Electronics

Radio

Television

Radar

UNITED ELECTRONICS LABORATORIES

LOUISVILLE KENTUCKY

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ELEMENTARY ALGEBRA FOR ELECTRONICS **ASSIGNMENT 10**

ELEMENTARY ALGEBRA FOR ELECTRONICS

In the mathematics studied in the previous assignments, we have dealt with the solution of arithmetic problems. These are problems involving addition, subtraction, multiplication and division of positive numbers. We have also studied roots and powers.

In algebra, we will apply these same operations to two new types of problems. These algebra problems will have negative number and they will use letters to represent numbers.

Actually, we have been using both of these things in our everyday life, but when we see them in a mathematical problem, they look as if they would be very difficult to handle. We shall see that there is nothing difficult about algebra, and that it shall be a very great help in our progress through electronics. If it so happens that you have not studied algebra previously, or if you have "forgotten all you knew" about algebra, do not become discouraged at this point. If you will just study through this assignment, a step at a time, you have a very pleasant surprise in store for you. You will find that algebra is a relatively simple subject and that you will master it easily!

Negative Numbers

In our study of algebra, let us first consider negative numbers.

Negative numbers are numbers which are less than zero, while positive numbers are numbers which are more than zero. Negative numbers are distinguished from positive numbers by the minus sign. Thus, —10 means 10 below zero, while +10 means 10 above zero. In most cases, +10 is written just 10. The plus sign is understood. Any number with no sign indicated is a positive number. So 17 means +17, 3 means +3, and 19.28 means +19.28. Negative numbers are used in everyday life. For example, a weather report lists a temperature of 10° below zero as —10°. An airplane pilot might think of a ten mile an hour tail wind as +10 miles an hour, and a ten mile an hour headwind as —10 miles an hour. A mechanical engineer working with steam and water pressure at a power plant might list a suction force of 2 pounds as —2 pounds per square inch of pressure.

In electronics work, we will have a great need for negative numbers. In Figure 1, for example, the wire lead connected to the battery may be either 6 volts above (+6V) or 6 volts below (—6V) ground potential depending on the way the battery is connected. In Figure 1(A), the wire lead is 6 volts more positive than ground potential, so the voltage in respect to ground would be +6V. In Figure 1(B), the voltage would be —6V. In each case we have 6 volts of electrical pressure. The + or — sign in front of the 6 volts tells us the direction of the electrical force or pressure. We know that the direction of the 6 volts is important. In all electrical circuits, the direction of the voltage determines the direction of current flow. Vacuum tube and transistor circuits will not function at all if the voltages applied to the different elements are not in the proper direction. There are many other instances when we will have to distinguish between plus (+) and minus (—) in electronics work.

The absolute value of a number is its value without reference to its sign. Thus, the absolute value of +6 is 6, and the absolute value of —6 is 6.

Now that we have found out what a negative number is and why we need be concerned with negative numbers, let us proceed to find out how to perform the mathematical operations of addition, subtraction, multiplication and division using these numbers.

Algebraic Addition

Let us consider the temperature reading of 10° below zero mentioned previously. This could be written algebraically as —10°. What would the thermometer read if the temperature dropped 5° more? Of course, we know that the thermometer would read 15° below zero or —15°. Notice that —5 added to —10 gives —15.

We already know how to add two positive numbers; +15 added to +10 gives +25. This demonstrates how we should add numbers with the same sign.

To add numbers with the same sign, add the absolute value of the numbers, and place the common sign in front of the answer. Thus, to add —70 and —30, add 70 and 30. This gives 100 for an answer. Then put the sign of both number (—) in front of the answer. The complete answer, then, is —100.

Examples of addition of numbers with the same sign are given below:

1.	192	2.	—71	3.	— 6.3	4.	—1603.1	5.	17.2
	46		—19		—18.2		— 21.4		9.6
	+238		—90		—24.5		—1624.5		+26.8
6.	—14.7	7.	—43.06	8.	15.20	9.	—43.15	10.	55.55
	— 6.07		— 7.02		40.06		—26.09		44.45
	—20.77		—50.08		+55.26		—69.24		+100.00

Again referring back to the thermometer which is reading 10° below zero, let us find what temperature would be indicated if the weather warmed up 5 degrees. We know from general knowledge that it would now be 5 below zero. To state this mathematically, —10 +5 = —5.

Also if the weather warmed up 15 degrees, from the 10° below zero point, we know that the thermometer would indicate +5 degrees. Stated mathematically this is —10 +15 = +5.

Let us state this mathematical operation in the form of a rule.

To add numbers of unlike sign, subtract the smaller absolute number from the larger absolute number, and place the sign of the larger number in front of the answer.

To add —10 and +5 we write 10 —5 = 5, then since 10 is the larger absolute number, we put (—) before the answer (—10 +5 = —5). Also, to add —10 and +15 we write 15 —10 = 5. Since the 15 is the larger number, we have a +5 for the answer. Several examples of adding numbers with unlike sign follows:

1.	2.	3.	4.	5.	6.	7.
—10	+5	—90	+16	+72	—66.6	—30
+25	—8	+10	—17	—72	+73.4	+12
+15	—3	—80	— 1	0	+ 6.8	—18

These mathematical rules can be applied directly to electronics circuits. In Figure 2, we have two batteries, one an 8 volt battery, and the other a 6 volt battery. We wish to know the potential difference between point x and ground. In adding voltages in a circuit such as Figure 2, we start at the point whose potential we wish to know, and list the voltages as we go around the circuit.

