SPECIALIZED TELEVISION ENGINEERING

TELEVISION TECHNICAL ASSIGNMENT

MENSURATION

703E

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TECHNICAL ASSIGNMENT

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In this technical assignment it is desired to give the student an insight into the elements of algebra, geometry, and trigonometry. The assignment will deal only with the fundamental principles of each subject, more advanced material being presented in later assignments. Complete understanding of the fundamentals is essential, if the later assignments are to be mastered without difficulty.

ELEMENTS OF ALGEBRA

Algebra, like arithmetic, treats of numbers but it goes one step further by treating numbers in a general way. In arithmetic absolute numerical values are always used. In algebra literal quantities may be used in addition to numerical values, and since the literal quantity may represent any desired numerical value, the scope of algebra is much greater than that of common arithmetic. In the preceding assignment it was shown a number like 3,595 could be written:

$$3,595 = 3,000 + 500 + 90 + 5$$

In algebra this could be written as

$$3,595 = a + b + c + d$$

where
$a = 3,000$
$b = 500$
$c = 90$
$d = 5$

In arithmetic 3,595 can have only one absolute value, that is, 3,595 units, but in algebra $a + b + c + d$ can be used to represent any desired number. For example, a might equal 1,525, b 35, c 34.5 and d .5 In this case, $a + b + c + d$ would equal $1,525 + 35 + 34.5 + .5 = 1,595$. Thus, algebra, by using literal quantities (quantities consisting of letters that may be assigned numerical values), extends the field of mathematics beyond the limits of arithmetic.

ALGEBRAIC SYMBOLS

Many of the mathematical symbols used in arithmetic are carried over to algebra, although some of the symbols are given more extended meanings. In algebra, the arithmetical signs of addition and subtraction, + and –, are used as signs of operation as in arithmetic, but the same signs can also be used to indicate positive and negative quantities, as explained in connection with positive and negative numbers. In algebra ANY NUMBER OR LETTER WRITTEN WITHOUT A SIGN, + OR –, PRECEDING IT IS UNDERSTOOD TO BE A POSITIVE QUANTITY. Thus, 3, 7, A, AB, etc., are all understood to be positive quantities; the plus sign is understood to precede the numbers or letters, although it is not always written.

The sign of multiplication in arithmetic, x, is not used to any great extent in algebra because of the possible confusion with the letter x, which is used so frequently to represent an unknown quantity. As a general rule, the first letters of the alphabet are used in algebraic equations to represent known quantities while the last letters are used to indicate unknown values that must be solved for. There are several possible ways to indicate multiplication in algebra. To indicate the multiplication of numerical values, such as 2 times 5, it is customary to use a slightly elevated dot between the numbers, thus, $2 \cdot 5$. In the case of letters multiplication is indicated by simply writing the letters in sequence; thus, abc means a times b times c, 3xyz means 3 times x times y times z. In the latter example the number 3 is called the numerical coefficient of xyz, while xyz is the literal coefficient of the term. Similarly, 3x is the coefficient of yz, 3xy the coefficient of z, etc.

Multiplication can also be indicated by use of the parentheses, (), the brackets, [], the braces, {}, and the vinculum. The vinculum is a straight line drawn over a number or group of numbers, thus, $\overline{a + b}$. An important point to remember in algebra is that any quantity enclosed in parentheses, brackets, braces or under the vinculum must be considered as a complete term, and special rules must be applied to remove these symbols. The rules will be explained later in this assignment.

The arithmetical sign of devision, ÷, is not used to any great extent in algebra, it being more customary to indicate division by means of a bar or slant sign as explained in the previous assignment.

The sign of equality, =, the radical sign, √, and the exponents are used in algebra in the same way as they are used in arithmetic. The vinculum is usually used with the radical sign to indicate what part of a quantity is to be included within the radical. For example, $\sqrt{a}b$ means the square root of a times the number b, but $\sqrt{ab}$ means the square root of the *product* of $a \cdot b$.

ALGEBRAIC OPERATIONS

To understand complicated algebraic equations the student must understand the grouping of terms and the order of operations. For example, the expression $2 + 4 \cdot 3$ might be taken to mean $6 \cdot 3$ or $2 + 12$ depending on whether the addition or the multiplication is performed first. Certain rules are therefore necessary to indicate the order of operations. When the expression is written $2 + 4 \cdot 3$, it is understood that the multiplication always precedes the addition, so the correct solution is $2 + 12 = 14$. If the expression was such that the sum was to be performed first, it would be written as $(2 + 4)3$.

When parentheses, brackets, braces or the vinculum are used, the quantity enclosed must be solved before these symbols can be removed. In the expression $(4 + 6)\ (5 - 2)$, the parentheses can be removed only by first adding 4 and 6, which gives the quantity in the first parentheses a value of 10, and then subtracting 2 from 5 to obtain 3 in the second parentheses. The multiplication is then performed, $10 \times 3 = 30$. Where a choice of operations exists in the solution of a problem, the order of performance should be multiplication, division and then the additions and subtractions. This order need not necessarily be followed if the problem is complicated by the signs of aggregation, the parentheses, braces, brackets, and vinculum, but must always be followed in reducing quantities within these symbols.

The expression abc must never be confused with $a + b + c$. The first is a product, the second a sum. In sums or subtractions the plus or minus signs between terms are never omitted. For example, if $a = 2$, $b = 3$, $c = 4$ then abc equals

$2 \times 3 \times 4 = 24$, but $a + b + c = 2 + 3 + 4 = 9$, a very different result.

In arithmetic, it is customary to complete all operations as they appear in a given solution, but in algebra solutions are often considered complete when the operations are only indicated. For example, if x is equal to the product of 2a and 3b, the expression can be written $x = 2a3b$. Since it is possible to multiply the numerical coefficients, the expression can be simplified to $x = 2 \cdot 3ab = 6ab$. The solution for x is completed by performing one numerical multiplication and *indicating* the multiplication of the literal factors. If x equals the sum of 2a plus 3b, it may be written $x = 2a + 3b$. Here an *indicated sum* satisfies the solution for x. No further simplification of this expression is possible until numerical values are assigned to a and b. Note particularly that in this case the *numerical coefficients of a and b cannot be added;* that is, $2a + 3b$ cannot be written as $(2 + 3)ab$ or $(2 + 3)\ (a + b)$. Check this by substituting values for a and b. If $a = 2$ and $b = 3$, then $x = 2a + 3b = 4 + 9 = 13$. But $(2 + 3)ab = 5 \cdot 2 \cdot 3 = 30$ and $(2 + 3)\ (a + b) = 5 \times 5 = 25$.

Exercises

Express the *products* and *sums* of the following terms:

1. a, 2b, c.

2. 3a, 4b, 6c.

3. a, b, c, 4d.

4. a/2, b/3, c/4, d/5.

5. 1.5ab, 4cd, $\frac{3a}{2}$

6. to 10. In the problems 1 to 5 evaluate the products and the sums if $a = 2$, $b = 3$, $c = 4$ and $d = 5$.

POWERS AND ROOTS

The laws of involution and evolution, as previously explained, also apply in algebra, but they are extended to include literal as well as numerical values, and hence greater care is needed in using them. Just as $2^2 = 2 \times 2 = 4$ and $\sqrt{4} = 2$, so does $a^2 = a$ times a and $\sqrt{a^2} = a$. The exponent of a number indicates how many times the number, be it literal or numerical, is to be used as a factor, so any term with an exponent can always be reduced to its factors. $a^3 = aaa$, $a^2b^2 = aa \cdot bb$, $(ab)^2 = a^2b^2$, etc. In the last example note particularly that the exponent applies to the entire expression within the parentheses. In the term $3a^2b^3$ the exponent 2 applies only to a, and exponent 3 applies only to b. The exponent of the numerical coefficient 3 is one and hence is not written. Enlarging $3a^2b^3$ gives 3aabbb.

The difference between exponent and coefficient must be thoroughly understood. $3b = 3$ times b but $b^3 = bbb$. If $b = 3$, then $3b = 3 \times 3 = 9$, but $b^3 = 3 \times 3 \times 3 = 27$, a very different result.

In evolution, the square root of ab is written $\sqrt{ab}$ and read "the square root of the *product* of a and b." $a^2b^2 = aabb$ and $\sqrt{a^2b^2} = \sqrt{aabb} = ab$. Similarly, the square root of $a^2/b^2 = \sqrt{a^2/b^2} = a/b$. If an algebraic expression under a radical indicates a sum or difference, then the expression is in its simplest form, and cannot be further evaluated until numerical values are assigned to the literal terms. For example, $\sqrt{a^2 + b^2}$ cannot be written $\sqrt{a^2} + \sqrt{b^2} = a + b$. This is easily proved. Let a = 3, and b = 4, then $\sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$, but $\sqrt{a^2} + \sqrt{b^2} = \sqrt{3^2} + \sqrt{4^2} = 3 + 4 = 7$, a different answer. Similarly, $\sqrt{a^2 - b^2}$ cannot be further reduced until numerical values are assigned to the literal terms.

Methods of extracting roots of algebraic expressions involving addition and subtraction will be taken up in more advanced assignments.

Exercises

Factor:

11. $x^4 = ?$

12. $y^3 = ?$

13. $2a^2 = ?$

14. $17a^2b = ?$

15. $ab^2c^3 = ?$

16. $x^2y^2 = ?$

17. $x^2y^0z = ?$

18. $a^0b^0c^0 = ?$

Solve:

19. $\sqrt{a^2y^2} = ?$

20. $\sqrt[3]{a^3b^6} = ?$

21. $\sqrt{x^2 + y^2}$,
if x = 16, y = 64.

22. $\sqrt{a^2 + b^2 - c}$,
if a = 11, b = 5, and c = 3.

23. $\sqrt{ab^2 - c}$,
if a = 25, b = 5, and c = 200.

24. $\sqrt{ab} + \sqrt{a^2b^2}$,
if a = 16, b = 4.

25. $\sqrt{ab/c}$,
if a = 40, b = 20, and c = 2.

ALGEBRAIC EXPRESSIONS

An algebraic expression is any quantity written with algebraic symbols. Examples of such expressions are x, $3a^2$, $x - y$, $2a + b - c$, and $x^2 + 2xy + y^2$. Each part of an algebraic expression, separated from any other part by a plus or minus sign, is called *a term of the expression*. In the last example above, x^2, $2xy$, and y^2 are the three terms of the expression, $x^2 + 2xy + y^2$. Any expression involving only indicated multiplication or division is a single term and must always be treated as such, even when the term is separated into its individual factors.

Terms like $4ab^2x$, $9ab^2x$, and ab^2x are called *LIKE* terms because each term has identical literal coefficients. Note that the numerical coefficients of the terms need not be alike.

Algebraic expressions of one term are called *monomial* while those of two or more terms are called *polynomial* or *compound* expressions. Quite often polynomials of two terms are called *binomials*, and those of three terms *trinomials*.

The algebraic sign of a term is determined by the sign PRECEDING the term and the same rules as explained in the preceding assignment for positive and negative numbers also apply in algebra, in fact addition and subtraction of terms, wherein the sign is considered as part of the term, is called algebraic addition and subtraction. If no sign is written ahead of the algebraic term, it is understood to be positive. A minus sign must always precede a negative term. If two terms are exactly alike and of opposite sign, they nullify each other just as 6 - 6 = 0, so does 3A - 3A = 0.

