by laboratory tests for satisfactory welding performance under a wide range of conditions.

Special Types—Several types of welders have been developed for specialized applications. The most important is the *tungsten inert-gas* (Tig) welder. It is usually an ac-dc unit to make it adaptable to any metal or alloy. In the Tig process, direct current with straight polarity (electrode negative) is used for welding practically all metals except aluminum and magnesium. With this polarity, the greater part of the heat is directed into the work instead of into the

CONSTANT-CURRENT INDUSTRIAL DC WELDER (top) is made in a wide range of current ratings to suit various applications. This is a Type-WSH 500-ampere unit. The ac welders of comparable class and rating are similar to it in appearance. CONSTANT-VOLTAGE DC WELDER (middle), Type RCS 300 ampere, has controls to adjust a dynamic reactor and to vary the slope of the volt-ampere characteristic for precise "tuning" of the power supply. Voltage also is adjustable. This type is generally used for gas-shielded consumable-electrode welding. In the semiautomatic operation shown, wire electrode is fed into the arc at a constant rate. The cabinet on top of the power-supply unit houses the wire drive and the controls for gas and cooling water.
CONSTANT-VOLTAGE DC WELDER (bottom), Type RS 200-ampere, has fixed volt-ampere slope to simplify adjustment. A wire drive and gas control are housed in the cabinet on top of the power-supply unit.

electrode; consequently, much higher current can be used with a given size of tungsten electrode, resulting in better and faster welding. Some reverse-polarity current (electrode positive) is required for welding aluminum and magnesium because it has a cleaning action on the oxides of those metals. Alternating current provides a half wave of reverse-polarity current for cleaning and also permits use of higher current density in the electrode. For these reasons, it is used for most welding of these two metals.

The Tig welder is similar to the conventional constant-current ac welder, with the addition of a rectifier and a dc inductor. In addition, it usually includes a high-frequency oscillator, gas and water controls, and relays for selecting the desired sequence of operation.

The oscillator superimposes a voltage of sufficient magnitude across the arc to start the arc without making contact between the tungsten electrode and the work. In ac welding, the high frequency must remain on continuously while welding to reignite the arc at each half-cycle current reversal. This high-frequency voltage is harmless to the operator. The oscillator is of the spark-gap type, producing a broad-band output in the radio-frequency range. Special attention is given to the design, construction, and installation of these welders to conform with the radio-interference regulations of the Federal Communications Commission.

In ac Tig welding of aluminum, welders with movable-core reactors give best performance. However, conditions usually encountered in Tig welding require hand or foot current control while welding, and this factor narrows the welder design to use of the saturable-core type of reactor. The result is a good compromise, aided somewhat by auxiliary devices. For example, the saturable reactor is supplemented by a resistor in the Westinghouse type TR welder. The added resistance in conjunction with the reactance improves the current wave form, insuring instantaneous arc reignition each half cycle for improved welding performance and sounder welds.

Stud welders are another special type. Standard industrial constant-current dc welders of the rectifier type could be used for supplying power to stud-welding equipment. However, this is usually an uneconomical application of power apparatus. The duty-cycle rating of standard industrial welders is much higher than required for stud welding; welding time per stud is only about one second, with an overall duty of 10 percent or less. Also, because the maximum ampere output of standard welders is limited, two or more units are required to supply the high current of short duration demanded for stud welding. The size of welding cable used for stud welding is determined by the allowable voltage drop for the length of cable required, rather than by current capacity based on heating. At stud-welding currents of 750 amperes and higher, the voltage drop is an important factor in limiting the maximum amperes; this tends to increase further the size and number of welders required. The situation is one of mismatched power-supply and load impedances.

Stud welders are specially designed three-phase dc power supplies of the rectifier type, matched to the load conditions peculiar to stud welding. They have a nominal current rating of 1000 amperes at 10-percent duty cycle but will deliver up to 1500 amperes under average load conditions. They are much smaller and lighter than conventional welders of the same ampere capacity because of the lower duty cycle and the type of current control (adjustable series resistance). To compensate for the wider range of load voltage encountered, including voltage drop in the cables, these units are designed for 100 volts open circuit.

Because of the high current output and relatively low duty-cycle capacity, stud welders are inherently more vulnerable to abuse than other types. Therefore, a most important feature is reliable built-in protection against accidental short circuit and overloading.

Carbon air cutting and gouging employs a carbon arc and an air jet. The process has become so common in industry that most industrial welders can be expected to meet its requirements.

Although the purpose of the air jet is to blow out the molten metal, it also extinguishes the arc at rapid intervals. The arc either self-ignites or is reignited by contact. Extinction of the dc arc by the air jet is equivalent to the action of a high-speed circuit breaker. When a direct current is suddenly interrupted, the inductive reactance in the power supply reacts in the direction to keep the current flowing and consequently builds up a peak voltage, or inductive "kick." The repetitive high peak voltages can cause damage to certain components, such as rectifiers and generator commutators.

Developments in recent years in the art of dissipating or suppressing the peak energy have successfully overcome the problem of inductive kick. Westinghouse power supplies employ a combination of capacitance and resistance. The capacitance is of sufficient magnitude to store the peak energy and release it slowly through a resistor.

Application of the *plasma arc* for cutting and welding is one of the more recent developments. The process requires power units of the constant-current dc type, with the load voltage and open-circuit voltage largely determined by the type of gas used. In general, the open-circuit voltage and the load voltage are higher than that of standard industrial welders.

While a combination of series-connected standard welders can sometimes be employed, the trend is toward use of special designs that provide the proper electrical features and auxiliary equipment. Such units are made in various ratings to suit the different torch and gas requirements. For example, a unit that supplies one of the larger torches, requiring 50 kw or more, is rated 250 amperes continuous, 320 amperes at 60-percent duty cycle, 400 volts open circuit, and 200 volts load.

Multiple-operator systems are used extensively in shipyards, field construction, and other places where there is a concentration of welders in an area. The dc system employs one or more constant-voltage rectifier power supplies (in parallel) supplying individual operator resistor panels. The system is planned to take advantage of the low duty cycle and diversity of loading of the individual operators, so apparatus cost for a number of welding outlets is comparatively small. The system has the disadvantages of inflexibility in location and low electrical efficiency. The low efficiency is the result of using resistor panels for current control or as arc ballast, instead of using reactance as in conventional single-operator welders.

The Brushless Excitation System for Large AC Generators

*D. B. **Hoover**, Manager, D. C. Electrical Section, Large Rotating Apparatus Division, Westinghouse Electric Corporation, East Pittsburgh, Pennsylvania.*

A completely self-contained excitation system is accomplished by mounting rectifiers on the generator shaft, thereby eliminating any need for movable current-collecting parts.

As the excitation requirements for electric utility ac generators increase, a brushless excitation system is rapidly gaining acceptance in the utility industry. In this relatively new excitation system, the usual commutator, collector, and brushes required with conventional dc exciter systems have been eliminated. Instead, a permanent-magnet pilot exciter, an ac main exciter, and a rotating rectifier are mounted on the same shaft as the field of the ac turbine generator. Thus, the total excitation power requirements, including the power supply for the regulator, are provided directly from the generator shaft. As a result, the exciter system is more reliable, and maintenance is simpler.

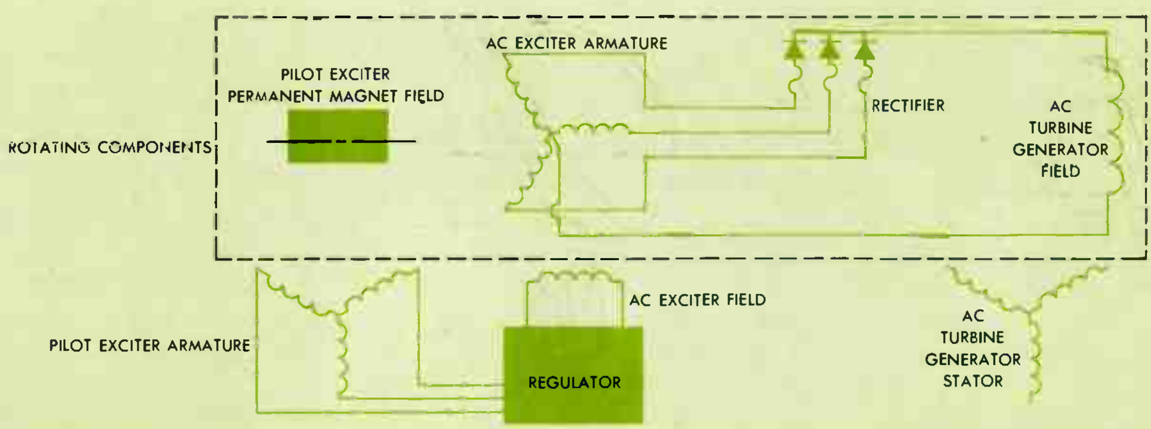
In medium and larger ratings (above 400 kw), the brushless excitation system is also smaller and lighter than the conventional gear-driven exciter with ac collector.

Brushless Excitation System

A simplified circuit diagram of the brushless excitation system is shown in Fig. 1a. The permanent-magnet pilot exciter is a stationary-armature type. It generates 420-cycle, three-phase power, which feeds the regulator. The regulator supplies regulated dc power to the stationary field of a rotating-armature-type ac exciter. The output of the ac exciter is rectified by diodes and fed to the field of the ac turbine generator. The system is protected against diode failure by series-connected fuses.

The exciter and rectifier are arranged in one of two basic circuits: a wye-connected ac exciter feeding diodes

1a



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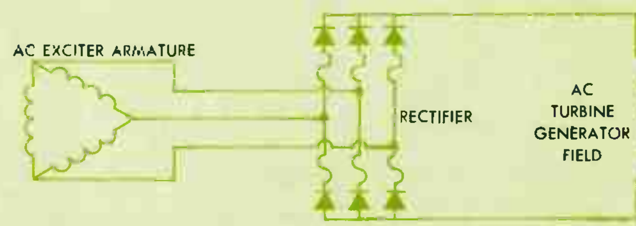
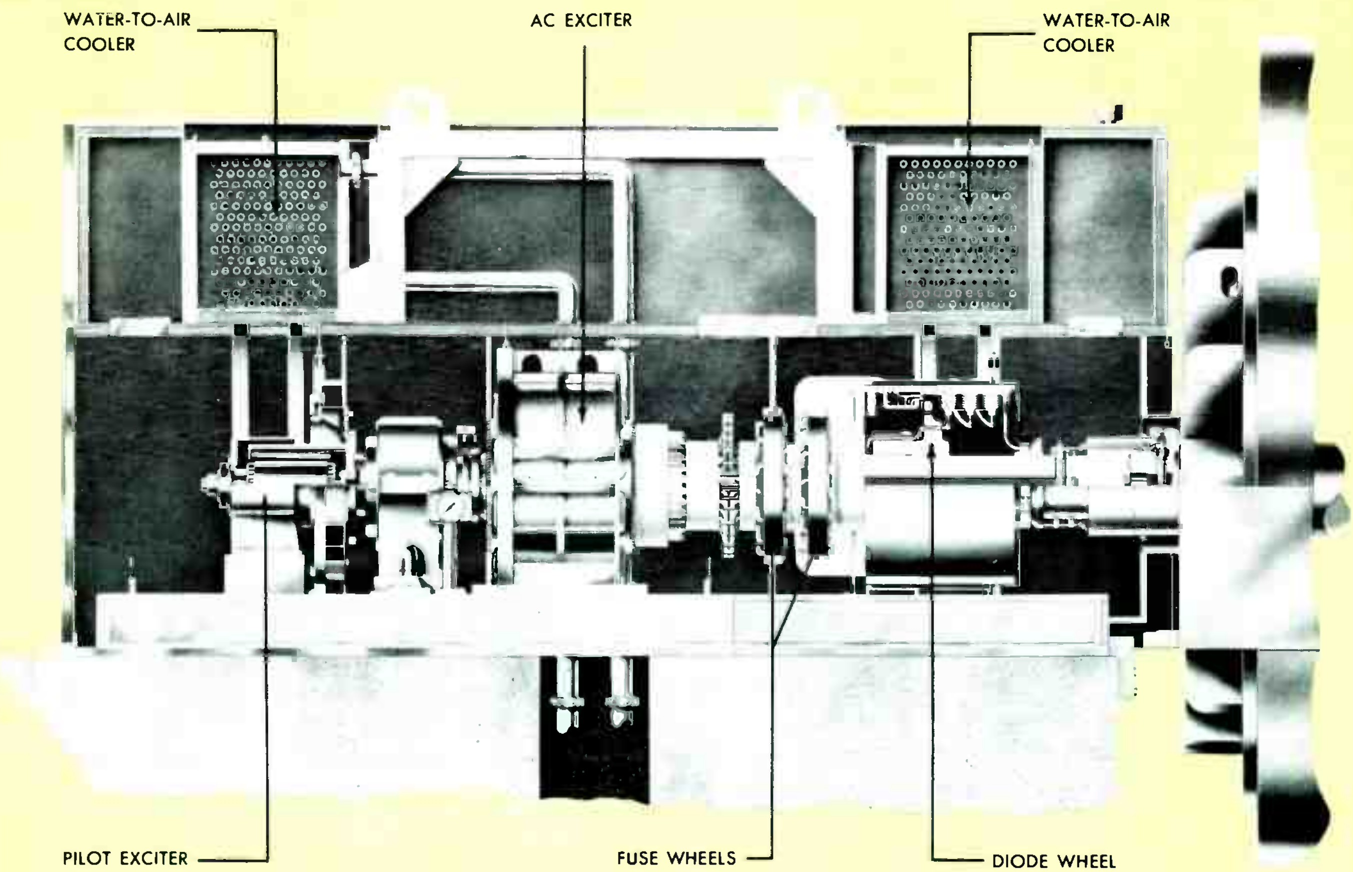


BRUSHLESS EXCITATION SYSTEM keeps all excitation power on the rotating shaft by mounting the rectifiers and protecting fuses on the same shaft as the pilot exciter, ac exciter, and turbine generator field. A wye-connected arrangement is shown in (a), and a delta-connected arrangement in (b).

2



COMPONENT ARRANGEMENT of the brushless excitation system is shown in this artist's drawing of the 1350-kw unit.

in each phase with a neutral return, or a delta-connected ac exciter feeding a full-wave bridge. The wye with neutral return requires less wiring and fewer fuses but increases the peak-inverse-voltage requirements on the diodes. The bridge circuit (Fig. 1b) requires more fuses and wiring, but has lower peak-inverse-voltage requirements per diode. The voltage rating determines which circuit is better. (Voltage ratings range from 160 to 500 volts.)

