





RESONANCE AND THE OSCILLATORY CIRCUIT

IMPEDANCE

A coil of wire possesses a property called INDUCTANCE, and it may be made of such a wire that it offers a measurable OHMIC RESISTANCE to current flow. A condenser possesses the property called CAPACITY; if the conductivity of the leads and plates is not good, the OHMIC RESISTANCE may be of measurable value.

If we connect a coil of wire, first to a source of direct current and then to a source of alternating current, both of the same voltage, the flow of resulting current measured in amperes is greater when the coil is connected to the direct current source than when connected to the alternating current source. This result is obtained because the counter-electromotive force of self-induction in the direct current circuit is only of momentary duration, the effect being noticed only when the current is turned on or off. The effect produced when the coil is connected to an alternating source, however, is quite different. Since the current intensity is continually changing the effects of self-induction are found to be continuous. This results in a back pressure or counter-electromotive force, which must always be taken into account in determining the net voltage which causes the current to flow.

The flow of steady direct current through the coil is opposed only by its ohmic resistance, $(I = E \div R)$. The flow of alternating current is impeded not only by the ohmic resistance, but by an additional obstacle due to self-induction and called REACTANCE. This is measured in equivalent ohms, and is considered positive when due to an inductance coil, and for this we use the term inductive reactance.

When a condenser is connected across a source of direct current, it will charge up to the line voltage, but will not pass any current then unless its dielectric insulation is defective. If the leads to the condenser plates have a measurable resistance, the charging rate will be slower, but the condenser voltage will finally equal the line voltage. When the condenser is connected across a source of alternating current, the flow of current is impeded not only by the ohmic resistance but by the additional obstacle called REACTANCE, which is due to the back pressure or counter e.m.f. caused by the opposition of the condenser to any change in the quantity and polarity of its charge. Since this counter e.m.f. is opposite in direction to the counter e.m.f. which would be developed in an inductance

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coil at that point, the reactance of the condenser or capacitive re-actance. is considered negative with respect to inductive reactance.

Reactance Values. The fundamental formulas for (1) Inductive reactance and (2) Capacitive reactance are as follows:

(1) Coil reactance formula $X_{L} = 6.28 \text{ x}$ Frequency x Inductance. (2) Condenser reactance formula $X_{C} = \frac{1}{6.28 \text{ x Frequency x Capacity}}$

Where: Reactance is measured in ohms, 11 " cycles per second, Frequency " " henries, Inductance " 11 11 " farads. ** Capacity

Impedance Values. Any inductance coil will have some value of re-sistance, though sometimes small, and in effect the coil resistance is in series with the coil inductance. When these two properties are present in the same material, their combined effect is termed IMPEDANCE. The relation between the values of the impedance (Z). reactance (X) and resistance (R) is given in the equation:

 $z^2 = R^2 + x^2$

Now this relation may be conveniently shown by what is known as the "impedance triangle", for the equation above represents Z as the hypotenuse of a right-angled triangle having X and R as its other



Fig. 1 - SERIES INDUCTANCE AND RESISTANCE

two sides, as shown in Fig. 1. This is for the combination of a resistance with an inductive reactive(X_L) which, as we stated before, is considered positive, and is drawn upwards from the resistance line in the figure.



Fig. 2 - SERIES CAPACITY AND RESISTANCE

The same general equations hold good for the condenser having poor conductors, which is in effect a perfect capacity in series with a resistance. The impedance triangle for this condition is shown in Fig. 2, with the capacitive reactance (X_{C}) graphed downward from the resistance line because capacitive reactance is exactly opposite in effect to inductive reactance.

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We have shown the resistance as being part of the coil or condenser construction. Usually the reactance of a choke coil is very high compared to its resistance; likewise the resistance of a condenser is usually low compared to its reactance. Even if they were perfect the rules would still hold for any external resistance in series with the terminals of the pure reactances.



Fig. 3 - SERIES INDUCTANCE, CAPACITANCE AND RESISTANCE

Consider the impedance of a path containing a condenser, a coil and a resistance in series, as in Fig. 3. The equation for this is:

$$Z^2 = R^2 + (X_L - X_C)^2$$

because the net reactance is the difference between the inductive and the capacitive reactances.