The use of a letter to represent any numerical value, greatly extends the possibilities of algebra. For any definite value of the letter, the algebraic expression containing the letter assumes a definite value. The result obtained by assigning a numerical value to the letter is known as *the numerical value of an algebraical expression.* In the expression 5b, if b is assigned the value of 2, then the expression becomes 5×2, which equals 10. If b has some other value, then the expression has some other numerical value. In the expression ab + bc + cd, if a = 1, b = 2, c = 3, and d = 4, the expression becomes $(1 \times 2) + (2 \times 3) + (3 \times 4) = 20$. If m = 2, n = 3, and p = 4, the expression $5m^2n/3p$ becomes

$$\frac{5 \times 2 \times 2 \times 3}{3 \times 4} = 5$$

The terms of an algebraic expression may be arranged in any sequence, likewise the factors in a term may be arranged in any order. In the expression $ax^2 + bx + c$, the terms could be written $c + bx + ax^2$ or $bx + c + ax^2$, and the terms themselves could be written $x^2a + xb + c$ or in any other combination so long as the validity of the expression is not changed. However, a factor of any term may not be written in any other term, nor may terms be changed from numerator to denominator in a fraction, unless certain provisions are carried out. These provisions belong to more advanced algebra, and will not be considered here.

Exercises

If a = 2, b = 3, c = 4, x = 5, y = 6:

26. a + 2b = ?

27. 4a + 3b = ?

28. a + b + 2c = ?

29. $a^2 + x - y = ?$

30. $a + b^2 - x + y = ?$

31. $x^3 = ?$

32. xy - a = ?

33. ab + 2xy = ?

34. $a^3 + bc + xy = ?$

35. abc + xy = ?

SIGNS OF AGGREGATION

The use of parentheses, braces, etc., to enclose certain terms in an algebraic expression, is of immense value and use. In simplifying an expression which has several terms enclosed by parentheses, it is necessary to remove the parentheses. Consider the expression a + (b + c). The quantity in parentheses b + c is a single term of the expression as long as the parentheses are retained. IT IS OF GREAT IMPORTANCE THAT THE STUDENT REALIZE THAT (b + c) ACT-

UALLY MEANS 1 TIMES THE QUANTITY (b + c). Just as 8 means 1 × 8, so does (b + c) mean 1(b + c), although it is customary to omit writing the 1. When the parentheses are removed, every term within the parentheses must be multiplied by 1. In the expression a + (b + c) both b and c must be multiplied by +1 when the parentheses are removed, and a + (b + c) becomes a + b + c.

Now consider the expression a - (b + c). When the parentheses are removed every term within them must be multiplied by -1 and in so doing the rule for multiplying numbers of unlike signs must be followed. -1(b + c) = -b - c, so a - (b + c) = a - b - c. From the above, the following rules can be devised.

When a quantity in parentheses is preceded by a plus sign, the parentheses may be removed by multiplying every term within by plus one.

When a quantity within parentheses is preceded by a minus sign the parentheses may be removed by multiplying every term within by minus one.

It is understood that the above rules also apply in the case of the other signs of aggregation, the braces and the brackets.

When several signs of aggregation are used in the same expression, apply the above rules starting with the innermost quantity. For example, simplify

$$a^2 + 2\{a^2 + b^2 - [c^2 + b^2 - (a^2 - c^2)]\}$$

The first step is to remove the parentheses from the quantity $(a^2 - c^2)$, and since this quantity is to be multiplied by -1, the first simplification yields

$$a^2 + 2\{a^2 + b^2 - [c^2 + b^2 - a^2 + c^2]\}$$

The next step is to eliminate the brackets and since these are preceded by a minus sign, all the terms within the brackets must have their sign changed. This yields

$$a^2 + 2\{a^2 + b^2 - c^2 - b^2 + a^2 - c^2\}$$

The quantity in braces can be somewhat simplified before proceeding with the removal of the last sign of aggregation. Plus a^2 occurs twice within the braces, so these terms may be added to obtain $+2a^2$. Similarly, $-c^2 - c^2$ becomes $-2c^2$. The remaining terms are $+b^2$ and $-b^2$, and since these are equal but opposite in sign, they reduce to zero. Hence, the quantity within the braces is equal to $2a^2 - 2c^2$, and the above expression reduces to

$$a^2 + 2\{2a^2 - 2c^2\}$$

To remove the braces all that is necessary is to multiply every term within by 2, because in this case the quantity within the braces is preceded by 2 instead of 1, and multiplication is indicated. This gives

$$a^2 + 4a^2 - 4c^2$$

Since $4a^2$ can be added to a^2 to obtain $5a^2$ the final reduction yields

$$5a^2 - 4c^2$$

The student in removing signs of aggregation is urged at this time to proceed step by step. After long practice, it is not difficult to learn how to remove all the aggregation signs in one operation, but until one is an expert it is very easy to make a miscue.

Exercises

Simplify:

36. $-[a + b - c\ (a + b)]$

37. $a - b\ (a + b)$

38. $2a - 4(a - c) - 4c$

39. $2(a + b) - 3(a + b) + (b - a)$

40. $a + (b - c - d)$

41. $a^2b - a(ab - abc)$

42. $2z(x^2 - y + 1)$

43. $10(8 + 3 - 2)$

44. $xy(2 + 8) + 2z(xy + 4)$

45. $4a - b - (2b + 2a)$

EQUATIONS

One of the greatest uses of algebra lies in the solution of equations. The operation most frequently performed in solving equations is transposition, and the importance of mastering this operation cannot be too heavily stressed.

An equation is a symbolical statement that two expressions are equal and hence represent the same thing. The equation $4x + 2 = 10 + 2$ implies that $4x + 2$ and $10 + 2$ represent the same number. The term "left member of an equation" refers to the entire expression to the left side of the sign of equality. Similarly, the term "right member" is applies to the expression on the right side of the sign of equality. In the equation given above, $4x + 2$ is the left member and $10 + 2$ is the right member.

An equation is frequently employed to solve for an unknown number from its relation to known values. Solving the equation will result in the discovery of the numerical value of the unknown quantity. An equation is considered solved when one member contains only the unknown as a single term having a coefficient and exponent of +1, and the other member contains only known values reduced to their most simple form. Thus, $x + 5 = 7$, $2x = 16$, $x^2 = 9$, $x = 25 + 2x$, $\sqrt{x} = 2$, and $-x = 15$ are all unsolved equations, while $x = 5$, $3 = x$, $x = -7$, and $x = 2/3$ are examples of solved equations. It is customary but not mandatory to write the unknown in the left member of the equation. Thus, the equation $3 = x$ is usually (but not necessarily) changed to the form $x = 3$.

In solving equations, certain axioms are utilized and these must be memorized and thoroughly understood. An axiom is a statement whose truth is self-evident.

AXIOM 1: *If equal numbers are added to or subtracted from both members of an equation, the condition of equality is maintained. Example:*

$$x + 3 = 5$$

If +3 is added to both sides of the equation, then:

$$x + 3 + 3 = 5 + 3$$

$$x + 6 = 8$$

If −3 is added to both members of the original equation, then:

$$x + 3 - 3 = 5 - 3$$

$$x = 2$$

If $x = 2$, then

$$x + 3 = 2 + 3 = 5$$

and

$$x + 6 = 2 + 6 = 8$$

By performing similar operations on any number of equations, the truth will become evident that, whenever the same quantity is added to or subtracted from both sides of an equation, the result is always an equality.

AXIOM 2: *If both members of an equation are multiplied by the same number, the equality is not changed.*
Example:

If $\quad x = 2$

then $\quad x + 3 = 5$

If both members are multiplied by 5, then:

$$5(x + 3) = 5 \cdot 5$$
$$5x + 15 = 25$$

Since $x = 2$

then
$$5x + 15 = 5 \cdot 5$$
$$5 \cdot 2 + 15 = 25$$
$$10 + 15 = 25$$
$$25 = 25$$

If the equation $x + 3 = 5$ is multiplied by -2, then:

$$-2(x + 3) = -2 \cdot 5$$

$$-2x - 6 = -10$$

Since $x = 2$, $-2x - 6 = -2 \cdot 2 - 6 = -4 - 6 = -10$.

AXIOM 3: *If both members of an equation are divided by the same number, the equality is not changed.*
Example:

If $\quad x = 6$

then $\quad 4x + 6 = 30$

Dividing both members by 2, gives:

$$\frac{4x + 6}{2} = \frac{30}{2}$$

$$2x + 3 = 15$$

To check, substitute 6 for x:

$$2 \cdot 6 + 3 = 15$$

$$12 + 3 = 15$$

Or, if both members of the original equation $4x + 6 = 30$ are divided by -4, then:

$$\frac{4x + 6}{-4} = \frac{30}{-4}$$

$$-x - 1.5 = -7.5$$

To check, substitute 6 for x again:

$$-6 - 1.5 = -7.5$$

AXIOM 4: *The same root of both members of an equation may be extracted without destroying the equality.*
Example:

$$16x^2 = 144$$

Extracting the square root of both sides

$$4x = 12$$

Dividing both sides by 4,

$$x = 3$$

Substituting 3 for x in the original equation:

$$16x^2 = 16 \cdot 3^2 = 16 \cdot 9 = 144$$

AXIOM 5: *Both members of an equation may be raised to the same power without disturbing the equality.*
Example:

If $x = 12$

Then $6x = 72$

Squaring both sides, then:

$$(6x)^2 = 72^2$$

$$36x^2 = 5{,}184$$

Since $x = 12,\ 36x^2 = 36 \cdot 12^2$

$$= 36 \cdot 144 = 5{,}184$$

AXIOM 6: *If two quantities are equal to the same number, they are equal to each other.*
Example:

If $x = 6$

and $y = 5$

then $x + 2 = 8$

and $y + 3 = 8$

By this axiom,

$$x + 2 = y + 3$$

Substituting 6 for x and 5 for y:

$$6 + 2 = 5 + 3$$

$$8 = 8$$

The above axioms will permit the successful solution of practically all the equations used in this course. The axioms should be memorized and *applied one at a time in* solving any equation.

The following examples deal with the solution of simple equations. The student is urged to follow this method of solving equations, if algebra appears to be a difficult subject.

Example 1:

Solve: $3x - 7 = 14 - 4x$

In this equation x is the unknown term and it is desired to isolate all the x terms on one side of the equation and all the known terms in the other member. It is customary but not mandatory to transpose the unknown terms to the left member, and all the known terms to the right member. $-4x$ can be transposed from the right member to the left member by Axiom 1.

If $+ 4x$ is added to both members then:

$$3x - 7 + 4x = 14 - 4x + 4x$$

and since $-4x + 4x = 0$

then $3x - 7 + 4x = 14$

In the left member, $3x + 4x = 7x$, so the equation reduces to:

$$7x - 7 = 14$$

To transpose -7 to the right side of the equation, use Axiom 1 again, and add $+7$ to both members of the equation

$$7x - 7 + 7 = 14 + 7$$

and since $-7 + 7 = 0$

then $7x = 21$

By Axiom 3, both members can be divided by 7:

$$\frac{7x}{7} = \frac{21}{7}$$

$$x = 3$$

The problem has now been solved, and the solution can be checked by substituting the numerical value of x in the original equation.

$$3x - 7 = 14 - 4x$$

If $x = 3$

then $3 \cdot 3 - 7 = 14 - 4 \cdot 3$

$$9 - 7 = 14 - 12$$

$$2 = 2$$

The expression 2 = 2 is called an IDENTITY, and any equation is considered proved if it reduces to an identity.

Example 2.

