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CONTENTS

The Institute of Radio Engineers announces with regret the death of

Jesse Edgar Baker.

Mr. Baker was born at Springfield, Illinois, on February 14, 1888. He was educated at the School of Arts and Crafts, at Berkeley, California, and then entered in June, 1913, the McCloud School of Art and Design in Los Angeles, California.

Having been a student of electricity for some time, he enlisted on January 1st, 1917, in the United States Naval Reserve Force, and, because of special qualifications, was sent to one of the Naval Radio Schools. He was thereafter assigned as a Radio Electrician to several of the United States superdreadnaughts.

While in service he contracted scarlet fever, and died on March 6th, 1918. He was accorded a military burial at Inglewood Park Cemetery, Los Angeles, on March 20th. A delegation of sailors from San Pedro acted as pallbearers.

Mr. Baker was an Associate member of The Institute of Radio Engineers since September 9th, 1917.

SOME ASPECTS OF RADIO TELEPHONY IN JAPAN*

By

EITARO YOKOYAMA

(ENGINEER OF THE MINISTRY OF COMMUNICATIONS, TOKIO, JAPAN)

OUTLINE OF THE EVOLUTION OF RADIO TELEPHONY IN JAPAN

The investigation of radio telegraphy in Japan has been carried on for some twenty years, since the year following the basic invention of Marconi. Great advances both in theory and practice have been made. However, systematic research in radio telephony was not begun until 1906 at the Electro-technical Laboratory of the Ministry of Communications, under the direction of Professor Dr. Osuke Asano, Ex-Director of the Laboratory. Under his direction, and more lately under the direction of Dr. Morisaburo Tonegawa, present Director, Dr. Wichi Torikata, Chief of the Radio Section, and his staff have devoted themselves to exhaustive researches continuing up to the present.

The primary object of the investigations carried on in the Laboratory at first was to obtain steady continuous electrical oscillations by any simple means, and various detailed researches were initiated. Nearly all the devices already described in publications, (including the Poulsen arc, and the Lepel and new Telefunken gaps) were tried; but they led to no useful results in the Laboratory and had no practical success. Good articulation was not obtained nor that simplicity and compactness of the apparatus which are of vital importance in devices intended for public use. The mercury vapor gap, and revolving gaps of special design, were also tried, but in vain. In 1912, after the lapse of six years, the staff of the Laboratory finally devised a new and special kind of oscillation gap which turned out to be excellent for the purpose and was, therefore, patented in Japan and several other countries. The title "T-Y-K" was given to the system of radiotelephony involving the use of these special oscillation gaps as its main feature, the initials

*Received by the Editor, August 16, 1917.

of the three inventors' names being thereby represented.† The system has been of practical utility in Japan.

Research in radio telephony is still being continued in the Laboratory in the search for perfection. There was recently invented a kind of rarefied gas discharger, which was developed thru the co-operation of Mr. Noboru Marumo and the writer. This will be described at some length below.

Meanwhile, Mr. Tsunetaro Kujirai, Assistant-Professor in the Engineering College, Tokio Imperial University, made some valuable contributions to the field. He developed a kind of arc generator in 1910, and a static frequency transformer in 1915, both of which are suitable for radio telephonic purpose and, therefore, were patented in Japan. These devices will also be considered below.

In addition to the above-mentioned gentlemen, Mr. Hidetsugu Yagi, Professor in the College of Engineering, Tohoku Imperial University, Sendai, Mr. Mitsuru Sayeki, Radio Engineer of the Ministry of Communications, and other radio engineers in the Japanese Navy and Army have also rendered much service in furthering progress of the work.

"T-Y-K" System of Radio Telephony

This is a spark system of radio telephony, and intended especially for short distance communication service, the essential features being compactness, simplicity, and cheapness of the apparatus.

The utility of this radiophone system has been sufficiently proved by its practical employment on both land and sea. The system is now in daily commercial use at the three land stations in the Bay of Ise, which is located in the central portion of the coast of Japan. Tho the distance between the farthest two of the above stations is merely eight miles (13 km.), experimental communication was established over a distance of more than 70 miles (110 km.) between shore and ship.

A wall-set type of the apparatus is shown in Figure 1. It was evolved by the joint effort of Messrs. W. Torikata, Masajiro Kitamura and the writer, and is now used in Japan.

Many publications have already been made relative to the system, certain features of this and a further description will not be necessary in this paper.

†(The inventors being Messrs. Wichi Torikata, Eitaro Yokoyama, and Masajiro Kitamura.—Editor.)

FIGURE 1—A Wall-set Type of "T-Y-K" Radio Telephone Apparatus

A Rarefied Gas Discharger—Evolution of the Discharger

The investigation was begun with a glass bulb discharger containing rarefied air as shown in Figure 2, which was inserted in the position of discharger in an ordinary oscillation producing

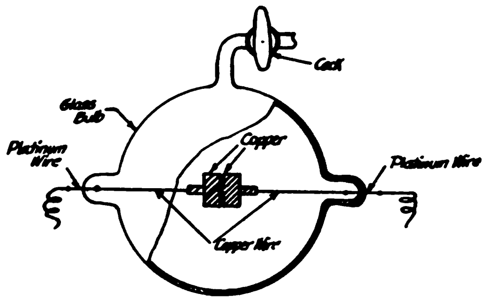


Figure 2—First Form of the Rarefied Air Discharger

circuit as shown in Figure 3. In Figure 2, the glass bulb was of spherical form with a diameter of about 10 cm. (4 inches), and the electrodes made of copper, flat cylindrical in shape, 12 mm. (0.3 inch) in diameter, facing each other wth a clearance as small as a fraction of a millimeter. In Figure 3, *G* is a direct

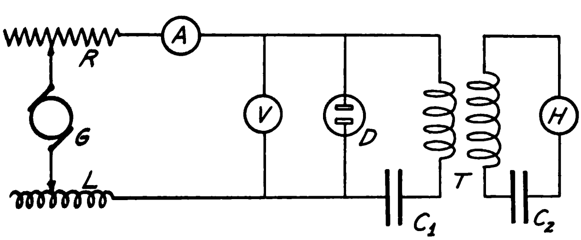


Figure 3—Oscillation Producer with Rarefied Air Discharger

current generator of 500 volts, *R* a resistance, *L* an inductance, *A* a milliammeter, *V* a static voltmeter, *D* a rarefied air discharger, C_1 a condenser in the primary oscillatory circuit, C_2 the same in the secondary, *T* an oscillation transformer and *H* an ammeter for radio frequency current. When the system was in adjustment and the degree of rarefaction of the bulb properly adjusted, the generation of oscillation current in the secondary was indicated by the deflection of ammeter H; but it was noticed that there was objectionable irregularity in the discharge thru the gap and consequently in the oscillation current in the secondary, and, furthermore, the oscillations died out after a couple of seconds. When the test was repeated with an aluminum electrode in place of one of the copper ones, the other parts of the bulb remaining the same, it was found that the discharge was not only very steady but also that the oscillations lasted much longer. As the discharge was smoother than any obtained previously in a usual spark transmitter, the research was pressed in many details for the purpose of utilizing the new discharger as a source for radio telephone transmission or radio frequency measurements.

It was, however, noticed that the oscillations would not last more than a few minutes with a bulb of the above construction. In the test a copper electrode was used as anode and aluminum as cathode. If the polarity were changed, it could be seen distinctly that the discharge became irregular and the oscillations faded away very quickly.

It was found that when the discharge was continued until the oscillations in the secondary died away entirely and the bulb was then allowed to cool for some time, (the power source being disconnected) and then the oscillations were started again, strong oscillations took place in the secondary as steadily and of the same power as the first time. This led the investigators to the theory that the temperature rise in the electrodes might play an important part in causing the decay of oscillating current. Another bulb with large metallic stems attached to the electrodes was then made as shown in Figure 4. Besides this, the bulb was so constructed that the electrodes could be easily replaced which enabled the investigators to make tests with several different forms and materials of electrodes. In tests with this bulb connected in the circuit of Figure 3, there were obtained not merely a steady discharge in the gap and smooth oscillations in the secondary, but also a remarkable increase in the duration of the oscillations, for example, as much as thirty or forty minutes. After further investigations dealing with the construction of the bulb and the shape of the electrodes, the investigators developed a rarefied air tube of the form shown in Figure 5, with which they succeeded in producing exceedingly steady currents in the secondary for hours continuously.

The tube of Figure 5 is suitable only for small powers, for example, for 100 watts. There was, therefore, developed another form of discharger suitable for fairly large power, which latter form is shown in Figure 6. The latter has a thick-walled metallic case in place of the glass bulb in the former, the interior construction remaining almost the same.

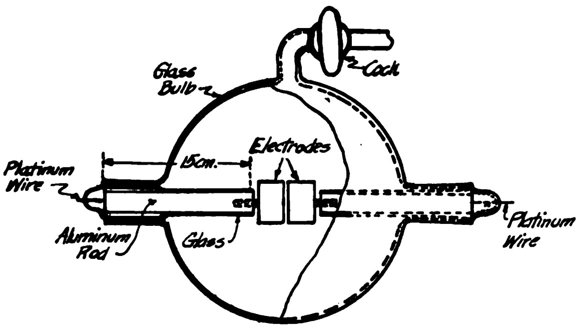


FIGURE 4—Second Form of Rarefied Air Discharger

The dischargers, as finally constructed, were used in a radio telephone transmitter circuit which was the same as the circuit of Figure 3 except that the secondary closed circuit was replaced by a real antenna, and a very steady and pure wave, suitable

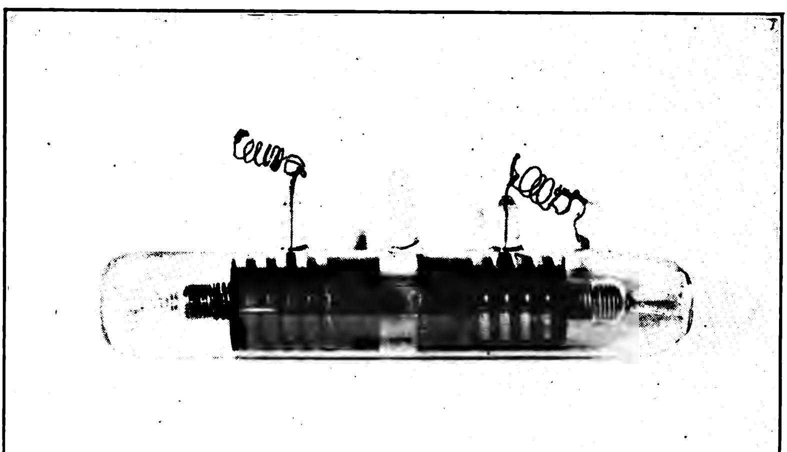


FIGURE 5—Latest Form of Rarefied Air Discharger for Small Power

for radiophone use was obtained in the antenna circuit with a very close coupling such as about 60 per cent. in the oscillation transformer. This indicated a good quenching action in the discharger.

FIGURE 6—A Form of Rarefied Air Discharger for Larger Powers

The above descriptions apply to a discharger having an atmosphere of rarefied air, but the kind of atmosphere used has a perceptible influence on its operation and life. For instance, the experiments indicated that ammonia gas was very effective, carbonic acid gas fairly good, while alcohol, ether, and benzene in the state of vapor were all useless.

It is very interesting to note that Professor R. A. Fessenden[1] and Professor Max Wien[2] made similar investigations with vacuum dischargers.

ARRANGEMENTS FOR THE TESTS

The writer desires to describe briefly the results of the tests dealing with the influence of air pressure in the discharger, length

[1] United Kingdom Patent, Number 28,647, 1907.
[2] "Jahr. der draht. Telegraphie u. Telephonie," volume 4, page 135.

of the discharge gap, dimensions, shape and materials of the electrodes, and supply voltage to discharger, and the effect of these on its life. Thruout all the tests, there was almost always used the arrangement shown in Figure 3.

To exhaust the dischargers a Gaede rotary pump was used as shown in Figure 7, which pump is claimed to reduce 6 liters of air from atmospheric pressure to 0.006 mm. of mercury in 15 minutes. For a vacuum gauge a manometer was used as shown in Figure 8 which enabled measuring roughly any pressure between a little below 1 mm. and 220 mm. of mercury.

