Book #3

SPECIALIZED TELEVISION ENGINEERING

TELEVISION TECHNICAL ASSIGNMENT

GEOMETRY - TRIGONOMETRY

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TELEVISION TECHNICAL ASSIGNMENT

GEOMETRY—TRIGONOMETRY

FOREWORD

The average radioman, thinking back to his high school study of Geometry, remembers many hours spent in memorizing proofs of theorems. In the majority of cases the only thing that really "stuck" was how to find the hypotenuse of a right triangle. This is something he could learn in ten minutes—and fortunately is the basis of almost all he has to know about Geometry to handle most practical electrical and radio problems. Alternating current problems are solved by means of "vectors" which in turn represent such quantities as current, voltage, reactance, resistance and impedance. All vector solutions may be broken down into right triangles, expressed in simple algebraic form, and solved by the simplest applications of Geometry.

The geometric solution is further simplified by the use of Trigonometry which, in its application to electrical and radio problems, is one of the simplest of the mathematical processes. Trigonometry deals with angles and "functions" of angles. A function of an angle is simply a numerical ratio or relationship between two sides of a right triangle which includes the angle under consideration. If the angle is known, the function is found from a set of tables. If the function is known, the angle is likewise found from the tables.

These terms are directly applicable to electrical problems. In an alternating current circuit, the current may *lead* or *lag* the voltage, depending upon whether the circuit is capacitive or inductive. The power in such a circuit is, $P = EI \cos \theta$. Cos is an abbreviation for cosine, one of the trigonometric functions. θ = angle of lead or lag. Thus the power is found by multiplying the voltage by the current and then by the Cosine of the angle of lead or lag. If the angle is known, the Cosine is found by reference to a set of tables. If the switchboard contains voltmeter, ammeter and wattmeter, Cosine of the angle of lead or lag is found simply by $\cos \theta = P/EI$. From $\cos \theta$, θ is found by reference to the tables.

This assignment is highly practical. It is not at all difficult, and it will lead you directly into the practical solutions of radio and television problems.

E. H. Rietzke,
President,

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INTRODUCTION

In the study of geometry the conventional method of proving the various geometric theorems has been discarded. The average student, on looking back several years on his study of geometry, finds that his workable useful knowledge of the subject consists mainly of the ability to find the hypotenuse of a right triangle. Geometry, as applied to electrical problems, can be very easily mastered if the principles and applications of the right triangle are thoroughly understood.

Geometry, as applied to electrical and radio problems, deals with the solution of two or more forces (also voltages, currents, or impedance) which may be considered as acting at varying angles upon a common point. The addition of these forces, to find the total resultant force, is called geometric addition. Right-angle solutions are particularly applicable to radio and electrical problems because the angular difference between the effects of circuit capacity and resistance, and inductance and resistance, is 90°. Such effects *must* be considered when dealing with the relations between voltages, currents, and impedances, in series and parallel radio frequency circuits.

GEOMETRY

ADDITION.—Arithmetic, Algebraic, Geometric: the various forms of addition include: First, arithmetic addition, in which no cognizance is taken of negative numbers; second, algebraic addition which recognizes both positive and negative numbers thus allowing the addition of forces, or values, acting in the same direction or in exact opposition; third, geometric addition which permits the addition of forces acting other than in the same direction or in exact opposition. Just as algebraic addition overlaps arithmetic addition, so does geometric addition overlap both arithmetic and algebraic addition.

For example, consider two forces, A equal to 8 and B equal to 10, acting on a point o. See Fig. 1.

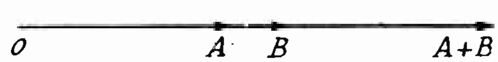


Fig- 1.—**Arithmetic addition.**

In this case the arithmetic, algebraic and geometric sums are the same and point o will move toward the right under the influence of the total force, A + B = 18. This is easily apparent because both A and B are acting on the common point in the same direction.

For example, consider two forces A and B acting on a point o. See Fig. 2.

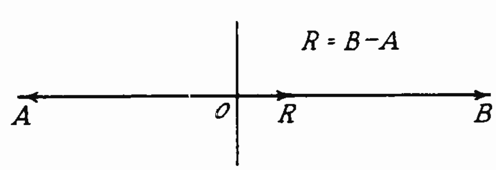


Fig. 2.—**Algebraic addition.**

A is equal to 8, B is equal to 10. The two forces, A and B, are acting on the common point in exact opposition and the total resulting force will be their algebraic sum (taking into consideration the difference in direction) which is equal to 2. In other words, the tendency will be for point o to move in the direction of the greater force due to the resulting force which is the arithmetic difference between the two forces. In this case the geometric sum equals the algebraic sum.

TRUE GEOMETRIC ADDITION.—Now consider an example of two forces acting on a common point at right angles to each other. (A right angle = 90°.) In Fig. 3, it is shown that since the two forces,

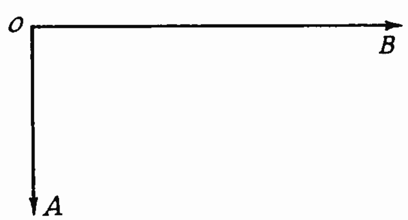


Fig. 3.—Two forces at right angles.

A and B are acting on point o in neither the same direction nor in exact opposition, both arithmetic and algebraic addition must be discarded as a solution for the total resulting force.

Point o will tend to move vertically due to force A. It will also tend to move horizontally to the right due to force B. Due to the combined influence of both forces, the point will move in a direction *between* forces A and B, the greater force having the greater effect on the direction of movement. If the two forces were of equal value the movement would be in a direction midway between the two. Neither arithmetic nor algebraic addition will give the resultant of these two forces. It is necessary therefore to make use of true geometric addition which is based on the solution of the right triangle.

The right triangle consists of two sides at right angles to each other, the extremes connected by a straight line called the hypotenuse. This is shown in Fig. 4. The hypotenuse must *always* be longer than either of the other two sides.

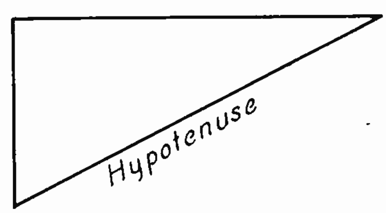


Fig. 4.—Right triangle showing hypotenuse.

It has been found that the *resultant of two forces acting at right angles on a common point is equal to the hypotenuse of a right triangle, the other two sides of which are the forces in question.* See Fig. 5.

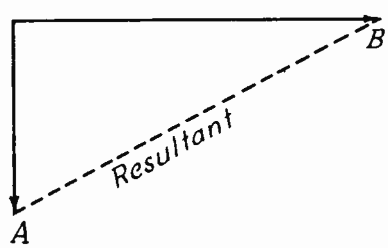


Fig. 5.—Resultant of two forces at right angles, by Rule 1.

RULE 1: *The hypotenuse of a right triangle is equal to the square root of the sum of the squares of the other two sides.*

This is the basic rule of geometric addition. Applying this rule to the solution of two forces acting on a point at right angles,

$$\text{Resultant} = \sqrt{A^2 + B^2}$$

Making A equal 3 and B equal to 4, the equation becomes,

$$\text{Resultant} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16}$$

$$= \sqrt{25} = 5$$

The total force acting on point o is 5 and the point will tend to move in a direction somewhere between A and B.

Use the two sides, A and B, as a base and from them form a rectangle. (See Fig. 6.) Since this is a parallelogram A = D and B = C.

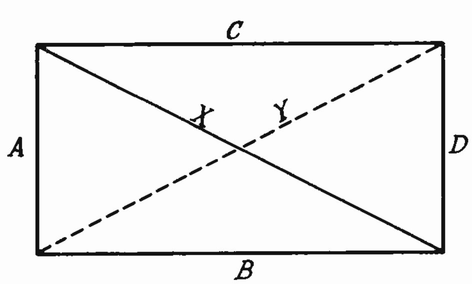


Fig. 6.—Forming a parallelogram by Rule 2.

Two right triangles can now be formed by drawing diagonals X and Y. One triangle will have sides A and C and hypotenuse Y while the other will be formed by sides B and D and hypotenuse Y. If A = D and C = B the two right triangles will be identical and X will equal Y.

RULE 2: *The resultant of two forces acting on a point and separated by any angle is equal to a straight line drawn between the two opposite corners of a parallelogram, of which the two forces form adjacent sides, one of the opposite corners being the point of intersection of the two forces.* (A parallelogram is a geometric figure having four sides in which opposite sides are equal and parallel. If the parallelogram encloses four 90° angles it is called a rectangle. If all enclosed angles are 90° and all sides are equal it is called a square.)

The straight line drawn from the point on which the two forces are acting to the opposite corner of the parallelogram indicates the direction and force of the resulting movement. The direction of the resultant force will *always* be *between* the two individual forces and the angle may be measured with a protractor.

When the two forces are equal and are acting at right angles on a point, the direction of the resulting movement will be half-way between the two, or forty-five degrees from the direction of either force.

Example: A = 5, B = 5. (Apply Rules 1 and 2) Fig. 7 applies to this example.

$$\text{Resultant} = \sqrt{A^2 + B^2} = \sqrt{5^2 + 5^2}$$

$$= \sqrt{50} = 7.07$$

$$\text{Resultant} = 7.07$$

Since forces A and B are equal, both exert an equal influence on point o and the direction of movement will be between forces A and B and forty-five degrees from each.

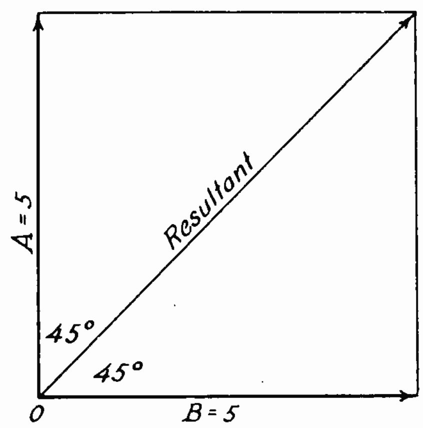


Fig. 7.—Rectangle using equal forces.

Example: A = 5, B = 10. See Fig. 8.

$$\text{Resultant} = \sqrt{5^2 + 10^2} = \sqrt{25 + 100}$$

$$= \sqrt{125} = 11.2$$

In this example the resultant is more nearly in the direction of B than in the direction of A. This is due to the fact that force B is greater than force A and therefore

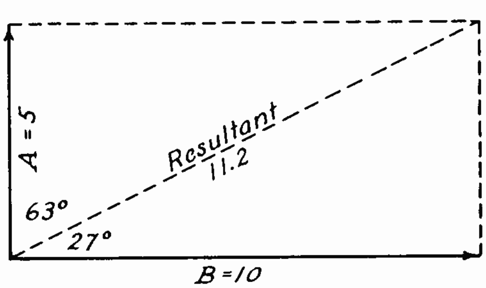


Fig. 8.—Rectangle using unequal forces.

has a greater influence on the direction of movement. The angle from the resultant in this instance measures 27° to force B, and 63° to force A.

Addition of Forces Not at Right Angles.—The preceding examples have assumed that the angles between the two forces were 90° or right angles. But referring to Rule 2, it will be seen that the rule specifically refers to *any* angle. In the discussion of Rule 1, it is stated that the basis for geometric addition is the right triangle. Taking both rules into consideration, it will be seen by Rule 2, that if two forces acting on a point are separated by *any* angle, it is possible to draw the parallelogram to scale and obtain the resultant by measuring the straight line between the point on which the two forces are acting and the opposite corner. For example, see Fig. 9.

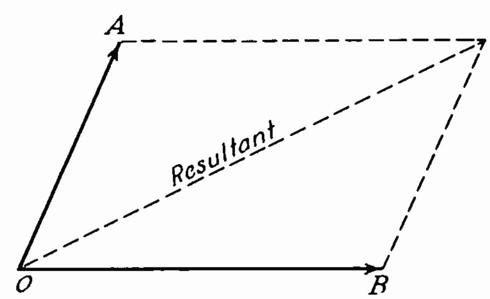


Fig. 9.—Parallelogram without 90° angles.

