VOLUME 3 JUNE, 1915 NUMBER 2

# PROCEEDINGS
*of*
# The Institute of Radio Engineers
(INCORPORATED)

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## TECHNICAL PAPERS AND DISCUSSIONS

EDITED BY
ALFRED N. GOLDSMITH, Ph.D.

PUBLISHED QUARTERLY BY
THE INSTITUTE OF RADIO ENGINEERS, INC.
ONE HUNDRED AND ELEVEN BROADWAY
NEW YORK

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In view of the kind cooperation of Messrs. John Hays Hammond and John Hays Hammond, Jr., the Board of Direction of the Institute of Radio Engineers, on the occasion of the removal of the office of the Institute to 111 Broadway, decided on the action expressed below:

April 19, 1915.

Mr. John Hays Hammond,
71 Broadway, New York City.

Dear Mr. Hammond:

The Board of Direction of the Institute has instructed me to express its thanks, and sincere appreciation as well as that of the membership for the very kind and material assistance that you have been good enough to extend to us during our existence. and particularly during the past year when we were favored with the free use of your offices and its facilities.

The Institute has advanced to a position of prominence and recognition in the engineering world, and in its building up, you and your son have been most helpful.

I feel it a privilege to extend to you our grateful thanks.

Very truly yours,

David Sarnoff,

Secretary

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Mr. John Hays Hammond responded as follows:

April 21st, 1915.

Gentlemen:

Thanks for your appreciative letter of April 19th.

I wish the Institute of Radio Engineers great success.

Yours very truly,

John Hays Hammond.

The Institute of Radio Engineers,
71 Broadway,
New York City.

# SEASONAL VARIATION IN THE STRENGTH OF RADIOTELEGRAPHIC SIGNALS*

By

LOUIS W. AUSTIN, PH.D.

(*Director of the United States Naval Radiotelegraphic Laboratory*)

In 1912, experiments were begun at the Bureau of Standard on the measurement of the strength of the receiving antenna current produced by signals sent from the radio stations in the Philadelphia and Norfolk Navy Yards. The object of the experiments was the determination of the variation in the strength of the signals at different times of the year.

It had been known qualitatively that the winter signals in general were stronger than those of summer, especially when the transmission took place overland. The reason ordinarily given for this was the absorption of the waves during the summer, due to the vegetation.

The conditions of the experiments were as follows: The sending wave length was 1,000 meters, and the spark frequency was approximately 1,000 per second, the sending antenna current being kept not far from 10 amperes, and care being taken that waves of only one frequency were emitted. The height to the center of capacity of the Philadelphia antenna was 39 meters, and of the Norfolk antenna 52 meters. The antenna at the Bureau of Standards is a harp 55 meters high, having an effective height to the center of capacity of 30 meters. The capacity is 0.0014 microfarad. The distance from the Bureau of Standards to the Philadelphia station is 185 kilometers, and to the Norfolk station 235 kilometers. The method of measuring the received antenna current has been described in another place.†

The observations are shown in the accompanying figure. The ordinates represent microamperes of received current reduced to a constant sending antenna current of 10 amperes. The total receiving antenna resistance, including that of coupling,

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* Delivered before The Institute of Radio Engineers, New York, December 2, 1914.

† Bulletin, Bureau of Standards 7, p. 295, 1910. Reprint No. 157.

was 69 ohms. The figure shows a well marked difference between the summer and winter intensities, but the great variation among the individual values makes it difficult to draw quantitative conclusions; observations on succeeding days in several instances differing from each other in a ratio of more than two to one, while the errors of observation are certainly less than 10 per cent. Rough curves have been drawn among the individual points of observation, indicating the general course of the changes. The Philadelphia values in general lie higher than the Norfolk values, with the exception of those taken in the Autumn of 1912 before certain changes were made in the Philadelphia antenna which appear to have increased its efficiency. No observations were taken in Norfolk after November, 1913, as changes in that station made it impossible properly to make comparison between the observations before and after that time. Notwithstanding the irregularities among the observations, a few facts appear fairly certain: The seasonal variations seem to be different in different years, the minimum of 1912 being higher than that of 1913. The rise in the curves in the Autumn of 1912 appears to be steeper than that of 1913, the practical maximum being attained by November 1st in 1912 and not until the middle of December in 1913. It has not been found possible definitely to connect the strength of signal with the changes in foliage conditions, altho it is possible that this is an important factor in the variations. Contrary to the ideas previously held, there seems to be no very marked connection between rainfall and the transmission of the signals. This was especially noticeable in the Autumn of 1912, when after a dry period, rain set in and fell heavily for four days. This, however, caused no certain increase in the strength of the received signals.

This preliminary series of observations shows that for a thoro study of the subject it will be necessary to observe at least twice a week, and preferably every day, for a long period of time. From these observations it will then be possible to derive average values from which the general course of the phenomena can be deduced with some degree of accuracy. It may then be possible by comparison with the curves of meteorological and magnetic phenomena to find relations which will help to explain the seasonal changes, and also the irregularities among the single observations.

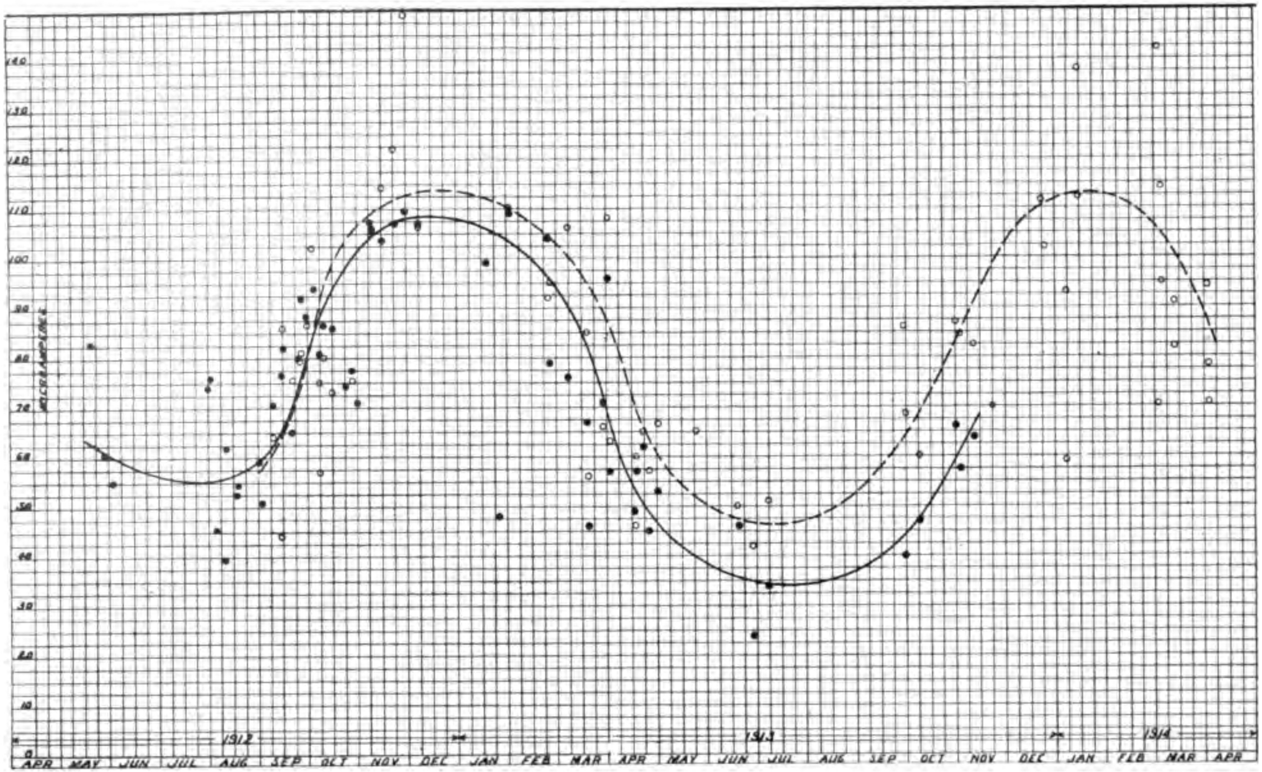
Most of the observations have been taken by my assistant: H. J. Meneratti, Chief Electrician, U. S. N

**SUMMARY:** **The strength of received signals from two stations was measured at the Bureau of Standards over a period of about two years. The transmitting wave length was 1,000 meters, spark frequency, 1,000, and sending antenna current about 10 amperes for each of the transmitters. Their distances were respectively 185 and 235 kilometers. The curves giving variation in intensity of received signals are shown and discussed.**

## DISCUSSION

**Robert H. Marriott:** It will be found interesting and instructive to compare what Dr. Austin has found with the results I described in my paper on "Radio Range Variation" before the Institute, (PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, Volume 2, Number 1, page 37) and especially with chart 3, Figures 1 and 12 of that paper.

**Alfred N. Goldsmith:** It is evident from Dr. Austin's results that for the particular stations under consideration, the day of best transmission is close to January 1st, and the day of most difficult transmission to July 15th. The average ratio of received energy in winter to received energy in summer (for the extreme cases) is found to be 6.3. However, this last result is not very accurate, since the individual values of the ratio lie between 3.9 and 10.



**Strength of Signals Received at Washington**
○ = Signal from Philadelphia
● = Signal from Norfolk

# RESONANCE PHENOMENA IN THE LOW FREQUENCY CIRCUIT OF RADIO TRANSMITTERS*

By

HENRY E. HALLBORG

It is the purpose of this paper to outline briefly the principal low (audio) frequency circuit characteristics common to all radio transmitters using alternators and transformers for charging the condensers of the radio frequency circuit. By low frequency we mean frequencies of the order of 60 to 500 cycles as commonly used.

The transformer is one of the important units of all radio stations, except in those of the arc or reflector alternator type. A practical study, therefore, of the phenomena occurring in the alternator-transformer circuit cannot fail to be of interest. In working with this circuit are to be found some of the most perplexing experiences of the experimenter and of the engineer. Strangely enough, many engineers who calculate freely the important constants of radio frequency circuit combinations entirely overlook the fact that the low frequency circuit combinations are equally numerous, and their proportioning equally important. Possibly more cases of inefficiency in radio transmitters are due to improper alternator-transformer circuit adjustments than to any other one cause. To sum up briefly, in the radio circuit resonance plays the master role, from generator slip rings to aerial.

In presenting this paper, the writer realizes that the subject has had much mathematical treatment, and that many empirical expressions covering particular phases and conditions of circuits have been derived. Unfortunately much of this work has been presented in such a way as not to appeal to the average engineer. It is the writer's hope so to cover the subject that its treatment may have more practical applications than heretofore. The expressions and circuit relations given are for the most part fundamental, or easily derived. The methods of taking these

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*Delivered before The Institute of Radio Engineers, New York City, November 4, 1914.

resonance observations were devised by the writer, and the curves shown are nearly all actual graphs of measurements on circuits of various types and sizes.

Resonance readings in the alternator-transformer circuit can be obtained by several methods. Since we can readily make quantitative measurements of the variation of current and voltage, two methods immediately present themselves. The first is a method which we shall call the **primary ampere method**, and the second a method which we shall term the **secondary voltage method**.

The **primary ampere method** consists simply in plotting relations between the current in the generator circuit, and capacity load in the high tension circuit, the latter being varied step by step. It is evident, since the circuit constants on the high and low tension sides of a transformer bear a definite relation to each other, that if the point of resonance in the primary circuit is determined, the constants of the entire circuit may be closely calculated. The only equipment necessary for obtaining this data is an ammeter, a frequency meter, and a widely adjustable field rheostat. The connections for taking measurements by the **primary ampere method** are shown in Figure 1.

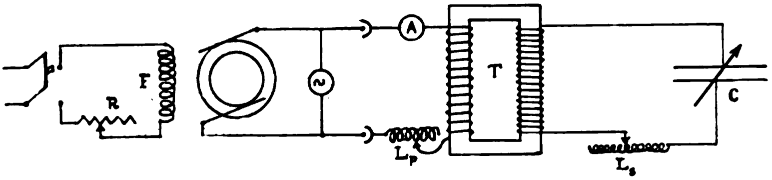

FIGURE 1—Connections for Primary Ampere Method

Here (F) represents the alternator field, and (R) a resistance inserted in this field of such a value, determined by trial, that the primary ammeter (A) has less than full scale deflection when the point of resonance is reached. (T) represents the transformer, and ($L_p$) and ($L_s$) series connected primary and secondary inductances. These are not essential to making the measurements; but are shown to cover general conditions. (C) is the condenser which is to be varied in known steps.

A plot may be made between any of the variables; capacity, frequency, or amperes. The most practical method is to hold the frequency (f) constant, and to determine the relation between primary current and the capacity load. When the exact value of (C) at which maximum current occurs is found, the value of the effective inductance of the secondary circuit is calculated by the well known relation:

$$L_2 = \frac{10^6}{4\pi^2 f^2 C} \text{ Henrys (C being in microfarads).}$$

This value of $L_2$ is especially useful from the point of view of the designer, since the maximum secondary current value may be obtained from it by the relation:

$$I_2 = \frac{E}{2\pi f L_2}.$$

E is the potential applied to the condensers determined by the usual power relation.

Several curves taken by the primary ampere method are shown later in the paper. In making this measurement with a closed core transformer, an error may be introduced by the low magnetic density of the iron. Ordinarily this error is not large, since high resistance silicon steel cores are now almost universally used. With open core transformers, the error is negligible since their saturation characteristic is a straight line. Slight error may also be introduced by a low saturation effect in the generator, but this error has not been found to be appreciable.

The **secondary voltage method** consists in determining the relation between generator open-circuit voltage,, and the discharge voltage of a calibrated ball or sphere gap connected in parallel with the secondary condenser. The connections are somewhat similar to the primary method, and the apparatus required is no more elaborate. The connections are shown in Figure 2.

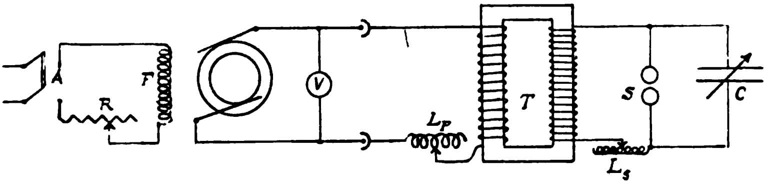

FIGURE 2—Connections for Secondary Voltage Method

Here (A) represents the alternator field switch, and (R) the rheostat of the alternator field, capable of varying its excitation thru a wide range. (V) is a voltmeter connected to read the alternator open-circuit voltage, and (S) is a calibrated discharge gap adjusted to breakdown at a point which will not endanger the transformer insulation. (C) is the capacity load as before.

The process of taking readings consists in varying (C) by known steps, and finding the alternator excitation which just causes a discharge across (S) when the field switch (A) is closed. The point of resonance is found by noting the condenser setting (C) which leads to a discharge of (S) at the least alternator excitation. The order of the resonance effect is found by dividing the known sparking voltage of (S) (which remains fixed) by the primary voltage (V) required to discharge it. A curve may be plotted from these values, showing the secondary voltage obtainable for any applied constant primary voltage as the value of (C) is varied. While open to criticism due to transient effects, this method gives information regarding the secondary potential under conditions that make static voltmeters unavailable. Curves taken by this method are shown below.

By reference to the vector diagrams of the ideal transformer, as given in most text books, we obtain three important relations between primary capacity, inductance, and resistance, and their equivalent values when transferred to the secondary of the transformer. These relations are useful enough in conjunction with transformer resonance to be here stated.

If we call the ratio of transformation (i. e., the number of secondary turns divided by the number of primary turns) of the transformer G, the relations are:

$$C_1 = G^2 C_2$$

$$L_2 = G^2 L_1$$

$$R_2 = G^2 R_1$$

Given a transformer ratio of 10, for example, these expressions may be interpreted as follows: The total capacity inserted in the primary to have the equivalent effect of a capacity $C_2$ inserted in the secondary is 100 $C_2$. Similarly an inductance $L_1$ inserted in the primary has an equivalent effect of 100 $L_1$ inserted in the secondary. Likewise a resistance $R_1$ inserted in the primary has a secondary equivalent effect of 100 $R_1$. The curves presented are evidence enough of the importance of these relations in connection with low frequency resonance. The writer has made several predeterminations of resonance characteristics in fair agreement with later measurement by transferring circuit constants by this means. For the predetermination of the primary current, the fundamental formula was used, namely:

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

The values of $\omega L$ and $\frac{1}{\omega C}$, where L and C are the total circuit inductance and capacity respectively referred back to the primary by the relations above shown, were obtained on both sides of the resonance value and plotted. Similarly, the equivalent value of R was obtained. In finding an equivalent primary value of R for this formula, a difficulty is experienced in determining a proper value of the total secondary resistance of the condenser circuit. This resistance is a function of the applied frequency, the number of condensers connected, and their manner of connection. Tests made at the Naval Radio Telegraphic Laboratory in Washington (See PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, Volume 1, part 2, page 35) indicate that this resistance for a single plate glass condenser of 0.002 microfarad capacity is of the order of 50,000 ohms at 60 cycles. This figure agrees quite well with values obtained by the writer.

A most useful and practical method of obtaining the low frequency characteristics of a radio set is by measurement of the percentage reactance of the transformer, the alternator, and the other circuit inductances. This method consists in observing the voltage drop at the terminals of each inductance in question: alternator, transformer, etc., when rated full load current is flowing. The percentage reactance is the percentage voltage drop on each unit in terms of the rated voltage. For instance, if a 500-volt generator has a synchronous impedance of 10 ohms at normal frequency, and if the rated full load current is 10 amperes, the impedance voltage of the machine is 100 volts, and its percentage reactance is 100–500, or 20 per cent. Similarly two or more reactances connected in circuit are added arithmetically to obtain the percentage total circuit reactance. This is the reactance value having direct bearing on the resonance characteristic. From it may be obtained the total primary inductance value $L_1$ as well as the total secondary inductance value $L_2$, as follows:

$$L_1 = \frac{(\text{Percentage Reactance of Total Circuit}) \cdot (\text{Normal Primary Volts})}{2\pi f \cdot (\text{Normal Primary Amperes})}$$

$$L_2 = \frac{(\text{Percentage Reactance of Total Circuit}) \cdot (\text{Normal Secondary Volts})}{2\pi f \cdot (\text{Normal Secondary Amperes})}$$

where (f) is the frequency of the generator.

Having these inductance values the capacity required for resonance is easily computed from the formula:

$$C = \frac{10^6}{4\pi^2 f^2 L} \text{ microfarads.}$$

In this formula, L, calculated as shown above, is given in henrys.

The capacity value is usually fixed by considerations other than those of transformer resonance, and the problem is one of adjusting the circuits properly for the specified capacity values. A few experimentally determined facts tend to simplify this adjustment. Nearly all spark transmitters operate most efficiently when the natural frequency of the alternator-transformer circuit, that is,

$$F = \frac{1}{2\pi\sqrt{LC}}$$

is lower than the impressed circuit frequency (f) of the alternator. The choice of the percentage difference between F and f depends on the type of spark gap used. The writer has found that a value of inductance 30 per cent. greater than the resonating value is a proper value for synchronous rotating gaps, and for quenched gaps a value 40 per cent. in excess of the value to give transformer-alternator resonance. The natural frequency of the circuit therefore must be 12 to 15 per cent. lower than the impressed frequency (f). In some cases it is necessary to detune to the extent of 20 per cent. or more; but wide detuning always results in loss of efficiency. In the case of quenched gaps, the choice usually lies between a clear note with lower efficiency, and a "medium" note with higher efficiency. The value of L for quenched spark sets given above as 40 per cent. above the resonance value, is a compromise choice between the limits just mentioned.

The transformers for the American Marconi high power stations were successfully adjusted by the methods above outlined. No condensers were required for test purposes, and the available test frequency was only 60 cycles, whereas the rated frequencies of the several equipments covered a wide range. All of these transformers are of the closed core, oil cooled type. One of them is shown in Figures 3 and 4. The total station capacity 300 kilowatts is obtained by paralleling four 75 kilowatt units, and supplying one spare unit. The complete breakdown of the transformer equipment is thereby made quite remote.

FIGURE 3

FIGURE 4

A transformer of the closed core type with alternate primary and secondary windings lends itself well to wide reactance variation. The design is not unlike the "tub" arc lighting transformer. The difference between the two lies in the fact that the flux leakage of the tub type is a function of the load, while the leakage of the radio transformer is fixed, and is made sufficient to suppress arcing and excessive wattless current when spark discharge occurs. With this type the required leakage is obtained by proper separation of the primary and secondary coils. The exact amount of leakage in the transformer is apportioned in accordance with the total circuit inductance found necessary, and is high or low as the condition may require.

In the case of the transformers for the Marconi high power stations, it was necessary to adjust precisely the reactance of each unit to insure proper division of the load when four units were operated normally in parallel. When similar adjustment of all the units for one station had been made, actual reactance readings on one unit were found to suffice, since the combined inductance value for normal operation could be obtained by dividing the single unit value by the number of units it was desired to operate in parallel. Actual measurement on four units in parallel checked this assumption exactly. The problem of reactance adjustment in a circuit consisting of alternator, several transformers in parallel, and a series of secondary loading coils is to determine the combined transformer inductance which, with the alternator and the secondary loading coils, gives a total secondary circuit inductance 30 per cent. in excess of the inductance calculated for resonance with the specified capacity.

Figure 5 illustrates a method used by the writer for charting a low frequency circuit, and thereby obtaining a complete graphical record of its inductance characteristics. It shows the

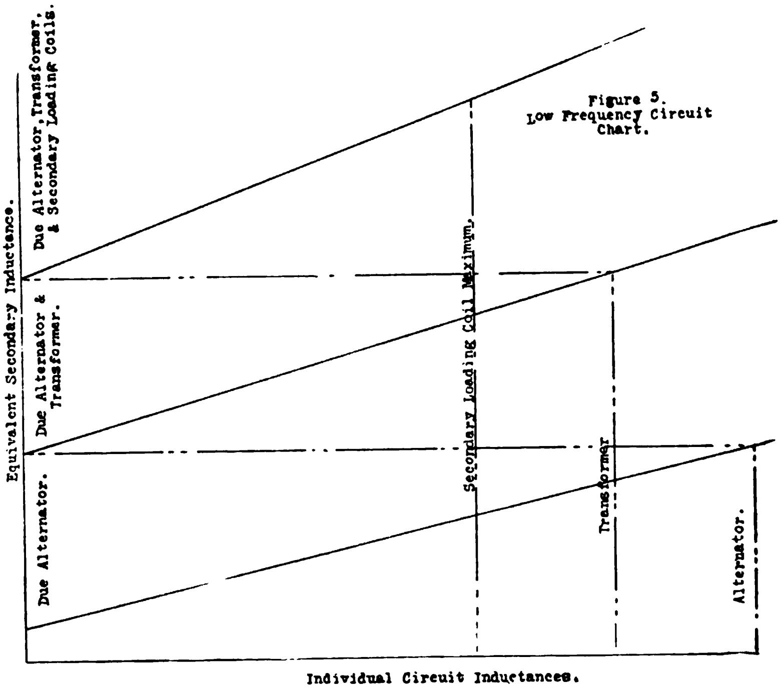

FIGURE 5

relation between total secondary inductance as ordinates, and series connected inductances (primary and secondary), as abscissas. The three curves shown give respectively, the value of the alternator inductance referred to the secondary, the value of alternator and transformer referred to the secondary, and the value of alternator, transformer, and secondary loading coils referred to the secondary. Data on any condition of the circuit is at once available. When the iron is worked at moderate densities, as in the units above mentioned, it was found that the curves are nearly straight lines, and only a few readings were necessary to locate the entire curve. With this data at hand, the value of condenser for resonance, the point of best operation, and even the general shape of the resonance curve can be closely approximated.

Figure 6 taken by the primary ampere method shows the actual tuning curves of the 300 kilowatts alternator-transformer circuit at New Brunswick, N. J., for various settings of the secondary loading coils. It will be noted that as the circuit is stiffened by adding loading coils, the primary current amplitude falls, and the resonance effect is sharpened. The decrease in primary current amplitude is probably partly due to added resistance as coils are inserted, and to the large increase in resistance due to the smaller number of condensers required.