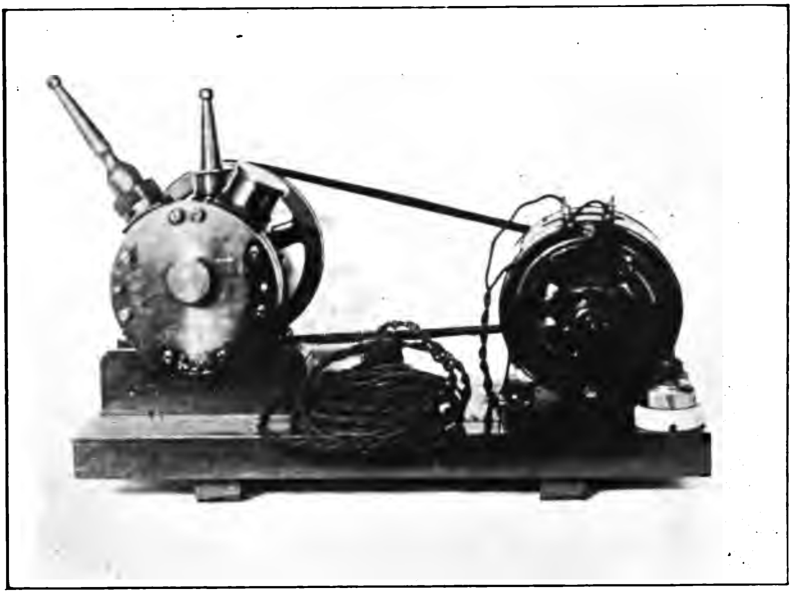


FIGURE 7—Gaede Rotary Pump Used in the Tests

In the tests the wave length was adjusted at about 2,000 meters thruout, with a condenser of capacity of about 0.065 microfarad in the primary and about 0.02 microfarad in the secondary. As a final test, however, the result was confirmed with a circuit of some 500 meters wave length, using a condenser of rather small capacity in the primary and a real antenna in the secondary.

Influence of Air Pressure on the Discharge

The discharge conditions in the rarefied air discharger largely depend on the air pressure. A change in the type of discharge was observed with variation of pressure. Copper was used as the positive electrode of the discharger and aluminum as the negative, the clearance between being about half a millimeter

FIGURE 8—Vacuum Gauge Used in the Tests

(0.02 inch). At the center of discharge surface of the latter a pin-hole was bored, which made the discharge steadier. The voltage of the direct current power source was kept at nearly 440 thru the test.

The observations under these condition are summarized as follows:—

(1) On reducing the air pressure gradually, beginning with ordinary atmospheric pressure, it was noticed that the discharge suddenly bridged over between the electrode surfaces at a pressure somewhere near 70 mm. of mercury.

(2) Within the range from about 70 mm. down to about 40 mm.,

there will be quiet discharge in a straight line between the centers of the electrode surfaces. The discharge is not very steady tho the oscillations produced thereby in the secondary are apparently quite smooth.

(3) On further reducing the pressure to below 40 mm., the discharge comes to have an entirely different appearance. The discharge, which before was in the form of a single straight line, splits into many lines terminating over the whole surfaces of the electrodes like a brush, while sometimes it changes into the form of a glow discharge, these changes depending on the conditions in the oscillation circuit. In this case, the terminal voltage at the gap has its smallest value, and the oscillations in the secondary are very weak and irregular.

(4) At pressures between about 10 mm. and 2 mm., the discharge is again gathered into a single straight line, just as in case (2). In this case, the discharge is, however, remarkably steadier and accordingly the oscillations are also steadier and smoother.

(5) Below the above limit of pressure, the discharge tends to change into a diffused glow, and the oscillations rapidly die away.

It was noticed that good oscillations are produced only when the discharge is in stages (2) and (4), and that stage (4) is especially excellent for utilization as a power source in radio telephony.

Influence of Gap Clearance upon Discharge

Descriptions have been given above of the discharge phenomena, and the range of best pressure for the production of steady oscillations has been considered, but only in the case of a certain definite gap clearance namely, 0.5 mm. To investigate the relation between the gap clearance and the corresponding most desirable pressure, a series of tests were made, the gap clearance being varied from 2 cm. (0.8 inch) down to 0.1 mm. (0.04 inch), The results of these tests are shown in Figure 9. This figure shows that, in the case of short gap clearances, the useful range of air pressure is rather wide and extends to low vacua, while for longer gap clearances, it is closely limited to higher vacua. Furthermore, there is no abrupt change between stages (2) and (4) for clearances longer than about 2 mm. (0.08 inch), stage (3) disappearing entirely.

To obtain data on the best operating conditions, the primary supply current, the terminal voltage at the gap, and the secondary oscillation current were simultaneously measured for various gap clearances with the corresponding most suitable air pressure, the supply voltage being kept constant. The results of

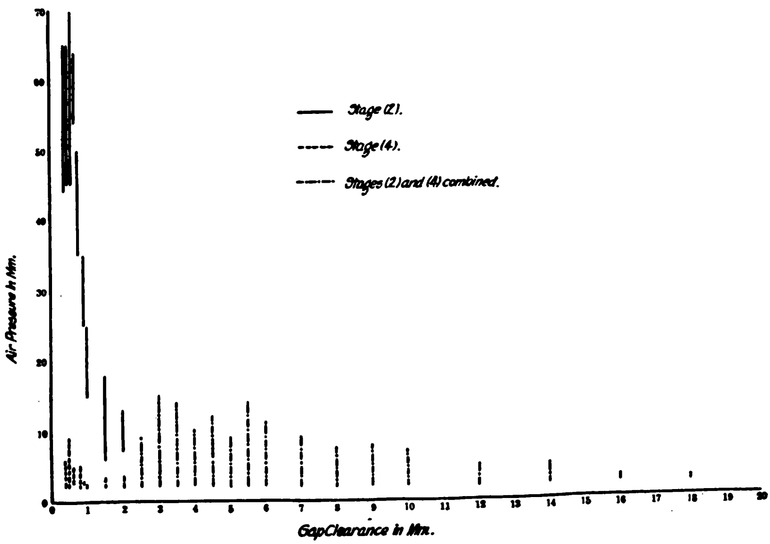


FIGURE 9—Relation Between Gap Clearance and Most Suitable Pressure

the tests in stages (2) and (4) are shown respectively in Figures 10 and 11. The data of the measurements for gap clearances longer than about 2 mm. (0.08 inch), (where the stages (2) and (4) cannot be differentiated), are all included in the curves of Figure 11. These two series of curves show that the increase in gap clearance decreases both the primary supply current and the secondary oscillation current, and increases the gap terminal voltage. The conclusion is, therefore, that it is most advantageous to use as short as possible a gap clearance. However, it must be remembered that there will be a lower limit to the shortest practical gap clearance because of other causes, such as heating of the electrodes which may melt them together in a very short gap.

Influence of Dimensions of Electrodes on Discharge

As stated above, the time of continuance of discharge in the gap (and accordingly the duration of the secondary oscillation current), depends largely on the dimensions of the discharger electrodes. The dischargers of Figures 2, 4, and 5 were compared in this connection, and the results are shown in Figure 12.

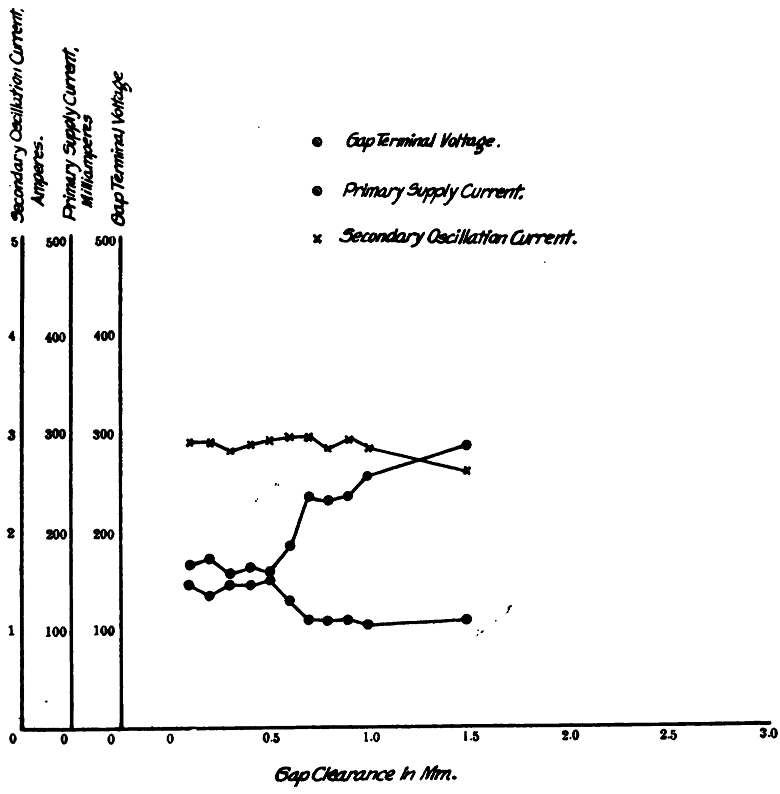


Figure 10—Relation Between Gap Clearance and Supply Current, Gap Terminal Voltage, and Secondary Oscillation Current. Stage (2)

With the discharger of Figure 2, the discharge conditions in the gap became highly unfavorable and the secondary oscillations remarkably feeble after only some thirty seconds of operation. Using the discharger of Figure 4, the time of continuance of the discharge and of the secondary oscillations was, on the contrary, prolonged as long as some thirty minutes. With the discharge tube in Figure 5, the operating condition of the discharger was still very good after two hour's continuous use.

As the supply voltage, the supply current, and the capacity of condensers, etc., were not exactly the same in each case shown by the curves of Figure 12, the magnitude of the secondary oscillation current cannot be fairly compared from these curves.

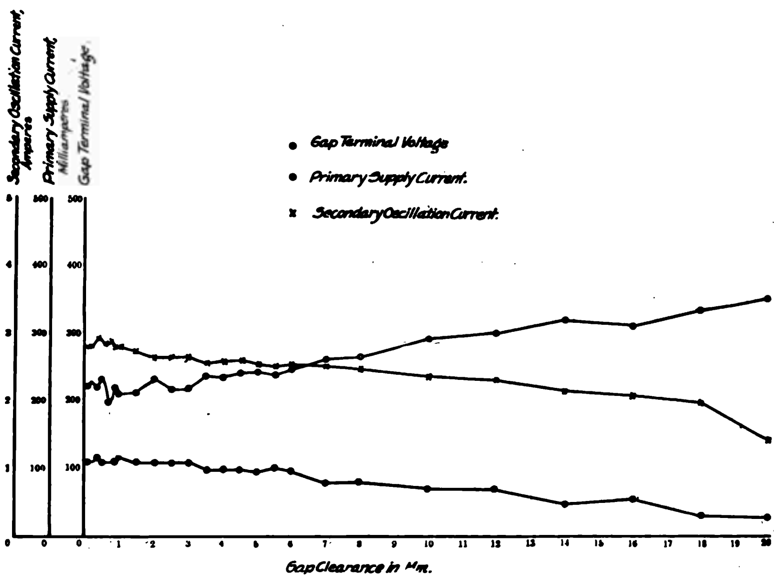


FIGURE 11—Relation Between Gap Clearance and Supply Current, Gap Terminal Voltage, and Secondary Oscillation Current. Stage (4)

In addition, since sufficient precautions were not taken as regards the perfect sealing of the dischargers (there being made for temporary experimental use), the air pressure within probably changed in the course of the test. Otherwise the tube of Figure 5 should have lasted far longer.

INFLUENCE OF SHAPE OF ELECTRODES ON DISCHARGE

At first, there were used electrodes of the form of Figure 13 (a). It was found that the discharge was not very regular and accordingly the oscillation produced not very steady. On attempting to use the electrodes having the form of Figure 13 (b), which were exactly the same except with a pin-hole at the center of their respective discharging surface, it was noticed that discharge and oscillation were remarkably improved. Comparing these two kinds of dischargers as the oscillation generator for radiophone work, the latter was found very satisfactory, while the former gave objectionable noise in the receiving telephone because of the irregularity of the discharge. It being

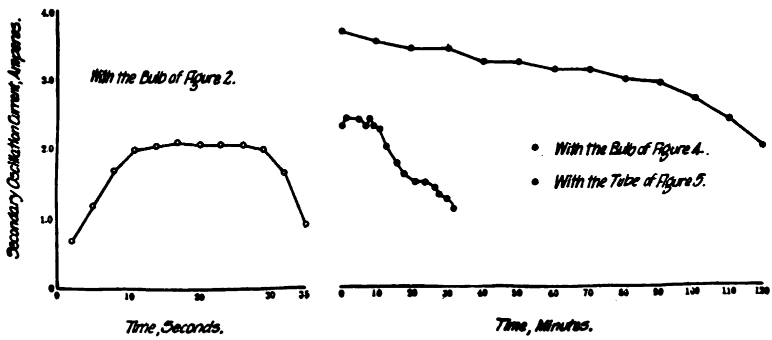


FIGURE 12—Effect of Dimensions of Electrodes on Time of Continuance of Discharge

evident that the shape of the electrodes had some influence on the nature and duration of the discharge, it was attempted to make a comparison using the four shapes of electrodes shown in Figure 13, all possible combinations of the four kinds being tried. In the test copper was used as anode and aluminum as cathode, the clearance between the electrodes being kept constant at 0.5 mm. (0.02 inch).

The best result was obtained by using the electrodes shown in Figure 13 (b) for both terminals. The discharge was irregular with the electrodes Figures 13 (a) or (d), as either one of the electrodes; this being due to the wondering of the discharge over the electrode surfaces. A discharger with (c) as either one of the electrodes also gave bad results in the long run, probably owing to excessive heating of the points.