In this example the parallelogram can be drawn to scale and the resultant found by measurement. However, Rule 1 cannot be applied to find the resultant from A and B, as the resultant is not the hypotenuse of a right triangle formed by the sides A and B, the angle in this case being less than ninety degrees.

Since it is very seldom convenient to draw all problems to scale by means of a protractor, and since such measurements are not accurate unless the drawing is made on a large scale, some means must be found for handling such problems mathematically. Further, since it is desired to use true geometric addition as stated in Rule 1, it is necessary to arrange the problem in the form of a right triangle.

Taking a horizontal line as a base and cutting it at right angles by a vertical bisector a background of coordinates can be formed for use as a reference in order to resolve the problem into the form of a right triangle.

In Fig. 10, force A is acting on the point of intersection o of

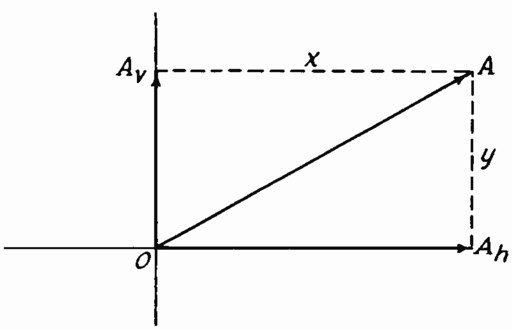


Fig. 10.—Finding components of force A.

the horizontal and vertical lines and parallel with neither. From the extreme end of A draw a horizontal line x to intersect the vertical bisector. Also drop a vertical line y to intersect the horizontal base line. It will now be seen that force A represents the hypotenuse of a right triangle the other two sides of which are A_v and A_h. A_v and A_h are called respectively the vertical and horizontal components of A, and added geometrically their resultant will equal A. The single force A may therefore be replaced with the two forces, A_v and A_h, acting at right angles to each other, and the same resultant force and direction obtained as were obtained directly from force A.

Using the same horizontal and vertical bisectors as a base, apply force B acting on the same point o but at a different angle. (See Fig. 11.) As in the diagram for force A, draw a horizontal q and a vertical p from the extremity of B. These

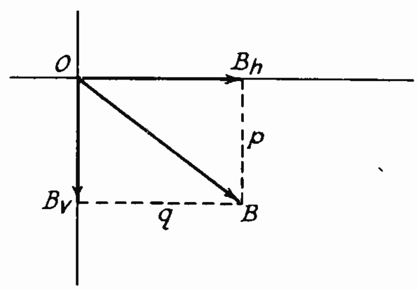


Fig. 11.—Finding components of force B.

lines intersect the vertical and horizontal base making B equal to the hypotenuse of a right triangle of which the other two sides are B_v and B_h. It is evident that force B could be replaced by its two components, B_v and B_h and the same results obtained.

If Figs. 10 and 11 are combined showing forces A and B acting on the same point, and showing also their respective horizontal and vertical components, Fig. 12 will result.

If the two original forces, A and B, are replaced with the component forces of each, there will be four forces, A_v, B_v, A_h and B_h, acting on point o instead of forces A and B. A_h and B_h are acting on point o in the same direction, along the base line to the right of the vertical bisector. Their effects therefore add algebraically and the total force in the horizontal plane is $A_h + B_h = H$. A_v and B_v are acting on point o in *opposite* directions along the vertical bisector. A_v tends to move the point upward; B_v tends to move the point below the base line. Their effects are opposite, the total effect therefore being the algebraic sum, $A_v - B_v = V$.

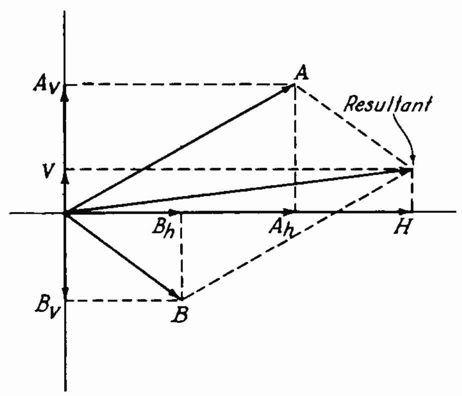


Fig. 12.—Combining forces A and B.

The four forces have again been resolved into two forces, *but* the two forces, H and V are *at right angles* to each other. The first rule for geometric addition will therefore apply and the total resulting force is equal to

$$\sqrt{H^2 + V^2}$$

Leaving the values of H and V in the form of their component parts, the equation for the solution of the two forces, A and B, becomes,

$$\text{Resultant} = \sqrt{(A_h + B_h)^2 + (A_v - B_v)^2}$$

Referring to Fig. 12, it will be seen that the resultant forms the hypotenuse of a right triangle, the other two sides of which are H and V. It will also be noted that if a parallelogram is drawn from the two forces, A and B, the diagonal, starting at point o and extending to the opposite corner, will coincide with the hypotenuse of a right triangle formed by H and the side opposite V. This indicates that Rules 1 and 2 combined provide the solution of two forces acting on a point at any angle with respect to each other. The use of the horizontal and vertical background permits an accurate statement of the direction of the resulting force with respect to some fixed point. This direction is usually expressed as the number of degrees in the angle starting at the base line to the right of the bisector and moving in a counterclockwise direction. This is plainly shown in Figs. 13 (A) and 13 (B).

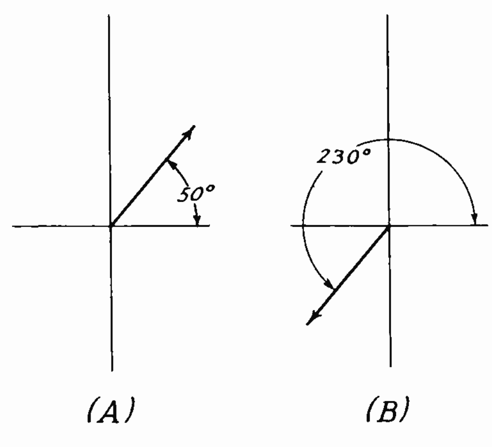


Fig. 13.—Vector force indicated by angle with base line.

Addition of Several Forces.— The resultant of any number of forces may be found by the same method as

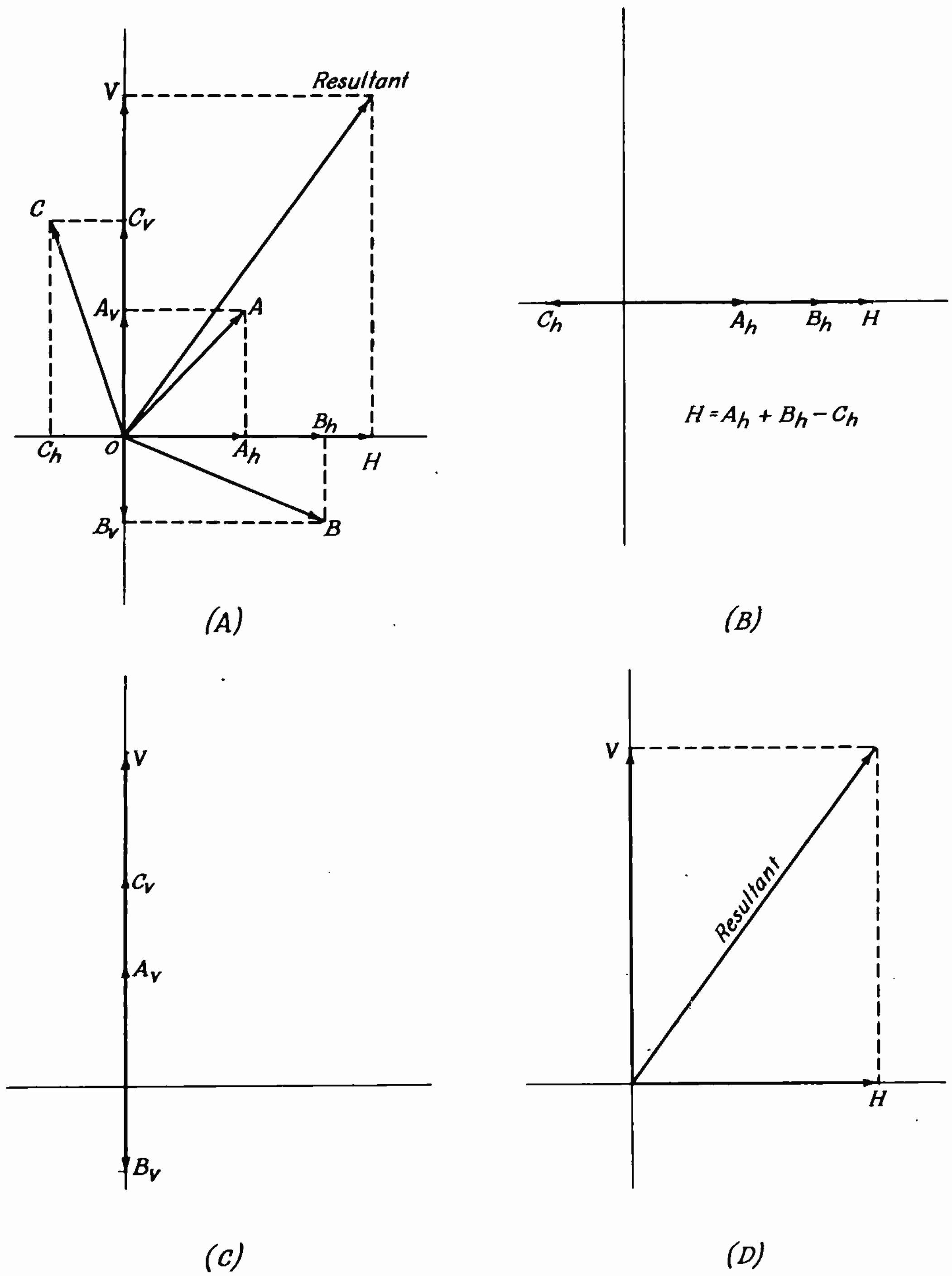


Fig. 14.—Combining three forces by geometric addition.

was used to find the resultant of forces A and B.

Figs. 14, 14(A), 14(B) and 14(C) show three forces, A, B and C, and the horizontal and vertical components of each. It will be seen that there are six component forces, three on the horizontal or base line and three on the vertical bisector. Of the horizontal components those of A and B are to the right of the bisector, therefore adding. C_h is to the left of the bisector in the opposite direction of A_h and B_h and must be subtracted from the sum of the latter, making the total horizontal component $H = A_h + B_h - C_h$. The vertical components A_v and C_v, being above the line, add. The other vertical component, B_v, acting in the opposite direction, subtracts, and the total vertical component becomes $V = A_v + C_v - B_v$. The three forces A, B and C, have, by means of their component forces, been resolved into two forces, at right angles to each other, which may now be added geometrically as

$$\text{Resultant} = \sqrt{H^2 + V^2}$$

Substituting for H and V their component parts, the equation becomes

$$\text{Resultant} = \sqrt{(A_h + B_h - C_h)^2 + (A_v + C_v - B_v)^2}$$

Any number of forces may be added in this manner, remembering that each force must be resolved into two components which act at right angles to each other, and that when the components are in the same direction they add arithmetically, and when in opposite directions they subtract.

In the solution of this type of problem a diagram should first be drawn showing the forces to be added and their various angles. Then an equation should be written as illustrated for the problem of Fig. 14 and following sketches. All that then remains is to find the components of the various forces, substitute those figures in the equation, and solve according to the equation. If the force and one component are known, the other component can be found by rearranging the equation to suit. For example, consider force A and its components, A_h and A_v.

$$A = \sqrt{A_h^2 + A_v^2}$$

$$A_h = \sqrt{A^2 - A_v^2}$$

$$A_v = \sqrt{A^2 - A_h^2}$$

If the force and the angle only are known, the only way to find the components, with the methods so far studied, is to draw the problem to scale with a protractor. It has already been explained why this is unsatisfactory. Simple trigonometry furnishes the additional processes needed to solve this type of problem easily and accurately. By knowing the value of the force and its angle it is possible, by means of trigonometry, to find either or both of the components. Or by knowing one of the components and the angle, the force and its other component may be determined.