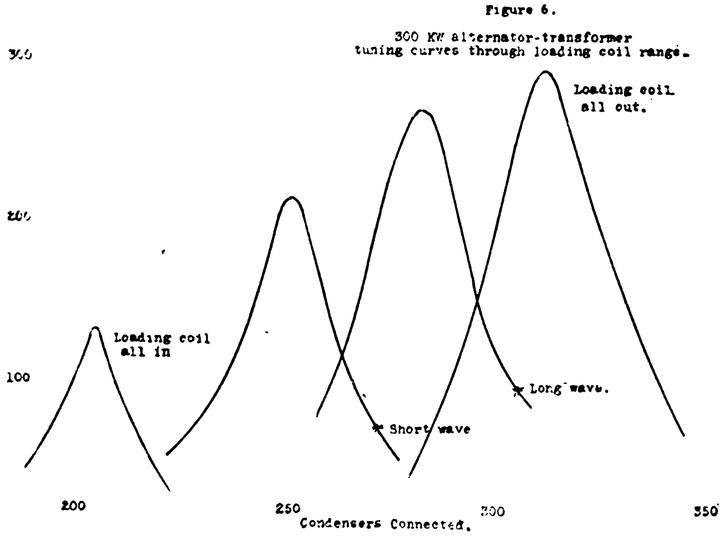

FIGURE 6

as the inductance is increased, as previously pointed out. The stars indicate actual operating points at different wave lengths. Since these operating points fall quite within the middle ranges of the loading coils, it is evident that it is possible to make factory adjustments as above outlined, with a high degree of accuracy.

Figure 7 is an application of the primary ampere method and the secondary voltage method to a 2 kilowatt, 500 cycle, 110–18,000 volt, open core transformer, designed for a synchronous rotary spark set. It will be noted that the point of resonance taken by the two methods does not occur at the same condenser value; but that the secondary voltage method gives an inductance value somewhat less than the value obtained by the primary voltage method. From the alternator-transformer constants of this particular circuit, namely 3.4 ohms synchronous impedance, and 2.8 ohms transformer impedance, we deduce for the equivalent secondary inductance by the "(ratio)$^2$ transformation" the value 19.7 henrys. The value of capacity for resonance should

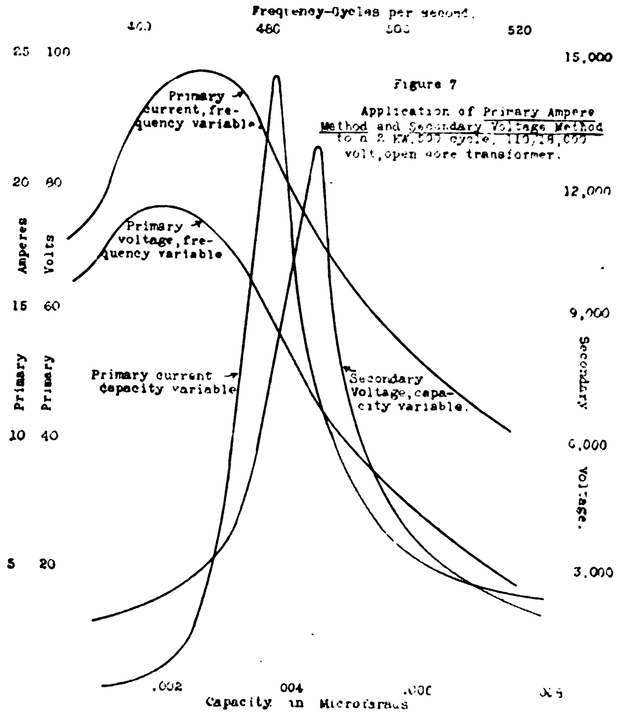

FIGURE 7

therefore be 0.0051 microfarad. However, the curves show this inductance to be about 30 per cent. greater than the figure deduced by the "(ratio)$^2$ method." For a loosely coupled transformer, such as the one under consideration, Seibt deduced the expression

$$L_2 = \frac{1}{\omega^2 C (1 - k^2)}.$$

We found the value of secondary inductance for resonance, with capacity C, and coupling factor K taken as equal to 0.7. Solving for K from the data above given we get a value of about 0.5. This figure is more nearly in conformity with the writer's experience and results on open core transformers. With closed core transformers, K is unity (or at least nearly enough so for all practical purposes). The operating point for best results is shown on the diagram by a star. The natural frequency of the circuit corresponding is 407 cycles, or 18 per cent. below the alternator frequency. The variation of primary voltage and current with frequency changes is also shown. These curves are quite similar, as is to be expected, since they are linked together by the relation

$$E = 2\pi f L_1 I$$

where $2\pi f_1$ L is the generator impedance, and $I_1$ the current flowing.

Figure 8 is of interest since it demonstrates quite conclusively that the alternator synchronous impedance has an effect on resonance similar to that of any inserted inductance of equivalent value, and must be considered as such. Curve A is the resonance characteristic when no reactance is inserted in series with a transformer and a 2 kilowatt, 500 cycle alternator of 0.5 ohm synchronous impedance. Curve B results from connecting a reactance of 3 ohms in this alternator-transformer circuit. Curve C is the result obtained by using the same transformer with another alternator, the synchronous impedance of which is 3.4 ohms, or roughly the sum of the impedances of curves A and B.

Figure 9 illustrates the effect on resonance of adding resistance in series with the secondary circuit of a 7.5 kilowatt open core transformer. The resistances inserted were carbon rods of 700 ohms each. The curves become rapidly flatter as the resistance, or damping, is increased. The amount of resistance required completely to wipe out resonance is approximately such that

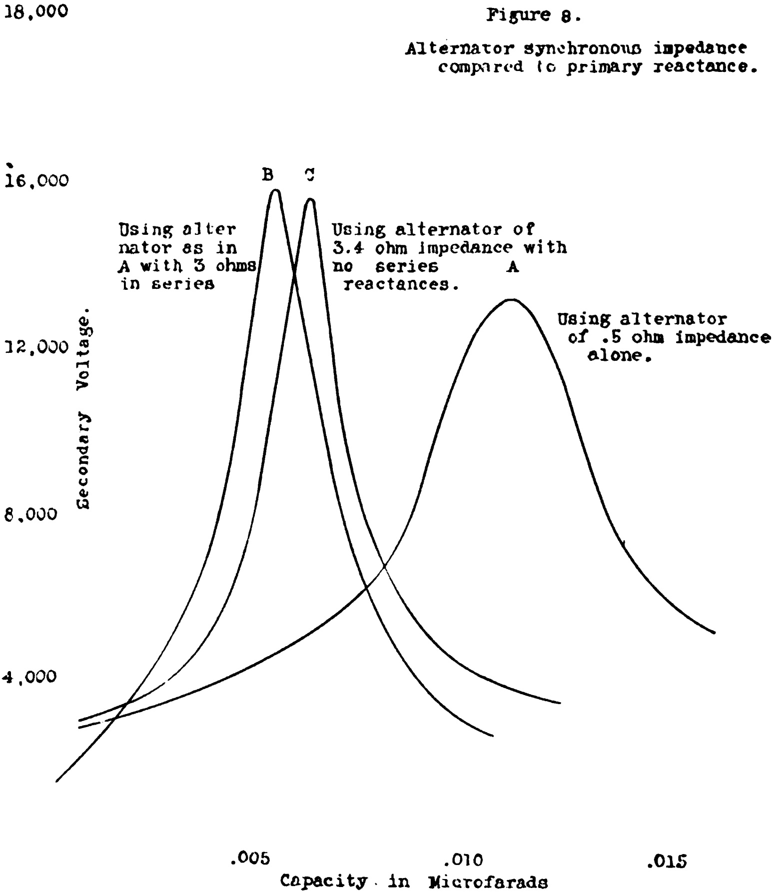

FIGURE 8

the rated output is all consumed in the resistance. Resistance has no effect on the resonance curve other than that of "broadening the tuning," so to speak.

Figure 10 shows the effect of resistance inserted in the primary of a 2 kilowatt, 500 cycle, 110–18,000 volt transformer circuit. These curves are striking examples of the correctness of the deduction that inserting a resistance $R_1$ in the primary circuit has an equivalent secondary effect of $G^2R_1$. Resonance is wiped out with astonishing facility. In this experiment, the point of resonance moved slightly to the left in the direction of increased inductance, since the rheostat used was slightly inductive. The curves were made by the secondary voltage method.

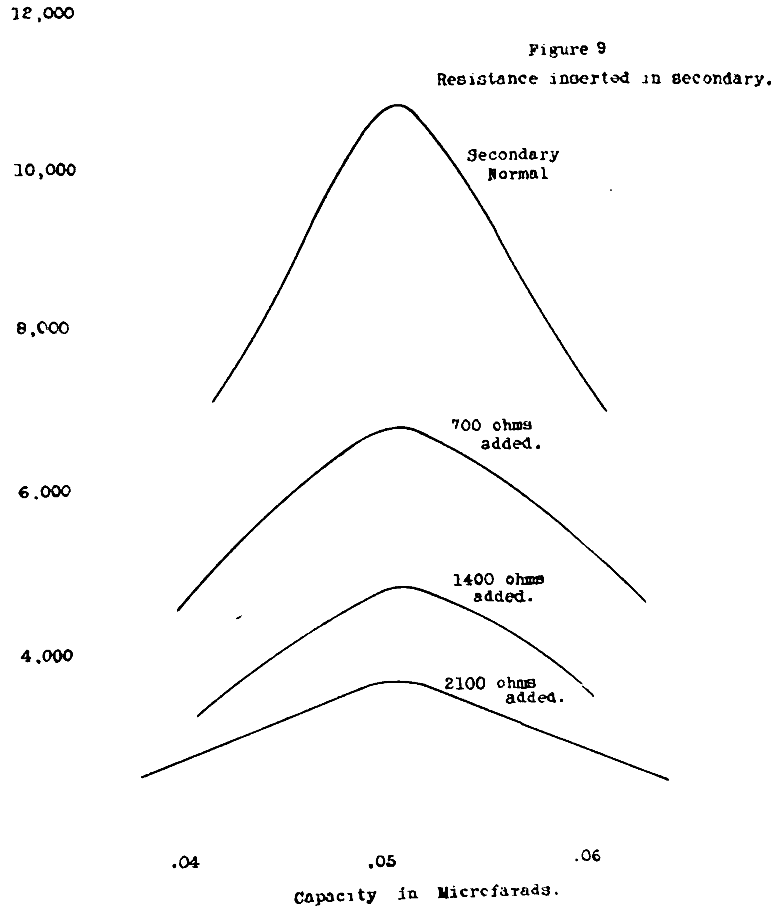

FIGURE 9

Figure 11 represents the conditions found to occur in a 5 kilowatt, 500 cycle, 110–12,500 volt, open core transformer tested by the secondary voltage method, when a step by step primary inductance was inserted. The secondary voltage rise becomes sharper, and its amplitude greater, as more primary inductance is inserted. We have noted in Figure 6 that the primary current diminishes with increased inductance, hence the secondary current must likewise drop. If the secondary voltage is to be considered as resulting simply by the building up of voltage across the inductance of the secondary, in accordance with the relation

$$E_2 = 2\pi f L_2 I_2,$$

it is evident that a condition such as that here shown can result only when the secondary inductance increase is more rapid than the secondary current decrease. This is probably the case with open core transformers having a liberal copper allowance, and a relatively weak coefficient of coupling.

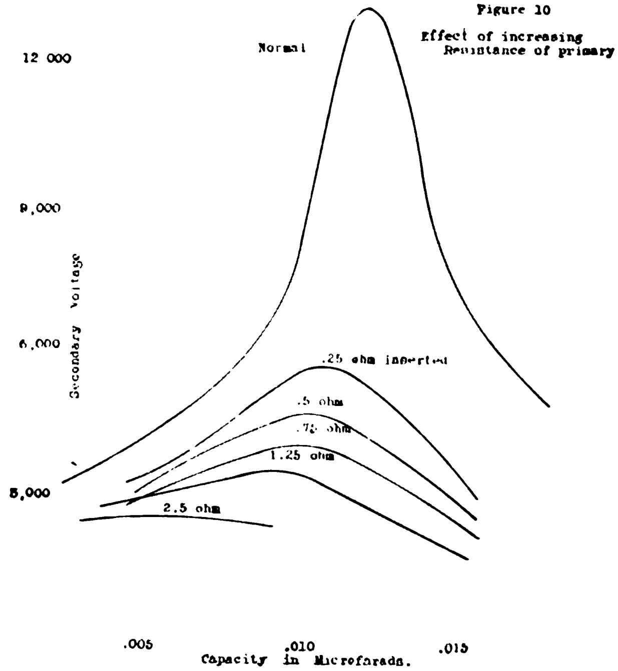

FIGURE 10

Figure 12 demonstrates the desirability of detuning the alternator-transformer circuit of a quenched spark transmitter with respect to the alternator frequency. The natural frequency of this circuit is seen to be 450 cycles, or 50 cycles lower than the alternator. This setting represents the working point nearest resonance, with this particular set, for a perfectly clear note. It was also the most efficient operating point. A detuning of 100 cycles or 20 per cent. is nearer the average condition for a good tone.

Figure 13 shows a simultaneous series of primary voltage and primary current readings for one of the settings made on the 300 kilowatt set at New Brunswick (and shown in Figure 6).

The primary voltage curve checks quite closely with the voltage calculated from the simple relation:

$$E_1 = 2\pi f L_1 I_1$$

or is merely the product of alternator synchronous impedance and the current flowing. A few calculated points are shown by circles.

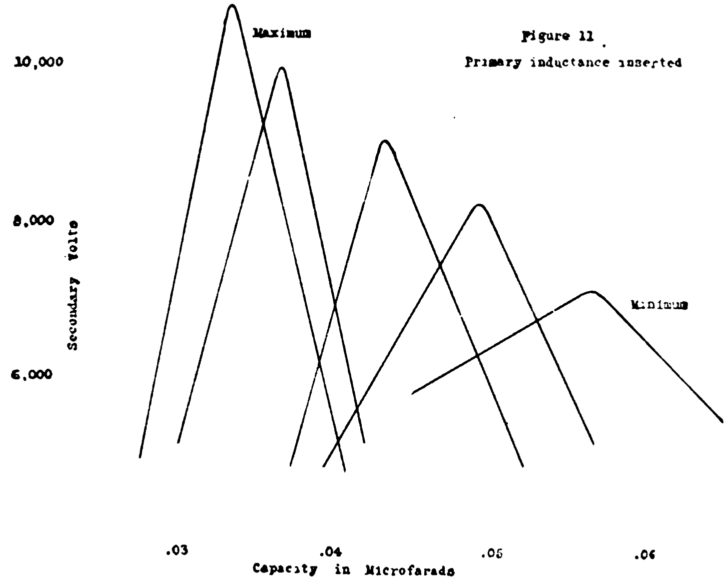

FIGURE 11

Figure 14 is a record of the simultaneous primary and secondary currents of the 300 kilowatt set at New Brunswick, using a resonance setting as above. These curves were taken to determine the extent of the variation of transformer ratio during resonance. The two curves are plotted against the same ordinates by multiplying the secondary current by the winding ratio of transformation and plotting primary amperes direct. It is apparent that no wide change of ratio occurs.

Some vital facts may be gleaned from the data presented in regard to the design of transformers for this class of work. Quite evidently, low resistance values in both primary and secondary are desirable. Further, it has been demonstrated that the total circuit inductance is the quantity of chief importance from the point of view of resonance.

We have also noted that this circuit inductance may be made up of a number of separate small inductances, or concentrated in the alternator and transformer alone. For a particular specified capacity value in the oscillating circuit, the most efficient arrangement is that in which the total inductance is concentrated in the alternator and transformer only. Both copper and iron losses are thereby reduced; but the arrangement lacks flexibility if a wide range of capacity is to be used. Usually this is not the case. Flexibility, if desired, is most easily obtained

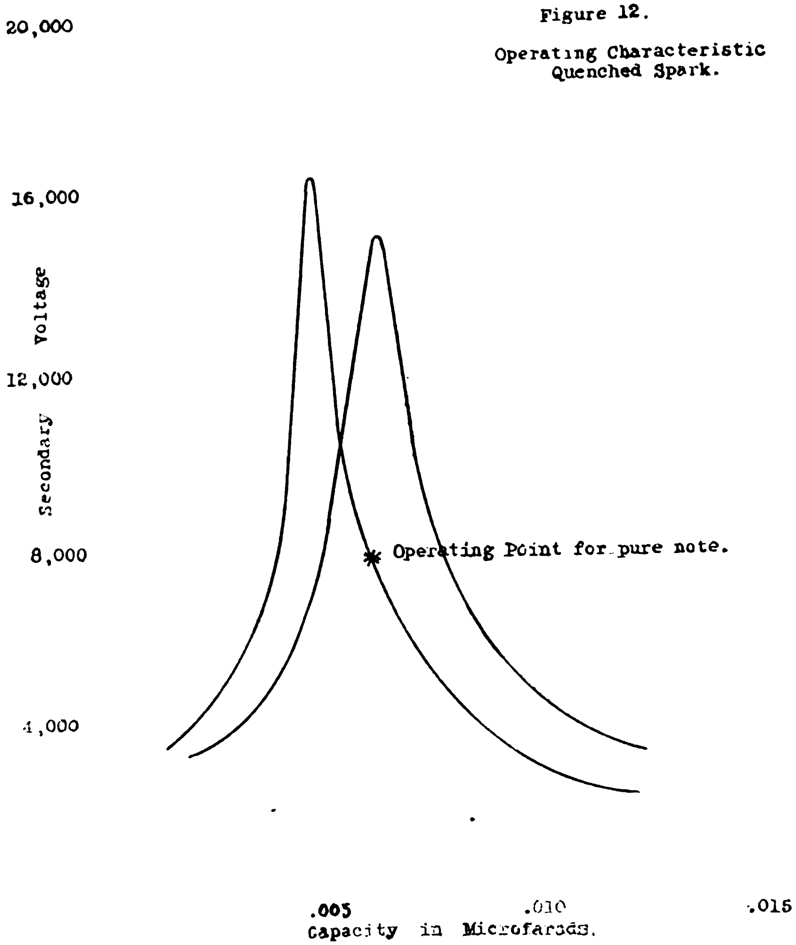

Figure 12

by means of primary variable reactance, or better still from the point of view of efficiency, by varying the mutual inductance of the transformer, thereby regulating its flux leakage.

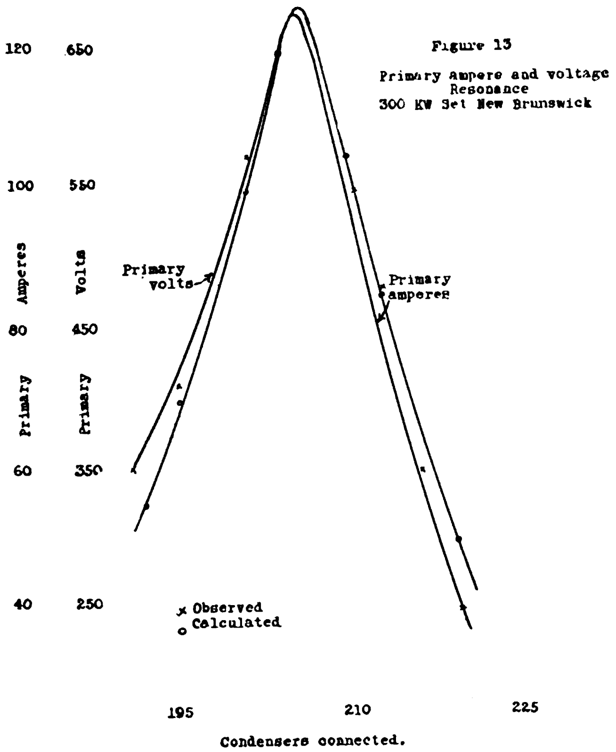

FIGURE 13

The choice between open and closed core transformers for radio work has long been a point in dispute. The open core transformer has the advantage of simplicity. It has also the inherent high leakage characteristic sometimes so desirable. It requires more iron and wire for a given output than the closed core unit. Assuming the same magnetic flux density in a similar unit of each type, the copper loss in the closed core unit will be less since less wire is needed, and for the same reason its iron loss is lower since the volume of iron is less altho the flux densities are the same. The closed core unit therefore is more efficient. A considerable saving in space in favor of the closed core type also results. This saving, as we have just noted, is effected in both core and coils. High leakage may be obtained in the closed core type by careful disposition of the windings without resort to magnetic shunts, or other devices. It is apparent for these reasons that the closed core transformer is the more economical type both electrically and mechanically.

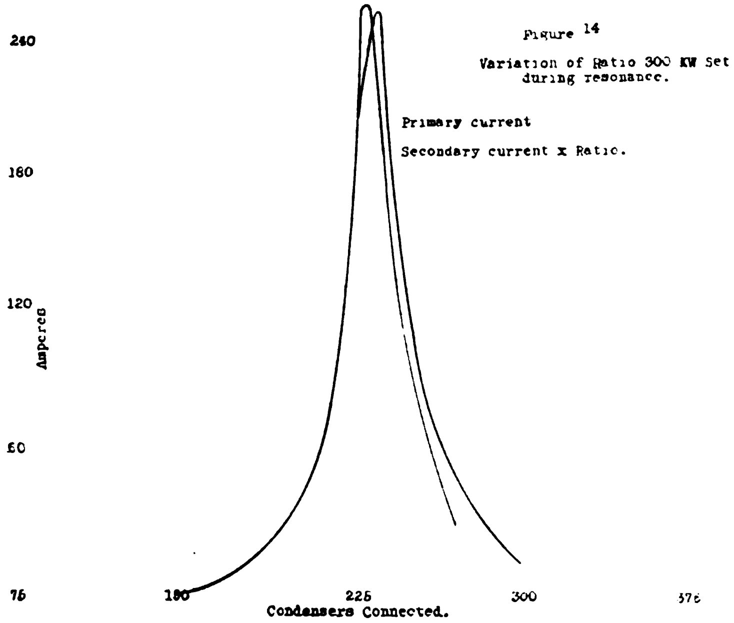

FIGURE 14

**SUMMARY: For determining the resonance characteristics of the audio frequency circuits of a radio transmitter, either the primary ampere method or the secondary voltage method may be used. These methods are described. The method of calculation of the total circuit reactance is given, and the important bearing of this quantity on the resonance effects is discussed. Percentage reactance of any portion of the circuit is defined. The extent of detuning the transformer circuit from the generator frequency in quenched spark work, namely about 15 per cent., is given and explained. The transformer circuit of the Marconi trans-Atlantic station at New Brunswick, N. J., is described. Curves giving the results of measurements by the above methods are shown. The advantages of low resistance in primary and secondary of transformers for quenched spark work, and the superiority of the closed core transformer are considered.**

# DISCUSSION

**Alfred S. Kuhn**: As a result of my experience with radio transformers, it seems to be that the diagram given by the author in which the relation between primary current and secondary capacity is shown would be more instructive if it were drawn as indicated in Figure A. In Figure A, the heavy lines show the primary current variation with change in secondary capacity; and the dotted lines the secondary voltage variation. The graph shows that at A, with zero capacity, the current is simply the transformer magnetising current. From A to B, the current lags, from B to C it leads, and from D to E it again lags behind the condenser current. R is the point of resonance.

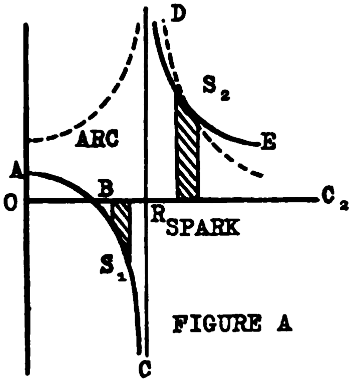

FIGURE A

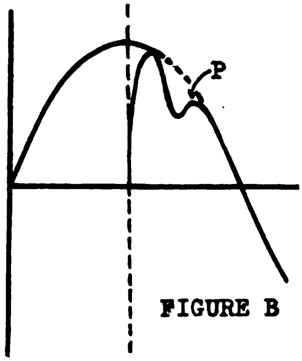

FIGURE B

In general, there are three possible regions of operation:

1. In the shaded region, $S_1$,

2. At R (tho operation at R causes a resonance rise resulting in what one might term a "smashing point." Such operation would therefore be very unstable and dangerous. Operation at R might be obtained by using a limiting resistance, but this would be uneconomical), and

3. In the shaded region, $S_2$.

By operating at $S_1$ a high power factor and antenna radiation are obtainable. The condenser discharge produces, however, an arc rather than a spark across the gap, as indicated by a hissing sound. The arc no doubt causes greater heating than the usual spark. Therefore operation at a natural circuit frequency greater than the impressed frequency is undesirable. Operation at R is very unstable. Therefore, to obtain the best operation, it is desirable to operate in the "spark" region, $S_2$;

that is, at a natural circuit frequency less than the impressed frequency. In order to reach this "spark" region, the inductance may of course be increased as well as the capacity. In some recent radio work with transformers, the closed core type have been used by me, and it was found unnecessary to use external reactances in either the primary or secondary of the transformer. High efficiencies were therefore obtained. Results obtained by the use of such equipment warrant my objection to the author's statement that in the case of quenched gap sets, "the choice generally lies between a clear note with diminished efficiency and a 'medium' note with higher efficiency." In my experience improving the note does not at all impair the efficiency but rather augments it.