INFLUENCE OF ELECTRODE MATERIALS ON DISCHARGE

A discharger with copper electrodes as shown in Figure 2 was used at first. The secondary oscillation current produced in this case was not only very unsteady, but lasted only a few seconds. By replacing the copper anode by an aluminum one, the secondary oscillations were greatly improved as mentioned above. It is interesting to note that, in the ordinary atmosphere, a discharger with copper—copper electrodes gives stronger secondary oscillations than one with copper—aluminum electrodes tho there is a little difficulty in starting the discharge in the former case.

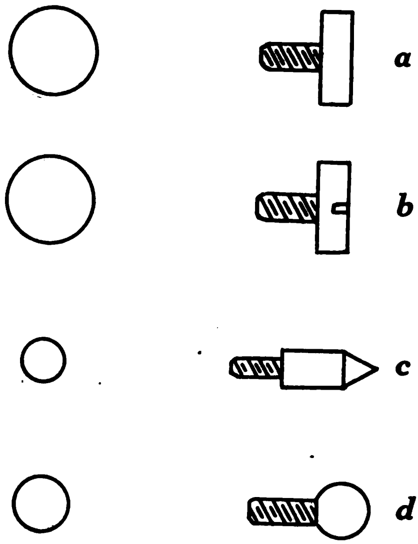


FIGURE 13—Various Forms of Electrodes Tested

A series of tests was made, several different metals being tried as electrodes, and it was finally found that aluminum was the best material for the negative electrode. With aluminum as a negative electrode and various metals as the positive electrode, the measurement of secondary oscillation current and corresponding primary supply current was made, other circuit condition remaining the same. The results of these measurements are plotted in the curves of Figure 14. It is noticeable in the curves that the combination of aluminum-aluminum electrodes gives the poorest result. This is probably due to the effect of polarity, dissimilar electrodes giving better result.

A series of experiments was also made with several different crystals (artificial and natural) such as silicon, carborundum, magnetite, zincite, etc., as electrodes. It is very interesting to find that there were several combinations of electrodes which gave very good results without the use of aluminum as the negative electrode. Generally speaking, the results with crystals were nearly the same as those with metals. However, it is very difficult to shape crystals into suitable forms; and since there is no necessity for the use of crystals as electrodes, the investigations were carried no further in this connection.

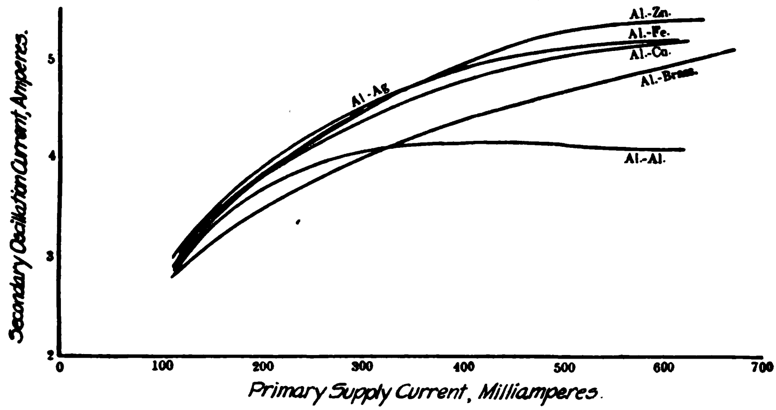


FIGURE 14—Oscillation Current Produced with Dischargers Having Combinations of Various Electrodes

INFLUENCE OF SUPPLY VOLTAGE ON OPERATION OF DISCHARGER

In the ordinary atmosphere a small clearance between electrodes is necessary to obtain continuously a discharge with a voltage less than 500. The use of so small a gap is likely to give rise to frequent short-circuits. In the case of the use of such a gap in air, it is necessary to insert a certain amount of resistance in series in the primary supply circuit in order to prevent short-circuits which might injure some parts of the apparatus because of the passage of abnormally large currents. As described above, the rarefied air discharger works well with a fairly wide clearance of electrodes in comparison with one at atmospheric pressure, tho the shorter the clearance the better operation can be obtained. As the operating conditions prevents short-circuiting, and the lack of air in the discharger doubtless greatly assists regular working, series resistance can be easily dispensed with, and a much lower voltage for the power source is sufficient for perfect functioning of the discharger. The experiment being made of varying the supply voltage from about 320 to 580, it was confirmed that equally good results were obtained at any point in this range of the supply voltage, the gap terminal voltage remaining nearly constant and in the vicinity of from 230 to 240.

Some Considerations Relative to the Life of Discharger

Descriptions have already been given of the proper construction for the rarefied air discharger. In the long run, deviations from good adjustment will occur even in a well constructed discharger, and the secondary oscillation current will gradually fall off. Some consideration will be given here to the causes of this effect and their remedies.

The life of the discharger depends largely upon its working in either stage (2) or (4). It has already been mentioned that stage (4) is much more suitable than stage (2) in a discharger with rarefied air as atmosphere, but the latter stage can be made equally as suitable by introducing a certain kind of gas in the discharger instead of air. Since the discharger working in stage (2) can be, moreover, operated thru a much wider range of air pressure than when in stage (4), satisfactory operation in stage (2) is less effected by variation of pressure. A discharger in stage (2) with a special kind of gas inside is, therefore, as good as, and has a longer life than a discharger in stage (4) with air.

Supposing a discharger to have been well constructed, and with the precautions observed which have been considered under the several headings above, it will still finally reach the end of its life because of the pressure variation of the contained air arising from the following well known causes: (a) imperfect elimination of occluded gas from the metallic bodies, and of water vapor from the surfaces of these metallic bodies and the glass wall; (b) disappearance of gas due to discharge; (c) changes in electrode surfaces due to discharge.

The defect (a) can be gotten rid of, to a certain extent, by submitting the glass and the metallic bodies of the discharger to high temperature when constructed. As regards the defect (b), there seems to be no means whereby it can be perfectly cured, tho it can be regulated by a method similar to that used for the adjustment of the "hardness" of X-ray tubes. As for the defect (c), the use of electrodes constructed as in Figure 13 (b) makes the discharge steady for some time from the beginning, but the irregularity gradually increases in the long run, the smooth straight line discharge altering into a poor discharge in the form of a brush. The discharge could be improved by covering the surface of the electrodes with a thin film of glass or enamel except at the center of discharge surfaces.

Rotary Gap for Radiophone Transmitter[3]

Mr. T. Kujirai has invented a kind of nearly sustained oscillation producer which is suitable for radio telephone transmitters.

His first device was made in public in Japan as early as 1910. The principal part of the apparatus consisted of a metallic or carbon rotary disc, directly driven by an electric motor, as one electrode, this being in light contact with a metallic or carbon brush, as the other electrode; direct current being used as the power source. His sustained oscillation producer was a combination of the rotary gap and an ordinary oscillation circuit in shunt.

The apparatus has been successively improved, until he finally modified it in 1912 producing a form which is more suitable for radio telephone purposes. The latest arrangement consists of a rotary brass disc and an aluminum point, which is shown in Figure 15. The circuit arrangements used by him in connection with his gap are shown in Figure 16.

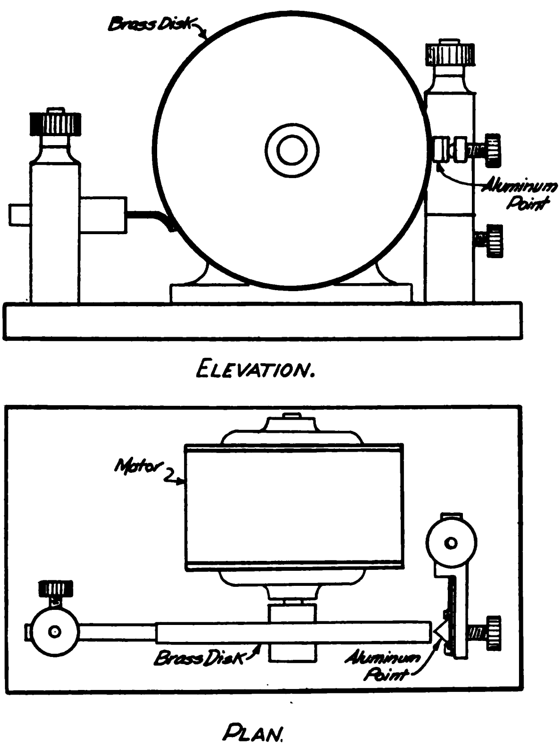


FIGURE 15—T. Kujirai's Rotary Gap

[3] For much of the information here given the writer is indebted to the inventor.

The power is supplied from a 500-volt direct current generator thru a resistance and an inductance to a gap not greater than 0.5 mm. (0.02 inch) in length. The supply current used in his arrangements varied 0.2 to 1.0 ampere, depending on the capacity in the oscillation circuit.

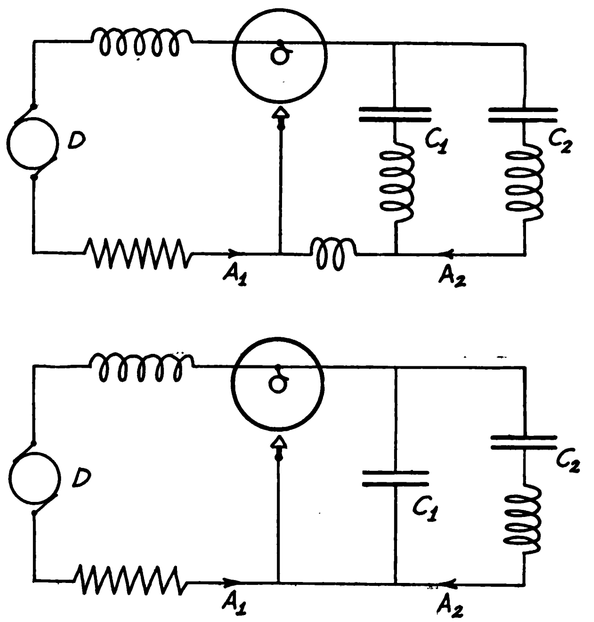


FIGURE 16—Circuit Arrangements for the Rotary Gap

The frequency of oscillation may be varied thru a wide range without affecting the stability of the discharge by varying one of the capacities in the oscillation circuit.

Figures 17 (a) and (b) are sample curves showing the variation of the oscillation current with the supply current.

Using this arrangement, he is said to have succeeded in communicating articulate speech more than 20 miles (32 km.).

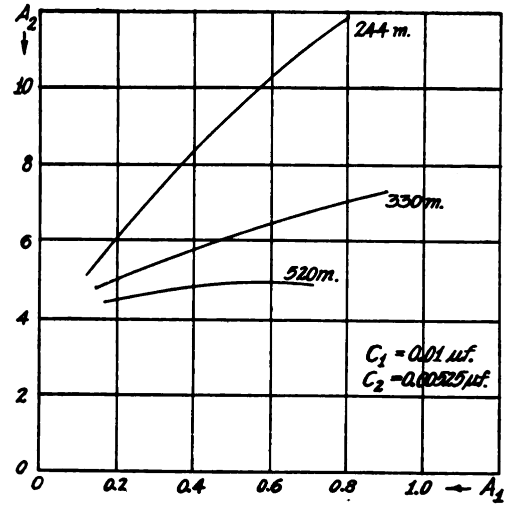


FIGURE 17 (a)—Variation of Oscillation Current with Supply Current

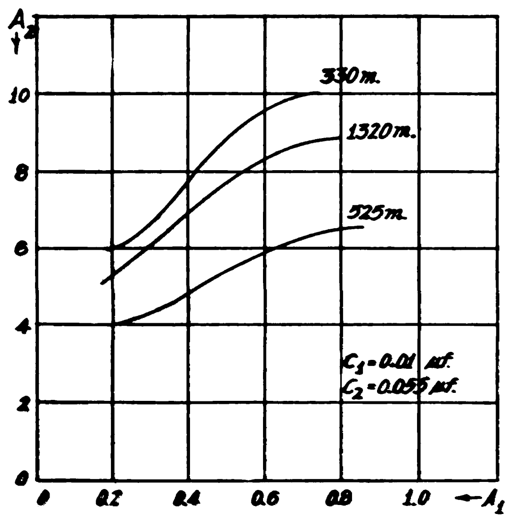


FIGURE 17 (b)—Variation of Oscillation Current with Supply Current

STATIC FREQUENCY TRANSFORMER[4]

Among others[5], Mr. T. Kujirai has invented a method of tripling the frequency of an alternating current in 1915, which

[4] For much of the information here given the writer is indebted to the inventor.

[5] See the paper "Radio Frequency Changers," by A. N. Goldsmith, "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS," March, 1915.

method was found to be very useful, not only in the common electrical engineering field, but also in radio telephony.

This static frequency transformer consists of three elements, two of which have their cores oppositely polarized by direct current thru an inductance X and windings D_A, D_B, Figure 18, while the third element is non-polarized.