Some method must be used to intelligently indicate the directions of the forces that are acting on a point. If the actual angle of a force with respect to the base line is not known, the general direction of the force can be indicated by stating the quadrant in which it lies. A quadrant is a quarter circle, or a ninety degree portion of a circle. In working takes in from 180° to 270° and the fourth from 270° to 360°. This is shown in Fig. 15.

Most geometrical errors can be

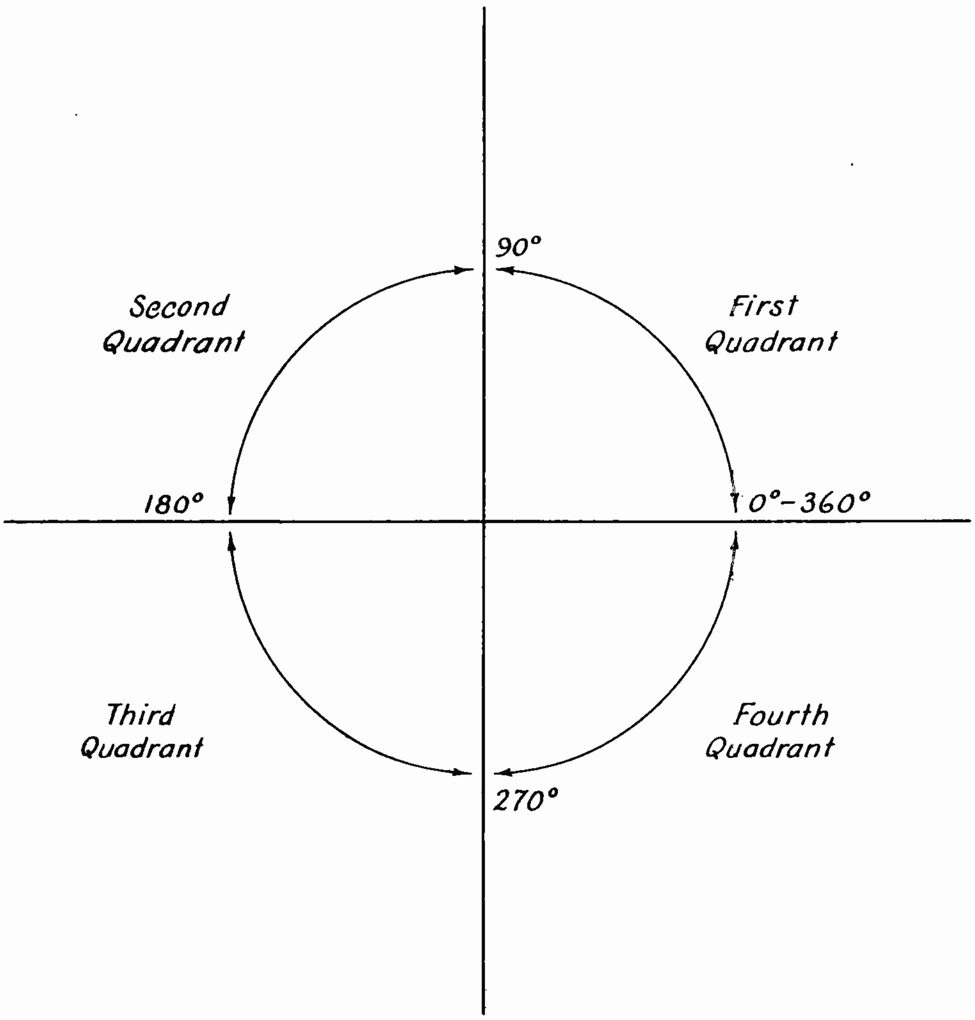


Fig. 15.—Quadrant designations to show the direction of forces.

with vector diagrams it is customary to start at the base line to the right of the bisector and proceed in a counterclockwise direction. The first 90° make up the first quadrant, 90° to 180° make up the second quadrant. The third quadrant

located by inspection. If a diagram of a problem is drawn approximately to scale it is usually not difficult to determine an approximate value for the resultant and the quadrant in which the resultant should lie. For example, A = 80,

A_v = 20, B = 60, B_v = 30; A in 1st quadrant, B in 4th quadrant. This may be shown approximately as in Fig. 16.

Mention was made in a previous assignment that the solution for the resultant force at any point of two charged bodies in space would be

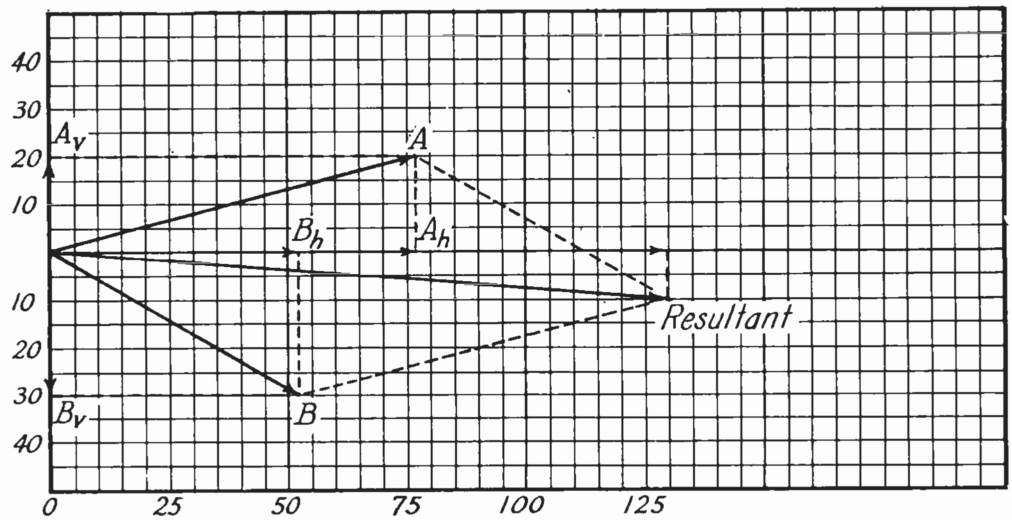


Fig. 16.—Forces shown to scale used as a check.

It is readily apparent from the sketch that since B_v is greater than A_v, the resultant should be in the 4th quadrant; that B_h should be slightly greater than 50 and that A_h should be a little less than 80, and that the resultant should be greater than either of the individual forces A or B. If such results are not obtained mathematically, the answer will evidently be in error. It can be seen that the resultant is approximately 130.

At this point it should be emphasized that the geometric sum of two forces can *never* exceed their arithmetic sum. This can easily be seen from all of the diagrams. As the angle between two forces approaches zero, the geometric sum approaches the arithmetic sum. As the angle between the forces approaches 180°, the geometric sum approaches the arithmetic difference.

discussed. Assume two protons and an electron in space. See Fig. 17.

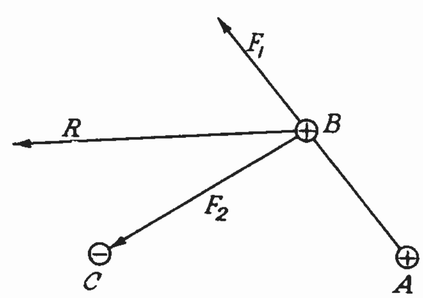


Fig. 17.—Two protons and one electron in space.

Proton A repels proton B as shown by the force F_1. Electron C attracts proton B as shown by F_2. Since force F_1 pulls toward the top of the page and force F_2 pulls toward the left side of the page, the resultant force, R, will lie somewhere between F_1 and F_2. If the force of repulsion of the two protons is greater than the force of attraction of the electron and proton, then the direction of the resultant force will be closer to F_1 than F_2.

Right-Angle Triangles.—If half of Fig. 14(D) be considered, it is clear that it would be a right triangle, with the resultant acting as the hypotenuse. Now to consider how to determine the unknown side of a right triangle from the known sides.

Assume the base of a right triangle is 20, its altitude is 15. What is the hypotenuse? From the formula to determine the resultant or the hypotenuse of a right triangle, the hypotenuse may be determined.

$$H = \sqrt{20^2 + 15^2} = \sqrt{400 + 225} = \sqrt{625}$$

$$= 25$$

Similarily by Algebra, the height or base may be determined, since

$$H = \sqrt{A^2 + B^2}$$

where A = altitude
B = base
H = hypotenuse

$$H = \sqrt{A^2 + B^2}$$

$$H^2 = A^2 + B^2 \text{ (squaring both sides)}$$

$$H^2 - B^2 = A^2 + B^2 - B^2$$

$$H^2 - B^2 = A^2$$

$$\sqrt{H^2 - B^2} = A$$

or, in the above problem, the altitude may be found by,

$$A = \sqrt{25^2 - 20^2} = \sqrt{625 - 400} = \sqrt{225}$$

$$= 15$$

In an exactly similar manner, the base B may be solved for. There is obtained:

$$B = \sqrt{H^2 - A^2}$$

By substituting original values into this equation,

$$B = \sqrt{25^2 - 15^2} = \sqrt{625 - 225} = \sqrt{400}$$

$$= 20$$

Exercises

1. Right triangle:
   Altitude = 40
   Base = 30
   Find hypotenuse.

2. Right triangle:
   Altitude = 74
   Base = 15
   Find hypotenuse.

3. Right triangle:
   Hypotenuse = 70
   Base = 30
   Find altitude.

4. Right triangle:
   Hypotenuse = 90
   Base = 55
   Find altitude.

5. Right triangle:
   Altitude = .76
   Hypotenuse = 2.3
   Find base.

6. Right triangle:
   Altitude = 102
   Hypotenuse = 274
   Find base.

In the following problems find the resultant and state in which quadrant the resultant lies. (Figs. 10, 11 and 12 show the method of handling these problems.)

In each of the problems a vector diagram should be drawn (not necessarily to scale) to show the various forces with their vertical and horizontal components, as well as the total resultant force.

7. $A = 100$, $A_h = 30$, $B = 90$, $B_h = 50$. A and B in 1st quadrant.

8. $A = 80$, $A_v = 25$, $B = 100$, $B_v = 40$. A and B in 1st quadrant.

9. $A = 120$, $A_h = 45$, $B = 30$, $B_v = 20$. A and B in 1st quadrant.

10. $A = 100$, $A_h = 90$, $B = 35$, $B_h = 10$. A in 2nd quadrant, B in 3rd quadrant.

11. $A = 90$, $A_v = 40$, $B = 100$, $B_v = 70$. A in 3rd quadrant, B in 1st quadrant.

12. $A = 150$, $A_v = 30$, $B = 70$, $B_v = 50$, $C = 100$, $C_v = 40$. A in 1st quadrant, B in 1st quadrant, C in 4th quadrant.

TRIGONOMETRY

In the solution of two or more forces acting on a point at various angles it was found that it is first necessary to resolve each of the individual forces into its two components, one along the horizontal or x axis and the other along the vertical or y axis. Then the algebraic sum of all the horizontal components forms one side of a right triangle, the other side is formed by the algebraic sum of all the vertical components, and the resultant of all the forces is equal to the hypotenuse of the right triangle.

This method of solution is called geometric or vector addition. But geometry by itself does not go quite far enough to make geometric addition of forces practical, because it provides no method of finding the exact mathematical values of the two components when only the forces and their respective angles are known.

To clear up this difficulty the use of trigonometry, which deals in angles and the sides of angles, is available. Trigonometry makes it possible to take any right-angle triangle and, knowing only one side and one angle, to find the other two sides. Or if two sides are known it is possible to find any angle and the other side. In other words, simple trigonometry makes practical the geometric use of the right triangle in the addition of forces.