Solve: $1 - 4(x - 2) = 7x - 3(3x - 1)$

The first step in this problem is to remove the parentheses in each member:

Thus, $-4(x - 2) = -4x + 8$

and, $-3(3x - 1) = -9x + 3$

So the original equation simplifies to:

$$1 - 4x + 8 = 7x - 9x + 3$$

In the left member +1 and +8 = +9, while in the right member $7x - 9x = -2x$, so:

$$-4x + 9 = -2x + 3$$

Adding +2x to both members by Axiom 1:

$$-4x + 2x + 9 = -2x + 2x + 3$$

$$-2x + 9 = 3$$

Adding -9 to both members by Axiom 1:

$$-2x + 9 - 9 = 3 - 9$$

$$-2x = -6$$

By Axiom 2, both members can be multiplied by -1:

$$-1(-2x) = -1(-6)$$

$$+2x = +6$$

By Axiom 3, divide both members by 2:

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

The solution may now be checked by substituting 3 for x in the original equation:

$$1 - 4(x - 2) = 7x - 3(3x - 1)$$

$$1 - 4(3 - 2) = 7 \cdot 3 - 3(3 \cdot 3 - 1)$$

$$1 - 4(1) = 21 - 3(9 - 1)$$

$$1 - 4 = 21 - 3(8)$$

$$-3 = 21 - 24$$

$$-3 = -3$$

Example 3:

Solve:

$$\frac{x}{3} - \frac{x}{4} + \frac{x}{6} = \frac{3x - 24}{4}$$

The lowest common multiple of the denominators, 3, 4, and 6 is 12. By Axiom 2, multiply both members of the equation by 12:

$$\frac{12x}{3} - \frac{12x}{4} + \frac{12x}{6} = \frac{12(3x - 24)}{4}$$

Cancellation reduces this to:

$$4x - 3x + 2x = 3(3x - 24)$$

Adding like terms in the left-hand member and removing parentheses in the right-hand member:

$$3x = 9x - 72$$

By Axiom 1, subtract 9x from both members:

$$3x - 9x = 9x - 9x - 72$$

$$-6x = -72$$

Multiplying both sides by -1 to change signs:

$$+6x = +72$$

By Axiom 3, divide both sides by 6:

$$\frac{6x}{6} = \frac{72}{6}$$

$$x = 12$$

To check the solution, substitute 12 for x in the original equation:

$$\frac{x}{3} - \frac{x}{4} + \frac{x}{6} = \frac{3x - 24}{4}$$

$$\frac{12}{3} - \frac{12}{4} + \frac{12}{6} = \frac{3 \cdot 12 - 24}{4}$$

$$4 - 3 + 2 = \frac{36 - 24}{4}$$

$$3 = \frac{12}{4}$$

$$3 = 3$$

Example 4:

Solve: $\frac{1}{x} + \frac{2}{x} + \frac{3}{x} = \frac{48 - 3x}{2x}$

In this problem, the LCM of all the denominators is 2x. Multiplying both sides of the equation by 2x:

$$\frac{2x}{x} + \frac{2 \cdot 2x}{x} + \frac{3 \cdot 2x}{x} = \frac{2x(48 - 3x)}{2x}$$

Cancelling:

$$2 + 4 + 6 = 48 - 3x$$

Collecting like terms:

$$12 = 48 - 3x$$

Subtracting 48 from both sides by Axiom 1:

$$12 - 48 = 48 - 48 - 3x$$

$$-36 = -3x$$

Dividing both members by -3:

$$\frac{-36}{-3} = \frac{-3x}{-3}$$

$$12 = x$$

Checking by substituting 12 for x in the original equation:

$$\frac{1}{x} + \frac{2}{x} + \frac{3}{x} = \frac{48 - 3x}{2x}$$

$$\frac{1}{12} + \frac{2}{12} + \frac{3}{12} = \frac{48 - 3 \cdot 12}{2 \cdot 12}$$

$$\frac{6}{12} = \frac{48 - 36}{24}$$

$$\frac{1}{2} = \frac{12}{24}$$

$$\frac{1}{2} = \frac{1}{2}$$

Example 5:

Solve:

$$4x^2 + 4(5x^2 - 40) = 16x^2 + 352$$

Removing parentheses:

$$4x^2 + 20x^2 - 160 = 16x^2 + 352$$

Subtracting $16x^2$ from both members:

$$4x^2 + 20x^2 - 16x^2 - 160$$

$$= 16x^2 - 16x^2 + 352$$

Collecting like terms:

$$24x^2 - 16x^2 - 160 = 352$$

$$8x^2 - 160 = 352$$

Adding 160 to both members:

$$8x^2 - 160 + 160 = 352 + 160$$

$$8x^2 = 512$$

Dividing both sides by 8:
$$8x^2/8 = 512/8$$

$$x^2 = 64$$

By Axiom 4, extracting the square root of both sides:

$$x = 8$$

The solution may be checked by substituting 8 for x in the original equation:

$$4x^2 + 4(5x^2 - 40) = 16x^2 + 352$$

$$4 \cdot 8^2 + 4(5 \cdot 8^2 - 40)$$
$$= 16 \cdot 8^2 + 352$$

$$4 \cdot 64 + 4(5 \cdot 64 - 40)$$
$$= 16 \cdot 64 + 352$$

$$256 + 4(320 - 40) = 1{,}024 + 352$$

$$256 + 4(280) = 1{,}376$$

$$256 + 1{,}120 = 1{,}376$$

$$1{,}376 = 1{,}376$$

Example 6:

Solve:
$$\frac{\sqrt{5x + 75}}{10} = 2$$

Multiply both sides by 10 to clear fractions:

$$\sqrt{5x + 75} = 20$$

Square both members:

$$(\sqrt{5x + 75})^2 = 20^2$$

$$5x + 75 = 400$$

NOTE: *The square of the square root of a number is the number.*

Thus: $(\sqrt{64})^2 = 8^2 = 64$

So $(\sqrt{5x + 75})^2 = 5x + 75$

Subtract 75 from both members:

$$5x = 325$$

Divide both members by 5:

$$x = 65$$

To check, substitute 65 for x in the original equation:

$$\frac{\sqrt{5x + 75}}{10} = 2$$

$$\frac{\sqrt{5 \cdot 65 + 75}}{10} = 2$$

$$\frac{\sqrt{325 + 75}}{10} = 2$$

$$\frac{\sqrt{400}}{10} = 2$$

$$\frac{20}{10} = 2$$

$$2 = 2$$

Example 7:

The sum of two numbers is 44. The difference between the numbers is 2. What are the numbers?

Let x = the larger number
y = the smaller number

Then $x + y = 44$

and $x - y = 2$

In the equation $x + y = 44$, if $-y$ is added to both sides:

$$x = 44 - y$$

In the equation $x - y = 2$, if $+y$ is added to both sides:

$$x = 2 + y$$

By Axiom 6, two quantities, each equal to x, will be equal to each other.

$$44 - y = 2 + y$$

Adding $+y$ to both members:

$$44 = 2 + 2y$$

Adding -2 to both members:

$$42 = 2y$$

Dividing both members by 2:

$$21 = y$$

or $y = 21$

Since $x = 2 + y$

then $x = 2 + 21 = 23$

Thus the two numbers are 23 and 21. To check, substitute 23 for x and 21 for y in the original equations:

$$x + y = 44$$

$$23 + 21 = 44$$

$$x - y = 2$$

$$23 - 21 = 2$$

Example 8: Solve for x:

$$z = \frac{4(xy + xz)}{5}$$

By Axiom 2, multiply both sides by 5:

$$5z = \frac{5 \times 4(xy + xz)}{5}$$

Cancellation reduces this to:

$$5z = 4(xy + xz)$$

By Axiom 3, divide both sides by 4:

$$\frac{5z}{4} = \frac{4(xy + xz)}{4}$$

Cancellation reduces this to:

$$\frac{5z}{4} = xy + xz$$

x maybe factored out of xy + xz, therefore xy + xz = x(y + z), then by Axiom 6,

$$\frac{5z}{4} = x(y + z)$$

By Axiom 3, divide both sides by (y + z):

$$\frac{5z}{4(y + z)} = \frac{x(y + z)}{y + z}$$

Cancellation reduces this to:

$$\frac{5z}{4(y + z)} = x$$

By Axiom 1, x may be subtracted from both sides:

$$\frac{5z}{4(y + z)} - x = x - x$$

Cancellation reduces this to:

$$\frac{5z}{4(y + z)} - x = 0$$

By Axiom 1, 5z/4(y + z) may be subtracted from both sides:

$$\frac{5z}{4(y + z)} - \frac{5z}{4(y + z)} - x = -\frac{5z}{4(y + z)}$$

Cancellation reduces this to:

$$-x = -\frac{5z}{4(y + z)}$$

By Axiom 2, multiply both sides by -1,

$$(-1)\ (-x) = (-1)\left[-\frac{5z}{4(y + z)}\right]$$

$$x = \frac{5z}{4(y + z)}$$

The above examples illustrate the use of the six axioms previously given for the transposition and solution of equations. Several corollaries to these axioms are also useful. (A corollary is a supplementary statement whose truth is immediately evident from the general statement of the axiom.)

Corollary to Axiom 1: *A term may be transposed from one member of the equation to the other by changing the sign of the term.*

Example:

$$3x - 8 = 2x + 14$$

The term 2x may be transposed from the right member to the left member by changing its sign. (This is equivalent to adding -2x to both members of the equation.) Thus:

$$3x - 8 - 2x = 14$$

The term -8 may be transposed from the left member to the right member by changing its sign. (This is equivalent to adding +8 to both members of the equation.)

Thus:

$$3x - 2x = 14 + 8$$

Simplifying by adding like terms:

$$x = 22$$

To check, substitute 22 for x in the original equation:

$$3 \cdot 22 - 8 = 2 \cdot 22 + 14$$

$$66 - 8 = 44 + 14$$

$$58 = 58$$

Corollary to Axiom 3: *If the sign of all the terms of both members of the equation are changed, the condition of equality will be maintained.*

Example:

$$-2x - 3 = -x + 7$$

The signs of all the terms of both members may be changed. (This is equivalent to multiplying both members by the quantity -1.) Thus:

$$2x + 3 = x - 7$$

Transposing x to the left member:

$$2x - x + 3 = -7$$

Add -3 to both members:

$$2x - x = -7 - 3$$

Simplifying by adding like terms.

$$x = -10$$

To check, substitute -10 for x in the orignal equation:

$$-2x - 3 = -x + 7$$

$$-2(-10) - 3 = -(-10) + 7$$

$$20 - 3 = 10 + 7$$

$$17 = 17$$

General Corollary: *Any operation performed on one member of an equation must also be performed on the other member or the equality is destroyed.*

Since most of the errors made in the transposition and solution of equations arise from a violation of this principle, it is especially important that it should be thoroughly understood to avoid violation.

PROCEDURE IN SOLVING SIMPLE EQUATIONS

The solution of simple equations is not difficult if the student will adopt the practice of *solving each equation step by step applying only one rule at a time.* As each operation is completed, the members of the equation should be carefully examined for the presence of like terms. Where like terms appear in the same member of the equation, they may be combined into a single term by algebraic addition.

Equations appear in a great variety of forms, and therefore no single procedure can be guaranteed to solve every equation that may be encountered. However, the following order of operations will be found to give a straight forward solution of the great majority of simple equations involving a single unknown, and is recommended as a guide for preliminary work in handling equations. When the student has gained more proficiency, he may occasionally find it advantageous to depart from this procedure in some respect; it should be understood that such a departure is entirely permissible provided operations are always carried out in accordance with the fundamental axioms already given.

