SPECIALIZED
TELEVISION ENGINEERING

TELEVISION TECHNICAL ASSIGNMENT

VECTOR ANALYSIS

Capitol Radio Engineering Institute
Washington, D. C.

713D

TELEVISION TECHNICAL ASSIGNMENT

VECTOR ANALYSIS

FOREWORD

Vector Analysis is the practical application of Geometry and Trigonometry as studied in a preceding assignment. A vector is simply a straight line drawn to some predetermined scale, the length of the line representing the numerical value of a current, voltage, resistance, reactance, or impedance in the circuit being analyzed. Vectors are drawn at given angles to some reference line, the angle expressing time in electrical degrees, or lead or lag of current with respect to voltage, or the phase relation between two voltages or two currents, or some similar relationship. Thus, if two vectors, one representing current and one representing voltage, are drawn from the reference point in exactly the same direction, E and I are said to be "in phase" and the circuit is purely resistive.

Vectors provide a means of graphically portraying the relations existing in an alternating current circuit, at power, audio, video or radio frequencies. If the vectors are carefully drawn to scale, the numerical relations existing may be measured by means of a scale and a protractor. (The protractor is a device for measuring angles.)

Usually it is simpler, by means of Geometry and Trigonometry, to calculate the numerical values than to carefully draw the problem to scale and solve by measurement. Interpretation of the calculations is simplified, however, if the vectors are drawn approximately to scale, the approximate graphical result being a check on the calulations.

Students find this assignment extremely interesting because it shows them graphically conditions they have known about theoretically but possibly have never really understood. This assignment is the basis of all electrical circuit calculations, whether for power, video, or radio frequencies.

E. H. Rietzke,
President.

- TABLE OF CONTENTS -

TECHNICAL ASSIGNMENT

VECTOR ANALYSIS

	Page
THE RADIAN	14

VECTOR ANALYSIS

A vector is defined as, "a directed magnitude, a line segment, a force or a velocity; the magnitude whose addition to a point in space transposes that point to another definite point." It particularly should be noted that a vector implies both amplitude and direction. Applied to electrical problems or expressions a vector may represent a current or voltage; the length of the vector represents the amplitude of the current or voltage; the direction of the vector represents the polarity of the voltage, direction of current flow, or the phase relation between a current and a voltage. A vector may be used electrically to represent a resistance, reactance or impedance in the sense that such factors have a definite effect on the relation between a current and voltage in a circuit. In other words, a resistance or a reactance may be thought of as a force vector which opposes the flow of current due to the effect of a voltage.

The most simple vector relations in electrical work occur in direct current circuits. For example, consider the circuit of Figure 1 in which two battery voltages E_1 and E_2 are applied in series across a resistance. Since the battery polarities are additive, the vectors representing E_1 and E_2 add arithmetically. Two methods of constructing the vectors are shown. In the upper arrangement both vectors are shown as starting at point o, extending in the same direction, being designated respectively at their extremes as E_2 and E_1, the amplitude of E_2 being less than that of E_1. The sum of the vectors $E_1 + E_2$ is then shown by adding the distances both extend from point o. The second method as shown in the lower arrangement consists of simply adding one vector to the extreme of the other, designating the vectors along the vectors instead of at their extremes, and then showing the sum E_1 +

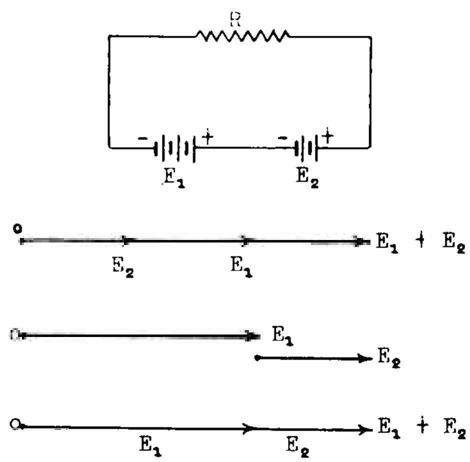


Fig. 1.

E_2 at the extreme. The writer favors the first method since it designates each factor operating on the point as starting directly at that point. This tends to somewhat simplify the conception of the more complex vector problems. $E_1 + E_2$ of course represents the total voltage applied across the resistance.

Figure 2 represents the circuit and voltage conditions if the connections of one battery, E_2, are reversed. The two methods are again shown. Since E_2 has been reversed in polarity, it is shown as $-E_2$. + directions are shown to the right of o with - directions to the left. It will be seen that with either method of representing the vector directions and amplitudes, the vector amplitude $E_1 + (-E_2)$ is to the right of point o. In direct current circuits the voltages either add or subtract arithmetically so that ordinary algebraic addition is sufficient to handle any type or problem. It is customary to show vectors expressing direct currents or voltages, the effects of resistance, or alternating currents or voltages *in purely resistance circuits, as operating along the horizontal axis.*

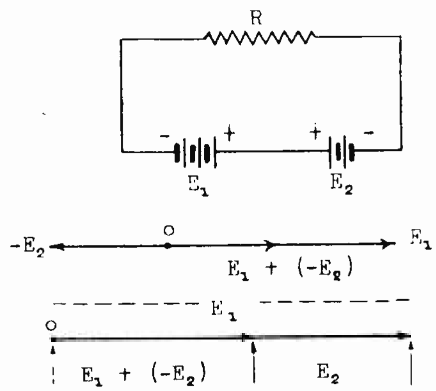


Fig. 2.

In most radio frequency circuits factors other than resistance are acting on the current and voltage relations. These factors are inductive and capacitive reactance. Neglecting the theory, at this point it is sufficient to state that the effect of reactance on the current-voltage relation is displaced 90° from that of resistance, the effect of capacity being to cause a 90° current lead, that of inductance being to cause a current lag of 90° behind the voltage. Thus in a circuit containing both reactance and resistance, the usual condition, vector directions along both the vertical and horizontal axes must be considered, as well as vectors at intermediate angles. Also in complex circuits or conditions, more than one voltage may be applied across a circuit at any given instant. Thus quite complex arrangements of vectors may result.

If the principles of vector analysis are fully understood, and if orderly processes are followed, the most complex vector problem may be simplified to *the point where it is only necessary to calculate the hypotenuse of a right triangle.*

Figure 3 illustrates again the two methods of representing vector addition. In the upper sketch vectors a and b operating on point o at right angles are shown as starting at point o. (In vector work it is customary to refer to point o as the pole.) In the lower sketch vector b extends in the proper direction from the extremity of a. In both cases the total force acting on the pole is equal to $\sqrt{a^2 + b^2}$ because a right triangle is formed in which the resultant force becomes the hypotenuse. However, the upper sketch, although it shows the magnitude of the resultant force acting on the pole, does not show the direction in which the pole would tend to move with forces a and b acting simultaneously. The lower sketch does not have this disadvantage since it shows both the magnitude and the angle θ of the resultant in respect to the horizontal base line. θ indicates the direction in which the pole would tend to move due to forces a and b.

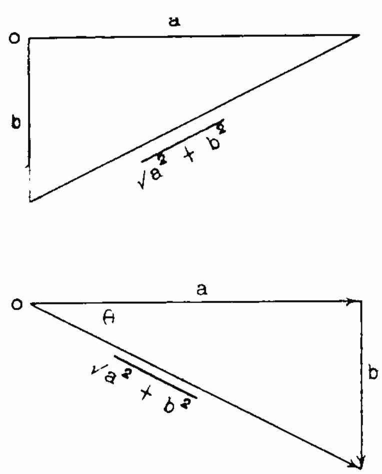


Fig. 3.