There will be three conditions possible as follows:

1. X_L is greater than X_C as in Fig. 4. 2. X_L is less than X_C as in Fig. 5. 3. X_L is equal to X_C as in Fig. 6. 3

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An inspection of the foregoing conditions shows that the impedance becomes smallest when it is equal to R, which happens when X_L and X_C have exactly the same numerical value as in Fig. 6; then $(X_I - X_C)$ becomes zero, and

$$Z^2 = R^2$$

Whence Z = R

Returning to the paragraph on reactance values, we find that lowering the frequency of an applied alternating current will decrease the inductive reactance of a coil, and increase the capacitive reactance of a condenser. Applying this principle, we can change Fig. 4 into Fig. 6 by merely lowering the frequency of the supply line. We could also change the impedance triangle of Fig. 5 by raising the frequency of the line, which decreases X_C and increases X_L until they are equal and this condition would be as shown in Fig. 6.

SERIES RESONANCE

From the foregoing we see that whenever an alternating voltage is applied to a circuit consisting of a capacitance and inductance in series there will be some frequency at which their reactance values are exactly equal, and being of opposite signs, will balance out. The alternating current flow is then determined only by the ohmic resistance of the path. This condition is called RESONANCE, and the particular frequency at which it occurs for a given circuit is called the "resonant frequency" of that circuit.

To determine this frequency in terms of circuit constants we note that the conditions at resonance may be expressed as follows:

$$X_L = X_C$$
 which is the same as: 6.28fL = $\frac{1}{6.28fC}$

Whence
$$f = \frac{1}{6.28\sqrt{LC}}$$

For currents of that frequency, the series arrangement is known as an "acceptor circuit."

Voltages in Series Resonant Circuit. If we let E equal the line voltage then line current I equals $E \div Z$. At resonance, I equals $E \div R$.

The voltage across the coil:
$$E_L = IX_L = \frac{X_L}{R}$$

The voltage across the condenser: $E_{C} = IX_{C} = \frac{X_{C}E}{R}$

From these equations we realize that in a series resonant circuit the voltage across the coil is numerically equal to the voltage across the condenser, and that each may be many times greater than the line voltage, depending on the ratio $X_{C} \stackrel{\cdot}{\to} \mathbb{R}$.

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PARALLEL RESONANCE. In Fig. 7 we show a diagram of an inductance and a condenser connected in parallel with respect to the applied voltage. This is known as an "anti-resonant" or "rejector circuit" because this combination offers an extremely high impedance to the flow of line current at the frequency at which the inductive and capacitive reactances are numerically equal.

The difference between series and parallel resonance is determined by whether the same current set up by the e.m.f. at the source flows





Fig. 7 - PARALLEL RESONANT CIRCUIT Fig.8 - SERIES RESONANT CIRCUIT INDUCTIVELY COUPLED TO CURRENT SOURCE

through both the coil and condenser in succession, or whether the current from the source divides into two separate currents, part flowing through the coil and part through the condenser. A comparison of Figs. 7 and 8 will help you to understand this point.

Voltage in Parallel Resonant Circuit. The voltage across the coil, and likewise the voltage across the condenser, is identical with the line voltage. But we find that the current through the coil may be quite high, as shown by an ammeter connected between one side of the coil and one side of the line. In the same fashion we may insert an ammeter between one side of the condenser and the adjacent side of the line, and find a very high current there indicated. This occurs in spite of the fact that an ammeter in the other supply lead will indicate that the net current coming from the source is quite low. This sounds curious, but no more so than the case of the series resonant circuit, in which the voltage across a reactance was apprecia-bly higher than the supply voltage. In the parallel resonant circuit, the currents through the coil and through the condenser are each about 1/4 cycle different in phase from the impressed electromotive force. The coil current lags behind the voltage, and the condenser current leads the voltage. The currents are in an opposite sense to each other, and the net current from the line can be found either by mathematical calculations or by graphing the values in a "vector diagram," of which the impedance triangle is one form.

What causes the high current we mentioned, and where does it flow? The counter-electromotive forces developed by the coil and by the condenser are opposite in direction, or 180 degrees different in phase. This is illustrated on the circuit diagram of Fig. 7 by drawing an arrow pointing up alongside the coil, to indicate the direction of the counter e.m.f. there at a given instant, and drawing an arrow pointing down alongside the condenser, indicating the opposite direction of its counter e.m.f. Forgetting the presence of the supply

line, the coil and the condenser form a continuous series path, in which two counter e.m.f.'s are being simultaneously developed, and in the same direction around the coil-and-condenser circuit. This addition of counter-electromotive forces causes the flow of a quite considerable circulating current, which is limited by the resistance of the circuit, since the reactances balance out from the series standpoint. So you see that the circulating current is opposed by very little series impedance, while the line current is opposed by quite a high parallel impedance.