As to the curve in Figure B, which was drawn by Mr. Simon and apparently verified by Mr. Hallborg's oscillograms showing the current thru the quenched gap circuit, the reasons for a rise at P are not clear to me unless there is a reaction from the open or radiating circuit back to the closed or oscillating circuit. This latter condition would indicate poor quenching. With a perfect note and complete quenching, however, this rise should not occur.

**Julian Barth**: The necessity for working radio power transformers above the resonance frequency might be explained on the basis of armature reaction. The current will lead, be in phase with, or lag behind the e. m. f. depending on whether the transformer is worked below, at, or above this resonance point. There is thus caused "building up," no effect, or dropping off of the generated voltage as the load increases; because leading currents cause the armature reaction to aid the generated e. m. f. while lagging currents cause the opposite effect. Of course, a building up of e. m. f. will tend to cause more than one discharge at the peak of the wave, while a dropping of the e. m. f. will tend to prevent it. It is obvious that for a clear note, the condition is one discharge occurring regularly for each peak; hence from the standpoint of clearness of tone, working well above resonance is an advantage. However, the explanation just given should be taken with a grain of salt. In the course of an intricate mathematical analysis of the subject, several other explanations were found as well. These explanations stand the test of practice, and the analysis referred to gives conditions which enable working the transformer directly on the resonance point. I hope in the future to explain this more fully.

The transferring of the primary external reactances into the secondary circuit by multiplying by the square of the ratio of secondary turns to primary turns, as given by Mr. Hallborg, is not a completely accurate procedure. We shall consider the circuits shown in Figure 1. We shall also take

$L_2 = L_5 + L_6$,
$L_1 = L_3 + L_4$,
$k$ = over-all coupling coefficient of the circuits, and
$k_1$ = coupling coefficient of transformation.

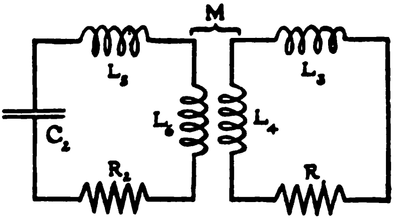

FIGURE 1

We have then:

$$L_1 \frac{d\,i_1}{d\,t} + R_1 i_1 + M \frac{d\,i_2}{d\,t} = 0 \tag{1}$$

$$L_2 \frac{d\,i_2}{d\,t} + R_2 i_2 + M \frac{d\,i_1}{d\,t} + \frac{1}{C}\int i_2\,d\,t = 0 \tag{1'}$$

which are the equations of the potentials if the circuits are permitted to oscillate freely.

Differentiate each equation twice:

$$\begin{cases} L_1 \dfrac{d^2 i_1}{d\,t^2} + R_1 \dfrac{d\,i_1}{d\,t} + M \dfrac{d^2 i_2}{d\,t^2} = 0 & (2) \\ L_2 \dfrac{d^2 i_2}{d\,t^2} + R_2 \dfrac{d\,i_2}{d\,t} + M \dfrac{d^2 i_1}{d\,t^2} + \dfrac{1}{C} i_2 = 0 & (2') \end{cases}$$

$$\begin{cases} L_1 \dfrac{d^3 i_1}{d\,t^3} + R_1 \dfrac{d^2 i_1}{d\,t^2} + M \dfrac{d^3 i_2}{d\,t^3} = 0 & (3) \\ L_2 \dfrac{d^3 i_2}{d\,t^3} + R_2 \dfrac{d^2 i_2}{d\,t^2} + M \dfrac{d^3 i_1}{d\,t^3} + \dfrac{1}{C} \dfrac{d\,i_1}{d\,t} = 0 & (3') \end{cases}$$

To separate the variables, we perform the following algebraic additions:

1. [Equation (3′) × (− M) + Equation (3) × ($L_2$) + Equation (2) × ($R_2$) + Equation (1) × $\frac{1}{C}$]

2. [Equation (3′) × ($L_1$) + Equation (3) × (− M) + Equation (2′) × ($R_1$)]
Thus we obtain:

$$\frac{d^3 i_1}{d t^3}(L_1 L_2 - M^2) + \frac{d^2 i_1}{d t^2}(L_2 R_1 + L_1 R_2) + \frac{d i_1}{d t}\left(R_1 R_2 + \frac{L_1}{C_2}\right) + \frac{R_1}{C_2} i_1 = 0 \tag{4}$$

$$\frac{d^3 i_2}{d t^3}(L_1 L_2 - M^2) + \frac{d^2 i_2}{d t^2}(L_2 R_1 + L_1 R_2) + \frac{d i_2}{d t}\left(R_1 R_2 + \frac{L_1}{C_2}\right) + \frac{R_1}{C_2} i_2 = 0 \tag{4'}$$

The equations are identical in $i_1$ and $i_2$, and hence the currents obtained by solving them must have the same period. We need therefore solve only one of them, and we may omit the subscript number of i.

Mr. Hallborg's curves showing the effect of resistance on the position of the resonance point prove that even far beyond the limit of working conditions resistance plays no part in determining the period of the circuit. My own observations substantiate this. Hence, in determining the period of the circuit, we can neglect all resistance terms in equation (4), leaving:

$$\frac{d^3 i}{d t^3}(L_1 L_2 - M^2) + \frac{d i}{d t}\frac{L_1}{C_2} = 0$$

Dividing thru by $L_1 L_2$, and remembering that

$$\frac{M^2}{L_1 L_2} = k,$$

$$\frac{d^3 i}{d t^3}(1 - k^2) + \frac{d i}{d t}\frac{1}{L_2 C_2} = 0 \tag{5}$$

This is a well-known differential equation form, and has a solution of the type

$$i = A \sin (pt + \theta) \tag{6}$$

where $$p = \frac{2\pi}{T},$$

and T is the period of the circuit. From equation (6), we have

$$\frac{d^3 i}{d t^3} = - p^2 \frac{d i}{d t} \tag{7}$$

Substituting in (5), and dividing by $\frac{di}{dt}$

$$-p^2(1-k^2)+\frac{1}{L_2C_2}=0$$

$$p^2=\frac{1}{L_2C_2(1-k^2)}.$$

and since $p=\frac{2\pi}{T}$,

$$T=2\pi\sqrt{L_2C_2(1-k^2)} \tag{A}$$

This agrees with the results obtained by other methods by Seibt, Blondel, and others. Now

$$k=\frac{M}{\sqrt{L_1L_2}}$$

$$k_1=\frac{M}{\sqrt{L_4L_6}}$$

$$\therefore \quad k=k_1\sqrt{\frac{L_4L_6}{L_1L_2}}$$

Substituting in (A), we obtain

$$T=2\pi\sqrt{L_2C_2\left(1-\frac{k_1^2L_4L_6}{L_1L_2}\right)}$$

$$=2\pi\sqrt{L_2C_2\frac{L_1L_2-k_1^2L_4L_6}{L_1L_2}}$$

but

$$L_1=L_4+L_3$$
$$L_2=L_6+L_5$$

$$\therefore \quad T=2\pi\sqrt{C_2\frac{L_3(L_4+L_3)+L_6[L_4(1-k_1^2)+L_3]}{L_1}}$$

If we call $(1-k_3^2)$ the leakage coefficient of the transformer, and $L_4$ the primary reactance, we have

$$L_4(1-k_3^2)=L_8$$

where $L_8$ is the leakage reactance of the transformer measured on the primary side.

$$\therefore \quad T=2\pi\sqrt{C_2\left[L_5+\frac{L_6}{L_1}\left(L_8+L_3\right)\right]} \tag{B}$$

In closed core transformers, $L_6=\rho^2L_4$ where $\rho$ is the ratio of turns.

$$\therefore \quad T=2\pi\sqrt{C_2\left[L_5+\rho^2\frac{L_4}{L_1}\left(L_8+L_3\right)\right]}$$

Now $\frac{L_4}{L_1}$ is of the nature of a coupling coefficient, and may be represented by $k_2^2$, where $k_2$ is defined as the coupling coefficient of the circuit considered as having no leakage in the transformer nor any external secondary inductance.

$$\therefore \quad T = 2\pi\sqrt{C_2\left[L_5 + \rho^2 k_2^2 (L_8 + L_3)\right]} \qquad \text{(C)}$$

Equation (A) gives the period of the power circuits in a spark radio outfit, for any kind of transformer. Equation (B) does the same, but shows more clearly the effects of external and leakage inductances. Equation (C) gives the period when a closed core transformer is used, and shows the effects of external and leakage inductances.

Equation (C) also shows Mr. Hallborg's method of transferring $L_8 + L_5$ into the secondary circuit by multiplying by $\rho^2$ not to be completely accurate, since the factor $k_2^2$ should also be used. However, in the usual practical cases, $k_2^2$ is greater than 0.9 and even as high as 0.97; and since T is a square root function of $L_8 + L_5$, it is seen that the error in calculating T is not very great when $k_2^2$ is neglected.

Another point of much interest is the doubtful value of power factor readings unless properly taken. For this purpose, let us consider the circuit shown in Figure 2.

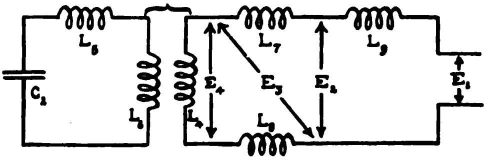

FIGURE 2

Here $L_5$ is any secondary choke coil, $L_9$ the generator inductance, $L_7$ the primary choke coil, and $L_8$ the leakage reactance of the transformer considered as a coil in series with the primary. For a given value of $L_3 + L_8$, where $L_3 = L_7 + L_9$, the circuits are identical no matter how the individual values of $L_8$, $L_7$, and $L_9$ are varied, as seen from equation (B). In the figure $E_1$ is the generated e. m. f., $E_3$ is the voltage at a transformer having leakage, $E_2$ is the terminal voltage of the machine, and $E_4$ is the actual voltage maintaining the transformer load and reactances. All these e. m. fs. are different, and give rise to different power factors when used. The usual method of measuring power factor is at the terminals of the machine. But the clever designer will so juggle $L_8$, $L_7$, and $L_9$ that $L_9$ will have a value which will give minimum e. m. f. reading at the generator terminals. The only power factor that means anything is that calculated from the generated e. m. f. This can be measured by getting a reading of the machine open circuit voltage when its field is adjusted for load conditions, provided the machine has no armature reaction. Otherwise the voltage generated under load conditions cannot be measured at all. I have seen a power factor measured at the transformer of 85 per cent., while the power factor measured back of the choke coils (that is, at the machine), was 40 per cent. If the choke coils had been introduced into the machine, the power factor measured at the machine terminals would have been 85 per cent.

**Alfred N. Goldsmith:** The conception of "entire circuit resonance" is well illustrated in this paper. It is very desirable that the radio engineer should regard the complex audio or low frequency circuit (consisting of the alternator, choke coils, primary of the transformer, secondary of the transformer, and capacity load of the secondary) as an equivalent simple circuit. It is this equivalent circuit which is to be tuned to resonance; or rather, as Mr. Hallborg has explained, to a frequency somewhat off the alternator frequency. The constants of this equivalent circuit are obtained, as shown, by transferring all capacities, inductances, and resistances from the primary to the secondary circuit or vica versa. The method of representing such complex circuits sometimes used by telephone engineers is also applicable. In this case it would consist, in brief, in joining the primary and secondary circuits thru a single inductance equal to the mutual inductance between the primary and secondary.

The entire treatment of the resonance problem by Mr. Hallborg has been dependent on the assumption, not always the case, that all currents in the circuits described are in phase. As Mr. John Stone Stone shows in one of his earlier papers on "Maximum Current in the Secondary of Coupled Circuits," the condition for maximum secondary current is also the condition for unity power factor in the primary. The importance of working near the resonance setting, so far as economy of copper in the alternator-transformer primary circuit is concerned, is therefore evident.

In measuring the secondary potential difference, a spark gap method has been used thruout. An ordinary Braun electrostatic voltmeter can be satisfactorily employed for the same purpose, and possibly with greater ease of manipulation, safety, and accuracy. Because of its intrinsically small capacity, it gives an accurate R. M. S. value of the secondary voltage at such moderate frequencies as are employed. Its readings are also more nearly independent of the wave form than is the case for a spark gap.

The effect of a resonance setting on sparking at the relay key contacts is also of interest. Will Mr. Hallborg outline his experience in this regard?

**Henry E. Hallborg**: The explanation given by Mr. Simon of the necessity for working above the resonance point, and the theoretical curve he has drawn to demonstrate the resulting circuit conditions are noteworthy in connection with the oscillograph records made by us on quenched gap and rotary synchronous gap sets during the spark discharge. For both types, the primary current and voltage waves show a sudden dip and a slight subsequent rise.

As regards Mr. Hill's suggestion that the field current in testing by the secondary voltage method should be gradually raised instead of suddenly closing the field switch, this suggestion is quite feasible. The method would be most suitable when the resonance rise is not abrupt, or in other words, when the tuning is broad. With high inductance values in the transformer, as is usual in small units, it is better to work with both the field switch and field rheostat.

In connection with Dr. Goldsmith's question regarding the comparative amount of arcing at key or relay contacts when operating near resonance or over it, I have found, in general, that there is less arcing when working above resonance; altho it is difficult to draw general conclusions since the results are largely dependent on local conditions. I recall an attempt to shunt a reactance across the relay key of the 100 kilowatt 500 cycle synchronous rotary gap set at Brant Rock, breaking about 400 amperes. It was accidentally of a value just sufficient to tune the generator-transformer circuit. A most violent arcing resulted each time the contact was broken accompanied by a noise almost as deafening as that of the spark itself. On replacing the reactance by a non-inductive water rheostat, excellent results were obtained. When the fixed and movable relay contacts had become burned into a good fit, satisfactory operation was obtained without a relay shunt of any kind. In general, the higher the generator frequency, the simpler the problem of operating a relay; since the current passes thru the zero value more frequently and materially assists in quenching the arcing.

The mathematical treatment of this problem given by Mr. Barth is largely covered by Dr. Seibt's work in 1904. Dr. Seibt derived various transformer relations for both loose and close couplings using the coupling coefficient of various combinations. One of the most important of the relations found is that used in the paper. This particular coupling coefficient is of importance only with open core transformers. I quite agree with Mr. Barth that for this type of transformer the value of the transformation ratio given in my paper is only approximate. With a closed core transformer, on the other hand, the value of k is unity, or very nearly so; even when the percentage reactance of the transformer itself varies between wide limits. For this latter case, therefore, the relation depending on the transformation ratio are quite exact. Readings made recently by the writer in conjunction with the engineers of the American Transformer Company checked these relations with remarkable closeness. They are surely accurate enough for practical purposes.

The transformer mentioned by Mr. Hill has previously been considered in the discussion on Mr. Kolster's paper on "The Effect of Distributed Capacity of Coils Used in Radio Telegraphic Circuits" (PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, Volume 1, Part 2, April, 1913, page 31). I had occasion to carry on a series of experimental tests on this transformer when connected to the radio frequency circuit operating at 1,500 meters and 3,750 meters. When the transformer was operated at 3,750 meters (80,000 cycles), no internal trouble was experienced even when the secondary windings were unprotected. However, operating at 1,500 meters (200,000 cycles) with the secondary unprotected, a certain coil within the transformer was always immediately punctured. The secondary coils of this transformer were wound with copper strip, spirally, and in two parallel pancakes per unit. A winding of high distributed capacity resulted. Puncture was undoubtedly due to resonance at the 200,000 cycle setting. The voltage rise must have been enormous. An air core reactance of about 5 per cent. inserted in series with the secondary, and between it and the radio frequency circuit, prevented further breakdown by detuning the resonating circuit previously formed. But transformer coils can be designed to withstand these radio conditions without the use of series choke coils by so winding them that the voltage between layers is a minimum and using comparatively few turns per layer. The distributed capacity is then of practically negligible magnitude.

# DESIGN AND CONSTRUCTION OF GUY-SUPPORTED TOWERS FOR RADIO TELEGRAPHY*

By

Roy A. Weagant

The purpose of this paper is to develop methods of determining the stresses in the guy-supported type of radio telegraph tower. These methods and their applications are illustrated in a complete design of a 625-foot (190 meters) structure of cylindrical form. Since the function of a radio telegraph tower is to support an aerial of chosen type and size at a desired height above the earth's surface, the determination of the stresses in, and due to, this aerial forms a necessary part of the whole problem. It may be stated as a general proposition in the design of a structure of this kind that we must start by making a number of assumptions; on the basis of which a set of calculations is carried thru. Guided by the results of the first set of assumptions, we continue the process until a satisfactory design is obtained.

## DETERMINATION OF STRESSES IN THE TOWER STRUCTURE AND GUYS

Assume that the tower is to be H feet high and that it is supported with guys arranged in sets of four at each guy point in vertical planes 90 degrees apart. Assume also that the tower has an external diameter of D feet and an internal diameter of d feet, and that the distance between the points of attachment is about 33 times D. Let the distance from the center of the tower be $H_2$. From Figure 1 the distances between the anchorages and the points of attachment of the guys may be calculated. Let the guys be constructed of an elastic material and let $d_1$ be the diameter. The various stresses in the structure are obviously due to two separate forces, viz: gravity and wind pressure. Assume that the wind acts in the direction indicated by the arrow in Figure 1, and that its maximum velocity produces a pressure of P pounds per square foot on a flat surface perpendicular to the direction in which it acts, or P/2 per square

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* A paper presented before The Institute of Radio Engineers, New York, January 6, 1915.

foot projected area if acting on a cylindrical body. With respect to the wind pressure, treat the tower as a series of beams of length h, supported at the ends and having a uniformly distributed load, but neglecting the effect of continuity. With respect to all vertical stresses, treat the tower as a series of

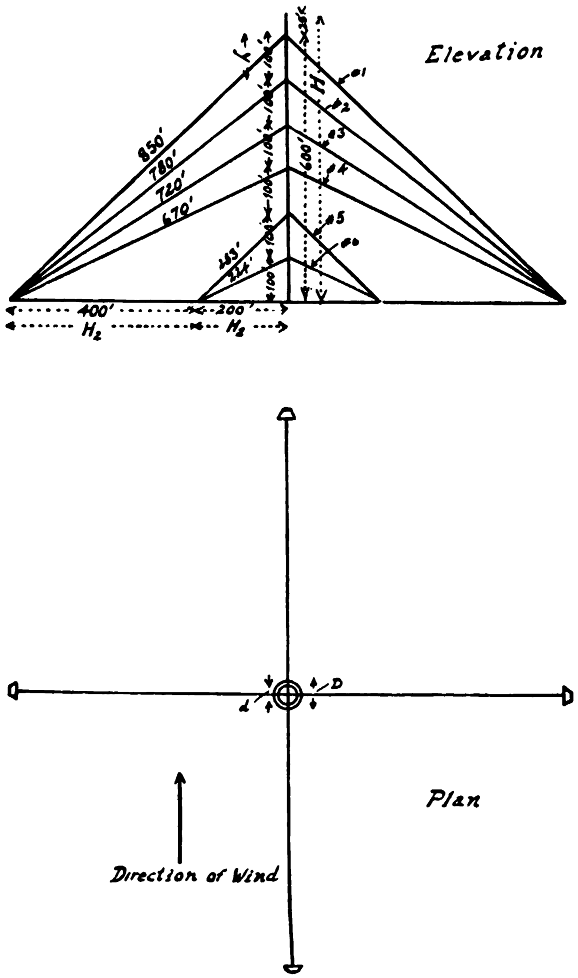

Figure 1

columns of length h and consider the points of attachment of the guys as being equivalent to pin bearings. Obviously the vertical stresses acting on the tower are composed of those due to the weight of the tower itself, and those due to the vertical component of the tensions in the guys. We must, therefore, determine these latter as accurately as possible. Let us, therefore, consider the condition of affairs when the tower is erected, but when no wind pressure is acting. We see that the tower must be vertical, that the guys on opposite sides must be drawn up with equal tightness, and that they must be drawn up with sufficient tightness so that the tower and the guys will not move unduly when the wind pressure acts. Next suppose that the wind pressure is acting. Obviously readjustments in the stresses in the guys take place. The windward sets have to sustain the full effect of the wind pressure. Therefore, on account of this added load they must stretch, and the tower will incline to leeward. It is also apparent that, to avoid weakening the tower, it must maintain a straight line as shown in Figure 2. If it does not, we have the condition shown in exaggerated form in Figure 3. In order that the conditions of Figure 2 may be

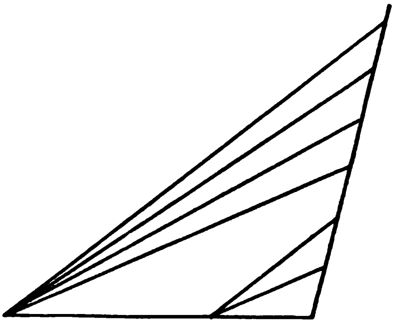

Figure 2

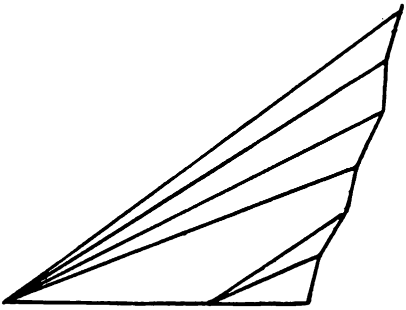

Figure 3

realized, it is necessary that each windward guy span must increase an amount proportionate to the movement of its point of attachment, which condition can only be realized when each set of guys is stressed initially to a definite amount. When a guy is stretched between two points, it forms the curve known as the catenary, and the tensile stress produced in it is given by the following equation:

$$T = \frac{Y^2 W}{8x} + \frac{x}{6} \qquad (1)$$

Where T = tension in pounds

Y = length of span in feet

x = sag or deflection at center of span in feet

W = load per foot length, uniformly distributed, and acting perpendicular to Y.

Since x is generally small, we may assume that the curve is a parabola and omit the second term in this equation. When the guy is inclined at an angle to the horizontal, the component of its weight perpendicular to the line of span must be used in this equation. If w equals the actual weight per foot length of guy, then $W = w \cos b$; where $b$ = inclination of guy to horizontal. Again if $w_1$ = total wind pressure per foot of guy, then W (for wind pressure) equals $w_1 \sin b$, when the vertical plane containing the guy is parallel to the direction of the wind. For the windward guys, weight and wind loads are added. For leeward guys, their difference is used. For those guys whose vertical planes are perpendicular to the direction of the wind, the loads are added, bearing in mind that they are acting at right angles to each other. We note that after the wind acts, the value of Y in the above equation is different for the windward and lee guys, and that the value W for all guys is changed. The length of span of the guys whose vertical planes are perpendicular to the direction of the wind is not changed, but it is apparent that due to the increase of the load W, these guys must stretch, and that this stretch appears as an increased value of the sag x. Similarly, the value of x for the windward and leeward guys changes, tending to become greater on account of the increased values of W and Y and smaller on account of the increased value of t for the windward guy; and tending to become smaller on account of the decreased values of W and Y, and greater on account of the decrease in value of T, for the leeward guys. We must, therefore, obtain a relation between these quantities which will take account of these changing conditions, and will enable us to determine the necessary initial guy tension to keep the tower straight after the application of the wind load. We must also find a method of determining the final stress in a guy which has been set up at a definite initial tension, and which has subsequently undergone a change of load. The method is as follows: Assume that the wind is acting, that the tower has inclined to leeward, and remains in a straight line, and that the base rests on a ball and socket joint.

Let $l$ = total distance along windward guy from anchorage to tower;

$l_1$ = length of guy if stress were removed (unstretched length of guy)

Then $l - l_1$ = stretch of guy

and $(l - l_1)\,k$ = tension in guy, where k is a constant, depending on the material and dimensions of the guy.

$$k = \frac{E\Delta}{l_1}$$

where E = coefficient of elasticity.

$\Delta$ = sectional area of guy.

All dimensions should be in inches, since values of E as given in the usual tables are based on this unit. Assuming now that the total horizontal pressure which the windward guy has to withstand is known, and therefore the windward guy tension T, we may determine the initial tension. Since $l_1$ must be the same in both the initial and the above conditions, we proceed as follows:

FOR INITIAL CONDITIONS

$$\frac{(l_2 - l_1)\,E\Delta}{l_1} = T_1 = \text{initial tension.}$$

$l_2$ = length along guy in initial condition,

$l_1$ = unstretched length of guy.

Also $$T_1 = \frac{Y_1^2 W_1}{8\,x_1},$$

where $Y_1$ = initial span length in feet,

$W_1$ = initial load per foot length,

$x_1$ = initial sag in feet.