In Figure 2A, from point x to ground, we have +6 and +8 volts. In 2A, point x is 14 volts positive in respect to ground, or 14 volts above ground. In Figure 2B, from x to ground, we have —6V and —8V.

$$\begin{array}{r} -6 \\ -8 \\ \hline -14 \end{array}$$

Point x is —14 volts in respect to ground in Figure 2B.

In Figure 2C, from point x to ground, we have —6 and +8 volts.

$$\begin{array}{r} -6 \\ +8 \\ \hline +2 \end{array}$$

Point x is 2 volts positive in respect to ground.

In Figure 2D, from x to ground we have +6 and —8 volts.

$$\begin{array}{r} +6 \\ -8 \\ \hline -2 \end{array}$$

Point x is —2 volts in respect to ground.

This same process has been applied to the circuits in Figure 3. Check each of these by adding the battery voltages, and see if you agree that the voltage indicated across each resistor is correct.

If more than two numbers are to be added, first add all of the numbers of like signs and then add the two sums. For example, if we wish to add —2, 4, —8, 7 and 10. First we add the numbers with like sign.

$$\begin{array}{r} -2 \\ -8 \\ \hline -10 \end{array} \qquad \begin{array}{r} 4 \\ 7 \\ 10 \\ \hline 21 \end{array}$$

Now add the two sums:

$$\begin{array}{r} 21 \\ -10 \\ \hline 11 \end{array}$$

(answer)

Add: 42, —16, 33, 14, —18

—16	42	89
—18	33	—34
—34	14	55 (answer)
	89	

Add: —73, 44, —21, 16, —51

—73	44	—145
—21	16	60
—51	60	— 85 (answer)
—145		

For practice, add the following numbers:

1. —7, —12 = −19
2. 27, —3 = + 24
3. —9, 17 = + 8
4. —25, —10 = - 35
5. 4, 8, 12 = 24
6. 16, —5, 4 = 15
7. 14, —5, —9 = 0
8. —17, 8, —10 = - 19
9. —2, 14, —6 = 6
10. 72, —66, —18, 23, —9 = 2

Algebraic Subtraction

The rule for algebraic subtraction is:

To subtract one number from another, change the sign of the number to be subtracted and then proceed as in addition.

Examples:

Subtract —6 from 8 | Subtract 8 over —6 | Change —6 to +6 and add. 8 + +6 = 14 (answer)

	Subtract		
Subtract —6 from 8	8 —6	Change —6 to +6 and add.	8 +6 14 (answer)
Subtract —6 from —8	—8 —6	Change —6 to +6 and add.	—8 +6 —2 (answer)
Subtract 6 from —8	—8 +6	Change +6 to —6 and add.	—8 —6 —14 (answer)

Subtract 6 from 12	12 —6 6	(Note: sign changed) (answer)
Subtract —6 from 14	14 +6 20	(answer)
Subtract 3 from —12	—12 —3 —15	(answer)

Subtract —7 from —12

$$\begin{array}{r} -12 \\ +7 \\ \hline -5 \end{array}$$ (answer)

Subtract —12 from —7

$$\begin{array}{r} -7 \\ +12 \\ \hline 5 \end{array}$$ (answer)

For practice, solve the following problems. Subtract the lower number from the upper number. You should be able to "mentally" change the sign of the lower number and add.

1. 25	2. 4	3. —19	4. —6	5. —8	6. —927	7. .006	8. —18.7
11	—2	18	—7	—6	—427	.93	4.27
14	6	-37	1	-2	-500	[redacted] -.924	-22.97

Multiplication and Division

Multiplication and Division with positive and negative numbers is very simple. One easy rule tells us whether the answer is plus (+) or minus (—).

In multiplication and division, **if both numbers have the same sign** (either positive or negative), **the answer will be positive, and if the two numbers have opposite signs,** (one positive and one negative), **the answer will be negative.**

For purpose of illustration, let us divide this into four parts. First let us consider multiplication of numbers with like signs. **If two numbers have the same sign** (both positive or both negative) **their product will be positive.**

Examples: (1) 8 x 6 = 48 (2) (—8) x (—6) = 48

If two numbers have different signs (one positive and one negative) **their product will be negative.**

Examples: (3) (—8) x 6 = —48 (4) 8 x (—6) = —48

These four examples are easy to understand. Multiplication is a short-cut for addition. In example 1, we have added 8 six times. In example 2, we have subtracted —8 six times. In example 3, we have added —8 six times. In example 4, we have subtracted 8 six times.

Ten more examples of multiplication of numbers are shown below.

(5) 16 x —2 = —32	(8) —4 x 3 = —12	(12) —72 x 2 = —144
(6) —9 x —5 = 45	(9) 4 x —3 = —12	(13) —72 x —2 = 144
(7) 7 x 5 = 35	(10) —6 x —4 = 24	(14) 72 x 2 = 144
	(11) 72 x —2 = —144	

The rule for division is identical. **If the two numbers we start with are both of the same sign, the answer will be positive.**

Examples: (1) $\frac{48}{6} = 8$ (2) $\frac{-48}{-6} = 8$

If the two numbers we start with have different signs, the answer will be negative.