In trigonometry one uses what are known as the "trigonometric functions" of the angle. A trigonometric function of an angle is simply the ratio between two sides of the angle. All of this work is based on the right triangle. Consider Fig. 18.

The horizontal line A may extend out to any distance. The line H extends out in a straight line and forms an angle θ with the horizontal line. If any point is selected along the side H, and a perpendicular line D is dropped to cut side A at right angles, there will be formed a right triangle having three sides, h_1, a_1, and D. If the sides of this triangle are measured and their lengths compared, there can be written three different ratios between the various sides, that is: the ratio of D to h_1, the ratio of a_1 to h_1 and the ratio of D to a_1.

If another point along H is selected and another perpendicular, C, is dropped cutting side A at right angles, another right triangle will be formed from point O having the sides h_2, a_2 and C. Measuring the three sides, three more ratios may be written: The ratio of C to h_2, the ratio of a_2 to h_2 and the ratio of C to a_2. (It should be understood at this point that all distances referred to are from the apex of the angle θ. Thus a_1 represents the distance from O to D; a_2 represents the distance from O to C; A represents from O to B. Likewise h_1 is from O to D along the hypotenuse; h_2 is from O to C along the hypotenuse; H is the total hypotenuse from O to B.

Comparing these ratios it will be found that the ratio of D to h_1 is equal to the ratio of C to h_2; the ratio of a_1 to h_1 will be equal to the ratio of a_2 to h_2; and the ratio of D to a_1 will be equal to the ratio of C to a_2.

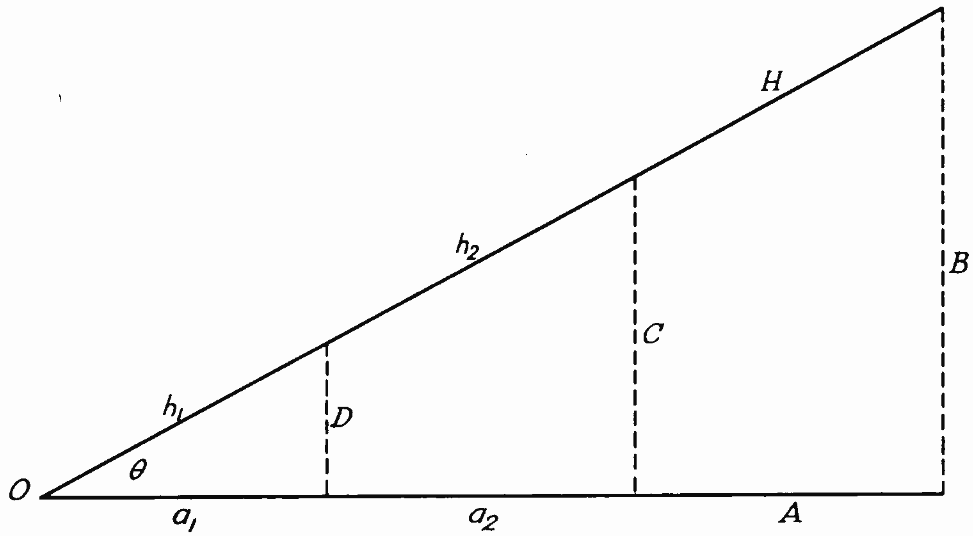


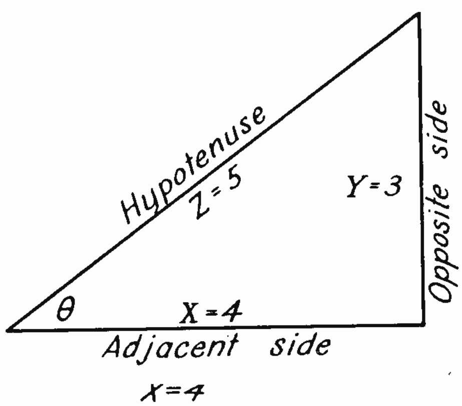
Fig. 18.—Trigonometry applied to the right triangle.

Referring to Fig. 18 the reason for this can be seen. Side A is a straight line in the horizontal plane. Side H is also a straight line, therefore the distance above A of any point along H varies in the same proportion per unit of distance that side H extends to the right. This will be true no matter how far the sides A and H extend out into space so long as the angle θ does not change. It will be seen then, that the *ratios of the various sides of any right triangle depend directly upon some angle*. Regardless of the difference in the areas of two right triangles, the *ratios* between any two corresponding sides will be the same if the corresponding angles of the two triangles are similar. This is clearly shown in Fig. 18.

Given two values, or forces, and knowing the ratio between them and the value of one, it is possible to find the other from the known value and the ratio existing between the two. For example, suppose it is known that X is 3 times as large as Y, that is, the ratio of X to Y is 3. If Y is equal to 15, then, since the ratio of X to Y is 3, X must equal 45.

If the ratio of A to B is one-fourth or .25 and the value of B is known to be 36, then A must be one-fourth of 36 or 9. Since vector addition is based on the right triangle, it will be seen that for any given angle, if the ratios of the various sides for that angle are known, it will be a simple matter to take any known side and select a ratio that will allow the determination of either of the other sides.

Fig. 18 demonstrates that if two straight lines, starting at some common point and extending to any distance with any given angle between them, are connected by a third straight line in such a manner that one of the two sides is cut by the third side at right angles, a right triangle is formed. With reference to the angle, the right triangle is made up of a hypotenuse, an adjacent side (the side that with the hypotenuse forms the angle) and an opposite side (the third side that was drawn to complete the right triangle or the side opposite the angle). It has been shown that, regardless of the length of the three sides, if they form a right triangle there is some definite ratio between each two of the three sides, and *these ratios are fixed by the angle formed by the intersection of the adjacent side and the hypotenuse*. Therefore if the ratios and the length of one side are known, it is very easy to find the lengths of the other two sides. Since the area of the triangle is not important, only the *relative* dimensions of the sides being required, it is only necessary in finding the ratios for any given angle, to use a protractor and form a right triangle, one angle of which is equal to the given angle. Measure the three sides of the triangle and work out the ratios between each pair of sides: The ratio of the opposite side to the hypotenuse, the ratio of the adjacent side to the hypotenuse, and the ratio of the opposite side to the adjacent side. Then, knowing the angle and the ratios, it is possible to compute the lengths of each side of any other right triangle having the same angles provided that one side of the second triangle is known. See Fig. 19.



Ratio Y/Z = 3/5 = .6
Ratio X/Z = 4/5 = .8
Ratio Y/X = 3/4 = .75

Fig. 19.—Given two angles; find the sides by ratios.

Given another right triangle in which one angle equals angle θ and the hypotenuse has length of 20, to find the other two sides. See Fig. 20.

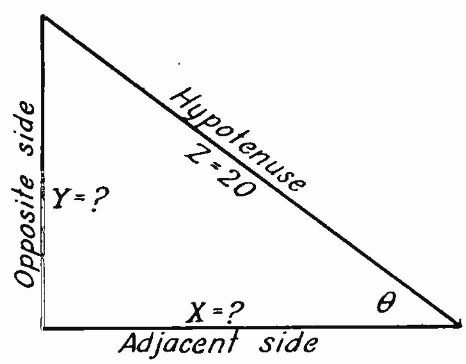


Fig. 20.—Given angle θ and the hypotenuse.

Referring to the preceding example, it is seen that the ratio of side Y to side Z is .6. This means that Y is equal to .6Z. Since in the second triangle the angle θ is the same as in the first triangle, the ratios between the sides must be the same. Therefore in the second triangle Y must also equal .6Z. Since Z equals 20, then $Y = 20 \times .6 = 12$.

Similarly the ratio of X to Z equals .8 in both triangles, therefore in the second triangle $X = 20 \times .8 = 16$.

To prove this, take the ratio of Y to X which is equal to .75. $Y = .75X$. X has been found to equal 16. Therefore $Y = .75 \times 16 = 12$, which is correct.

Regardless of the area of the triangle, so long as one angle is equal to angle θ and one side is known, it is a simple matter to solve for the remaining sides by using the known ratios. Anyone could, very easily, with only a protractor and ruler, compute the three ratios of all the angles between zero and ninety degrees and arrange them in the form of a table, thus having the means of solving all right triangles. This is not necessary as the ratios have been very accurately computed and placed in all mathematical tables under the heading of "Trigonometric Functions of the Angle," or "Natural Sines, Cosines and Tangents."

It will be seen that *the Trigonometric Functions of the angle are merely the ratios of combinations of two sides of the angle*, assuming that the two lines which intersect to form the angle also form two sides of a right triangle, the third side of which is called the opposite side. The opposite side may be only assumed or imaginary.

In order to facilitate the use of trigonometric tables, and to form a consistent or standard system of reading the functions or ratios, they have been given certain names, as follow:

Sine = Ratio of Opposite Side to Hypotenuse

$$= \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \text{Sin}$$

Cosine = Ratio of Adjacent Side to Hypotenuse

$$= \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \text{Cos}$$

Tangent = Ratio of Opposite Side to Adjacent Side

$$= \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \text{Tan}$$

There are also three other functions of the angle, the reciprocals of the three defined above:

Cosecant = Csc

$$= \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{1}{\text{Sine}}$$

Secant = Sec

$$= \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{1}{\text{Cosine}}$$

Cotangent = Cot

$$= \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{1}{\text{Tangent}}$$

As the Sine, Cosine and Tangent are sufficient to solve all right triangles, the three reciprocal functions need not be discussed in detail. If any one of the reciprocal functions should be given in an equation, however, it is only necessary to obtain its value from a table of trigonometrical functions and use as directed. Very often an equation will be written in the form of a multiplication by a Cotangent instead of division by a Tangent, or the reciprocal of a Sine or Cosine used is some similar manner. The use of the Cosecant or Secant is equally as simple as that of the Sine or Cosine.

A comparison between the functions of the angle will help to make their use clear. Assume a hypotenuse of constant value, one end fixed at point "o," the other end moving in a counterclockwise direction over a quarter circle, or ninety degrees, beginning at a point along the horizontal axis, The results are as shown in Fig. 21. Keeping the hypotenuse the same length, and sweeping through an

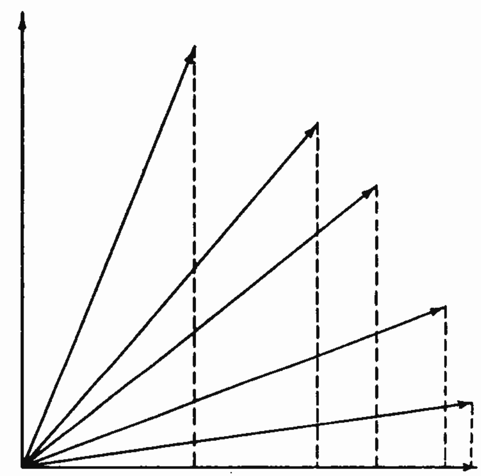


Fig. 21.—Comparison of the functions of an angle.

arc of ninety degrees starting at the horizontal line to the right of point "o," as the angle is increased above the base line the length of the opposite side (shown by dotted line) increases, until *at ninety degrees the opposite side and the hypotenuse coincide making both of equal length.* The ratio is then equal to 1. At the starting point, position 1, the hypotenuse coincides with the horizontal axis or adjacent side, forming a zero degree angle. *In this position the opposite side is equal to zero.* The ratio of the opposite side to the hypotenuse (Sine) is then equal to Zero/Hypotenuse = Zero.

(Zero divided by any value equals zero.) It may be stated that the Sine of a zero degree angle is equal to zero, and the Sine of a ninety degree angle is equal to one. The Sine of any angle between zero degrees and ninety degrees *is equal to some decimal between zero and one, INCREASING with the angle.*

On studying the ratio of the Adjacent side to the Hypotenuse (the Cosine) for various angles between zero and ninety degrees, exactly opposite conditions are found to exist. At zero degrees the hypotenuse coincides with the adjacent side, the two being equal. The ratio is equal to 1. As the angle is increased, the length of the adjacent side, between point "o" and the opposite side, becomes shorter and shorter, until at ninety degrees the adjacent side becomes zero. The ratio then becomes Zero/Hypotenuse = Zero.