The primary (P_A, P_C, P_B) and secondary (S_A, S_C, S_B) windings are respectively connected in series, but the secondary (S_C) of the non-polarized transformer element is connected in opposition to those (S_A, S_B) of the other two polarized elements.

The function of this arrangement is that the induced electromotive forces E_A and E_3 (Figure 19) in the secondary windings of the polarized transformer elements are asymmetrically distorted owing to the magnetic saturation of their iron cores, while the induced e. m. f. E_C in the non-polarized transformer element remains entirely symmetrical but remarkably peaked

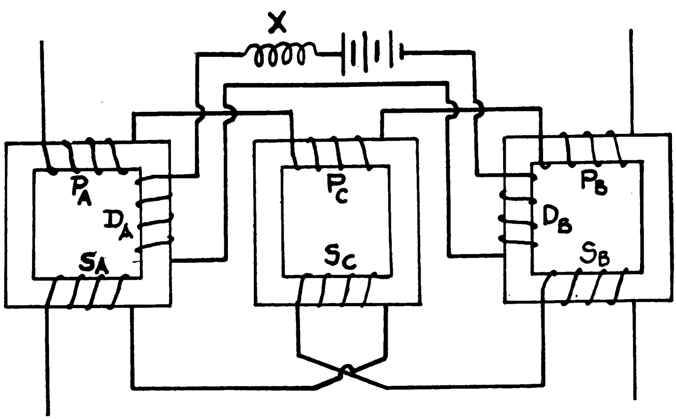


FIGURE 18—Arrangement of the Static Frequency Transformer of T. Kujirai

owing to the low magnetic density in its iron core. The distorted e. m. f.'s being superposed in opposition to the symmetrical one, the resultant of these e. m. f.'s will have such a form as E_z (Figure 19) which has a weak component of the fundamental frequency and a strong third harmonic, as well as higher harmonics.

Figures 20 (a), (b), and (c) are oscillograms showing the constitution of the secondary e. m. f. (a) is an induced e. m. f. in the non-polarised core transformer, (b) the resultant e. m. f. of the two polarised core transformers, and (c) the resultant of the e. m. f.'s of the three transformers. The oscillograms were taken by the inventor at University College, London.

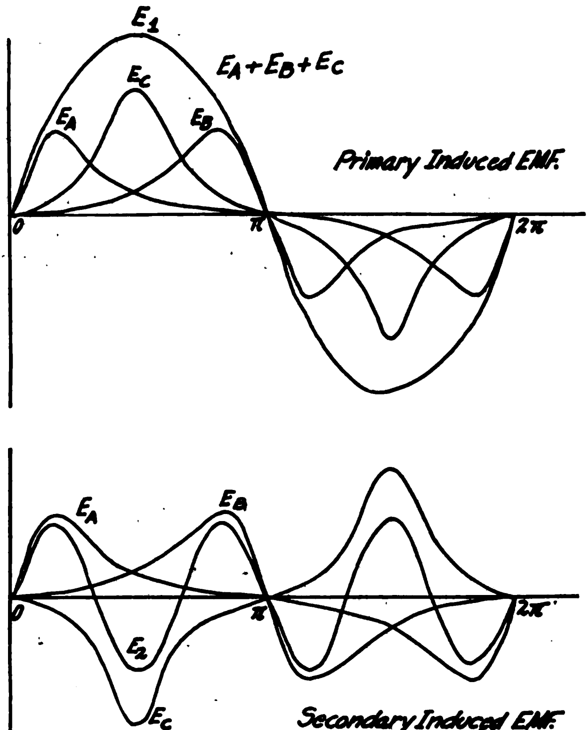


FIGURE 19—Diagram Showing the Principle of Kujirai's Transformer for Tripling Frequency

Figure 21 shows an example displaying the construction of a static frequency transformer which is now being used for experiments in radio telephony in connection with a radio frequency current alternator of the Alexanderson type, in the Electrical Engineering Laboratory of Tokio Imperial University. The principal electrical data of the transformer are as follows:

(a) (b) (c)

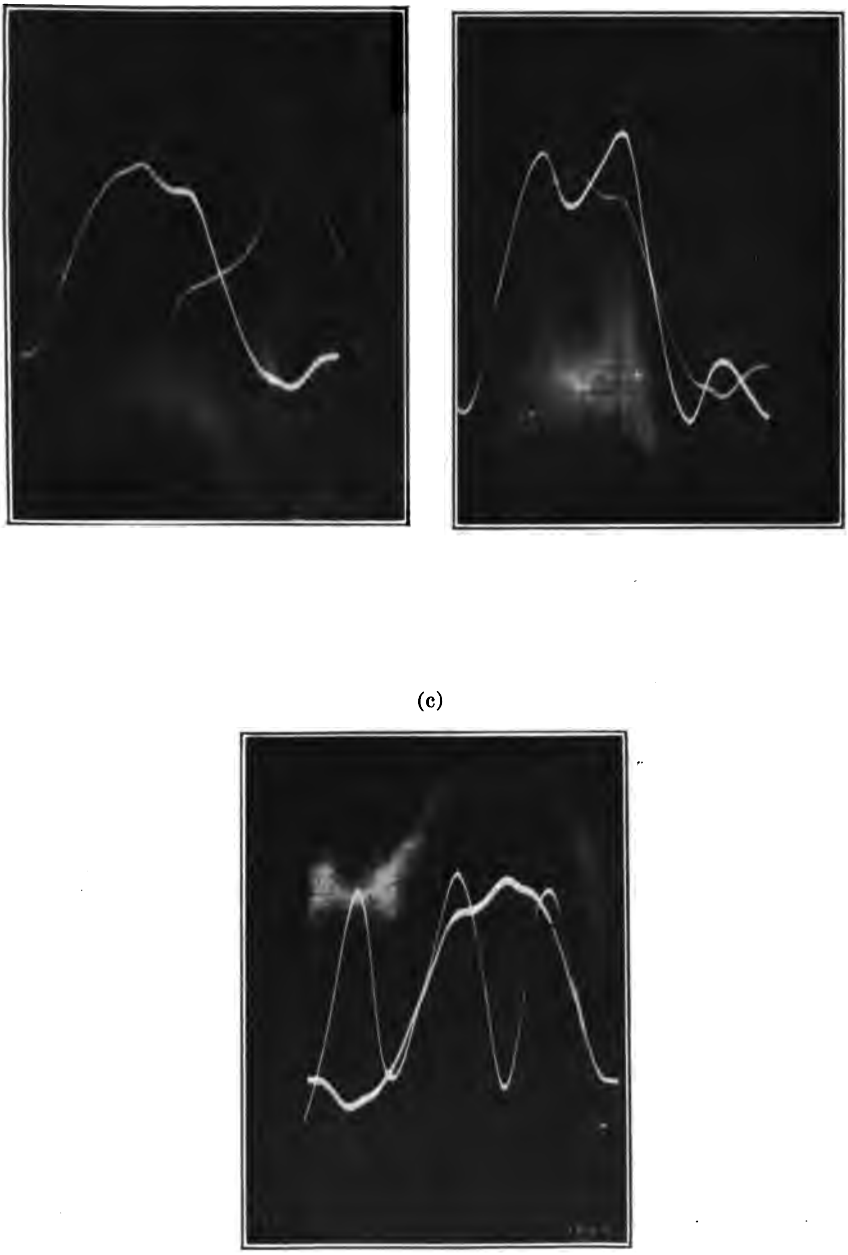


FIGURE 20—Oscillograms Showing the Constitution of Secondary E.M.F. in the Static Frequency Transformer of Kujirai

Primary capacity.......	1.3	k. v. a.
Primary frequency......	40,000	cycles
Secondary frequency....	120,000	"
Primary voltage........	260	volts
Secondary voltage......	120	"
Primary current........	5	amperes
Secondary current......	3	"

New York, August, 1917.

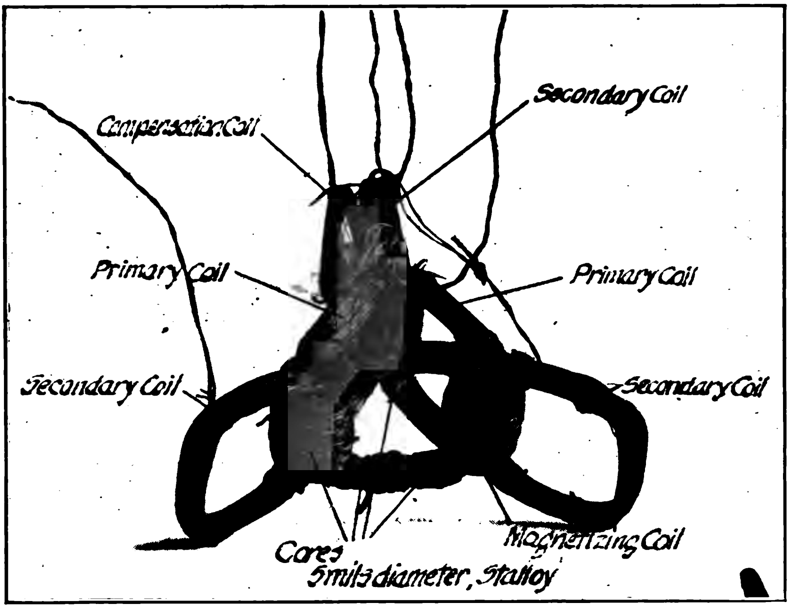


FIGURE 21—Static Transformer, with Case Removed

SUMMARY: After outlining the experiments in radio telephony carried out in Japan since 1906, the "T. Y. K." system is cited. A newer system using, in the most favorable case, copper and aluminum electrodes about 0.2 inch (0.5 mm.) apart in rarefied air or other gas at a pressure of about 40 to 70 mm. as the discharger is then described in great detail. The influence of electrode design, gas pressure, electrode material, and other factors is considered. Means of securing steady and continued operation are given.

Mr. T. Kujirai's earlier system of radio telephony is then described. This involves a direct current, moderate voltage discharger having a rotating brass disc and a fixed aluminum point as electrodes.

A ferromagnetic frequency tripler due to Mr. T. Kujirai is discussed and operating data for its use in conjunction with an Alexanderson radio frequency alternator are given.

A DYNAMIC METHOD FOR DETERMINING THE CHARACTERISTICS OF THREE-ELECTRODE VACUUM TUBES*

By

John M. Miller

(Bureau of Standards, Washington, D. C.)

In three-electrode vacuum tubes, such as the audion or pliotron, we are concerned with two circuits, that between the grid and filament or input circuit and that between the plate and filament or output circuit. The current which flows in the grid circuit is of importance in determining the power input and the detecting action of the tube, but in the use of the tube as a relay or amplifier it is usually negligible and will not be considered in the following.

Characteristic Surface and Curves of the Plate Current

In tubes, with a high vacuum, the value of the plate current, at a given instant, is a function of the values of the plate and grid voltages at the same instant. A surface, called the characteristic surface, is required to represent this function. Langmuir[1] has given the equation

$$i_p = A\ (v_p + k\, v_g)^{3/2}$$

as representing the equation of the characteristic surface. In this equation i_p is the plate current, v_p the plate voltage, v_g the grid voltage, and k is a constant for a given tube construction and is a relative measure of the effects of grid and plate voltages upon the plate current.

In investigating the functioning of the tubes experimentally, it is customary to determine the static characteristic curves of the plate current. These curves are the intersections of the characteristic surface with the plane surfaces v_p = constant, or v_g = constant. The first of these represents the variation of the plate current with the grid voltage when the plate voltage is

* Received by the Editor, May 9, 1918.

[1] I. Langmuir, "Proc. Inst. Radio Engrs.," 3; page 261, 1915.

constant, the second, the variation of plate current with plate voltage with the grid voltage held constant. These characteristic curves are usually obtained by varying in steps the battery voltages applied between the elements of the tube and reading the plate current corresponding to the applied voltages. The slopes of these curves, being the ratio of current to voltage, have the dimensions of a conductance. This method is very slow and inaccurate.

In many cases the operation of a tube takes place about a point in the characteristic surface where the surface is nearly plane. Also when the variations are confined to a small area about the operating point and we are not concerned with distortion, the characteristic surface may be considered to be a plane. Thus the equation of the characteristic surface about an operating point (i_p, v_g, v_p) may be written, as given by Vallauri[2] in the form

$$i_p = a\,v_g + b\,v_p + c$$

where a is the slope of the plate-current grid-voltage characteristic curve passing thru the operating point while b is similarly the slope of the plate-current plate-voltage characteristic. The quantities a and b are fundamental in determining the behavior of a tube as an amplifier and oscillator as has been shown by Vallauri.

Amplification Constant and Internal Resistance

It is, however, more convenient to deal with the quantities $\frac{a}{b}$ and $\frac{1}{b}$. The first of these is the same as k in Langmuir's equation; and, as mentioned before, is a constant for a given tube. It is called by H. J. van der Bijl[3] the amplification constant. The quantity $\frac{1}{b}$ is the internal a. c. resistance of the tube in the plate circuit and will be designated R_i. Its value depends upon the plate and grid voltages and to some extent upon filament temperature.