1. *Remove the signs of aggregation, if any, in each member of the equation.* The rules for the removal of parentheses, brackets, etc., have been discussed earlier in this assignment. Begin with the innermost pair and continue until all the signs of aggregation have been removed. (See *Example Col. 1, Page 6.)*

2. *Clear the equation of fractions, if any are present.* This involves multiplying both members of the equation by the Least Common Multiple of all the given denominators and reducing the terms in the resulting equation to the most simple form. (See *Examples 3 and 4,* page 11.)

3. *Clear the equation of radicals, if any are present.* Transpose one term containing a radical to the left member and all other terms to the right member. Raise both members to whatever power will clear the left member of its radical. If other radical terms are remaining in the right member, repeat the process until all radical terms have been eliminated from the equation. (See *Example 6,* page 12.)

4. *Transpose all terms containing the unknown to the left member and all terms containing known values to the right member of the equation.* After transposition the like terms in each member should be combined into a single term by algebraic addition.

5. *Divide both members of the equation by the numerical coefficient of the single unknown term in the left member.* For example, in the equation $5x = 15$, divide both members by the numerical coefficient of x, that is, by 5. In the resulting equation the coefficient of the unknown term will be +1.

6. *If the exponent of the unknown term in the left member is not +1, then extract the root of both members that will make the exponent of the unknown equal to +1.* For example, if the single term in the left member is x^2, then the square root of both members of the equation should be extracted, giving x in the left member. This completes the determination of the numerical value of the unknown quantitiy.

7. *Check the solution of the equation by substituting the numerical value of the unknown in the original equation.* No solution of an equation should be accepted as correct until the value of the unknown has been shown to satisfy the original equation.

Exercises

In the following equations, *solve for the unknown quantity:*

46. $x + 5 = 21$

47. $x - 15 = 12$

48. $43 = 25 + x$

49. $a + 5 - 3 = 20$

50. $16 - b + 6 = 24$

51. $5 - 4x = 29$

52. $10x - 15 = 7x + 24$

53. $10x - 4(2x + 1) = 2$

54. $12 - 4(4x - 5) = 0$

55. $\frac{x}{3} + \frac{x}{2} = 20$

56. $\frac{3x}{5} + \frac{x}{2} = 2x - 9$

57. $\frac{x}{4} - (x + 5) = 7$

58. $\sqrt{2x - 3} = 5$

59. $E = \sqrt{PR}$ *Find P*

60. $P = I^2R$ *Find I*

The most difficult part of the solution for an unknown is generally the setting up of the equation. When the numerical value is unknown, it should be set down as x. Then all the possible things known about it should be formulated as the shape of an equation. A simple illustration of the use of the equation is as follows. The problem: Three times a certain number equals the number with twenty added. What is the number? Let x be the number since it is unknown. Then from the statement above, 3x will be three times the number. Also, from the statement, this is equal to the number with 20 added or to x + 20. The equation may then be set down:

$$3x = x + 20$$

Subtracting x $\quad 3x - x = 20$

Combining $\quad 2x = 20$

Dividing by 2 $\quad x = 10$

The value x = 10 is the desired number.

Two ships start from port traveling in the same direction. One makes 200 miles a day, the other 300 miles a day. In how many days will they be 350 miles apart? In this problem, the number of days is required. Let it be equal to x. The distance travelled will then be the product of the distance per day times x. For the first ship, it will be 200x and for the second 300x. From the statement, the distance apart in x days is 350. Then the equation may be set up:

$$300x - 200x = 350$$

Subtracting $\quad 100x = 350$

Dividing by 100 $\quad x = 3.5$ days

The principles involved in solving equations are very valuable and necessary in the solution of formulas which are in themselves equations. The use of letters to stand for the general conditions is well-known. In specific cases, all the letters except one will have numerical values and the solution for the one unknown letter is desired. In the formula for the area of a rectangle, the general statement is: Area = Length x Width. Expressed by letters: A = LW

Applied to a specific case, the length and width will be given. A field is 50 ft. long and 30 ft. wide. What is the area in sq.ft.? Here L equals 50 and W equals 30. Substituting these values,

$$A = 50 \times 30 \text{ or } 1{,}500 \text{ sq.ft.}$$

A formula frequently encountered in electrical work is Ohm's Law, which may be stated I = E/R, where I is the current, E is the voltage, and R the resistance. If E equals 10 and R equals 5, these values are substituted and the value of I may be found.

$$I = \frac{E}{R} = \frac{10}{5} = 2 \text{ amperes}$$

Another type of formula involving somewhat different arrangement of letters is $A = 1/2(B + b)a$, which is the formula for the area of a certain kind of geometrical figure, the trapezoid, where A is area, B is the length of one base, b the length of the other base, and a is the altitude. Assume B = 20, b = 16, and a = 10. Substituting these values for the letters,

$$A = \frac{1}{2}(20 + 16)\,10$$

adding, $= \frac{1}{2}(36)\,10$

Simplifying by multiplication,

A = 180 sq. units of area

Another extensive use of algebra is in the rearranging of a formula so that it may be solved for some other letter than perhaps the letter for which it is set up. Going back to the first formula considered, given the area and the length, it is desired to find the width. The original equation A = LW is to be changed so that W is by itself on one side. L and W are factors of the term on the right-hand side. By *division*, it will be possible to get rid of L, but both sides must be divided. Dividing both sides by L will give the desired result.

$$A = LW$$

Dividing by L

$$\frac{A}{L} = W$$

or

$$W = \frac{A}{L}$$

Ohm's Law may also be changed in any desired way by these same laws. From the expression I = E/R it is desired to solve in terms of E. To do this, E must appear alone on one side. Multiplying both sides by R will remove the R from under the E. Multiplying by R,

$$IR = \frac{ER}{R}$$

Cancelling the R's on the right-hand side IR = E or

$$E = IR$$

If it is desired to solve for R from the original expression, it is performed as follows:

$$I = \frac{E}{R}$$

Multiplying both sides by R

$$IR = E$$

Dividing by I

$$R = \frac{E}{I}$$

In the trapezoid formula, it is desired to solve for the altitude, and the formula is to be changed accordingly.

$$A = \frac{1}{2}(B + b)\,a$$

The term (1/2) (B + b)a has three factors: 1/2, (B + b) and a. Only the factor a is to be left on that side. The other quantities will be treated exactly like any other term in the transformation.

First Step:
Multiply both sides by 2 to eliminate the fractional form.

$$2A = (B + b)a$$

Second Step:
Divide both sides by (B + b).

$$\frac{2A}{(B + b)} = a$$

or

$$a = \frac{2A}{B + b}$$

From the work above, it will be noticed that when a term appears in the numerator on one side of an equation, it may be placed in the denominator on the other side without any change in sign, and if in the denominator on one side, will reappear in the numerator on the other side. This is *not* transposition which arises from addition or subtraction, and hence involves no change in sign. In transposing terms, a sign change is always involved; when *transferring a factor* from the numerator on one side to the denominator on the other, and vice versa, no change in sign results.

Exercises

Solve:

61. $6x - 10 = 3x + 2$

62. $12x - (2x - 1) = 38 + 7x$

63. The sum of three successive even numbers is 54. What are the numbers?

64. A room is 3 times as long as wide. Its perimeter (distance around the room) is 48 feet. Set up the expression algebraically and find the length and width.

65. If a man has twice as many ten dollar bills as five dollar bills, and total money of seventy-five dollars, how many of each kind has he?

ELEMENTS OF GEOMETRY

The practical applications of the principles of geometry are included in what is known as mensuration or the measurement of lines, surfaces, volumes, and angles.

The first consideration is that of the measurement of lines or distances. A line is considered as having only one dimension, length. A straight line is one which is usually defined as the shortest distance between two points. A curved line is one, no part of which is straight. When two straight lines in the same plane do not meet, they are called parallel lines.

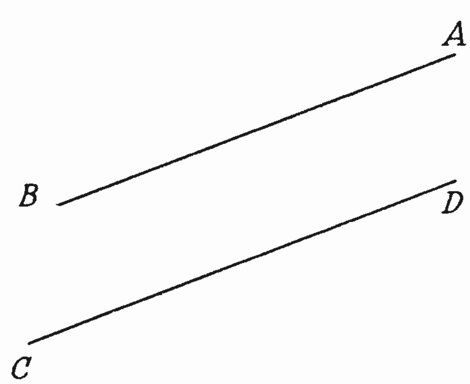

Fig. 1.—Parallel lines.

could also be drawn between the opposite corners to those already illustrated.

sides are the lines enclosing the right angle. The sides are also called the legs of the triangle. If

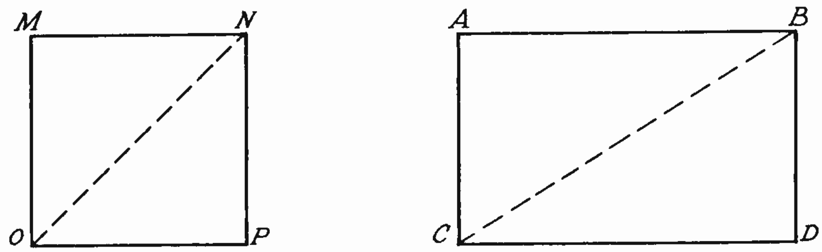

Fig. 7.—Two important quadrilaterals.

It is frequently desirable to determine the distance around any of the figures discussed. This is done by adding up the lengths of the individual sides, the resulting sum being called the *perimeter.* If a triangle is scalene, the perimeter of one whose sides are 30, 60, and 70 would be 30 + 60 + 70 which equals 160. The same method may be applied to any triangle if all the sides are known. In the case of the equilateral triangle, where all the sides are equal, the perimeter is found by multiplying the length of one side by three. The perimeter of an equilateral triangle of side 15 is $3 \times 15 = 45$.

The right triangle has one very important mathematical property. It may be stated as follows: *The length of the hypotenuse equals the square root of the SUM of the squares of the other two sides.* In Fig. 8 is pictured a right triangle whose sides are a and b, and whose hypotenuse is c. Expressed in the language of mathematics from the above rule, $c = \sqrt{a^2 + b^2}$. This may also be stated—the square of the hypotenuse is equal to the sum of the squares of the other two sides, or $c^2 = a^2 + b^2$. The hypotenuse is the longest side, and the other two sides and the hypotenuse are given numerical values as in Fig. 8, it will be seen that $50^2 = 40^2 + 30^2$ or

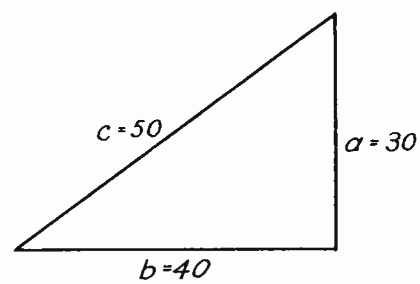

Fig. 8.—The right triangle.

$2{,}500 = 1{,}600 + 900$. This is a very important and useful fact about right triangles. If the expression is made as follows with regard to any right triangle with legs designated as shown above,

$$c^2 = a^2 + b^2$$

Transposing a^2,

$$c^2 - a^2 = b^2$$

Extracting square root,

$$b = \sqrt{c^2 - a^2}$$

The same is true for the other leg, and this may be stated as a rule.

