# SPECIALIZED TELEVISION ENGINEERING

TELEVISION TECHNICAL ASSIGNMENT

## THE SERIES LCR CIRCUIT



723D

# THE SERIES LCR CIRCUIT

## FOREWORD

This is the first approach to the applications of L, C and R in tuned circuits and to the phenomena associated with circuit "resonance." The approach is mathematical but only simple mathematics is required. Since the vast majority of radio frequency circuits are operated at, or very close to resonance, it is necessary to understand just what changes take place in the circuit voltage and current relations as resonance is approached. Similar conditions apply in television receivers, particularly in filters by which the several components of the picture and sound signals are separated.

To the technician who has not previously analyzed a tuned r.f. circuit mathematically, it is rather startling to find that in a series LCR circuit at resonance, an applied voltage of, for example, 1,000 volts may result in a voltage across the inductance of possibly 20,000 volts with a similar voltage across the condenser. Many a condenser has had its dielectric punctured, and many good r.f. choke coils have been burned up by engineer and technicians who did not have the proper conception of the effects of series resonance.

The effects of series resonance can be used in a filter to provide a short-circuit for some frequency it is desired to suppress while passing signals at all desired frequencies with negligible attenuation. The effect of series resonance is used to provide voltage gain between the antenna coil and the grid of the first r.f. amplifier tube in a receiver, which may be several times greater than that obtained by the amplifying action of the tube itself, with subsequent reduction of receiver noise level. A transmitting antenna is operated at resonance in order that the antenna current may be maximum for given power input. The ratio of L/C in a tuned circuit at a given frequency is highly important and will be discussed. This is the factor which determines, in conjunction with the circuit resistance whether the circuit tunes sharply or broadly—a very important consideration in many television and other applications.

You will find this assignment extremely interesting. It will answer many of the questions that have puzzled you in the past.

E. H. Rietzke,
President.

## - TABLE OF CONTENTS -

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# *THE SERIES LCR CIRCUIT*

*INTRODUCTION.*—Up to this point impedance has been considered as the resulting opposition offered by inductance and resistance or capacity and resistance in series. It has been found that in either case the impedance is equal to $\sqrt{R^2 + X^2}$, and, that the angle of the current lag or lead depends upon the ratio of the reactance to the resistance, i.e., $\text{Tan } \theta = X/R$; also, that this equation holds true whether the reactance is $X_L$ or $X_c$, the only difference being that in the case of $X_L$, $\theta$ represents an angle of LAG, while with $X_c$, $\theta$ represents an angle of LEAD.

Consider the case of a circuit composed of inductance, capacity and resistance in series. (See Fig. 1). The effect of inductive reactance is to cause a current LAG of exactly 90°. Capacitive reactance has the effect of causing a current LEAD of exactly 90°. These effects are 180° out of phase or in exact opposition. One therefore cancels an equivalent amount of the other, and the total reactance of a series circuit is the difference between $X_L$ and $X_c$. In a condition such as this the smaller always counteracts an equivalent amount of the larger, and the remaining reactance assumes the characteristics of the larger.

For example, with an inductance and a capacity in series, (temporarily neglecting the resistance), and the frequency such that $X_L = 40$ ohms and $X_c = 15$ ohms, the total reactance will equal $X_L - X_c = 40 - 15 = 25$ ohms. The resulting 25 ohms of reactance will be $X_L$ and the circuit may be spoken of as an inductive circuit. This condition is shown in Fig. 2 (a).

Fig. 2 (b) shows the inverse of 2 (a). The frequency is now such that $X_c = 40$ ohms and $X_L = 15$ ohms. The total reactance is thus equal to $X_c - X_L = 40 - 15 = 25$ ohms. The numerical value of reactance is the same as that previously obtained, but in the second case the resulting reactance is $X_c$ and the circuit is a capacitive circuit.

It may be stated: THE TOTAL

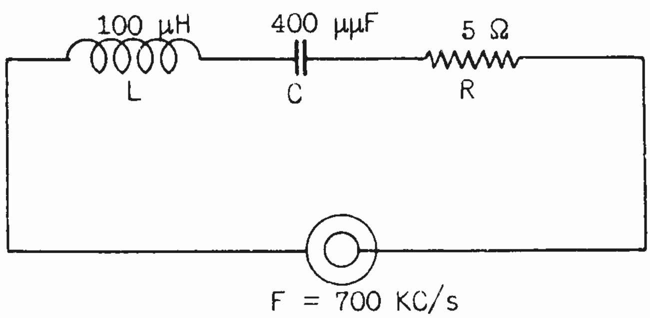



Fig. 1.—Series circuit containing inductance, resistance and capacity.

REACTANCE OF A SERIES CIRCUIT IS EQUAL TO THE DIFFERENCE BETWEEN THE INDUCTIVE REACTANCE AND THE CAPACITIVE REACTANCE, AND THE RESULTING REACTANCE ASSUMES THE CHARACTERISTICS OF THE LARGER.

In other words, $X = X_L - X_c$ or

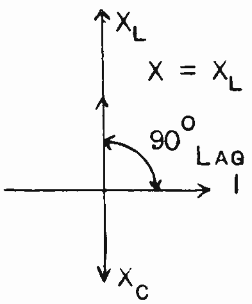



Fig. 2(A).—In a series L-C circuit, when $X_L > X_c$ the circuit is inductive in nature.

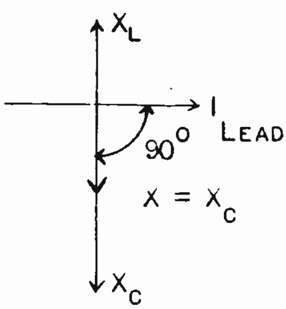



Fig. 2(B).—In a series L-C circuit, when $X_L < X_c$ the circuit is capacitive in nature.

$X_c - X_L$ depending on which, $X_L$ or $X_c$, is the larger. In the event the two are equal, $X_L = X_c$, the total reactance of the circuit is zero, this condition being called *series resonance*.

The above condition exists regardless of the resistance of the circuit. However, the resistance cannot be neglected, particularly in radio-frequency calculations where the circuit is almost always operated at or near a condition of resonance.

Assume that in the circuit considered in Fig. 1 the resistance is equal to 40 ohms. The effect of the resistance is to bring the current into phase with the voltage. It is therefore plotted on the horizontal axis 90° from both $X_L$ and $X_c$. Fig. 3 shows the condition of the circuit with $X_L = 40$ ohms, $X_c = 15$ ohms, and $R = 40$ ohms.