Assume that the plate circuit is closed thru an impedance Z, which may be the impedance of a pair of telephones in the case of ordinary use or may take the form of a pure resistance in the case of a resistance-coupled amplifier or the primary of a transformer in the case of a transformer-coupled amplifier.

[2] G. Vallauri; "L'Elettrotecnica," IV, 3, page 43, 1917.
[3] Unpublished paper of September 17, 1917.

Assume also that an alternating e. m.f. e_g is impressed between the grid and filament. It then follows that the alternating current in the plate circuit will be the same as that which would flow in a simple a. c. circuit in which the impressed e. m. f. is $k\,e_g$ and which contains a resistance R_t in series with the inserted impedance.[4]

To prove this theorem let us introduce the additional notation

I_p = d. c. component of the plate current.
i = a. c. component of the plate current.
E_g = voltage of grid battery.
e_g = a. c. impressed e. m. f. on grid.
E_p = voltage of plate battery.

and assume an impedance $z=(x+j\,y)$ inserted in the plate circuit which has a d. c. resistance x'.

Then
$$i_p=I_p+i$$
$$v_g=E_g+e_g$$
$$v_p=E_p-I_p x'-i\,(x+j\,y).$$

By Vallauri's equation we have

$$i_p=a\,v_g+b\,v_p+c$$

and substituting in this the values of i_p, v_g, and v_p,

$$I_p+i=a\,(E_g+e_g)+b\,[E_p-I_p x'-i\,(x+j\,y)]+c.$$

But the steady current

$$I_p=aE_g+b\,(E_p-I_p x')+c,$$

hence

$$i=a\,e_g-b\,i\,(x+j\,y)$$

$$\frac{a}{b}e_g=i\frac{1}{b}+i\,(x+j\,y)$$

or

$$k\,e_g=i\,R_t+\,i\,(x+j\,y)$$

This latter is the equation of e.m.f.'s for an impressed e.m.f $k\,e_g$ in a circuit with a resistance R_t in series with an impedance $(x+j\,y)$.

As an example take the case of a resistance-coupled amplifier, in which case an alternating e.m.f. e_g is impressed between the grid and filament of one tube and the e. m. f. across a resistance R in the plate circuit of that tube is introduced into the grid circuit of the next tube. The voltage amplification is the

[4] The author is indebted to Mr. H. H. Beltz of the Bureau of Standards for suggesting this theorem.

ratio of the e. m. f. handed on to the next tube, iR, to the impressed e. m. f., e_g.

We have

$$k e_g = i R_i + i R,$$

hence

$$\frac{i R}{e_g} = k \frac{R}{R_i + R}$$

This shows that the voltage amplification increases as the coupling resistance R is increased, approaching as a maximum the value of the amplification constant k. This assumes that R_i and, therefore, the actual voltage between plate and filament, remains unchanged. Other amplifier problems and the conditions for oscillation of numerous circuits may be readily worked out by the use of this simplifying theorem.

Experimental Method for Determining the Amplification Constant and Internal A. C. Resistance of Three-Electrode Vacuum Tubes

Instead of deriving the values of the amplification constants and internal resistance indirectly from the static characteristic curves, they may be determined directly and rapidly in a very simple manner using alternating current of audio frequency. The circuit is that shown in Figure 1, in which $c d$ is a slide wire supplied with a small audio frequency current from an alternator thru a step-down transformer or by coupling to a tube source. A variable ground connection not shown in the figure is likewise put on a rheostat which is in parallel with the slide wire in order to obtain a better minimum in the phones. R is a dial resistance box going up to about ten thousand ohms which may be connected in circuit by means of the switch S.

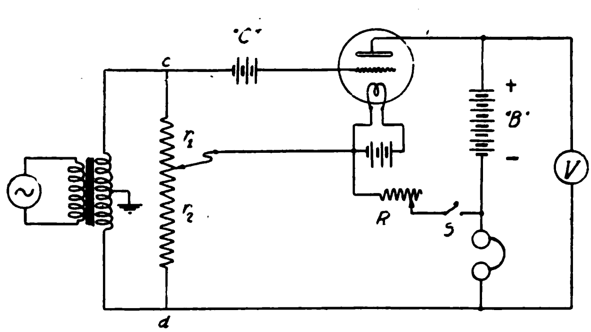


Figure 1

First, the amplification constant k is determined with the switch S open, by adjusting the slider until the telephones are silent when

$$k = \frac{r_2}{r_1}.$$

To determine the internal resistance R_i, the switch S is then closed and the slider set at, say, the middle point and silence is again obtained by varying the resistance R. Then

$$R_i = (k-1)\,R$$

Any other definite ratio of $\frac{r_2}{r_1}$ (less than k) can be used in which case

$$R_i = \left(\frac{r_1}{r_2}k - 1\right)R$$

The above expressions are readily proven. In the first determination an e. m. f. e_g which is proportional to r_1 is introduced between the grid and filament. As pointed out before, this has the effect of impressing an e. m. f. $k\,e_g$ in the plate circuit. The e. m. f. across r_2 which is $\frac{r_2}{r_1}\,e_g$ is 180° out of phase with the e. m. f. $k\,e_g$ and will balance it when $\frac{r_2}{r_1}\,e_g = k\,e_g$. Hence

$$k = \frac{r_2}{r_1}.$$

In the second case an e. m. f. e_g proportional to r_i is likewise impressed between grid and filament, which will produce an alternating current $\frac{k\,e_g}{R_i+R}$ in the plate circuit provided no current flows thru the phones. The e. m. f. across the resistance R is $\frac{k\,e_g\,R}{R_i+R}$. This is balanced by the e. m. f. $\frac{r_2}{r_1}\,e_g$ across r_2 when

$$\frac{r_2}{r_1}\,e_g = \frac{k\,e_g\,R}{R_i+R} \quad \text{or} \quad R_i = \left(\frac{r_1}{r_2}k - 1\right)R,$$

and in the case when the slider is set at the middle point $(r_1 = r_2)$,

$$R_i = (k-1)\,R.$$

As noted before, the amplification constant is very nearly a constant for a given tube, but the internal resistance R_i varies with plate and grid voltages and to some extent with filament current. Curves may be obtained for R_i as a function of these variables obtaining directly and accurately all of the data furnished indirectly and inaccurately by the static characteristic curves excepting the grid current characteristics.

In the actual set-up, the slider $c\,d$ of Figure 1 was seven ohms in resistance and consisted of ten turns of resistance wire inductively wound on a marble cylinder, each turn corresponding to one hundred divisions on the scale. It would be preferable to use a straight wire in order to reduce the inductance. This can be marked to read amplification constant directly. The current in the slide wire and hence the voltages acting on the tube should be kept so small that the operation of the tube takes place over a portion of the characteristic so limited that it is practically a straight line. In the measurements described herein, a current of 50 milliamperes or less was used. This current was supplied by either an alternator or tube source and up to 2,000 cycles per second, and for the tubes investigated no change due to frequency was observed. A buzzer source of interrupted current will suffice in many cases.

The dial resistance box R consisted of non-inductively wound coils and, as noted before, went up to ten thousand ohms. In some cases the internal resistance of the tube may be so high that it becomes necessary to use a ratio of $\frac{r_1}{r_2}$ greater than unity in order that a balance can be obtained with this box.

In obtaining values of the internal resistance for given plate voltages, it must be remembered that since the direct current flowing thru the tube must flow thru the telephones and R in parallel, the actual voltage applied to the tube will be somewhat less than that of the plate battery. In order to measure the actual voltage on the tube, the voltmeter should be connected across the battery and telephone receivers as shown in Figure 1, and the measurement made with the voltmeter key closed. It is desirable to use a high resistance voltmeter and telephones of moderately low resistance, since then the applied voltage when the measurement of k is made, with the switch S open, will differ only slightly from that acting when R_t is measured.

In Figures 2 and 3 are shown curves of the internal resistance with varying plate voltages for two types of tubes, each intended for use as amplifiers or detectors. Tube 1 has an amplification constant of 14.5 while Tube 2 has only 7.5. The resistance of the former is, however, very much greater than that of the latter even when the plate voltages are, respectively, 100 and 20. This shows that, for a given type of tube, the associated apparatus should be designed so as to fit the tube characteristics or, with given apparatus, a tube should be chosen which most nearly fits the apparatus.

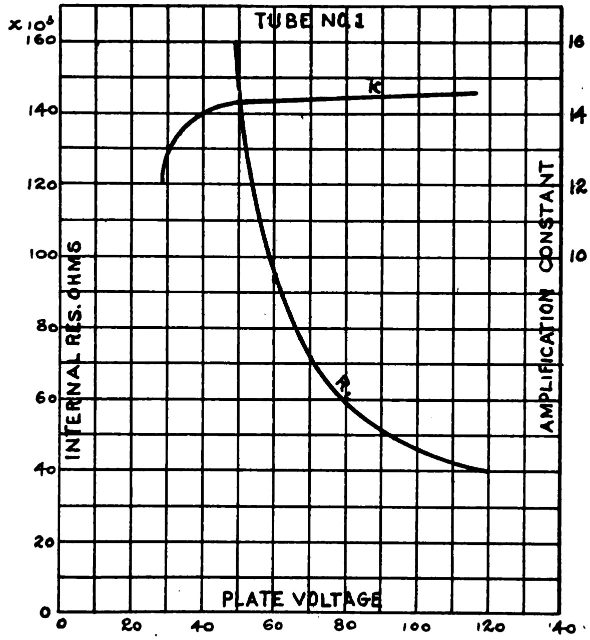


FIGURE 2

The above method has been applied for audio frequencies and to tubes of sufficiently high vacuum and operated at such

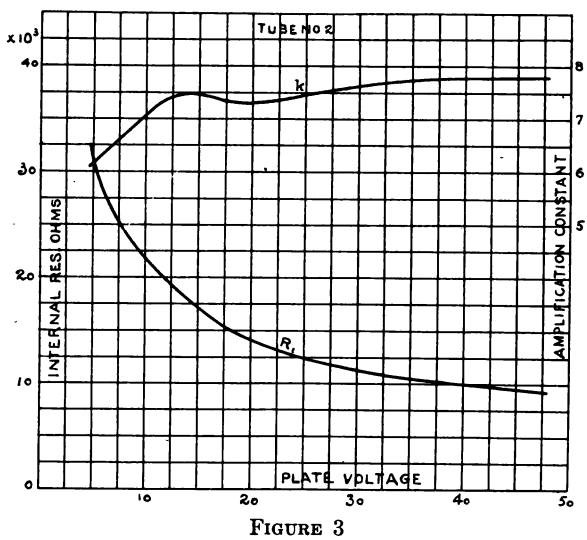


FIGURE 3

voltages that there was no lag of the plate current with respect to the applied grid and plate voltages. The method can, without doubt, be modified so as to measure the phase angle of the lag if such exists and to determine the dynamic values of the tube constants at radio frequencies.

Washington, D. C.
May 3, 1918.

SUMMARY: After considering the characteristics and "amplification constants" of three-electrode vacuum tubes, the author describes the theory and practice of a new method for determining the amplification constant and internal resistance of such tubes directly and rapidly. Examples of results thus obtained are given.

EDISON STORAGE BATTERIES FOR ELECTRON RELAYS*

By

Miller Reese Hutchison, E.E., Ph.D.

It is a well-known fact that the "wing" circuit of an electron relay must be energized by a source of electrical energy entirely free from pulsations of electromotive force.

Notwithstanding the splendid work which has been done in "ironing out" the commutator ripples of dynamo electric machines, there are frequent periods when, owing to any one of a number of causes, non-periodic pulsations result, which seriously affect the operation of the relay.

It is for this reason that batteries, both primary and secondary, have been found the most satisfactory sources of electrical energy for the wing circuit.

Until recently, batteries of miniature dry cells have been employed and have proven fairly satisfactory when absolutely new ones could be readily obtained from the factories; but such cells have a comparatively short period of usefulness, produce a "frying" sound in the receiver when polarization of the elements occurs, and are relatively expensive because of the necessity of frequent substitution by new ones, etc. These disadvantages are pronounced on shipboard, where the dampness makes the life of such a battery particularly short and uncertain, where the unreliability of any piece of apparatus is emphasized, and where a reserve stock of dry cells cannot be depended upon because of their rapid deterioration at sea. On long cruises this uncertainty is of considerable moment.

About a year ago, radio engineers and those upon whom devolves the responsibility of maintaining radio apparatus at remote land stations and aboard ship, cast about for a more dependable and more economical battery for this service.

My attention was first called to this demand by Professor Alfred Goldsmith, who, having used Edison storage batteries in the Radio Telegraphic and Telephonic Laboratory of the College of the City of New York, was familiar with their ruggedness and dependability.

* Received by the Editor, September 7, 1917.