Any side of a right triangle equals the square root of the hypotenuse of the right triangle squared minus the other leg squared. Practically these facts are made use of as follows:

The two sides of a right triangle are 5 feet and 12 feet. What is the length of the hypotenuse?

$$\text{Hypotenuse} = \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$$

$$= \sqrt{169} = 13 \text{ feet}$$

To find a side when the other side and the hypotenuse are given, the second condition is used. The hypotenuse of a right triangle is 46 inches and one of the legs is 13 inches. What is the length of the other leg?

$$\text{Leg} = \sqrt{46^2 - 13^2} = \sqrt{1947} = 44.1 \text{ in.}$$

The area of a triangle is found from the rule which states that *area equals one-half the product of the base and the altitude.* In letters, A = 1/2 ba where b is the base and a is the altitude. The altitude of a certain triangle is 14 and its base is 24. What is the area?

$$A = 1/2 \times 14 \times 24 = 168$$

Knowing the base and altitude, the area of any triangle is found in exactly the same way. If the triangle is a right triangle, one of the sides may be considered as the base and the other the altitude in determining the area, neither side in this case being the hypotenuse. This is true regardless of the actual position of the triangle.

Exercises

Consider the following right triangles in which a = altitude, b = base, h = hypotenuse, A = area.

66. a = 15, b = 20.
Find h, perimeter and area.

67. a = 24, b = 10.
Find h, perimeter and area.

68. a = 120, b = 150.
Find h, perimeter and area.

69. a = 60, h = 110.
Find b, perimeter and area.

70. h = 65, b = 20.
Find a, perimeter and area.

71. b = 240, h = 300.
Find a, perimeter and area.

72. h = 80, a = 50.
Find b, perimeter and area.

73. a = 110, b = 150.
Find h, perimeter and area.

74. A = 240, a = 20.
Find b and h.

75. A = 180, b = 18.
Find a and h.

To find the area of the special types of quadrilaterals which have been mentioned, the square and the rectangle, the area is equal to the product of the base times the altitude. Either edge may be considered the base and either the altitude. This rule applies to both the square and the rectangle.

Areas = ba

where b is the base and a is the altitude.

Usually the length and width are referred to instead of the base and altitude but the reasoning is exactly the same.

Determine the area of a rectangle 20 inches by 15 inches.

$$\text{Area} = 20 \times 15 = 300 \text{ sq. in.}$$

Another kind of plane figure important in electrical work is the circle. A circle is defined as a closed curve all parts of which are equidistant from a point called the center. The shortest distance from the center to any point on the curve is called the *radius*, and the shortest distance from any point on the curve through the center to a point directly opposite is called the *diameter*. The diameter is thus seen to be equal to twice the radius. The distance around the circle is called the circumference. These are illustrated in Fig. 9. The ratio of

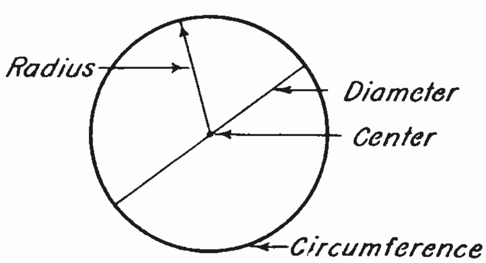

Fig. 9.—The circle, showing important dimensions.

the length of the circumference to the diameter is a constant quantity regardless of the size of the circle and is called π (pi). Its numerical value to two decimal places is 3.14 a value which should be remembered. Expressed mathematically, this constant π is stated as follows:

$$\pi = \frac{\text{circumference}}{\text{diameter}} = 3.14$$

The circumference is abbreviated c and the diameter d. Since the radius is one-half the diameter, it may also be written

$$\pi = \frac{\text{circumference}}{2 \times \text{radius}} = 3.14$$

The expression may also be written as a formula

$$c = \pi d \text{ or } c = 2\pi r$$

where r is the radius.

A circle has a diameter of 20 inches. Determine the circumference.

$$c = 3.14 \times 20 = 62.8 \text{ in.}$$

A circle has a circumference of 120 feet. Determine the diameter and the radius.

$$d = \frac{c}{3.14} = \frac{120}{3.14} = 38.2 \text{ ft.}$$

Note that the radius is one-half the diameter. The radius is therefore 1/2 × 38.2 or 19.1 feet.

It will thus be seen that the diameter equals the circumference divided by 3.14 and the circumference equals the diameter multiplied by 3.14.

The area of a circle is obtained from the formula:

$$\text{Area} = \frac{1}{4}\pi d^2 \text{ or } \pi r^2 \text{ or } .7854 d^2$$

Since the use of the radius formula is simpler, it is to be preferred. The radius may always be obtained by dividing the diameter by 2.

Determine the area of a circle whose diameter is 12 inches. The radius will equal 1/2 the diameter or 6 inches. Substituting in the formula

$$\text{Area} = 3.14 \times 6^2 = 3.14 \times 36 = 113 \text{ sq. in.}$$

An important use of the area of a circle formula is illustrated in the following practical problem. In Fig. 10 is illustrated a flat washer. It is desired to determine the area of the actual metal in the washer. The outside diameter is 3 inches and the inner diameter (or the diameter of the hole) is 2 inches. A little thought will show that the total area based on the outer circumference minus the area of the hole is the area of the actual metal.

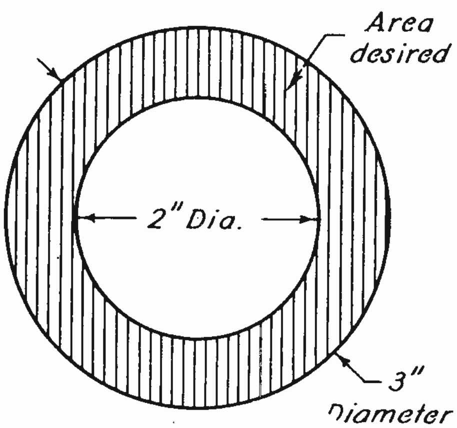

Fig. 10.—A flat washer involves two concentric circles.

The outside area:

$$A = 3.14 \times 1.5^2 = 7.06 \text{ sq. in.}$$

The inner area:

$$A = 3.14 \times 1^2 = 3.14 \text{ sq. in.}$$

Actual area is:

$$7.06 - 3.14 = 3.92 \text{ sq. in.}$$

Exercises

Given the following circles in which c = circumference, d = diameter, r = radius, and A = area:

76. d = 15
Find r, c and A

77. r = 20
Find d, c and A

78. r = 120
Find d, c and A

79. c = 250
Find r, d and A

80. c = 200
Find r, d and A

81. A = 2,000
Find r, d and c

82. A = 800
Find r, d and c

83. d = 120
Find r, c and A

84. See Fig. 10
Outer d = 10, Inner d = 4
Find A.

85. See Fig. 10
Outer d = 25, Inner d = 20
Find A.

Any part of the circumference of a circle is called an *arc* and a line connecting the ends of an arc is called a *chord*. The largest possible chord in a circle is a diameter, but the arc determined by this diameter is not the largest arc. This illustrated in Fig. 11. It is seen that a chord determines two arcs, the larger being called the *major* *arc* and the smaller the *minor arc*. Arcs are measured in units the same as the circumference and also in degrees, determined by the central angle formed by radii drawn from the ends of the arc. A *sector* is that portion of the circle enclosed by the arc and the terminal radii. In Fig. 11, the area ABCO is a sector in the left hand figure. A *segment* is the area enclosed between the arc and its chord, such as the area ACB in either the right or left hand figure. A quadrant is a sector whose area is one-fourth the area of the circle and whose arc is one-fourth of the circumference. Since there are 360° in the circle, then *there are 90° in the arc of a quadrant*. A line drawn perpendicular to a radius at the junction of the radius with the circumference and touching the circle at but one point is said to be tangent to the circle. The line MN in Fig. 11 is tangent to the circle at point C.

The length of an arc is easily determined if the number of degrees in the angle measuring it is known. If the angle at the center of the circle is 40°, then the length of the arc is 40/360 of the circumference. Suppose the circle in Fig. 12 has a radius of 4 in. Then

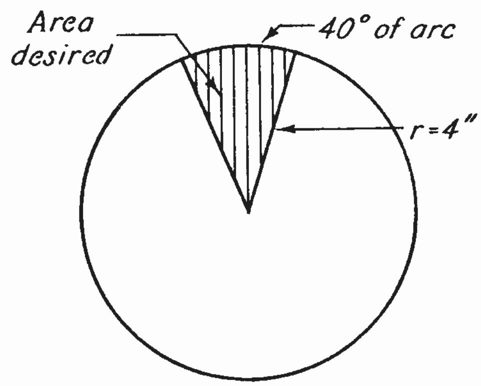

Fig. 12.—Sector of a circle.

the circumference is 2 × 3.14 × 4 and the arc of 40° on this circle is

$$\text{Arc} = \frac{40}{360} \times 2 \times 3.14 \times 4 = 2.79 \text{ in.}$$

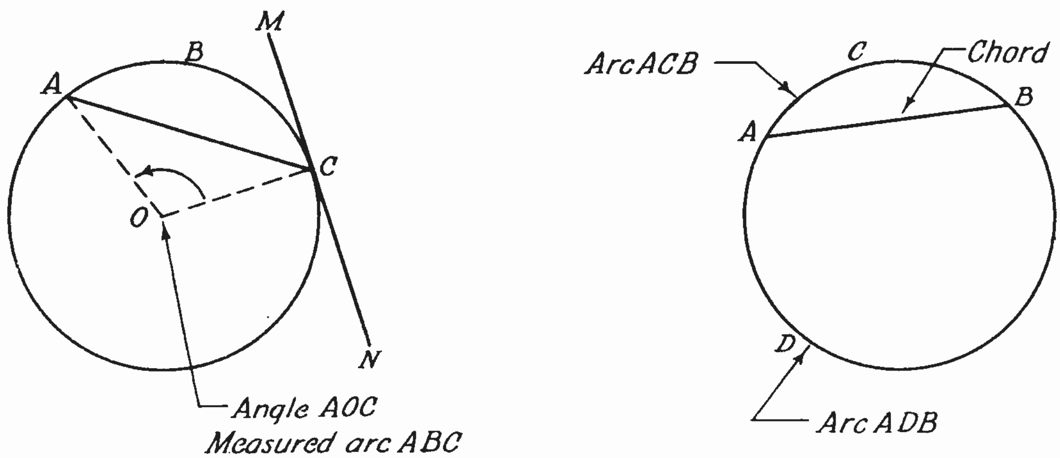

Fig. 11.—Arcs and chords of a circle.

The area of the sector is likewise proportional to the length of arc in degrees to the total length of the circumference in degrees and the area of the sector may be found by multiplying the area of the circle by the ratio of the number of degrees of arc to the number of degrees in the circle. In Fig. 12 this ratio is 40/360. The area of the circle is:

$$A = 3.14 \times 4^2 = 50.24 \text{ sq. in.}$$

$$40/360 \times 50.24 = 5.58 \text{ sq. in.}$$

the area of the sector.

THE MIL

Because practically all electrical conductors are circular in nature, electrical engineers have adopted a more convenient measure of the cross-section of a conductor than the usual square units. The area of a circle in sq. units necessitates the use of 1/4 × 3.1416 or .7854 as a constant affecting the diameter squared. It is this awkward constant that is to be avoided in the measurement of circular areas, such as the cross-section of a round conductor. The inch is divided into one thousand parts and each part is called a mil from the Latin word meaning thousand. Thus 1,000 mils = 1 inch. Conversion from mils is done by shifting the decimal point respectively either to the left or right three places. 1.5 inches equals 1,500 mils, .234 inches equals 234 mils and, going in the opposite direction, 1,600 mils is 1.6 inches, 64 mils is .064 inches, etc.