The impedance of any series circuit is equal to $\sqrt{R^2 + X^2}$. The

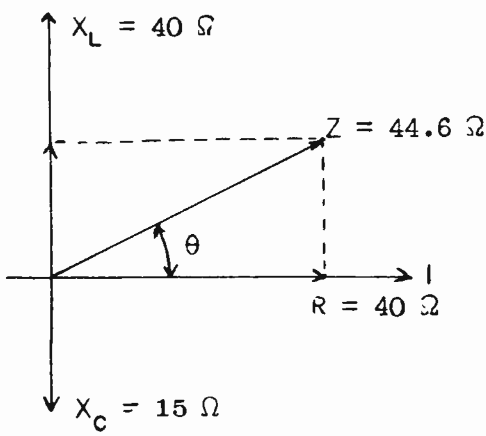



Fig. 3.—Vector diagrams for a series circuit when $X_L = 40\Omega$, $X_c = 15\Omega$, and $R = 40\Omega$.

total reactance in this case is $X_L - X_c$ and is therefore 40 - 15 or 25 ohms of inductive reactance. The equation $Z = \sqrt{R^2 + X^2}$ can then be enlarged and solved,

$$Z = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{40^2 + 25^2}$$

$= \sqrt{1600 + 625} = \sqrt{2225} = 47.2$ ohms

Any series circuit can be solved in this manner, the only difference being a condition where $X_c$ is larger than $X_L$; in that case the equation will become

$$Z = \sqrt{R^2 + (X_c - X_L)^2}$$

and the result will be a leading current.

If $X_c$ is larger than $X_L$, the equation will be written,

$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$

$$Z = \sqrt{R^2 + (-X)^2}$$

$$Z = \sqrt{40^2 + (-25)^2}$$

$$= \sqrt{1600 + 625} = 47.2 \text{ ohms}$$

When such a procedure is followed, it must be understood that $X_L$ is to be considered as a positive reactance and $X_c$ as a negative reactance. Thus if a negative reactance is obtained when solving for X, it is apparent that the circuit contains a preponderance of capacitive reactance. The numerical value of impedance is not changed by this procedure because the square of a negative number is always positive. Thus $(-25)^2 = (-25) \times (-25) = +625$.

In the problem of Fig. 3, the current will meet with a total opposition of 47.2 ohms, the component parts of which are $X_L$ and R. R tends to bring the current in phase, $X_L$ tends to cause a lag of 90°, the result being a lag somewhere between 0° and 90°; and since R is greater than $X_L$ it will be nearer 0° than 90°. The exact angle is determined by solving for the Tangent of θ.

$$\text{Tan } \theta = X/R = 25/40 = .625$$

Inspection of a table of trigonometric functions shows that a tangent of .625 represents an angle of 32°. In this circuit at the frequency specified, the current will lag 32° behind the voltage. If the frequency is lowered to such a point that the reactance is as shown in Fig. 2(b) and the resistance remains at 40 ohms, the total impedance will still be 47.2 ohms but angle θ of 32° will be an angle of LEAD. This is shown in Fig. 4.

In this problem the resistance

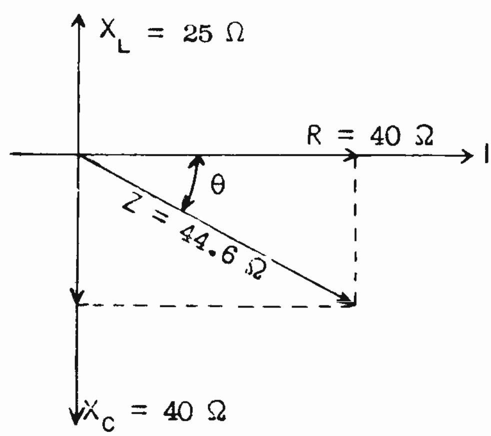



Fig. 4.—Vector diagram for a series circuit when $X_L = 25\Omega$, $X_c = 40\Omega$, and $R = 40\Omega$.

was found to be large compared to the reactance. In most radio-frequency circuits, except at a condition of resonance where $X_L = X_c$, the resistance is small compared to the values of reactance encountered. For example, assume a circuit having a capacity of 400 μμF, an inductance of 100 μH, and resistance of 5 ohms distributed throughout the circuit. (See Fig. 5.) This circuit is to be connected across a voltage source of 700 kc/s, (in this case the incoming signal from a distant transmitter);

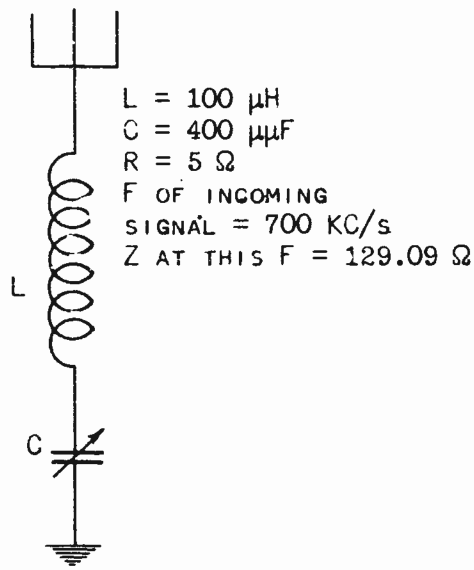



Fig. 5.—Typical series circuit wherein R is small compared to $X_L$ and $X_c$.

to find the impedance at that frequency. (See Fig. 1 for the equivalent simple electrical circuit). It is necessary to first calculate $X_L$ and $X_c$ at this frequency, and then combine X with R to find Z.

$$R = 5 \text{ ohms}$$

$$X_L = 2\pi FL$$

$$2\pi = 6.28$$

$$F = 700 \text{ kc} = 700{,}000$$

$$= 7 \times 10^5 \text{ cycles}$$

$$L = 100\ \mu H = 10^{-4}\ H.$$

$$X_L = 6.28 \times 7 \times 10^5 \times 10^{-4}$$

$$= 6.28 \times 7 \times 10^1 = 439.6 \text{ ohms}$$

$$X_c = 1/2\pi FC$$

$$2\pi = 6.28$$
$$F = 7 \times 10^5 \text{ cycles}$$
$$C = 400\ \mu\mu F = 4 \times 10^{-10} F$$

$$X_c = \frac{1}{6.28 \times 7 \times 4 \times 10^{-5}}$$

$$= \frac{10^7}{17584} = 568.6 \text{ ohms}$$

$$R = 5 \text{ ohms}$$

$$X_L = 439.6 \text{ ohms}$$

$$X_c = 568.6 \text{ ohms}$$

$$Z = \sqrt{R^2 + (X_c - X_L)^2}$$

$$= \sqrt{5^2 + (568.6 - 439.6)^2}$$

$$Z = \sqrt{5^2 + 129^2}$$

$$= \sqrt{16666} = 129.09 \text{ ohms}$$

$$\text{Tan } \theta = X/R = 129/5 = 25.8$$

$\theta = 88°$ Lead

Since $X_c$ is greater than $X_L$, the current LEADS the voltage, and since X is very large compared to R the angle is large, almost 88°. This is shown in Fig. 6. (In the vector the actual length of R is