It may be stated that the *Cosine of a zero degree angle is equal to 1 and the Cosine of a ninety degree angle is equal to zero.* The Cosine of any angle between zero degrees and ninety degrees *is equal to some decimal between zero and one, DECREASING as the angle increases.*

Consider the ratio of the opposite side to the adjacent side (the Tangent). At zero degrees the opposite side being equal to zero, the ratio is equal to Zero/Adjacent = Zero. At forty-five degrees the opposite and the adjacent sides are equal, making the ratio 1. As the angle approaches ninety degrees the adjacent side approaches zero. No numerical value can be assigned to the tangent of 90° because Opposite/Zero requires division by zero which is impossible.

It may be stated that the tangent of any angle between zero degrees and forty-five degrees is equal to some decimal between zero and one, *increasing with the angle*; and that the tangent of any angle between forty-five degrees and ninety degrees is between one and infinity, *increasing with the angle.*

It is interesting to note that in the cases of the Sine and the Cosine, the opposite and adjacent sides can equal but can *never* exceed the Hypotenuse. The ratio of either to the hypotenuse can therefore never exceed one. In the case of the Tangent, however, a similar condition does not exist. For all angles of less than forty-five degrees the adjacent side is longer than the opposite side, *but* for angles of greater than forty-five degrees the opposite side is longer than the adjacent. Also in the case of the tangent, one side increases in length while at the same time the other side decreases in length, making the change per degree much more rapid than in the case of either the sine or cosine where one value in each ratio, the hypotenuse, remains constant while the other value is varied. In the case of very small angles, *less than about 6°*, the sine and the tangent are almost identical, so that in dealing with certain types of problems, such as the phase angles of condensers, the two are sometimes interchanged in equations.

The changes in the function of the angle with angular variation can be seen very clearly if reference is made to a table of the Trigonometric Functions of an Angle, commonly called a "Trig Table." The natural functions listed in this table are the actual numerical values of the Sines, Cosines, Tangents, and Cotangents of angles for each tenth of a degree from 0° to

90°.

In using the trig tables, the student should be careful to distinguish between the tables of natural functions and the tables of logarithms of the functions. In certain types of work where involved problems are encountered, the logarithmic functions are specified in the equations and may be taken directly from the tables. In most problems in electrical and radio work the natural functions will be used and these will be understood unless otherwise stated. The table of natural functions shown in the Mathematical Tables will give an accuracy of .1 per cent and is recommended to the student for use in all work involving trigonometry in this course.

The arrangement of the table is quite simple. See portions of the trig table printed in Table I. These are taken from the Mathematical Tables. A few examples will serve to show how the table is used.

To find the Tangent of an angle of 40.8°. In the left-hand column headed "Deg." read downward to 40.8, the given angle. From this point move to the right to the column headed by "Tan" (the function desired) and read .8632. This is the Tangent of the angle 40.8° to four decimal places.

To find the Sine of 49.4°. Reading down the left-hand column headed "Deg." it is seen that the table apparently terminates at

TABLE I

Deg.	Sin	Cos	Tan	Cot	Deg.
40.5	0.6494	0.7604	0.8541	1.1708	49.5
.6	.6508	.7593	.8571	1.1667	.4
.7	.6521	.7581	.8601	1.1626	.3
.8	.6534	.7570	.8632	1.1585	.2
.9	.6547	.7559	.8662	1.1544	.1
41.0	0.6561	0.7547	0.8693	1.1504	49.0
.1	.6574	.7536	.8724	1.1463	.9
.2	.6587	.7524	.8754	1.1423	.8
*	*	*	*	*	*
44.5	0.7009	0.7133	0,9827	1.0176	.5
.6	.7022	.7120	.9861	1.0141	.4
.7	.7034	.7108	.9896	1.0105	.3
.8	.7046	.7096	.9930	1.0070	.2
.9	.7059	.7083	.9965	1.0035	.1
45.0	0.7071	0.7071	1.0000	1.0000	45.0
Deg.	Cos	Sin	Cot	Tan	Deg.

45.0°. The table must therefore be used in a different manner in finding the functions of angles between 45° and 90°.

The sum of all the angles of any triangle is equal to 180°. All work in trigonometry is based on the right triangle, one angle of which is always equal to 90°. It follows that the acute angles of any right triangle are complementary, that is, their sum is 90°.

If one acute angle is 49.4°, then the other acute angle will be 90 - 49.4 or 40.6°. See Fig. 22.

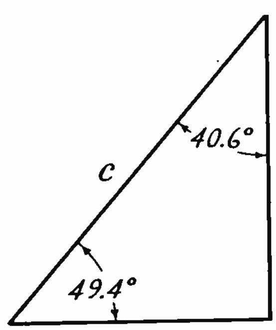


Fig. 22.—Illustration of the relation of angles and the sides of a right triangle.

The Sine of 49.4° is Opposite Side ÷ Hypotenuse or a/c. The Cosine of 40.6° is Adjacent Side ÷ Hypotenuse or again a/c, because the Opposite Side of the 49.4° angle is identical with the Adjacent Side of the 40.6° angle. Thus Sin 49.4° = Cos 40.6°. By reference to the table, Cos 40.6° = .7593 and this is also the value of Sin 49.4°.

This example illustrates an important principle by means of which the table of functions for angles between 0° and 45° is extended to include angles between 45° and 90° without the tabulation of additional function values: *The function of an angle is equal to the co-function of its complementary angle.* Note in this connection that the Sine and Cosine are co-functions of each other. Similarly, the Tangent and the Cotangent are co-functions. The following examples will clarify the application of the principle stated above.

Sin 70° = Cos (90° - 70°)

= Cos 20°

Cos 50.5° = Sin (90° - 50.5°)

= Sin 39.5°

Tan 15° = Cot (90° - 15°)

= Cot 75°

Cot 84.3° = Tan (90° - 84.3°)

= Tan 5.7°

In determining the function of any given angle between 45° and 90° from the tables, the operation of calculating the complimentary angle and looking up its co-function can be avoided by referring to the right-hand "Deg." column for the given angle and along the base of the table for the name of the desired function.

For example, to find the Cosine of 45.4°. See Table I. Find 45.0 in the right-hand "Deg." column and read up to .4. From this point move to the left to the column whose base is marked "Cos." In this column read the tabulated value .7022 which is the Cosine of 45.4°.

Note that the tabulated value .7022 is also the Sine of 44.6°. This is in accordance with the principle that the function of an angle is equal to the co-function of its complementary angle. The student should practice the use of the trig table in the Mathematical Tables until he can find any desired function of any angle between 0° and 90° with complete facility.

In a previous assignment dealing with logarithms the number corresponding to a given logarithm in the tables was called the "Antilog." It is sometimes abbreviated to "$\log^{-1}$.." and read "the number whose logarithm is.." In Trigonometry there is somewhat similar method of expressing the angle corresponding to a given function in the tables. For example, Tan^{-1} .4877 is read as "the angle whose Tangent is .4877." The designation $^{-1}$ in both cases is merely a method of expressing a statement and does *not* mean that the function is raised to a -1 power. Tan^{-1} is sometimes shown in text books as arc tan. The two expressions are an abbreviation for "the angle whose tangent is."

In trigonometric calculations it is frequently necessary to find the value of an angle, given the magnitude of one of its functions. The trig table is used for this purpose in a manner somewhat similar to that used in finding an antilogarithm. The column headed "Sin" is marked "Cos" at the base, while that headed "Cos" is marked "Sin" at the base. Therefore if the Sine or Cosine function is given, the nearest tabulated value is located in one of these two columns. If the title at the head of the column in which the nearest tabulated value is found agrees with the name of the given function, the value of the angle will be read in the left-hand "Deg." column. If the title at the foot of the column agrees with the name of the given function, then the angle is found in the right-hand "Deg." column.

For example, to find Sin^{-1} .7100 (the angle whose Sine is .7100). See Table 1. By searching in the "Sin" and "Cos" columns, the nearest tabulated value is found to be .7096. The title at the head of the column is "Cos" which differs from the name of the given function, the Sine. The title at the foot of the column is "Sin" which agrees with the name of the given function. Therefore move to the right from the tabulated figure .7096 and in the "Deg." column read .2 or angle of 45.2°. This is the angle to the nearest tenth of a degree whose Sine is .7100. In other words, Sin^{-1} .7100 = 45.2°.

When the Tangent or Cotangent function is given, the same procedure is employed, except that the nearest tabulated value is located in one of the two columns headed "Tan" or "Cot."

The most important fact to remember in the use of the trig table is that the labels at the top of the function columns are to be used in conjunction with the angles in the left-hand "Deg." column. Similarly, the labels at the bottom of the function columns and the angles in the right-hand "Deg." column go together.

It will be observed that the Trigonometric Tables cover only angles between 0° and 90°. As the hypotenuse describing an arc as in Fig. 21 moves past the 90° position it moves into the second quadrant.

In this quadrant for purposes involving the trigonometric function, the angle is taken to the *nearest horizontal axis*. The nearest horizontal axis in the second quadrant is at 180°. Thus for purposes of finding the trigonometric function of an angle of, for example 110°, use the angle 180° - 110° or 70°.

In the third quadrant use the angle in question minus 180°, and in the fourth quadrant 360° minus the given angle. Thus at no time is it necessary to have tables for angles greater than 90° because the necessary function can always be determined from a tabulation to 90°.

Exercises

Find the magnitude of the indicated functions:

13. Sin 29.8°

14. Tan 30.2°

15. Cos 11.6°

16. Cot 12.2°

17. Sin 46.1°

18. Cot 78.8°

19. Sin 60.6°

20. Tan 84.0°

21. Cot 88.9°

22. Sin 168.5°

23. Sin 103.0°

24. Tan 267.7°

25. Cos 295.5°

26. Cos 270.0°

27. Tan 245.0°

Find the acute angle to the nearest tenth of a degree.

28. Tan^{-1} .1228

29. Cos^{-1} .8500

30. Cos^{-1} .4454

31. Tan^{-1} 1.3319

32. Cot^{-1} 4.000

33. Sin^{-1} 3/5

34. Cot^{-1} .23

35. Sin^{-1} .8729

36. Cot^{-1} 5/11

It will be remembered that geometry provided no practical method of calculating the horizontal and vertical components of the various forces to be added geometrically. In order to calculate the value of the vertical component of a force, it was necessary first to know the value of the horizontal component. Similarly, the horizontal component could be determined only when the value of the vertical component was given.

Trigonometry removes this limitation of the processes of geometry. With the use of a table of trigonometric functions, the horizontal and vertical components of any force acting at any angle may be easily determined. In fact, trigonometry will completely solve any *right* triangle in which two sides, or one side and one acute angle, are known.

STANDARD TRIANGLE.—In order to avoid confusion in the use of trigonometric formulas, a uniform procedure for designating the sides and angles of a triangle has been adopted. This is called the Standard Triangle and is shown in Fig. 23.

The angles of the triangle are designated by the capital letters A, B, and C. The sides are denoted by lower case letters, "a" referring to the side opposite Angle A, "b" to the side opposite Angle B, and "c" to the side opposite Angle C. In the case of a *right* triangle, the right angle is *always* designated by C, in which case "c" will always be the hypotenuse, since it is the side opposite the right angle. The following formulas are based on a standard triangle having one right angle.