The unit of area in the system of circular areas is taken as the area of a circle 1 mil in diameter and this area is called a circular mil (abbreviated C.M.) Note that a circular mil is a measure of area, a mil a measure of linear distance. Now, if a circle 1 mil in diameter has an area of 1 C.M., then a proportion may be established between this diameter and area, and the diameter and area of any circle. Thus, $1 : d^2 = 1 : A$, since the area is proportional to the diameter squared. This simply means that to find the area in circular mils of a round object, all that is necessary is to square the diameter. There is nothing else to do. If a wire has a diameter of 50 mils, its circular mil area is 2,500. Conversely, if a wire has a circular mil area of 6,400, then its diameter is the square root of 6,400 or 80 mils. Fig. 13 shows a representation of a

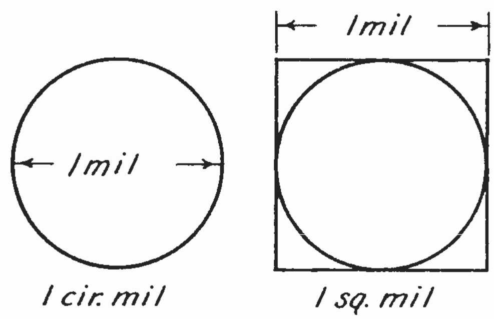

Fig. 13.—Relation of a circular to square mil.

circular mil, (not to scale because it would be too small), and also a square mil with a circular mil inscribed. It will be observed that the circular mil is smaller than the square mil. The relation between the two is 1 C.M. = .7854 S.M. Thus a wire of area 1,000 C.M. would be equal in area to a wire of area 785.4 S.M.; (.7854 × 1,000 C.M.). Similarly 1,000 S.M. = 1000/.7854 = 1/.7854 × 1000 = 1.273 × 1000 = 1273 C.M.

To convert C.M. to S.M. multiply by .7854.

To convert S.M. to C.M. multiply by 1.273.

Exercises

86. No. 8 Copper Wire has:
d = 128.49 mils
What is A in C.M.?

87. No. 20 Copper Wire has:
d = 31.961 mils
What is A in C.M.?

88. No. 38 Copper Wire has:
d = 3.965 mils
What is A in C.M.?

89. No. 28 Copper Wire has:
A = 159.79 C.M.
What is d in mils?

90. No. 14 Copper Wire has:
A = 4106.8 C.M.
What is d in mils?

91. A rectangular copper conductor has width of 1/2 inch and thickness of 3/16 inch. What is the cross-section area in circular mils? What would be the diameter of a round conductor containing the same copper area?

VOLUME

Besides the determination of lengths and areas, it is frequently necessary to determine the capacity or volume of an object, such as the volume of a box or bin, a can, a tank, etc. Volumes are measured in cubic units, as previously explained in "measurements". In practical work, the principal objects whose volumes must be determined are tanks, bins, cans, spheres, etc. The volume of a rectangular or square bin is obtained by the product of its three dimensions, as explained in the discussion of weights and measures. The other object of importance is the circular cylinder or can, the bottom of which is perpendicular to the upright wall. This volume is equal to the product of the height of the can times the area of the base. Such a cylinder is illustrated in Fig. 14.

The area of the base is determined by the formula

$$A = \pi r^2 = 3.14r^2$$

The area is then multiplied by the height and the answer is determined

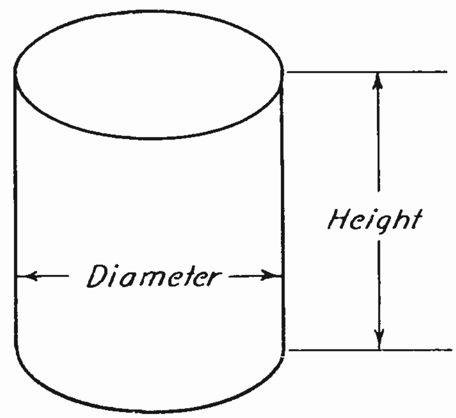

Fig. 14.—Circular cylinder, showing important dimensions.

in cubic units. As an illustration, the volume of a can 20 inches high and 6 inches in diameter is found as follows: The radius is one-half the diameter or 3 inches.

$$A = 3.14 \times 3^2 = 28.26 \text{ sq. in.}$$

$$\text{Volume} = 28.26 \times 20 \text{ (height)}$$
$$= 565.2 \text{ cu. in.}$$

The surface area of a sphere is given by the formula

$$\text{Area} = 4\pi r^2$$

or in terms of the diameter

$$\text{Area} = \pi d^2$$

The volume of a sphere is given by

$$V = \frac{4}{3}\pi r^3$$

or in terms of the diameter

$$V = \frac{1}{6}\pi d^3$$

If the diameter of the earth is 8,000 miles and it is taken as a sphere, the surface area is:

$$A = \pi d^2 = 3.14 \times 8000^2$$
$$= 201{,}000{,}000 \text{ sq. miles}$$

The volume of the earth is:

$$\frac{1}{6} \times 3.14 \times 8000^3 = 268 \times 10^9 \text{ cu. mi.}$$

GEOMETRICAL CONSTRUCTIONS

To erect a perpendicular to a line at a given point, the process is a follows: (See Fig. 15).

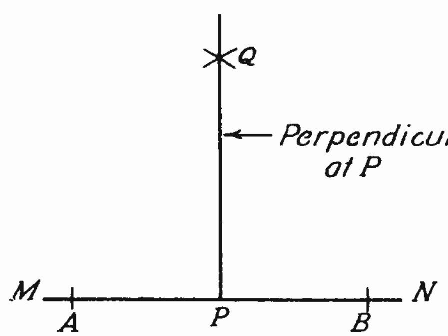

Fig. 15.—Geometrical constructions.

STEP 1: The perpendicular is to be erected at point P. With P as a center, take any convenient radius such as PA and lay off points A and B on the line MN.

STEP 2: With A and B as centers and with any convenient radius greater than either the distance PA or PB, describe arcs intersecting at point Q.

STEP 3: Draw a line between Q and P. This is the perpendicular required.

To draw a line perpendicular to a line from a point outside the line proceed as follows: (See Fig. 16.)

STEP 1: The line is MN and the point is P outside the line MN. With any convient radius greater than the shortest distance from P to MN, describe an arc intersecting MN at A and B.

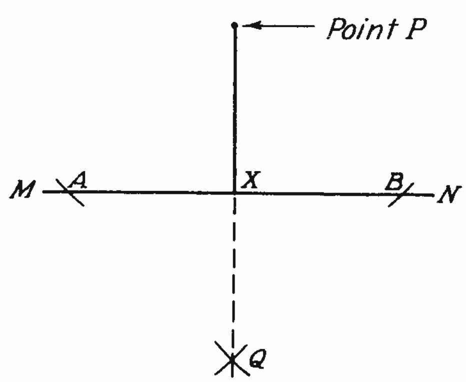

Fig. 16.—Another geometrical construction.

STEP 2: With A and B as centers, and with any radius greater than half the distance from A and B, draw arcs intersecting at Q.

STEP 3: Draw a line between P and Q intersecting the line MN at X. This line PX is the perpendicular required.

These are two very useful geometrical constructions and should be thoroughtly understood.

Exercises

92. The side of a square transmitter lot is 85 yards. What is the area in acres?

93. A antenna base in the shape of a right triangle has sides of 75 and 85 feet respectively. At $1.20 per square yard what will be the cost of paving the base?

94. The foot of a ladder is placed 12 feet from the base of an antenna. The upper end is 30 feet above ground. How long is the ladder?

95. An antenna guy wire 60 feet long is used with the bottom end 20 feet from the antenna base. How far outward must the end of the guy wire be moved to lower the upper end 5 feet?

96. If the diameter of a tree increases an average of 1/4 inch per year, how old is a tree whose girth is 18 feet 4 inches?

97. How many acres are covered by a quarter mile circular antenna ground mat?

98. The drivers of a locomotive are 5 feet in diameter. The tender wheels are 6.28 feet in circumference. Neglecting slippage how many more revolutions must the tender wheels make than the drivers in travelling 10 miles?

99. If a cubic foot of iron is formed into a rectangular rod 1" by 1/4" without waste, how long will the rod be?

100. A pyramid made up of 4 equilateral triangles has sides of 25 feet. At a cost of 25 cents per square yard, what is the cost of painting the pyramid?

101. What is the least common multiple of $4, $21, and $72?

102. Find the *lateral* area of a cylinder 3 feet high whose base is 9 inches in diameter.

103. A cylinder 5 inches long has a lateral area of 200 square inches. Find the radius of the base.

104. What is the surface area and volume of a sphere 4 inches in diameter?

TRIGONOMETRY

In this assignment only the fundamentals of trigonometry as applied to the right triangle will be discussed. Trigonometry is a powerful tool to the mathematician in solving alternating current problems and will be of considerable value in later assignments. The subject is easy once the basic idea is understood.

It has been explained previously that if two sides of a right triangle are given, the third side can be found from the Pythagorean theorem which states what the hypotenuse of a right triangle is equal to the square root of the sum of the squares of the two sides. In Fig. 17(a) is shown a right triangle with sides a, b, and c. If a = 3 units and b = 4 units then c, the hypotenuse, equals

$$\sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16}$$

$$= \sqrt{25} = 5$$

By simple transposition of the equation it is possible to find any side of a right triangle if two sides are known. The three forms in which the equation appears are:

$$c = \sqrt{a^2 + b^2}$$

$$a = \sqrt{c^2 - b^2}$$

$$b = \sqrt{c^2 - a^2}$$

Particular note should be taken of the method of labelling the triangle in Fig. 17. This is the standard triangle of trigonometry. Capitol letters A, B, and C are placed at the apex of the angles and are used as names in referring to the angles. Note that C always is placed at the apex of the right angle. Small letters a, b and c are used to designate the sides with side a being OPPOSITE angle A, side b OPPOSITE angle B, and c, the hypotenuse, opposite the right angle. Side b is said to be ADJACENT to angle A while side a is ADJACENT to angle B. This nomenclature is very important and must be memorized before studying the fundamentals of trigonometry.

In Fig. 17(a) suppose that only angles A, B, C and one side of the triangle are known. The other two sides of the triangle cannot be found by the Pythagorean formula because two sides must be known in order to determine the third side. Since various instruments such as the theodolite, the sextant, and others are available for measuring angles very accurately, it is very desirable to have some method of solving triangles that depends on the angles. Trigonometry is very important in the field or surveying, map making, and navigation as well as in engineering branches of science.

Consider Fig. 17(a) again and compare the ratios of the various sides of the triangle.