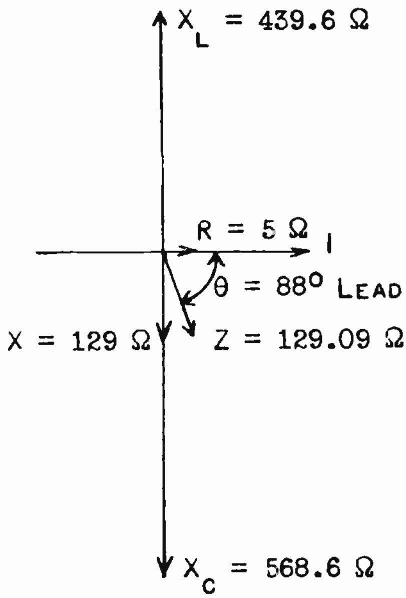



Fig. 6.—Vector Diagram for Fig. 5.

exaggerated to make it visible along the horizontal axis). Under this condition where R is very small as compared to X, the resistance may be neglected when computing Z. This will be the condition existing in a series radio-frequency circuit tuned to a frequency considerably higher than the frequency of the impressed voltage; in a receiver circuit this impressed voltage is due to the incoming signal. A total opposition of 129.09 ohms will be offered to the signal voltage of 700 kc/s by this circuit and very little current will flow in the circuit. I = E/Z = (E/129.09) amperes.

If, however, the frequency of the transmitter is increased so that the frequency of the incoming signal is such as to cause $X_L$ to equal $X_c$, then the two reactances will cancel and the total opposition to current flow will be the 5 ohms resistance. With this condition a comparatively large current can flow in the circuit. As in the preceding paragraph, I = E/Z. Z, however, is now equal to R, so I = E/5 amperes. The current will be approximately 26 times greater than before and will be in phase with the voltage. This is shown in Fig. 7 where $X_L = X_c$ and Z = R. (R is again exaggerated on

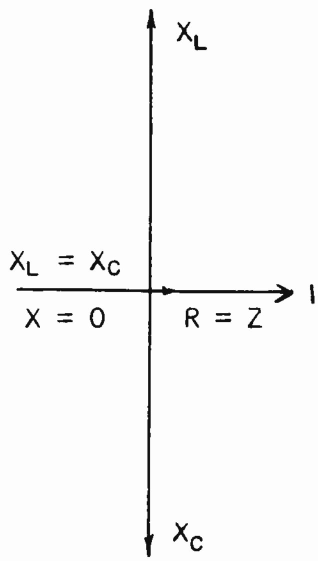



Fig. 7.—Vector diagram for Fig. 5, when resonant.

the vector).

If the frequency is increased beyond this point, $X_L$ will be further increased and $X_c$ further decreased, and X will then be equal to $X_L - X_c$ and no longer equal to zero. Z will again become greater than R.

It may be stated: IN A SERIES CIRCUIT AT RESONANCE THE IMPEDANCE IS EQUAL TO THE RESISTANCE AND IS AT ITS LOWEST VALUE, THE CURRENT THEREFORE BEING AT MAXIMUM AND IN PHASE WITH THE VOLTAGE.

A series circuit, at a frequency HIGHER than its resonant frequency acts as an INDUCTANCE, and at a frequency LOWER than its resonant frequency acts as a capacitor. This is due to the fact that $X_L$ increases with an increase of frequency while $X_c$ decreases as the frequency is increased, and vice versa. This is illustrated in Fig. 8, where the current is shown at maximum and the impedance at minimum for the condition of resonance. The derivation of the equation for the resonant frequency of a circuit was well covered in Assignment 3. The equation itself is

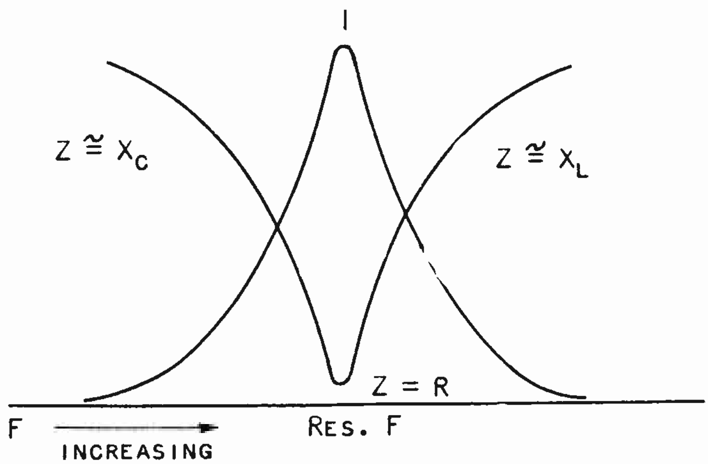



Fig. 8.—Resonance curves for a series circuit.

$$F = \frac{1}{2\pi\sqrt{LC}}$$

L in Henries
C in Farads
F in Cycles

Applying the resonant frequency equation to the circuit as shown in Figs. 1 and 5, where L = 100 μH and C = 400 μμF; converting L to henries and C to farads:

$$L = 100\ \mu H = 10^{-4} H$$

$$C = 400\ \mu\mu F = 4 \times 10^{-10}\ F$$

$$2\pi = 6.28$$

The equation for the resonant frequency becomes

$$F = \frac{1}{6.28\sqrt{4 \cdot 10^{-10} \cdot 10^{-4}}}$$

$$= \frac{1}{6.28\sqrt{4 \cdot 10^{14}}}$$

$$= \frac{1}{6.28 \cdot 2 \cdot 10^{-7}}$$

$$= \frac{1}{12.56 \cdot 10^{-7}} = \frac{10^7}{12.56}$$

= 796,178 c/s or 796.178 kc/s

At this frequency $X_L$ equals $X_c$ and the total opposition Z is equal to R or 5 ohms.

Another method of showing the characteristics of a series inductance-capacity circuit is illustrated in Fig. 9. At zero frequency, that is, direct current, $X_L$ will be zero and $X_c$ will be infinite, therefore no current will flow. As the frequency is gradually increased, $X_L$ increases in a straight line in direct proportion to the increase in F. At the same time $X_c$ is decreasing in the form of a curve. At the frequency at which $X_L = X_c$ the condition of resonance exists. Beyond that point $X_L$ predominates, and the circuit changes from a capacitive circuit, (as below the resonant frequency), to an inductive circuit.

The third curve shown in Fig. 9 is the Reactance curve X. This curve at any point is equal to $X_L - X_c$ or $X_c - X_L$ and very clearly demonstrates the actual operating characteristics of the circuit at any frequency. At the resonant frequency where $X_L = X_c$ the reactance curve X passes through zero and at that point Z = R. It must be clearly understood that the reactance curves as plotted on Fig. 9 do not take into consideration the resistance of the circuit which may be high or low.