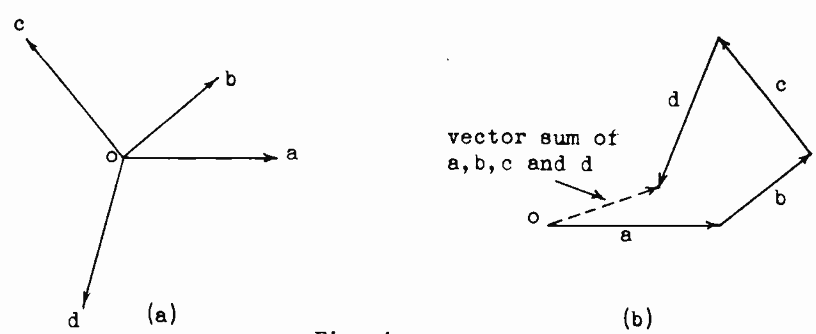


Fig. 4.

Figure 4(a) and 4(b) represent the same problem in two different ways. In both cases the forces a, b, c and d have the same amplitudes and directions. The advantage of the method in 4(b) is at once apparent because if the diagram is drawn to scale with the correct angles, when the last vector is drawn its extremity places the amplitude and direction of the resulting force acting on point o. This is shown by the dotted line in 4(b). From 4(a) it is difficult to state with any accuracy even the approximate amplitude and direction of the resultant. However, when it comes to writing the mathematical equation to express the vector sum of all the forces acting on the point, 4(a) will give a much clearer conception of the problem. Thus where the problem is to be solved only by graphical methods, or where a sketch is to be drawn with only approximate accuracy required to explain the operation of a circuit, the method as shown in 4(b) or the lower example in Figure 3 is to be preferred. Where an exact mathematical solution is required, the method of 4(a) is preferable and will be used in most problems in this and following lessons.

Revolving Vectors: In electrical work vectors are often spoken of as "revolving" or "rotating" around a point. Such vectors are found in alternating current problems at both power and radio frequencies. In such a problem a very exact mechanical analogy can be shown. Figure 5(a) represents one conductor in the armature of a generator rotating in a counter-clockwise direction under a magnetic field. At position 1 corresponding to 0^o the voltage is zero; at position 2, 90^o, the voltage is maximum; zero at position 3 or 180^o; maximum in the reverse direction in position 4 or 270^o; and zero again at the end of the revolution at 0^o. At any point in the revolution the *instantaneous voltage e is equal to maximum voltage E times the Sine of the angle made with respect to the horizontal axis.* Thus when the conductor is at position x, $e = E \sin \phi$ as shown.

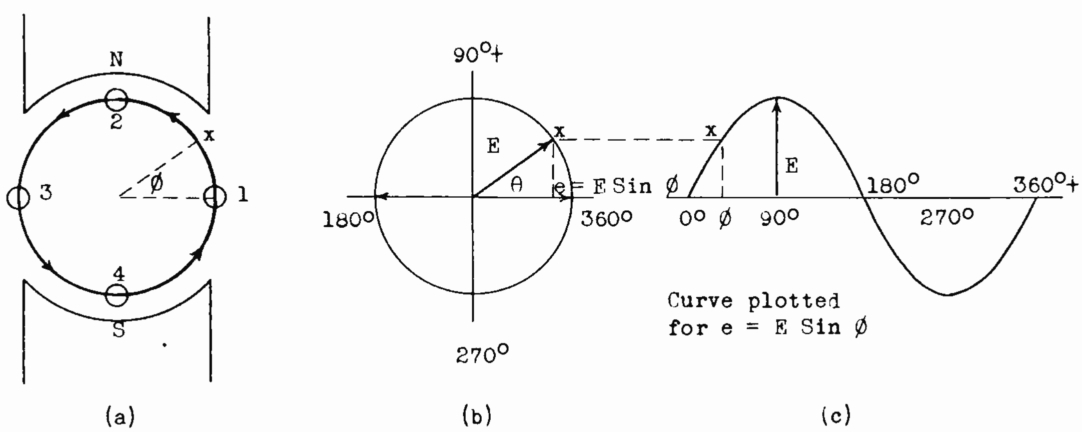


Fig. 5.

In Figure 5(b) the conductor is replaced with a vector, the length of which is made equal to the amplitude of the ***maximum*** voltage generated across the conductor at the 90° and 270° positions. As the vector rotates, the distance the vector extremity extends above or below the horizontal axis at any instant equals the voltage, + or -, generated across the conductor at that instant. Thus at ϕ degrees, $e = E \sin \phi$. If a point had been taken ϕ° below the horizontal axis, the equation would have been $-e = E \sin \phi$. It will be seen that on such a vector diagram *the instantaneous value e always represents the opposite side of the angle ϕ while E represents the hypotenuse.*

If the angular distance is plotted along a horizontal axis as in 5(c) with amplitudes e along the vertical axis, and enough points are plotted over a full cycle, the *sine curve* of Figure 5(c) will result. Thus any voltage in the form of a perfect sine curve may be thought of as being represented by a vector E rotating at constant velocity around a point. This vectorial conception of a voltage is very helpful in an analysis of the operation of an electrical circuit.

A number of alternating voltages may be applied across a circuit, or a number of current components may be flowing in a circuit. These voltages or currents may be out of phase, that is, may not reach maximum or minimum values simultaneously. In the following problems the currents or voltages will simply be called forces, A, B, C, etc., and methods of solving for the total or resultant force will be discussed in detail.

A quite detailed study having been made of the principles of Geometry and Trigonometry, both will be applied to the solution of forces acting on a point at various angles. Consider two forces acting upon a point; Force A at an angle of 60 degrees above the horizontal axis and Force B at an angle of 20 degrees above the horizontal axis. (See Figure 6.)

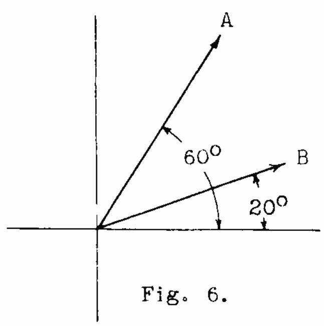


Fig. 6.

In the study of Geometry it was shown that in order to find the resultant of two forces acting on a point, if the forces are acting in the same direction or in exact opposition, the resultant force can be found by Algebraic Addition; if the forces are acting at an angle of exactly 90 degrees from each other, the solution requires simple geometric addition in which the resultant is equal to the square root of the sum of the squares of the two forces; but if the angle between the forces differs from zero, 90 or 180 degrees, the two forces must be resolved into their vertical and horizontal components, the proper combination of which will result in a right angle.

In Figure 6 are two forces, A and B, with an angle between them *not* equal to 90°. From geometry, force A may be resolved into two component forces, A_h which lies on the horizontal axis, and A_v which lies on the vertical axis. (See Figure 7.) In a similar manner force B may be resolved into its two component forces, B_h on the horizontal axis and B_v on the vertical axis. (See Figure 8.) Further inspection of Figures 7 and 8 shows that A_h and B_h are in the same direction therefore adding algebraically, and that A_v and B_v are also in the same direction therefore adding. The sum of A_h and B_h is H which lies along the horizontal axis while $A_v + B_v = V$ along the vertical axis. (See Figure 9.)

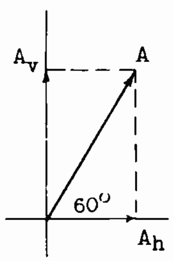


Fig. 7.