Examples: (3) $\frac{-48}{6} = -8$ (4) $\frac{48}{-6} = -8$

These examples are easy to understand if we consider the division problems as "check work" for our four previous multiplications.

Some more examples of division are given below:

(5) $\frac{16}{4} = 4$

(6) $\frac{-16}{-4} = 4$

(7) $\frac{72}{-24} = -3$

(8) $\frac{-72}{24} = -3$

(9) $\frac{-72}{-24} = 3$

(10) $\frac{100}{-10} = -10$

For practice, solve the following problems:

(1) $12 \times -3 =$ -36

(2) $-6.4 \times 3 =$ -19.2

(3) $-9 \times -.03 =$ .27

(4) $12 \times -14 =$ -168

(5) $-.08 \times .02 =$ -.16

(6) $\frac{12}{3} =$ 4

(7) $\frac{-6}{3} =$ -2

(8) $\frac{-36}{-6} =$ 6

(9) $\frac{300}{-15} =$ -60

(10) $\frac{-2}{-8} =$ $\frac{1}{4}$ = .25

Algebraic Terms

An algebraic term is made up of three definite components.

a. Sign. The sign of the term may be (+) or (—). We have already covered the rules for obtaining the correct sign in a problem.

b. Coefficient. The coefficient is merely a number that tells us how many units we have in the term. We worked with both signs and coefficients in the preceeding examples.

c. Literal factors. The literal factors are usually letters from the alphabet used to represent certain numbers whose value may be unknown.

In the term —3 ABC, the sign is (—), the coefficient is 3 and the literal factors are A, B, and C. We can read this term —3ABC, as minus three

ABC. The term actually means —3 times A times B times C.

In the term RP, the sign is (+), the coefficient is 1, and the literal factors are R and P. When no other coefficient is shown, it is understood to be one (1). This term is read RP, and means R times P.

In the term —XYZ the sign is (—), the coefficient is 1, and literal factors are X, Y and Z. The term means, —1 times X times Y times Z.

In the term 77XYP, the sign is (+), the coefficient is 77, and the literal factors are X, Y and P.

In the term $\frac{2X}{3Y}$, the sign is (+), the coefficient is $\frac{2}{3}$, and the literal factors are X and Y. This term could be read as two X divided by three Y, or as two X over three Y, or as $\frac{2}{3}$ X over Y. The term actually means 2 times X all divided by 3 times Y.

Thus, we see that 3A means 3 times A, and AB means A times B.

In algebra, the letters or literal factors, are generally used in place of numbers. Sometimes the numbers for which these letters are used are known, and sometimes they are unknown. Let us find the value of some terms when we know the value of the literal terms.

Assume, a = 6, b = 2, c = 3

Example 1. 3abc = 3 x a x b x c. Now substituting the numbers which each letter is equal to in the problem we have: 3abc = 3 x a x b x c = 3 x 6 x 2 x 3 = 108.

Example 2. $\frac{2bc}{5a} = \frac{2 \times 2 \times 3}{5 \times 6} = \frac{\overset{2}{\cancel{12}}}{\underset{5}{\cancel{30}}} = \frac{2}{5}$ or .4

Example 3. —6bc = —6 x 2 x 3 = —36

Example 4. $\frac{-3ab}{-ac} = \frac{-3 \times 6 \times 2}{-6 \times 3} = \frac{-36}{-18} = 2$

Using the same values for $\overset{6}{a}$, $\overset{2}{b}$, and $\overset{3}{c}$ as in the Example 1, 2, 3, and 4, find the value of the terms in the four problems below.

1. 7ac = 126
   7x6x3
2. $\frac{72}{-abc} = -2$
3. $\frac{-6a}{bc} = -6$
4. —5bc = -30

In the next four problems, assume that f = 2, g = —3, h = 7, k = —4. Find the numerical value of each of these terms.

5. $\frac{gk^{\,12}}{f_{\,2}} = 6$
6. $\frac{49f^{\,98}}{gk_{\,12}} = 8\frac{1}{6}$
7. 3 kgfh = 504 [redacted]
8. $\frac{-2k^{\,8}}{g} = -2\frac{2}{3}$

Kinds of Algebraic Expressions

The algebraic expressions we have dealt with so far have been **Monomials.** Monomial is a fancy way of saying **one term.** Any number of numbers and literal factors may be multiplied together, or divided and still remain a monomial, but any time addition or substraction occurs in an algebraic expression, the expression is no longer a monomial. For example, 6XYZQ is a monomial and 7BCDEF is also a monomial, but 7BC + DEF and 7BC — DEF are **not** monomials. In the expression 7BC + DEF, we have two terms. The two terms are 7BC and DEF. This algebraic expression is called a **Binomial.**

In the expression 5c —3ab + 27, we have three terms. This expression is a **Trinomial.**

All expressions having more than one term may be called **Polynomials.** Polynomial means many terms.