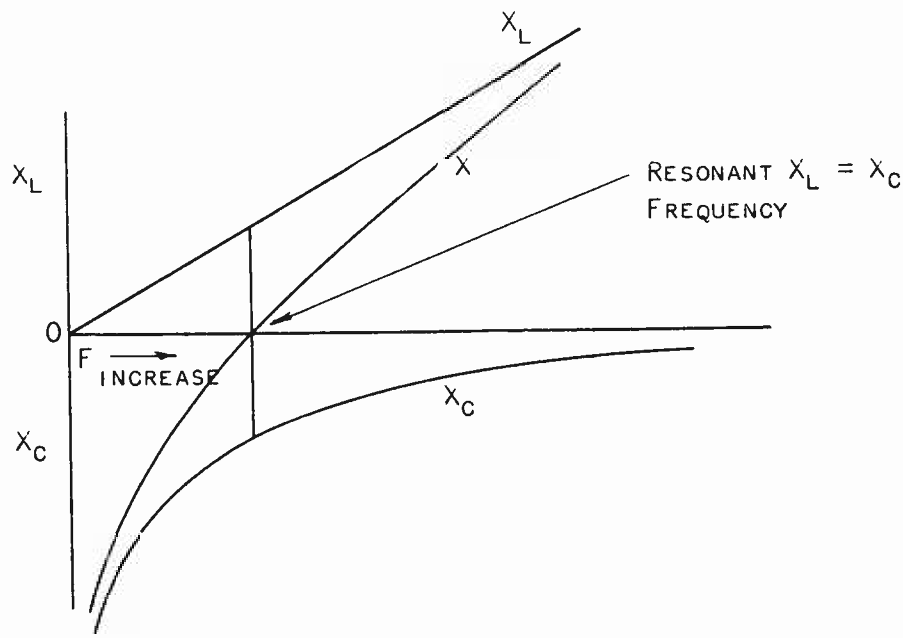



Fig. 9.—Reactance curves for a series circuit.

In a series circuit the current is the same in all parts of the circuit and the total voltage across a series circuit is equal to the current times the total opposition Z. In other words, E = IZ.

In a circuit composed entirely of resistance the total R is equal to the algebraic sum of the individual resistances, and Z is equal to R. The total voltage will then equal the algebraic sum of the voltage drops across the individual resistances.

However, in a series circuit composed of L and C as well as R, that is NOT true. In that case the total impedance Z is the VECTOR SUM of the individual resistances and reactances and may be LESS THAN EITHER OF THE INDIVIDUAL REACTANCES. In fact, as demonstrated in the problem corresponding to Fig. 7, both $X_L$ and $X_c$ may be equal to several hundred ohms, and the total impedance of the circuit at resonance be only 5 ohms, or equal to R. This means that if that particular circuit is connected across a source of power of 50 volts at the frequency to which the circuit is resonant, the current will be equal to E/Z = 50/5 = 10 amperes, even though both of the individual reactances are large.

Consider an example where $X_L$ = 200 ohms, $X_c$ = 200 ohms, and R = 10 ohms. An alternator is delivering 100 volts r.m.s. at frequency $F_r$, the resonant frequency of the circuit. The circuit is shown in Fig. 10(a). The impedance vector will be as in Fig. 7 where $X_c$ cancels $X_L$, making X equal to zero and Z equal to R or 10 ohms. I = E/Z = 100/10 = 10 amperes r.m.s., as will be indicated by the ammeter in the circuit.

The voltage drop across any piece of apparatus is equal to the impedance of that piece of apparatus times the current through it. In the inductance only, Z = $X_L$. $X_L$ = 200 ohms, and the ammeter indicates a current of 10 amperes which must flow through all parts of the circuit. A voltage $IX_L$ = 10 × 200 = 2000 volts is therefore built up across the inductance alone.

Since the impedance of the capacitor is equal to $X_c$ or 200 ohms, a voltage $IX_c$ = 10 × 200 = 2000 volts is also built up across the capacitor.

The voltage across the resistance is equal to IR = 10 × 10 = 100 volts. Thus a condition exists in the circuit such that there are three voltage drops, 2000 volts, 2000 volts, and 100 volts, with a supply voltage of ONLY 100 VOLTS. At first thought this may seem impossible. However, the relation between the current and voltage in the different parts of the circuit must be considered. Through the purely inductive portion of the circuit the current must lag the voltage by 90°. But since this is a series circuit the current must be the same at any instant in *all* parts of the circuit. The voltage built up across the inductance must therefore be such that it leads the current by 90°, and across the capacity a voltage must exist that is 90° behind this same current. See Fig. 10(b). This will place the two voltages 180° out of phase or in exact opposition at every instant. The two voltages, therefore, being equal and opposite, cancel and have no effect on the circuit as a whole. The total applied voltage of the alternator is expended in forcing the current through the resistance, and this current and the applied voltage will be in phase. The total voltage across the circuit will equal the vector sum of the individual voltages, i.e.,

$$E = \sqrt{E_R^2 + (E_L - E_c)^2}$$

At resonance this will be, $E = E_R$.

It is evident that at series resonance very high voltages may be

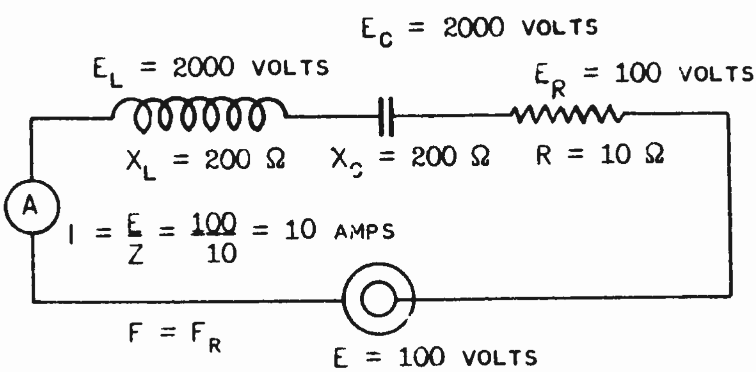



Fig. 10(A).—Typical series circuit.

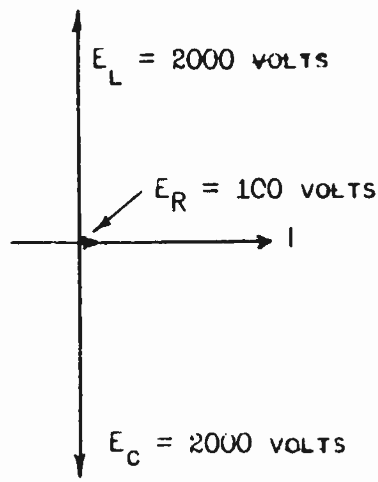



Fig. 10(B).—Vector diagram for Fig. 10(A).

built up across the various components in a radio-frequency circuit while the applied voltage is comparatively low. A good example of this is a circuit sometimes used to improve the low-frequency response of an audio-frequency transformer over a limited range. A capacitor of the proper value is inserted in series with the primary of the transformer (interstage) and the plate voltage is fed through a resistor on the plate side of the series capacitor. Then, the tube may be considered as a voltage source feeding a series circuit composed of the capacitor and the transformer primary; this series circuit will be made to resonate at some low frequency, say 40 c/s, and the resulting "resonant rise" of voltage across the primary will prevent low-frequency attenuation that might otherwise take place.