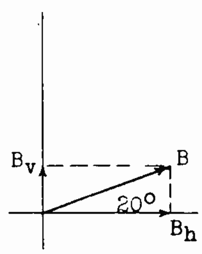


Fig. 8.

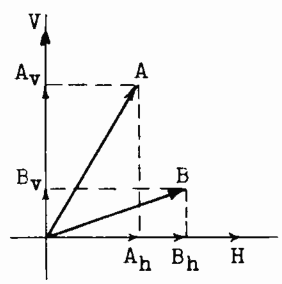


Fig. 9.

The resultant of the four component forces will be equal in all respects to the combined force of A and B. It is also apparent that the resultant F of forces H and V will equal the resultant of forces A and B.

H and V being at right angles to each other may now be added geometrically and the following equation will be obtained: $R = \sqrt{H^2 + V^2}$.

Enlarging and replacing H and V with their equivalents,

$$H = A_h + B_h$$

$$V = A_v + B_v$$

Then $$R = \sqrt{(A_h + B_h)^2 + (A_v + B_v)^2}$$ (Shown in Figure 10.)

This provides the basic equation for the solution of two forces, but geometry provides no practical method of determining mathematically the values of A_h, B_h, A_v and B_v. These values may be found very easily by the use of Trigonometry.

Consider force A: Referring to Figure 7 it is seen that force A forms the hypotenuse of a right triangle, the other two sides of which are A_h and A_v. In the study of trigonometry it was shown that, knowing one side and one angle, it is possible by using a table of Trigonometric functions, to find the other two sides of the triangle. In the present example force A forms the hypotenuse, A_h forms the adjacent side, and A_v forms the opposite side. The factors given are force A and the Angle 60°. Knowing the hypotenuse and the angle, from definition,

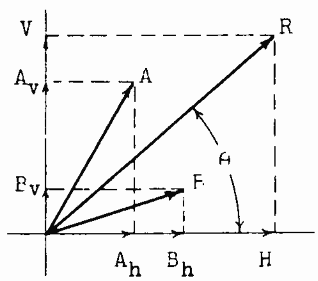


Fig. 10.

Adjacent = Hypotenuse times Cosine Angle.

$$A_h = A \cos 60^\circ$$

Opposite = Hypotenuse times Sine Angle.

$$A_v = A \sin 60^\circ$$

The same method is used to find the components of force B. (Figure 8.)

$$R = \sqrt{(A_h + B_h)^2 + (A_v + B_v)^2}$$

Replacing the fundamental components with their equivalents as determined by trigonometry:

$$R = \sqrt{(A \cos 60^\circ + B \cos 20^\circ)^2 + (A \sin 60^\circ + B \sin 20^\circ)^2}$$

This equation may be solved by simply substituting the numerical values of forces A and B and the values of the Cosines and Sines of the respective angles as obtained from the table.

From Figure 10 it is seen that $(A \cos 60^\circ + B \cos 20^\circ)$ forms the adjacent side of a right triangle of which the resultant is the hypotenuse. $(A \sin 60^\circ + B \sin 20^\circ)$ forms the opposite side of the same triangle. By definition, the Tangent Angle equals $\frac{\text{Opposite Side}}{\text{Adjacent Side}}$. Replacing with equivalent values,

$$\tan \theta = \frac{(A \sin 60^\circ + B \sin 20^\circ)}{(A \cos 60^\circ + B \cos 20^\circ)}$$

$$\tan^{-1} \frac{(A \sin 60^\circ + B \sin 20^\circ)}{(A \cos 60^\circ + B \cos 20^\circ)} = \theta$$

In other words, the total sine value divided by the total cosine value, (*not squared*), equals the Tangent of the angle of the resultant force to the horizontal axis. Knowing the Tangent of the angle it is only necessary to refer to the Trig table to find the angle.

Having determined the resultant of forces A and B and the angle of the resultant, the problem is completely solved.

Consider as an example two forces, A and B, acting on point o; force A at an angle of 50° *above* the horizontal axis and force B at an angle of 60° *below* the horizontal axis. Assume that the amplitude of B is 30 and the amplitude of A is 10. See Figure 11. The angles could also be expressed as Angle A = 50°, Angle B = 300°, both counter-clockwise from the same position.

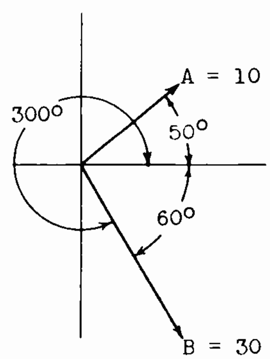


Fig. 11.

Again two forces are given whose general directions are both to the right of the vertical axis, but unlike the problem illustrated in Figure 6, the two forces are not both above the horizontal axis, force B being below the reference axis.

As in the preceding problem the first step is to find the horizontal and vertical components of the two forces, A and B.

A represents the hypotenuse of a right triangle of which A_h is the adjacent side and A_v is the opposite side. By trigonometry,

$$A_h = A \text{ Cos } 50^\circ \text{ and } A_v = A \text{ Sin } 50^\circ$$

B represents the hypotenuse of a right triangle of which B_h is the adjacent side and B_v is the opposite side. By trigonometry,

$$B_h = B \text{ Cos } 60^\circ \text{ and } B_v = B \text{ Sin } 60^\circ$$

From inspection of Figure 12 it is determined that the combined force acting along the horizontal axis is equal to $H = A_h + B_h$ and is the adjacent side of the triangle of which resultant R is the hypotenuse. Enlarging, this becomes

$$H = A \text{ Cos } 50^\circ + B \text{ Cos } 60^\circ$$

Figure 12 shows also that the combined force acting along the vertical bisector is the *difference* between the vertical components of A and B and is in the direction of the greater, which in this case is B_v. V is then equal to $B_v - A_v$ and is *below* the horizontal axis. Enlarging, $V = B \text{ Sin } 60^\circ - A \text{ Sin } 50^\circ$.

Since the resultant force is the hypotenuse of a right triangle, the other two sides of which are H and V, the resultant R is given by,

$$R = \sqrt{H^2 + V^2}$$

Enlarging and replacing H and V with their equivalent values,

$$R = \sqrt{(A \cos 50^\circ + B \cos 60^\circ)^2 + (B \sin 60^\circ - A \sin 50^\circ)^2}$$

Replacing in the above equation the numerical values of A and B and using the mathematical tables to find the Cosines and Sines of the given angles, it is not at all difficult to find the resultant force. A = 10 and B = 30.

$$R = \sqrt{(10 \cos 50^\circ + 30 \cos 60^\circ)^2 + (30 \sin 60^\circ - 10 \sin 50^\circ)^2}$$

From the Trig Tables,

$\cos 50^\circ = .643$	$\sin 50^\circ = .766$
$\cos 60^\circ = .5$	$\sin 60^\circ = .866$

Then

$$A \cos 50^\circ = 10 \times .643 = 6.43$$
$$B \cos 60^\circ = 30 \times .5 = 15.$$
$$B \sin 60^\circ = 30 \times .866 = 25.98$$
$$A \sin 50^\circ = 10 \times .766 = 7.66$$

$$R = \sqrt{(6.43 + 15)^2 + (25.98 - 7.66)^2}$$

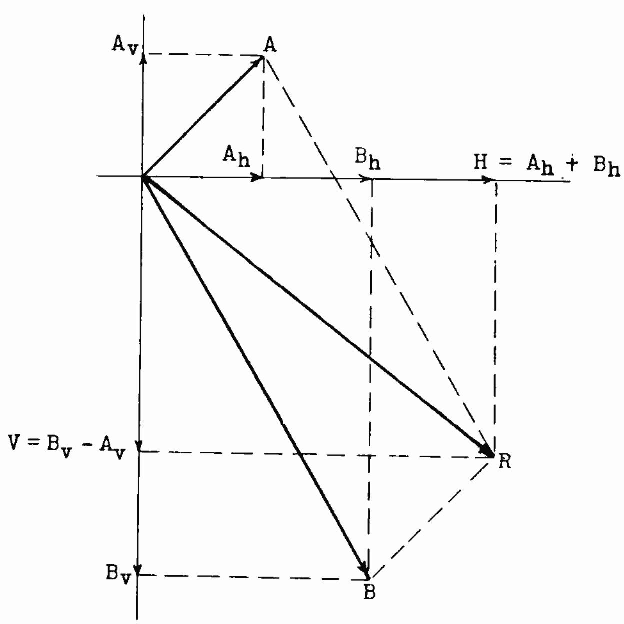


Fig. 12.