Algebraic terms frequently have exponents in them. We worked with exponents when we studied Powers of Ten. Remember, 10^5 meant 10 x 10 x 10 x 10 x 10. In the term, 10^5, the base is 10 and the **exponent** is 5. The exponent indicates the number of times the base is to be used as a factor. In the term, A^5, A is the base and 5 is the exponent. This term means A x A x A x A x A. Likewise, B^6 means B x B x B x B x B x B. The term C has an exponent of 1 understood, and can be written C^1. The term ab^2 means a x b x b, and the term $m^2n^2y^3$ means m x m x n x n x y x y x y.

When we were studying powers of 10, we found that to multiply terms, we added the exponents. Thus $10^3 \times 10^2 = 10^{3+2} = 10^5$. Likewise, the terms $a^3 \times a^2 = a^{3+2} = a^5$. This is logical when we consider what a^3 and a^2 means. The term a^3 means a x a x a, and a^2 means a x a. Then $a^3 \times a^2$ means a x a x a times a x a, or a x a x a x a x a or a^5. To state this in the form of a mathematical law we could say, when the bases are the same (the base is the number which is to be multiplied by itself), we add exponents in multiplication. Then $b^5 \times b^2 = b^7$ and $y^2 \times y^6 = y^8$. But a^2 times b^3 has to be written as a^2b^3. Since the bases are not the same, we cannot combine exponents.

In division, literal factors of the same base are combined by subtracting their exponents. Thus $X^3 \div X^2 = X^{3-2} = X^1$ or X. Remember we did the same thing when dealing with powers of 10. For example,

$$10^3 \div 10^2 = \frac{10^3}{10^2} = 10^{3-2} = 10^1 \text{ or } 10.$$

Also, $$\frac{X^4Y^3}{X^2Y} = X^{4-2}\,Y^{3-1} = X^2Y^2.$$

Several examples involving the multiplication and division of terms containing exponents follow:

Examples: 1. $C^2 \times C = C^{2+1} = C^3$
2. $2D^5 \times D^2 = 2 \times D^{5+2} = 2D^7$
3. $ab \times a^2b^2 = a^{1+2}\, b^{1+2} = a^3b^3$
4. $abc \times a^2b^2d = a^{1+2}\, b^{1+2}\, cd = a^3b^3cd$

5. $D^5 \div D^2 = \dfrac{D^5}{D^2} = D^{5-2} = D^3$

6. $X^2Y^2 \div XY = \dfrac{X^2Y^2}{XY} = X^{2-1}\, Y^{2-1} = XY$

7. $\dfrac{a^2b^2c^2}{ab} = a^{2-1}\, b^{2-1}\, c^2 = abc^2$

8. $\dfrac{x^2y^2z^2c}{y^2} = x^2y^{2-2}\, z^2c = x^2z^2c$

For practice, solve the following problems:

1. $b \times b =$ b^2
2. $b \times b^2 =$ b^3
3. $c^2 \times c^8 =$ c^{10}
4. $XYZ \times 2XY =$ $2X^2Y^2Z$
5. $a^2b^2 \div b =$ a^2b
6. $XY^2Z^2 \div XYZ =$ YZ
7. $c^3d^2e \div c^2e =$ cd^2
8. $y^5x^3 \div y^3x^3 =$ y^2

The term $2y^2$ means $2 \times y \times y$. Notice that only the literal factor (y in this example) is squared. The term $(2y)^2$, means to square the entire term 2y. This is equal to $2y \times 2y = 2 \times 2 \times y \times y$ or $4y^2$.

In addition and subtraction we can only combine terms whose literal factors are identical (the same letter and the same exponent).

Examples: (1) $2X + 3X = 5X$ (2) $4A - 7A = -3A$
(3) $2B + 5C - 6C = 2B - C$ (4) $3A + 5A^2 - A^2 = 3A + 4A^2$

The process of combining the like terms in an algebraic expression is called **combining terms.**

Examples of combining terms are given below.

(1) Add 3XY, 4abc, 2XY, —2abc, 10

The 3XY and the 2XY can be combined since they have identical literal factors. $3XY + 2XY = 5XY$. The 4abc and the —2abc can be combined: $4abc - 2abc = 2abc$. The entire expression is then equal to $5XY + 2abc + 10$.

(2) Add —6E, 14R, 3E, —5R $= -3E + 9R$

The easiest way to combine such terms is to place them in columns, placing terms with identical literal factors in the same column, and then adding.

Solution: $-6E$
$3E$
$14R$
$-5R$
$-3E + 9R$ (answer)

(3) Add $3A + 5B$, $2A - 7B$, $B + C$
$3A + 5B$
$2A - 7B$
$B + C$
$5A - B + C$ (answer)

(4) Add $13XYZ - 2XY^2Z$, $5XY^2Z - 27XYZ$
Solution: $13XYZ - 2XY^2Z$
$-27XYZ + 5XY^2Z$
$-14XYZ + 3XY^2Z$ (answer)

(5) Subtract $3A + 5B$, from $2A - 7B + C$
Solution: $2A - 7B + C$
Subtract: $3A + 5B$

Remember that algebraic subtraction is performed by changing the sign of the lower quantity in the problem and then adding.