And $$L_2 = Y_1 + \frac{8\,x_1^2}{3\,Y_1}, \quad \text{therefore}$$

$$\frac{Y_1^2 W_1}{8\,x_1} = \left(Y_1 + \frac{8\,x_1^2}{3\,Y_1} - l_1\right) k,$$

whence $$x_1^3 + \frac{3}{8} x_1 (Y_1^2 - l_1 Y_1) = \frac{3 Y_1^3 W_1 l_1}{64 E \Delta}. \qquad (2)$$

This last equation is a cubic in x, which is easily soluble by trial, or by the use of hyperbolic functions. Solution of this equation gives the value of the initial sag of the guy, and substitution of this value in Equation 1 gives the value of the initial tension (at the bottom of the guy). It is to be noted that the value of $l_1$ is the same for all guys in a particular set, and, therefore, we may determine the tension in the leeward guy by substituting in this equation the proper values of Y and W. Similarly, by substituting the proper value of W, the tensions in the perpendicular guys* may be determined. The application of these methods is made clear in the example given. Returning then to the tower structure, we can proceed to determine in detail various stresses acting upon it, and we will begin with the top section.

### HORIZONTAL FORCES

$D \times h_1 \times P/2 = Q$ = total pressure due to wind on top section of tower.

$l \times d_1 \times P/2 = Q_1$ = horizontal pressure due to wind on two perpendicular guys.

$Q_2$ = horizontal stress due to antenna, parallel to direction of wind.

$l$ = total length of perpendicular guys.

### VERTICAL FORCES

W = total weight above center of top section of tower, including weight of tower, top set of guys, and the antenna.

$V_1$ = vertical component of windward guy tension.

$V_2$ = vertical component of leeward guy tension.

$V_3$ = vertical component of two perpendicular guy tensions.

### GUY TENSIONS

$$T_1 = \left(\frac{Q}{2} + Q_1 + Q_2\right) \frac{1}{\cos b} + T_2 = \text{tension in windward guy.}$$

$T_2$ = tension of leeward guy.

$T_3$ = tension of perpendicular guys.

These are all determined from Equations 1 and 2, and their vertical components are T sin b. (Strictly speaking, the direc-

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* The term "perpendicular guys" is used to designate those guys which lie in vertical planes perpendicular to the direction of the wind.

tion of the guy tension at the point of attachment of the guy is a tangent to the curve at this point, but the above assumption is sufficiently accurate for practical purposes.) It will be noted from the above that to determine $T_1$, we must know $T_2$, but we cannot attempt to determine $T_2$ without a knowledge of $T_0$ (initial tension). This makes it necessary to assume a value for $T_2$ (which in actual practice is always small), and after determining $T_0$ to calculate the value of $T_2$ from Equation 2, repeating this process until the assumed and calculated values substantially agree.

Having determined the horizontal and vertical forces acting on the top section, we assume a thickness of the wall of the tower and determine the stresses produced per unit of area. Let V = total vertical force; A = cross section of area of the metal in the tower. Then $V/A$ = unit stress due to vertical loads. (This is assuming that the vertical force acts thru the center of gravity of the tower section. If a guy is attached to the edge of the tower, the load is eccentric, and the unit stress is

$$\frac{V}{A}+\frac{V\,d\,C}{I}$$

where d = distance from point of application of load to center of gravity of section.)

The wind pressure produces a bending moment of maximum value

$$M = \frac{1}{8} W\,h_1 \text{ (at center of section),}$$

where W = total wind pressure.
The maximum stress in this section is

$$S = \frac{M\,C}{I}$$

Where C = distance of the fiber most remote from the neutral axis, and equals in this case $D/2$, and

$I$ = moment of inertia $= \frac{\pi}{64}\left(D^4 - d^4\right)$ for a hollow cylinder.

This stress is a compression on the windward side of the tower, and is tensile on the leeward side. Therefore, the total maximum stress produced at the middle section is:

$$\frac{V}{A} + S.$$

If the ultimate strength of the material to be used (usually steel), divided by the factor of safety (say, four to five), agrees substantially with the value obtained above, the assumption of thickness of the tower wall at this point is correct. Otherwise, a new value must be chosen and the process repeated. The value used for the ultimate strength of the material must be corrected by an amount depending upon the relation of length to diameter of the portion of the tower between guy points. Let U = ultimate compressive strength per square inch of material. Then

$$U_1 = \frac{U}{1 + \frac{(12\,h_1)^2}{1{,}800\,r^2}} = \text{corrected value of the ultimate strength of}$$

the material,

where $r$ = radius of gyration of cylinder = $\sqrt{\frac{I}{A}}$ in inches, and

$h_1$ = height of column in feet.

The necessary sectional area of a guy for a given load is found most simply by reference to the table of working strength supplied by the makers. Shearing forces have been neglected as they are of importance only in such details as rivets, flanges, etc. By continuing this process, the stresses in all parts of the tower will be found. Also the necessity of revising some of our preliminary assumptions will be made evident. Various arrangements of design may be made, and it is only after considerable work that the most satisfactory one can be chosen. For instance, we may decrease the diameter of the tower if we increase the number of sets of guys used, the choice between these being determined by the relative cost and the facility of erection.

## ANTENNA

Two types will be considered, viz., the flat top and the umbrella; the former being supported by two or more towers, the latter usually by a single tower. In the flat top type, the maximum tension is developed when the direction of the wind is perpendicular to the length of the antenna. The most commonly used conductor for antenna construction is composed of seven strands of No. 20 or 22 silicon-bronze wire,* whose safe working load is about 150 pounds. Therefore, for any antenna whatsoever, the maximum permissible tension is n times a hundred and fifty, where n equals the number of wires. The problem, then, in constructing the antenna is to insure that this maximum permissible stress is not exceeded, and this may be

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* Diameter of No. 20 wire = 0.032 inch = 0.081 cm.
Diameter of No. 22 wire = 0.025 inch = 0.064 cm.

accomplished in one of two ways: first, by allowing one end of the wire to pass over a pulley, and attaching thereto a weight. Providing the pulley does not stick, this arrangement will give a constant tension under all conditions, but is generally used only for large aerials at land stations. Secondly, we may determine by use of Equation (2) an initial tension such that the occurrence of maximum load conditions will not cause the production of a stress in excess of the permissible value.

As regards the umbrella antenna, the method of determination of the stresses in this type of aerial is the same as for the determination of the guy stresses.

No account has yet been taken of the effect of sleet. This adds to the total weights and increases the surface exposed to wind pressure. If, in making our calculations, we assume simultaneously maximum sleet and maximum wind velocity, almost any structure within practicable limits will fail. Fortunately these conditions seldom occur simultaneously, and the most practicable method of dealing with the sleet is to provide means for its easy removal. All stresses have been determined so far on the assumption of a steady wind pressure. Actually, however, this quantity varies continually and thru a wide range, and we do not know the exact way in which this variation takes place. We can, however, set a maximum limit by assuming that a change takes place instantly from zero to its maximum value. The resultant stress in any part of the structure effected is exactly twice that of the same load applied steadily.

## EFFECTS OF TEMPERATURE VARIATION

This factor is of rather negligible consequence, since the material in the guys and tower is generally the same, namely, steel; and because the value of $l_1$ of the guys undergoes variations of temperature which are proportional to the change in length of the tower. In the case of an aerial, rigidly fastened at both ends, temperature effects may be very important, and this effect is calculable. The value of $l_1$ is corrected for the temperature change, and the resultant value substituted in Equation (2).

## FOUNDATIONS

Two general types will be considered, viz., the insulated and the uninsulated. The former is shown in Figure 4, which is the type of construction employed in the tower erected at Brant Rock. In this sketch a and b form a ball and socket joint, the purpose of which is to permit the tower to move without undue stresses at the base. The casting of the socket is flanged out to distribute the load to a ferro-concrete block d. Under block d is a set of porcelain "flower-pot" insulators, which rest on a slab of ferro-concrete, under which is a second set of insulators,

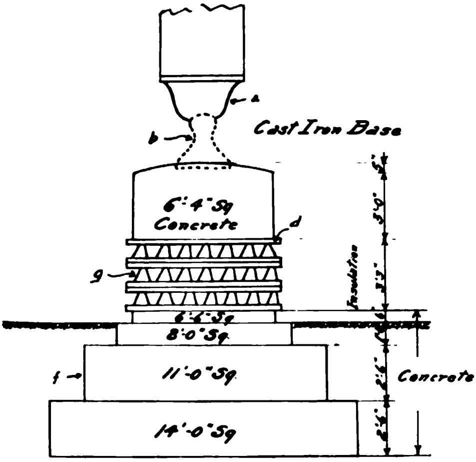

FIGURE 4

FIGURE 5

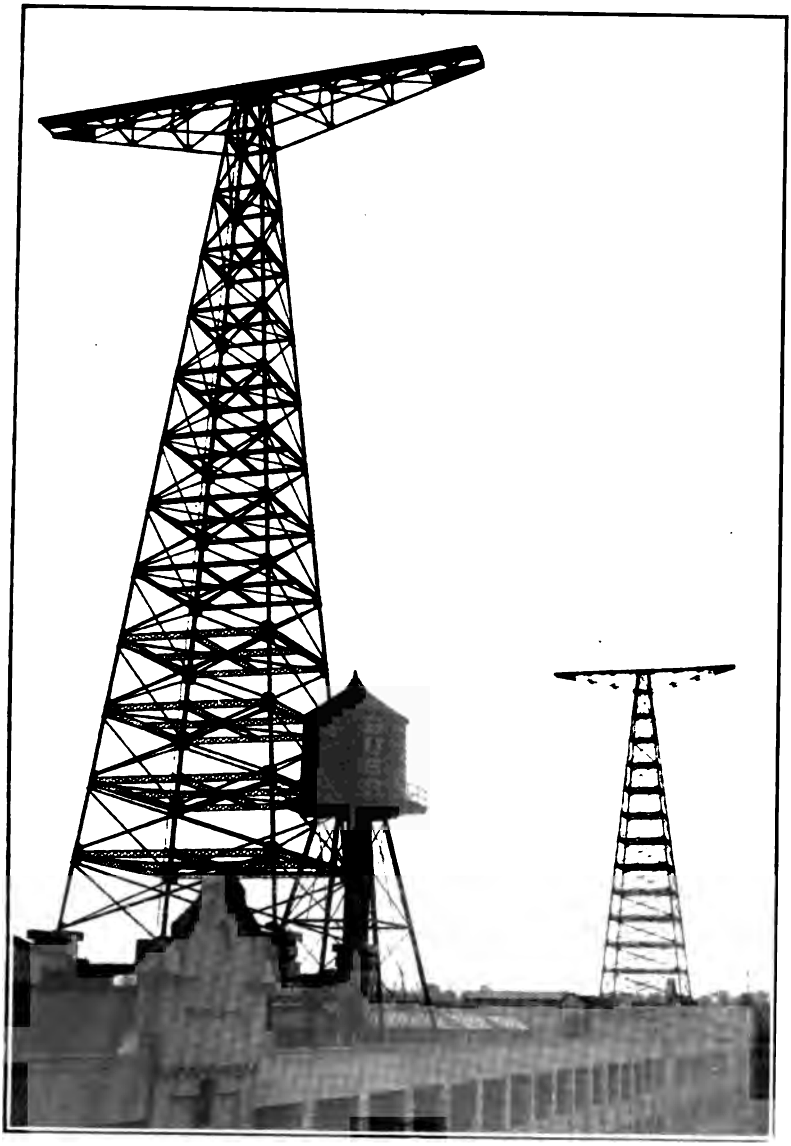

Figure 6

resting on the foundation f. The insulators g are the most interesting feature of this construction. They are about nine inches high, seven inches outside diameter at the base, and three inches in diameter at the top, by about three-quarters of an inch thick, and each has an ultimate strength of about 50,000 pounds compression. There is also an uninsulated type in which the base of the tower is fastened rigidly to its concrete foundation. In both cases the principal stress is a vertical one, altho there is in addition a small horizontal stress equal to one-half the wind load on the lowest tower section. Figure 5 shows the base of one of the supports of the insulated rigid towers at Bush Terminal. An idea of the strength of the "flower-pot" insulators can be had from Figure 6, which shows the entire towers.

## ANCHORAGES

Figure 7 shows a common form of guy anchorage made of ferro-concrete. The weight of this must equal the vertical components of the guy tensions times the factor of safety, say two to three. It must also have a vertical face on the side facing the tower of sufficient area to distribute the horizontal components of the guy tensions over a sufficient area of the soil in which it is buried. This area will vary with the location chosen for the erection of a tower, and the nature of the soil encountered. Embedded in these foundations are the anchor rods of the guys, which must be attached to suitable distributing plates embedded in the concrete. Their upper ends usually terminate in turn buckles to permit of tightening the guy.

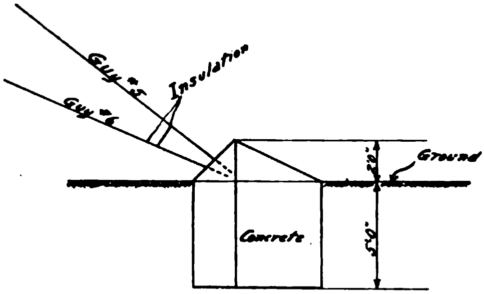

FIGURE 7

To illustrate the above methods of calculation, the design of a 625-foot cylindrical tower is given in detail. Figure 1 shows in plan and elevation the principal dimensions of the structure, and the attached tables give the complete data. The base of the tower is assumed to be a ball and socket joint, and the antenna of the umbrella type.

## STRESSES IN TOP SET OF GUYS

Total horizontal pressure supported by top guy equals pressure on section projecting above top guy plus one-half the pressure on section between first and second guys, plus pressure on perpendicular guys, plus force in direction of wind due to wind pressure on antenna. Reaction of guy for equilibrium equals $14{,}800 \text{ lbs.} = (1{,}875 + 3{,}750 + 1{,}520 + 3{,}355) \times \frac{850}{600}$, the figure 3,355 being assumed.

Assume the tension of the lee guy to be 500 pounds; then the total tension of the windward guy equals 15,300 pounds.

$$\text{Then} \qquad x = \frac{(851.41)^2 \times 2.378}{8 \times 15{,}300} = 14.1 \text{ ft.} = \text{sag.}$$

$$l = 851.41 + \frac{8 \times (14.1)^2}{3 \times 851.41} = 852.035 = \text{length along curve of guy.}$$

$$l_1 = \cfrac{852.035}{\cfrac{15{,}300}{15 \times 10^6 \times 4} + 1} = 849.892 \text{ feet} = \text{unstretched length of guy.}$$

Then for the initial conditions we have

$$Y - l_1 = 0.169, \text{ and}$$

$$x^3 + 7{,}800\,x = 7{,}800{,}000. \quad \text{Therefore}$$

$$x = 185 \text{ inches} = 15.4 \text{ feet} = \text{initial sag.}$$

$$T_0 = \frac{(850)^2 \times 1.113}{8 \times 15.4} = 6{,}700 \text{ lbs.} = \text{initial tension.}$$

Similarly, for the lee guy,

$$Y - l_1 = 0.125, \text{ and}$$

$$x^3 + 59{,}000\,x = 1{,}070{,}000. \quad \text{Therefore}$$

$$x = 255 \text{ inches} = 21.2 \text{ feet.}$$

$$T_2 = \frac{(748.59)^2 \times 15.2}{8 \times 21.2} = 500 \text{ lbs.} = \text{tension of the lee guy.}$$

If the value of $T_2$ as calculated above does not check with the value assumed, it is necessary to repeat these calculations until an agreement is obtained.

## STRESS IN PERPENDICULAR GUYS

The conditions for these guys differ from the initial condition in that the load per foot is greater, since it is

$$\sqrt{(\text{wind force})^2 + (\text{weight})^2}.$$

$$x^3 + 7{,}800\,x = \frac{7{,}800{,}000 \times 2.11}{1.13} = 14{,}600{,}000.$$

$$x = 19.5 \text{ feet, and } T_3 = 9{,}850 \text{ lbs.}$$

We next determine the stresses at the section of the tower midway between the first and second guys. Assume that the thickness at this point is one-quarter inch, the vertical components of the guy tensions will then be as follows:

$$\text{Windward} = \frac{15{,}300 \times 600}{850} = 10{,}500 \text{ lbs.}$$

$$\text{Lee} = \frac{500 \times 600}{850} = 353 \text{ lbs.}$$

$$\text{Perpendicular} = \frac{2 \times 9{,}850 \times 600}{850} = 13{,}900 \text{ lbs.}$$

Total = 24,753 lbs.

Weight of tower above this point = 14,750 lbs.

Weight of guys above this point = 5,360 lbs.

Weight of antenna = 2,000 lbs. (assumed)

Total vertical load = 46,863 lbs.

Cross sectional area of metal = 9 square inches.

Compression per square inch due to vertical load = 5,200 lbs.

$$\text{Ultimate strength of column} = \frac{60{,}000}{1 + \dfrac{(1{,}200)^2}{18{,}000 \times 625}} = 45{,}500 \text{ lbs.}$$

In the determination of the vertical stress due to guy tensions, it is assumed that their vertical components act thru the center of gravity of the tower section. If attached to the edge of the tower, then:

Total vertical component due to windward guy = 10,500 lbs.

Total vertical component due to leeward guy = 353 lbs.

The resultant acts 1.18 inches from the edge of the tower where the windward guy is attached, and unit stress due to this eccentricity of load is:

$$\frac{10{,}853}{9} + \frac{10{,}853 \times 16.82 \times 18}{11{,}000} = 1{,}510 \text{ lbs.}$$

Since the vertical components of the perpendicular guy tensions are equal, they act thru the center of gravity of the tower when the guy is attached to the edge of the tower, and therefore the total increase of the unit stress is only 300 lbs.

## STRESSES DUE TO BENDING MOMENTS IN SECTION OF TOWER BETWEEN FIRST AND SECOND SET OF GUYS

Bending moment $= \frac{1}{8} \times 7{,}500 \times 100 \times 12 = 1{,}125{,}000$ lb.—inches.

Stress due to bending moment =

$$\frac{1{,}125{,}000 \times 18}{5{,}400} = 3{,}700 \text{ lbs. per square inch.}$$

Therefore,

Total compression $= 5{,}200 + 3{,}700 = 8{,}900$ lbs. per square inch.

$$\text{Factor of safety} = \frac{45{,}500}{8{,}900} = 5.12$$

For that section of the tower above the top guys:

Wind load = 1,850 lbs.

Bending moment due to wind load =

$$\frac{1}{2} \times 1{,}850 \times 25 \times 12 = 277{,}000 \text{ lb.-inches}$$

Load parallel to wind due to wind on antenna = 3,350 lbs.
Bending moment due to antenna load =

$$3{,}350 \times 25 \times 12 \times 1{,}010{,}000 \text{ lb.-inches.}$$

$$S = \frac{1{,}287{,}000 \times 18}{11{,}000} = 2{,}110 \text{ lbs. per square inch.}$$

$$\text{Factor of safety} = \frac{45{,}500}{2{,}110} = 21.6.$$

This factor of safety is unnecessarily large, but good engineering practice does not permit the use of metal of less thickness, and decreasing the diameter of the cylinder does not cause sufficient saving to make it worth while. Data for the remaining guys and tower sections are given in the tables.

## GENERAL COMMENTS

The value of E used in the above calculations is $15\ (10)^6$ in inch units, which is one half that for solid steel. This is based on tests of plow steel ropes with wire centers which show the elongation under test to be twice that of solid metal of equal actual cross sectional area. The radius of the outer guy anchorages might well be reduced to 450 feet. Inspection of the data will show that guys of the same diameter could be used and a considerable saving in cost effected. If the guys be broken into sections with insulators, the weight and wind pressure on these must be considered as tho uniformly distributed. The effect of rigidly fastening the base of the tower to a concrete base is of interest, and can be determined as follows.

Assume that all points of attachment of the guys to the tower remain in a straight line after the wind acts, but that that portion of the tower between the lowest guy point and the foundation bends when the whole structure moves to leeward. This portion of the tower will become a parabola, and

$$S = 8dEC$$

where d = horizontal movement of the point of attachment of the bottom set of guys. In the example given:

$$S = \frac{8 \times 0.298 \times 12 \times 18 \times 15 \times 10^6}{2 \times (100)^2 \times 144} = 2{,}660 \text{ lbs. per sq. in.}$$

This is a rather large stress which, however, could be reduced by allowing a smaller movement of the tower. A reaction at the point of attachment of the lowest windward guy results from fixing the base, which reaction must be subtracted from the total horizontal force held in equilibrium by this guy. In the example given the reaction =

$$P = \frac{3dEI}{h_1^3} = \frac{3 \times 0.298 \times 12 \times 15 \times 10^6 \times 2 \times 10^4}{(100)^3 \times 1728} = 1{,}880 \text{ lbs.}$$

If two towers support a flat-top aerial, which does not use the weight and pulley arrangement for insuring constant antenna tension, then the wind pressure acting on it produces at the towers a horizontal force perpendicular to the direction of the wind; and the perpendicular guys do not then have equal stresses since the towers deflect in the direction of this force. If this force is large, it is necessary to determine the initial tensions for the perpendicular guys.

## TEMPERATURE EFFECTS

Assume guy number 1 to be stressed initially to 6,700 lbs. at a temperature of 70° Fahrenheit (21° C), and that at some subsequent time a temperature of 0° Fahrenheit (– 18° C.) obtains. Then $L_1$ becomes less, and may be called $L_1'$. Then

$$L_1' = L_1(1 - et) = L_1 - etL_1$$

where $e$ = temperature coefficient of expansion of the material used, and $t$ = temperature change. In the case under consideration,

$$e\,t\,L_1 = 0.000{,}0065 \times 70 \times 859{,}429 = 0.387 \text{ feet, and}$$

$$L_1' = 849.842 \text{ feet.}$$

Similarly, for the tower, $H$ becomes $H_1$, and

$$H_1 = 600 - 0.000{,}0065 \times 70 \times 600 = 599.725 \text{ feet,}$$

and the span $Y$ is reduced by 0.25 feet. Then, for this condition

$$Y - L_1 = 0.308,$$

and the initial tension becomes 7,100 lbs., an increase of only 400 lbs. for a 70 degree change in temperature. The tension of the windward guy is essentially independent of the temperature, except in so far as the tension of the lee guy is affected by it.

## NECESSARY ACCURACY OF CALCULATIONS

The slide rule may be used for the determination of most of the quantities except $L_1$. Since the second term of the equation involving $L_1$ contains the factor $(Y - L_1)$, $L_1$ must be determined arithmetically to about three decimal places.

**SUMMARY:** Proceeding on the assumption that a guyed tower inclined bodily by the pressure of the wind should have the points of guy attachment remain in a straight line, methods are given for calculating the following quantities: guy tensions (for windward, leeward and perpendicular guys), horizontal and vertical forces acting on the top section of the tower, stresses in the middle section, stresses in the bottom section due to bending, design of flat-top and umbrella antennas, change in stresses due to temperature variation, and dimensions of foundations and anchorages.

These methods are fully illustrated by the complete calculation of a 625-foot (190 meters) high hollow cylindrical guyed steel tower. Tables of all guy weights, tensions, and sags are given, together with the quantities which determine them.