Adding,

$$R = \sqrt{21.43^2 + 18.32^2}$$

Squaring Values

$$R = \sqrt{460.24 + 335.64}$$

Adding squared values

$$R = \sqrt{795.88}$$

Extracting square root

$$R = 28.2$$

The Tangent of angle θ made by R with respect to the horizontal axis is equal to $\frac{\text{Opposite}}{\text{Adjacent}}$, in this case the $\frac{\text{Sine Value}}{\text{Cosine Value}}$. From the equations above the total Sine value is 18.32 and the Cosine value is 21.43. Therefore Tangent $\theta = \frac{18.32}{21.43} = .854$.

$$\theta = \text{Tan}^{-1}\ .854 = 40.5^\circ$$

Referring to a Trig table and looking up the angle corresponding to a tangent of .854 it is found to be 40.5° *below* the horizontal axis, because the vertical component of B which is below the horizontal is greater than the vertical component of A above the horizontal. This is clearly shown in Figure 12.

A thorough understanding of the principles involved in these problems will provide means for solving the most complex alternating current problems in which the forces added vectorally will be voltages, currents, or impedances, as well as many other types of problems encountered in the solution of forces acting on a point.

In the event of three or more forces acting on a given point at various angles, the problem will be handled in a similar manner with the exception that both the Cosine and Sine values will be made up of three or more component parts. For example,

Force A, Angle 30°
Force B, Angle 70°
Force C, Angle 345°

In the case of C, Angle 345°, it is necessary to first find the angle to the nearest horizontal axis. Thus $\theta_C = 360 - 345 = 15^\circ$, as shown in Figure 13.

$$R = \sqrt{(A \text{ Cos } 30^\circ + B \text{ Cos } 70^\circ + C \text{ Cos } 15^\circ)^2 + (A \text{ Sin } 30^\circ + B \text{ Sin } 70^\circ - C \text{ Sin } 15^\circ)^2}$$

If the problem consists of solving for three forces as shown in Figure 14, the equation for the resultant R will be

$$R = \sqrt{(B \text{ Cos } 60^\circ + C \text{ Cos } 80^\circ - A \text{ Cos } 40^\circ)^2 + (A \text{ Sin } 40^\circ + B \text{ Sin } 60^\circ - C \text{ Sin } 80^\circ)^2}$$

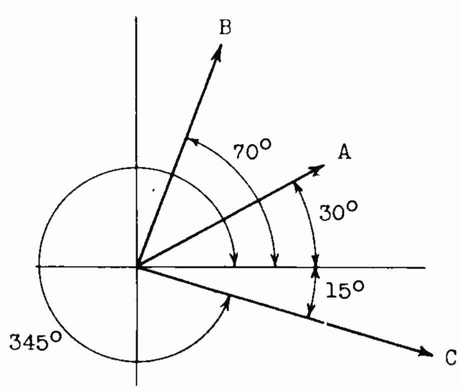


Fig. 13.

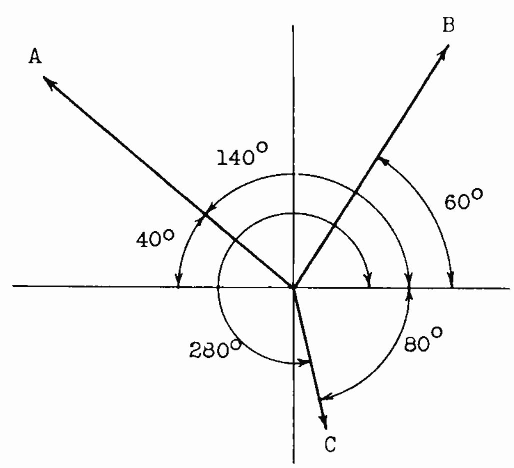


Fig. 14.

It will be observed that with respect to the horizontal axis to the right of the vertical bisector, and for counter-clockwise rotation, the angle of B = 60° is used in the equation. The angle of A is given as 140° with respect to the same point and the angle actually used is the angle to the nearest horizontal line, that is, $180^\circ - 140^\circ = 40^\circ$. In the case of C, the angle is given as 280°, this subtracted from 360° giving the angle to the nearest horizontal as 80° which is used in the equation.

In this problem the horizontal or Cosine components of B and C are in the same direction (to the right) therefore adding, while the horizontal component of A is to the left thus subtracting from the sum of the Cosine components of B and C. If the Cosine component of A is less than the sum of the Cosine components of B and C, the resultant will be to the right of the vertical axis. If the cosine component of A is greater than the sum of the cosine components of B and C then the resultant will be to the left of the vertical axis.

The vertical or Sine components of A and B are both above the horizontal thus adding; the Sine component of C is below the horizontal axis thus subtracting from the sum of the Sine components of A and B. Whether the resultant is above or below the horizontal line is determined by the relative values of the Sine components. In the problem shown in Figure 14 it is evident by inspection that the resultant will be above the horizontal axis and to the left of the vertical axis. Actual measurement shows that R in this case will be in the second quadrant, that is, above the horizontal axis to the left of the vertical bisector.

The actual angle of the resultant *to the nearest horizontal line* will be found from the tangent.

$$\mathrm{Tan}^{-1}\ \frac{A \sin 40^\circ + B \sin 60^\circ - C \sin 80^\circ}{B \cos 60^\circ + C \cos 80^\circ - A \cos 40^\circ} = \theta$$

Expressing the angle corresponding to this tangent as θ and assuming that R is in the second quadrant, then the actual angle with respect to the zero position (the horizontal axis to the right of the bisector), operating in a counter-clockwise direction and calling this angle ϕ, is

$$\phi = 180^\circ - \theta$$

A typical electrical problem consisting of three branches of a parallel circuit is shown in Figure 15. (In this case neglect the theory involved and work according to the vector diagram and mathematical equation.) According to the laws of electrical circuits, in the first branch which contains L and R the current will lag behind the voltage. In the second branch which contains resistance only, the current and voltage will be in phase. In the third branch which contains capacity and resistance the current will lead the voltage. In each case the circuit components and the generator voltage and frequency are such as to cause the currents and phase angles as specified. The vector diagram of this circuit is shown in Figure 16.

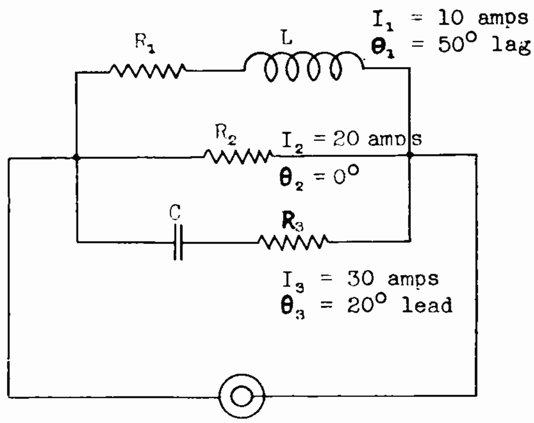


Fig. 15.