Another interesting example, although not pertaining to radio, is the voltage rise that occurs across the winding of an electric razor or fan when a suitable value of capacity is placed in series. When this is done, the razor or fan will run at a higher speed than when connected directly across the power line.

Resistance, while it has very little effect when the circuit is being operated at some frequency other than its resonant frequency, becomes the deciding factor at resonance. For example, in the problem illustrated vectorally in Fig. 6, the five ohms of resistance may be practically neglected. The reactance of 129 ohms determines the impedance and the addition of 1 ohm, 5 ohms, or 10 ohms, will cause only a very small increase in the impedance and therefore very little decrease in current.

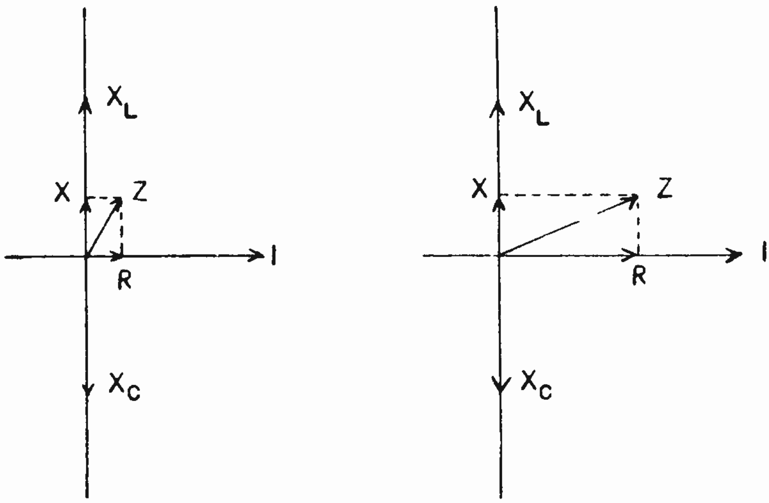
When the same circuit is operated at its resonant frequency, the resistance is the only factor of importance so far as the impedance and current are concerned. The reactances may be 10 ohms or 1000 ohms; so long as both are equal their effect on the impedance is zero. If the resistance is varied the impedance varies directly as the variation of resistance, and the current varies inversely as the variation in resistance. In the problem corresponding to Fig. 7, $X_L = X_c$ and may be neglected. R is 5 ohms and the voltage is 50 volts. The current is therefore 10 amperes. Now, if the resistance is decreased to 1 ohm the current will be increased to 50 amperes. If the resistance is increased to 10 ohms the current will be decreased to 5 amperes. It is apparent that in tuned radio-frequency circuits, which are nearly always operated at or very near resonance, the resistance of the circuit is the current limiting factor. If the circuit is in a receiver, the receiver which has the lowest resistance circuits, everything else being equal, may be expected to produce the greatest signal amplitude and the greatest consistent reception distance. In a transmitter the most efficient circuits are those having low values of resistance; (with the exception of the antenna circuit.)

The resistance of a tuned radio-frequency circuit is also a determining factor in the selectivity of that circuit. Fig. 11(a) represents a vector for a low resistance circuit, operated somewhat off resonance, showing the values of X, R and Z. In 11(b), are shown the conditions existing in a circuit of the same reactance characteristics operated the same amount off resonance, but having a larger value of resistance.

At resonance, where Z = R, the current in the circuit represented by 11(a) will be large because R is small. When the circuit is tuned sufficiently off resonance to produce the condition as represented in Fig. 11(a) the impedance is increased about three times, the current decreased to about one-third. Those effects are indicated in the receiver circuit by a very strong signal at resonance and the signal cutting out sharply when the circuit is detuned.

In the condition shown in Fig. 11(b), where R is about four times as large as in 11(a), the signal at resonance will be weak due to the high circuit resistance. When the circuit is detuned the same amount as in 11(a), the condition as shown in 11(b) exists. The vector addition of R and X results in an impedance but slightly greater than at resonance. The signal when the circuit is slightly detuned will be almost as strong as at exact resonance. Such a circuit condition existing in a receiver will result in very weak signals from distant stations, lack of volume on signals from nearby stations, and very broad tuning of the signals of nearby stations with corresponding interference between stations.

The two circuit conditions are shown in Fig. 12, (not drawn accurately to scale). $I_1$ is the current curve for the low resistance circuit, the vector for which is illustrated in Fig. 11(a). The current is high at resonance falling off sharply as the circuit is detuned. Curve $I_2$ represents the current of the higher resistance circuit, the vector for which is quantity called the LC value. This value is the product of the inductance and the capacity of the circuit




(A) (B)

**Fig. 11.—Diagrams illustrating effects of R on the selectivity of a series circuit.**

illustrated in Fig. 11(b). The current at resonance reaches only a small fractional part of the current in the lower resistance circuit and on weak signals the reception will be very poor. On nearby stations when sufficient amplification is used to obtain good volume, the circuit will tune very broadly making it impossible to tune in weak signals at frequencies anywhere near that of the local station without interference. This will be considered a little later in the discussion of the Q, ($\omega L/R$,) of series circuits.

Another factor which enters into the determination of the selectivity of the series circuit is the ratio L/C. In every circuit that is operated at resonance, the resonant frequency is determined by a and must be the same in any number of circuits that are to be resonant to the same frequency. However, in all of these circuits operated at the same resonant frequency the values of L do not have to be the same; nor do the values of C have to be identical. The PRODUCTS of L and C, however, MUST BE EXACTLY THE SAME IN ALL OF THE CIRCUITS IF THEY ARE TO HAVE THE SAME RESONANT FREQUENCY. Thus one circuit may have a given amount of L and a given amount of C; another circuit may have one-half as much L and twice as much C; another circuit may have three times as much L and one-third as much C, etc. If the PRODUCTS of L and C are equal the resonant frequencies of the circuit will be identical.

To obtain a high degree of selectivity the ratio of L/C should be made as great as practical. The reason for this can be seen from an inspection of Fig. 9. If L is larger the slope of the line representing $X_L$ will be steeper; this means that a given change in frequency will cause a greater change in the value of $X_L$. In a similar manner, if the capacity is made smaller the curve representing $X_c$ will be more abrupt, a given change in frequency thus making a greater change in $X_c$. These effects will combine to make the inclination of the reactance curve X steeper as it passes through zero at the resonant frequency.

It is the percentage of change in the impedance of the circuit for a given change in frequency that determines the selectivity of the circuit; except at or very near resonance, the impedance is almost entirely a function of the amount of reactance. This of course assumes that the circuits in question are properly designed to have a reasonably small resistance, a necessary requirement in all efficient radio-frequency circuits.