In explaining the use of the functions of the angle for solution of right triangles, they will be defined and treated as simple algebraic equations involving three factors:

Sine Angle	$= \dfrac{\text{Opposite Side}}{\text{Hypotenuse}}$	$\text{Sin A} = \dfrac{a}{c} = \text{Cos B}$
Opposite Side	= Hypotenuse × Sine	$a = c \text{ Sin A}$
Hypotenuse	$= \dfrac{\text{Opposite Side}}{\text{Sine}}$	$c = \dfrac{a}{\text{Sin A}}$
Cosine Angle	$= \dfrac{\text{Adjacent Side}}{\text{Hypotenuse}}$	$\text{Cos A} = \dfrac{b}{c} = \text{Sin B}$
Adjacent Side	= Hypotenuse × Cosine	$b = c \text{ Cos A}$
Hypotenuse	$= \dfrac{\text{Adjacent Side}}{\text{Cosine}}$	$c = \dfrac{b}{\text{Cos A}}$
Tangent Angle	$= \dfrac{\text{Opposite Side}}{\text{Adjacent Side}}$	$\text{Tan A} = \dfrac{a}{b} = \text{Cot B}$
Opposite Side	= Adjacent × Tangent	$a = b \text{ Tan A}$
Adjacent Side	$= \dfrac{\text{Opposite Side}}{\text{Tangent}}$	$b = \dfrac{a}{\text{Tan A}}$

Consideration of the three sets of equations above will demonstrate

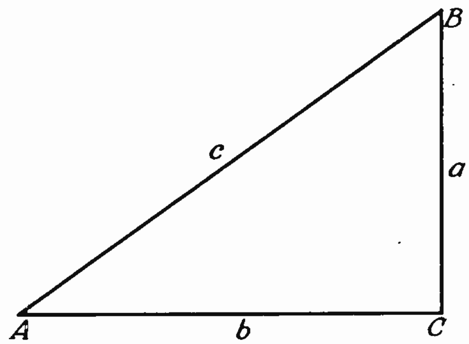


Fig. 23.—**Standard triangle.**

their use in the solution of a triangle for all three sides and the angles. If any two sides of the right triangle are given, they may be used in one of the above forms to find one function of one angle. When the function of the angle is determined, it will only be necessary to refer to the table to find the angle.

It is customary to use the designations of the Standard Triangle in stating problems involving the solution of triangles by trigonometry. In this manner, the given conditions are fully described without recourse to the construction of a diagram.

For example: Given; a = 5, b = 10. To find; c, A.

It is required to find the acute angle A and the hypotenuse of a right triangle when the side opposite angle A is five and the side adjacent is ten. The equivalent triangle is shown in Fig. 24.

Since for angle A, a is the opposite side equal to 5, and b is the adjacent side equal to 10, inspection of the three functions of the angle indicates that the Tangent is the only function dealing with both a and b. The definition for the Tangent is:

$$\text{Tan } A = a/b$$

Using this equation and replacing the designating letters with figures:

$$\text{Tan } A = 5/10 = .5$$

Inspection of the Tangents indicates that a tangent of .5 refers to an angle of 26.6°.

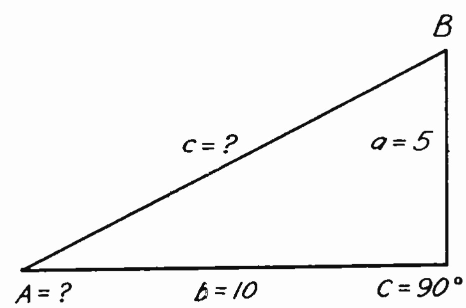


Fig. 24.—**Figure for illustrative example problem.**

Knowing that angle A = 26.6° it is possible to use either of the known sides with the correct function to find hypotenuse c.

By definition, the Sine of the Angle deals with both the opposite side a and the hypotenuse c; c = a/Sin A. Substituting figures, c = 5/Sin 26.6° = 5/.4478 = 11.16.

Since the length of the adjacent side b is known, it could have been used with the Cosine. The equation then becomes: c = b/Cos A = 10/Cos 26.6° = 10/.8942 = 11.16.

It should be noted that the same result is obtained by the use of both methods. Starting with two sides of the right triangle, the other side and one of the unknown angles have been determined.

The acute angles of a right triangle are complementary, that is, their sum is 90°. Angle A having been found, it follows that Angle B must equal 90 degrees minus Angle A. Thus B = 90° - 26.6° = 63.4°.

To solve any *right* triangle it is only necessary to know one side and one angle. The proper use of the Trig tables will allow the determination of the other two sides and angles.

The following example illustrates the procedure in a typical case:

Given: b = 20, A = 40°.

Find: a, c, and B.

According to the equations defining the functions of angles, the hypotenuse equals Adjacent/Cosine or c = b/Cos A. Thus:

c = b/Cos 40° = 20/.766 = 26.1

Again referring to the equations, it is seen that Opposite Side equals Hypotenuse × Sine Angle or a = c Sin A. Thus:

a = c Sin 40° = 26.1 × .6428 = 16.78

Using the tangent to prove the work: The tangent of the angle equals Opposite/Adjacent = 16.78/20 = .839. By inspection of the table, .839 is the Tangent of an angle of 40°.

Since the acute angles of a right triangle are complementary the other angle, B, must be 90° - 40° = 50°.

In this example one side and one angle were given and, by the use of the trigonometric tables, the other two sides and angles were calculated and the correctness of the answers proved. The answers may also be checked by drawing the triangle to a fairly large scale using a protractor and ruler, and measuring the sides and angles. Heretofore, discussion has been limited to determining the resultant. However, knowing the resultant and an angle, the component forces composing the resultant may be determined.

Problems are quite frequently stated in these terms. A typical example is an antenna. As discussed in an earlier assignment, a moving electric field in space produces a magnetic field at right angles to it. Assume an antenna whose base forms a 65° angle with the ground and a vertically polarized electric field approaches the antenna as in Fig. 25.

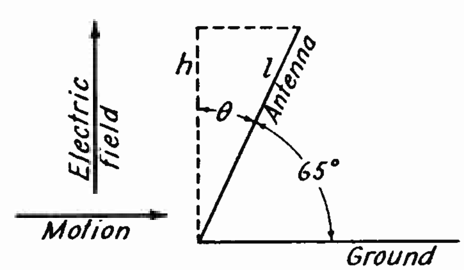


Fig. 25.—Effective length of an antenna to a vertical electric field.

h represents the effective part of the antenna length or of the induced voltage. If the length of the antenna is 140 ft., then h = l Cos θ and θ = 90 - 65° = 25°. Then h = 140 × Cos 25° = 140 × .906 = 126.84 ft. This represents the effective part of the antenna.

It might be well to state here that for maximum induced voltage the antenna should be parallel to the electric field as may be seen from the above example.

Note that the resultant may

be determined by knowing the component parts that constitute the resultant, or by knowing one component and one acute angle. On the other hand, by knowing the resultant and an angle; or the resultant and one component, the other components and angles may be determined. Various problems involving both of the above methods will be found. The problem must first be analyzed in terms of what is given, or stated in the problems and what is to be found. Then choose the method that corresponds to the given data and solve for the unknowns.

Exercises

Solve the following problems by the use of trigonometry and construct right triangle to scale showing the values of all sides and angles.

37. c = 50, A = 38°.

38. b = 100, A = 70°.

39. a = 60, A = 20°.

40. c = 30, b = 25.

41. c = 130, a = 60.

42. a = 80, b = 150.

Other values should be substituted in each of the above problems and the problems reworked for the new values. This should be done until the student is thoroughly familiar with each type of problem.

LAW OF COSINES.—It will be found useful in certain problems, such as in considerations of noise in Frequency Modulation, to be able to use the law of cosines to solve for forces where no right angle exists; i.e. oblique triangles.

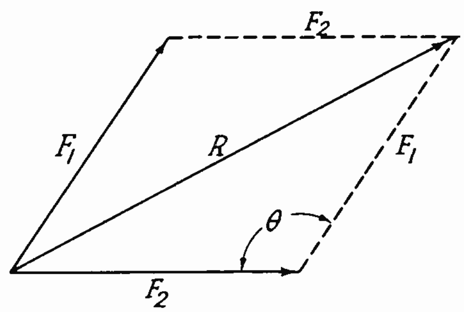


Fig. 26.—Oblique triangle.

The law is stated: *In any triangle, the square of any side equals the sum of the squares of the other two sides minus twice the product of these two sides times the cosine of the angle between them.* Stated algebraically, based on Fig. 26,

$$R^2 = F_1^2 + F_2^2 - 2F_1F_2 \cos \theta$$

$$R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2 \cos \theta}$$

If θ = 90°, we have a right triangle as shown in Fig. 27.

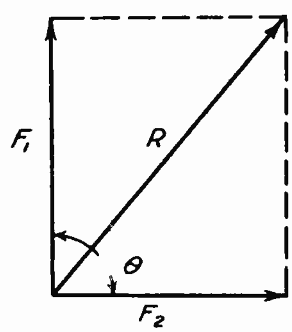


Fig. 27.—Special case of the oblique triangle.

$$\cos \theta = \cos 90° = 0$$

so that,

$$R = \sqrt{F_1^2 + F_2^2 + 0}$$

which is the previous theorem given for a right triangle.

Illustrative Example: In Fig. 26, if $F_1 = 10$, $F_2 = 12$, and $\theta = 150°$, find R.

$$\cos 150° = -\cos 30°$$

$$R = \sqrt{100 + 144 - 2[120(-\cos 30°)]}$$

$$= \sqrt{244 - 2[120(-.866)]}$$

$$R = \sqrt{244 + 2[103.9]} = 21.25$$

In Fig. 28 we see another use of trigonometry in finding unknown values. Suppose we are given θ, E_1 and E_2.

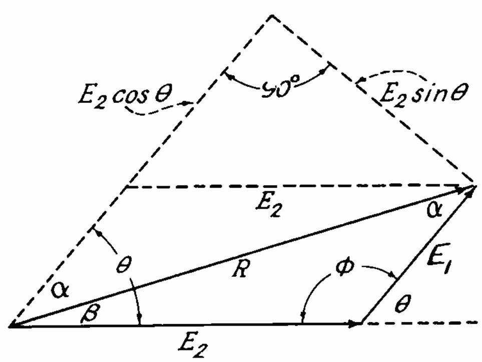


Fig. 28.—Solution of an oblique triangle and the unknown angles α and β.

First we can find α by the values E_2 and θ and $E_2 \sin \theta$, constructed as shown in Fig. 28

$$\tan \alpha = \frac{E_2 \sin \theta}{E_1 + E_2 \cos \theta}$$

Since θ is known we can find β,

$$\beta = \theta - \alpha$$

Now to find R,

$$R^2 = E_1^2 + E_2^2 - 2E_1E_2 \cos \phi$$

$$\frac{R^2 - E_1^2 - E_2^2}{-2E_1E_2} = \cos \phi$$

Since the sum of the angles in any triangle equals 180 degrees,

$$\phi = 180 - \alpha - \beta$$

$$\cos \phi = \cos [180 - (\alpha + \beta)]$$

From Trigonometrical identities (Math tables) we find that

$$\cos (x - y) = \cos x \cos y + \sin x \sin y$$

substitute for cos x, cos 180; and for cos y, cos $(\alpha + \beta)$ etc;

$$\cos 180° = -1; \ \sin 180° = 0$$

$$\cos [180 - (\alpha + \beta)]$$

$$= (-1) \cos (\alpha + \beta) + 0$$

$$= - \cos (\alpha + \beta)$$

therefore

$$\frac{R^2 - E_1^2 - E_2^2}{2E_1E_2} = \cos (\alpha + \beta) = \cos \theta$$

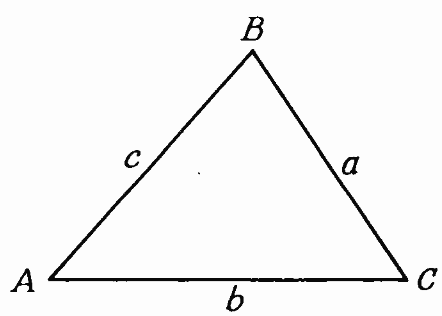
θ and ϕ are called supplementary angles since they add to make 180 degrees or a straight line as shown in Fig. 28.

From the equation for R^2 we can find by algebra that,

$$E_1^2 = R^2 - E_2^2 + 2E_1E_2 \cos \phi$$

Note that this is of no use since E_1 appears on both sides of the equation and is not readily simplified. Later we will show formulas that simplify solutions for E_1 and E_2.