Changing the sign of the lower quantity we have:
$2A - 7B + C$ (note we had to change the sign of
$-3A - 5B$ **each term** in the lower number)
$-A - 12B + C$ (answer)

(6) Subtract $6XY$ from $3XY$
$3XY$
$-6XY$
$-3XY$ (answer)

(7) Subtract $17a - 4b$ from $3a - b$
Solution: $3a - b$
$-17a + 4b$
$-14a + 3b$ (answer)

(8) Subtract $3a^2b - d$ from $7xy + d$
Solution: $7xy + d$
$-3a^2b + d$
$-3a^2b + 7xy + 2d$ (answer)

For practice, solve the following problems:

1. Add $6A$, $9B$, $-3A$, $3B$ = 3A + 12B
2. Add $13a + 5b$, $-7a - 10b$, $a - 4b$ 7a − 9b
3. Add $16XY + 3ab$, $6ab - 4XY$ 12XY + 9ab
4. Subtract $7X$ from $19X$ 12X
5. Subtract $6a^2b + 3ab^2$, from $11a^2b + ab^2$ 5a²b − 2ab²
6. Subtract $16yx - 3mn$ from $13yx - 4mn$ −3yx − 1mn

Signs of Operation

Before we can conveniently use binomials, trinomials and polynomials, certain symbols or Signs of Operation are needed. These Signs of Operation are used merely to reduce the amount of written work in the solution of algebraic problems.

The most commonly used signs of operations are:

a. Parentheses ( ) b. Brackets [ ] c. Braces { }

Notice that we always use these symbols in pairs. Parentheses, Brackets and Braces all have the same meaning. They indicate that all the terms inside are to be considered as one quantity.

Thus: $(7A)$ means $7A$, and $4B + (2C)$ means $4B + 2C$.

$-(3A + 4B - C)$ indicates that we are to subtract $3A + 4B - C$ from some other quantity, or from zero. Remember our rule for subtraction: "Change the sign and add". If we remove this set of parentheses we change **each** sign.

Thus: $-(3A + 4B - C) = -3A - 4B + C$

Also: $3(3A + 4B - C)$ indicates that we are to multiply $3A + 4B - C$ by 3.

Thus: $3(3A + 4B - C) = 9A + 12B - 3C$.

Likewise, $-2(4A - 6C + 2) = -8A + 12C - 4$. Notice that all we really did in this last case was to multiply the three terms inside the parentheses by -2.

If we have $-8\ (A - 3B + 4A + B - C + 2A)$ it is best to collect terms within the parentheses before we multiply by -8.

Thus: $-8(A - 3B + 4A + B - C + 2A) = -8(7A - 2B - C) = -56A + 16B + 8C$.

In the algebraic expression: $2 + 3\ (4 - 2a + b - 3a)$ we first collect terms inside the parentheses and we have $2 + 3(4 - 5a + b)$. Next multiply the terms within the parentheses by 3. This gives $2 + 12 - 15a + 3b$. Again collecting terms, the final answer is $14 - 15a + 3b$.

Notice that the 2 was not involved with the parentheses.

Another example indicating the use of parentheses follows.

$6B + 7C - (4B + 5C - D + C) =$ First collect terms within the parentheses.

$6B + 7C - (4B + 6C - D) =$ Removing the parentheses, we have to change the signs of each term in the parentheses due to the minus sign before the parentheses. $6B + 7C - 4B - 6C + D =$ Then collect terms; $2B + C + D$ (Answer.)

We will now work a longer problem containing parentheses and brackets. Notice that in removing signs of operation we start with the innermost signs.

Example: $2[3a + 2(a + b)] =$

First we start with terms in parentheses.

$2[3a + 2a + 2b] =$

Now we collect terms

$2[5a + 2b] =$

Now we remove the brackets.

$10a + 4b$ (answer)

Example:

$$3 + 2a\{5 - b + 2[3 - 2(a + b) - 4 + 3(a + b - 3a) + 2] - 3a\}$$

Collecting terms:

$$3 + 2a\{5 - b + 2[3 - 2(a + b) - 4 + 3(-2a + b) + 2] - 3a\}$$

Removing parentheses:

$$3 + 2a\{5 - b + 2[3 - 2a - 2b - 4 - 6a + 3b + 2] - 3a\}$$

Collecting terms: $3 + 2a\{5 - b + 2[1 - 8a + b] - 3a\}$

Removing brackets: $3 + 2a\{5 - b + 2 - 16a + 2b - 3a\}$

Collecting terms: $3 + 2a\{7 + b - 19a\}$

Removing braces: $3 + 14a + 2ab - 38a^2$ (answer)

For practice solve problems 1 through 10. Simplify the expression by removing signs of operation. Answers have been given for the first three problems. Check the answers before proceeding with the remaining 7 problems. The terms in your answers need not appear in the same sequence as shown in Problems 1, 2 and 3. Thus $5a + 3b - 13c$ could also be written as $3b + 5a - 13c$ or $-13c + 5a + 3b$ etc.