The horizontal axis to the right of o is designated E and the relations of the currents with respect to E are shown. It is specified in the circuit diagram that E and I_2 are in phase so that the angle between them is 0°; I_1 lags E by 50° and since the rotation is assumed, unless otherwise stated, to be counter-clockwise, I_1 will be shown below the horizontal axis or lagging E by 50°; I_3 leads E by 20° so is shown above the E axis.

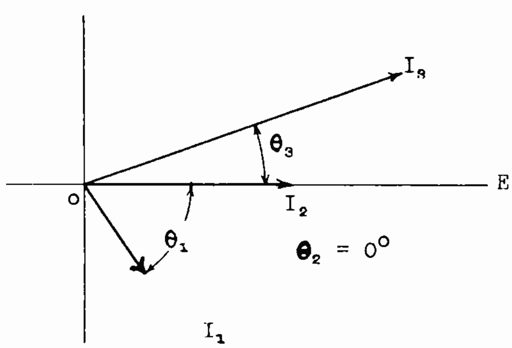


Fig. 16.

Inspection of the vector diagram shows that the Cosine values of I_1, I_2, and I_3, are all to the right of the pole and hence positive. The sine component of I_3 is +, of I_1 -, and of I_2 zero. Since Cos 0° = 1 the cosine component of I_2 = I_2 and the equation may be written as follows:

$$I = \sqrt{(I_2 + I_3 \cos 20^\circ + I_1 \cos 50)^2 + (I_3 \sin 20^\circ - I_1 \sin 50^\circ)^2}$$

While the vector diagram clearly shows that the resultant vector I will be to the right of o, it is not certain whether it will be above or below the horizontal axis, that is, whether the resultant current I will lead or lag the voltage. This is determined as follows: If the resultant Sine value is + the current vector will be above the horizontal axis and the current will lead E; if the resultant Sine value is - the current vector will be below the horizontal axis and the current will lag behind E. Evaluating I_1, I_2 and I_3,

$$I = \sqrt{(\underset{I_2}{20} + \underset{I_3}{30 \text{ Cos } 20^\circ} + \underset{I_1}{10 \text{ Cos } 50^\circ})^2 + (\underset{I_3}{30 \text{ Sin } 20^\circ} - \underset{I_1}{10 \text{ Sin } 50^\circ})^2}$$

From tables: Sin 20° = .342 Cos 20° = .94
Sin 50° = .766 Cos 50° = .643

$$I = \sqrt{[20 + (30 \times .94) + (10 \times .643)]^2 + [(30 \times .342) - (10 \times .766)]^2}$$

$$I = \sqrt{(20 + 28.2 + 6.4)^2 + (10.3 - 7.7)^2} = \sqrt{54.6^2 + 2.6^2}$$

$$I = \sqrt{2988} = 54.7 \text{ amperes.}$$

It will be seen that the Sine value is positive, therefore the angle of current with respect to the horizontal or voltage axis will be + and the resultant vector I will be in the first quadrant.

$$\text{Tan } \theta_I = \frac{\text{Sine Value}}{\text{Cosine Value}} = \frac{2.6}{54.6} = .0476$$

$$\text{Tan}^{-1} .0476 = \theta_I = 2.7^\circ$$

The current I in the external generator circuit, which is the vector sum of all branch currents, equals 54.7 amperes and this current leads the voltage by 2.7°. The electrical theory of such a circuit will be discussed in detail in later lessons.

The student should draw vector diagrams to complete Figures 13 and 14 showing the forces and their respective angles, and plotting out the component forces and the resultant as shown in Figures 10 and 12.

The following exercise problems should be carefully worked out, finding resultant and angle of resultant, drawing diagrams approximately to scale.

1. Force A = 40, Angle 20°
   Force B = 60, Angle 70°
2. Force X = 100, Angle 37°
   Force Y = 100, Angle 280°
3. Force X = 150, Angle 335°
   Force Y = 200, Angle 60°
4. Force X = 50, Angle 40°
   Force Y = 100, Angle 65°
   Force Z = 80, Angle 325°
5. Force A = 100, Angle 75°
   Force B = 100, Angle 310°
   Force C = 80, Angle 25°

6. Force A = 25, Angle = 98°
Force B = 50, Angle = 175°
Force C = 40, Angle = 295°

7. Force X = 115, Angle = 60°
Force Y = 73, Angle = 343°
Force Z = 155, Angle = 265°

8. Force A = 60, Angle = 22.5°
Force B = 58, Angle = 121°
Force C = 71, Angle = 225°

9. Force W = 100, Angle = 115°
Force X = 330, Angle = 350°
Force Y = 150, Angle = 190°
Force Z = 200, Angle = 20°

10. Force A = 3.5, Angle = 270°
Force B = 1.5, Angle = 215°
Force C = 1.25 Angle = 148.7°
Force D = 2.7, Angle = 72.3°
Force E = 1.5, Angle = 300°

THE RADIAN: To express angles in degrees, minutes and seconds is often rather inconvenient. In electrical work, particularly in the derivation of formulas, it is often much more convenient to express an angle in terms of *frequency and time.* An electrical cycle, since it consists of 360 electrical degrees, may be compared to a circle. (See Figure 5.) One cycle is represented by a complete revolution of a vector, the length of the vector representing the maximum value of the force. The instantaneous value is represented by the opposite side of a right triangle having the maximum value as a hypotenuse *with angle ∅ representing the time in degrees* at which the instantaneous value is desired. Figure 5(c) shows the magnitude of the instantaneous value plotted vertically against time in degrees along the horizontal axis.

In Figure 5(b) it is shown that the instantaneous value e = E Sin ∅ where E = maximum value attained during the cycle and ∅ is the angle to the nearest horizontal axis. Frequency is expressed in cycles per second and it is apparent that some means of expressing ∅ for *any* time would be more convenient than limiting ∅ to 360° or the time of one cycle. The radian provides a method of indicating the value for ∅ for any value of time.

The radian is defined as that angle which, if placed at the center of a circle, would intercept an arc of the circumference equal to the radius of the circle. In Figure 17 the distance along the circumference from A to B equals the radius OA so ∅ equals one radian. Do not confuse the radian with the radius. A radian is a *unit angle* whereas the radius is a linear measure. The circumference of a circle is equal to π times the diameter, and since the diameter equals twice the radius, $C = 2\pi R$. But one radian is subtended by a distance along

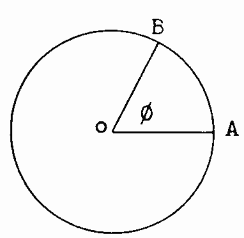


Fig. 17.

the circumference equal to R. Since R divides into C 2π times there must be 2π radians in one circle. One circle = 360 degrees, from which one radian equals $360/2\pi = 360/6.2832 = 57^\circ\ 17'\ 44''$ or approximately 57.3°. One degree equals $1/57.3 = .01745$ radian.