In either circuit arrangement, each phase has several diode paths in parallel, the number depending upon current requirements; diodes are connected in series depending upon peak-inverse-voltage requirements. Resistors and capacitors (where required) in the paralleled circuits insure proper voltage division between series-connected diodes, and suppress diode spike voltages during the diode commutation period.

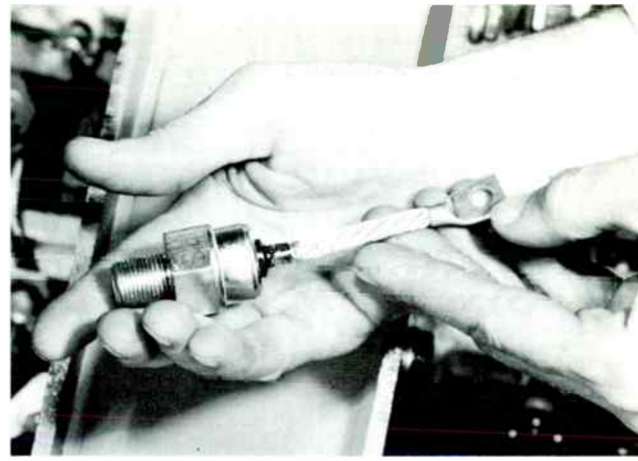
Rectifier circuits ranging from two to twenty parallel paths per phase, depending on the rating of the exciter, have been designed. A 1350-kw exciter, for example, that uses a three-phase neutral return circuit, with ten paths in parallel per phase, has been designed and installed. Each path has a fuse and three diodes in series.

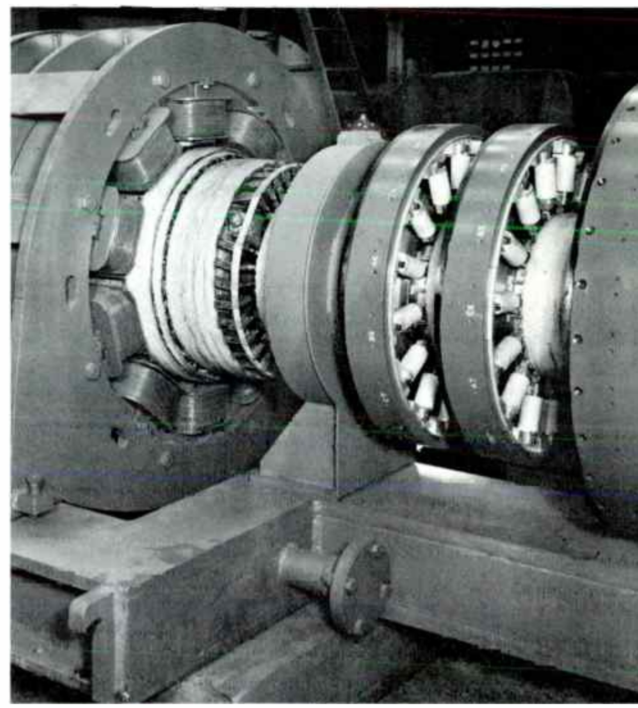
The whole rotating element is designed to withstand the forces of rotation, torsional vibration, and critical speeds. Both the diodes and fuses are mounted in wheel rings, in a position that minimizes the effect of high-speed rotation on lead and base connections.

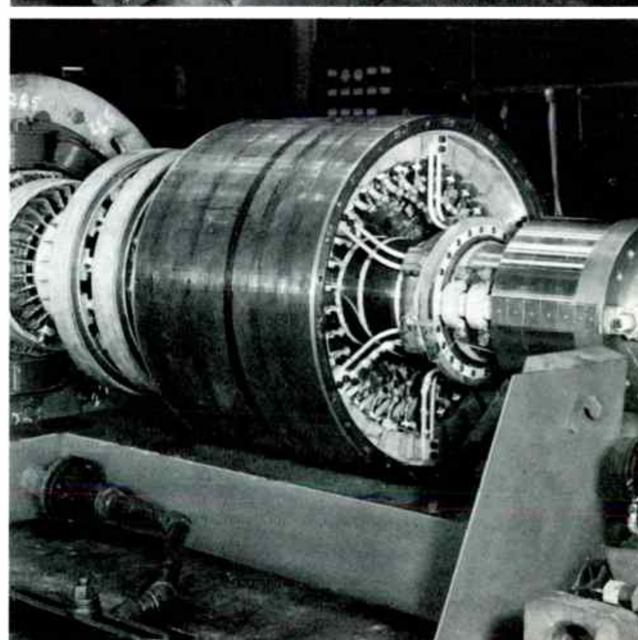
Component Development

Brushless excitation systems of this type have been used extensively in aircraft applications. Silicon diode rectifiers have proven ideal in the application. However, the higher-rated silicon diodes required in electric utility power systems were originally designed for stationary application, and therefore underwent extensive design analysis and testing for 3600-rpm operation. Available designs were studied to determine the best axis of rotation, and special lead connections, and base mountings, were developed to withstand the forces of high-speed rotation. The resulting designs were tested in a special "spin" tester to prove the design both at rated 3600 rpm and also at 20 percent overspeed.

The fuses must prevent any connected circuit from feeding into a shorted diode. However, a fuse must not blow for an operating condition, such as short circuit on the ac generator, or during field forcing. Mechanically, the fuse must withstand the same high forces of rotation as the diode. It must also be possible to easily detect any open fuses while the unit is in operation so that a shutdown can be planned after a given number of fuses have opened.



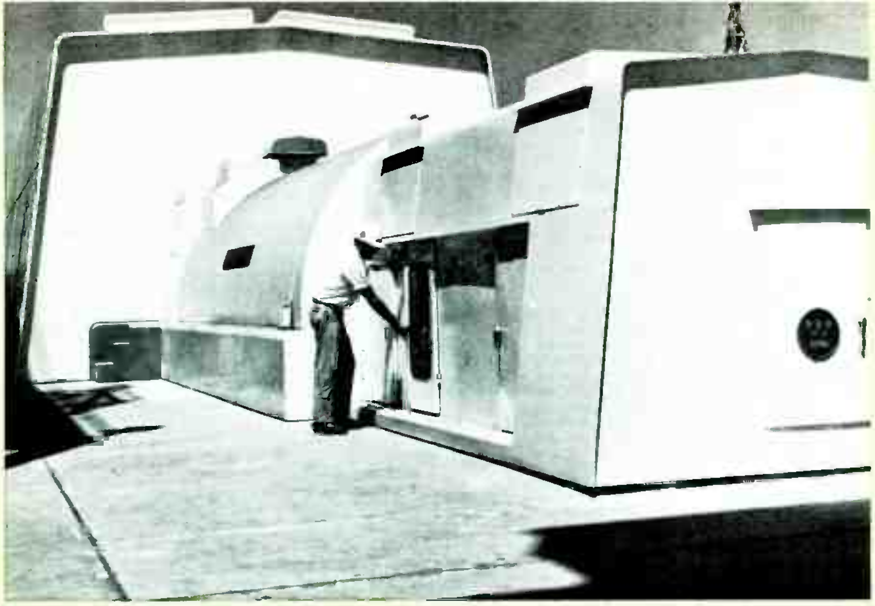




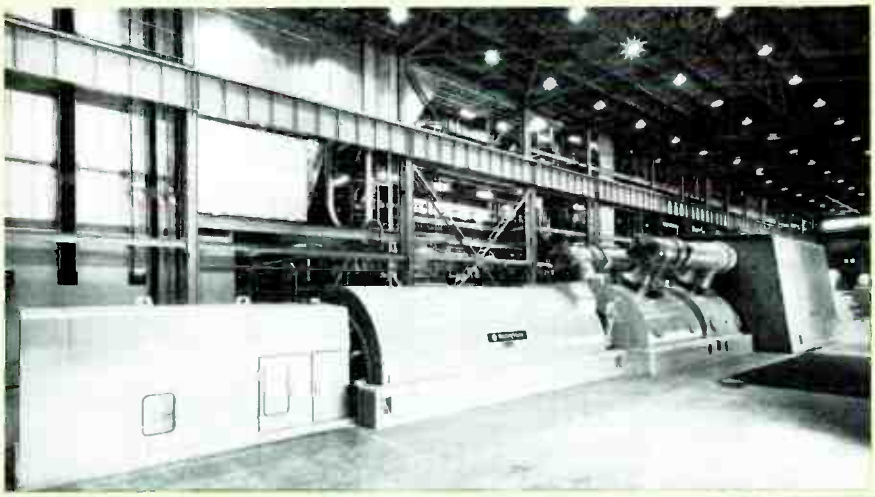
TYPICAL SILICON DIODE RECTIFIER (top) used in the brushless exciter is rated at 225 amperes.

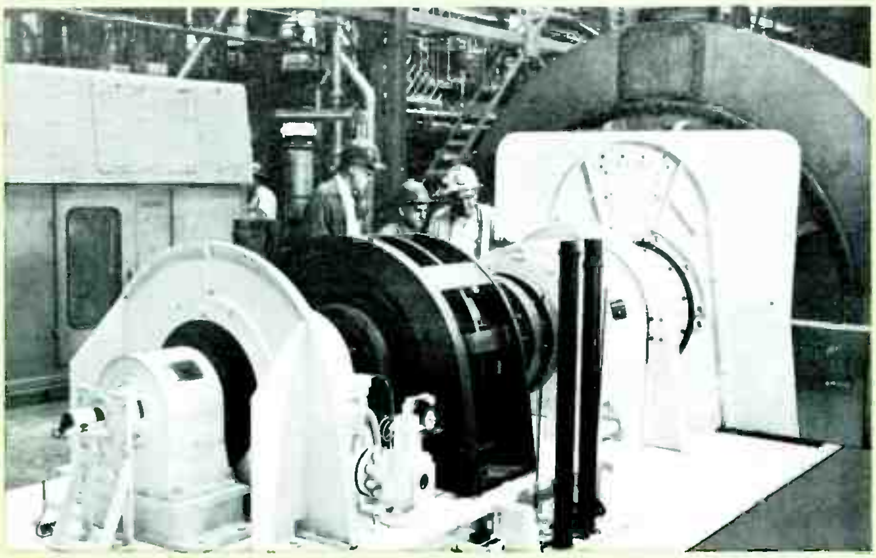
FUSES ARE MOUNTED (center) on wheels in a position to best withstand rotational forces. If a fuse blows, an indicator is released and is easily detected under stroboscopic light.

SILICON DIODES (bottom) are also mounted within a wheel ring to withstand rotational forces. The mounting arrangement is such that every diode is accessible without removing any other diode.



1150-KW BRUSHLESS EXCITER provides excitation power for 243-mva turbine generator at the Asheville Steam Electric Generating Plant of Carolina Power & Light Company.





TURBINE GENERATOR UNIT THREE of West Penn Power Company's Mitchell Station is rated at 352 mva. The housing on the left end of the turbine generator contains the 1350-kw brushless excitation system. Brushless exciter system is shown with the housing removed (below). From left to right, major components are pilot exciter, ac exciter, fuse wheels, and diode wheel.

The fuse design was developed with an indicating device that is normally held back by the fuse element. If the main fuse element opens, the indicating device is released and moves out by centrifugal force. The released indicating device is easily detected under stroboscopic light while the machine is in operation. Intensive rotational tests including mechanical checks, current-carrying capacity, and interrupting capacity have verified fuse performance.

In most cases, a totally enclosed ventilation system is provided to insure proper cooling of all components. The larger units use two water-to-air heat exchangers. Smaller units need only one heat exchanger. Two paths of air circulation are provided within the housing, one for the ac exciter and the other for the diode wheel. With this ventilation arrangement, foundation design is simple because only cooling water pipes and small external electrical connections are required. The electrical connections are the small leads from the stationary pilot exciter armature to the voltage regulator and from the voltage regulator to the stationary ac exciter field.

Reliability

Because the rotating rectifier excitation system derives all of its power (including excitation) from the shaft of the main generator, it does not depend on any other rotating machines, such as a motor-generator exciter set or pilot exciter.

With this feature, plus components of proven reliability, designers hope to eliminate the need for reserve excitation. All components are applied on the basis that continuous conservative operation can be achieved with only 80 percent of the diodes per phase in operation. The excitation system will operate successfully through any transient conditions imposed by the ac generator, including operation at ceiling voltage during forcing with only 80 percent of the diodes per phase in operation.

The elimination of possible outages because of collector and commutator brushes, the elimination of the adverse effect of carbon dust in the system, and the elimination of the field breaker and motor-operated field rheostat provide further reliability advantages over conventional dc exciter systems.

Maintenance

The brushless excitation system requires no maintenance or inspection other than checks of oil flow, temperatures, visual fuse checks, and an occasional check of the insulation resistance to ground. Since there are no collector or commutator brushes, carbon dust is eliminated. The absence of carbon dust lengthens the time between cleanings and lessens the possibility of low insulation resistance to ground.

The system is not affected by atmospheric contamination since all of the components are sealed. This is a major advantage over the present operation of carbon brushes in contaminated air.

The elimination of brush replacement and maintenance and of collector and commutator maintenance is a major advantage. Brush life for dc commutators on large conventional dc exciters is normally somewhat less than a year under usual operating conditions; ac collector brushes, because of the higher operating speed, have an even shorter life. Usually at intervals of every few years, surfaces of commutators and collectors must be resurfaced.

The brushless exciter need not be shut down until 20 percent of the diodes (per phase) are out of service, and with the use of stroboscopic inspection, shutdowns can be planned well in advance. The diodes are arranged for easy access and any diode may be changed without removing any other diode. This is also true of the fuses.

Large access doors are used during shutdown inspection. During operation, the doors remain closed and inspection is made by stroboscopic light through double-glass windows.

The insulation resistance-to-ground is checked by taking readings obtained from two rings on the exciter; one ring is connected to ground and the other to the windings. Brushes are pressed onto these rings by a solenoid device only while resistance to ground is being checked.

Operating Experience

The initial application of the brushless excitation system was a 180-kw, 250-volt unit for a 50-mva generator of the West Penn Power Company. This developmental installation has now been in operation about 4½ years. Early design improvements were made to eliminate fatigue of solder between the wafers of the diode, and to eliminate loosening of the fuse element where it is attached to the fuse base. Four diodes and two fuses were replaced because of the original construction, which has been changed for future units. In the last four years, only one fuse and three diodes have been replaced (because of normal failures) out of a total of 36 diodes and 18 fuses. There has been no replacement of diodes or fuses for over a year. None of these replacements required emergency shutdown since all work could be scheduled during normal maintenance shutdowns. The reliability of future units can probably be based upon this experience record.