For angle A

$$\frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{a}{c} = \frac{3}{5}$$

$$\frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{b}{c} = \frac{4}{5}$$

$$\frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{a}{b} = \frac{3}{4}$$

For angle B

$$\frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{b}{c} = \frac{4}{5}$$

$$\frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{a}{c} = \frac{3}{5}$$

$$\frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{b}{a} = \frac{4}{3}$$

In Fig. 17(b) is shown a right triangle with the same angles as were used in 17(a) but with side a increased from 3 to 6 units. Sides b and c must also be increased to maintain the right triangle. By measurement b = 8 and c = 10. As a check:

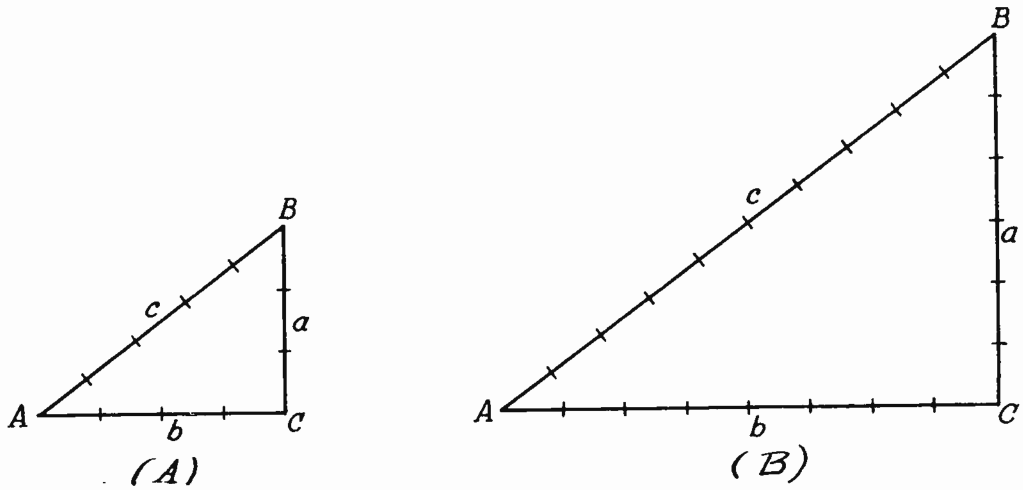

Fig. 17.—Right triangles and their trigonometrical relationships.

$c = \sqrt{a^2 + b^2} = \sqrt{6^2 + 8^2} = \sqrt{100} = 10.$

Now compare the ratios of the sides of the larger triangle. For angle A

$$\frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{a}{c} = \frac{6}{10} = \frac{3}{5}$$

$$\frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{b}{c} = \frac{8}{10} = \frac{4}{5}$$

$$\frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{a}{b} = \frac{6}{8} = \frac{3}{4}$$

For angle B

$$\frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{b}{c} = \frac{8}{10} = \frac{4}{5}$$

$$\frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{a}{c} = \frac{6}{10} = \frac{3}{5}$$

$$\frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{b}{a} = \frac{8}{6} = \frac{4}{3}$$

Note particularly that the ratios of the sides of the large and small triangles do not change with an increase or decrease in the length of the sides *if the angles all remain constant.* This is the fundamental point of trigonometry. No matter how large or how small the right triangle is made the ratios of the sides are constant if the angles are constant.

Assume that a third triangle similar to those in Fig. 17 is to be constructed with side a equal to 10. (The angles in similar triangles are identical). If side a = 10 and it is known that

$$\frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{a}{c} = \frac{3}{5}$$

then $$\frac{10}{c} = \frac{3}{5}$$

$$5 \times 10 = 3c$$

$$c = \frac{5 \times 10}{3} = \frac{50}{3} = 16.67$$

If c = 16.67 and it is known that

$$\frac{b}{c} = \frac{4}{5}$$

then

$$\frac{b}{16.67} = \frac{4}{5}$$

$$5b = 4 \times 16.67$$

$$b = \frac{4 \times 16.67}{5} = 13.34$$

Since $c = \sqrt{a^2 + b^2}$

$$c = \sqrt{10^2 + 13.34^2}$$

$$= \sqrt{100 + 178}$$

$$= \sqrt{278}$$

$$c = 16.67$$

This checks with the calculated value for c. A further check is

$$\frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{a}{b} = \frac{10}{13.34} = \frac{3}{4}$$

The solution can also be obtained if angle B instead of angle A is used as a reference. The student should perform this second solution.

If a table is calculated to show the ratios of the sides of a right triangle for all angles from 0 to 90 degrees, then the length of the sides of any right triangle as well as all the angles can be calculated, if one acute angle and one side are known.

In order to facilitate mathemathical calculations, the ratios of the sides of a right triangle have been given abbreviated names. These are shown in Table 1 and should be memorized. The letters refer to the angles and sides of the standard triangle.

The sides of a triangle are small case letters and the opposite angles are large case letters. The hypotenuse is the largest side and is always opposite the 90° angle.

At the end of this assignment is shown a trigonometric table of sines, cosines, and tangents in steps of 1 degree from 9° to 90°.

TABLE I

	Written
Sine A	$\text{Sin } A = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{a}{c}$
Cosine A	$\cos A = \frac{\text{adjacent side}}{\text{Hypotenuse}} = \frac{b}{c}$
Tangent A	$\text{Tan } A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$
Sine B	$\text{Sin } B = \frac{\text{opposite side}}{\text{Hypotenuse}} = \frac{b}{c}$
Cosine B	$\text{Cos } B = \frac{\text{adjacent side}}{\text{Hypotenuse}} = \frac{a}{c}$
Tangent B	$\text{Tan } B = \frac{\text{opposite side}}{\text{Adjacent side}} = \frac{b}{a}$

Examples of how the table is used are shown below.

1. Side b of a standard right triangle equals 45 units and angle A = 30°. Find the lengths of sides a and c and angle B.

$$b = 45$$

$$A = 30^\circ$$

By reference to Table 1 it is noted that:

$$\text{Cos } A = \frac{b}{c}$$

From the trigonometric table, the

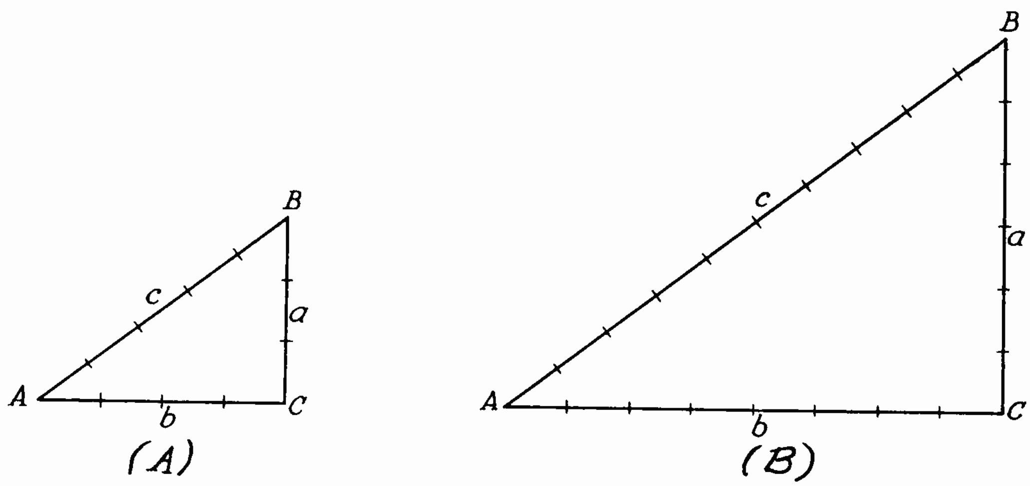

Fig. 17.—Right triangles and their trigonometrical relationships.

cosine of 30° is .866 so:

$$\text{Cos } 30^\circ = \frac{b}{c}$$

$$.886 = \frac{45}{c}$$

$$.866c = 45$$

$$c = \frac{45}{.866} = 52$$

If b = 45 and c = 52 side a can be found from the formula

$$a = \sqrt{c^2 - b^2}$$

However using the trigonometric table

$$\text{Tan } A = \frac{a}{b}$$

A = 30° and Tan 30° = .577 from table.

$$.577 = \frac{a}{45}$$

$$a = .577 \times 45 = 26$$

Since $c = \sqrt{a^2 + b^2}$

Then $c = \sqrt{26^2 + 45^2} = \sqrt{676 + 2025}$

$$= \sqrt{2701} = 52$$

Angle B is easily found from the rule that the sum of all angles in any triangle is 180°. Angle c = 90, Angle A = 30° so:

$$\text{Angle B} = 180 - 90 - 30 = 60^\circ$$

In a right triangle angles A and B are always complementary, that is, their sum equals 90°.

2. Angle B = 50° and c = 140 Find sides a and b.

The cosine of B involves the ratio of a to c. So;

$$\text{Cos } B = \frac{a}{c}$$

$$\text{Cos } 50^\circ = \frac{a}{140}$$

Cos 50° = .643 from table.

$$.643 = \frac{a}{140}$$

$$a = 140 \times .643 = 90$$

$$\text{Tan } B = \frac{b}{a}$$

$$\text{Tan } 50° = \frac{b}{90}$$

Tan 50° = 1.192 from table.

$$1.192 = \frac{b}{90}$$

$$b = 90 \times 1.192 = 107$$

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{107^2 + 90^2} = \sqrt{19549} = 140 \text{ check}$$

The sines, cosines and tangents are said to be functions of the angle because they depend entirely on the angle for their numerical value. *In solving any right triangle from given values, it is only necessary to select a function of the angle which will permit setting up an equation that will have only one unknown.* The sides of a triangle are small case letters and the opposite angles are large case letters. The hypotenuse is the largest side and is always opposite the 90° angle. The student should strive to think of sines, cosines and tangents as nothing more than ratios of the sides of a triangle using some angle as a reference point.

There are other functions of the angles in addition to those explained in this assignment, but they are of little importance at this time. A more detailed discussion of trigonometry will be taken up in a later assignment.

Exercises

105. Angle A = 20°
c = 40
Find a and b.

106. a = 75, c = 150
Find angles A, B, and side b.

107. A = 30°, b = 65
Find angles B and sides a and c.

108. c = 600, Angle B = 45°
Find a, b and angle A.

109. Angle B = 75°, a = 210
Find b, c and angle A.

110. A tree casts a shadow 100 feet long. The angle between the earth and a line of sight from the end of the shadow to the top of the tree is 37°. How high is the tree?

111. From a tower 100 feet high on the bank of a river the angle of depression of the opposite bank is 23°. How wide is the river?

112. To find the height of a radio tower a surveyor paces off a distance of 300 feet and sets up his transit. He levels the transit and then measures the angle of elevation to the top of the tower. This measures 65°. If the transit telescope is 5 feet above ground how high is the tower?

113. How high is a radio tower if the length of the tower shadow is 200 feet and the sun's rays are striking the earth at an angle of 61°?

114. A 20-foot ladder on level earth is placed against a vertical wall so that the base of the ladder makes an angle of 53° with the earth. If the base of the ladder is moved 2.5 feet further from the wall, what angle would it make with the surface of the earth?