Since one cycle equals 360° or 2π radians, then in F cycles per second the vector will rotate through $2\pi F$ radians for every second of time. In electrical work the quantity $2\pi F$ is usually indicated by the Greek letter ω (omega). If the vector completes ω radians in one second, then in t seconds it will complete ωt radians. Therefore $\phi = \omega t$ radians. Thus ϕ is measured in terms of time and frequency instead of degrees, minutes and seconds. Since $e = E \sin \phi$,

then $$e = E \sin \omega t = E \sin 2\pi Ft$$

where
E = maximum value of voltage in the cycle
F = frequency in cycles per second
t = time in seconds
2π = constant = 6.28

Example: Find the instantaneous voltage .003 second after it passes through zero degrees if the maximum value of E is 200 volts, F = 400 cycles/second.

$$E = 200 \text{ volts}$$
$$F = 4 \times 10^2$$
$$t = 3 \times 10^{-3} \text{ second}$$
$$e = E \sin 2\pi Ft$$
$$= 200 \sin (6.28 \times 4 \times 10^2 \times 3 \times 10^{-3})$$
$$= 200 \sin 7.536 \text{ radians}$$
$$1 \text{ radian} = 57.3^\circ$$
$$\phi = 7.536 \text{ radians} = 7.536 \times 57.3 = 431.8^\circ$$

Since in .003 second E has completed 431.8° it is apparent that the vector has completed one cycle and 431.8 - 360 or 71.8 degrees of the second cycle. Figure 18 shows the position of E at .003 second after passing through zero. ϕ then must reduce to $\phi_1 = 71.8^\circ$ and

$$e = 200 \sin 71.8^\circ$$
$$\sin 71.8^\circ = .95$$
$$e = 200 \times .95 = 190 \text{ volts.}$$

190 volts is the instantaneous voltage .003 second after it passed through zero degrees.

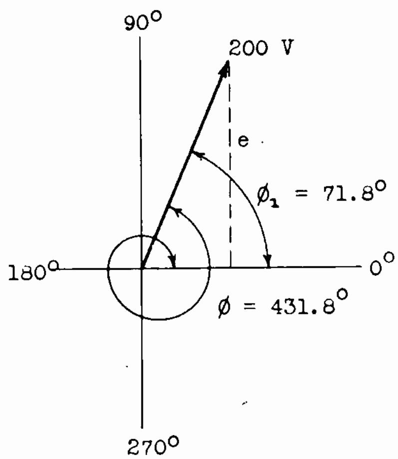


Fig. 18.

In dealing with relations between current and voltage it is often necessary to consider the time elapsing between maximum values of current and voltage in a circuit, expressed in terms of angular measure as the angle of lag; the phase difference between two voltages impressed across a circuit, etc.

Example: At a frequency of 60 cycles per second find the angle in radians between two voltages displaced by .001 second.

As previously shown $\phi = \omega t = 2\pi Ft$ radians

$$\phi = 6.28 \times 60 \times .001 = .3768 \text{ radian.}$$

By multiplying by 57.3 this becomes 21.6°.

The two voltages are 21.6° apart.

By expressing the angle in terms of radians as being equal to ωt, the angular relations between any number of voltages or currents at any number of frequencies may be expressed in the form of equations. In the practical use of such equations it simply becomes necessary to evaluate with the proper values of f and t. Of course the above equation may be easily rearranged.

$$\text{Radians} = \omega t$$

$$t = \frac{\text{Radians}}{\omega}$$

$$f = \frac{\text{Radians}}{2\pi t}$$

An example of the practical use of the radian as an angular measure is in the following type of equation:

$$e_1 = E_1 \text{ Sin } \omega_1 t$$

$$e_2 = E_2 \text{ Sin } \omega_2 t$$

These equations state that the instantaneous voltages e_1 and e_2 are functions of their respective frequencies at some specified time. If it is assumed that the two voltages have different frequencies as expressed by ω_1 and ω_2, and that both are applied across a common load, it may be desired to find the instantaneous voltage across the load at some given time after the two pass through zero simultaneously. In that case,

$$e = e_1 + e_2$$

$$e = E_1 \text{ Sin } \omega_1 t + E_2 \text{ Sin } \omega_2 t$$

As a practical example, consider the equation for e, when E_1 = 100 volts, f_1= 1000 cycles, E_2 = 200 volts, f_2 = 1100 cycles, t = .0006 second after the instant at which the two voltages reached zero values simultaneously. To find the voltage e at time t.

Angle $E_1 = \theta_1 = \omega t = 2\pi f_1 t = 6.28 \times 1000 \times .0006 = 3.77$ radians $= 216^\circ$

Angle $E_2 = \theta_2 = \omega t = 2\pi f_2 t = 6.28 \times 1100 \times .0006 = 4.145$ radians $= 237.5^\circ$

The positions of the two voltage vectors are shown in Figure 19. It will be observed that since f_2 is greater than f_1, E_2 has completed a greater portion of its cycle in .0006 second than has E_1. The length of the vector E_2 is twice as long as that of E_1 because the peak values of E_1 and E_2 are given as 100 and 200 volts respectively.

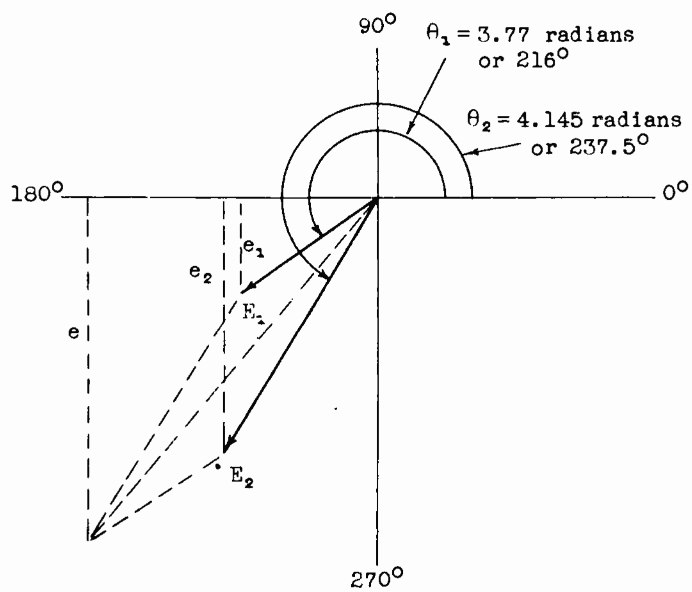


Fig. 19.

It should also be noted that the equation for e is one of *simple algebraic addition of the sine or vertical components of two forces* as shown by the dotted lines in Figure 19. Thus E_1 Cos $\omega_1 t$ and E_2 Cos $\omega_2 t$ form the *adjacent* or *horizontal* components of E_1 and E_2 respectively, and E_1 Sin $\omega_1 t$ and E_2 Sin $\omega_2 t$ respectively form the *vertical* components of E_1 and E_2.

It has been stated that the angles used in such calculations are always to the ***nearest horizontal axis.*** Therefore to actually evaluate the equation, the angle of E_1 is taken as $216^\circ - 180^\circ = 36^\circ$, and the angle of E_2 is taken as $237.5^\circ - 180^\circ = 57.5^\circ$.

From the tables: $\text{Sin } 36^\circ = .5878$

$\text{Sin } 57.5^\circ = .8434$

Then, $e = E_1 \text{ Sin } \omega_1 t + E_2 \text{ Sin } \omega_2 t$ becomes,

$$e = (100 \times .5878) + (200 \times .8434) = 58.8 + 168.7 = 227.5 \text{ volts.}$$

Measurement of e on the vector diagram which is drawn quite closely to scale gives a quick check on the accuracy of this work. The vector diagram shows quite clearly that the two vertical components--Sine values--add, because both voltage vectors are in the same quadrant. It will be observed however that since the vector E_2 is rotating considerably more rapidly than is the vector E_1 at certain instants one will be in one quadrant and one in another. To provide for such cases so that equations similar to the above may be written without regard to quadrants and still be algebraically correct, the following procedure is adopted.