THE OSCILLATORY DISCHARGE - ANALYSIS

Fig. 9 represents a condenser of fairly high capacitance connected to a direct current charging source. After charging in this way,

CHARGING Source Protective Lamp P

Fig. 9 - CHART A CONDENSER the condenser may be discharged within a reasonable time by making a metallic connection across the two plates of the condenser as shown in Fig. 10.

When making the connection at plate A first, and then bringing the free end of the wire to plate B, a spark will be seen to pass between the free end of the wire and plate B just prior to the actual contact being made. This spark is only momentary and upon completion of the spark the condenser will be found to be discharged.

In watching the discharge during the short interval of time the spark is visible, it would seem CHARGING that there had been but one rush of current. In reality there were many backward and forward rushes

of current while the spark was in evidence. Starting with the positive plate this discharge moves through the connecting wire to the opposite plate making it positive. This plate then discharges back into the first plate. This backward

then discharges back into the first plate. This backward and forward rush of current continues, giving up some of its energy at each reversal, which is manifested by light, heat, and sound, until the energy originally stored in the condenser has been dissipated. This back and forward motion of current (or oscillations) finally ceases.

The number of oscillations taking place with a given charge will in this case depend mainly upon the resistance of the path between the two plates. When this path is of low resistance, there will be several cycles of current with each succeeding cycle becoming smaller in amplitude than the preceding cycle until the current dies out entirely. This gradual dying out of the oscillations is called damping.

A high resistance path will noticeably affect the discharge taking place between the two plates because the initial discharge may produce few or no reversals of current since the energy is dissipated in heat in flowing through the resistance.

For simplifying our explanation again charge the condenser as indicated in Fig. 9 and place it in series with a coiled conductor and a spark gap as in Fig. 11. A spark discharge will occur across the gap if it is properly adjusted for the condenser voltage. This dis-

Fig. 10 -DISCHARGING CONDENSER

charge will consist of a series of cycles of alternating current of constantly decreasing amplitude. The events taking place during one complete cycle are explained with the aid of sketches as follows:

as the original impressed e.m.f.



Fig. 11

Fig. 11. Just prior to the first spark, the charge in the condenser takes the form of an electrostatic field stored up between the plates. When the gap breakdown voltage is equal to the condenser terminal voltage, a spark will occur across the gap, causing current to flow through the inductance.

Fig. 12. The flow of discharge current has set up

rent reaches its maximum, and the magnetic field flux is at its maximum. This field collapses, inducing an e.m.f. which is opposite to its original

counter e.m.f. and therefore in the same direction

Fig. 13. The e.m.f. induced in the inductance by the collapse of the magnetic field causes a current to flow in the same direction as the original discharge current. This continued current charges the condenser to the opposite polarity, but with somewhat less than the original charge, due to pow-

a strong magnetic field around the inductance. As the electrostatic field reaches zero, the cur-

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Fig. 12



Fig. 13



Fig. 14



er losses in resistance, and the creation of sound and light energy at the gap. Fig. 14. The first spark heated the air in the gap, lowering its resistance and lowering the critical breakdown voltage; so the gap will break down under the lessened voltage to which the condenser is now charged. The current flow resulting will be in the opposite direction to its first flow, and will build up a magnetic field again about the inductance coil but opposite in polarity to Fig. 12.

Fig. 15. When the condenser is completely discharged and the current reaches its maximum, the collapse of the magnetic field will continue this second discharge current in the same direction. This charges the condenser again to its original polarity, but with a somewhat lessened charge.

This action continues, with each reversal of current becoming smaller in amplitude than the preceding one, as shown in Fig. 16.

The foregoing may be summed up by saying that when an isolated charge of electricity is applied to a condenser and the plates are connected together by an external circuit the charges do not completely neutralize at the first instant of discharge but, in fact, several alternations of current take place before a state of equilibrium

is restored. When we have an inductance coil connected in series with a charged condenser as shown in Fig. 17, in which the inductance and capacity can be adjusted to various values, we can, by making the proper adjustments, cause the current to oscillate many times before it finally ceases.



Fig. 16 - DAMPED OSCILLATIONS -EACH REVERSAL OF CURRENT IS SMALLER IN AMPLITUDE THAN THE PRECEDING ONE

In the circuit just described resistance has a marked effect upon the damping of the oscillating circuit.