1. $4a + 5b - 10c - (3c + 2b - a) = 5a + 3b - 13c$
2. $3(a - b - 4a + 5) + 2 - 6a(b + 3) = -27a - 3b - 6ab + 17$
3. $4 + 3[b - 2a(a + b - 3a + 3) + 2b] = 12a^2 - 18a - 6ab + 9b + 4$
4. $7(2a) + 5(3b) =$ $14a + 15b$
5. $3(a + c) - 4(a - b - c) =$ $-1a + 4b + 7c = -a + 4b + 7c$
6. $5 + 2(10 - 6a + 5 - 4b + 3 - a) =$ $77 - 28a - 16b$
7. $3[a + 2(b + c)] =$ $3a + 6b + 6c$
8. $2(a + b) + 6(b + c) - 4(a - c) =$ $2a + 8b + 10c$
9. $6 - 4[5 - 3(a - 8) + 2(6 - a - 10) + 2] =$ $-86 + 20a$
10. $7a - 4[4 - (a - 3) + 2(b + 3) + 4a] =$ $-52 - 5a - 8b$

Multiplication of Polynomials

The term 2abc means 2 x a x b x c. Also the term (2) (a) means 2 x a or 2a. (3b) (4c) means 3b x 4c or 12bc.

It follows that $(3 + 4)(6 - 2)$ means $(3 + 4) \times (6 - 2)$. $(3 + 4) = 7$. and $(6 - 2)$ equals 4, so $(3 + 4)(6 - 2)$ is equal to $(7)(4) = 28$.

We could also obtain the answer, 28, in the following manner:

Multiply the 3 by the 6 and then by -2.

Multiply the 4 by the 6 and then by -2.

Sum up the four products.

Thus: $(3 + 4)(6 - 2) = 18 - 6 + 24 - 8 = 28$

Notice that we multiply the first number in the first parentheses by **each** of the numbers in the second parentheses. Then we multiply the second number in the first parentheses by **each** number in the second parentheses. Then we sum up the four products.

This will be demonstrated in the examples below.

Example 1. $(4 - 3)\ (7 - 2) = 28 - 8 - 21 + 6 = 5$

Example 2. $(6 + 2)\ (3 - 4) = 18 - 24 + 6 - 8 = -8$

We will use this same method in multiplying algebraic expressions containing more than one term.

Thus: $(a + 6)\ (a - 3) = a \times a - 3 \times a + 6 \times a - 18 = a^2 - 3a + 6a - 18 = a^2 + 3a - 18.$

Also: $(a^2 - 3)\ (7 - b) = 7a^2 - a^2b - 21 + 3b$

In these examples we have used the same method outlined above.

Additional problems of this nature are given below:

Example 1. $(4C - b)\ (7 - 2d) = 28C - 8Cd - 7b + 2bd$

Example 2. $(3a^2 - b)\ (2a - b) = 6a^3 - 3a^2b - 2ab + b^2$

Example 3. $(9X + 7Y)\ (X + Y) = 9X^2 + 9XY + 7XY + 7Y^2 = 9X^2 + 16XY + 7Y^2$

Example 4. $(a + 7)\ (10 - a^2 + a) = 10a - a^3 + a^2 + 70 - 7a^2 + 7a = 17a - a^3 - 6a^2 + 70$

Notice that in this problem there were three terms in the second parentheses, but we applied the same rule and multiplied each term in the first parentheses times **each term** in the second parentheses. Example 5 shows another such problem.

Example 5. $(3 - M)\ (10 + M^2 + M^3) = 30 + 3M^2 + 3M^3 - 10\ M - M^3 - M^4 = 30 + 3M^2 + 2M^3 - M^4 - 10M$

Note in Example 4 and 5, the answers are correct, but are not considered to be in the best form. To state the answers to Example 4, $17a - a^3 - 6a^2 + 70$, we should write it $-a^3 - 6a^2 + 17a + 70$. This has the literal factors arranged with powers in descending order. The $-a^3$ is the highest power in the term so should be written first. The $-6a^2$ is the next highest power so should come next. The next highest power is the $17a$ and last of all is the $+70$.

The answer to Example 5 should be written $-M^4 + 2M^3 + 3M^2 - 10M + 30$.

Example 6. $(a + b)\ (a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$

For practice solve the following problems:

1. $(6 + b)\ (a + 3) =$ $6a + 18 + ab + 3b$
2. $(c - 4)\ (5 - c) =$ $9c - c^2 - 20$
3. $(a^2 + 3)\ (a + 6) =$ $a^3 + 6a^2 + 3a + 18$
4. $(X + Y)\ (X - Y) =$ $X^2 - Y^2$
5. $(a + 6)\ (a^2 + a + 1) =$ $a^3 + 7a^2 + 7a + 6$

Division of Polynomials

A method quite similar to long division in arithmetic may be used when dividing expressions containing several terms.

For example, let us divide $(a^2 + 2ab + b^2)$ by $(a + b)$.
Write the problem in the form of a long division problem.

Example 1. $$a + b\ \overline{)\,a^2 + 2ab + b^2}$$

Then we see how many times the first term in the divisor will go into the first term in the dividend.

In this case, a will go into a^2, a times, so a is the first term in the answer.

$$\begin{array}{rl} & \,a \\ a+b & \overline{)\,a^2+2ab+b^2} \\ & \,\underline{a^2+ab} \end{array}$$

Multiply a times $a + b$, and put the product below the terms in the dividend as shown.

$$\begin{array}{rl} & \,a \\ a+b & \overline{)\,a^2+2ab+b^2} \\ & \,\underline{a^2+ab} \\ & ab+b^2 \end{array}$$

Subtract the product $(a^2 + ab)$ from $a^2 + 2ab + b^2$.