In August 1963, the second brushless excitation system was put into operation, a 1350-kw, 375-volt brushless exciter on a 352-mva generator at the Mitchell Station of the West Penn Power Company. This exciter contains 90 diodes and 30 fuses. Two diodes have been replaced since installation. Both diodes were replaced because of cracked ceramic insulation, which may have been caused during installation or perhaps from a slight flaw in the ceramic material. However, the trouble did not cause either diode to fail and did not cause a shutdown.

The third unit, a 1150-kw exciter on a 243-mva generator at the Asheville Steam-Electric Generating Plant of the Carolina Power & Light Company, has been in operation approximately six months. In addition, four other units have been shipped and are either being installed or are already in operation. Eighteen more brushless exciters are on order in ratings from 185 to 3500 kw. The 3500-kw brushless exciter will be installed in a 733-mva tandem compound generator unit, the largest rated ac generator on order.

The successful design, installation and operation of these inital brushless excitation systems has resulted in continued acceptance of this new type of excitation system. The advantages of the brushless excitation system should be even more significant on the larger ac generators of the future.

Wide-Range Speed Control for Small Motors

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A simple and versatile thyristor drive system gives good control at low cost.

Speed control of dc fractional-horsepower motors has been a problem because the traditional variable-speed drives involve compromise. Control has been possible only at the expense of torque loss at low speeds, poor speed regulation, or excessive size and weight, in addition to high cost of installation, operation, and maintenance.

Now, however, the economic availability of the solid-state control device known as the silicon controlled rectifier (thyristor*) has made it possible to design dc drives that provide good performance, are inexpensive to install and operate, are extremely compact, and have high reliability. The new thyristor drive reduces or eliminates the problems encountered in applying wide-range speed control to small dc motors.

These characteristics suit the drives to a host of fractional-horsepower applications, such as conveyors, test stands, portable hand tools, machine tools, processing machinery, winding machines, and materials-handling machinery.

Motor Control Problems

The speed of a conventional series-wound dc motor is a direct function of the armature current. However, using a rheostat in series with the series winding to drop the armature current causes torque fall-off at low speeds and poor system efficiency. Another method bypasses some of the current from the armature and increases the field current, thus reducing the motor speed. Although this technique provides better torque characteristics than does the straight resistance method, efficiency still is poor. As the speed of the motor approaches zero rpm, the power dissipation around the armature circuit approaches the rated power of the motor.

The classic adjustable-speed motor is the dc shunt-wound unit with the speed controlled by varying a resistance in series with the field or the armature. However, if only armature resistance is varied, widely varying torque loads will cause motor speed to vary widely. Regulation at low speed settings, where the resistance is highest, is poor; power dissipation and poor efficiency also present problems.

For small motors, the field flux can be changed by varying the shunt-field current and the reluctance of the magnetic circuit. This approach provides a wide range of speeds with good regulation throughout all speed settings. The major objections to it are the derating, the instability of the motor due to main pole weakening, and the dissipation in the field resistance. Also, it cannot provide control below the basic motor speed.

The Thyristor Drive

In contrast, the thyristor drive supplies continuous control through a desired range of speed ratios. It does not have the power losses incurred with conventional dissipation circuits, and it allows control of the armature voltage and current without sacrificing torque characteristics. The drive also provides IR compensation for better regulation of speed versus torque as the speed is reduced. Moreover, the drive gives constant-speed operation under varying loads, has speed regulation of five percent at one-eighth speed, and gives high torque at low speeds. It can control dc shunt, series, and compound motors, as well as universal motors.

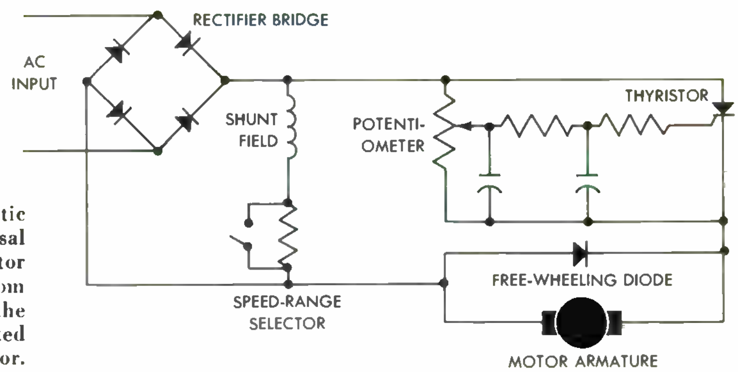
To get all of the performance advantages possible from the drives, motors can be obtained with armature and shunt field voltages specifically matched to the outputs of the drives.

The thyristor drive is simple in design and operation (Fig. 1). Its ac input is rectified by a full-wave silicon-rectifier bridge. The output of this bridge is controlled by a thyristor, which in turn is controlled by a potentiometer. This circuit provides an adjustable voltage to the armature of a dc or universal motor. The shunt field of the motor is energized at a constant value from the load side of the rectifier bridge.

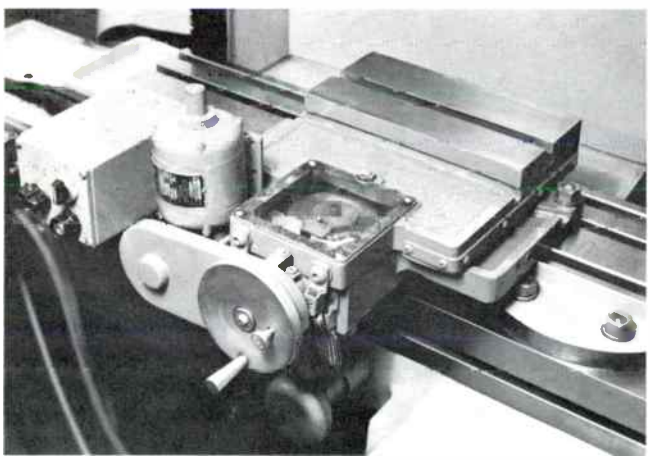
The inductive dc nature of the load requires the inclusion of a free-wheeling diode to maintain current flow when the

"Thyristor" is the term accepted by the IEEE standards committee for the silicon controlled rectifier. It replaces the terms "SCR" and "trinistor."

1



THYRISTOR DRIVE SYSTEM, shown in simplified schematic form, has a rectifier bridge, a thyristor, and a dc or universal motor. A potentiometer controls the firing time of the thyristor to vary the voltage to the motor armature, and feedback from the armature to the thyristor tends to keep speed constant at the set value. A resistor in series with the shunt field can be shorted out, providing two control ranges with a shunt-wound motor.



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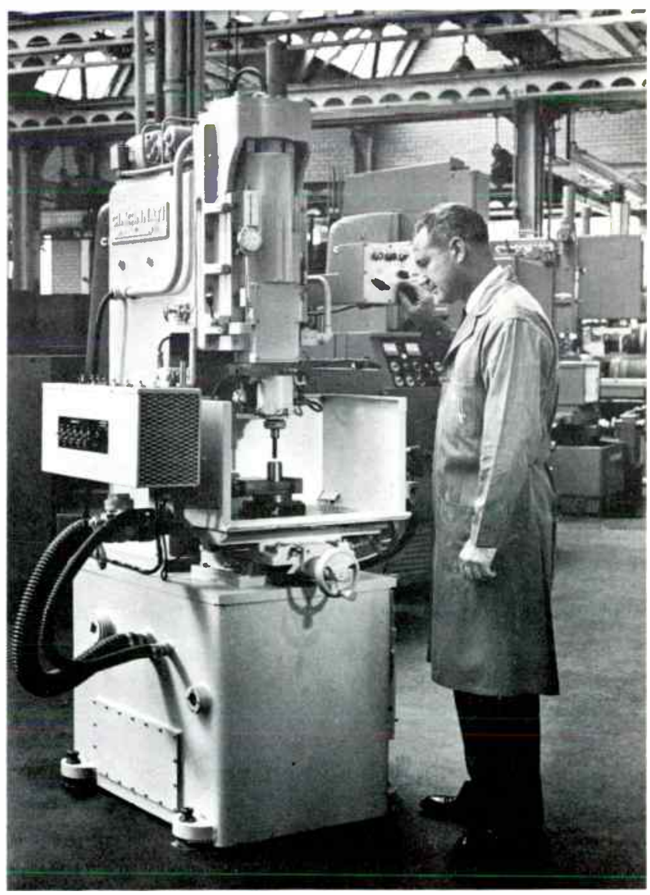
ELECTROLYTIC PLUNGE-GRINDING ATTACHMENT for an electrical machining tool is powered by a thyristor drive. The compact drive is completely mounted on the attachment, so both it and the attachment can be removed when not in use.



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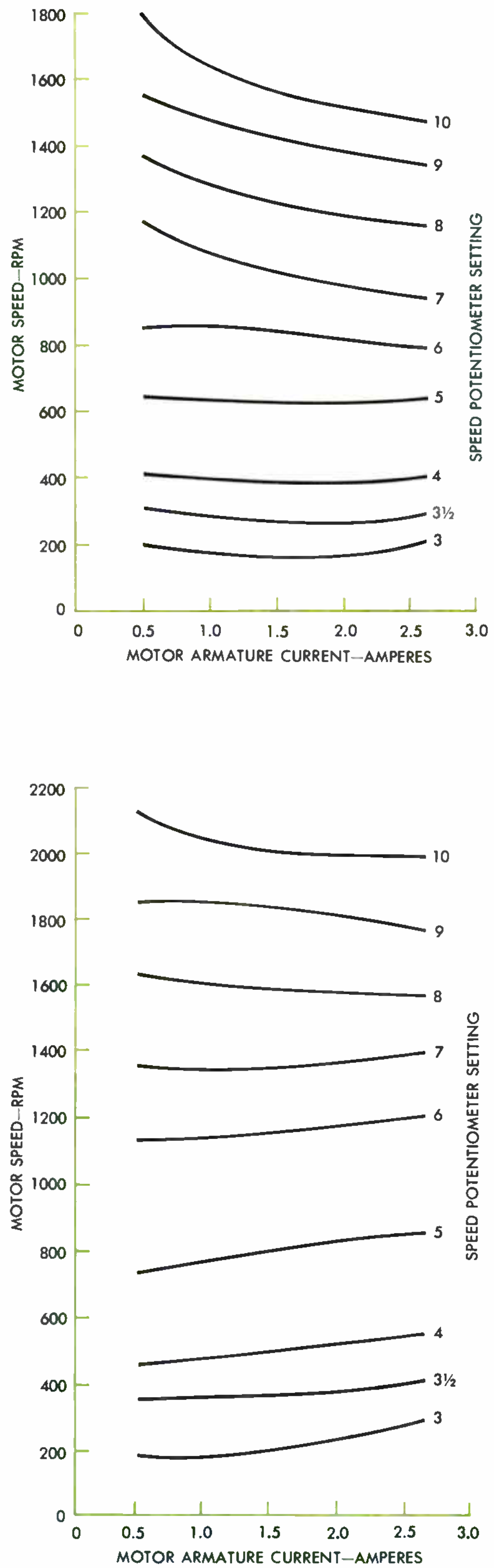
ELECTRICAL DISCHARGE MACHINING TOOL (right) has a thyristor drive to rotate electrodes of various sizes at controlled speeds.

4



SPEED REGULATION with the type 902 thyristor drive in the *low* speed range (top) and the *high* range (bottom). Excellent control is provided over a range of seven to one or greater. The curves are based on the performance of a dc shunt-wound motor rated at 1/4 horsepower, 2.5 amperes, and 1725 rpm.

thyristor is turned off. (This inductance causes a holding current to flow through the thyristor, preventing commutation, or complete turnoff. The free-wheeling diode suppresses this current, allowing the thyristor to turn off.)

A feedback circuit from the motor armature to the thyristor provides high torque at low speeds. Armature counter-emf supplies the feedback signal. If a heavy load starts to reduce the motor speed, the induced voltage decreases and the thyristor fires earlier in the cycle; the additional current furnished to the motor supplies the necessary torque to handle the increased load. A light load tends to increase armature counter-emf and speed; this action retards the firing and reduces the voltage supplied to the motor, thus maintaining stable operation.

Because the thyristor drive is simple, it is compact and reliable. Heat generated by power loss is not a problem in this drive because it does not have a transformer nor a rheostat. The solid-state devices, including the thyristor, are protected against voltage transients by a selenium surge suppressor connected across the line side of the silicon-rectifier bridge. Because of these factors, maintenance costs of thyristor drives are negligible. In various ratings, they are now replacing conventional units in industrial applications.

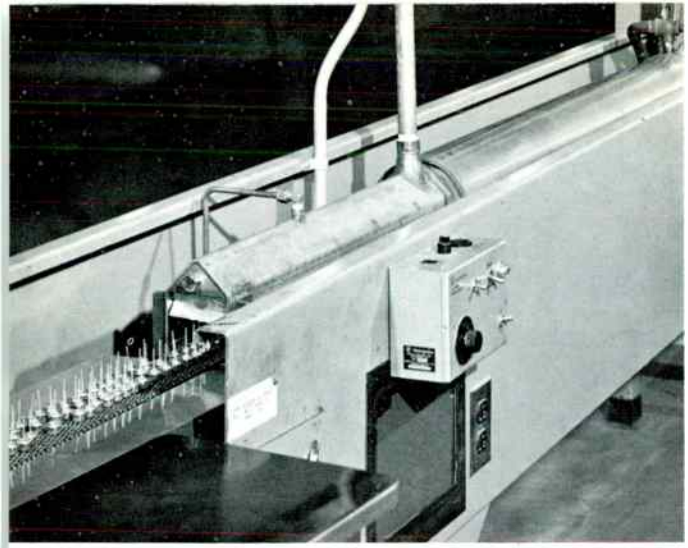
For example, the Westinghouse type 901 thyristor drive, for motors of 1/15 horsepower and smaller, is being used in electrical machining equipment manufactured by the Meta-Dynamics Division of the Cincinnati Milling Machine Company. In an electrolytic plunge-grinding attachment, a slide operated by a cam is driven by a dc shunt motor (Fig. 2). The cycle of operation is rapid advance, feed, dwell, and rapid retract, with feed rate controlled by varying the speed of the motor. Because the thyristor drive is completely mounted on the attachment, both attachment and drive can be removed from the machine when not in use. The drive provides wide speed range, and high torque at low speeds, with low cost.