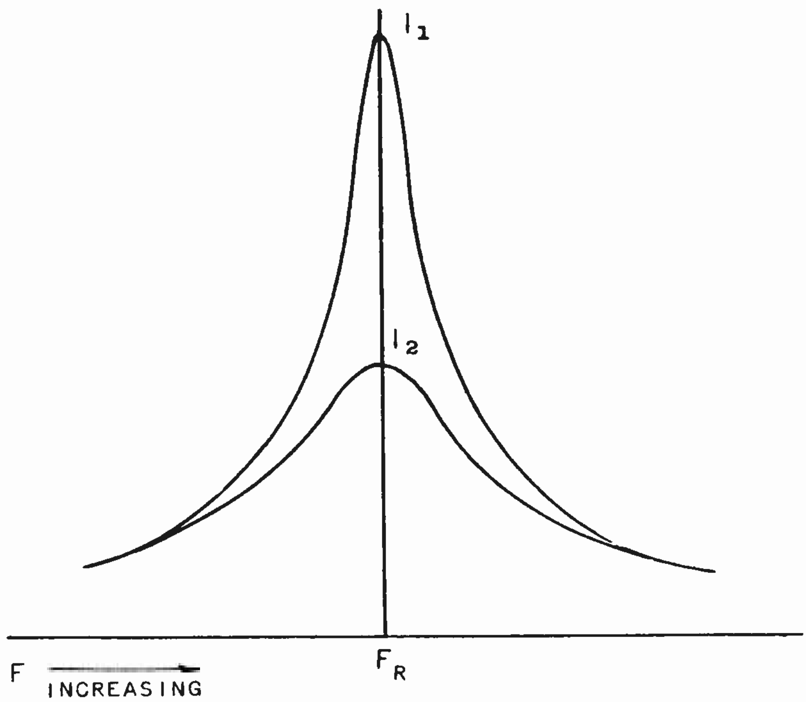



Fig. 12.—Current curves for Fig. 11.

How far the ratio L/C can practically be increased is determined by several factors. First, most of the resistance in a circuit, if the connections are so made that connection losses are negligible, is in the inductance of the circuit; thus an increase of inductance due to a coil of more turns will increase the circuit resistance. This means that L can practically only be increased to the point where a further increase will bring R to such a value that it will counteract the good effects of the increase in the L/C ratio. This was discussed in detail in an earlier assignment on the Q of coils.

Second, most radio-frequency circuits must be capable of being adjusted or tuned over a considerable frequency range, usually by varying the capacity. If the capacity used is very small, an extremely small variation in that capacity will cause too great a frequency change and introduce mechanical difficulties in the construction of the variable capacitors and their controls by which the variations in the frequency of the circuit are made.

In some circuits, such as those used in broadcast receivers, too high a degree of selectivity is not desired due to the distortion in the musical programs that would result. All of these factors must be taken into consideration in the design of the series circuit.

The mechanical requirements will, as a rule, determine the limits at which L/C may be increased. If, with the circuit so designed as to take full advantage of the highest ratio of L/C permitted mechanically, the resistance of the circuit is found to nullify the effects of this increase in L/C, it should be possible in many cases to redesign the inductance in order to lower the effective resistance. This may be done by the use of a different type of conductor, a larger size of conductor, the use of a lower loss material for the framework on which the winding is placed, a better grade of insulation or less insulating material around the conductors themselves, or any of the many methods of decreasing the radio-frequency resistance of the coil which forms the inductance of the circuit. The development of specially designed iron-core coils for r-f and i-f circuits has been an important contribution in this direction. The iron core of course allows larger L with fewer turns and hence lower wire resistance. The core consists of a very low-loss composition made up of extremely small particles of iron dust, each particle individually insulated from the others. Perhaps a lower loss type of capacitor could also be selected. Any means of decreasing the resistance of the circuit will permit the use of a higher ratio of L/C and the combination of low R and high L/C will operate to increase the selectivity of the circuit.

In working with series circuits, the factor Q is very convenient. $Q = \omega L/R$ where R is the total resistance of the circuit. Q has been used in an earlier assignment as a coil factor. However in many radio-frequency circuits, particularly in the case of receivers, almost all of the circuit resistance is in the coil winding, in which case the coil Q practically becomes the circuit Q. Consider the case of the tuned-grid circuit of a receiver first r-f amplifier, this circuit being coupled to the antenna. (See Fig. 13). Assume that the Q of coil L is 75 and that this also practically represents the Q of the circuit. This means that $\omega L = 75R$ where R is the resistance of the series circuit LC. Since circuit LC is to be operated at resonance,

$$X_L = \omega L = X_c = \frac{1}{\omega C} = 75R$$

and the voltage developed across C is the voltage impressed between grid and filament of the first tube.

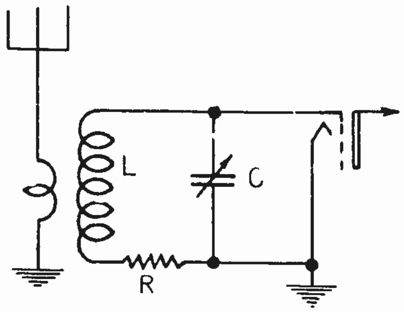



Fig. 13.—Typical r-f amplifier circuit.

Assume that a voltage of 15µv is developed across the antenna from an incoming signal and that by means of inductive coupling 8µv is impressed across circuit LC. This is just as if an alternator delivering a voltage of 8µv were connected in series with LC. (See Fig. 14.) But as has previously been shown, at resonance all of voltage E will be consumed across the circuit resistance R and R will limit the current in the circuit. The current will be the same in all parts of the circuit:

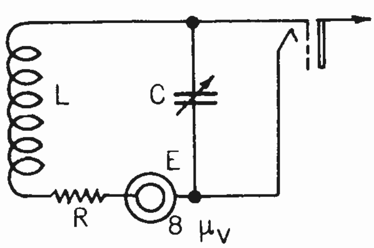



Fig. 14.—Equivalent circuit of Fig. 13.

So:

$$E_R = IR = 8\mu v$$

$$E_L = IX_L = I\omega L$$

$$Ec = IXc = \frac{I}{\omega C}$$

But,

$$\omega L = 75R = QR$$

So,

$$I\omega L = IQR = 75IR$$

And,

$$E_L = Ec = I\omega L = 75IR$$

$$= 75 \times 8 = 600\mu v$$

Thus although the signal voltage impressed across LC from the antenna coupling is only 8µv, the voltage developed at the grid of the tube is 600µv. This is called the "resonant rise of voltage" due to series resonance and is a very important factor in receiver design. Actually such a high voltage rise as shown between the antenna and the grid of the first tube will hardly be realized in practice because of coupling losses, tube losses, etc. However a voltage rise of 20 or more in such a circuit is entirely practical. The importance of this factor in contributing to the decrease in the receiver noise level will be brought out in a later study of receiver design. It should be noted that the circuit resonant voltage rise is equal in magnitude to the circuit Q.

There are several important cases where the coil Q is not the effective circuit Q. This is true in all coupled circuits where resistance from one circuit is reflected into another. It has been shown that where optimum coupling is employed, the reflected resistance is equal to the circuit resistance, so that in such a case the effective Q of the circuit is decreased to one-half that of the Q of the circuit when considered alone.