When the vector is in the 1st or 2nd quadrant, the Sine function is +; when in the 3rd or 4th quadrant, the Sine function is -. Thus in the above example the Sine functions of both E_1 and E_2 are negative and $e = -227.5$ volts. In this case the numerical value is not changed.

When the vector is in the 1st or 4th quadrant the Cosine function is +; when in the 2nd or 3rd quadrant the Cosine function is -. Thus in Figure 19, both Sine and Cosine functions would be given - signs.

Examples:	at 45°,	Sin = +.707	Cos = +.707
	at 135°,	Sin = +.707	Cos = - .707
	at 225°,	Sin = -.707	Cos = - .707
	at 315°,	Sin = -.707	Cos = +.707

Where the angles in degrees are known before an equation is written, the equation may be written with these facts in mind and the signs of the functions properly arranged. Where the angle is given in terms of t and f, it is usually not possible in advance to predict the angular position of the vector, even approximately, so that the equation must be written disregarding the signs of the functions. When the equation is then evaluated, great care must be exercised in properly writing the signs. Figure 20 shows the signs of the sine and cosine functions for each quadrant.

II Sin + Cos -	I Sin + Cos +
III Sin - Cos -	IV Sin - Cos +

Fig. 20.

It is sometimes desirable in the solution of a problem to express one function in terms of another. By substitution and transposition the basic trigonometric functions may be converted into many different forms. Often by proper substitution of these equivalent formulas the solution of a problem is greatly simplified. **Pages 266 and 267 of Chemical Handbook Mathematical Tables lists a large number of these substitute formulas. For example:**

$$\text{Sin } \theta = \sqrt{1 - \text{Cos}^2 \theta} = \frac{\text{Tan } \theta}{\sqrt{1 + \text{Tan}^2 \theta}}$$

$$\text{Cos } \theta = \sqrt{1 - \text{Sin}^2 \theta} = \frac{1}{\sqrt{1 + \text{Tan}^2 \theta}}$$

$$\text{Tan } \theta = \frac{\text{Sin } \theta}{\sqrt{1 - \text{Sin}^2 \theta}} = \frac{\sqrt{1 - \text{Cos}^2 \theta}}{\text{Cos } \theta}$$

The actual derivation of these formulas is of minor importance but the realization that an equivalent formula may be substituted in an equation will frequently simplify the problem.

For example, if a problem involves the use of Sin θ and Cos θ is known it is a simple matter to substitute the value $\sqrt{1 - \text{Cos}^2 \theta}$ for Sin θ. $\text{Cos}^2 \theta$ means $(\text{Cos } \theta)^2$. If Tan θ is required and Sin θ is given, then $\text{Sin } \theta/\sqrt{1 - \text{Sin}^2 \theta}$ may be substituted for Tan θ.

The student should check several of these substitute formulas by solving them for various angles and comparing the results obtained with those given in the Mathematical Tables.

For example prove $\text{Cos } A = \frac{1}{\sqrt{1 + \text{Tan}^2 A}}$. Let $A = 30^\circ$. $\text{Cos } 30^\circ = .866$ and $\text{Tan } 30^\circ = .5774$.

If the formula is true then

$$.866 = \frac{1}{\sqrt{1 + .5774^2}}$$

$$.866 = \frac{1}{\sqrt{1 + .3334}}$$

$$= \frac{1}{\sqrt{1.3334}}$$

$$= \frac{1}{1.155}$$

$$.866 = .8658 \text{ or } .866$$

This result is close enough to prove the equation is correct.

ANSWERS TO EXERCISE PROBLEMS

1. $R = 91$ at 50°
2. $R = 104$ at 338°
3. $R = 260$ at 25°
4. $R = 165$ at 28°
5. $R = 171$ at 19°
6. $R = 37$ at 191°
7. $R = 137$ at 326°
8. $R = 35$ at 137°
9. $R = 332$ at 13.2°
10. $R = 2.55$ at 253°

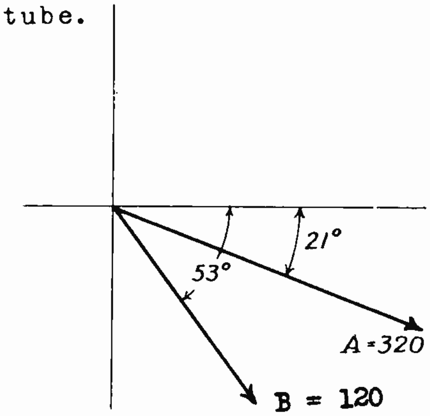
V-9174-33

TELEVISION TECHNICAL ASSIGNMENT

VECTOR ANALYSIS

EXAMINATION

The following forces act on an electron in the beam in a cathode ray tube.



1. Find resultant and angle of resultant.

$H = A_H + B_H = A\cos 21° + B\cos 53°$
$= 320 \times .93358 + 120 \times .60182$
$= 298.75 + 72.385 = 371.235$

$V = A_V + B_V = A\sin 21° + B\sin 53°$
$= 320 \times .35837 + 120 \times .79864$
$= 114.68 + 95.835 = 210.515$

$R = \sqrt{H^2 + V^2} = \sqrt{(371.235)^2 + (210.515)^2} = \sqrt{137{,}830 + 44{,}314}$
$= \sqrt{182{,}144} = 426.8$ units

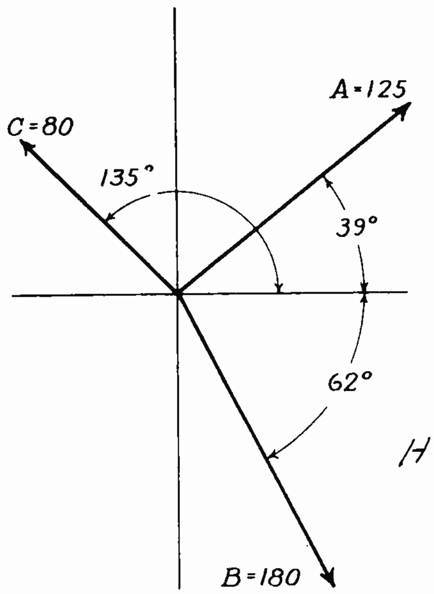
$\theta = \tan^{-1} \frac{210.51}{371.23} = \tan^{-1} .56705$

$\theta = 29° 33.3'$

2. Solve problem 1 graphically by drawing all vectors *to scale* on enclosed cross-section paper.

EXAMINATION, Page 2.