# GUY DATA

**Material of Guys:** Plow steel, galvanized with wire center, seven strands, nineteen wires to the strand

| No. | Length | Total Weight | Diameter | Weight per ft. | Weight per ft. ⊥ to Line of Span | Wind pressure per ft. ⊥ to Line of Span | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | Windward | Lee | ⊥ |
| 1 | 850 | 5360 | 1 in. | 1.58 | 1.113 | 1.265 | 1.265 | 1.79 |
| 2 | 780 | 3750 | ⅞ " | 1.20 | .923 | .970 | .970 | 1.51 |
| 3 | 720 | 3450 | ⅞ " | 1.20 | 1.000 | .840 | .840 | 1.51 |
| 4 | 670 | 2360 | ¾ " | .88 | .787 | .600 | .600 | 1.34 |
| 5 | 283 | 990 | ¾ " | .88 | .620 | .945 | .945 | 1.34 |
| 6 | 224 | 540 | ⅝ " | .60 | .535 | .525 | .525 | 1.12 |

| No. | Total load per ft. ⊥ to Line of Span | | | Guy Tensions | | | | Guy Sags | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Windward | Lee | ⊥ | Windward | Lee | ⊥ | Init. | Windward | Lee | ⊥ | Init. |
| 1 | 2.378 | .152 | 2.11 | 15300 | 500 | 9850 | 6700 | 14.6 | 21.2 | 19.5 | 15.4 |
| 2 | 1.893 | .047 | 1.775 | 11690 | 470 | 7750 | 4770 | 13.1 | 7.65 | 18.7 | 14.8 |
| 3 | 1.840 | .160 | 1.810 | 10822 | 572 | 6950 | 4700 | 11.8 | 17.9 | 17.2 | 14 |
| 4 | 1.387 | .187 | 1.555 | 10386 | 1356 | 5000 | 5850 | 7.6 | 7.85 | 15.6 | 7.66 |
| 5 | 1.565 | .325 | 1.475 | 9400 | 700 | 5000 | 4000 | 1.62 | 4.5 | 3.16 | 1.54 |
| 6 | 1.060 | .010 | 1.731 | 6130 | 279 | 3420 | 2770 | 1.14 | 2.16 | 2.13 | 1.21 |

# TOWER DATA

Height = 625 feet. Weight = 143,890 lbs. External Diameter = 3 feet

Movement of top under maximum wind pressure = 2 feet

| Section | Thickness of Wall | Section Area | I | Wind Pressure | M | Stress Due to M |
|---|---|---|---|---|---|---|
| 1 | $\frac{1}{4}$ in. | 9 sq. in. | 5,400 | 7,500 lbs. | $1.12\,(10)^6$ | 3,700 lbs. |
| 2 | $\frac{5}{16}$ " | 11.3 " " | 6,800 | 7,500 " | $1.12\,(10)^6$ | 2,950 " |
| 3 | $\frac{11}{32}$ " | 12.4 " " | 6,900 | 7,500 " | $1.12\,(10)^6$ | 2,900 " |
| 4 | $\frac{3}{8}$ " | 13.6 " " | 7,850 | 6,300 " | $0.94\,(10)^6$ | 2,150 " |
| 5 | $\frac{13}{32}$ " | 14.6 " " | 8,800 | 5,400 " | $0.81\,(10)^6$ | 1,640 " |
| 6 | $\frac{1}{2}$ " | 18 " " | 9,800 | 4,500 " | $0.61\,(10)^6$ | 1,220 " |

| Section | Total Compression Per Sq. In. | Ultimate Strength of Column | Factor of Safety | Wind Pressure Per Sq. Ft. of Projected Area |
|---|---|---|---|---|
| 1 | 8,275 | 45,500 lbs. | 5.4 | 25 |
| 2 | 10,030 | 45,000 " | 4.5 | 25 |
| 3 | 11,350 | 41,500 " | 3.65 | 25 |
| 4 | 11,706 | 43,500 " | 3.75 | 21 |
| 5 | 12,540 | 45,000 " | 3.60 | 16 |
| 6 | 11,420 | 41,000 " | 3.57 | 15 |

The tower is built of sections eight feet long, which are joined by steel castings. Weight of castings, bolts, rivets, etc., approximately 1,000 lbs. for each section.

## DISCUSSION

**Henry E. Hallborg:** I have calculated in round numbers the stresses that occur in one of the masts of the trans-Atlantic station at Belmar or New Brunswick, N. J. when a 75-mile per hour (120 kilometers per hour) breeze is blowing at right angles to the center line of the directive antenna. These masts are of steel, 425 feet (130 meters) in total height: and of a mean diameter of 3 feet (0.91 meter). Each mast is supported by 32 stays. These stays are anchored to concrete blocks, at a distance from the mast of approximately one-half its height.

All the figures given are based on mean values, and the average angle between the guys and the mast is taken as 45 degrees. The actual tension in the guys under normal conditions was known by dynanometer tests to be 4 tons (18,000 kilograms) per stay on the average; and the total weight of the steel masts 60 tons (265,000 kilograms). The figures are as follows.

### HORIZONTAL WIND PRESSURES:

At 75 miles (120 km.) per hour, the pressure is 30 lbs. per square foot (0.061 kg. per sq. cm., or 610 kg. per sq. m.).

Average length of guys = 350 feet (103 m.).

Average diameter of guys = 0.07 foot (1.78 mm.).

Effective number of guys = 16.

Total area of exposed guys = 350 x 16 x 0.07 = 392 square feet (36.3 sq. m.).

Total force on guys = (392 x 30) ÷ 2,000 = 5.9 tons (26,000 kg.).

Height of mast = 425 feet (130 m.).

Average diameter of mast = 3 feet (0.91 m.).

Total area of mast exposed = 425 x 3 = 1,275 square feet (118 sq. m.).

Total force on mast = (1,275 x 30) ÷ 2,000 = 19.0 tons (84,000 kg.).

Total length of aerial = 5,000 feet (1,520 m.).

Length of aerial per windward mast = 830 feet (253 m.).

Number of wires = 32. Diameter of wires = 0.02 feet (0.61 cm.).

Total exposed wire area per mast = 830 x 32 x 0.02 = 530 square feet (49.1 sq. m.).

Force on wires = (530 x 30) ÷ 2,000 = 8.0 tons (35,300 kg.).

Therefore *Total Horizontal Force* = 5.9 + 19.0 + 8.0 = 32.9 tons (145,000 kg.). Resolving the horizontal force into guy tensions, and then into mast compressions (taking 45 degrees as the angle at which the forces are applied to produce the resulting mast compression, we have for this last:

Mast compression = 0.7 x 0.7 x 32.9 = 16.1 tons (71,000 kg.).

The normal mast compression due merely to its weight and the guy tensions is found as follows.

Weight of mast = 60 tons (265,000 kg.).

Average tension in the guys = 4 tons (18,000 kg.).

Aggregate guy tension = 32 x 4 = 128 tons (562,000 kg.).

Resulting compression in the mast = 128 x 0.7 = 89.6 tons (395,000 kg.).

Therefore the normal compressing force at the foot of the mast = 60 + 89.6 = 149.6 tons (660,000 kg.).

Hence the increase of compression in the mast due to a 75-mile (120 km.) per hour wind is 16.1 ÷ 149.6 or 10.8 per cent.

If, under conditions similar to the above, the diameters of the guys and aerial wires are doubled by the accumulation of sleet, the total horizontal pressures will be increased to the following values.

Force on the guys = 11.8 tons (52,000 kg.).

Force on the aerial wires = 16.0 tons (70,000 kg.).

Force on the masts (as before) = 19.0 tons (84,000 kg.).

Therefore, total horizontal force = 19.0 + 16.0 + 11.8 = 46.8 tons (207,000 kg.). The resulting compression in the mast as previously indicated now becomes 0.7 x 0.7 x 46.8 = 22.9 tons (101,000 kg.).

Hence the increase of mast compression due to the 75-mile (120 km.) per hour breeze and 1 diameter of sleet on aerial and guys is 22.9 ÷ 149.6 = 15.3 per cent.

One of the effects noted at the trans-Atlantic station at New Brunswick, N. J. when operating at 12,000 meters (the fundamental of the antenna being 8,000 meters), was the setting up of interference with commercial stations at wave lengths of 600 and 1,200 meters. The cause of this interference has not yet been definitely traced; but an attempt is being made to account for it. Since any free insulated wire, if set into electrical vibration, has a wave length of radiation of about 4 times its length in meters (or 1.31 times its length in feet), it follows that the lengths of steel cables which when vibrating electrically would radiate waves of 600 and 1,200 meters must be 150 meters (480 feet) and 300 meters (960 feet), respectively. These lengths correspond quite well to the height of the steel masts with short guy lengths attached, and to certain lengths of steel supporting wires used at New Brunswick. It remains to be considered how radiation from these members can be set up.

If the main antenna were excited by an arc, it would be plausible to ascribe the effects observed to the presence of overtones in the arc current, which might excite the guys and lead to re-radiation. In the case of excitation by a rotary synchronous gap, it is conceivable that the reasons are similar, since lower harmonics than those reported have been previously noted.

**Lester L. Israel:** Two possible explanations exist for the presence of short waves in the radiation of high powered long wave stations.

In one, it is assumed that slight sparking or brushing occurs at one or more of the insulators in the guy wires or along the length of the antenna itself. The resulting sudden change of potential is equivalent to impulse excitation of the near-by guy wire or section of the antenna. The guy wire would then oscillate at its natural frequency, or the antenna would oscillate in some harmonic. For example, if the natural wave length of the antenna were 12,000 meters, the exciting spark frequency would be 50,000; so that if sparking occurred at the insulation at the top of a grounded 100 meter guy wire, there would be 50,000 trains of 400 meter waves per second. The wave trains would be damped. I should like to ask Mr. Cohen if any decrement measurements of the harmonic oscillations have been made at Washington.

The second explanation applies to the Tuckerton station. It is thought that the harmonic oscillations observed may be due to the presence of the Poulsen arc in the antenna circuit. It is well known that the voltage characteristic of the arc is rich in harmonics. Under certain conditions, the energy output in any harmonic may be half that of the fundamental. These conditions might easily be approached in a complicated antenna, mast, and guy wire system. Harmonic oscillations produced in this way would be undamped.

**Henry E. Hallborg:** Mr. Israel's suggestion in regard to sparking in the guy wires does not appear to me to be the probable solution in view of the fact that all sparking had been eliminated as far as could be determined before the interference was reported. It is also questionable if sparking at the guys would not be picked up in the receiver as is static, since a spark occurs only when sufficient time has elapsed for the insulated section to accumulate a charge. The result would be an irregular sequence of spark discharges for the various stays. If sparking in the stays is responsible for this short wave disturbance, and if the spark frequency at the stay is a function of the insulated length, it would be a simple matter to check up the phenomenon by paralleling the stays with suitable condensers and determining the alteration in the emitted short waves.

**Louis Cohen:** The subject under discussion is of considerable interest to radio engineers. In connection with the cubic equations at which Mr. Weagant has arrived in the calculation of the sag in the guy wires, it is evident that it is not a very simple matter to obtain the values of the roots of this equation. If a number of computations must be made, the labor involved must be considerable. There are a number of graphical methods available for the calculation of the sag in transmission lines, which are discussed in books dealing with the subject, and the same methods can be applied to the problem under discussion. It is preferable, and certainly simpler to use a graphical method in place of the analytical method since the results can be obtained more quickly and with less labor.

The question raised by Mr. Stone regarding the oscillation periods of the guy wires is certainly one which merits consideration and investigation. The same question was brought up at a recent meeting of the Washington Section of the Institute of Radio Engineers during the discussion of a paper by Mr. George H. Clark. At that meeting, Mr. Clark presented his results of an important investigation of an arc system, and he has shown that the oscillations excited in an antenna circuit contain harmonics of a very high order. The explanation was offered that possibly free oscillations were produced in the guy wires by the first impulse of the waves, which were afterward re-radiated with a frequency corresponding to the natural period of the guy wires. Of course, in setting up free oscillations in a complicated electrical system such as that of a loaded antenna, we may expect that the oscillations will also be of a complex character, but we should also expect the lower harmonics to be of greater intensity than the higher ones, which was not the case in the experiments cited above. The theory of re-radiation from the guy wires was therefore offered as a possible explanation.

**George S. Davis:** While Mr. Weagant's paper deals primarily with the design of a particular type of steel mast, the subjects of field construction and the maintenance of such masts are of equal importance, and it would be very interesting to hear from him again on this subject.

My own experience in the field on this particular type of mast has been limited to two stations, the Tropical Radio Telegraph Company's 50 kilowatt station at New Orleans, Louisiana, and the United Fruit Company's 50 kilowatt station at Santa Marta, Colombia. At the former we erected 295 feet (89 meters, 29 full sections), in 30 working hours, and averaged about 36 working hours on each of the 4 masts. This speed was in a large measure made possible by the very ingenious method of erection devised by the designing engineers which briefly is as follows:

A wooden mast 45 feet (13.7 meters) in length, about 8″ x 8″ (20.3 x 20.3 centimeters) at the butt, tapering to 5″ (12.7 centimeters) diameter (round) at the top, and rigged for carrying an erection cage and hoisting gear, is placed on the concrete mast pier, and the first two steel sections bolted in place around it. The erection cage (in 2 sections) is then bolted together and suspended by means of chain hoists from steel out-riggers from the top of the mast. A steel cable is made fast to the flange of the top steel section and passed down thru a sheave in the heel of the wooden mast, and then up and out over the opposite side of the steel section to a winch on the ground. The mast is then hoisted 10 feet (2.55 meters), (1 steel section) and rested on an iron fid* passing thru the steel section and thru the heel of the mast. The erecting cage is then lowered to a point just below the flange of the top steel section, and the next steel section hoisted and bolted in place. The cage is again raised by means of chain hoists and the next section bolted in place and then the entire mast is raised 20 feet (5.1 meters) (2 sections), and the next steel section hoisted, and so on.

When the steel work is finished, and the guys in place, the erection cage is unbolted and sent down, and the mast rested on the diafram plate in the next to the top section of the mast. In erection, four men were used aloft and four on the ground to handle the winches and hoist the steel.

The proper maintenance of the masts and rigging is quite important, especially in the Tropics, and this type of mast has its disadvantages in that there is no way to paint the inside after it is in place, and the problem of renewing the wooden masts, which will eventually be necessary owing to the action of the

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* Crosspiece supporting a topmast.

elements, or to damage by lightning, is a rather difficult one, and worthy of considerable attention.

As compared with the self-supporting type of tower, the maintenance costs of the tubular steel masts are much greater. In the former there are no guys to be renewed or to be tarred down and taken up, and the only maintenance cost is the cost of painting (which is also an expense in the maintenance of the tubular steel masts). Personally I favor the self-supporting steel tower over any other type, for the reason that it can be made to, and does, bear stresses equally as great as those of the tubular steel type, the cost of maintenance is considerably less, the initial cost is very little, if any, greater than the tubular steel type, and it can be very easily taken down and moved to another point if it is desirable to do so. There is also the cost of ground for guying purposes to be considered in the case of the tubular steel masts, which is not the case with the self-supporting tower. I recall one instance where $10,000 was paid for the ground necessary for guying the steel masts.

In our practice, we have been using self-supporting steel towers at our principal stations in the Tropics for upwards of ten years. Some of these towers have been taken down and transported to distant points and re-erected. Within the last year, the towers of the old New Orleans station, which had been up for seven years, were taken down and sent to Swan Island, and two other towers on Swan Island moved to a different location. A careful examination of these towers showed they were in just as good condition as the day they were first erected.

In the Tropics these towers are gone over once in every two or three years and painted, at a cost of approximately $200.00 per tower, depending upon their location which in turn determines the cost of labor. The paint used is "karbonkote," which withstands the tropical climate exceedingly well. This painting, as well as the scraping, is taken care of by ordinary day laborers, but in the case of the tubular steel masts it is necessary to have two or three expert riggers to take care of the guys and also to furnish a transit and dynamometer, all of which are not required in the case of the self-supporting towers.

**Alfred N. Goldsmith:** It is well known, in connection with the design of the wire guys in aeroplanes, that when in motion thru the air the guys are kept in a state of continual vibration. As a result of this, elastic fatigue of a marked sort appears; and guys which originally had a strength far in excess of the requirements speedily become dangerously weak and must be replaced. In the case of guys in radio work, it may well be that a similar effect exists. It would be interesting to test the effect on the strength of a stretched guy cable of keeping it in continual and fairly violent vibration over considerable periods of time. A number of previously unexplained guy failures may thus be accounted for.

# WOODEN LATTICE MASTS*

BY

CYRIL F. ELWELL

This paper is not intended to cover all the various types of masts used in past and present radio installations, but rather to give details of the design and erection of one type; developed by the author from the original design advanced by Professor C. B. Wing of Stanford University; and many examples of which have been erected.

In the present state of the radio art some form of antenna supporter is a necessity, and a large proportion of the cost of an installation is in most cases incurred thereby. It is also a fact that the results obtained vary greatly with increased height of antenna. This has caused some comparatively high structures to be erected in the past few years. There have been relatively few failures of these supporting structures, but those which have occurred have taught their lessons.

There are, broadly considered, two main types of antenna support; viz.: self-supporting and guyed structures. These may be of steel or wood. To meet radio requirements, wooden structures are more suitable, but are not always permissible for climatic reasons. Self-supporting structures are more expensive than guyed ones, but in some cases this expense may be more than offset by the saving in the cost of the land required. Wooden structures designed along the lines which will presently be outlined are cheaper than equally strong steel structures. In steel towers the stresses in some cases call for smaller steel members than would be consistent with long life, hence more material must be used than is required. The erection of steel structures is a more costly undertaking than the erection of wooden ones constructed along the lines to be described.

Since large radio stations are rather uncertain as to length of tenure for many reasons, it is the author's opinion that wooden masts, being cheaper and more easily taken down, are the most suitable. If the radio installation proves after a number of

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* Delivered before The Institute of Radio Engineers, New York, February 3, 1915.

years to be quite correctly located from all points of view, then the question of the renewing or the replacing of the wooden structures can be taken up. There is also the possibility that the radio installation of the future will be without the high structures now thought to be a necessity.

Having decided on the height and number of masts for a given installation the first step is to determine the load which the antenna in a high wind, and perhaps covered with ice, will develop. This will in most cases be in the form of an almost horizontal pull on the mast in one or more directions. With large spacing of masts, and the elimination of sag in the messengers, these loads can be quite high. All recent structures erected by the author have been designed for a horizontal pull at the top of 30,000 pounds (14,000 kg.). If under some conditions this is too great, a small saving can be made by reducing the size of the top set of guys which are to resist this pull; but a reduction in the cross section of the timber would hardly be warranted.

The next step is to decide on the number of guys to be used for the height of the mast selected. The fewer the guys, the more expensive the structure; and if too few guys are used, the failure of one of them will cause dangerous stresses to be developed in the others, and possibly cause their failure. This is borne out by the failure of the 400-foot (122-meter) tower at Macrihanish, of the 600-foot (184-meter) tower at Nauen, and of the 492-foot (150-meter) tower at Ballybunion. The last mentioned tower failed during construction by the parting of one of the lower set of guys before the upper, and only other set of guys, could be stretched. It was a steel tube tower, with three supporting columns on a 20-foot (6.6-meter) triangle, and should have had at least four sets of guys. Too few guys make for long spans between guy points, with correspondingly high stresses due to beam action and long column action. Long spans call for heavier guys, and the difficulty of effectively breaking heavy guys into the well insulated sections demanded by radio engineers is much greater with guys over 1 inch (2.5 cm.) in diameter. It is therefore a sound policy to keep all guys less than 1 inch in diameter.

We come now to the question of anchorages. It can very readily be shown that the most economical point at which to guy any mast is such that the guy makes an angle of 45 degrees with the point guyed. There are two causes in radio work which diminish the economy of this choice. One is the necessity of breaking up the guys into well insulated sections, and the other is the area of land required. The saving in steel rope, on a tower guyed out to a point equal to its height, as compared with one guyed out to a point say two-thirds of its height, may easily be absorbed by the cost of the extra number of insulators and of the extra area of land required. For high structures, it is not economy to attach all guys to the same anchorage, as the lower guys become too flat and too long.

Having decided the height, the horizontal load, the number of guy points, and the position of anchorages, the assumed wind load must be decided. All the towers erected by the author have been designed on the assumption of a wind load of 40 pounds per square foot (200 kg. per square meter) of exposed surface; which is high and on the side of safety. Each section of the masts shown in the accompanying illustrations is a 6-foot (2-meter) square; and it is considered, that with wind from a certain angle, 30 of the 36 square feet would be effective as wind-opposing surface, so that a linear load of 200 pounds per foot (300 kg. per meter) of mast is used in computing guy stresses. The guy stresses can be readily calculated; and to them a substantial addition to allow for initial stress is made. A reduction of the actual angle is also made to compensate for the sag of the guy.

Having computed the vertical components of the wind loads, the load at the center of each span can be obtained by adding the dead weight of the mast above to the stresses due to the beam and long column actions. Having these figures, it is an easy matter to arrive at the size of the column necessary, when the strength of the material and the factor of safety to be used have been decided on.

Simplicity should be kept in mind at all times. For example, the tapering down of successive columns to meet the reducing stresses does not pay for the milling, sorting, handling, etc. In all the 300-foot (92-meter) masts erected, the columns are the same size thruout the height. In the 440-foot (144-meter) masts, two, and in some cases three sizes were used; e. g., one size to 258 feet (85 meters), and another size to the top. The same statement applies to brace frames, which, if graded to meet the varying stresses between the guy points and the centers of the spans, would lead to a confusing number of sizes.

The design of the foundation for the mast is a simple matter, and some reinforcing should be put in to take the bending action of the load imposed by the mast in a heavy wind. Sufficient area

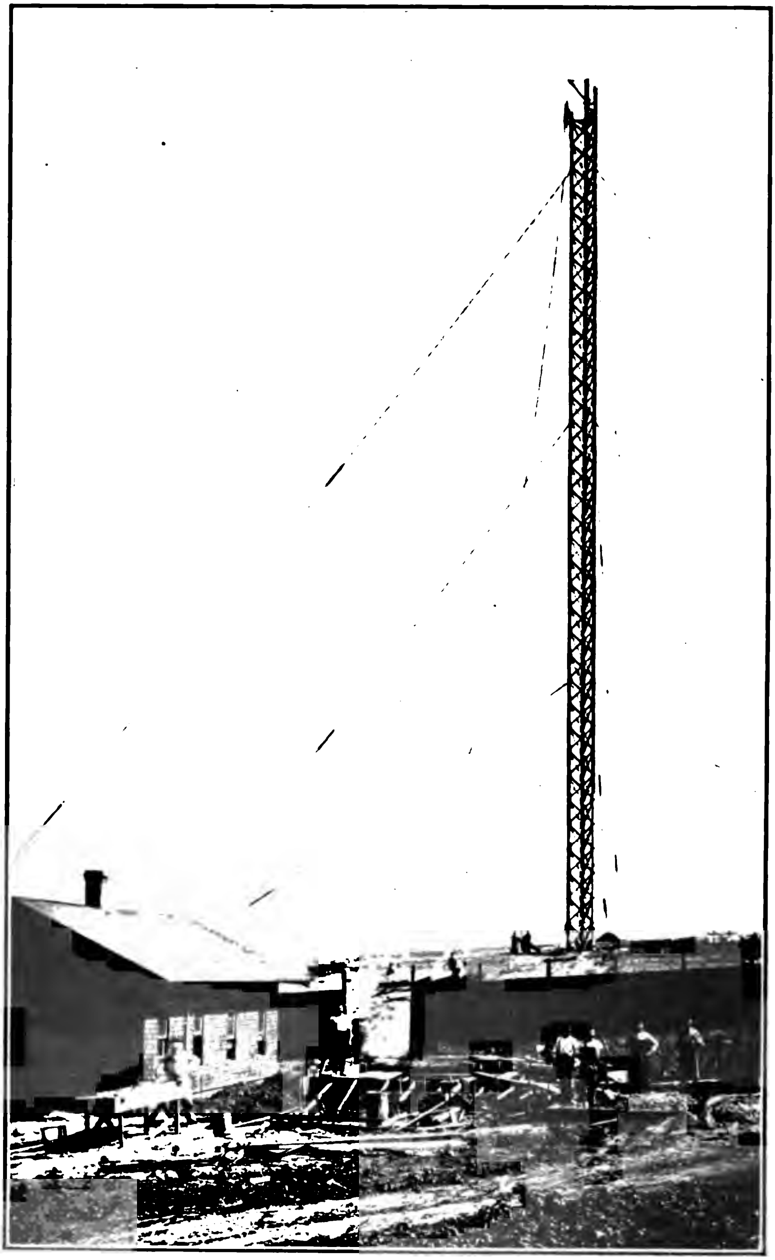

FIGURE 1

to take care of the total load, and not to impose too great a load per square foot on the soil at the site must be allowed for.

The anchorages are of concrete, with ample tension rods of steel completely incased in concrete. The wire rope guys are brought around cast iron sheaves placed on a large pin. This is a great help in stretching the guys. Turnbuckles are not used, and are not considered necessary with steel center wire ropes in which the stretch is minimized.

The method of constructing the masts may prove of interest. The first columns are respectively 8, 14 and 20 feet (2.6, 4.6, and 6.6 meters) long respectively. They are placed in position by hand, and a set of brace frames put in. This constitutes 6 feet (2 meters) of the completed mast. A light hoisting derrick is temporarily attached to one of the holes in the 20-foot (6.6-meter) column, which will afterwards take a steel tie rod, and an 18-foot (5.9-meter) column is hoisted and placed on the top of the shortest column. A set of brace frames and tie rods is inserted. The last erected column has now become the longest; and the hoisting pole is moved to it and another 18-foot column hoisted and placed upon the shortest column. This in turn is used for the erection of the next column. All columns are made up of 18-foot sticks except the bottom and top of the tower. A wooden platform in two sections is used by the men to work upon, and is passed up as each 6 feet (2 meters) of tower is completed. Two men can comfortably work aloft, and with three to five men on the ground, they can easily erect from 36 to 54 feet of tower per day. The erecting of such a tower is shown in Figure 1.