Another case is that of the antenna circuit. In the well designed transmitting antenna circuit, it is desired to make the radiation resistance high and all non-useful resistances as low as possible. (Radiation resistance will be explained in detail in a later assignment. Briefly, it represents the power actually radiated from the antenna). Thus in such a circuit the coil resistance may be a fraction of an ohm and the effective antenna resistance a number of ohms. The Q of such a circuit would be very small compared with the coil Q.

In some r-f transmitting circuits, resistance is deliberately introduced in order to broaden the circuit tuning. Such a case is the tank circuit of a modulated amplifier of a broadcast transmitter where the frequency response is to be essentially flat over a considerable range of frequencies, say 10,000 cycles above and below resonance. The resistance introduced in such a circuit will ordinarily be several times greater than that of the coil, so that the circuit Q will be considerably less than the coil Q.

It is interesting to note how the selectivity of a circuit operated at resonance varies with Q. In order to specify the degree of circuit selectivity, it is customary to calculate the frequency band in which the current reduces to a certain proportion of the maximum. Or to express it in another manner the amount by which the frequency must be varied from resonance to cause the circuit current to be decreased to some given percentage of maximum. In such a calculation the current is usually assumed to be decreased to .707 maximum which means that the power in the circuit is reduced by one-half.

If $f_r$ is the resonant frequency of the circuit, $f_1$ is the lower frequency for one-half power, and $f_2$ is the higher frequency for one-half power, R is the circuit resistance and L the circuit inductance, then

$$(f_2 - f_1) = \frac{R}{2\pi L} *$$

Dividing both sides by $f_r$

$$\frac{(f_2 - f_1)}{f_r} = \frac{R}{2\pi f_r L} = \frac{R}{\omega_r L}$$

But

$$\frac{\omega L}{R} = Q \text{ and } \frac{R}{\omega L} = \frac{1}{Q}$$

Therefore,

$$\frac{(f_2 - f_1)}{f_r} = \frac{1}{Q}$$

*See appendix for derivation.

As a practical example, assume that a circuit operating at 1200 kc/s has a Q of 60. How far must the frequency be varied on each side of resonance in order for the circuit current to be reduced to .707 maximum?

$$\frac{(f_2 - f_1)}{f_r} = \frac{1}{Q}$$

$$f_2 - f_1) = \frac{f_r}{Q} = \frac{1200}{60} = 20 \text{ kc/s}$$

Since the resonance current curve may be assumed to be symmetrical on both sides near resonance, the frequency must be varied 10 kc/s in either direction from resonance, that is, to 1190 or 1210 kc/s in order to reduce the circuit current to .707 maximum.

If the circuit has essentially the same value of Q when tuned to 600 kc/s,

$$(f_2 - f_1) = \frac{f_r}{Q} = \frac{600}{60} = 10 \text{ kc/s}$$

The selectivity in this case is twice as great as the lower frequency. This is somewhat typical of tuned r-f circuits employing a single coil and a variable capacitor. Of course in practice Q will not remain constant over such a wide range of frequencies. However with proper coil design the value of Q may not vary over such very wide limits for quite a frequency variation, so that with a given coil and capacitor combination, the frequency discrimination *expressed in terms of kilocycles* may be expected to be sharper at the lower frequencies and broader at the higher frequencies.

It should be noted at this point that while series resonance is often a desired condition, as shown in the examples above, there are circuits in which it is extremely undesirable and must be avoided. A number of such cases will be pointed out from time to time in following assignments. Fundamentally, series resonance is undesirable wherever a high impedance circuit is required, because at series resonance the circuit impedance drops to just that of the circuit resistance. Several typical examples of where resonance must be avoided: in the design of a low pass filter between a rectifier and a load; if resonance occurs at the fundamental or at a strong harmonic in any filter section, that section loses its filter effect for that frequency. In a transformer-coupled audio-frequency amplifier, the high frequency amplifying limit is usually set by the frequency at which series resonance occurs due to the inductance of the transformer secondary and the inter-winding capacity, this effective series circuit acting to short-circuit the transformer windings. A little thought will bring out numerous cases in which series resonance will be undesirable.

## APPENDIX

### BANDWIDTH

The selectivity of a series tuned circuit is determined by the bandwidth obtained, when the power in the circuit at points $f_1$ and $f_2$ (See Fig. 1) is equal to one-half the power in the circuit at reso-

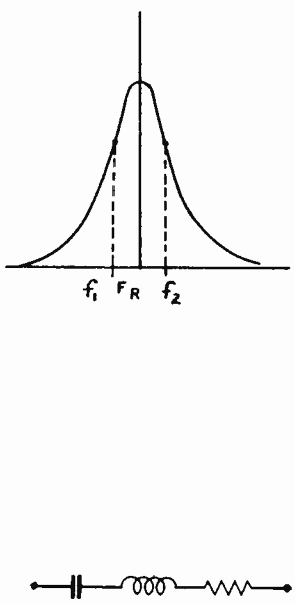



Fig. 1.

nance. If $P_r$ represents the power at resonance, then $P_r/2$ is the power at points $f_1$ and $f_2$. Under this condition the current at points $f_1$ and $f_2$ is reduced to .707 its value at resonance.

For example, let $P_f$ = the power at points $f_1$ and $f_2$, and $I_f$ the current at these two points, then:

$$P_f = \frac{P_r}{2} = \frac{I_r^2R}{2} = I_f^2R$$

and,

$$I_f^2R = .5\ I_r^2R$$

So,

$$I_f = .707\ I_r$$

With the current at $f_1$ and $f_2$ reduced to .707 times its value at resonance, the impedance of the circuit must have increased to

$$\frac{1}{.707} = 1.414$$

times its value at resonance. Now if Z = R = impedance at resonance (purely resistive), then at $f_1$ (or $f_2$) Z = 1.414 R (which will be capacitive at $f_1$ and inductive at $f_2$; however, only the magnitude of the impedance is of interest here).

So, at $f_1$ (or $f_2$)

$$Z = 1.414\ R$$

$$= \sqrt{2}\ R$$

$$= \sqrt{2\ R^2}$$

$$= \sqrt{R^2 + R^2}$$

But Z is also equal to $\sqrt{R^2 + X^2}$ at $f_1$ (or $f_2$), so:

$$Z = \sqrt{R^2 + R^2} = \sqrt{R^2 + X^2}$$

hence the difference (X) between the reactances $[(1/\omega_1C) - (\omega_1L)$ for $f_1$, or $(\omega_2L) - (1/\omega_2C)$ for $f_2]$ is equal to the circuit resistance (R) at resonance.