Now the first term in the divisor, a, goes into ab, $+b$ times, so $+b$ is the next term in the answer.

$$\begin{array}{rl} & \,a+b \\ a+b & \overline{)\,a^2+2ab+b^2} \\ & \,\underline{a^2+ab} \\ & ab+b^2 \end{array}$$

Then multiplying $(a + b)$ times b, enter the product as shown and subtract. There is no remainder, so the problem works out evenly.

$$\begin{array}{rl} & \,a+b \\ a+b & \overline{)\,a^2+2ab+b^2} \\ & \,\underline{a^2+ab} \\ & ab+b^2 \\ & \underline{ab+b^2} \end{array}$$

The answer is $a + b$.

To check the answer, multiply the answer by the divisor, and the dividend should be the product.

Check: $(a + b)(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$.

Example 2. Divide $(14 + X^3 - 2X^2 + 4X)$ by $(X - 3)$

$$X - 3 \overline{\big)\, X^3 - 2X^2 + 4X + 14}$$

Notice that we rearranged the terms and placed X^3, the highest power of X first, and so on down to 14, the term containing no powers of X. This greatly simplifies our work.

$$\begin{array}{r|l} & X^2 \\ \cline{2-2} X - 3 & X^3 - 2X^2 + 4X + 14 \\ & \underline{X^3 - 3X^2} \\ & \qquad\quad X^2 + 4X + 14 \end{array}$$

The first term in the answer is X^2, since X will go into X^3, X^2 times. We multiply X^2 times $X - 3$, and subtract the product.

$$\begin{array}{r|l} & X^2 + X \\ \cline{2-2} X - 3 & X^3 - 2X^2 + 4X + 14 \\ & \underline{X^3 - 3X^2} \\ & \qquad\quad X^2 + 4X + 14 \\ & \qquad\quad \underline{X^2 - 3X} \\ & \qquad\qquad\quad 7X + 14 \end{array}$$

The next term in the answer is X. We multiply $X - 3$ times X and subtract the product.

$$\begin{array}{r|l} & X^2 + X + 7 \\ \cline{2-2} X - 3 & X^3 - 2X^2 + 4X + 14 \\ & \underline{X^3 - 3X^2} \\ & \qquad\quad X^2 + 4X + 14 \\ & \qquad\quad \underline{X^2 - 3X} \\ & \qquad\qquad\quad 7X + 14 \\ & \qquad\qquad\quad \underline{7X - 21} \\ & \qquad\qquad\qquad\quad 35 \end{array}$$

The next term in the answer is $+7$. We multiply $(X - 3)$ times 7, and subtract the product.

There is a remainder. It is the same as a remainder in long division and can be shown by a fraction.

Answer: $X^2 + X + 7 + \dfrac{35}{X - 3}$

Example 3. Divide $(x^3 - y^3)$ by $(x - y)$

$$\begin{array}{r|l} & x^2 + xy + y^2 \\ \cline{2-2} x - y & x^3 \qquad\qquad - y^3 \\ & \underline{x^3 - x^2y} \\ & \qquad x^2y - \qquad y^3 \\ & \qquad \underline{x^2y - xy^2} \\ & \qquad\qquad xy^2 - y^3 \\ & \qquad\qquad \underline{xy^2 - y^3} \\ & \qquad\qquad\qquad 0 \end{array}$$

Since there are no x^2y or xy^2 terms, we leave blank spaces for the missing terms. Check the answer by multiplying $(x^2 + xy + y^2)\ (x-y)$.

Example 4. Divide $(12x^2 - 36y^2 + 11xy)$ by $(-4x - 9y)$

Do not be discouraged if you find these long division problems difficult. Study them carefully to learn the proper method of working them, and you will discover that they are much easier than they appear at a glance. Try solving Example 4 without looking at the solution given, and compare your work with the example. Do this also for a couple of the other examples.

$$
\begin{array}{r|l}
 & -3x + 4y \\
\cline{2-2}
-4x - 9y & 12x^2 + 11xy - 36y^2 \\
 & \underline{12x^2 + 27xy} \\
 & - 16xy - 36y^2 \\
 & \underline{- 16xy - 36y^2} \\
 & 0
\end{array}
$$

Notice the $-3x$ in the answer. We have to multiply $-4x$ by minus $3x$ in order to obtain plus $12x^2$. Check the answer by multiplying $(-4x - 9y)(-3x + 4y)$

For practice, solve the following problems:

1. Divide $(m^2 - 2mn + n^2)$ by $(m - n)$ Ans. $(m - n)$
2. Divide $(X^2 - Y^2)$ by $(X + Y)$ $= X - Y$
3. Divide $(X^3 - Y^3)$ by $(X - Y)$ $= X^2 + XY + Y^2$
4. Divide $(a^2 + 2ab + b^2)$ by $(a + b)$ $= a + b$

In this assignment, we have learned how to perform the fundamental operations of algebra. We have learned what negative numbers are and how to use them. We learned how to add, subtract, multiply and divide algebraic terms containing letters in place of numbers. Perhaps you have noticed that most of the problems in this assignment were exercises in handling algebraic quantities. Most of them cannot be applied directly to electronic circuits, but were presented in this assignment for the purpose of familiarizing you with the various operations of algebra.