THE LAW OF SINES.—In any triangle, the sides are proportional to the sines of the opposite angles. By this law many triangle problems can readily be solved. Stated algebraically with reference to Fig. 29, we can solve for the fourth unknown when three values are known by using the equation which is applicable.



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Fig. 29.—Oblique triangle.

Solution of Oblique Triangles.— Four general cases are encountered where the preceeding laws may be used. See Fig. 23 (repeated here for your convenience).

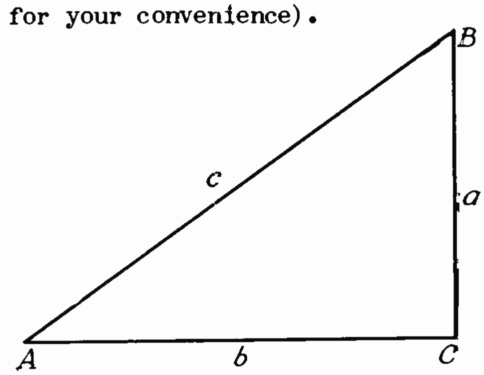


Fig. 23.—Standard triangle.

1. One side and two angles are given.

2. Two sides and an angle opposite one side are given.

3. Two sides and the included angle are given.

4. Three sides are given.

From the law of cosines we can work out the following equations,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Select the equation above or under

the law of sines which fits the given data and solve for the unknown element. Work can be checked by a careful drawing or by another equation.

The proof will not be shown here for the laws of sines and cosines and their applications. This can be found at your local library in Palmer & Bibbs Practical Mathematics for Home Study and many other texts. Oblique triangles can be broken up into right triangles by construction but direct solution is usually quicker and less trouble.

Conclusion.—The formulas given as defining the functions should be studied and used in problems until they are thoroughly understood and as easy to handle as the ordinary multiplication table.

The solution of a right triangle is the type of problem most frequently encountered in radio and electrical work. However, it should be understood that this is but one of the many uses of trigonometry in the broad field of engineering. A complete study of trigonometry would involve the derivation of many interesting formulas showing the relationships between the functions. The attention of the student is directed to the Mathematical Tables where the more important theorems regarding the functions of angles have been recapitulated in algebraic form. The student may test the validity of these equations by assigning any value to the angle x, substituting its functions as taken from the trig table, and confirming the equality of both members of the equation. The equations given in the tables are useful for the solution of triangles other than right triangles, and for spherical triangles. While used to a great extent in other types of work such as civil engineering and navigation, they are of interest to the radio engineer only in the more specialized branches of the profession.

ANSWERS TO EXERCISES

GEOMETRY — TRIGONOMETRY

1. 50
2. 75.5
3. 63
4. 71.2
5. 2.17
6. 254
7. 188 1st quad.
8. 180 1st quad.
9. 147 1st quad.
10. 100 2nd quad.
11. 31.4 2nd quad.
12. 291 1st quad.
13. .4970
14. .5820
15. .9796
16. 4.625
17. .7206
18. .1980
19. .8712
20. 9.51436
21. .0192
22. .1994
23. .9744
24. 24.8978
25. .4305
26. 0
27. 2.145
28. 7°
29. 31.8°
30. 63.5°
31. 53.1°
32. 14.0°
33. 36.9°
34. 77°
35. 60.8°
36. 65.5°
37. a = 30.8
b = 39.4
B = 52°
38. C = 292
a = 275
B = 20°
39. C = 175
b = 165
B = 70°
40. A = 33.6°
B = 56.4°
a = 10.6
41. A = 27.5°
B = 62.5°
b = 115
42. A = 28.1°
B = 61.9°
c = 170

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TELEVISION TECHNICAL ASSIGNMENT

GEOMETRY — TRIGONOMETRY

EXAMINATION

1. Given three vectors:

$$A = 40,\ A_h = 10,\ \text{1st Quadrant}$$
$$B_v = 70,\ B_h = 30,\ \text{2nd Quadrant}$$
$$C = 90,\ C_v = 50,\ \text{4th Quadrant}$$

Find the common resultant of all three vectors combined, and state the quadrant in which the resultant lies.

$$A_v = \sqrt{A^2 - A_h^2} = \sqrt{1600 - 100} = \sqrt{1500} = 38.7$$

$$C_h = \sqrt{C^2 - C_v^2} = \sqrt{90^2 - 50^2} = \sqrt{8100 - 2500} = \sqrt{5600} = 74.8$$

$$R_h = A_h + B_h + C_h = 10 - 30 + 74.8 = +54.8$$

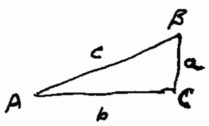
$$R_v = A_v + B_v + C_v = 38.7 + 70 - 50 = +58.7$$

$$R = \sqrt{54.8^2 + 58.7^2} = 80.3$$ First Quadrant.

2. Prove your work in Problem 1 by drawing all the forces to scale on the enclosed graph paper. Use the *largest* practicable scale within the limits of the graph.

3. Solve the right triangle and *construct* to scale on graph paper showing the values of all sides and angles.

(A) a = 240, A = 32°



$$c = a/\sin A = \frac{240}{\sin 32°} = \frac{240}{.52992} = 452.9$$

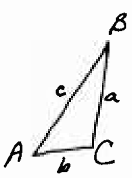
$$b = a/\tan A = \frac{240}{\tan 32°} = \frac{240}{.62487} = 384.2$$

$$C = 90° \quad A = 32°$$

$$B = 90° - 32° = 58°$$

/

(B) c = 100, b = 20



$$\cos A = \frac{20}{100} = .2 \quad A = \cos^{-1} .2 = 78° 28'$$

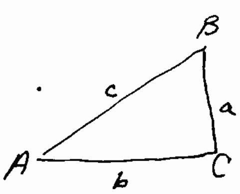
$$B = 90° - 78° 28' = 11° 32'$$

$$a = c \sin A = 100 \cdot .9798 = 97.98$$

/

4. Solve the right triangle and *construct* to scale on graph paper showing the values of all sides and angles.

(A) c = 72, a = 30



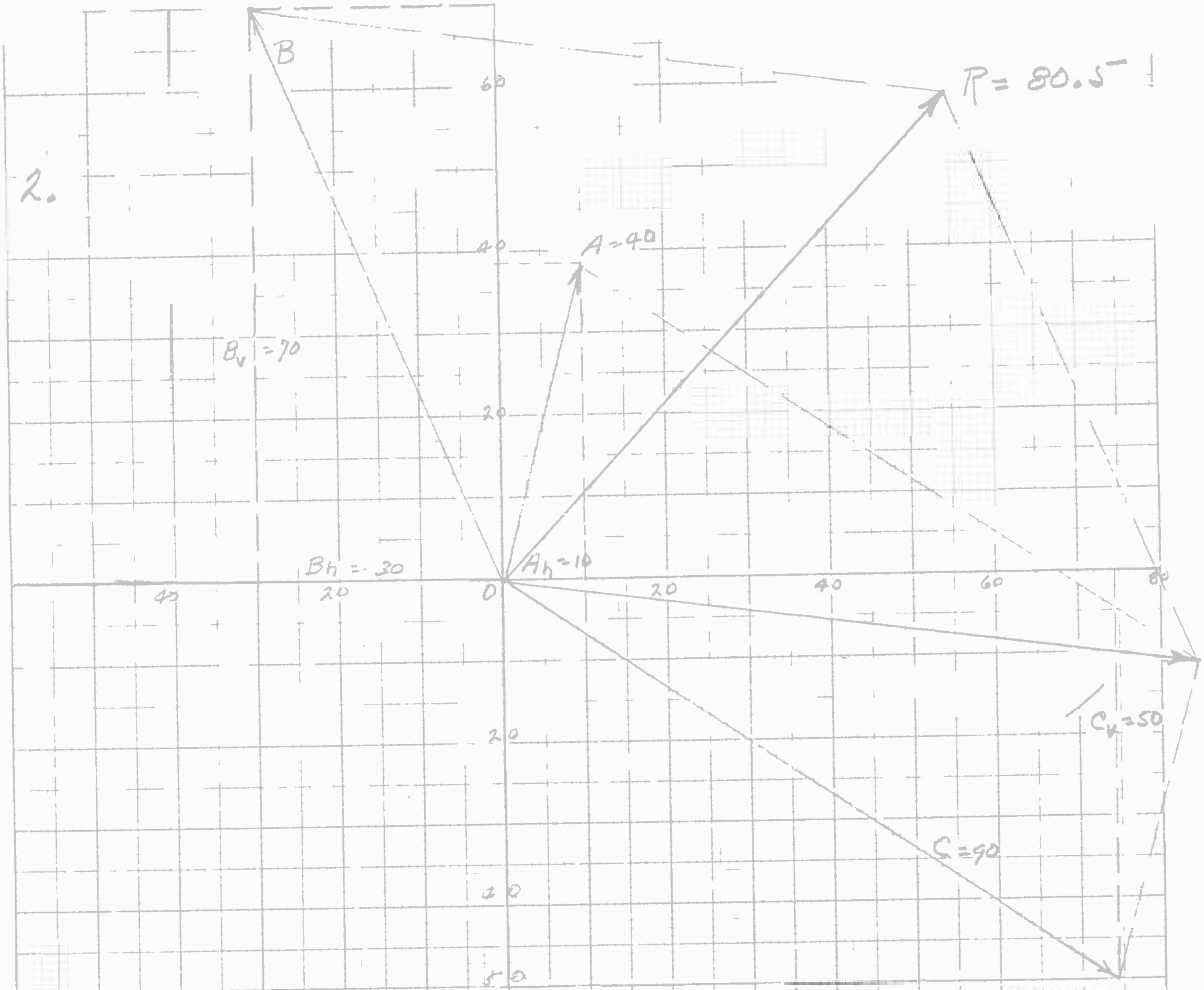
$$\sin A = \frac{30}{72} = .41667 \quad A = \sin^{-1} .41667 = 24° 37'$$

$$C = 90° \quad B = C - A = 90° - 24° 37' = 65° 23'$$

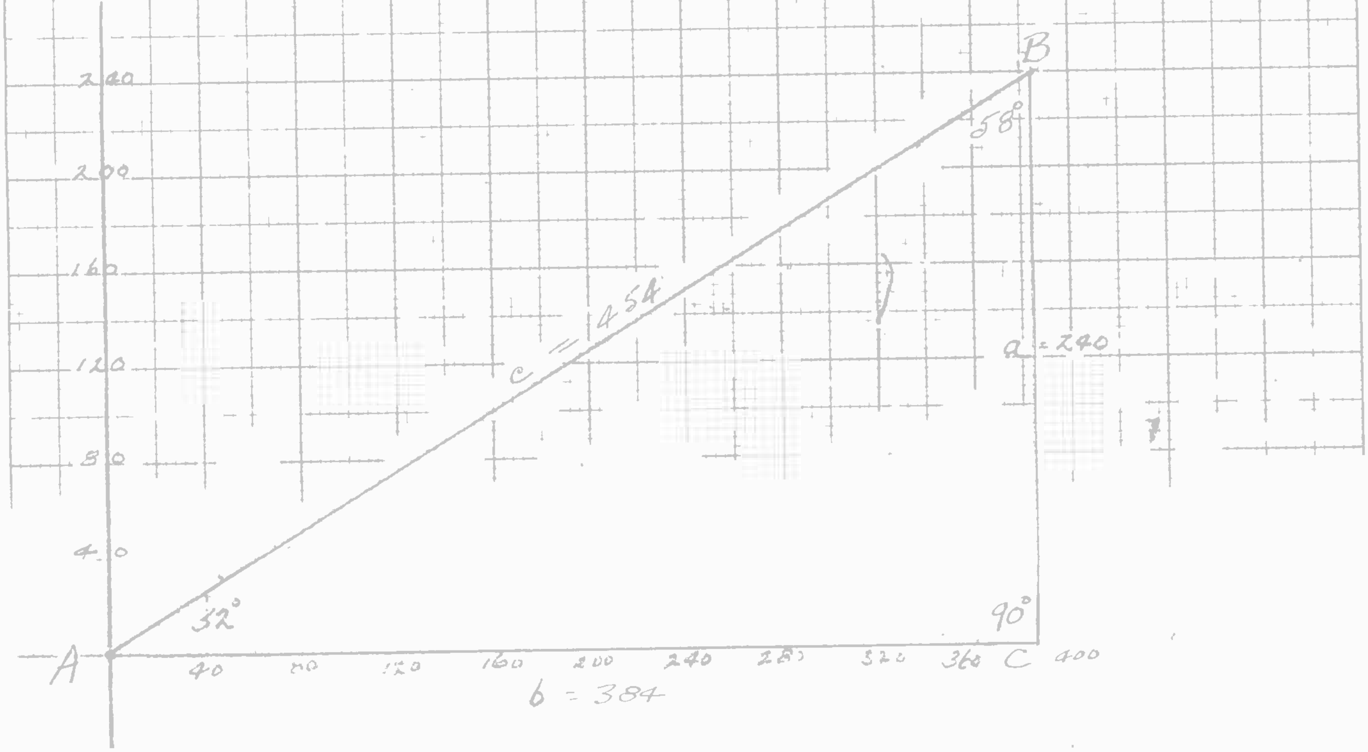
$$\cos A = \frac{b}{c} \quad b = c \cos A = 72 \cdot .90911 = 64.46$$

/

2.