The same manufacturer also uses the drive in an electrode-rotating attachment for electrical discharge machining (Fig. 3). This process improves dielectric flow conditions and permits the machining of precise round holes. A variable-speed drive with a dc shunt motor rotates electrodes of different sizes at the required speeds. Again, the thyristor drive has wide speed range, high torque at low speeds, and low cost.

The type 902 thyristor drive is designed for motors up to 1/3 horsepower. To permit two ranges of speed control for shunt-wound dc motors, a resistor with a shorting *high-low* toggle switch is inserted in series with the supply to the shunt field (Fig. 1). For the *low* speed range, the resistor is shorted out by the toggle switch and the shunt field operated at full rated value. For the *high* range, the resistor is inserted in series with the shunt field; the motor then operates at a top speed higher than that obtainable with the field at full rated value (Fig. 4).

A type 902 drive is being used at the Westinghouse Semiconductor Division to power a conveyor belt for a continuous gas-atmosphere brazing furnace (Fig. 5). Speed control previously was obtained by changing belts and pulleys, which was both time-consuming and expensive in maintenance costs.

In another application at the Semiconductor Division,



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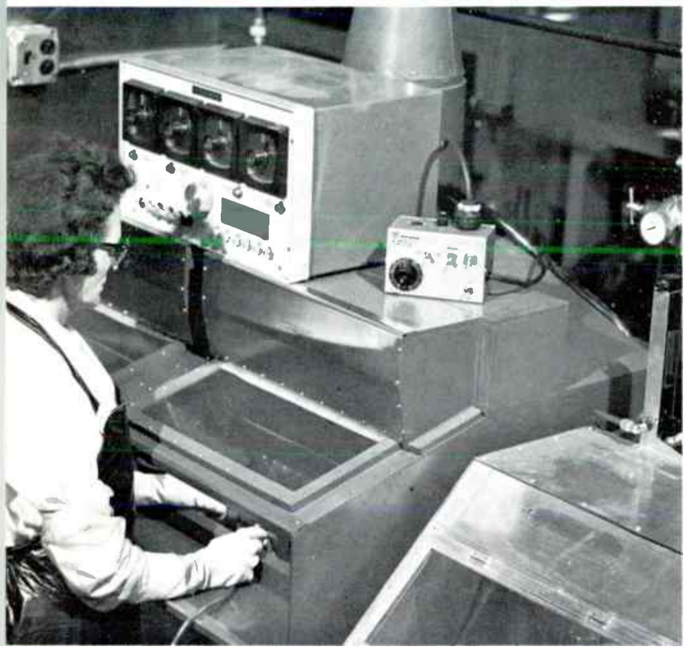

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5) CONVEYOR BELT for a continuous furnace is powered by a thyristor drive, with a dc shunt motor, operating from 115 volts ac. This system replaced an inefficient belt and pulley arrangement. Its speed range is eight to one.

6) SPIN-ETCH MACHINE used for processing silicon junctions for semiconductor devices is powered by a thyristor drive. The drive replaced a variable transformer-rectifier system and eliminated the problem of insufficient starting torque at low speeds.

7) SABER SAW is powered by a thyristor drive for precision cutting of stainless steel. Precise speed control at low speeds prevents tool breakage and tearing of the material.

a type 902 drive with a dc shunt motor powers a spin-etch machine used in the processing of the basic silicon junctions for high-power semiconductors (Fig. 6). It eliminated the problem of insufficient starting torque at low speeds encountered with the previous variable transformer-rectifier type of control.

Yet another application example is a portable saber saw controlled by a thyristor drive for cutting stainless steel (Fig. 7). The controlled speed prevents tool breakage and allows precision processing.

A larger thyristor drive, the type 903, is for motors up to one horsepower. It operates from 115 or 220 volts, single phase, 50 or 60 cycles. When operating from 115 volts, the output is adjustable from 0 to 90 volts for the motor armature and 100 volts fixed dc for shunt-field excitation. With 220-volt input, the output is adjustable from 0 to 180 volts dc for the motor armature and 190 volts fixed dc for the shunt field.

Speed range of the type 903 drive is normally eight-to-one or greater, and regulation is approximately five percent after stabilization. (Both speed range and regulation vary with the application and with the type of motor driven.) Applications for this drive include textile-machine control and washing-machine cycling control.

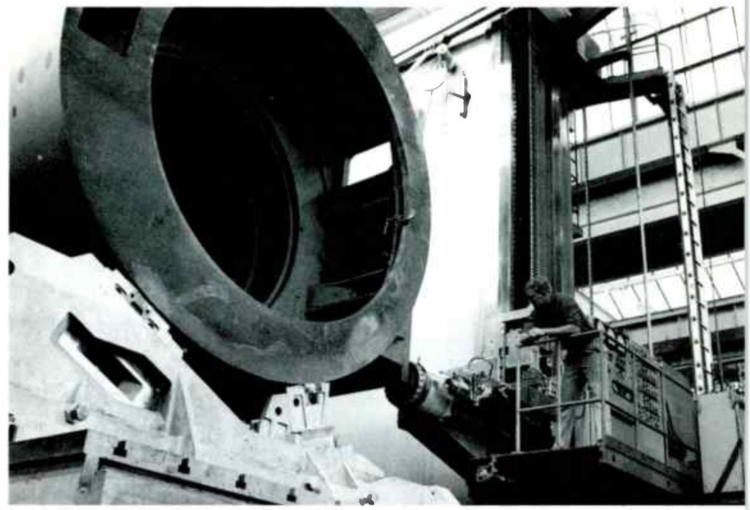
Other Uses

The thyristor drives can also be used as general-purpose adjustable dc power sources for applications such as saturable-core reactor controls and other applications where close regulation is not required. The type 901 can supply up to 1.5 amperes resistive or capacitive load; the 902, 5 amperes. The 903 operating from 115 volts ac can supply adjustable voltages from 0 to 90 volts dc, at up to 6, 8.5, and 12 amperes maximum. Operating from 220 volts ac, it can supply dc voltages from 0 to 180 with maximum amperage of 3, 4.5, and 6.

Conclusions

Thyristor controls for small motors have advantages, for many applications, that the older drives do not have. They give good control at lower cost, and they permit a smaller motor to be used in some applications than would otherwise be possible.

Control cycles can be set up directly from sensing devices for cycling speeds higher than those attainable with other inexpensive electromechanical controls. These systems can be as complex as desired. When a tachometer feedback system is used, for example, speed regulation as low as 0.01 percent can be obtained.



TECHNOLOGY IN PROGRESS

Computer Program to Simulate Rapid-Transit System Operation

With rapid-transit systems becoming ever more complex and more important in urban life, it is increasingly desirable to be able to predict the performance of a proposed system before any part of it is built. Modern high-speed computers offer the means for doing just this, and a program for system simulation is now being devised.

The program will simulate the movements of trains and passengers as well as other factors that make up the total operation of a rapid-transit system. With it, a digital computer will be able to simulate hours of operation of the full-scale rapid-transit system in a few minutes. The computer will give answers to questions concerning, for example, the number of passengers moving on and off trains, passenger waiting time for boarding trains, "hold" time of trains during passenger loading, delay in arrival and departure of trains at crowded stations, changes in transit time of trains between stations, and transfer time at intermediate points and at final destination.

This simulation will give the system designer a powerful new tool for evaluating his work quickly and at low cost. The technique will automatically generate data now laboriously produced by hand, actually printing out such data as time schedules and graphs of train movement from station to station. Because the technique will be applicable to any proposed transit system and train schedule, the designer will be able to study the effects of system or schedule changes just as though he were working on a real transit system. He will be able to analyze the results according to any performance criteria he wants—such as passenger waiting time—simply by making the criteria a part of the computer program.

With a computer programmed in this manner, the effects of automatic control schemes can be studied against the background of various system arrangements and passenger loadings. A designer will even be able to insert random occurrences into the simulation, such as fluctuations in passenger arrival rates or various types of interruptions that might occur on the system. Thus, the rapid-transit simulation will allow for the various irregularities to be expected of any system that caters to the fluctuating habits and needs of human beings.

The computer language that is used reduces the operation of the system under study to specific events occurring at particular times, with the interval between events variable. A series of simulated events occurs as a train approaches, stops, and leaves a station, during which time the movement of trains and passengers is closely followed. Travel time of trains between stations is calculated separately from the other parts of the program; it is extended if an arriving train is not free to enter a crowded station. Any sort of delay will be handled by the program.

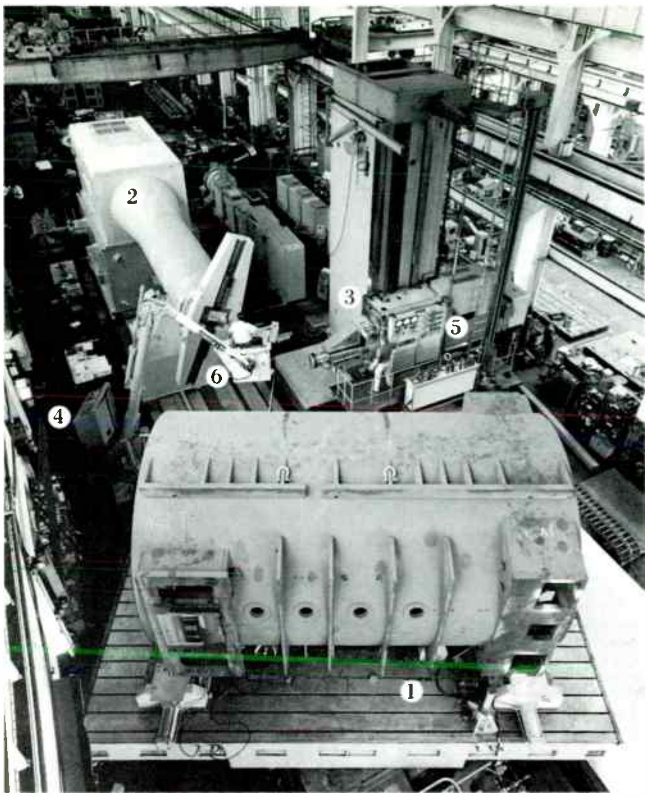
The output of the simulation will be a series of notices that certain events have occurred at specific times. Thus, system designers will be able to trace the motion of a train throughout the transit system by picking out the notices that announce its arrival, stop, departure, and passenger handling at the various stations.

An ultimate goal in the program development is the ability to simulate any transit system by inserting a deck of cards, punched with data on the physical description of the system, into a computer. A New York subway and a San Francisco train system, for example, could then be studied in two successive runs simply by removing one data deck and inserting another.

Large Generator Frames Machined at One Station

A giant machining center makes possible a completely new generator manufacturing technique, that of performing all machining operations on large generator frames at one work station. The technique will improve machining accuracy, with consequent quality improvement. Numerical control, preset tooling, special fixtures, and optical set-up add to the accuracy.

The machine accepts frames as large as 16 feet wide, 38 feet long, and 160 tons in weight. It performs milling, slotting, boring, facing, drilling, tapping, spotfacing, counter-sinking, and slotting in any desired sequence. The machine was photographed being tested in the plant of its manufacturer, Farrel Corporation, Rochester, New



MACHINING CENTER for large generator frames (above) performs all machining operations at one work station. Its side unit (left) machines the sides of a frame, while its end unit (right) works on the two ends.

York, before being installed at the Westinghouse Large Rotating Apparatus Division, East Pittsburgh, Pa.

An indexing table (1, in center photograph) holds the generator frame. It can be rotated 180 degrees, and it positions the frame so that its ends and sides can be machined by the end unit (2) and the side unit (3). These two units are controlled from separate stations (4 and 5); however, they can operate simultaneously. The Prodac numerical positioning and control system at each station can be commanded by punched tape, by manual setting of dials, or by manual pushbuttons and switches.

The end unit indexes and drives five interchangeable heads—a 16-foot double-slide facing head, a drill head, and three combination boring and milling heads. Its boring spindle is driven by a 50-horsepower dc shunt motor. A portable control station gives an operator limited manual control of the unit while setting up the generator frame or while using the lifting device (6) to change tools.

The side unit has a milling spindle and a drilling spindle. The milling spindle is driven by a 100-horsepower dc motor, operated over a four-to-one speed range at constant horsepower through shunt-field control.

Fluorescent Lamp Performance Improved

The mercury-vapor pressure in ordinary fluorescent lamps fluctuates with changes in the temperature of the surrounding air and, as a consequence, light output of these lamps changes widely with temperature. The lamps provide maximum light output only within a narrow temperature range, and they lose up to 30 percent of their light output over the normal range of temperature changes.

Now, however, a fluorescent lamp that has practically constant light output over a temperature range of 40 to 130 degrees F has been developed by the Westinghouse Lamp Division. The lamp can be expected to produce over 25 percent more light outdoors and over 15 percent more light indoors, depending on temperatures.

Mercury pressure inside the new lamp is controlled by the addition of a small amount of a rare metal. This material attracts and releases mercury atoms inside the lamp as temperature varies, controlling pressure.

Oceanographic Survey Ship Has Integrated Electric System

A complete ship's electrical and electronic system is being supplied as a coordinated package for the first time. The ship, designated AGS-26, is one of the Navy's newest oceanographic survey ships. It is being built by American Ship Building Company, Lorain, Ohio. It will have a full-load displacement of 2550 tons, a 285-foot length, and a 48-foot beam, and it will accommodate 12 officers, 32 crewmen, and 34 scientists. Westinghouse will supply the electric propulsion system, the ship's service power and distribution systems, and navigation and communications equipment.

The main propulsion system will consist of two main 1260-kw diesel generator sets and a double-armature 3000-horsepower dc propulsion motor. An electric control system will provide control of the ship's speed and course from a pilot-house stand and from bridge-wing stands.

A second propulsion unit, located in the bow, will be powered by a gas turbine driving a 300-kw dc generator. It will give the ship the precise maneuverability and positioning ability required for its work. If the main propulsion generators become inoperative, the bow-propulsion generator can be connected into the main system to supply emergency propulsion power.

The ship's communications equipment includes radar, sonar, radio direction finders, radio telephone, radio telegraph, facsimile, and intercommunication systems. Navigation equipment includes a course computer, automatic pilot, gyrocompass, and underwater log system.