A few examples of towers erected according to designs above outlined are shown. Figure 2 shows the first of this type erected in San Francisco in 1909. They are of the four-column type, 300 feet (92 meters) high, and guyed in four directions. After a number of this type were erected, a three-post design was adopted as shown in Figure 3. Figure 4 shows a 440-foot (144-meter) example, as erected at San Francisco and Honolulu, where 606-foot (200-meter) examples have also been erected. More recent examples are three 440-foot masts for the British Admiralty at Portsmouth, England, and one 492-foot (150-meter) mast at Ballybunion, Ireland. This last is shown in Figure 5.

**SUMMARY:** For radio work, the use of wooden guyed structures is advocated on the grounds that they are inexpensive and suited to the doubtful permanency of some radio stations. The horizontal pull of the antenna at the top of the masts is assumed to be about 14,000 kg. (30,000 lbs.); thus taking account of wind pressure, ice covering of the antenna wires, and the tension due to elimination of wire sag. The antenna should be guyed at a

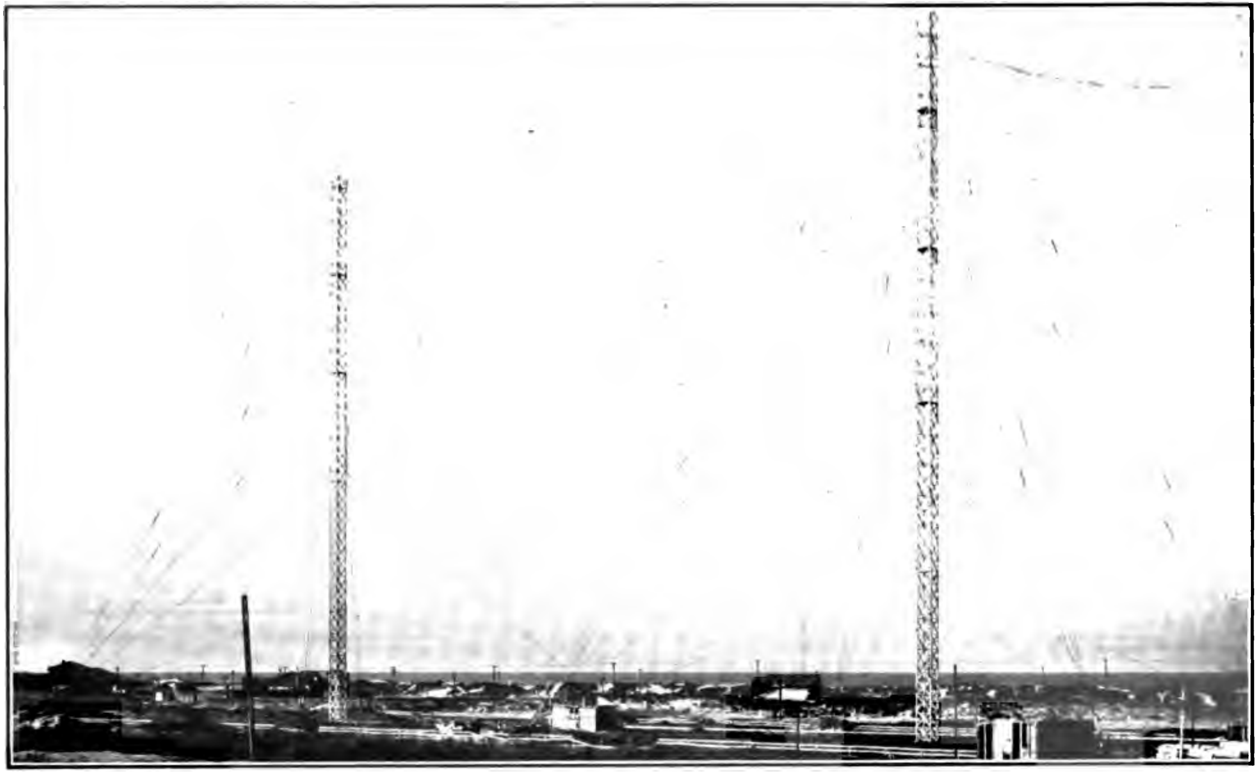

**Figure 2**

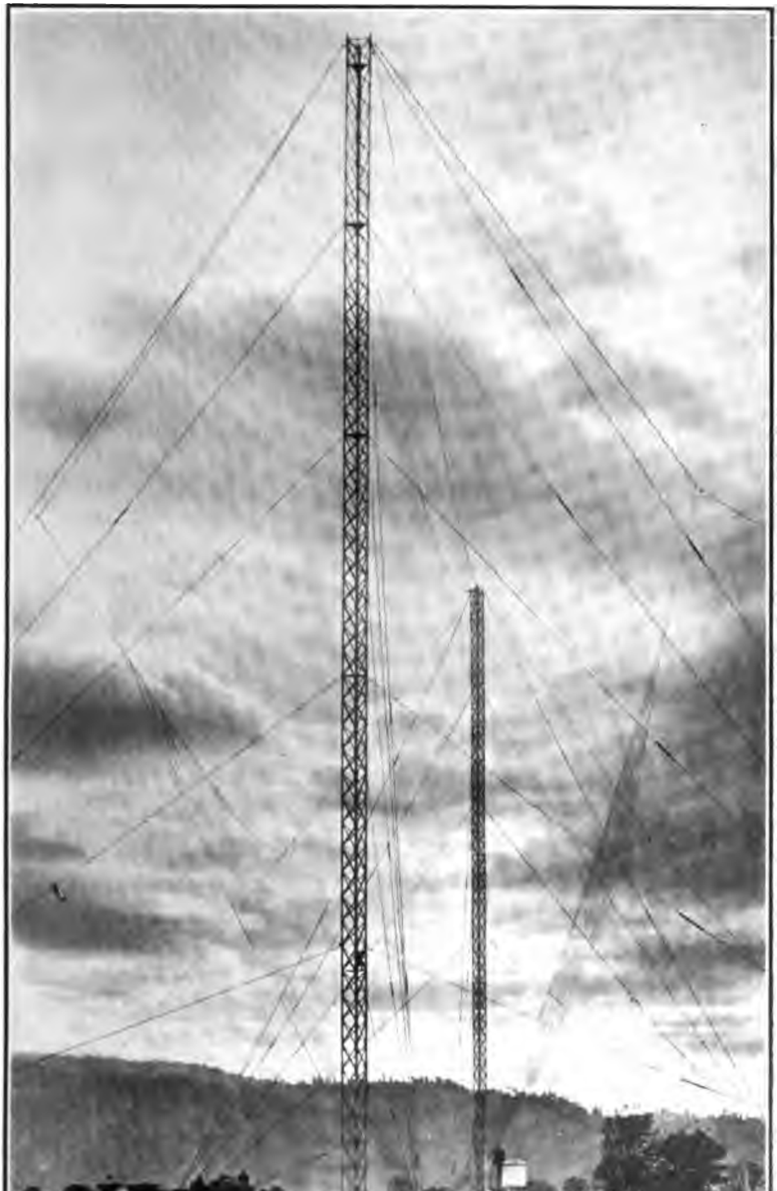

FIGURE 3

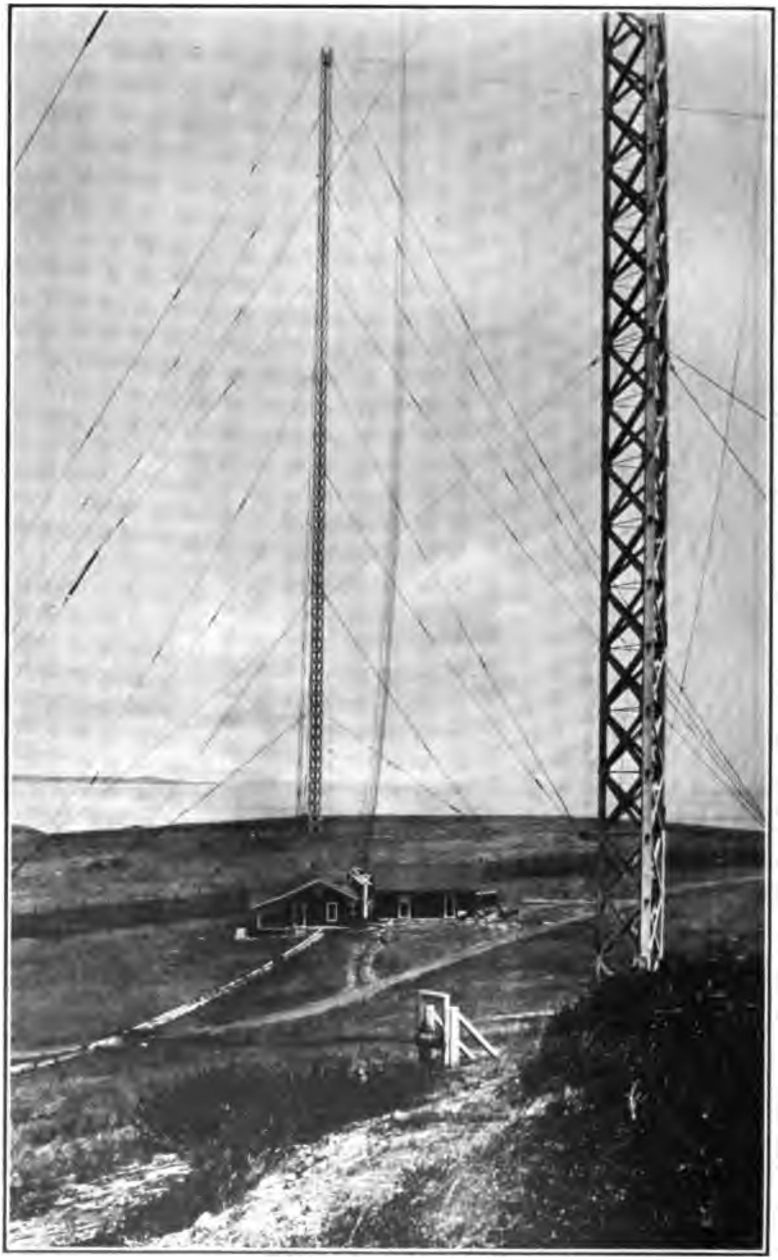

FIGURE 4

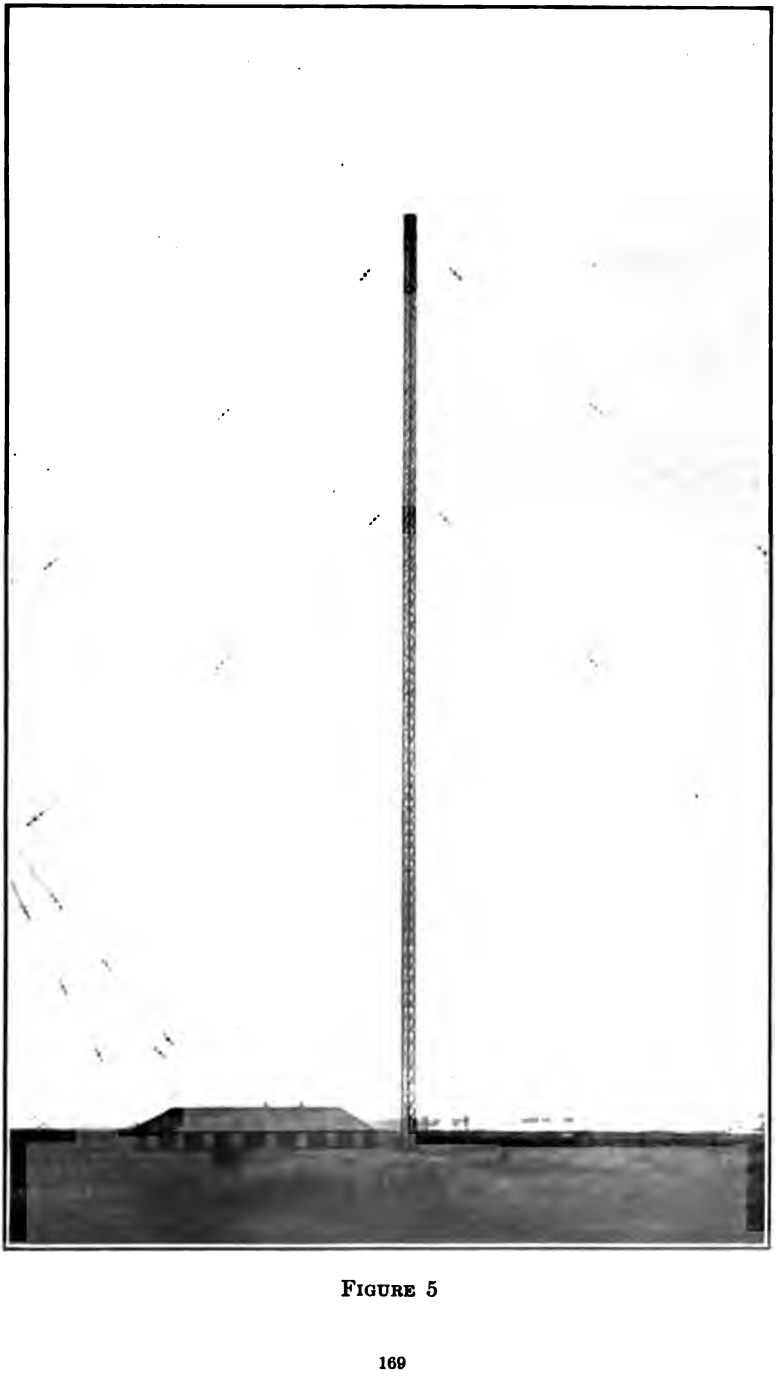

Figure 5

number of points to avoid long spans with high stresses due to beam and column action. The use of many light guys is advisable because of the difficulty of breaking up heavy guys by insulators. The position of the anchorages is determined by considerations of guy economy and of value of surrounding land. In calculating wind stresses, an assumed wind load on the masts of 200 kg. per sq. meter (40 lbs. per square foot) is used; and about 0.8 to 0.9 of the mast surface is regarded as effective in opposing the wind. The load at the center of each span is equal to the dead weight of the masts above added to the stresses due to beam and long column action. This load enables calculating the cross section of the masts. For masts up to about 100 meters (300 feet) high it is recommended to use only one size of timber to avoid complication. Above that height, a second lighter timber may be used. The construction of the anchorages is described and the method of erecting the masts is considered. Several examples of this type of mast are then shown.

# DISCUSSION

**George S. Davis:** My discussion on the tubular steel masts described by Mr. Weagant, with the possible exception of the initial cost of material will apply with equal force to the wooden lattice type of mast which is so ably described by Mr. Elwell. The wooden lattice masts may possibly have certain advantages in certain localities, but within the last six or seven months we have had an example of what will occur to this type of mast when it has been in use for any length of time. I refer to the accident at the United States Naval Radio Station at Colon, Panama, of a few months ago, in which two men were killed while engaged in dismantling one of the wooden masts installed about 1904. The aerial equipment of this station originally consisted of three wooden lattice masts, one of which I believe blew down during a high wind a year or two after it was erected. The station has since been using only two masts, and judging from the description of the accident both had been considered unsafe for some time. The height of these masts, as near as I can remember, was 250 feet (80 meters). Of course these masts were subjected to the unusual climatic conditions of the Tropics, and possibly to the ravages of insects; and for these reasons the use of wooden lattice masts in hot countries appears to be undesirable. I recall also that the old United Fruit Company's station at New Orleans was originally equipped with a wooden lattice mast 200 feet (60 meters) in height, and that when the self-supporting towers were substituted and the wooden mast thrown down, it was found that the wood in a number of places was rotting badly; and it is safe to say that it would not have withstood the elements, or the antenna strain much longer.

The point that Mr. Elwell makes in regard to using wooden lattice masts temporarily, or until such a time as the station site is definitely decided upon, has been brought up at different times but it does not seem to have received any very great consideration, probably owing to the fact that once a station is erected it has only in a very few instances been found desirable to move it. It is safe to say that if such stations had been equipped with self-supporting steel towers they could have been just as readily taken down and re-erected on a new site at less waste perhaps than would occur in taking down and re-erecting a wooden lattice mast.

**Alfred N. Goldsmith:** There are a number of portable and semi-portable types of masts in use. In one of these portable masts, a steel tape is used which, with a number of circular or square flanges surrounding it, can be raised by means of a windlass to a considerable height and then guyed in place. It can be used either as a support for the observer in military work or as a support for an antenna in connection with radio communication. Such masts can be raised to a height of from 75 to 100 feet. (See Jahrbuch für drahtlose Telegraphie, etc., Vol. 3, 1910, page 521.) Among other types of masts which can be rapidly assembled is the Rendahl mast, much used in Germany, and a number of the other similar types of masts which have been employed in this country and elsewhere.

# LONG RANGE RECEPTION WITH COMBINED CRYSTAL DETECTOR AND AUDION AMPLIFIER*

By

Haraden Pratt

In the early part of the present year, an antenna was erected at the University of California to receive the time signals from Arlington, which station was then engaged in the Paris-Arlington longitude difference tests. The writer was thus enabled to conduct some interesting experiments in long distance reception during February, March, and April.

On the western coast of the United States, the radio conditions appear to be unusually favorable during the winter months. This is particularly the case in the San Francisco Bay region, where the almost complete absence of thunder storms and, in fact, atmospherics of any strength during the time mentioned makes the reception of signals from very distant stations nearly continuously possible.

The first antenna used in these experiments consisted of a wire 750 feet (230 meters) long supported at one end on a 300-foot (92-meter) steel tower, and by an 80-foot (24.5-meter) stack at the end nearer the receiving apparatus. The results obtained were so encouraging that two extra wires, spaced about 3 feet (92 cm.) apart were subsequently added. The strength of the signals was practically doubled by their presence. The fundamental or natural wave length of the antenna system was about 1,170 meters.

The receiving apparatus differed from the usual type only in that a galena-audion amplifying combination was used. The latter arrangement was based on the following principles.

It is at present understood that the audion detector possesses two distinct and separable properties in connection with currents of radio frequency. Firstly: because of the unilateral conductivity of the region around the heated filament, oscillating currents in passing across any portion of this region suffer a

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* Delivered before The Institute of Radio Engineers, New York, December 2, 1914.

partial rectification. Thus one current impulse per wave train is produced, and an ordinary polarized telephone receiver can be affected by the transformed energy. Secondly: in the audion, the potential gradient across the rarefied gas, from the filament outward, is not linear. And the total potential difference across this space is brought to a critical value such that any further increase in it will cause a large current to flow. We have thus an amplifier and rectifier combined. The amplifying quality differs considerably in different bulbs, the shape of the E-I curve being an individual characteristic of the bulb.

A very good galena crystal and an audion bulb of fair sensitiveness were available in these experiments. When the two were used in combination, the audibility of received signals was enormously increased. It is interesting to note that there are many possible combinations of these detectors, and that a large number were tried before a successful one was found.

Five changeable features exist which affect the sensitiveness of the combination. They are:

1. Polarity of the filament battery,
2. Polarity of the secondary ("B") battery,
3. Interchanging of the galena detector terminals,
4. Interchanging of the grid and plate, and
5. Interchanging of the terminals of the secondary of the receiving transformer.

Each of these features was found to affect the strength of the signals received in the telephone. There was, indeed, one good combination out of 120 possibilities. The proper arrangement under conditions 1, 2, and 4 could be quickly found, however, because of the extreme effects produced when the adjustment was incorrect. The reason for including condition 5 is that the capacity between the primary and the secondary of the receiving transformer and the capacity of the audion apparatus to ground were different for the two modes of connection.

The results obtained with this combination of galena and audion were very satisfactory, previously inaudible signals becoming perfectly readable. To determine the amount of the amplification numerically, and to show the constancy of adjustment, a number of measurement of the audibility of signals were taken. The galena detector was used as the basis of comparison in these measurements, not only because of its steadiness and ease of adjustment, but also because it is representative of the best ordinarily used rectifying detectors. The shunted telephone method of measuring audibility was employed. Constancy of the galena detector adjustment was controlled from day to day by noting the audibility of a buzzer signal kept constant in intensity thruout the experiments.

The values tabulated below give the mean of more than 200 independent values taken over a period of 30 days. The stations ranged in distance from close at hand to over 5,000 miles (8,000 km.) away. They lay in all directions from the receiving station, their wave lengths were between 600 and 3,200 meters, and their spark frequencies between 100 and 1,000 per second.

In the columns headed "1," are given the audibility using galena alone, in columns "2" the audibility using galena and the audion. Columns "3" give the ratio of these audibilities, that is, the amplification ratio.

| 1 Audibility with Galena | 2 Audibility with Combination | 3 Ratio | 1 Audibility with Galena | 2 Audibility with Combination | 3 Ratio |
|---|---|---|---|---|---|
| 507.0 | 3541.0 | 7.0 | 5.0 | 60.0 | 12.0 |
| 3.5 | 36.4 | 10.3 | 18.3 | 273.0 | 15.1 |
| 4.5 | 51.6 | 11.4 | 26.2 | 254.0 | 9.7 |
| 2.7 | 30.5 | 11.3 | 6.0 | 51.8 | 8.6 |
| 1.5 | 18.7 | 12.5 | 90.5 | 895.0 | 9.9 |
| 9.8 | 118.6 | 12.0 | 13.0 | 102.0 | 7.8 |
| 4.5 | 60.0 | 13.2 | 26.0 | 355.0 | 13.6 |
| 8.0 | 72.0 | 9.0 | 13.0 | 119.0 | 9.1 |
| 28.0 | 253.0 | 9.0 | 51.0 | 505.0 | 9.9 |

AVERAGE AMPLIFICATION = 10.6

The average value of the amplification is, therefore, about 10. In order to note the effect of changing the "B" battery potential across the audion when using the adjusting telephone shunt, another pair of telephones was kept in series with the measuring pair and shunt. By listening in this extra pair of receivers, it was found that adjusting the shunt for audibility measurements did not disturb the signals. No single set of observations on one signal was averaged for more than one half hour's readings.

In carrying on these experiments it was noticed that audion bulbs which served excellently as detectors when used alone were not necessarily of value when used in combination with the galena as an amplifier. To determine the proper characteristics of an audion bulb for use in connection with the galena, some further experiments were tried.

It was noted first that, with certain values of the "B" battery potential of the audion, the clicks in the telephone while adjusting the galena crystal became weak, and the ease of manipulation was much increased. On the other hand, at other values of the "B" battery voltage, the clicks were loud and the crystal adjustment could be made only with considerable trouble.

A microvoltmeter was placed across the galena crystal, and it was found that for a certain value of the "B" battery voltage of the audion, no potential difference existed across the crystal. It was with this adjustment that the crystal could be adjusted and used most easily. Using one of the crystals, the relation between the "B" battery voltage and the potential across the crystal was determined. A curve showing this relation is shown in Figure 2. (Figure 1 shows the normal arrangement of the circuits.) It is interesting to note that for zero potential across the crystal, the "B" battery voltage for the particular audion used was nearly the critical value for that particular bulb. The point of zero potential across the crystal could be brought to coincidence with the critical "B" battery value of the voltage by slightly varying the filament (or "A" battery) voltage.

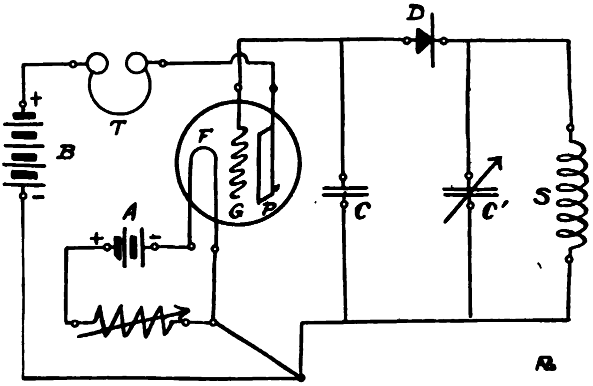

FIGURE 1

With poor audion bulbs, unsuited for use as amplifiers in conjunction with galena crystals, these conditions were not found to exist. The "B" battery potential could be varied, but the crystal potential never fell to a very low value. When the "B" battery potential was increased, the crystal potential difference diminished, but the blue light would appear in the bulb showing that the critical potential value had been passed, and saturation reached. No variation in the value of filament current could alter this condition.

It appears that unless the "B" battery potential can be brought to a critical value and at the same time the crystal potential difference is zero, satisfactory operation of this device cannot be secured.