3A.



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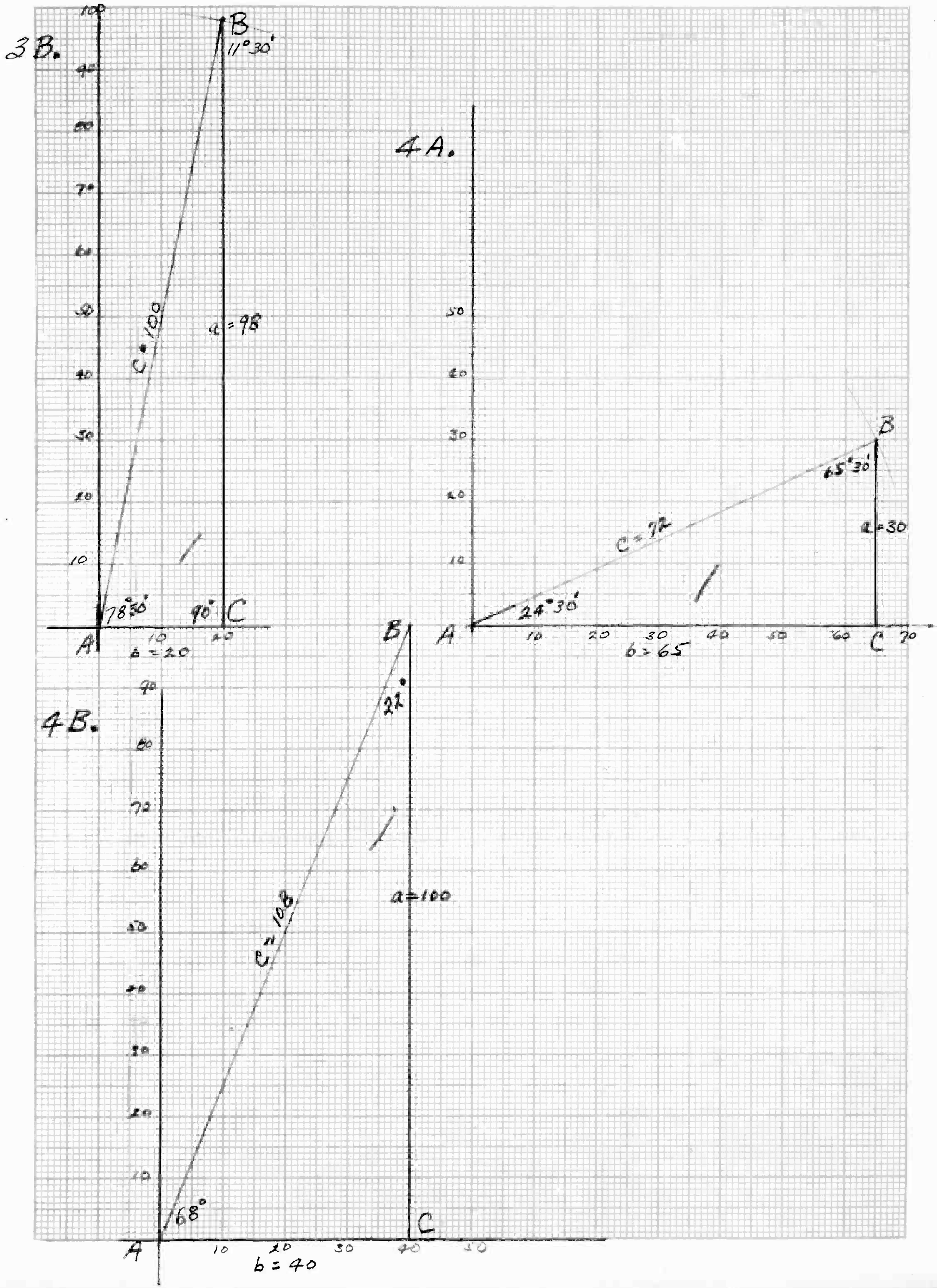
3B.
B
11°30'
c = 100
a = 98
78°30'
90°
C
A
b = 20
4A.
B
65°30'
c = 72
a = 30
24°30'
A
C
b = 65
4B.
B
22°
c = 108
a = 100
68°
C
A
b = 40

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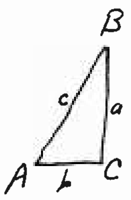
GEOMETRY — TRIGONOMETRY

EXAMINATION, Page 3.

4. (B) a = 100, b = 40



$\tan A = \frac{a}{b} = \frac{100}{40} = 2.5 \quad A = \tan^{-1} 2.5 = 68°12'$

$C = 90° \quad B = C - A = 90° - 68°12' = 21°48'$

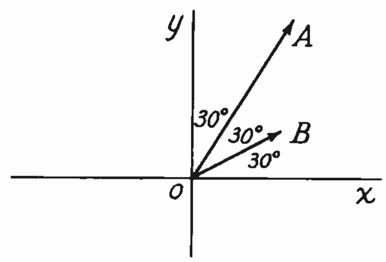
$\sin A = \frac{100}{c} \quad c = \frac{100}{\sin A} = \frac{100}{.92849} = 107.7$

5. *Given:* A = 10

B = 5

∠YOA = ∠AOB = ∠BOX = 30°

Find: The Resultant



$\sin BOX = .5 = \frac{B_y}{5} \quad B_y = 2.5$

$\sin AOX = .866 = \frac{A_y}{10} \quad A_y = 8.66$

$Y = 11.16$

$\cos BOX = .866 = \frac{B_x}{5} \quad B_x = 4.33$

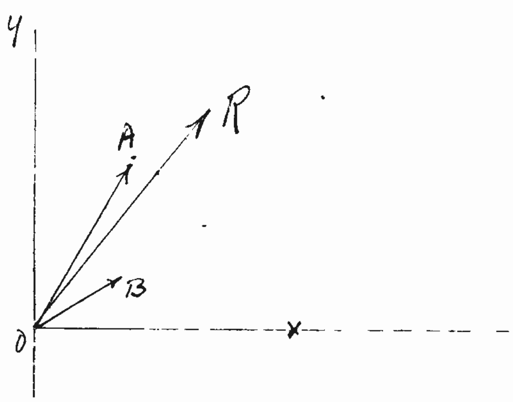
$\cos AOX = .5 = \frac{A_x}{10} \quad A_x = 5.00$

$X = 9.33$

$R = \sqrt{X^2 + Y^2} = \sqrt{9.33^2 + 11.16^2} = 14.55$ (slide rule)

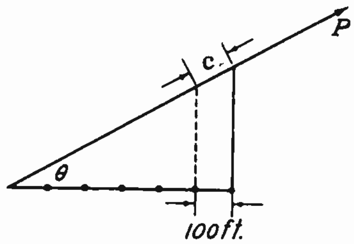
$\tan ROX = \frac{Y}{X} = \frac{11.16}{9.33} = 1.2$

$\tan^{-1} 1.2 = 50°12'$



$R = 14.55$ at $50°12'$

8.



(A) Given a right triangle with an angle θ of 28° 40′. Antennas along the base are spaced 100 ft. apart. Find the distance (C), which is a measure of the difference between each antenna and the next antenna, with regard to a point P some distance away.

$\cos$ ~~Sin~~ $\theta = \frac{100}{C}$ $\quad C = \frac{100}{\cos\theta}$

$\cos\theta = \cos 28°40' = .87743$ ~~.47971~~

$C = \frac{100}{.87743} = 113.97$ ft.

✓

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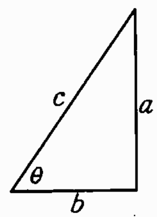
GEOMETRY — TRIGONOMETRY

EXAMINATION, Page 7.

8\.



(B) Given the following values for a right triangle:

$\frac{a}{c} = \sin\theta$, a = 33.75, c = 45.72

$\log a - \log c = \log \sin\theta$

By means of logarithms find θ.

$\log a = \log 33.75 = 1.52827$

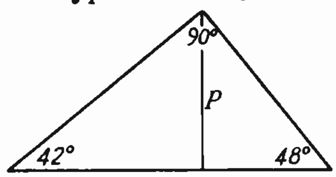
$\log c = \log 45.72 = 1.66011$

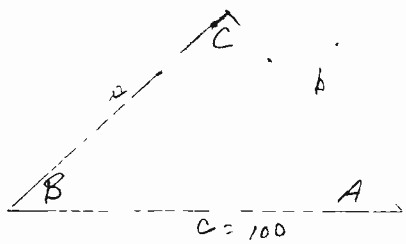
$\log a - \log c = \log \sin\theta = \bar{1}.86816$

from tables "Logarithms of the functions"

$\log \sin\theta^{-1} = 17° 5' 20.3''$

9. The hypotenuse of a right triangle is equal to 100 and the acute angles are 42° and 48°. What is the length of a perpendicular (P) dropped from the apex of the right angle to the hypotenuse?

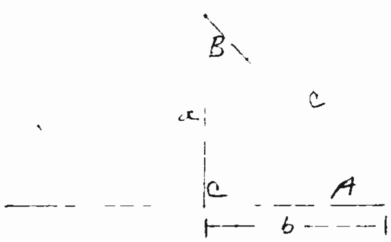




$\frac{b}{c} = \cos A = \cos 48° = .66913$

$b = .66913\, c$

$= 66.913$ units.



$\frac{a}{c} = \sin A = \sin 48° = .74314$

$c = 66.913$

$\therefore a = 66.913 \times .74314$

$= 49.723$

$P = 49.723$ units

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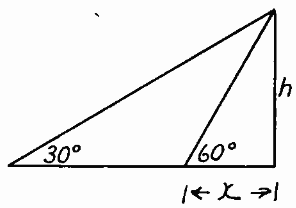
GEOMETRY — TRIGONOMETRY

EXAMINATION, Page 9.

10. At a certain distance, the angle of elevation of a vertical antenna tower is measured as 30°. At a point 300 ft. nearer the tower, the angle is again measured and found to be 60°. What is the height of the antenna?



$$\frac{h}{300+x} = \tan 30° = .57735$$

$$\frac{h}{x} = \tan 60° = 1.7321 \qquad h = 1.7321x$$

$$\frac{1.7321x}{300+x} = .57735$$

$$1.7321x = 173.205 + .57735x$$

$$(1.7321 - .57735)x = 173.205$$

$$1.15475x = 173.205$$

$$x = \frac{173.205}{1.15475} = 150.06 \text{ ft.}$$

$$h = 1.7321x = 1.7321 \times 150.06 = 259.92 \text{ ft.}$$