The Edison type "WI-T" cell was thereupon developed and several batteries of these cells were sent to Professor Goldsmith for an extensive series of tests. After due time and a few minor changes, the commercially available battery appeared, incorporated with a standard Edison storage battery for heating the filament. Both have proven highly satisfactory in practical service and have been adopted as the standard for electron relay service by at least one Government.

Because of the prominent position which the new battery occupies in radio engineering, I have been invited to prepare an illustrated description for the radio profession.

A battery of *any* kind, for use in radio work, must, above all things, be *dependable*, even when subjected to the greatest of all abuse—neglect. Of course, when under the eye of a trained battery expert, almost any kind of a storage battery will give good service, if the demand upon it is such as does not necessitate ruggedness; but small units, widely scattered and in the hands of many who may be entirely unskilled in storage battery practice, seldom receive more attention than an occasional charge (which may be a prolonged overcharge at excessively high rates), and the replenishment of the solution with distilled water from time to time. In very small cells, the total liquid content is not sufficient to fill the smallest hydrometer; therefore the specific gravity of such a cell cannot well be ascertained. To remove all the electrolyte from *any* storage battery cell, when the cell is in a charged condition, will positively injure it; so that hydrometer readings of the electrolyte which are so necessary to keep some types of storage battery in condition are a practical impossibility in miniature cell operation. It is therefore requisite that a type of battery which requires no hydrometer readings should be used, and that such a battery should also incorporate the virtue of not being injured by oft-repeated overcharging, or by standing idle for protracted periods in a charged, semi-charged or, in some cases, totally discharged condition. Also if, by chance, the battery be charged in a reverse direction—a frequent error in small battery practice—the battery should not be injured.

When used on an aeroplane, it is of great importance that vibration and concussion should not injure the battery. It should also continue to operate for a short period, at least, even when completely inverted, and should not lose enough electrolyte by such temporary inversion as to affect its operation when right side up.

It should also be of as light weight as is consistent with rugged constructions.

All these and many more characteristics, so necessary for absolute dependability, are possessed by the Edison storage battery.

As is well known, the Edison storage cell consists of a plurality of inter-connected positive tubes and a plurality of inter-connected negative pockets, immersed in a solution of caustic potash. The positive tubes are of perforated, nickel-plated sheet steel. These are loaded with alternate layers of nickel hydrate and pure nickel flake. The negative pockets are likewise made up of perforated nickel-plated sheet steel, and are loaded with iron oxide. For a description of the very interesting processes of manufacture and of the chemical phenomena which take place in the Edison cell, the reader is referred to the various engineering bulletins and published articles on the subject.

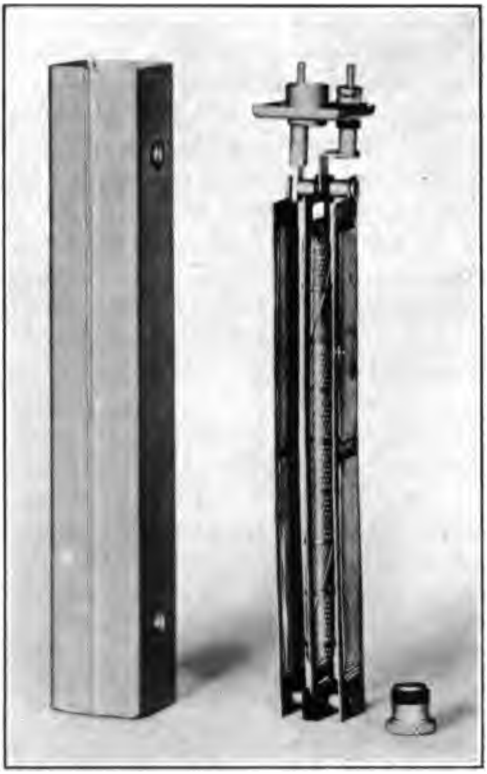
The number of tubes and pockets that make up the elements of a cell depends on the capacity required. In the wing circuit of ordinary electron relays the current rarely exceeds a few milli-amperes, and a single positive tube with four negative pockets provide ample capacity. The manner of uniting these parts to form the cell elements is illustrated in Figure 1, which also shows the steel container. Figure 2 shows the completed cell. All steel parts, it should be noted, are nickel-plated, the plating being *welded on*, in accordance with Edison standard practice.

Two of these cells are combined to form a "twin" cell by connecting their containers together, and grounding the inner positive of one cell and the inner negative of the other to their respective cans. The completed twin is known as type "WI-T."

At the low discharge rates used in electron relay work, the average voltage during discharge of a twin cell is 2.56, and the final voltage, when re-charging becomes necessary, is 2.4. Hence, 16 twin cells are employed for a 40-volt battery and 42 for a 110-volt battery.

The ampere-hour capacity of one of these cells is 1.25, and the normal charging rate 0.25 amperes. Hence, under normal conditions, the time of charge is five hours; but in an emergency, the normal charging rate can be greatly exceeded, without injuring the cell. The low discharge rate in the wing circuit enables a battery to run from several hundred to several thousand hours continuously, on one charge, depending upon the characteristics of the relay bulb.

To correspond with the different sizes of electron relay which have been standardized, two sizes of Edison storage battery units are furnished. For ship and shore work the unit is known as type "42-WI-T—5-B-4." It consists of 42 cells of



Metal Container Elements

FIGURE 1—Edison "WI" Cell Parts

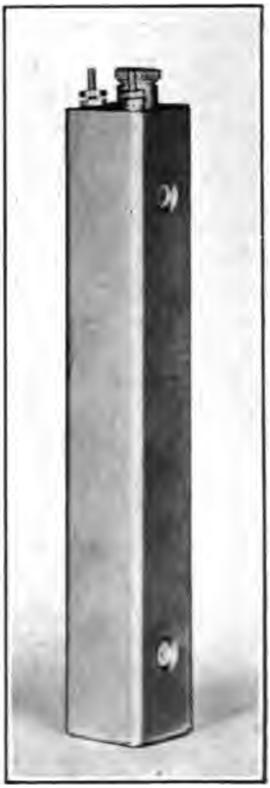


FIGURE 2—Edison Type "WI" Cell, Complete

the twin type described above (type "WI-T"), contained in the same case with five cells (6 volts) of the "B-4" type for heating the filament. The average voltage of the 42 twin cells at the low rate of discharge is 107.5, which is suitable for the wing circuit of the ship and shore relays.

For aeroplane service, the unit is known as Type "16-WI-T—5-M-20," and consists of 16 twin cells in the same container with five cells of type "M-20" for furnishing the filament current.

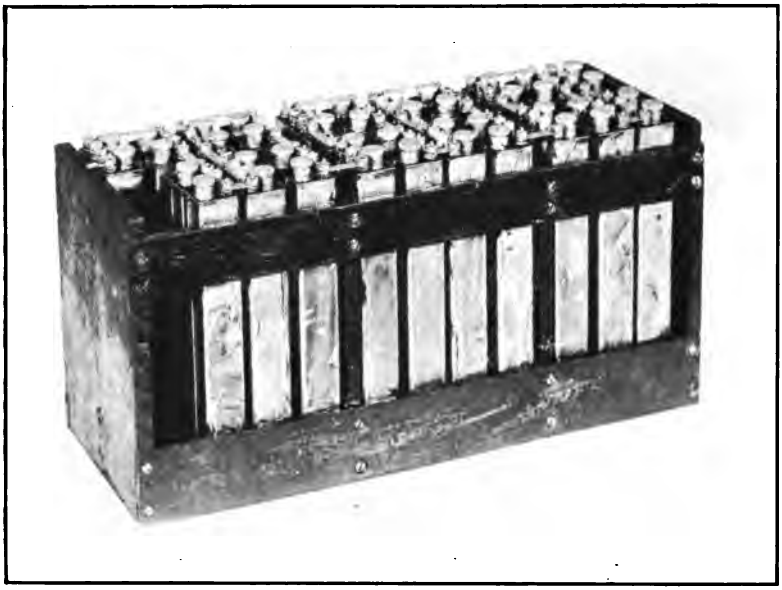


FIGURE 3—A Tray of 21 Twin Cells

The average voltage of the 16 twins is 40, as required for aeroplane relays.

The more important specifications of these two units are given in the following tables:

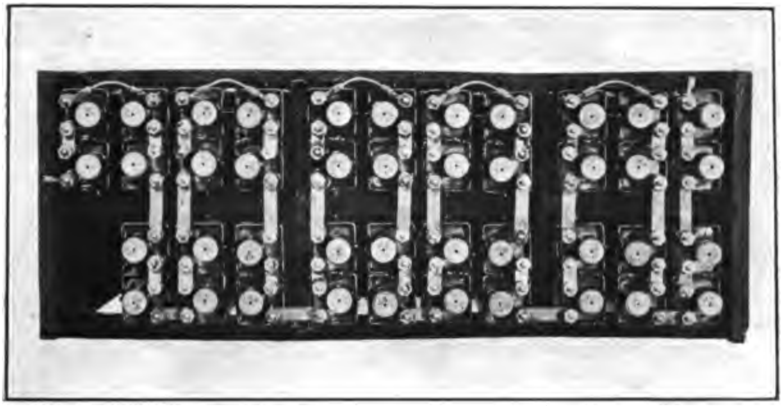


FIGURE 4—Plan View of a Tray of 21 Twin Cells

TABLE I*

Edison Storage Battery Unit

Type "42-WI-T–5-B-4." (Figure 5.)

For Electron Relays for Ship and Shore Stations.

"42-WI-T" Battery:

Ampere-hour capacity	1.25
Watt-hour capacity	135.
Discharge rate in wing circuit of electron relay, milliamperes	0.2 to 5.
Average voltage on discharge at this rate to 100 volts	107.5 ?
Normal charging rate, amperes	02.5

.25

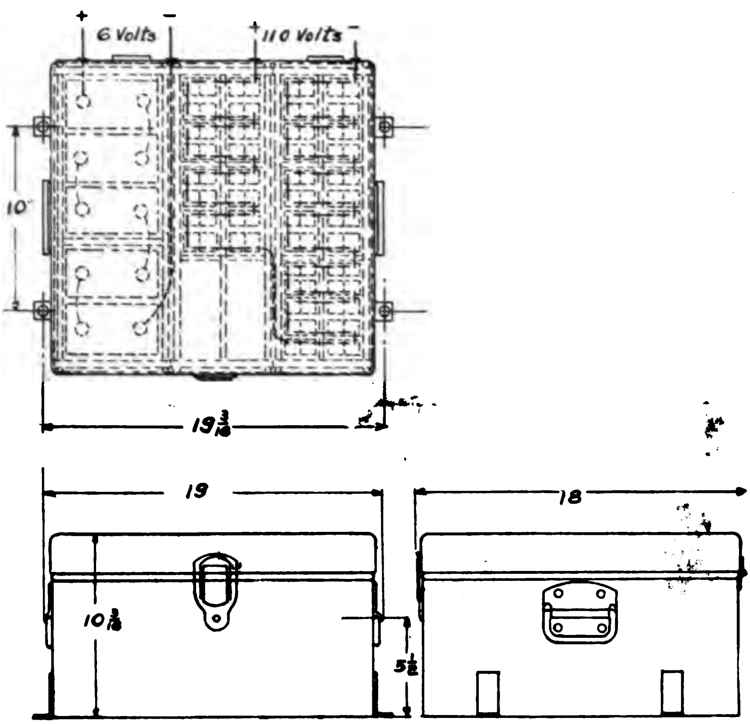


FIGURE 5—42 Type "WI-T" (110 Volts) and 5 Type "B-4" (6 Volts) Edison Cells Mounted in Steel Container

* Further information relative to these cells and their distribution may be obtained from Miller Reese Hutchison (Inc.), Orange, New Jersey.

"5-B-4" Battery:

Ampere-hour capacity	75.
Watt-hour capacity	450.
Normal charging rate, amperes	15.
Normal discharging rate, amperes	15.
Average voltage on discharge at normal rate to 5 volts	6.

Weight of unit complete, contained in heavily-japanned weather-proof steel box, equipped with carrying handles and securing means...........100 lbs. (45 kg.)

Over-all dimensions of steel container, inches,
$18 \times 19 \times 10\frac{3}{16}$. (45.7×48.3×25.9 cm.)

Edison Storage Battery Unit

Type "16-WI-T-5-M-20" (Figure 6)

For Electron Relays for Aeroplane Service.

"16-WI-T" Battery:

Ampere-hour capacity	1.25
Watt-hour capacity	51.
Discharge rate in wing circuit of electron relay, milliamperes	0.2 to 3.
Average voltage on discharge at this rate, to 38 volts	41.
Normal charging rate, amperes	0.25

"5-M-20" Battery:

Ampere-hour capacity	12.
Watt-hour capacity	72.
Normal charging rate, amperes	2.5
Normal discharging rate, amperes	2.5
Average voltage at normal discharge rate to 5 volts	6.

Weight of unit complete, contained in heavily-japanned weather-proof steel box, equipped with carrying handles and securing means...............35 lbs. (16 kg.)