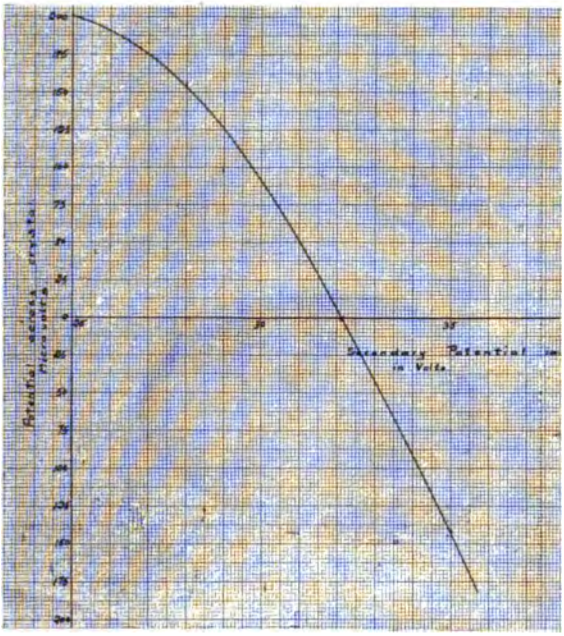

FIGURE 2

During the month of February, 1914, using the amplifier described, signals were received from Sayville (Long Island, New York), Arlington (Virginia), Key West (Florida), Colon (Panama), stations in Alaska, and others including a station in eastern Siberia. One night, signals were heard from two Telefunken stations, one in the Marshall Islands, and the other on Yap Island in the Caroline group. The distances of these stations from San Francisco are respectively 5,100 miles (8,200 km.) and 6,100 miles (9,800 km.). These signals were heard every night for three months thereafter with a nearly steady audibility of 25. The strength of signals varied but slightly from night to night. Nevertheless these stations experienced great difficulty in working with each other, altho they were but 2,100 miles (3,400 km.) apart. The more distant station of the two could with difficulty be heard using either the audion or galena alone, but with the combination, signals were always readable up to the month of May, when the summer atmospherics began to interfere. Local signals caused no interference as the wave length of these stations was above 3,000 meters.

**SUMMARY:** If an audion bulb is used as an ordinary receiver, across the "stopping" condenser in series with a galena detector, an amplification of signals of about ten times is attained. The audion bulb used, to be effective for this purpose, must have certain definite voltage-current characteristics, which are described. The circuit diagram is given. Working with an antenna of an approximate fundamental wave length of 1,200 meters, a number of observations on stations up to 8,000 km. away were made.

## DISCUSSION

**Lee De Forest:** Unfortunately, I find myself this evening in the unpleasant position of being explicitly directed by my patent attorneys not to give out any information as to my own recent work with the audion, so far as such information relates to any practical application.

In answer to a question, I wish to say that in 1905 and 1906 I investigated the effect of a magnetic field on the audion, and found that with certain bulbs a magnetic field, carefully disposed, could so localize and "focus" the cathodic discharge (probably on edges or corners of the "wings") as to increase considerably the sensitiveness of those bulbs as detectors. By this means, a diminution in filament current could be effected without loss in original sensitiveness. However, the same degree of sensitiveness could generally be attained without the magnet, by a different adjustment of the "B" battery voltage and the filament current.

**Alfred N. Goldsmith:** I believe that those who have worked with the extremely sensitive receivers of the audion amplifier type will agree that the sensitiveness of the detector, at least for work in the summer months, cannot be profitably increased because atmospheric disturbances already produce sounds many times louder than the desired signal. Until we learn to overcome the problem of static, any further increase in detector sensitiveness is of no practical advantage. And some idea of the difficulty of the problem of eliminating static may be gathered from an analogy in the field of mechanics. Required a tuning fork, which shall respond loudly to a note of a given pitch, but shall not respond to a sound of the same pitch simultaneously accompanied by vibrations of other pitch in phase with it, and which shall not respond, or but feebly, to a hammer blow!

**J. H. Morecroft:** It seems to me highly regrettable that a member, called upon to discuss the electrical actions taking place in such a device as the audion, a device not new, but well known to all radio men, should feel that his speaking in such a way is subject to the dictates of a patent attorney. Surely the art of radio telegraphy is not going to progress very rapidly if all the men working in this field are so restricted in dealing with the information which their work yields.

I hope that in future writings on radio work Mr. Armstrong will be given the credit due to him for the conscientious and careful work he has done on the audion, which work was carried out to a large extent in the radio laboratory of Columbia University. Until taking up the audion with him and applying the oscillograph to explain its action, I knew but little about it. In fact it is impossible to find in the literature a careful study of the operation of this wonderful device. But, thanks to the oscillograph, its actions are now clear and the reasons for the different behavior of the bulbs under different conditions is known.

The study of the audion leads one into advanced questions in modern physics and is extremely interesting. Altho we could not exactly simulate actual conditions of radio work (as we had to use frequencies of the order of 100 cycles) it must be remembered that, with proper discretion, the laws deduced for low frequency phenomena, are directly applicable to high frequency phenomena. The study of the audion is being continued in our laboratory and I hope will yield results interesting to the members of the Institute.

I think the results given in Dr. Austin's paper are extremely interesting, as they, too, give us some exact information on a very hazy subject. If the Editor of the PROCEEDINGS can continue his good fortune in getting papers of this class, we may soon regard the PROCEEDINGS as our most valuable reference on radio subjects.

**Alfred N. Goldsmith:** In Mr. Tesla's lectures, delivered before the Institution of Electrical Engineers in London in 1904, and published as "Experiments with Alternate Currents of High Potential and High Frequency," he describes on page 43 *et seq.* the phenomenon of the "rotating brush." This glowing electron stream was extraordinarily sensitive to electrostatic forces; so much so that the approach of the observer at a distance of several meters would deflect it noticeably. Its sensitiveness to magnetic forces is shown by its rotation under the influence of the earth's magnetic field. I should like to ask Dr. de Forest if he has ever noticed any at all similar effect with the audion.

**Lee De Forest:** I have frequently observed a somewhat similar phenomena in bulbs where the "B voltage" is made sufficiently high to produce the "globular" blue aura around the edge or the back face of the wing of an audion. Then, by adroit manipulation of filament current, this aura can be brought to a state of unstable equilibrium where the approach of the hand to the bulb, or the reception on the grid of strong radio signals, will produce a flickering or dancing of this glow. The phenomenon is a beautiful one, but I have never found a sensitiveness approaching that described by Mr. Tesla.

I believe that Professor Morecroft's confidence in the correctness of his dictum that electrical laws and phenomena must necessarily hold for high frequencies exactly as for low frequencies is not sufficient to warrant him in passing a 60 cycle, 20,000 volt current thru his body as readily as he would one of 100,000 cycles and like voltage!

But unless he is thus ready to admit of no exception to his rule he should not be so certain that oscillograms of audio phenomena at 100 cycles describe what takes place at 100,000 cycles.

I, myself, am by no means so sure that Mr. Armstrong's very interesting exposition really tells us all that transpires, whether the audion is used as a detector or amplifier, especially when we remember that there are many varieties of bulbs.

**E. F. W. Alexanderson:** In connection with the discussion of the mercury valve as a detector, I should like to call attention to an observation which I have made that a mercury rectifier shows considerable sluggishness at radio frequencies; and tho it is possible to use a mercury rectifier for doubling 100,000 into 200,000 cycles, the action is much less efficient than it is at ordinary frequencies. The sluggishness would probably exist in any vacuum detector containing mercury or an ionized gas. The usefulness of such a valve detector will therefore largely depend upon the extent to which it would respond to high frequencies. This is what I had in mind in my question to Mr. Armstrong as to the method for making the oscillograph measurements, but from his answer I gather that he used a vacuum valve which is not sluggish.

**Roy A. Weagant:** Mr. Pratt's paper is of considerable interest giving, as it does, quantitative measurements of the amplifying power of the audion when used in receiving actual radio signals. I note that Mr. Pratt depends upon the shunted telephone method of measurement, and while he has taken precautions to prevent the shunted resistances disturbing the adjustments in the wing circuit, I think there is always a liability to error in this method, and it would seem preferable to make use of a sensitive galvanometer for this work. Of course, this instrument could not be connected directly in the wing circuit, because of the direct current flowing therein, but could be coupled to it by a suitable transformer. I am particularly interested in the explanation of the actions taking place within the audion as described by Mr. Armstrong in his discussion; as this is the first time to my knowledge that these phenomena have been properly investigated and analyzed; previous explanations have been made to the effect that imposing either positive or negative potential upon the grid, resulted in the decrease of current flow thru the wing circuit. This explanation may seem valid to experimenters who have made the test, because, unless certain, not specially evident precautions are taken this result will be obtained. I hope that Mr. Armstrong will, at some future time, fully explain these points. That this former explanation of the audion phenomena is invalid, was pointed out by me during the discussion of Dr. de Forest's paper on the "Audion Amplifier" when I stated that on the basis of this explanation the audion would be a frequency changer; that is, that the action of electromotive forces upon the grid circuit would give rise to a current in the wing circuit, having a frequency twice their own. The action described by Mr. Armstrong may be termed the pure electron phenomenon and is the result of the bombardment of the molecules of rarified gas by the negative electrons in their passage from the hot filament to the cold plate. This action gives visible evidence of its existence in the so-called "blue arc" which appears when the voltage of the wing circuit is raised above a certain critical value, and is accompanied by a very greatly increased flow of current thru the wing circuit. Ordinarily when the audion is in this condition, its sensitiveness is of very low order, but by the employment of valves of special characteristics and the use of suitable circuits, von Lieben and Reisz have been able to render the valve very sensitive when in this condition, and to secure thereby a greater degree of amplification than is possible by the usual arrangement. This is due to the fact that the volt-ampere characteristic of a vacuum valve detector is very steep just at the point where the ionization begins to appear. With the audion as usually constructed this curve is so excessively steep that it is impossible to take advantage of this property, stable adjustments being impossible to secure.

**Lester L. Israel:** In Mr. Pratt's measurements, audibilities were determined by the shunted telephone method. I believe that great caution must be observed in using this method with the audion. Dr. de Forest has recently pointed out to me that if this method is used as is customary with crystal detectors, the efficiency of the audion is varied. I have found that for audibilities greater than four, serious inaccuracies appear with most bulbs, due to the variation of plate potential. Bulbs sometimes flash over when using the shunt resistance.

Mr. Pratt has attempted to meet this objection by placing a high resistance in series with the telephone making the variable potential drop across the telephones negligible. This certainly keeps the audion efficiency at a nearly constant value but now the theory of the shunt method no longer applies in the same manner as with crystal detectors.

With crystal detectors the generated E. M. F. varies so that the energy output is constant when the absorbing resistance is varied.

Thus with a shunt resistance $R_s = \frac{R_t}{4}$, we have

$$\begin{aligned} \text{telephone energy} &= (R_t)\,(i_t)^2 \\ \text{shunt energy} &= \left(\frac{R_t}{4}\right)(4\,i_t^2) \\ \text{total energy} &= 5\,R_t\,i_t^2 \end{aligned}$$

and the audibility is properly taken as

$$\frac{R_s + R_t}{R_s} = 5.$$

With a galvanometer in series with the detector, the current increases only as the square root of the audibility so that the energy measured on a galvanometer ($R_g\,i_g^2$) is proportional to the audibility $\frac{R_s + R_t}{R_s}$.

When using Mr. Pratt's method, it must be remembered that the value $\frac{R_s + R_t}{R_s}$ gives only the ratio of current thru the series resistance R to the audible telephone current.

The actual energy output of the audion is

$$R\,i^2 \text{ or } R\left(\frac{R_s + R_t}{R_s}\right)^2 i_t,$$

so that audibilities should be taken as proportional to the square of the audibility meter reading, when $R_t$ is negligible in comparison with R. In order to make comparisons with crystal detectors, the audibility of audion signals should be taken as

$$\frac{R}{R_t}\left(\frac{R_s + R_t}{R_s}\right)^2$$

since this value gives the ratio of total energy output to $R_t\,i_t^2$.

# THE THEORY OF HETERODYNE RECEIVERS

## (A DISCUSSION ON "THE HETERODYNE RECEIVING SYSTEM"[1] BY JOHN L. HOGAN, JR.)

BY

BENJAMIN LIEBOWITZ

Certain misconceptions seem to be current regarding the mode of amplification in receivers of the heterodyne type, in which a local radio frequency current is made to produce beats in conjunction with the received current. It is usual to assume, for example, that the maximum energy present in the antenna due to both currents is proportional to $(i_1 + i_2)^2$, and the minimum to $(i_1 - i_2)^2$, giving an energy fluctuation of $4i_1 i_2$; whereas the energy due to the received current alone would be proportional to $i^2_1$; from which it is deduced that the ratio of amplification is $2\frac{i_2}{i_1}$. The incorrectness of this view will appear from the following discussion.

Suppose the received and local currents to be simple harmonic, the first expressed by

$$i_1 = A \sin pt,$$

and the second by

$$i_2 = B \sin qt.$$

Let L denote the effective inductance of the antenna, and W the instantaneous value of the energy present in L. Then

$$W = \frac{1}{2} L (i_1 + i_2)^2 = \frac{1}{2} L (A \sin pt + B \sin qt)^2$$

$$= \frac{1}{2} LA^2 \cdot \sin^2 pt + \frac{1}{2} LB^2 \cdot \sin^2 qt + ABL \cdot \sin pt \cdot \sin qt$$

$$= \frac{1}{2} L [A^2 \sin^2 pt + B^2 \sin^2 qt + AB \cos (p - q) t$$
$$- AB \cos (p + q) t].$$

The instantaneous value of the energy has, therefore, four com-

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[1] A paper printed in THE PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, 1913, Volume 1, Part 3, page 75, *et seq.*

ponents, which for convenience may be denoted by $W_1$, $W_2$, $W_3$, and $W_4$. In dealing with energy, however, it is necessary to consider not the instantaneous values, but the average values. Thus, it would be incorrect to assume that $W_3$ represents the energy available for producing signals because it fluctuates with audible frequency, without regard to its average value; for, if that were the case, the received current acting alone would have no energy available for producing signals. (It must be borne in mind that the energy present in the antenna is under consideration, without regard to the manner in which that energy is utilized.) It is, therefore, only the average value of W which is of importance, and to find this average value we have merely to find the average values of the four components and add them. The period of $W_1$ is $\frac{\pi}{p}$, and if we integrate $W_1$ from any instant, t, to an instant one period later, i. e., to $t + \frac{\pi}{p}$, and then divide by the period, we get:

Average of

$$W_1 = \frac{p}{\pi} \cdot \frac{LA^2}{2} \int_t^{t+\frac{\pi}{p}} \sin^2 pt \cdot dt = \frac{p}{\pi} \cdot \frac{LA^2}{4} \int_t^{t+\frac{\pi}{p}} (1 - \cos 2pt)\, dt$$

$$= \frac{p}{\pi} \cdot \frac{LA^2}{4} \left(t - \frac{1}{2p} \sin 2pt\right)_t^{t+\frac{\pi}{p}}$$

$$= \frac{LA^2}{4} = \frac{1}{2} L \frac{A^2}{2} = \frac{1}{2} L I_1^2,$$

where $I_1$ is the effective value of $i_1$.

Similarly, it can be shown that

$$\text{Average of } W_2 = \frac{LB^2}{4} = \frac{1}{2} L \frac{B^2}{2} = \frac{1}{2} L I_2^2,$$

where $I_2$ is the effective value of $i_2$. Turning now to $W_3$, the period is $\frac{2\pi}{(p-q)}$ and if $W_3$ is integrated from any instant, t, to $t + \frac{2\pi}{(p-q)}$, and the result divided by $\frac{2\pi}{(p-q)}$, we get:

$$\text{Average of } W_3 = \frac{p-q}{2\pi} \cdot \frac{ABL}{2} \int_t^{t+\frac{2\pi}{p-q}} \cos(p-q)t \cdot dt$$

$$= \frac{1}{2\pi} \cdot \frac{ABL}{2} \Big[\sin(p-q)t\Big]_t^{t+\frac{2\pi}{p-q}} = 0.$$

Similarly, Average of $W_4 = O$.
Hence the average value of the energy present in the antenna is given by

$$\text{Average value of } W = \frac{1}{2} L (I_1^2 + I_2^2).$$

In other words, when currents of different frequencies are present in a circuit, the average value of the energy present is equal to the sum of the average values of the energy due to each current separately. In fact, the law of conservation of energy demands this; and furthermore, it is a well-known theorem in electrical theory, that if a number of currents of different frequencies and of effective values $I_1$, $I_2$, $I_3$ · · · are present in a resistance R, then the average rate of heat development is equal to

$$R (I_1^2 + I_2^2 + I_3^2 + \cdots).$$

It is clear, therefore, that receivers of the heterodyne type do not amplify by increasing the energy component of the received currents in the antenna. Before considering the true mode of amplification in such receivers, it is necessary to distinguish between two types of amplification, namely: (1), by infusing new energy into the received oscillations, and (2), by increasing the efficiency of the receiving apparatus. As an example of amplification by the infusion of new energy into the incoming oscillations, consider receivers which employ an electron stream acted on by the currents to be amplified. In such receivers the resulting variations in the electron current can be made many times greater than the amplitude of the original current, so that here we have actually reproduced the original currents, but with greater energy. As an example of the other type of amplification, consider the ordinary telephone receiver. That the presence of the permanent magnet produces an enormous increase in the amplitude of the vibrations of the diaphragm is too well known to require mention, but it cannot be said that the permanent magnet puts new energy into the system. This is clearly amplification by increasing the efficiency of the receiving apparatus; the energy in the sound can never exceed the energy in the received current.

The theory of ordinary telephone receivers, as usually presented, is worthy of further scrutiny in this connection. The force of attraction between a magnet and a piece of iron is directly proportional to the square of the flux. If this flux has a constant component $\phi_1$, and a variable component $\phi_2 \sin pt$, the force at any instant is proportional to

$$(\phi_1 + \phi_2 \sin pt)^2 = \phi_1^2 + 2\,\phi_1\,\phi_2 \sin pt + \phi_2^2 \sin^2 pt.$$

$\phi_1$ is usually very large compared with $\phi_2$; hence, neglecting the last term, the variable force is proportional to

$$\phi_2\,\phi_1 \sin pt,$$

and since $\phi_2$ is proportional to A, the amplitude of the received current, the variable force is proportional to

$$A\,\phi_1 \sin pt.$$

Hence, the larger the permanent flux the larger the useful force.

This theory is correct, however, only so long as the motion of the diaphragm is very small; i. e., only so long as the efficiency of the receiver is very low. The telephone receiver is, after all, a synchronous motor, and the excursions of the diaphragm produce a back e. m. f. in the coils, just as in any other motor. This back e. m. f. is ordinarily negligible, because the efficiency of the telephone receiver is ordinarily very low. For higher efficiencies, however, this back e. m. f. would attain values of the same order of magnitude as the resistance reaction and inductance reaction, and the effect of this would be to diminish the incoming current. Hence, the useful force cannot be indefinitely increased by increasing the permanent flux; the best that can be attained is an increase in the efficiency. This phenomenon is analogous to the events in an ordinary motor; as the motor speeds up the back e. m. f. becomes increasingly important, and, at full speed, is the largest reaction in the circuit if the motor is efficient.

The theory of the electrostatic telephone receiver in which a constant difference of potential is maintained between the plates is entirely analogous. If $V_2$ represents the constant e. m. f. and $V_1 \sin pt$ a superimposed variable e. m. f., then, since the force between the plates varies as the square of the e. m. f., the force at any instant is proportional to

$$(V_2 + V_1 \sin pt)^2 = V_2^2 + 2\,V_1\,V_2 \sin pt + V_1^2 \sin^2 pt.$$

Again neglecting the last term, the variable component of the resulting force is proportional to

$$V_1\,V_2 \sin pt.$$

This could be indefinitely increased by increasing $V_2$ indefinitely; but here again a back e. m. f. is produced as soon as the device becomes appreciably efficient, and this back e. m. f. results in a decrease of $V_1$ when $V_2$ is increased. In this receiver, the battery which maintains the constant voltage does not supply any useful energy. It acts in a manner entirely analogous to the permanent magnet in the ordinary telephone receiver. The acoustic energy of such a device can never exceed the energy in the received currents.

Turning now to receivers of the heterodyne type, consider, for example, the form shown in Figure 11 of Mr. Hogan's paper, "The Heterodyne Receiving System." (See these PROCEEDINGS, July, 1913.) No attempt will be made to give a rigorous theory of the problem presented by these circuits, but an approximation of the facts sufficiently close for practical purposes will be presented.

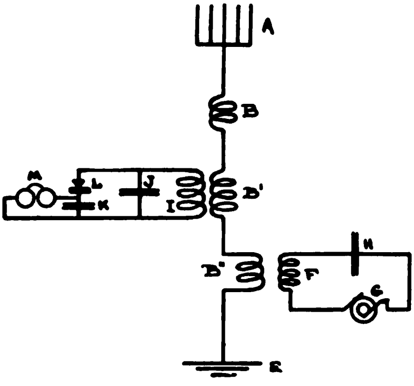


FIGURE 11
(OF ORIGINAL PAPER)

In the circuit IJ of Figure 11 of Mr. Hogan's paper, suppose that the two currents,

$$i_1 = A \sin pt \text{ and } i_2 = B \sin qt$$

are flowing. The voltage across the condenser J will be

$$v = \frac{1}{C}\int (i_1 + i_2) d\,t = a \cdot \cos pt + b \cdot \cos qt$$

where $a = -\frac{A}{p\,c}$, $b = -\frac{B}{q\,c}$, and C is the capacity of condenser J. It can be shown, in a manner entirely similar to the previous cases, that the average value of the energy present in this condenser is proportional to $(a^2 + b^2)$ and not to $(a + b)^2$. Suppose now that a is much smaller than b, as is the case in practice; then, denoting the difference in the amplitudes by h $(h = b - a)$, we may write v in the form

$$\begin{aligned} v &= a\,(\cos pt + \cos qt) + h \cos qt \\ &= 2a \cos\left(\frac{p-q}{2}\right) t \cdot \cos\left(\frac{p+q}{2}\right) t + h \cos qt. \end{aligned}$$

The voltage across the condenser J can, therefore, be resolved into two components,

$$v_1 = 2a \cos\left(\frac{p-q}{2}\right) t \cdot \cos\left(\frac{p+q}{2}\right) t \text{ and } v_2 = (b - a) \cos qt.$$

The first may be called the "beat" component, the second the "sustained" component. The graph of $v_1$ is of the form shown by Mr. Hogan (loc. cit.) in Figure 4, curve C; the graph of $v_2$ is a simple sine curve. The effects of these components in the rectifying detector circuit KLM will now be separately considered.

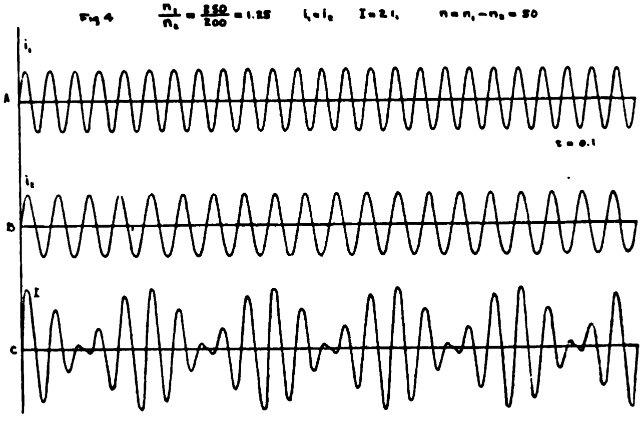


FIGURE 4
(OF ORIGINAL PAPER)

Owing to the rectifying and integrating action of the detector circuit, the rapidly varying voltages $v_1$ and $v_2$ give rise to constant or slowly varying unidirectional currents through the telephone receivers M. More specifically, the "beat" voltage component $v_1$ tends to produce a current in the detector circuit of the form shown in Figure 6, curve C of Mr. Hogan's paper, with the negative loops omitted, however. But owing to the high resistance of the detector and the large inductance of the telephone receivers, this series of unidirectional current loops is smoothed out into the form shown by Mr. Hogan's curve E of Figure 6. The maximum value of these smoothed

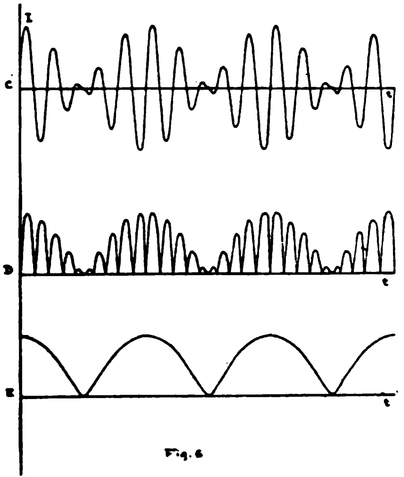


Figure 6
(OF ORIGINAL PAPER)

loops will obviously be proportional to the maximum amplitude of the "beat" component, i. e., to 2a, and since the minimum value is zero, the *amplitude* of the variable component of the telephone current will be directly proportional to a, which in turn is proportional to A, the amplitude of the received current. We see, therefore, that the amplitude of the variable telephone current is directly proportional to the amplitude of the received current.