Molecular-Electronic Radar and Infrared Tracking Systems

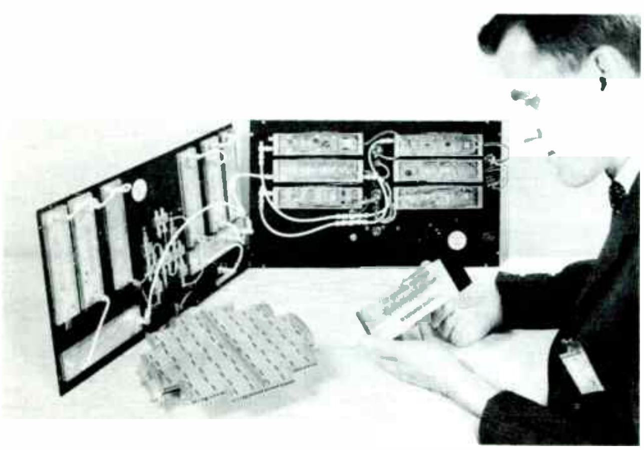
Molecular-electronic functional blocks have been used extensively in two complex aerospace electronic systems—an infrared tracking system and a doppler radar system. In both, the molecular-electronic techniques drastically reduce the number of components required; this reduces size and weight of the systems and also tends to improve reliability and reduce costs.

The *infrared tracking system* has 543 components, which are the electronic equivalent of 6300 conventional components. They include 135 specially developed molecular-electronic devices and 408 high-reliability standard components. The system was developed by the Westinghouse Aerospace Division under a contract from the Electronics Technology Division of the U. S. Air Force Avionics Laboratory.

The engineering model measures 18 inches in diameter and 30 inches long, and it weighs 95 pounds. Most of its weight is in the standard optics used; special optics could reduce size and weight in an operational system to 13 inches in diameter, 24 inches in length, and approximately 30 pounds in weight.

A mosaic of 100 infrared detection cells enables the tracking system to detect targets within a four-degree field of view, automatically acquire targets, and track without mechanical scanning. Each detector has its own channel amplifier. Each amplifier consists of two silicon wafers mounted in a package 3/8 by 3/4 by 1/32 inch in size. The amplifiers have a voltage gain of 45 000, stabilized within 10 percent from −40 degrees C to +80 degrees C, and a total input power requirement of one to four milliwatts. Noise figure is less than 4.5 decibels and input impedance more than 100 000 ohms.

DOPPLER RADAR CIRCUITRY built mainly from molecular-electronic functional blocks could be packaged in the mock-up held by the man. Major components of a bread-board model that has been operated are (left) a microwave and intermediate-frequency assembly and (right) a signal-processing and tracking assembly. The planar-array antenna lies between them.



The system detects heat radiated from any target within its field of view and generates a coarse error-tracking signal that can be used to center the system on the target. When the target is brought into the center of the field of view, the center four detectors in the mosaic generate a precise angle-tracking signal accurate to less than one milliradian.

The *semiactive doppler radar system* was built as an operating breadboard model to evaluate the practicality of using molecular-electronic techniques in radar applications and to serve as a dynamic test tool for molecular-electronic devices in radar circuitry. Called MODOR (for molecular doppler radar), the new unit operates at X-band frequency. It has a sensitivity of −110 dbm (decibels below one milliwatt) and a velocity-tracking bandwidth of 5 kc.

All receiver and signal-processing circuitry is made up of the Westinghouse commercial WM-1106 molecular-electronic functional blocks and of two types of developmental block—LAVA and MIRT. The LAVA is a universal linear block containing the equivalent of six transistors, two field-effect devices, four diodes, and 18 resistors on a single double-diffused planar passivated silicon substrate. The MIRT is a linear block designed for extremely high dc isolation and extremely low-power operation. It contains the equivalent of a transistor with all the necessary bias resistors and diodes for temperature-stabilized operation. The only nonmolecular devices used are the solid-state local oscillator and the planar-array antenna.

The receiver subsystem has two 30-megacycle preamplifiers, a 10-megacycle offset oscillator, a 40-megacycle filter amplifier, and a 10-megacycle signal i-f amplifier. WM-1106 molecular-block amplifiers and etched functional-circuit tuning boards permit all i-f functions to be achieved with only 45 discrete components.

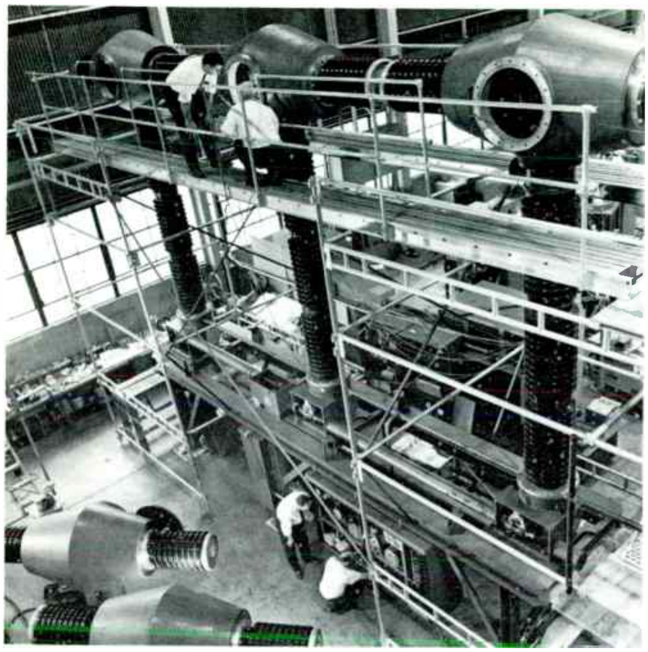
The signal-processing circuitry, consisting of velocity-tracking loop, automatic gain-control loop, and velocity-searching and -detection unit, has LAVA and MIRT blocks for all gain functions. It has 75 discrete components.

Electronic packaging studies indicate that all electronic and microwave functions in the developmental unit, exclusive of servomotors and rate gyros, can be packaged in a volume of two by two by six inches. The electronic package, without antennas, would weigh between 1¼ and 1½ pounds. (See photograph.)

Although the size and weight reductions demonstrated are significant, the principle advantages of the MODOR design are improvement of reliability and reduction of production and maintenance costs. These improvements result from the use of molecular-electronic functional blocks and compatible etched-circuit functional tuning boards to reduce the number of components required.

SF_6 Breakers Designed for Rating Flexibility

The sulfur hexafluoride (SF_6) power circuit breakers that will be used on the Virginia Electric Power Company's 500-kv project have modular live tanks, a significant advance in design for SF_6 breakers. Without the modular live tanks, each different BIL used for 500 and 700 kv would require costly new development of a suitable dead tank and bushing; the live-tank approach makes it possi-



ble to simply add modular units to increase voltage ratings.

The VEPCO breakers are rated at 35 000 mva. Each pole contains six interrupter units housed in three live tanks, each mounted atop a porcelain column. A single high-pressure SF_6 reservoir is located at the base of the unit. Each pole has a separate pneumatic operating mechanism, and all six breaks of each pole are mechanically interconnected for simultaneous operation. The three poles are synchronized electrically and pneumatically to assure opening and closing of individual phases within one-half cycle of each other.

Resistors are not needed to control the rate of rise of recovery voltage nor to help interrupt line-charging current. However, resistors can be applied in the closing operation to greatly reduce switching surge voltages; their use can permit system insulation to be reduced to the level required by lightning.

Mercury-Vapor Lamps Require No External Ballasts

Mercury-vapor lamps that will burn in ordinary lamp sockets without the customary ballasts and auxiliary wiring have been developed. The self-ballasted lamps contain an integral incandescent filament that substitutes for the external ballasts otherwise required. This incandescent filament also adds to the light emitted by the lamp and improves its color balance.

The lamps have an average rated life of 12 000 hours, compared with 1000 hours for standard incandescent lamps. Their efficiency also is greater. Initial production is in 450- and 750-watt sizes, 230 volts. The lamps are available clear or with a white phosphor coating.

The new lamps are especially useful in applications where it is desirable to switch to mercury lighting to obtain the benefits of increased light and longer life but where an investment in new fixtures, ballasts, and auxiliary wiring is not justified.

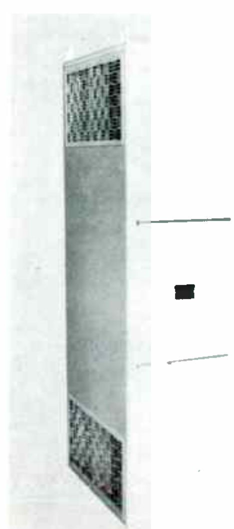
PRODUCTS FOR INDUSTRY

Electric prime-mover governor initiates a speed correction the instant a load change occurs. Major components of the type EFG governor are an electrical sensing and control unit, an electrohydraulic actuator, and a hydraulic power supply. It is an electrical frequency sensing device with circuits for stabilization, isochronous paralleling, and load sharing. It can control any type of prime mover driving ac generators or mechanical loads singly or in parallel. (When the prime mover drives a mechanical load, the electrical speed signal is furnished by a shaft-driven permanent-magnet generator.) *Westinghouse Systems Control Division, Buffalo, N. Y. 14240.*

Centrifugal chilling system is a completely packaged direct-expansion hermetic type for use singly or in multiple to provide air cooling for commercial and industrial buildings. It is available initially in the 100- to 140-ton range. The unit is shipped ready to run, with no field assembly or testing required. Factory assembly includes initial charge of refrigerant, insulation of chiller shell, mounting and wiring of control center, and operating test. No special concrete base is required, since the system is virtually vibration free. *Westinghouse Air Conditioning Division, P. O. Box 510, Staunton, Va.*

Housings for current-limiting reactors meet the increasing need for free-standing reactors in separate indoor metal enclosures. Vertical housings for three single-phase reactors and horizontal housings for one three-phase assembly are available for the full range of MSP reactor ratings. These current-limiting reactors provide short-circuit protection for low-voltage industrial power systems. They have been used extensively in standard control-center line-ups to limit fault current to molded-case breakers; in the new housings, they can be readily applied to protect control centers when the incoming service is increased or when fuses do not provide suitable protection. They can also serve other purposes than control-center protection. *Westinghouse Power Transformer Division, Muncie, Ind. 47305.*

Direct-connected valve lightning arresters with an external gap are for use where visible indication of the main arrester gap is desired and where the slightly higher impulse sparkover characteristics, inherent in the externally gapped design, are not significant. For most applications, sparkover values are well within safe limits for dependable equipment protection. The Autovalve type LVG units are available in ratings from 3 to 20 kv. All exposed terminal parts are made of nonferrous material for long life, and the bronze terminal connections are plated with tin to allow use of aluminum or copper conductors. *Westinghouse Distribution Apparatus Division, P. O. Box 341, Bloomington, Indiana 47402.*



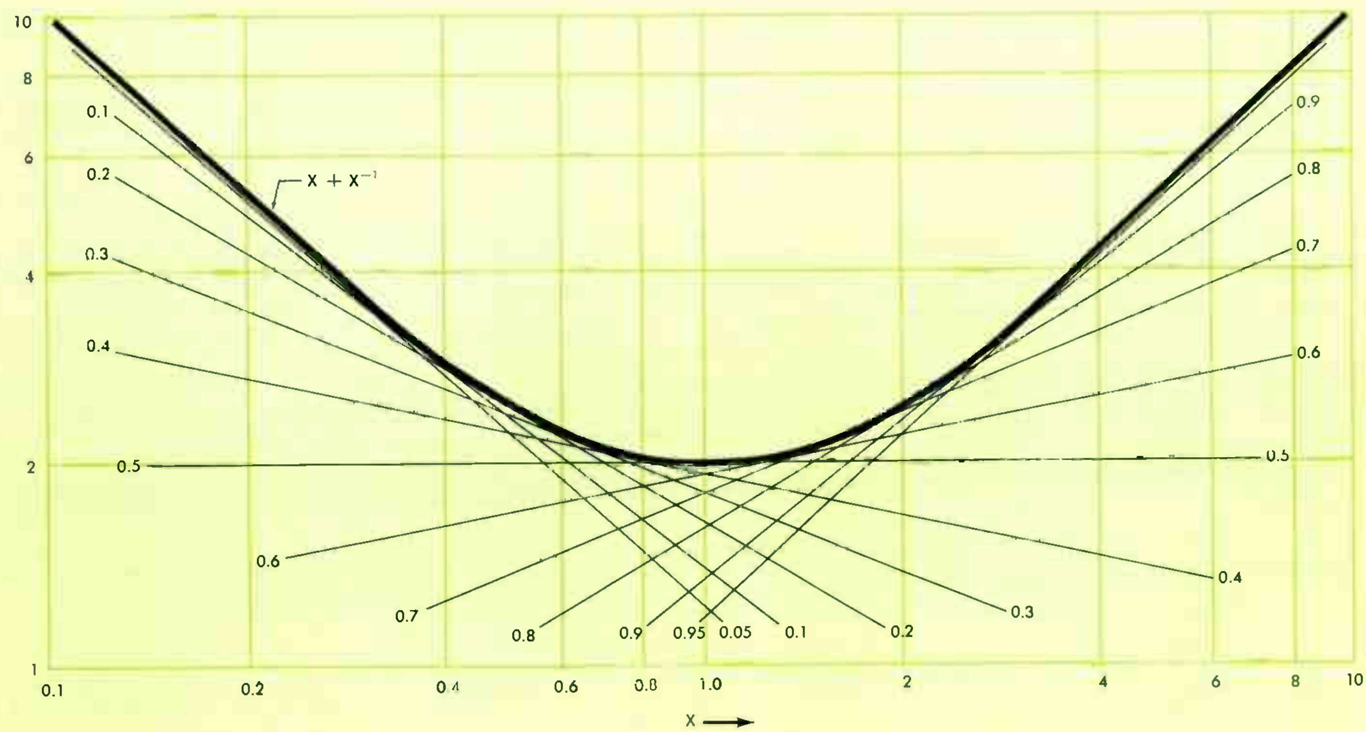
Optimization of Engineering Problems

C. Zener, Director of Science,
R. Duffin,
Research Laboratories,
Westinghouse Electric Corporation, Pittsburgh, Pennsylvania.

1

$$x+x^{-1}\geq (x/\delta_1)^{\delta_1}(x^{-1}/\delta_2)^{\delta_2}$$

IN THE ABOVE RELATIONSHIP, the right-hand side will be a family of straight lines on log paper with sets of values of (δ_1, δ_2) allowed by the relationship, $\delta_1+\delta_2=1$. The right-hand side of the equation will equal the left-hand side only when the values chosen for (δ_1, δ_2) render the right-hand member independent of *x*.