It follows then, that:

$$\frac{1}{\omega_1 C} - \omega_1 L = X = R$$

$$1 - \omega_1^2 LC = \omega_1 CR$$

$$1 = \omega_1 CR + \omega_1^2 LC$$

$$1 = C\ (\omega_1 R + \omega_1^2 L)$$

$$C = \frac{1}{(\omega_1 R + \omega_1^2 L)}$$

In a similar manner (using $\omega_2 L - \frac{1}{\omega_2 C}$) we can obtain:

$$C = \frac{1}{(\omega_2^2 L - \omega_2 R)}$$

Hence:

$$\frac{1}{(\omega_2^2\ L - \omega_2\ R)} = \frac{1}{(\omega_1^2\ \ L + \omega_1 R)}$$

inverting the fractions,

$$\omega_2^2\ L - \omega_2\ R = \omega_1^2 L + \omega_1 R$$

Rearrange and solve for L,

$$\omega_2^2\ L - \omega_1^2 L = \omega_1 R + \omega_2 R$$

$$L(\omega_2^2\ - \omega_1^2) = R(\omega_1\ + \omega_2)$$

$$R = \frac{L(\omega_2^2 - \omega_1^2)}{(\omega_1\ + \omega_2)}$$

$$R = \frac{L\ (\omega_2\ + \omega_1)\ \ (\omega_2\ - \omega_1)}{(\omega_1\ + \omega_2)}$$

$$R = L\ (\omega_2\ - \omega_1)$$

$$(\omega_2\ - \omega_1) = \frac{R}{L}$$

But $\omega = 2\pi f$, hence:

$$2\pi\ (f_2\ - f_1) = \frac{R}{L}$$

$$f_2\ - f_1 = \frac{R}{2\pi L}$$

Let $f_o$ represent the resonant frequency of the series circuit, then:

$$f_2\ - f_1 = \frac{Rf_o}{2\pi f_o L}$$

$$\text{and } Q = \frac{2\omega f_o L}{R}$$

then the bandwidth between half-power points is

$$f_2\ - f_1 = \frac{f_o}{Q}$$

V-9174-33

THE SERIES LCR CIRCUIT

EXAMINATION

1. An inductance and capacitor *in series* form part of a so-called series-peaking circuit used in one type of video (television) amplifier. The capacitor is the input capacity of the succeeding vacuum tube, and has a value of 20 μμf. The coil in series with it has an inductance of 19.75 μhenries. Calculate the resonant frequency of the two in series.

$$F = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.28\sqrt{20\times10^{-12}\times 19.75\times10^{-6}}}$$

$$= \frac{10^9}{6.28\sqrt{395}} = \frac{10^9}{6.28\times 19.87} = 8012\ kc$$

2. (A) If the coil in Problem 1 has a resistance of 20 ohms, what is the impedance of the circuit at 10 mc?

$$X_L = 2\pi FL = 6.28\times10^7\times19.75\times10^{-6} = 1240\ \Omega$$

$$X_C = \frac{1}{2\pi FC} = \frac{1}{6.28\times10^7\times20\times10^{-12}} = \frac{10^5}{125.6} = 796.2\ \Omega$$

$$X = 443.8\ \Omega$$

$$Z = \sqrt{R^2+X^2} = \sqrt{20^2+443.8^2} = 444.24\ ohms$$

7. *(Continued)*

low and being the total impedance at resonance the current is high. As voltage drop equals IX it is high with high circuit current values.

8. (A) What effect has resistance on the tuning characteristics of a series circuit at resonance? *Explain in detail.*

The resistance affects the selectivity of a tuned circuit. The impedance of a circuit is $\sqrt{X^2+R^2}$. If R is very small the impedance will increase rapidly with slight detuning of the circuit, thereby giving sharp tuning of a signal. If, however, R is comparitively large the circuit has to be detuned a relatively

8. (A) *(Continued)*

greater amount to get a comparabile increase in impedance, thus giving broad tuning.

(B) What is its effect on the signal output of a receiver? *Explain.*

Low resistance gives a strong signal at resonance with sharp cut off at slight detuning. Higher resistance causes a weak signal at resonance with a less rapid decrease at detuning.

9. (A) What is meant by the L/C ratio of a series circuit and how does it affect the selectivity of the circuit? *Explain in detail.*

The L/C ratio is the ratio of inductance to capacity in the circuit. Within limits, the larger L/C ratio the more selective the circuit.

## PROBLEM SOLUTIONS
## SERIES CIRCUITS

1. $$F = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.28 \times \sqrt{20 \times 10^{-12} \times 19.75 \times 10^{-6}}} =$$

$$\frac{1}{6.28 \times \sqrt{3.95 \times 10^{-16}}} = \frac{1 \times 10^{8}}{6.28 \times \sqrt{3.95}} =$$

$\text{Log } 3.95 = .59660$
$\text{Log } \sqrt{3.95} = .29830$

$\text{Log } 6.28 = .79796$
$\text{Log } \sqrt{3.95} = \underline{.29830}$
$1.09626$

$\text{Log } 10^{8} = 8.$
$\underline{1.09626}$
$6.90374$

AnLog 6.90374 = 8,012,000 cps = 8.012 Megacycles

2. (a)

$X_L = \omega L = 6.28 \times 10^{7} \times 19.75 \times 10^{-6} = 1240.3\,\Omega$

$X_c = 1/\omega c = 1/6.28 \times 10^{7} \times 2 \times 10^{-11} = 10^{3}/1.256 = 796.2\,\Omega$

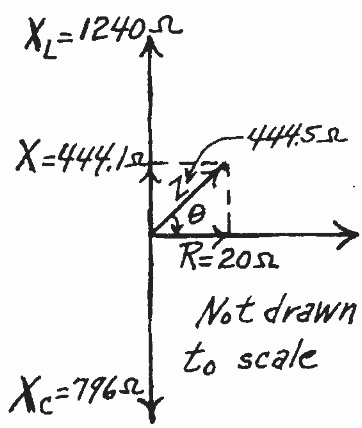
$X = X_L - X_c = 1240.3 - 796.2 = 444.1\,\Omega$

$Z = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{20^2 + 444.1^2} = \sqrt{400 + 197,200}$

$= \sqrt{197,600} = 444.5\,\Omega$

$\theta = \tan^{-1} X/R = \tan^{-1} 444.1/20 = \tan^{-1} 22.2 = 87.4^\circ$ Lag

(b)



## *EXAMINATION SOLUTION PROBLEM 6*
## *SERIES LCR CIRCUITS*

A generator of the proper frequency is connected across the circuit of problem 5. The peak voltage of the generator is 1500 volts.

(A) What would be the ammeter reading at resonance?

$$E_{rms} = .707\ E_{max} = .707 \times 1500 = 1061 \text{ volts}$$

$$I_{rms} = E_{rms}/Z = 1061/10 = 106 \text{ amps.}$$

(Since R.F. ammeters are of the thermocouple type, depending upon the average heat caused by the passage of the R.F. current through the resistance of the heater to generate a d.c. voltage in the couple to operate the meter, R.F. ammeters show the RMS value of R.F. current.)

(B) What would a voltmeter connected across the capacitor read?

$$X_c = \frac{1}{6.28 \times 1.125 \times 10^6 \times .0004 \times 10^{-6}} = 354 \text{ ohms}$$

$$E_c = IX_c = 106 \times 354 = 37500 \text{ volts, RMS}$$

or $E_c$ peak $= 1.41 \times 37500 = 53000$ volts, peak