Consider now the sustained voltage component (b − a) cos qt. It will obviously give rise to a practically constant, unidirectional current thru the telephone receivers. This might result in two improvements (1), a slight increase in the sensitiveness of the telephones resulting from a possible increase in the permanent flux, and (2) an increase in the detector sensitiveness resulting from working on a better part of its characteristic. Whether or not these improvements exist is immaterial from our present point of view, because if they do exist, the same results could be obtained by suitably placing a battery in the detector circuit. In any case it is clear that these last two improvements would be amplification by increase in efficiency, and not by infusion of new energy. Barring, therefore, possible improvements which could be obtained by the use of a battery, we see that no matter how large the amplitude of the local current may be, only that part is useful whose amplitude is equal to the amplitude of the received currents.

It will now be seen that the maximum true amplification, i. e., amplification by infusion of new energy, which the heterodyne receiving system can produce is four. To prove this, suppose that the local current is absent, that the same system of circuits is employed, and that a "chopper" in series with the telephones is used to break up the sustained received oscillations into trains of audible frequency. The maximum value of the voltage across condenser J will now be equal to $-\frac{A}{pc}$, assuming the current in I J to be expressible by A sin pt, as before. The resulting pulsating current thru the telephone receivers will, therefore, vary between O and a maximum value proportional to $\frac{A}{pc}$, i. e., proportional to A. Hence the amplitude of the telephone current will be proportional to $\frac{A}{2}$. But we have seen that when the local current is present, the amplitude of the telephone current is proportional to A, the factor of proportionality being the same in both cases; and since the acoustic energy is proportional to the square of the telephone current, it follows that the useful energy is four times as great when the local current is present as it is when the local current is not present. At the very most, therefore, the maximum true amplification which the heterodyne receiving system can produce is four. Any additional amplification which has been observed must be regarded as due to an improvement in the efficiency of the receiving system, and not to any particular virtue of the heterodyne principle. Such additional amplification is obtained, for example, by making the beat frequency equal to the natural frequency of the telephone receivers.

The form of the heterodyne receiver which we have been discussing, i. e., the form shown in Figure 11, is the most efficient of all those described in Mr. Hogan's paper. The other forms shown offer considerable mathematical difficulties when the energy relations are analysed, altho the principal forces acting may be readily found. Thus, for example, in Figure 10, suppose that the voltage v across the electrostatic telephone receiver D is expressible by

$$v = a \cos pt + b \cos qt,$$

the first term being due to the incoming, the second to the local oscillations. Since the force between the plates of the receiver is proportional to $(v)^2$, we have, denoting the force by F and a proportionality factor by K,

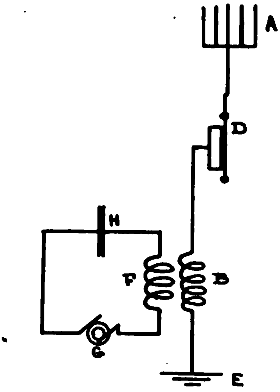


Fig. 10.

FIGURE 10
(OF ORIGINAL PAPER)

$$\begin{aligned} F &= K\,(a \cos pt + b \cos qt)^2 \\ &= K\,(a^2 \cos^2 pt + b^2 \cos^2 qt + 2ab \cos pt \cos qt) \\ &= K\,[a^2 \cos^2 pt + b^2 \cos^2 qt + ab \cos (p - q)\,t + ab \cos (p + q)t]. \end{aligned}$$

The force, therefore, has four components, only the third of which is useful in producing acoustic energy. Hence the useful force is given by

$$f = K\,a\,b \cos (p - q)\,t.$$

This shows that the greater the amplitude of the local oscillations, the greater is the useful force; but here again we should have back e. m. f.'s produced which, as soon as the device became efficient, would limit any further increase in the force due to a further increase in the local amplitude, by reducing the incoming amplitude proportionately. It is clear that the local oscillations perform the same function in this system that the permanent magnet does in the ordinary telephone receiver and that the constant impressed voltage does in the electrostatic telephone receiver. It is possible, however, that the local oscillations may give, besides, a limited amount of true amplification, as they do in the form discussed above. An investigation to determine whether or not this is the case would be difficult; the value, moreover, of such an investigation would be doubtful, since this form is not the most efficient and since it has been just shown that the maximum true amplification obtainable in the most efficient form of the heterodyne receiver is four.

**SUMMARY:** The necessity of viewing the energy relations in the heterodyne receiver from the standpoint of average, not of instantaneous values, is pointed out; and the average energy present due to two currents of different frequencies is studied.

A distinction is made between two general types of amplification; (1) by infusion of new energy into the received currents, and (2) by increase of efficiency of the receiving apparatus. Only the first may be regarded as true amplification. As examples typical of the second, the theory of the electromagnetic and of the electrostatic telephone receivers is sketched. Finally, it is shown that the maximum true heterodyne amplification is four.

# DISCUSSION

**Louis Cohen:** At the beginning of the article, we have a mathematical demonstration to show that receivers operating on the heterodyne principle do not amplify the energy of the received current. The result is summarised in the following statement: "It is clear, therefore, that receivers of the heterodyne type do not amplify the energy component of the received current in the antenna." Further on in the article, Mr. Liebowitz shows with equal mathematical skill that the heterodyne does produce an amplification of four times. To quote again: "It will now be seen that the maximum amplification, i. e., amplification by the infusion of new energy, which the heterodyne receiver can produce is four."

It is interesting to note that he arrives at these contradictory conclusions by using the same fundamental equations, but applying different trigonometrical transformations in either case. A little reflection should have convinced Mr. Liebowitz that a theory which may lead to different conclusions, depending solely on juggling with trigonometry, must be fundamentally wrong. The fact also that all the experimental evidences obtained by various observers contradict his theory, does not seem to concern him. Apparently he prefers to ignore entirely experimental facts.

In support of his argument, Mr. Liebowitz shows that the average value of the energy in the circuit is $\frac{1}{2} L (I_1^2 + I_2^2)$, where $I_1$ and $I_2$ are the amplitudes of the two currents, and not $\frac{1}{2} L (I_1 + I_2)^2$, as has been maintained by others. This is perfectly true, but it does not prove anything. The error he makes is in averaging the energy of each component separately for a different period as if the other component were entirely absent. This, however, is not the true condition. If we have two currents, A sin pt and B sin qt acting on the same circuit, and put $q = p + \beta$, the resultant current in the circuit at any instant of time is,

$$\begin{aligned} I &= A \sin pt + B \sin (p + \beta) t \\ &= A \sin pt + B \sin pt \cos \beta t + B \cos pt \sin \beta t \\ &= (A + B \cos \beta t) \sin pt + B \sin \beta t \cos pt \\ &= \sqrt{A^2 + B^2 + 2 A B \cos \beta t}\, \sin (pt + \phi). \end{aligned}$$

The amplitude of the resultant current is variable, of frequency $\frac{\beta}{2\pi}$ equal to the difference of the frequencies of the two currents. The average value of the square of the current for a period $\frac{2\pi}{\beta}$ is $(A^2 + B^2)$, but the variation in amplitude is proportional to 2 A B, and hence the amplification is proportional to $\frac{2A}{B}$; that is, the ratio of the local current to the received current.

In the discussion of the electrostatic telephone receiver, Mr. Liebowitz arrives at an equation somewhat similar to the one given above, which leads to the conclusion that the amplification must be proportional to the amplitude of the local current, but he is not willing to admit this, and he introduces the idea of the back e. m. f. of the telephone. What he may mean by the back e. m. f. of the infinitesimal displacement of the condenser plate is a little hard to see. The fact is that in the many experiments I have carried on using the electrostatic telephone, the sensitiveness continually increased as the local current was increased. We have obtained amplifications of a thousand times and more, and our only limitation was the arc noise of the local exciting circuit. In the case of the crystal detector greater difficulties were experienced, the irregularity of the arc current was a more disturbing factor, but under favorable conditions amplifications of twenty to fifty times were obtained.

**Benjamin Liebowitz:** Mr. Cohen's criticisms of the theory I have proposed are, with one possible exception, wholly without basis, as I shall show. It is true that integrating different energy components over different periods is not an entirely rigorous process, but since the result obtained is well known to be correct, and since the rigorous proof is somewhat more complicated than that given, a sacrifice of rigor for simplicity is quite justifiable. However, in order to remove all possible ground for argument, I shall now prove by a perfectly rigorous method that the average energy $W_m$ due to two currents of different frequencies present in a circuit is

$$W_m = \frac{1}{2} L \left(I_1^2 + I_1^2\right).$$

If the two currents are given by A sin pt and B sin qt, the instantaneous value of the energy is

$$W = \frac{1}{2} L \left(A \sin pt + B \sin qt\right)^2.$$

By applying the same transformations as are used in the article we get:

$$\begin{aligned} W = \frac{1}{4} L \Big\{ & A^2 + B^2 - A^2 \cos 2\,pt - B^2 \cos 2\,qt \\ & + 2\,AB \cos (p - q)\,t - 2\,AB \cos (p + q)\,t \Big\}. \end{aligned}$$

Integrate this between the limits $t_1$ and $t_2$, and then divide by $(t_2 - t_1)$. Remembering that

$$\text{Average of } W = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} W\,dt = W_m, \quad \text{we get}$$

$$(1)\quad \left\{ \begin{aligned} W_m = \frac{1}{4} L \Big( & A^2 + B^2 - \frac{L}{t_2 - t_1} \Big\{ \frac{A^2}{8\,p} \sin 2\,p\,(t_2 - t_1) \\ & + \frac{B}{8\,q} \sin 2\,q\,(t_2 - t_1) - \frac{AB}{2\,(p - q)} \sin (p - q)\,(t_2 - t_1) \\ & + \frac{AB}{2\,(p + q)} \sin (p + q)\,(t_2 - t_1) \Big\}. \end{aligned} \right.$$

If the time interval be so chosen that $2\,p\,(t_2 - t_1)$, $2\,q\,(t_2 - t_1)$, $(p - q)\,(t_2 - t_1)$, and $(p + q)\,(t_2 - t_1)$ are all integral multiples of $\pi$, then the bracketed term will vanish. But the time interval may always be so chosen. For, let k and $l$ be two integers such that

$$(2)\qquad \left\{ \begin{aligned} (p - q)\,(t_2 - t_1) &= k\,\pi \\ (p + q)\,(t_2 - t_1) &= l\,\pi. \end{aligned} \right.$$

Then, by addition and subtraction:

$$\begin{aligned} 2\,p\,(t_2 - t_1) &= (l + k)\,\pi \\ 2\,q\,(t_2 - t_1) &= (l - k)\,\pi. \end{aligned}$$

Hence, if equations (2) are satisfied, $2\,p\,(t_2 - t_1)$ and $2\,q\,(t_2 - t_1)$ will also be integral multiples of $\pi$. But equations (2) can be satisfied if the integers k and $l$ fulfill the condition

$$(3)\qquad \frac{p - q}{p + q} = \frac{k}{l}.$$

When $(p - q)$ and $(p + q)$ are commensurable numbers, condition (3) is obviously fulfilled. But even when $(p - q)$ and $(p + q)$ are not commensurable, the required integers can be found to an approximation as close as desired, for it is known that any fraction can be expressed as the ratio of two integers to any required degree of approximation. It follows that the time interval $(t_2 - t_1)$ can always be so chosen as to make the arguments of all the sines in equation (1) integral multiples of $\pi$, and hence so as to make the bracketed term vanish. There remains therefore

$$W_m = \frac{1}{4} L (A^2 + B^2) = \frac{1}{2} L I_1^2 + \frac{1}{2} L I_2^2,$$

if effective values $I_1$, $I_2$ are used instead of amplitudes A, B. Hence it is established with perfect rigor that the average value of the energy due to two currents present in a circuit is equal to the sum of the average values due to each current separately. The only possible ground for criticism of the theory is thereby removed.

Turning now to the other points raised, Mr. Cohen tries first to overthrow the validity of the theory by quoting two sentences from different parts of the article and declaring them to be contradictory; a little more careful consideration of the context shows, however, that there is no contradiction whatever. The first statement quoted by Mr. Cohen means that merely mixing two currents of different frequencies and different amplitudes in a circuit does *not* increase the energy component of the weaker one, since the average energy present is equal to the sum of the individual average energies. The theories of Mr. Cohen and of Mr. Hogan on the other hand, assert that by merely mixing a weak current with a strong one of different frequency, an amplification proportional to the ratio of the amplitudes is produced. The first of my statements quoted by Mr. Cohen flatly denies the existence of *such* amplification, but it does not deny that amplification in other ways; i. e., by properly utilizing the effect of the two currents in other circuits, is possible. In fact, immediately following this statement in the article will be found the sentence beginning, "Before considering *the true mode of amplification in such receivers,* it is necessary . . ." etc. The first quoted statement therefore does not say that *all* amplification is impossible, but only that such amplification as is called for by the older theories is impossible. Later on, I show that a limited true amplification may indeed exist. It is clear, therefore, that there is no contradiction whatever in the two statements quoted, and that Mr. Cohen's first criticism of the theory is therefore wholly without foundation.

Mr. Cohen's next objection is to the method I employed in proving that $W_m = \frac{1}{2} L (I_1^2 + I_2^2)$, altho he accepts the results. I have already removed this ground for criticism, but granting a lack of rigor in the first proof, the result is correct, as Mr. Cohen admits, and a faulty derivation thereof would not affect the validity of the body of the theory. Mr. Cohen cannot escape from the consequences of this result, therefore, by merely calling attention to a lack of rigor in the particular derivation employed.

As an additional reason for disregarding the aforesaid result, Mr. Cohen asserts that nothing is proved thereby. But something is most decidedly and very obviously proved thereby; viz., that in seeking for the amplification produced by heterodyne receivers we must look not for the energy of or the amplitudes of all the currents existing in the circuits, but for the energy of or the amplitude of the particular current which produces the signal. Thus, it is very definitely proved that in the best form of the heterodyne receiver (Figure 11 of Mr. Hogan's paper), for example, the only criterion for the amplification is the amplitude of the audible frequency current in the telephone receivers.

This conclusion has escaped Mr. Cohen, for he persists in attempting to deduce the amplification of the heterodyne by considering all the currents, instead of the useful current alone. Leaving this point aside, however, the proof which he offers that the amplification is proportional to the ratio of the amplitudes is easily shown to be fallacious. Mr. Cohen throws the expression for the sum of the two currents into the form:

$$I = \sqrt{A^2 + B^2 + 2\,A\,B \cos \beta t}\,\sin(p\,t + \phi).$$

where the angle $\phi$ is given by

$$\phi = \tan^{-1} \frac{B \sin \beta t}{A + B \cos \beta t}.$$

This expression for I is in the form of a single current of variable amplitude and variable phase. Mr. Cohen infers from this, presumably because the term 2 A B occurs under the radical, that there exists an amplitude variation proportional to 2 A B. Now, it is perfectly obvious that the maximum value which the radical can have is

$$\sqrt{A^2 + B^2 + 2\,A\,B} = A + B,$$

which value it takes when $\beta t = 2\,n\,\pi$, where n is any positive integer; also that the minimum value which the radical can have is

$$\sqrt{A^2 + B^2 - 2\,A\,B} = A - B,$$

which value it takes when $\beta t = (2\,n + 1)\,\pi$, where again n is any positive integer. The variation in amplitude is therefore

$$A + B - (A - B) = 2\,B,$$

and is not 2 A B. That the amplitude variation is 2 B follows directly from physical considerations, moreover, and from graphical construction; and no amount of mathematical transformation can change the physical facts. Mr. Cohen's proof that there exists an amplitude variation proportional to 2 A B, and hence that the amplification produced is proportional to $\frac{2A}{B}$ is therefore fallacious.

In the article, I have shown how the amplitude of the useful current in the telephone receivers may be arrived at by breaking up the currents into a "beat" component and a "sustained" component. By this process I have shown that the use of the local current makes it possible to double the amplitude of the audio frequency telephone current, and hence that the maximum true amplification is four. Additional amplification due to increase of efficiency does indeed exist, but that is not *true* amplification. Mr. Cohen ignores this part of the paper entirely. Unless he can show, however, that the reasoning employed here is fallacious, the conclusion that the maximum amplification of the best heterodyne receiver is four would still stand, even if all of Mr. Cohen's other criticisms were justifiable.

The rigorous theory of the electromagnetic or electrostatic telephone receiver is difficult, because it involves differential equations with variable coefficients, and such equations must be solved by complicated mathematical processes. I have not carried the theory farther than to show that *so long as the efficiency is very small*, the useful force acting on the diafram is proportional to the permanent magnetic flux in the electromagnetic telephone receiver, to the constant impressed force in the ordinary electrostatic receiver, and to the amplitude of the local current in the electrostatic form of the heterodyne receiver. In the first two of these receivers, no one will question that the constant force or the constant flux can not put additional energy into the system, and that therefore the amplification produced is due entirely to increased efficiency. From the closeness of the analogy between these two receivers and the electrostatic form of the heterodyne, and from the fact that the sensitiveness of the latter is, at best, no better than that of an ordinary detector (See Mr. Hogan's paper, page 86), it is clear that the amplification produced by the electrostatic form of the heterodyne must be due also to increased efficiency; if any true amplification exists, it is negligibly small. The fact is very significant that an amplification of 1,000 times can be produced by the electrostatic form of the heterodyne (according to Mr. Cohen), without increasing the sensitiveness of the arrangement beyond that of the ordinary detector (according to Mr. Hogan).

It is hardly worth while going further into the discussion of this form of heterodyne receiver, because of its admittedly low efficiency; but in reply to Mr. Cohen's remark concerning the e. m. f. due to the "infinitesimal motion of the condenser plate," I wish to remind him that in any device for converting electrical into mechanical energy, the energy w so converted is given by

$$w = \int e\,i\,dt,$$

where e is the counter e. m. f. due to mechanical motion. If this motion is infinitesimal, then the counter e. m. f. due to it is also infinitesimal, the electrical energy converted into mechanical energy is also infinitesimal, and the efficiency is likewise infinitesimal. And even after the converted energy has been multiplied 1,000 fold, it is still infinitesimal!

Mr. Cohen states that my theory is contradicted by the experimental facts, and cites amplifications of 1,000 times in the case of the electrostatic form, and of 20 to 50 times in the best form of the heterodyne. He gives no estimate, however, as to the extent to which the amplifications are due to increase of efficiency, and to true amplification. An approximate estimate may be made, however, with very little difficulty. In the electrostatic form, there is no evidence that the amplification is due to anything other than increased efficiency. Take away the permanent magnet of an ordinary telephone receiver, for example, and measure the volume of sound produced by a given e. m. f.; replace the permanent magnet and measure the sound again; I dare say that an amplification of several thousand times will be observed, but it will be due entirely to increased efficiency. The state of affairs in the electrostatic form of the heterodyne I have shown to be almost exactly similar. As regards the amplifications of 20 to 50 times observed in the most efficient form of the heterodyne, it should be borne in mind that adjusting the beat frequency to the natural frequency of the telephone diafram may easily produce an amplification of 6 to 10 times, and that adjusting the amplitude of the local current so as to work the crystal on the best part of its characteristic may produce an additional amplification of 2 or 3 times. These latter amplifications are due entirely to increased efficiency, so that an observed amplification of 50 times can readily arise from an apparent amplification of 25 times, say, and a true amplification of 2 times. There is therefore no experimental evidence that a *true* amplification greater than 4 times has ever been produced by a heterodyne receiver.

**Louis Cohen:** In his reply to my criticism, Mr. Liebowitz has gone to some trouble to work out a more elaborate proof to show that the average energy $W_m$ due to two currents of different frequencies present in a circuit is

$$W_m = \frac{1}{2} L (I_1^2 + I_2^2).$$

I can not see that this throws any further light on the subject; there was no argument about this point. I have stated in my first remarks, in discussing the paper, that this is perfectly true; but I maintained that it did not prove anything and gave my reasons for it. He evidently missed that point of my argument.

He further goes on to show that the expression I gave for the instantaneous value of the current

$$I = \sqrt{A^2 + B^2 + 2\,A\,B \cos \beta t}$$

leads to the conclusion that the variation in an amplitude is 2B and not 2AB. He makes the error in implicitly assuming that the effect on the detector is proportional to the first power, which is of course incorrect. The effect on the detector is proportional to the square of the current, hence it follows from the expression I gave, that the maximum value of the effect on the detector is $(A^2 + B^2 + 2\,A\,B)$, and the minimum value is $(A^2 + B^2 - 2\,A\,B)$, and the difference is $4\,A\,B$. The variation is therefore proportional to AB as I have stated before.

Finally, Mr. Liebowitz argues that the amplification may be due to an increase in sensitiveness of the telephone by adjusting the beat frequency to the natural frequency of the telephone diafram. To show that this argument is not valid, it is only necessary to call attention to the fact that the same amplification is obtained whatever the beat frequency within the range of audibility.

**Benjamin Liebowitz:** In his first criticism of my paper, Mr. Cohen, while admitting that $W_m = \frac{1}{2} L (I_1^2 + I_2^2)$, goes on to say that "The error he makes is in averaging the energy of each component separately. . . . " In other words, Mr. Cohen took the view that altho the result was correct, a lack of rigor in proving it invalidated the rest of the argument. Rather than dwell on the illogical nature of this view, I met the criticism by giving a rigorous proof.

Apparently Mr. Cohen still believes that this result proves nothing. I have already emphasized that something *is* proved thereby; namely, that the amplitude of the audible frequency current in the telephone receivers is the *only* criterion for amplification. This fact would seem to be beyond question, but Mr. Cohen refuses to admit it. Instead, he speaks of "the effect on the detector," as tho "the effect on the detector" were in some way connected with the operator's auditory nerve and thereby directly involved in the reception of signals. It is the conversion of electrical energy associated with the audible frequency telephone current into mechanical energy in the receiver, and not "the effect on the detector" which produces the signal. Mr. Cohen's discussion of "the effect on the detector" is meaningless.

It might be pointed out, furthermore, that the two versions of his theory presented by Mr. Cohen are contradictory; in the first version it is the first power of the amplitude, in the second it is the square of the amplitude, whose variation is determinative of the amplification. But further discussion on this point is unnecessary.

In concluding his remarks, Mr. Cohen states that "the same amplification is obtained whatever the beat frequency within the range of audibility." Doubtless he is referring here to the case where the received note is far from musical, so that a change in the beat frequency produces very little change in the pitch. If he refers to the case where the received note is musical, how can Mr. Cohen reconcile his remark with the well-known fact that the efficiency of the telephone receiver is very much greater at the frequency of mechanical resonance than at frequencies differing widely therefrom. And furthermore, if the received note is far from musical, does Mr. Cohen include the circumstance as one of the *favorable* ones under which amplifications of 20 to 50 times are obtained? Rather than discuss this point further, I shall conclude by quoting from Dr. Austin's paper on "Quantitative Experiments in Radio-Telegraphic Transmission" ("Bulletin of Bureau of Standards," April 1, 1914, page 84): "*The reports indicate that the heterodyne is somewhat more sensitive than the slipping contact, but that the difference is not very great.*" (The italics are mine.)

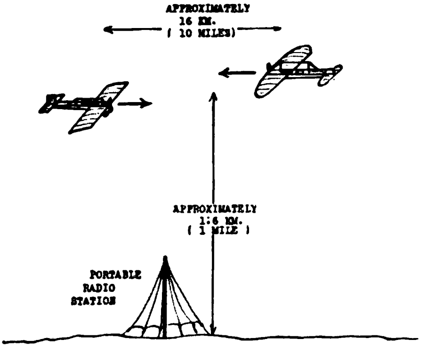
**(The discussion on this paper is left open till the date of publication of the September issue of the PROCEEDINGS. Members desiring to contribute thereto are invited to do so.—EDITOR.)**

# RADIO COMMUNICATION WITH AEROPLANES

(The following is a portion of a communication to the Editor by Mr. Robert A. Fliess. It is of interest in that it outlines one of the important problems of communication with aeroplanes.)

The complete solution of the problem of directing an aeroplane by radio transmission will have arrived only when the pilot or observer on the aeroplane is enabled to maintain constant intercommunication with the headquarters radio station, even when at great heights and at considerable distances from that station.

So far it has been found difficult to send radio messages with any degree of certainty more than approximately 50 kilometers (30 miles); while the reception of messages by the observer or pilot has been limited to even shorter distances.



The accompanying diagram shows a problem suggested by Colonel Mortimer Delano for solution by the Institute membership. It points out a definite line of experiments. Briefly, the problem may be stated thus: An aeroplane is to be enabled to signal to another one, while both are in full flight; and it also is to be in communication with a headquarters radio station. The limitation of the problem is that the antenna is not to be dropped from the aeroplane, tho aerials may be carried on the planes.