The solution to most engineering problems involves a compromise between many variables. The engineer seeks the ideal compromise, usually one that optimizes some specific characteristic. For example, the engineer may be called upon to design for such characteristics as maximum efficiency, minimum cost, or minimum weight.

Once the goal and the related variables are known, the selection of the best compromise usually becomes a mathematical problem. However, the solution of this problem is often a tedious, time-consuming process; more important, although the solution provides an answer to the specific problem, it does not necessarily give the designer any real insight into the relationship of the various factors involved.

However, a well-known, but heretofore not effectively applied, mathematical theorem can be applied to solve a wide class of optimization problems.

The recognition that this well-known mathematical theorem has applicability to a wide range of optimization problems has evolved in a series of papers.[1,2,3,4] In the present article, the application of this concept is described, and, through examples, the depth of its solutions is demonstrated. This article is in two parts. The first part presents the pertinent mathematical theorem, and its numerous extensions, with plausible but not rigorous justifications. More rigorous proofs are contained in the references. The first part also illustrates the use of the theorem and its extensions with elementary examples. The second part shows a typical example of the application of the theorem to a design problem.

Editor's Note: *Occasionally, a new approach to solving a common engineering problem is developed. This article describes a mathematical technique for optimization, a frequent task for many managers and engineers.*

Extensive mathematical treatments of any subject are not a common feature in this magazine, first, because our readers cover a broad spectrum of backgrounds and experience—and second, because we recognize that their time is limited. However, because of the potentially wide usefulness of this technique we have made an exception in this case.

As a time saver, we have asked the authors to omit most of the proof of their technique, and concentrate on its application. More rigorous proof can be found in the references listed.

We hope the article proves useful to many of our readers; any comments would be welcomed.

I—The Mean Theorem and its Generalizations

A mathematical theorem found in many texts says that if x_1, x_2 represent two positive numbers, then

$$\frac{x_1+x_2}{2} \geq \sqrt{x_1 \cdot x_2}. \quad (1)$$

Thus,

$$\frac{1+4}{2} > \sqrt{1 \cdot 4} \text{ and } \frac{2+2}{2} = \sqrt{2 \cdot 2},$$

but no pair of numbers can be found for which the left-hand side is less than the right-hand side. Actually, this relationship is a specific example of the mean theorem, which states that *an arithmetic mean is always equal to or greater than a geometric mean*. Using the weighted expressions for the means, the theorem can be written:

$$\delta_1 x_1 + \delta_2 x_2 \ldots \delta_n x_n \geq x_1^{\delta_1} x_2^{\delta_2} \ldots x_n^{\delta_n}, \quad (2)$$

where δ_1, δ_2 ... δ_n are weights that must satisfy the *normality* condition,

$$\delta_1 + \delta_2 + \ldots \delta_n = 1. \quad (3)$$

As might be suspected from the above example, the inequality in (2) becomes an equality if, and only if, all x_i terms are equal.

The essentially new feature in applying the mean theorem to optimization problems is the introduction of a new set of variables,

$$T_1 = \delta_1 x_1, \text{ and } T_2 = \delta_2 x_2.$$

This change gives the theorem that if T_1 and T_2 are any two positive numbers,

$$T_1 + T_2 \geq (T_1/\delta_1)^{\delta_1} (T_2/\delta_2)^{\delta_2} \quad (4)$$

where δ_1 and δ_2 are any other two positive numbers satisfying the normality condition $\delta_1+\delta_2=1$. The equality sign in (1) is valid when, and only when, $x_1=x_2$; thus, the equality sign in (4) is valid only when

$$T_1/T_2 = \delta_1/\delta_2\,. \quad (5)$$

The potential usefulness of (4) to optimization problems may best be appreciated by considering a simple example such as:

$$x + x^{-1} \geq (x/\delta_1)^{\delta_1} (x^{-1}/\delta_2)^{\delta_2}. \quad (6)$$

The right-hand side is a straight line when plotted on log paper. Fig. 1 shows the family of such straight lines given by those sets of values of (δ_1, δ_2) allowed by the normality condition (3). Note that each straight line touches the curve $x+x^{-1}$, but lies beneath this curve, in accordance with the inequality sign of (6). One of these straight lines, namely, the horizontal line, touches the minimum of $x+x^{-1}$. This horizontal line is given by that set of values (δ_1, δ_2) which renders the right-hand member of (6) independent of x. Thus, this set must satisfy the *orthogonality* condition, which for this example is

$$\delta_1 - \delta_2 = 0,$$

as well as the normality condition (3). Herein lies the key to the use of the mean theorem to solve optimization problems. *The minimum of the left-hand side of (6) is equal to that value of the right-hand side where the weights are such as to render the right-hand side independent of x, simultaneously, of course, satisfying the normality condition (3).*

Since the basic mean theorem (2) is applicable to a set of an arbitrary number of x's, the number of x's (or T's) and δ's can be extended from 2 to an arbitrarily large number. The mean theorem can be directly applied as above to obtain the minimum to any polynomial in n terms containing $(n-1)$ variables. Thus, for three terms in two variables, Fig. 1 is replaced by a family of planes which touch a surface. The one plane that is horizontal is at a level equal to the minimum value of the polynomial. Thus, in the example

$$ax^{-1}y^{-1}+bx^2y+cxy^2 \geq \left(\frac{ax^{-1}y^{-1}}{\delta_1}\right)^{\delta_1}\left(\frac{bx^2y}{\delta_2}\right)^{\delta_2}\left(\frac{cxy^2}{\delta_3}\right)^{\delta_3}$$

the minimum value of the left number is given by the right member when the set of weights $(\delta_1, \delta_2, \delta_3)$ will simultaneously satisfy the normality condition (3) and the orthogonality condition. The latter condition is satisfied if the sum of the exponents for each of the independent variables is zero. Thus,

$$-\delta_1+2\delta_2+\delta_3=0, \text{ for } x;$$
$$-\delta_1+\delta_2+2\delta_3=0, \text{ for } y.$$

Such a set of weights is uniquely given by $(3/5, 1/5, 1/5)$, as a solution of the three simultaneous equations will show.

Unfortunately, this geometrical insight must be abandoned when n is greater than three. The analyses presented in the references demonstrate that in the general case the minimum may be obtained by a procedure strictly analogous to that given above for three terms and two variables. Thus, when the left member of (4) has n terms in $(n-1)$ variables, the right member contains n factors:

$$T_1+T_2+\ldots+T_n \geq (T_1/\delta_1)^{\delta_1}(T_2/\delta_2)^{\delta_2}\ldots(T_n/\delta_n)^{\delta_n}. \quad (7)$$

The minimum value of the left-hand member is then given by that set of weights $(\delta_1, \delta_2, \ldots \delta_n)$ that satisfies the one equation of the normality condition $(\delta_1+\delta_2+\ldots\delta_n=1)$ and the $(n-1)$ linear equations expressing the orthogonality condition; namely, the condition that the product of the n terms of the type $(T_i/\delta_i)^{\delta_i}$ be independent of the $(n-1)$ independent variables.

Independent Variables Less Than $(n-1)$

A further generalization (demonstrated in the references) can be made. If the number of independent variables is less than $(n-1)$, the total number of equations given by the normality and orthogonality conditions is not sufficient to uniquely determine the δ's.

In this case,

$$\text{Min}(T_1+\ldots T_n)=\text{Max}(T_1/\delta_1)^{\delta_1}\ldots(T_n/\delta_n)^{\delta_n}, \quad (8)$$

the maximum being taken over that set of $\delta_1, \ldots \delta_n$ that satisfy the normality and orthogonality conditions.

Thus, consider the example,

$$ax^2+bx+cx^{-3} \geq \left(\frac{ax^2}{\delta_1}\right)^{\delta_1}\left(\frac{bx}{\delta_2}\right)^{\delta_2}\left(\frac{cx^{-3}}{\delta_3}\right)^{\delta_3}. \quad (9)$$

From the normality and orthogonality conditions,

$$\delta_1+\delta_2+\delta_3=1, \text{ and}$$
$$2\delta_1+\delta_2-3\delta_3=0,$$

two of the weights can be obtained in terms of the third, e.g.,

$$\delta_2=(3-5\delta_1)/4,$$
$$\delta_3=(1+\delta_1)/4. \quad (10)$$

The right member of (9) may thus be written as an explicit function of δ_1. The minimum of the left member, a function of x, is then equal to the maximum of the right member as a function of δ_1. That value of δ_1 that maximizes the right member is readily found to satisfy the equation*

$$\frac{ac^{1/4}}{4b^{5/4}}=\frac{\delta_1\,(1+\delta_1)^{1/4}}{(3-5\delta_1)^{5/4}}.$$

From the plot of $ac^{1/4}/4b^{5/4}$ as a function of δ_1 given in Fig. 2, δ_1 can be read off as a function of $ac^{1/4}/4b^{5/4}$, and hence also δ_2 and δ_3 through (10). All these functional relations are combined in Fig. 2. As an example, consider a particular case,

$$(a, b, c)=(1, 2, 3).$$

Then

$$\log\,(ac^{1/4}/4b^{5/4})=-0.856,$$

and from Fig. 2,

$$(\delta_1, \delta_2, \delta_3)=(0.260, 0.425, 0.315).$$

These weights, substituted into the right member of (9), give the value 5.59 for the minimum of the left member of (9). At this minimum the second term must contribute the fraction δ_2 to the minimum [see equation (5)]; hence, the value of x at the minimum is $(0.425\times5.59)/2=1.2$.

Inequality Constraints

The application of the mean theorem can be extended to problems in which a minimum is sought subject to certain inequality constraints. Basically, the method consists of putting the constraint into a form such that it can be combined directly with the mean theorem (7).

For example, suppose the problem is to find the minimum surface area of an open cylindrical tank, which must have a volume not less than unity. If r is the radius and h the height, the quantity to be minimized is

$$U=\pi r^2+2\pi rh \geq \left(\frac{\pi r^2}{\delta_1}\right)^{\delta_1}\left(\frac{2\pi rh}{\delta_2}\right)^{\delta_2}, \quad (11)$$

where δ_1 and δ_2 are any two positive numbers that satisfy the normality condition, $\delta_1+\delta_2=1$.

*The approach used to obtain this equation can be outlined briefly as follows. Since the expression to be maximized,

$$\left(\frac{a}{\delta_1}\right)^{\delta_1}\left(\frac{b}{\delta_2}\right)^{\delta_2}\left(\frac{c}{\delta_3}\right)^{\delta_3}$$

is inconvenient to differentiate, its logarithm is used. Thus, the quantity to be maximized becomes:

$$\delta_1 \ln\left(\frac{a}{\delta_1}\right)+\delta_2 \ln\left(\frac{b}{\delta_2}\right)+\delta_3 \ln\left(\frac{c}{\delta_3}\right).$$

This quantity is differentiated and set equal to zero. From (10), the relationships

$$d\delta_2=-(5/4)d\delta_1$$
$$d\delta_3=(1/4)d\delta_1$$

are obtained for use in the differentiation. The final differential expression simplifies to:

$$\ln\left\{\left(\frac{a}{\delta_1}\right)\left(\frac{b}{\delta_2}\right)^{-5/4}\left(\frac{c}{\delta_3}\right)^{1/4}\right\}=0.$$

Since a number whose logarithm is zero equals 1,

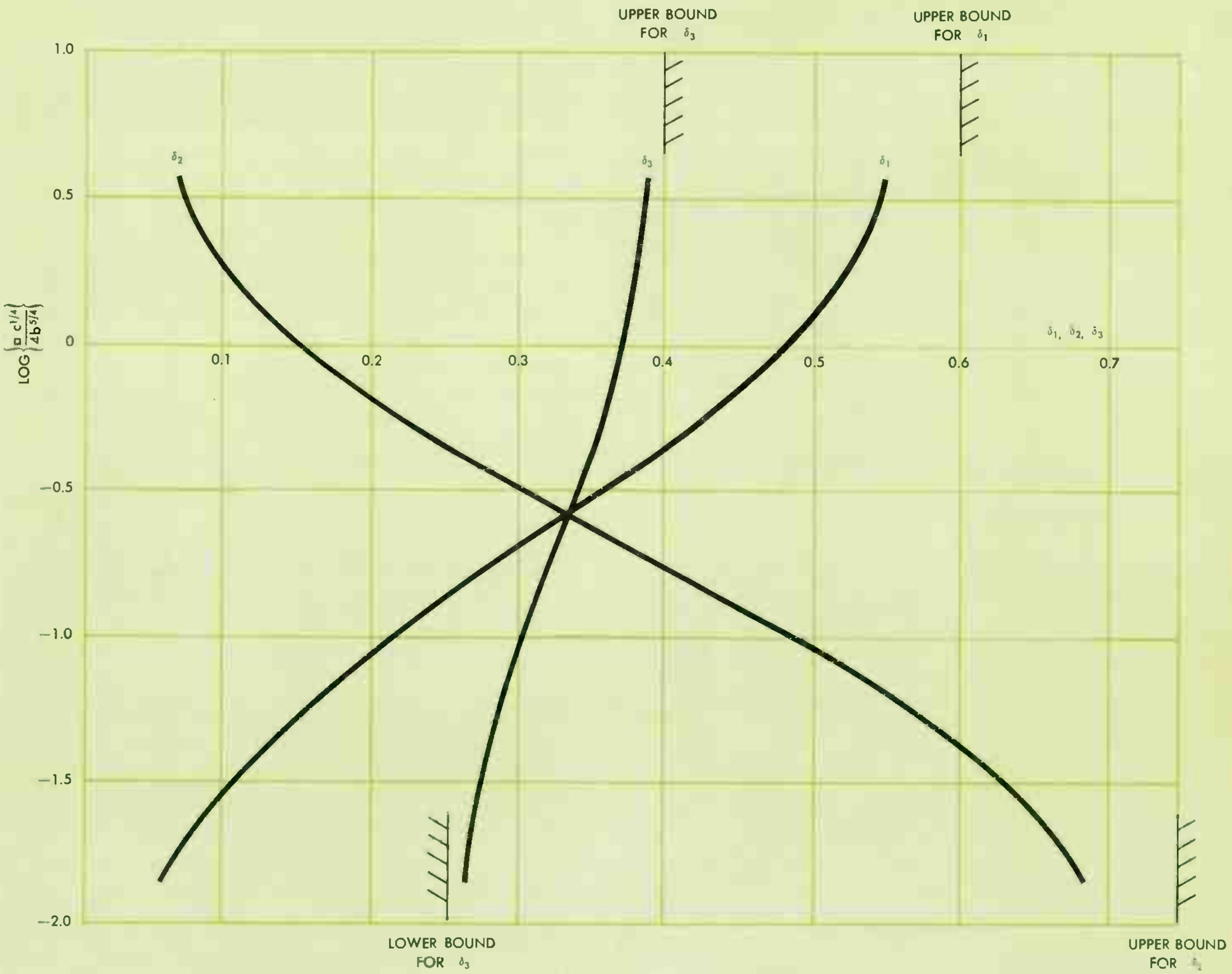
$$\left(\frac{a}{\delta_1}\right)\left(\frac{b}{\delta_2}\right)^{-5/4}\left(\frac{c}{\delta_3}\right)^{1/4}=1.$$

The final equation in terms of δ_1 is then developed by substituting equations (10) for δ_2 and δ_3 in the above expression.