EFFECT OF RESISTANCE ON OSCILLATIONS

It is clear that some work is done, with the expenditure of electrical energy, when a charge of electricity moves through a circuit such as Fig. 17. Heat is generated to a certain extent in the condenser plates, leads, and the coil; appreciably more heat is generated in the spark itself. The energy loss in sound and light radiation also represents an effective resistance.

The energy-consuming effect of all these resistances is evidenced in the fact that the current intensity for each successive half-cycle is less than the preceding one, because some of the charge is wasted during each flow from plate to plate. It is simple to see that the greater the resistance of the circuit, the less will be the charge that gets through on each reversal.

Feebly damped oscillations are desired as a general rule, and therefore the resistance of a circuit must be kept low. As the resistance is increased there are fewer and fewer cycles before the charge is so reduced that the condenser voltage becomes less than the voltage required to break down the gap.

When in any given circuit a certain critical value of resistance is exceeded, relative to the other factors of the circuit, the circuit will fail to oscillate. The first discharge of the condenser will be its last, for the charge will leak off the condenser at such a low rate that a negligible magnetic field is built up, and the condenser is not charged at all in the opposite direction.

On page 4 you were given the following formula for the frequency of a typical circuit of this kind:

$$f = \frac{1}{6.28 \sqrt{LC}}$$

This is sufficiently accurate for most of the circuits you will use, in which the demands of efficiency require low losses in resistance.

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Fig. 17 - AN OSCILLATORY CIRCUIT WITH VARIABLE INDUCTANCE

In laboratory work, when very close measurements are taken of an oscillatory circuit, the damping factor R/2L is always considered. The formula for frequency then becomes:

$$f = \frac{1}{6.28} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

This formula is also useful in determining whether the circuit will oscillate. The last term of the above equation is the square of the damping factor. The equation may be worked out to determine just how high a value of resistance may be used and still have an oscillatory condition. If R is greater than $2\sqrt{L/C}$, the circuit will not oscillate.

If the resistance R is barely less than $2\sqrt{L/C}$ the circuit is just oscillatory. When the resistance is appreciably less the circuit will oscillate and it can then be put to practical use.

In the foregoing equations the fundamental units were used, namely, the henry and the farad. Since the oscillatory circuits we are chiefly interested in are for providing radio-frequency currents, it is well to have a formula for frequency in terms of the micro-units generally used, and neglecting the damping factor we have

Frequency (in kilocycles) =
$$\frac{159.2}{\sqrt{LC}}$$

where L is in microhenries and C in microfarads.

RESONANCE APPLIED TO AN OSCILLATORY DISCHARGE

In order to bring out the idea of resonance as applied to radio-frequency circuits we will consider three types of systems.



Fig. 18 - AN OSCILLATORY DISCHARGE CIRCUIT COUPLED TO ANOTHER OSCILLATORY CIRCUIT

The first consists of an oscillatory discharge circuit coupled as in Fig. 18 to another circuit having a variable capacitance, a variable inductance and a current indicating meter. The coupling is secured by placing the inductance of the latter circuit in the field of the inductance of the discharge circuit.

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Let us assume the circuit LCG is set into oscillation by charging the condenser; the variable capacity C_1 is left set at some convenient value and the inductance L_1 carefully adjusted. A point will be found where the milliammeter will give a maximum deflection. At any other adjustment of the inductance the meter will indicate a lower reading. It can be shown that when the current indication is maximum, the values of L_1 and C_1 are such that their circuit has the same natural frequency as the discharge circuit. It is under such adjustments only that large values of current can be induced in the circuit L_1-C_1-A .

The same reasoning applies to the above when the inductance tap is left fixed at some value, and the condenser C_1 varied through the range of its values. Since a variable condenser provides a better means for obtaining a continuous change of values by small amounts than is afforded by the usual coil, a still more accurate determination can be made of the resonant condition of the circuit by varying C_1 . Due to this fact, it may be possible to bring the circuit so closely into resonance that the current will read higher than when the frequency change was made by connecting to different turns of the coil.

The flow of current through the coupled circuit is attended by a certain loss of energy in the resistance of that circuit. This energy, being supplied by the oscillatory discharge circuit, will cause



Fig. 19 - A LARGE CONDENSER IS FORMED, IN EFFECT, BETWEEN THE ANTENNA WIRES AND THE GROUND

the energy of each cycle of current in the latter to be decreased by that amount. This means that the damping of the oscillatory discharge will be greater than if the resonant circuit were removed from the influence of the discharge circuit.