3. Find the resultant and angle of resultant.



$A_H = 125 \cos 39°$

$= 125 \times .7771 = 97.137$

$B_H = 180 \cos 62°$

$= 180 \times .4695 = 84.51$

$C_H = -(80 \cos 45°)$

$= -(80 \times .7071) = -56.568$

$H = A_H + B_H + C_H = 97.137 + 84.51 - 56.568$

$= 125.079$

$A_V = 125 \sin 39° = 125 \times .6293 = 78.662$

$B_V = -(180 \sin 62°) = -(180 \times .8829) = -158.922$

$C_V = 80 \sin 45° = 80 \times .7071 = 56.568$

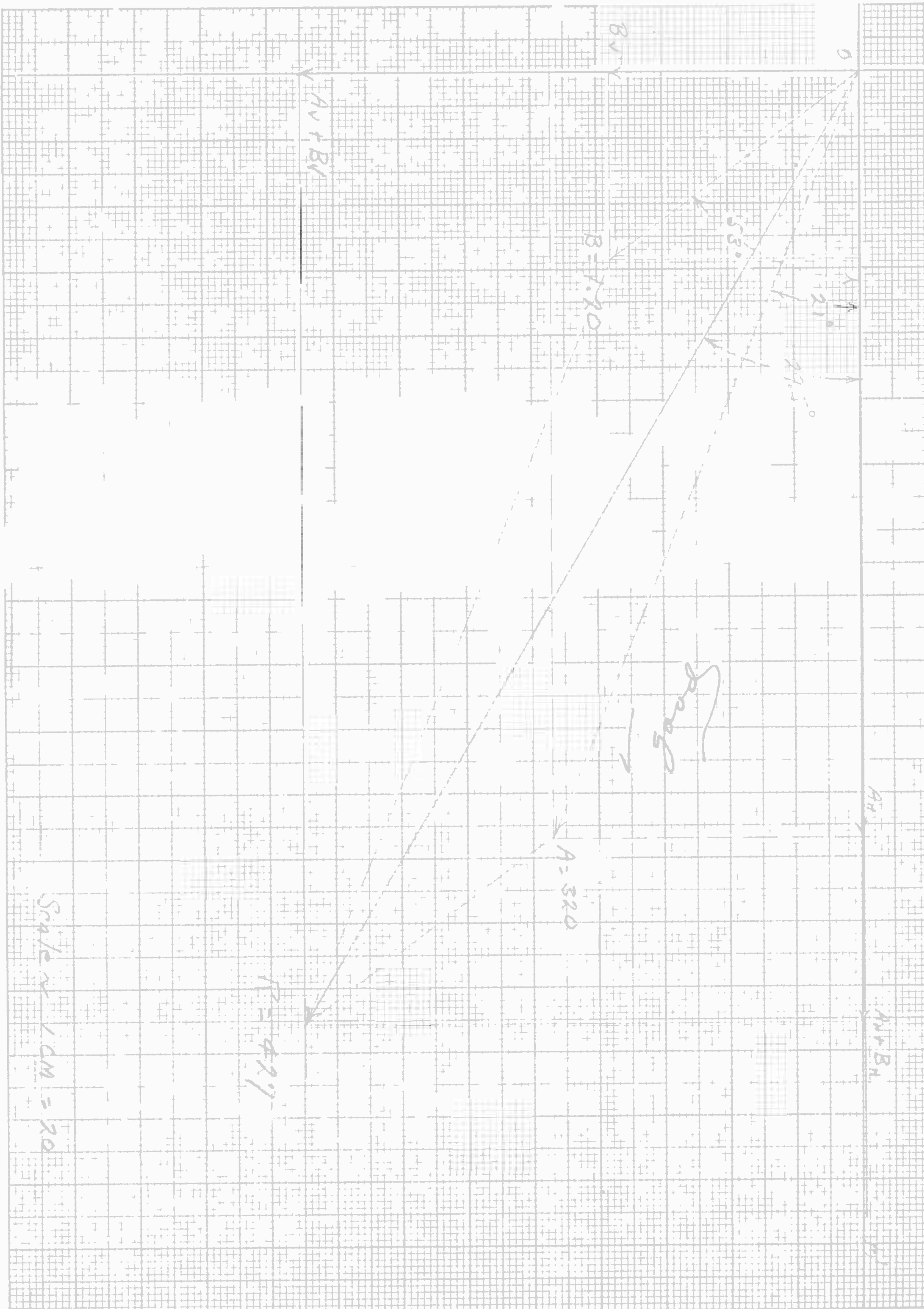
$V = A_V + B_V + C_V = 78.662 - 158.922 + 56.568 = -23.69$

$R = \sqrt{H^2 + V^2} = \sqrt{(125.079)^2 + (23.69)^2} = \underline{\underline{127.34}}$

$\theta = \tan^{-1} \dfrac{-23.69}{125.079} = \tan^{-1} -.1889 = -10° 42'$

4. Solve problem 3 graphically by drawing all vectors *to scale* on enclosed cross-section paper.

V-9174-33



2.

V-9174-33

VECTOR ANALYSIS

EXAMINATION, Page 3.

5. (a) Express the following angles in radians: 45°, 90°, 135°, 315°.

(1.) $45° = \frac{45}{360} \times 2\pi = \frac{\pi}{4} = \frac{3.1416}{4} = \underline{\underline{.7854}}$ rad.

(2) $90° = \frac{90}{360} \times 2\pi = \frac{\pi}{2} = \underline{\underline{1.5708}}$ rad.

(3) $135° = \frac{135}{360} \times 2\pi = \frac{3\pi}{4} = \underline{\underline{2.3562}}$ rad.

(4) $315° = \frac{315}{360} \times 2\pi = \frac{7\pi}{4} = \underline{\underline{5.4978}}$ rad.

(b) Express following in degrees and tenths of a degree: 1.26 radians, .372 radians, .145 radians, 8.91 radians, 13.6 radians.

(1) 1.26 rad. $= 57.3 \times 1.26 = \underline{\underline{72.2°}}$

(2) $.372$ rad $= 57.3 \times .372 = \underline{\underline{21.3°}}$

(3) $.145$ rad $= 57.3 \times .145 = 8.3°$

(4) 8.91 rad $= 57.3 \times 8.91 = 510.5°$ or $510.5 - 360 = \underline{\underline{150.5°}}$

(5) 13.6 rad $= 57.3 \times 13.6 = 779.3°$ or $779.3 - 720 = \underline{\underline{19.3°}}$

EXAMINATION, Page 4.

6. If a vector completes 8.43 radians in .006 second find the frequency in cycles per second.

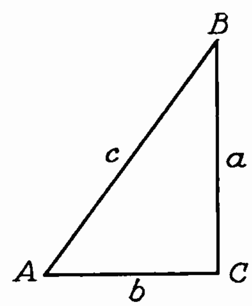
$$\text{rad/sec} = \frac{8.43}{.006} = 1405$$

$$\text{frequency} = \frac{1405}{6.2832} = \underline{\underline{223.6}}\ \text{cps}$$

7. a = 4, b = 3, c = 5.

Prove: $\sin(1/2)A = \sqrt{\dfrac{1 - \cos A}{2}}$

Show all work.



$$\sin A = \frac{4}{5} = .8 \qquad A = 53°8'$$

$$\tfrac{1}{2}A = 26°34'$$

$$\cos A = \frac{3}{5} = .6 \qquad \sin 26°34' = .44724$$

$$.44724 = \sqrt{\frac{1 - .6}{2}} = \sqrt{\frac{.4}{2}} = \sqrt{.2} = .44722$$

$$.44724 = .44722$$

V-9174-33

VECTOR ANALYSIS

EXAMINATION, Page 5.

8. Using the same values for a, b, and c as given in problem 7 prove

$$\text{Tan } B = \frac{1 - \text{Cos } 2\,B}{\text{Sin } 2\,B}$$

Show all work.

$B = 90° - 53°8' = 36°52'$

$2B = 73°44'$

$\tan B = \frac{3}{4} = .75$ $\cos 2B = .28011$
                                  $\sin 2B = .95997$

$$.75 = \frac{1 - .28011}{.95997} = \frac{.71989}{.95997} = .7499$$

$$\underline{\underline{.75 = .7499}}$$

The error is caused by taking angle A as 53°8'. Actually it is just about 53° 7' 49".

9. The following voltage is applied to the control grid of a television picture tube:

E = 150 volts, F = 800 cycles per second. Find e at .0015