2

$$\frac{ac^{1/4}}{4b^{5/4}}=\frac{\delta_1(1+\delta_1)^{1/4}}{(3-5\delta_1)^{5/4}}$$

GIVEN THE ABOVE EQUATION, the logarithm of the left-hand side of the equation can be plotted as a function of δ_1. Then, from the relationships given in (10), δ_2 and δ_3 can also be plotted.



The constraint condition is that for the minimum surface area, the volume must be at least equal (or greater than) unity. Thus, the inequality constraint is

$$1 \leq \pi r^2 h.$$

However, to use an inequality constraint with the mean theorem, the constrained quantity must have a less-than-or-equal relationship to unity; therefore, the constraint is rewritten for the *reciprocal* of volume, which is less than or equal to unity:

$$1 \geq \frac{1}{\pi r^2 h}. \tag{12}$$

For combining the inequality constraint with (11), the constraint is put in the form

$$1 \geq \left(\frac{1}{\pi r^2 h}\right)^{\delta_3}, \tag{13}$$

where δ_3 is any positive number whatsoever. The extreme sides of (11) and (13) are multiplied, to obtain the expression,

$$U \geq \left(\frac{\pi r^2}{\delta_1}\right)^{\delta_1} \left(\frac{2\pi r h}{\delta_2}\right)^{\delta_2} \left(\frac{1}{\pi r^2 h}\right)^{\delta_3}.$$

As in a previous example, the minimum value of the left side, namely the minimum area, is equal to the right member when the set (δ_1, δ_2) satisfies the normality condition, and the set $(\delta_1, \delta_2, \delta_3)$ satisfies the orthogonality condition that the right member be independent of r and h. This set is $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$, as found by simultaneous solution of the normality and orthogonality conditions.

The generalization of the mean theorem to several constraints of the type (13) is obvious. Not so obvious is the case where a constraint contains more than one term. Thus, suppose the following is to be minimized:

$$T_1 + T_2 \geq \left(\frac{T_1}{\delta_1}\right)^{\delta_1} \left(\frac{T_2}{\delta_2}\right)^{\delta_2}, \tag{14}$$

where $\delta_1 + \delta_2 = 1$, subject to the constraint

$$T_3 + T_4 \leq 1.$$

This constraint can be written in the form

$$(1)^{\delta_3+\delta_4} \geq (T_3+T_4)^{\delta_3+\delta_4} \geq \left(\frac{T_3}{\delta_3}\right)^{\delta_3} \left(\frac{T_4}{\delta_4}\right)^{\delta_4} (\delta_3+\delta_4)^{\delta_3+\delta_4}. \tag{15}$$

The last factor is required because the normality condition is not imposed on weights δ_3 and δ_4.[4] By multiplying (14) by the two extreme sides of the inequality (15),

$$T_1 + T_2 \geq \left(\frac{T_1}{\delta_1}\right)^{\delta_1} \left(\frac{T_2}{\delta_2}\right)^{\delta_2} \left(\frac{T_3}{\delta_3}\right)^{\delta_3} \left(\frac{T_4}{\delta_4}\right)^{\delta_4} (\delta_3+\delta_4)^{\delta_3+\delta_4}.$$

The minimum of $T_1 + T_2$ is then equal to the maximum of the right member over that range of weights $(\delta_1, \delta_2, \delta_3, \delta_4)$ that satisfy the normality condition for δ_1 and δ_2, and the orthogonality condition that the right member be independent of the variables.[4] This technique can be extended to treat any number of inequality constraints.

II—Optimizing Design by the Mean Theorem Method

The full power of the mean-theorem approach can best be appreciated by studying in detail a problem whose solution requires the extensions of the mean theorem discussed in Part I. The following problem has been chosen for illustrating the mean theorem, and has a high degree of idealization. The problem is to design a choke coil having a minimum total cost, the inductance and current having specified values.

By total cost is meant manufacturing cost plus the present worth of future power losses. The core is assumed to be an ideal magnetic material with no loss and with constant magnetic permeability. The cost may therefore be taken as:

$$u = C_1 V_i + C_2 V_c + C_3 j^2 V_c \tag{16}$$

where: V_i = volume of iron core;
V_c = volume of copper coil; and
j = current density in copper coil.

The choke coil is described further by the following notation:

A_i = cross section area of iron core;
A_c = cross section area of copper coil;
L_i = mean magnetic flux length;
L_c = mean turn length in copper coil;
N = number of copper turns; and
J = current in copper coil.

The problem has three restraints: the specified value of inductance, the specified value of current, and the relationship between the geometrical parameters of the choke coil.

The inductance restraint can be developed from the expression for inductance,*

$$Inductance \sim N^2 A_i / L_i.$$

Since higher inductance can be obtained only at a greater cost, the inequality constraint must express the condition that inductance must be equal to or greater than a given value. However, constraints used in conjunction with the mean theorem must contain a less-than-or-equal ($\leq$) relationship, so the constraint must say that the *reciprocal* of inductance is less than or equal to a given value. Therefore, the inductance restraint must be written:

$$\frac{C_4 L_i}{N^2 A_i} \leq 1. \tag{17}$$

Whereas total cost can always be made smaller by reducing the current density j, current density cannot be reduced below NJ/A_c. Thus, the second constraint should be written,

$$\frac{NJ}{A_c j} \leq 1. \tag{18}$$

The geometrical parameters V_i, V_c, A_i, A_c, L_i, L_c are interdependent, and to express this interdependence as a constraint, a certain type of geometry must be specified. For this problem, the choke is assumed to be of the form of

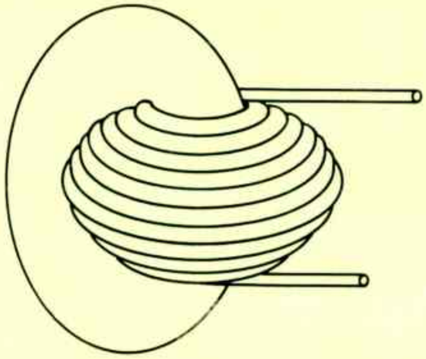
*The derivation of this constraint is obtained by observing that the induced electromotive force is given by

$$emf \begin{cases} N\dot{H}A_i \sim N^2 \dot{J} A_i / L_i \\ Inductance \times \dot{J} \end{cases}$$

and hence,

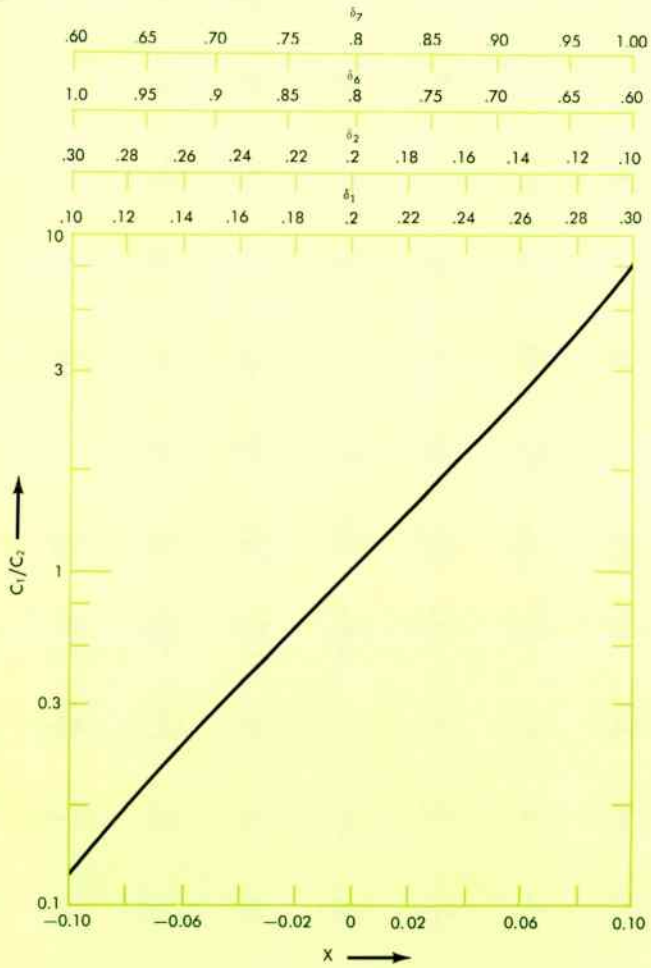
$$Inductance \sim N^2 A_i / L_i.$$

3



THE CHOKE configuration chosen for the illustrative problem is of the form of two interlocking doughnuts, the core comprising one doughnut, and a coil of N turns making up the other.

4



$$\frac{C_1}{C_2}=\left(\frac{\delta_1}{\delta_2}\right)\left(\frac{\delta_7}{\delta_6}\right)^2.$$

THE ABOVE EQUATION is the solution for the choke problem. By setting $\delta_1=(1/5)+x$ and using the relationships shown in (30), all δ's can be shown by a single curve with a separate axis for each δ.

two interlocking doughnuts, as illustrated in Fig. 3. For this configuration, L_i and L_c are identical, namely,

$$L=2\pi(r_i+r_c+\Delta),$$

where: r_i = cross sectional radius of iron core;
r_c = cross sectional radius of copper coil; and
Δ = clearance between core and coil.

Since the clearance Δ can be only positive, the geometrical constraint becomes:

$$2\pi(r_i+r_c)/L\leq 1. \qquad (19)$$

In this doughnut model the two volumes are given by:

$$V_i=\pi r_i^2L,\quad V_c=\pi r_c^2L. \qquad (20)$$

Therefore, (16) can now be written

$$U=C_1r_i^2L+C_2r_c^2L+C_3j^2r_c^2L, \qquad (21)$$

where U is equal to the u of (16) divided by π.

The design objective is to minimize this expression, subject to the three constraints,

$$\frac{C_4L}{\pi N^2r_i^2}\leq 1, \qquad (22)$$

$$\frac{NJ}{\pi r_c^2j}\leq 1, \qquad (23)$$

$$2\pi(r_i+r_c)/L\leq 1. \qquad (24)$$

Direct application of the mean theorem gives:

$$U\geq\left(\frac{C_1r_i^2L}{\delta_1}\right)^{\delta_1}\left(\frac{C_2r_c^2L}{\delta_2}\right)^{\delta_2}\left(\frac{C_3j^2r_c^2L}{\delta_3}\right)^{\delta_3}$$

$$\left(\frac{C_4L}{\pi N^2r_i^2}\right)^{\delta_4}\left(\frac{NJ}{\pi r_c^2j}\right)^{\delta_5}\left(\frac{2\pi r_i}{L\delta_6}\right)^{\delta_6}$$

$$\left(\frac{2\pi r_c}{L\delta_7}\right)^{\delta_7}(\delta_6+\delta_7)^{\delta_6+\delta_7} \qquad (25)$$

where $(\delta_1, \delta_2, \ldots \delta_7)$ is any set of positive numbers, not necessarily integers, which satisfy the normality condition,

$$\delta_1+\delta_2+\delta_3=1. \qquad (26)$$

The minimum of U is given by the maximum of the right side of (25) with respect to that range of $(\delta_1, \delta_2, \ldots \delta_7)$ which satisfies, in addition to the normality condition, the orthogonality condition. This condition restricts $(\delta_1, \delta_2, \ldots \delta_7)$ to those values that render the right member of (25) independent of the design parameters r_i, r_c, L, j, N. Since the parameter N occurs in only the 4th and 5th factors, the orthogonality condition requires

$$\delta_4=(1/2)\delta_5.$$

Further, since j occurs in only the 3rd and 5th factors, the orthogonality condition requires

$$\delta_5=2\delta_3.$$

Thus, the original factors for δ_3, δ_4, and δ_5 can be combined into a single factor in terms of δ_3, and (25) is reduced to:

$$U\geq\left(\frac{C_1r_i^2L}{\delta_1}\right)^{\delta_1}\left(\frac{C_2r_c^2L}{\delta_2}\right)^{\delta_2}\left(\frac{C_3C_4L^2J^2}{\pi^3r_i^2r_c^2\delta_3}\right)^{\delta_3}$$

$$\left(\frac{2\pi r_i}{L\delta_6}\right)^{\delta_6}\left(\frac{2\pi r_c}{L\delta_7}\right)^{\delta_7}(\delta_6+\delta_7)^{\delta_6+\delta_7}. \qquad (27)$$

It is interesting to note that (27) could also have been obtained if (22) and (23) had been changed into equalities, then j eliminated from (21) by use of (22) and (23), and finally, the mean theorem applied to the minimization of U subject to the single remaining constraint (24). Before applying the mean theorem, L could also have been eliminated from (25) by means of (24). However, the difficulty of solving an optimization problem by the mean theorem is essentially proportional to the excess of the total number of terms, including both terms in U and terms in the constraints, over the number of design parameters. Thus, the total number of terms was reduced by one, and the number of design parameters was reduced by one, when j was eliminated, and when N was eliminated. However, the elimination of L would lead to an increase in the number of terms with a reduction in the number of design parameters, and thereby would doubly increase the difficulty of obtaining a final solution.

One particular feature of the mean-theorem approach deserves special discussion at this point. In the standard approach to design problems, the design parameters are first solved at optimum, and these optimized parameters are then used to deduce various characteristics of the optimized design. The mean theorem leads to an essentially reverse approach. Here, certain characteristics of the optimized design are first found, and then the optimized design parameters are determined. An advantage of this approach is that many interesting properties of the optimized design fall out with almost no calculations.