The second illustration of resonance at radio frequency is given in the use of an inductance between an antenna and ground. Instead of a small condenser we have, in effect, a large condenser which is formed by the antenna wires as one plate, and the ground as the other plate. The air between constitutes the dielectric. As shown by Fig. 19 the condenser will be charged by the electrostatic field which is one component of the energy sent out into space in the form of radio waves from a transmitting station. In addition, there is an electromagnetic field component of those waves which induces currents in the antenna wires and leads due to their small inductance, even though they are straight wires. If the inductance coil has the proper value for resonating a receiving antenna with the frequency of the incoming waves, a maximum current will flow through the coil.

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As a third case, if we inductively couple to the antenna coil L another circuit containing a coil L_1 and a variable condenser C, as shown in Fig. 20, the magnetic field set up about the coil L will cut the coil L_1 and induce therein an e.m.f. This will cause current to flow in the circuit L_1 -C. The value of this current will be



Fig. 20 - MAXIMUM TRANSFER OF SIGNAL ENERGY WILL RESULT IF THE PRIMARY AND SECONDARY CIRCUITS ARE IN RESONANCE

small unless we make use of the principles of resonance again. A value may be found for the variable condenser which will make the natural frequency of its circuit the same as the frequency of the incoming signal. At this point of resonance the maximum current for that signal will flow; at the same time an appreciably high impedance is offered to currents of any other frequency which may exist in the antenna circuit. Tuning can be accomplished by using either a variable inductance or a variable capacitance, or both. Convenience in manufacturing and the fine degree of variation allowed has led most manufacturers to use variable condensers for tuning receivers. When a wide range of frequencies must be received, it is sometimes good practice to have the inductance variable in steps, with the condenser providing the fine variation for accurate tuning.

INDUCTANCES IN SERIES IN AN OSCILLATORY CIRCUIT

In practice we find oscillatory circuits which require more than one inductance coil for their operation. If the inductances have no



Fig. 21 - DIVIDING INDUCTIVE REACTANCE INTO TWO SECTIONS, AS IN THE ORIGINAL HARTLEY CIRCUIT

mutual field flux (zero coupling between them) the total inductance will be the sum of the separate inductances.

However, dividing the total inductance into separate units permits us to use these units in different ways. In Fig. 21 is shown the resonant circuit used in a particular vacuum tube oscillator known as the "Hartley" oscillator. The resonant voltages across the coils are numerically in direct proportion to their inductances. The usefulness of the circuit consists in the phase difference between the voltages across the coils. Whenever the top condenser plate is positive, the lower plate is negative. The upper lead is then positive with respect to the center lead, and the lower lead is negative with respect to the center lead. Opposition of phase of these voltages holds good throughout all the polarity reversals of the oscillating current.

CAPACITIES IN SERIES IN AN OSCILLATORY CIRCUIT

Just the opposite use of reactances is found in the resonant circuit of the "Colpitts" oscillator, shown in Fig. 22, in which a single inductance is used, but the net capacity for resonance is secured



Fig. 22 - DIVIDING CAPACITIVE REACTANCE INTO TWO SECTIONS, AS IN THE COLPITTS CIRCUIT

through the use of two condensers in series. When the charges of the condensers are such that the upper lead is positive with respect to the lower lead, then the upper lead is positive with respect to the center lead, and the lower lead is negative with respect to the center lead. Phase opposition of these two voltages holds good throughout the current reversals of the oscillatory circuit.

The voltages across the condensers are in direct proportion to their capacitive reactances, and therefore in inverse proportion to the capacities of the condensers.

EXAMINATION QUESTIONS

- 1. What conditions must be met in a series type alternating current circuit, which contains both inductance and capacity, to obtain a maximum current flow?
- 2. Will a perfect condenser retain its charge over a period of time?
- 3. When a condenser is discharged is there only one rush of current?
- 4. How will a high resistance path affect the discharging of a condenser?
- 5. Explain briefly in your own words the action of the discharging condenser when inductance is in the circuit.
- 6. In tuning a circuit to resonance with only one variable reactance what advantage has the variable condenser over the usual variable inductance?
- 7. Is it possible to tune a receiving set in which the capacity and inductance are both variable?
- 8. Which have you found by experience to be the most generally used method of tuning, fixed inductance with variable capacity, fixed capacity with variable inductance, or with both variable?
- 9. In what way does a receiving antenna and ground system respond to the electrostatic field of a transmitting station?
- 10. What rule of current flow determines whether a resonant circuit is of the series or the parallel type?

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