We have now mastered all of the fundamental operations of algebra, and in a future assignment, we will apply these fundamentals in solving some very practical electronic problems.

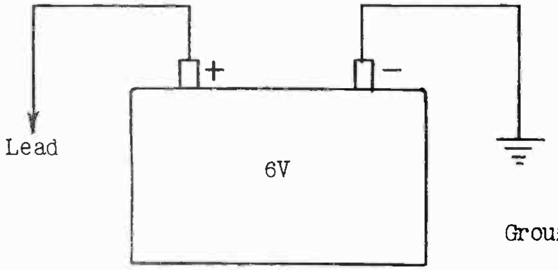
Test Questions

Be sure to number your Answer Sheet Assignment 10.
Place your Name and Associate Number on **every** Answer Sheet.

Submit your answers to this assignment immediately after you finish them. This will give you the greatest possible benefit from our personal grading service.

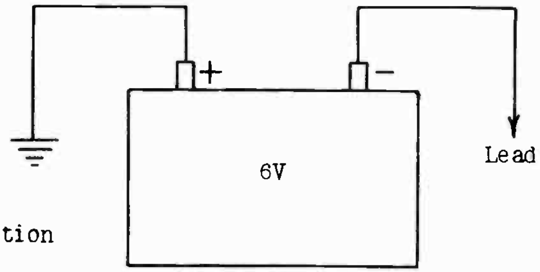
In answering these algebra problems, **show all of your work. Draw a circle around your answer.** Do your work neatly and legibly.

1. Add 70, —41, 23, —21. = 31
2. Multiply —63 by —27. = 1701
3. Divide 144 by —12. = −12
4. If a = 2, b = 3, and c = 4, what is the numerical value of 4abc? = 96
5. Add 6c + 7d, 9c — 6d. = 15c + d
6. Subtract 6a + 7b from 17a + 17b. = 11a + 10b
7. Simplify 3(a + b + c) + 4b. = 3a + 7b + 3c
8. Multiply (a + 3) (a — 3). = $a^2 - 9$
9. $B^3 \times B^7 =$ B^{10}
10. $B^7 \div B^3$ = B^4

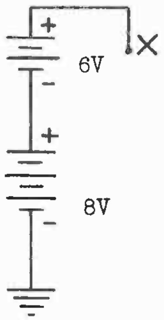


1A

Ground Connection



1B



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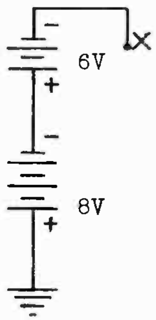

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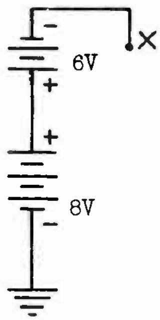

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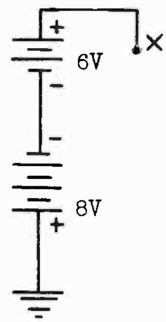
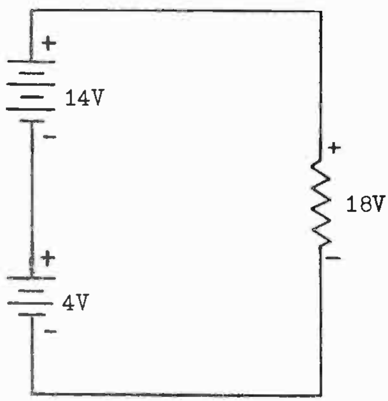


2D



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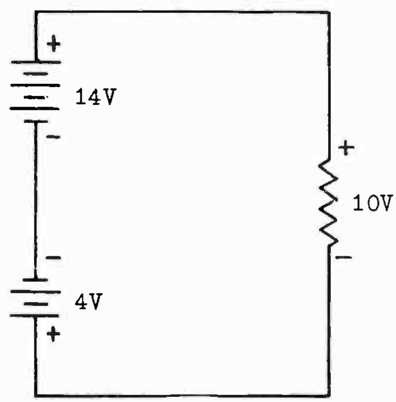

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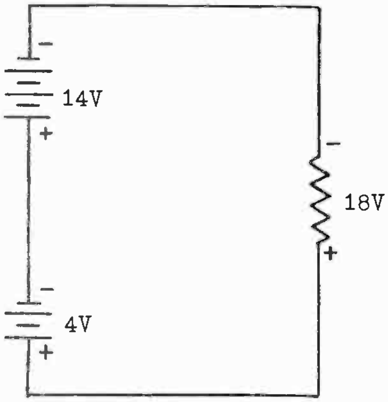

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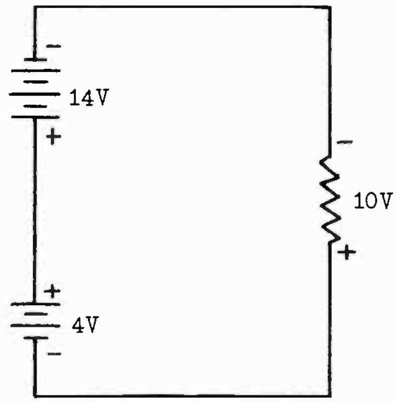

3D