Over-all dimensions of steel container, inches
$9\frac{5}{8} \times 11\frac{13}{16} \times 8\frac{1}{2}$ (24.5×30×21.6 cm.)

The lay-out of the ship and shore station unit is shown in Figure 5, and of the aeroplane unit in Figure 6.

These batteries for electron relays are also furnished in individual boxes, where desired. Figure 7 shows the lay-out of type "42-WI-T" battery and container, for the wing circuit of ship and shore electron relays; Figure 8, the lay-out of type "5-B-4" battery for the filament circuit of these relays; Figure 9, the "16-WI-T" battery for wing circuit of the aeroplane set; and Figure 10 the "5-M-20" battery for the filament circuit of the same.

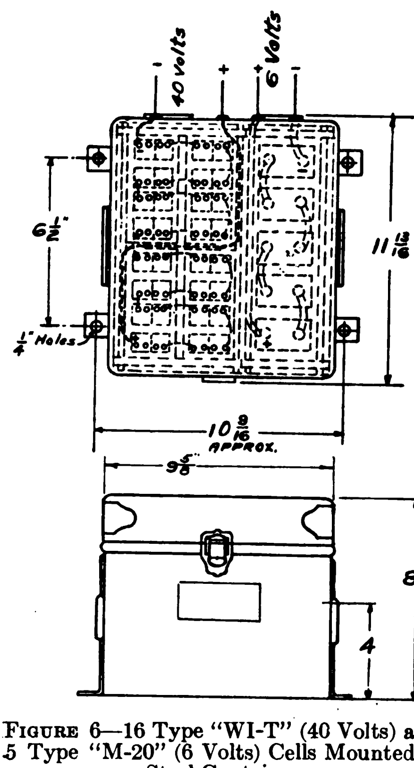


FIGURE 6—16 Type "WI-T" (40 Volts) and 5 Type "M-20" (6 Volts) Cells Mounted in Steel Container

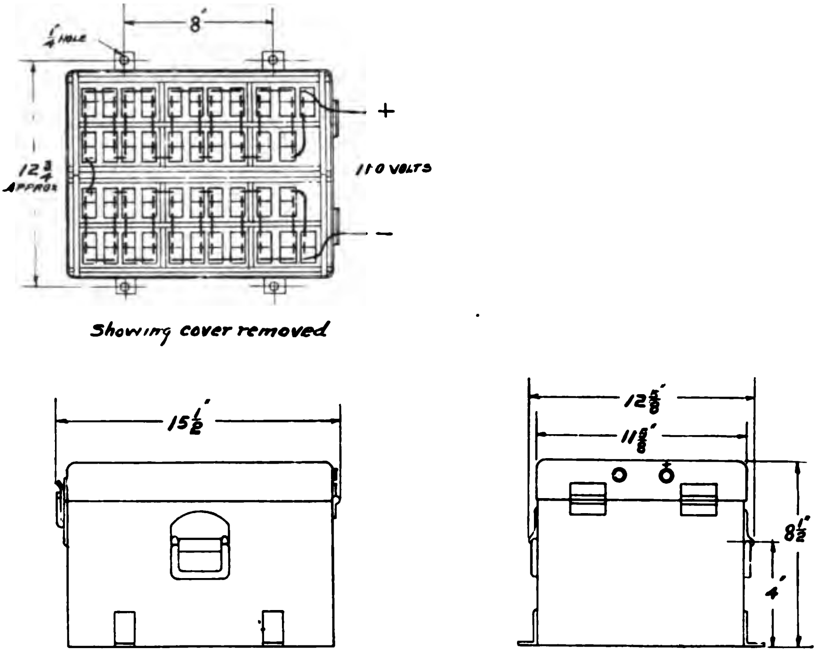


Figure 7—42 Type "WI-T" (110 Volts) Edison Cells Mounted in Container

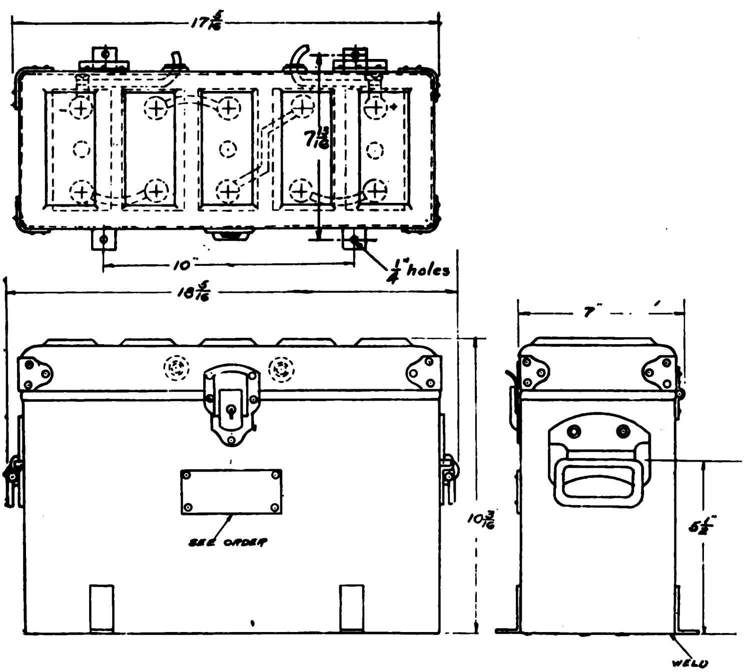


Figure 8—5 Type "B-4" (6 Volts) Edison Cells in Steel Container

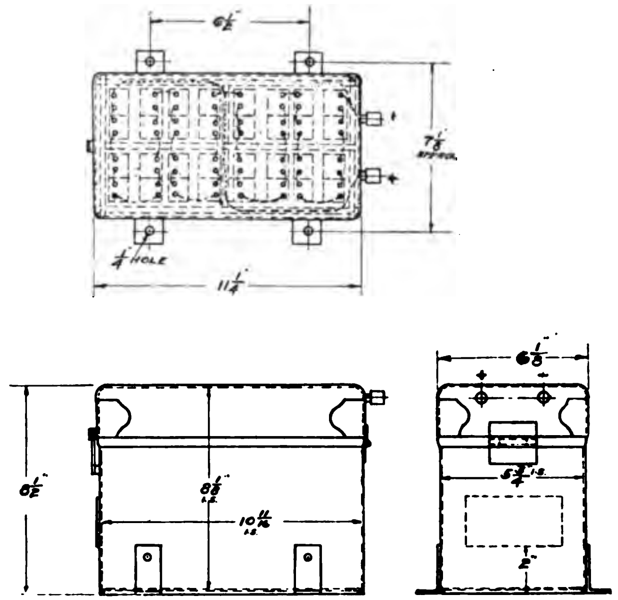


FIGURE 9—16 Type "WI-T" (40 Volts) Edison Cells, Mounted in Steel Container

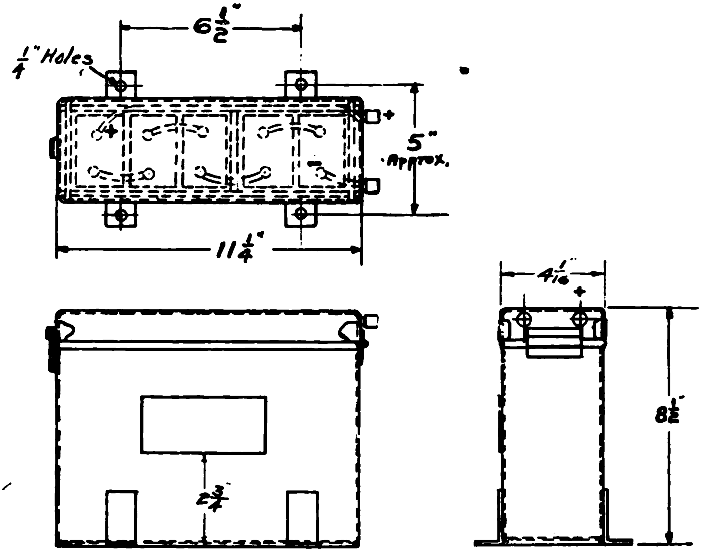


FIGURE 10—5 Type "M-20" (6 Volts) Edison Cells in Steel Container

SUMMARY: The development of a new type of Edison nickel-steel storage battery for use in the plate circuit of electron relays is described. This is a small 1.25 ampere-hour, 2.56-volt twin-cell. Normal discharge lasts several thousand hours.

Various assemblies of these for 40 and 110 volts, in conjunction with various filament-lighting Edison storage battery sets, are described in detail, with electrical operating data, weights, and dimensions

FURTHER DISCUSSION ON "ON THE USE OF CONSTANT POTENTIAL GENERATORS FOR CHARGING RADIOTELEGRAPHIC CONDENSERS AND THE NEW RADIOTELEGRAPHIC INSTALLATIONS OF THE POSTAL AND TELEGRAPH DEPARTMENT OF FRANCE" BY LEON BOUTHILLON

By

J. F. J. Bethenod

(Paris, France)

In the issue of June, 1917, of the "Proceedings of The Institute of Radio Engineers," page 199 and following, Mr. L. Bouthillon had published an interesting paper on the use of constant electromotive force for charging the radiotelegraphic condensers. I desire to mention that I have already indicated in British Patent 27,247 (1910), page 2, lines 14-30, the enormous advantages of the "periodic charging" versus the "aperiodic charging."

Moreover, the system described by Mr. Bouthillon is practically the same as the one experimented with by Major Fracque in 1910 at the Eiffel Tower station.

A very complete theory of this charging method has also been published by Major Fracque (see for instance "La Lumière Electrique," January 16, 1915, pages 45-46), so that I cannot agree with Mr. Bouthillon when he says that the above system is new.

In concluding, let me say that the use of constant electromotive force for charging does not give the *maximum maximorum efficiency* (greatest of the maximal efficiencies).

Let us suppose that a condenser is connected across the terminals of a direct-current supply by means of a self-inductance L and an ohmic resistance R (this resistance taking account of the dielectric losses in the condenser). We propose to find the wave of the charging current which will give the maximum maximorum efficiency, the time, and the increase of the charge having given values τ and ΔQ. It seems admissable to reduce to such a problem that of the most economical charging of a radiotelegraphic condenser. If the current is i at the time t, this problem consists in fact to determine the minimum value of the integral

$$W = R \int_0^\tau i^2 \, dt,$$

the integral

$$\Delta Q = \int_0^\tau i \, dt$$

having a fixed value.

In other words, we ought to try to find the shape of wave which gives the maximum square root of the mean square, or effective value, the average value being fixed.

It is possible to find the answer by means of the general method which is applicable to such a problem; but we prefer a very elementary solution, which has been already applied by M. Marius Latour for a problem concerning the electric transmission of energy.[1]

Put:

$$i = \frac{\Delta Q}{\tau} + x.$$

We can immediately write from (2):

$$\int_0^\tau x \, dt = 0.$$

Therefore, when substituted in (1):

$$W = R \frac{\overline{\Delta Q}^2}{\tau} + 2 R \frac{\Delta Q}{\tau} \int_0^\tau x \, dt + R \int_0^\tau x^2 \, dt,$$

the first integral of the second member is zero, and the minimum value of W manifestly occurs when $x = \text{constant} = 0$, since the second integral contains only essentially positive elements.

Consequently it may be said that the most economical charging for a radiotelegraphic condenser is obtained when the current has a constant value during the whole charging. This value is naturally:

$$I = \frac{\Delta Q}{\tau}.$$

If we call Q the initial charge of the condenser, the efficiency can be defined by the ratio: $\dfrac{\text{stored energy}}{\text{stored energy} + \text{wasted energy}}$ or:

[1] Cf. Marius Latour, "L'Eclairage Electrique," February 23, 1901, page 279, number 8, volume XXVI.

$$\eta=\frac{\frac{1}{2}\times\frac{(Q+\Delta Q)^2}{C}-\frac{1}{2}\times\frac{Q^2}{C}}{\frac{1}{2}\times\frac{(Q+\Delta Q)^2}{C}-\frac{1}{2}\times\frac{Q^2}{C}+R\frac{\overline{\Delta Q}^2}{\tau}}$$

$$=\frac{\tau\left(1+2\frac{Q}{\Delta Q}\right)}{\tau\left(1+2\frac{Q}{\Delta Q}\right)+2CR}$$

or finally

$$\eta=\frac{\tau\left(1+2\frac{Q}{\Delta Q}\right)}{\tau\left(1+2\frac{Q}{\Delta Q}\right)+\frac{\delta T}{\pi^2}},$$

δ being the logarithmic decrement, and T the natural period of the circuit.

It will be noted that with the Poulsen arc working with oscillations of the 2nd type of M. Blondel, one realizes in practice the charging with a current of constant value by means of the extremely high inductance inserted in the charging circuit.[2]

It is theoretically possible to determine the shape of wave for applied electromotive force which allows us to realize exactly such a method of charging. But it will not be of practical interest, because, for one thing, the initial conditions are always uncertain.