TABLE OF SINES, COSINES AND TANGENTS

Angle	Sin	Cos	Tan	Angle	Sin	Cos	Tan
0.	.000	1.00	.0000	46.	.719	.695	1.035
1.	.0175	.9998	.0175	47.	.731	.682	1.072
2.	.0349	.9994	.0349	48.	.743	.669	1.111
3.	.0523	.9986	.9524	49.	.755	.656	1.150
4.	.0698	.998	.0699	50.	.766	.643	1.192
5.	.0872	.996	.0875	51.	.777	.629	1.235
6.	.105	.995	.105	52.	.788	.616	1.280
7.	.122	.993	.123	53.	.799	.602	1.327
8.	.139	.990	.141	54.	.809	.588	1.376
9.	.156	.988	.158	55.	.819	.574	1.428
10.	.174	.985	.176	56.	.829	.559	1.483
11.	.191	.982	.194	57.	.839	.545	1.540
12.	.208	.978	.213	58.	.848	.530	1.600
13.	.225	.974	.231	59.	.857	.515	1.664
14.	.242	.970	.249	60.	.866	.500	1.732
15.	.259	.966	.268	61.	.876	.485	1.804
16.	.276	.961	.287	62.	.883	.469	1.881
17.	.292	.956	.306	63.	.891	.454	1.963
18.	.309	.961	.325	64.	.899	.438	2.050
19.	.326	.946	.344	65.	.906	.423	2.145
20.	.342	.940	.364	66.	.914	.407	2.24t
21.	.358	.934	.384	67.	.921	.391	2.356
22.	.375	.927	.404	68.	.927	.375	2.475
23.	.391	.921	.424	69.	.934	.358	2.605
24.	.407	.914	.445	70.	.940	.342	2.747
25.	.423	.906	.466	71.	.946	.326	2.904
26.	.438	.899	.488	72.	.951	.309	3.078
27.	.454	.891	.510	73.	.956	.292	3.271
28.	.469	.883	.532	74.	.961	.276	3.487
29.	.485	.875	.554	75.	.966	.259	3.732
30.	.500	.866	.577	76.	.970	.242	4.011
31.	.515	.857	.601	77.	.974	.225	4.331
32.	.530	.848	.625	78.	.978	.208	4.705
33.	.545	.839	.649	79.	.982	.191	5.145
34.	.559	.829	.675	80.	.985	.174	5.671
35.	.574	.819	.700	81.	.988	.156	6.314
36.	.588	.809	.727	82.	.990	.139	7.115
37.	.602	.799	.754	83.	.993	.122	8.144
38.	.616	.788	.781	84.	.995	.105	9.514
39.	.629	.77	.810	85.	.996	.0872	11.43
40.	.643	.766	.839	86.	.998	.0698	14.30
41.	.656	.755	.869	87.	.9986	.0523	19.08
42.	.669	.743	.900	88.	.9994	.0349	28.64
43.	.682	.731	.933	89.	.9998	.0175	57.29
44.	.695	.719	.966	90.	1.000	.000	Infinite
45.	.707	.707	1.000				

ANSWERS TO EXERCISES

1. $2abc$
   $a + 2b + c$
2. $72abc$
   $3a + 4b + 6c$
3. $4abcd$
   $a + b + c + 4d$
4. $\frac{abcd}{120}$

   $\frac{a}{2} + \frac{b}{3} + \frac{c}{4} + \frac{d}{5}$
5. $9a^2bcd$
   $1.5ab + 4cd + 1.5a$
6. 48, 12.
7. 1728, 42.
8. 480, 29.
9. 1, 4.
10. 2160, 92.
11. xxxx
12. yyy
13. 2aa
14. 17aab
15. abbccc
16. xxyy
17. xxz
18. 1
19. ay
20. ab^2
21. 65.9
22. 11.9
23. 20.6
24. 72
25. 20
26. 8
27. 17
28. 13
29. 3
30. 12
31. 125
32. 28
33. 66
34. 50
35. 54
36. $-a - b + ac + bc$
37. $a - ab - b^2$
38. $-2a$
39. $-2a$
40. $a + b - c - d$
41. a^2bc
42. $2x^2z - 2yz + 2z$
43. 90
44. $10xy + 2xyz + 8z$
45. $2a - 3b$
46. $x = 16$
47. $x = 27$
48. $x = 18$
49. $a = 18$
50. $b = -2$
51. $x = -6$

52. x = 13

53. x = 3

54. x = 2

55. x = 24

56. x = 10

57. x = -16

58. x = 14

59. $P = E^2/R$

60. $I = \sqrt{P/R}$

61. x = 4

62. x = 12 1/3

63. 16, 18, 20.

64. 18 by 6 ft.

65. 3 $5 Bills
6 $10 Bills

66. h = 25
P = 60
A = 150

67. h = 26
P = 60
A = 120

68. h = 192
P = 462
A = 9,000

69. b = 92.2
P = 262.2
A = 2,766

70. a = 61.8
P = 146.8
A = 618

71. a = 180
P = 720
A = 21,600

72. b = 62.4
P = 192.4
A = 1,560

73. h = 186
P = 446
A = 8,250

74. b = 24
h = 31.2

75. a = 20
h = 26.9

76. r = 7.5
c = 47.1
A = 176.6

77. d = 40
c = 125.6
A = 1,256

78. d = 240
c = 753.6
A = 45,216

79. r = 39.8
d = 79.6
A = 4,974

80. r = 31.9
d = 63.7
A = 3,195

81. r = 25.2
d = 50.4
c = 158

82. r = 15.9
d = 31.8
c = 100

83. r = 60
c = 377
A = 11,304

84. 65.94

85. 176.6

86. 16,510 C.M.

87. 1021.5 C.M.

88. 15.72 C.M.

89. 12.64 mils

90. 64.08 mils

91. *119,344 C.M.*, 345 *mils*

92. 1.5 acres

93. $425

94. 32.3 feet

95. 10.78 feet

96. 280 years

97. 3.18 acres

98. 5,045 revs.

99. 576 feet

100. $30

101. $504

102. 7.07 sq. ft.

103. 6.38 inches

104. 50.24 sq. in.
33.49 cu. in.

105. a = 13.68
b = 37.6

106. A = 30°
B = 60°
b = 130

107. B = 60°
a = 37.5
c = 75

108. a = 424.2
b = 424.2
A = 45°

109. b = 783.7
c = 811.3
A = 15°

110. 75.4 feet

111. 236 feet

112. 648.5 feet

113. 361 feet

114. 43°

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TECHNICAL ASSIGNMENT

MENSURATION

EXAMINATION

Show all work:

1. (A) Express the product of: c, x, 5, x^2y, 3cz.

$15c^2x^3yz$

(B) Express the sum of: 12, 3a, b, 2a, -5.

$5a + b + 7$

(C) If a = 10, x = 7, y = 15, z = 20:

$3a + \left[x^2 - (2y - z)\right] = 30 + [49 - (30 - 20)]$

$= 30 + [49 - 10] = 30 + 39 = \underline{\underline{69}}$

2. (A) Solve for x:

$2x + 5 = 23$

$2x + 5 - 5 = 23 - 5$

$2x = 18 \qquad x = \underline{\underline{9}}$

(B) Solve for b:

$16 = b^2 - 9$

$16 + 9 = b^2 - 9 + 9$

$b^2 = 25 \qquad b = \underline{\underline{5}}$

3. (A) Solve for x:

$\frac{2x}{3} - \frac{x}{2} = x + 10$

$\frac{4x - 3x}{6} = x + 10$

$4x - 3x = 6x + 60$

$4x - 3x - 6x = 60$

$-5x = 60$

$x = \underline{\underline{-12}}$

EXAMINATION, Page 2.

3. (B) Solve for a:

$y = 2a(b + c)$

$$\frac{Y}{b+c} = 2a \qquad a = \frac{Y}{2(b+c)}$$

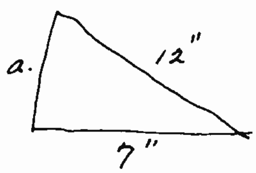
4. An ohmmeter and a voltmeter together cost $37.50. The ohmmeter cost $13 more than the voltmeter. Find the cost of each.

Let x = Cost of Voltmeter
and x + $13 = Cost of ohm-meter.

$$2x + \$13 = \$37.50$$
$$2x = \$37.50 - \$13 = \$24.50$$
$$x = \frac{\$24.50}{2} = \$12.25 \text{ Voltmeter}$$
$$x + 13 = \$12.25 + \$13 = \$25.25 \text{ ohm-meter}$$

5. A metal mounting bracket used to support the chassis in a television receiver is in the form of a right triangle, whose hypotenuse is 12 inches, and whose base is 7 inches.

(A) Find the altitude of the bracket.



$$a = \sqrt{12^2 - 7^2} = \sqrt{144 - 49}$$
$$= \sqrt{95} = 9.75 \text{ in}$$

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MENSURATION

EXAMINATION, Page 3.

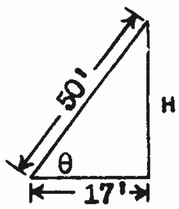
5. (B) Find the perimeter.

per = 12 + 7 + 9.75 = 28.75 in

(C) Find the area.

$A = \frac{7 \times 9.75}{2} = 34.125$ sq. in.

6.



A 50-foot ladder is placed against the wall of a television transmitter building. In order to form a secure setup, the base of the ladder is placed 17 feet from the building wall.

(A) What is the angle θ between the ladder and the level ground?

$\cos\theta = \frac{17}{50} = 0.34$.

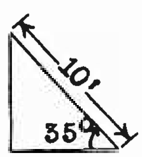
From table $\theta = 70^\circ$

(B) What is the height h of the top of the ladder above the ground?

$\tan 70^\circ = 2.747$

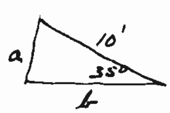
$2.747 = \frac{H}{17}$ $H = 2.747 \cdot 17 = 46.7$ ft. (Slide rule)

7.



In a triangular piece of television studio scenery (having one right angle), it is desired to put a piece of molding around its perimeter and to paper its surface. The hypotenuse of the piece is 10 feet, and one of its acute angles is 35 degrees.

(A) What is the total length of molding required?



$\cos 35^\circ = 0.819 = \frac{b}{10}$ $b = 0.819 \cdot 10 = 8.19$ ft.

$\sin 35^\circ = 0.574 = \frac{a}{10}$ $a = 0.574 \cdot 10 = 5.74$ ft

Length of moulding = 10 + 8.19 + 5.74 = 23.93 ft.

7. (B) How many square feet of wall paper are required (ignoring waste)? $\frac{8.19 \cdot 5.74}{2} = 23.5$ sq. ft.

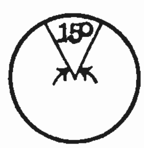
8. In a miniature studio scene, it is desired to represent a huge flywheel by a wooden disc 30 inches in diameter.

(A) What is the formula to find the area of the disc?

$A = \frac{\pi D^2}{4} = \pi R^2$

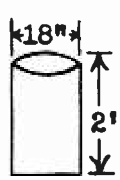
(B) What is the area of the disc?

$A = \pi R^2 = 3.14 \cdot (1.25)^2 = 4.9$ sq. ft



(C) Find the area of a 15° sector of the disc.

Area of sector $= \frac{15}{360} \cdot 4.9 = 0.204$ sq. ft

9. 

In another miniature set a silo is to be represented by a round metal container of cylindrical shape, having a flat top and bottom, and having a diameter of 18 inches and a height of 2 feet.

(A) What is the formula for finding the volume in cubic inches? $\pi R^2 h$.

(B) What is the volume in cubic inches? 6050 Cu. in

$h = 24$

$R = 9$

$\pi R^2 h = 3.14 \cdot 81 \cdot 24 = 6050$

6104 cu in

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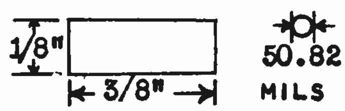
MENSURATION

EXAMINATION, Page 5.

10. A rectangular copper bus-bar (conductor) used to feed 60-cycle power to a transmitter has accidentally burned out, and as an emergency repair it is desired to substitute temporarily several strands of No. 16 wire. The bus-bar has a width of 3/8 inch and a thickness of 1/8 inch. No. 16 wire has a diameter of 50.82 mils.



(A) What is the diameter of the No. 16 circular wire in decimal parts of an inch? 0.05082

(B) What is the corresponding cross-sectional area of the No. 16 wire?

$= (50.82)^2 = 2582.7$ C.M.

(C) What is the cross-sectional area of the rectangular bus bar?

$\frac{1}{8}$ in = 125 mils

$\frac{3}{8}$ in = 375 mils

A in cm = 375 • 125 / 0.7854 = 59683 C.M.

(D) How many strands of No. 16 wire will be required to replace the rectangular conductor, the total cross section area of the stranded wire to be not less than that of the rectangular conductor?

No. of strands $= \frac{59683}{2582.7} = 23.15$

or 24 strands