The orthogonality condition for (27) is represented by three equations:

$$2\delta_1+0-2\delta_3+\delta_6+0=0 \text{ (for } r_i\text{)};$$
$$0+2\delta_2-2\delta_3+0+\delta_7=0 \text{ (for } r_c\text{)};$$
$$\delta_1+\delta_2+2\delta_3-\delta_6-\delta_7=0 \text{ (for } L\text{)}.$$

The sum of these three equations ($3\delta_1+3\delta_2-2\delta_3=0$) can be combined with the normality condition, (26), to determine δ_3:

$$\delta_3=3/5. \tag{28}$$

Equation (28) provides considerable insight into the optimized design. First, it says that at optimized design,

$$U_{\text{Min}}\sim(C_3C_4)^{3/5}.$$

Since C_3 contains resistivity as a factor, U_{Min} varies as the resistivity to the $3/5$th power; since C_4 contains the inductance as a factor, U_{Min} also varies as the $3/5$th power of the inductance. Second, the fact that δ_3 is a constant means that both δ_6 and δ_7 are independent of the constants C_3, C_4. But from (24) and (5), it can be seen that at optimized design

$$r_i/r_c=\delta_6/\delta_7. \tag{29}$$

Therefore, the ratio r_i/r_c is independent of the constants C_3 and C_4, and hence also of the various factors contained in C_3 and C_4. Thus, the ratio r_i/r_c is independent of copper resistivity, specified inductance, and of the manner in which power loss is converted into present worth.

The first step in obtaining the complete optimized design is to determine the unknown weights (δ_1, δ_2, δ_3, δ_6, δ_7) for the optimum design. However, the three equations representing the orthogonality condition and the one equation representing the normality condition are not sufficient to completely specify this set. If δ_1 is left unspecified, then from the orthogonality and normality conditions, the other δ's may be found in terms of δ_1:

$$\delta_2=2/5-\delta_1,$$
$$\delta_3=3/5$$
$$\delta_6=6/5-2\delta_1,$$
$$\delta_7=2/5+2\delta_1. \tag{30}$$

The minimum of U is then equal to that value of the right member of (27) when the weights (δ's) are such as to render a first-order variation of the right member equal to zero. Similarly to the previous example, the logarithm of the right member is taken, and its variation equated to zero, using the relationships derived from (30), ($d\delta_1$, $d\delta_2$, $d\delta_3$, $d\delta_6$, $d\delta_7$)$=(1, -1, 0, -2, 2)\epsilon$, where ϵ is a small order quantity. The expression finally obtained is:

$$\left(\frac{C_1}{C_2}\right)=\left(\frac{\delta_1}{\delta_2}\right)\left(\frac{\delta_7}{\delta_6}\right)^2. \tag{31}$$

By setting $\delta_1=1/5+x$ and using (30), (31) may be written:

$$\left(\frac{C_1}{C_2}\right)=\frac{1/5+x}{1/5-x}\left(\frac{2/5+x}{2/5-x}\right)^2. \tag{32}$$

Equation (32) contains the complete solution to the optimum design problem. Thus, by plotting C_1/C_2 as a function of x (Fig. 4), one can, by reading this plot backwards, obtain x, and hence all weights, as a function of C_1/C_2. Since all δ's are linear functions of x, the same curve can serve for all the δ's by shifting coordinate axes and in some cases changing scales. Such adjustments have been made in Fig. 4 so that the curve that gives x as a function of C_1/C_2 also gives all δ's as a function of C_1/C_2.

By using the previously found values for δ_3 and for the sum $\delta_6+\delta_7$, (25) can be written:

$$U_{\text{Min}}=\frac{2^{8/5}\,5^{3/5}}{3^{3/5}\pi^{1/5}}(C_3C_4)^{3/5}\frac{C_1^{\delta_1}\,C_2^{\delta_2}}{(\delta_1)^{\delta_1}\,(\delta_2)^{\delta_2}\,(\delta_6)^{\delta_6}\,(\delta_7)^{\delta_7}}.$$

Given the constants C_1, C_2, C_3, C_4, and making use of Fig. 4, the numerical value of U_{Min} can be computed. From (29), the ratio r_i/r_c can be calculated. Finally, by writing the first term in U, namely the iron cost, as equal to $\delta_1\,U_{\text{Min}}$, r_i can be calculated. The optimum design is thereby completely specified.

Conclusions

When the number of terms in the polynomial does not exceed the number of independent variables by more than one, the solution of a maximization problem by the mean-theorem approach is direct since all weights (δ's) can be uniquely determined from the normality and orthogonality conditions. The technique demonstrated in the design problem can be applied in cases where there are two more terms than variables. However, if the gap between the number of terms and the number of variables exceeds two, then a solution is not obvious. A method is presently being developed to handle this more general type of problem.

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ABOUT THE AUTHORS

C. R. Olson graduated from Carnegie Institute of Technology in 1943 with a BS in electrical engineering. He went to work for the Aluminum Company of America as a plant electrical engineer. The following year, he was called to active service in the U. S. Navy Reserve and served as a lieutenant (jg) in the maintenance and operation of radar and other airborne electronic equipment.

Olson joined Westinghouse on the graduate student course in 1946. He was assigned to Industry Engineering (now Industrial Systems Divisions), where he has helped develop electric power and control systems for mining locomotives, chemical production, petroleum refining, and the glass industry. His present field of major interest is electric glass melting, and he was responsible for the design of the power and control systems of the electric furnaces mentioned in his article in this issue. Olson is a prolific writer of technical articles and papers, a member of Eta Kappa Nu, and vice-chairman of the IEEE industry division committee.

Emil F. Steinert has been a major contributor to the development of arc-welding technology. His main responsibilities at the Westing-Arc Division are in the design and application of power supplies for arc welding and other arc applications. His influence is felt outside Westinghouse, too. As chairman of the NEMA technical committee on arc-welding apparatus, he has been instrumental in revising the NEMA standards. As a member of the American Standards Association sectional committee on arc-welding machines, he has helped prepare the American Standards on electric arc-welding apparatus. He also is a life member of the IEEE, and a member of its electric welding committee, and he belongs to the American Welding Society. He has been granted "about 25" patents.

Steinert graduated from the University of Minnesota in 1925 with a BS in electrical engineering. He joined Westinghouse on the graduate student course and then went to the Westinghouse design school. His first assignment was at the Transformer Division, where he helped design transformers of various types, constant-current regulators, and transformer-type welders. When the Welding Division (now the Westing-Arc Division) was formed in 1945, Steinert became its manager of apparatus engineering with responsibility for design of arc-welding apparatus. He moved with the division to Buffalo in 1946.

Dillon B. Hoover, author of the article on the brushless excitation system, graduated from the University of Florida in 1935. He joined Westinghouse on the student test program in 1936, and in 1939 transferred to electrical design engineering of large dc motors and generators. Hoover obtained his MS degree from the University of Pittsburgh in 1940.

Hoover has had wide experience in the design of large dc machines—vertical exciters for waterwheel generators, steel mill generators and motors, mine-sweeping generators, marine propulsion generators and motors, high-speed exciters, and most recently, the brushless excitation system. Hoover was appointed manager of the electrical design section for dc motor and generator engineering, his present position, in 1950.

J. C. Taylor graduated from Pennsylvania State University in 1950 with a BSEE degree. He has since taken graduate work there and also at the University of Pittsburgh and at New York University. He served in the U. S. Army from 1950 to 1952, assigned to research projects at the Signal Corps Laboratories, Fort Monmouth, New Jersey. He then worked for the Pennsylvania Railroad Company until 1955 as an application engineer in power apparatus.

Taylor joined the Westinghouse Semiconductor Division in 1955, where he has worked on the design, application, testing, and evaluation of semiconductor products. He is now a senior design engineer in the transistor—controlled-rectifier department, and he has been awarded a patent on the small-motor drive system described in this issue.

F. P. Weigold earned his BS in electrical engineering at Michigan Technological University in 1960 and an MBA at the University of Pittsburgh this year. He joined Westinghouse on the graduate student course in 1960 and was assigned to the Semiconductor Division. He is now an industry engineer in the marketing department, responsible for development of applications for semiconductor devices in the fields of industrial control, instrumentation, and medicine. He was one of the contributors to the *High Voltage Silicon Rectifier Designers Handbook* published by the Semiconductor Division earlier this year.

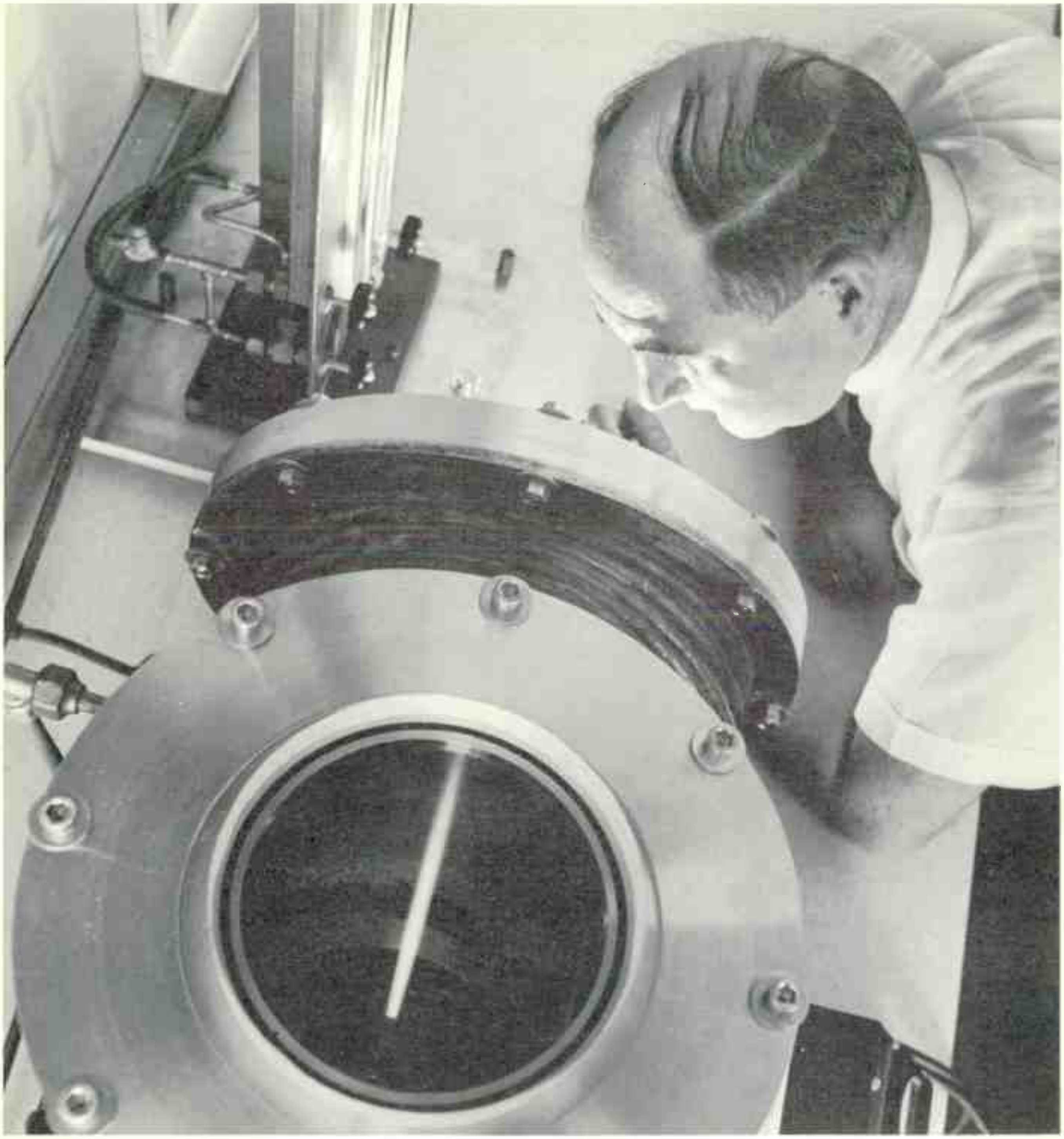
A theoretical physicist and a mathematician join forces in this issue to describe a new technique for optimizing engineering design problems. The physicist is **Dr. Clarence Zener**, Director of Science at the Westinghouse Research Laboratories; the mathematician is **Dr. R. J. Duffin**, Professor of Mathematics at Carnegie Institute of Technology and consultant to the Westinghouse Research Laboratories. The optimizing technique which they describe was originally suggested by Dr. Zener in 1961, and has been further developed by Dr. Duffin.

Before coming with Westinghouse in 1951, Dr. Zener spent most of his time as an educator. He obtained his B.A. in Physics from Stanford in 1926, and his Ph.D. in Physics from Harvard in 1929. He then accepted fellowships, first at the University of Leipzig (Germany), then Princeton University, and finally at Bristol University (England). From 1935 until 1942, he taught physics at Washington University (St. Louis), City College of New York, and Washington State University.

During World War II, Dr. Zener carried on research and development work on projectile design and armor penetration at Watertown Arsenal. In 1945, he returned to the campus as Professor of Physics at the University of Chicago. It was here that his work on metals gained wide recognition, and where he developed an entirely new theory for the causes of ferromagnetism.

Dr. Zener came with the Westinghouse Research Laboratories in 1951 as Associate Director. He has since had assignments of Acting Director, Director of Research, and presently, Director of Science. In addition to his recent work on applications of optimization to engineering design problems, Dr. Zener has done research on various aspects of metals, particularly internal friction, diffusion, and ferromagnetism; thermoelectricity; and oceanography.

Dr. Duffin obtained his B. S. degree in 1932, and his Ph. D. degree in 1935, both from the University of Illinois. In addition to teaching, he has done mathematical research in the areas of Fourier type integrals and series, functional inequalities, navigational devices, partial difference equations, linear programming, and electrical network theory. Dr. Duffin has also held positions of visiting professor at the Dublin Institute for Advanced Studies (Ireland), Physicist with the Department of Terrestrial Magnetism, Carnegie Institution of Washington, and Director of Special Research in Applied Mathematics at Duke University. Dr. Duffin has been a consultant to the Westinghouse Research Laboratories since 1956.



An arc plasma jet is being operated in a vacuum chamber here as part of an evaluation of its usefulness as an attitude-control device for space vehicles. The jet unit consists basically of a nozzle and two electrical terminals, between which an electric arc is maintained. When helium is fed into the arc, the great amount of heat imparted to it causes it to expand through the nozzle at supersonic velocity and thereby develop thrust. A heated cathode supplies free electrons for easy arc starting and stable operation. Less than 100 watts of power is required to start and maintain the arc.

The designers believe that a plasma-jet system could operate at least ten times longer, for a given amount of propellant, than the cold-gas jets now used for attitude control. This ability would be especially important for long-lived vehicles that have to be oriented constantly.