[2] H. Barkhausen, "Das Problem der Schwingungserzeugung," Leipzig, 1907.

NOTES RELATIVE TO THE IMMEDIATELY PRECEDING DISCUSSION OF MR. BETHENOD*

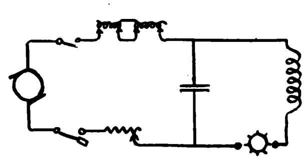
By

Leon Bouthillon

It was a pleasure to learn of the confirmation by Mr. Bethenod of what I consider to be one of the most important conclusions of my paper, dealing with the high efficiency of the system developed by me.

But I cannot agree with the statement that my system is practically the same as that which was experimented with by Commandant Fracque in 1910. The following passage, in which Commandant Fracque describes his experiments, is pertinent ("La Lumière Electrique," January 16, 1915, page 46):

"I had an opportunity to study the charging phenomena (using high voltage direct current), in 1910, at the radio station of the Eiffel Tower, using a Gramme dynamo (2,000 volts, 1.5 amperes), a paper condenser built according to the Boucherot system, two high voltage choke coils with taps on the windings from Mr. Blondel's laboratory, and a resistance made up of incandescent lamps (by means of which resistances up to 10,000 ohms could be obtained). (See the accompanying Figure.)



"The spark gap used was made up of a toothed disc which turned between two fixed discharging studs, and was driven by a small electric motor the speed of which could be very closely regulated.

"By choosing a speed of the wheel and distance between the electrodes in such a way that the time of passage of two consecu-

* Received by the Editor, May 8, 1918.

tive teeth past the fixed electrodes was equal to the time required for the generator to charge the condenser to the highest voltage

$$t = \frac{\pi}{m} = \frac{\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}},$$

I succeeded in obtaining very regularly spaced discharges, and, in consequence, a pleasant musical note.

"I attempted to choose such values of C, L, R, that, without ceasing to be a oscillatory circuit, the charging circuit had a large resistance, which much diminished the likelihood of an arc being produced in the discharger.

"A comparison of the energy taken from the dynamo with that appearing in the form of oscillations in the condenser discharge circuit, showed clearly that the efficiency, while not equal to unity, was nevertheless very high."

It can be seen from this quotation that one of the characteristics of the system employed by Commandant Fracque was the insertion into the charging circuit of a large resistance (namely, a rheostat capable of going up to 10,000 ohms), the presence of which rheostat would instantly reduce the efficiency considerably. My system is distinguished, on the other hand, by the care with which all resistances in the charging circuit are reduced to a minimum. Thus it happens, as explained by Commandant Fracque, that his charging circuit being of high resistance without ceasing to be oscillatory, the efficiency never seems to have been greater than 0.6; that is, of the same order of magnitude as that obtained when the charging is done by alternating current. In my system, efficiencies greater than 0.9 have frequently been obtained. An increase of efficiency of 50 per cent. seems to me a sufficiently interesting improvement, being an improvement of which Mr. Bethenod recognizes the importance since he claims the honor of having first described it.

As regards the theoretical part of my paper, I desire to state that in his paper in "La Lumière Electrique," Commandant Fracque considered only the particular case when the charging time of the condenser was equal to half the natural period of oscillation of the charging circuit. My theory was much more general and extends to all possible cases of musical tone systems, no matter what the duration of charge; and my theory further indicates that musical tone systems are the only systems which are stable as well as possible. The importance of this much more extended theory is indicated by the fact that the less complete discussion of Commandant Fracque led him to exaggerate the precision with which the discharger must be regulated.

"At the exact time when V reaches its greatest maximum; that is to say at the time $\frac{\pi}{m}$, it is necessary by appropriate arrangement to release the condenser charge."

On the other hand, from the complete theory which I have given, it can be seen (and this has been repeatedly verified by frequent experimentation) that the speed of the discharger can be varied between wide limits on each side of the best value without markedly diminishing the efficiency. For example, with the charging circuit having a decrement equal to 0.4, which corresponds to the maximum efficiency of 0.91, the efficiency is not diminished by more than 10 per cent. below the maximum value if the speed of the discharger is 0.4 less or 1.5 times greater than the best value.

(Translated from the French by the Editor.)

THEORY OF FREE AND SUSTAINED OSCILLATIONS*

By

H. G. Cordes

(Radio Research Engineer, Bremerton, Washington)

The amplitude of a free oscillation decreases with time because of the dissipation of energy in the circuit. The energy is generally considered as being dissipated by a series resistance or by the equivalent of a series resistance. It is often useful to differentiate between damping due to resistance in series and damping due to conductance in parallel with the capacitance of an oscillatory circuit. The current taken by a detector circuit will produce conductance damping in a receiving oscillatory circuit. In a transmitter the losses thru insulators, corona and dielectric losses cause conductance damping. During each oscillation the conductance loss is a maximum when the potential is a maximum while the resistance loss is greatest when the current is a maximum. The damping due to radiation is generally stated as an equivalent resistance damping altho its exact nature has not been determined experimentally.

Free Oscillations

Let Figure 1 represent a circuit having concentrated inductance, capacitance, resistance and conductance. The resistance r is in series with the inductance L and the conductance g is in parallel with the capacitance C.

Assume the direction of the arrows positive.

The equation of potential thru inductance L, resistance r and conductance g is

$$L\frac{d\,i}{d\,t}+i\,r-\frac{i_g}{g}=0 \tag{1}$$

Equating potentials in the circuit formed by capacitance C and conductance g gives

$$\frac{i_g}{g}+\int\frac{i_c\,d\,t}{C}=0 \tag{2}$$

* Received by the Editor July 1, 1917.

where g is the reciprocal of the insulation resistance.

The current relation is

$$i_c = i + i_g \tag{3}$$

Eliminate i_c from (2) and (3), and differentiate

$$\frac{1}{g} \cdot \frac{d\,i_g}{d\,t} + \frac{i}{C} + \frac{i_g}{C} = 0 \tag{4}$$

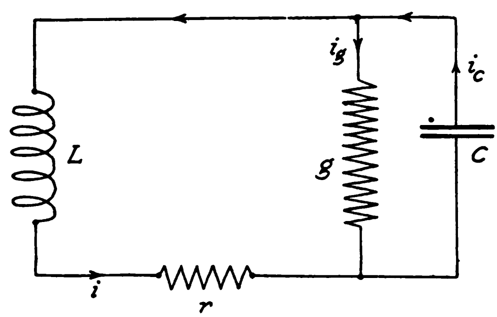


FIGURE 1

Solve (1) for i_g and substitute in (4)

$$\frac{d^2\,i}{d\,t^2} + \left(\frac{r}{L} + \frac{g}{C}\right)\frac{d\,i}{d\,t} + \frac{1+g\,r}{L\,C}\,i = 0 \tag{5}$$

The auxiliary equation of (5) is

$$m^2 + \left(\frac{r}{L} + \frac{g}{C}\right)m + \frac{1+g\,r}{L\,C} = 0 \tag{6}$$

Solving (6) for m

$$m = -\left(\frac{r}{2L} + \frac{g}{2C}\right) \pm j\sqrt{\frac{1}{LC} - \left(\frac{r}{2L} - \frac{g}{2C}\right)^2}.$$

Let $\quad a = \dfrac{r}{2L} + \dfrac{g}{2C}$ and let $\omega = \sqrt{\dfrac{1}{LC} - \left(\dfrac{r}{2L} - \dfrac{g}{2C}\right)^2}.$

When $t=0$, $i=0$, therefore the solution of (5) is

$$i = I\varepsilon^{-a\,t} \sin \omega\,t \tag{7}$$

The damping factor a shows the obvious fact that increasing $\frac{L}{C}$ decreases the resistance damping and that decreasing $\frac{L}{C}$ decreases the conductance damping. Conductance and resistance damping have the same effect upon the frequency when considered individually but when both are considered present they neutralize each other; so that when the resistance and conductance damping are equal, the frequency is independent of the damping.

A free discharge becomes non-oscillatory when $\left(\frac{r}{2L}-\frac{g}{2C}\right)$ is equal or greater than $\frac{1}{LC}$. It is seen that a non-oscillatory circuit may become oscillatory by the introduction of resistance or conductance damping.

The potential of the capacitance is

$$v=L\frac{di}{dt}+ri. \tag{8}$$

Substitute (7) in (8)

$$v=I\,\varepsilon^{-at}\left[\omega L\cos\omega t+(r-a)\sin\omega t\right] \tag{9}$$

When $t=0$, $v=E_o$, therefore

$$E_o=\omega L I \tag{10}$$

Substitute (10) in (9)

$$v=E_o\,\varepsilon^{-at}\left(\cos\omega t+\frac{r-a}{\omega L}\sin\omega t\right) \tag{11}$$

which may be written in the form

$$v=E\,\varepsilon^{-at}\cos(\omega t-\phi) \tag{12}$$

where $\qquad E_o=E\cos\phi$ and $\phi=\tan^{-1}\frac{r-a}{\omega L}$.

Sustained Oscillations

An alternating current in a circuit having comparatively large inductance and capacitance and small impedance constitutes a sustained oscillation. The circuit is in resonance when the reactance of the circuit is zero at a given frequency of the impressed e. m. f. The impedance of the circuit is generally stated in terms of resistance, inductance, capacitance, and frequency. The effect of conductance is not always negligible nor can an equivalent resistance always be substituted for it.

In Figure 1, let $v=$ the e. m. f. of the capacitance.

Then $$i_g=g\,v. \tag{13}$$

Eliminate i_g and i_c from (2), (3), and (13), by differentiating and rearranging

$$i=-C\frac{dv}{dt}-g\,v \tag{14}$$

Assume a sinusoidal e. m. f., $e=-E\sin\omega_1 t$, impressed upon the inductance L. Equation (1) will then become

$$L\frac{di}{dt}+ri-v=e \tag{15}$$

Let

$$\omega_2=\sqrt{\frac{1}{LC}-\left(\frac{r}{2L}-\frac{g}{2C}\right)^2} \tag{16}$$

which may also be written

$$\frac{1+gr}{LC}=\omega_2^2+a^2 \tag{17}$$

Substitute (14) in (15) and divide by LC

$$\frac{d^2v}{dt^2}+2a\frac{dv}{dt}+(\omega_2^2+a^2)v=\frac{E\sin\omega_1 t}{LC} \tag{18}$$

Differentiate (18) twice, solve for $E\sin\omega_1 t$, and substitute in (18)

$$\frac{d^4v}{dt^4}+2a\frac{d^3v}{dt^3}+(\omega_1^2+\omega_2^2+a^2)\frac{d^2v}{dt^2}+2a\omega_2^2\frac{dv}{dt}+\omega_1^2(\omega_2^2+a^2)v=0 \tag{19}$$

The auxiliary equation of (19) is

$$m^4+2am^3+(\omega_1^2+\omega_2^2+a^2)m^2+2a\omega_2^2m+\omega_1^2(\omega_2^2+a^2)=0 \tag{20}$$

which consists of the factors

$$m^2+\omega_1^2=0,\text{ and } m^2+2am+\omega_2^2+a^2=0 \tag{21}$$

Therefore the roots of the above are

$$m=\pm j\omega_1,\text{ and } m=-a\pm j\omega_2 \tag{22}$$

Hence the solution of (19) is

$$v=A\varepsilon^{j\omega_1 t}+B\varepsilon^{-j\omega_1 t}+C\varepsilon^{-a+j\omega_2 t}+D\varepsilon^{-a-j\omega_2 t} \tag{23}$$

which may be written in the form

$$v=V\sin(\omega_1 t+\phi)+V_1\varepsilon^{-at}\sin(\omega_2 t+\theta) \tag{24}$$

The transient component, $V_1\varepsilon^{-at}\sin(\omega_2 t+\theta)$, disappears in a very short time; therefore the sustained oscillation is represented by

$$v=V\sin(\omega_1 t+\phi) \tag{25}$$

in which V and ϕ are constants of integration to be determined for the sustained component of the oscillation. V is the maximum potential and ϕ represents the phase difference between the impressed e. m. f. amd the oscillatory e. m. f.

Substitute (25) in (18) and let $t=0$.

$$V\,\omega_1^2 \sin\phi + 2\,a\,V\,\omega_1 \cos\phi + (\omega_2^2 + a^2)\,V \sin\phi = 0 \qquad (26)$$

Substitute (25) in (18) and let $\omega_1 t + \phi = 0$

$$2\,a\,V\,\omega_1 + \frac{E \sin\phi}{LC} = 0, \text{ or } \phi = \sin^{-1}\left(-\frac{2\,a\,V\,\omega_1 LC}{E}\right) \qquad (27)$$

Eliminate ϕ from (26) and (27)

$$VLC\,[\omega_1^2 - (\omega_2^2 + a^2)] = -\sqrt{E^2 - (2\,a\,V\,\omega_1 LC)^2}$$

or

$$V = \frac{E}{LC\sqrt{(2\,a\,\omega_1)^2 + [\omega_1^2 - (\omega_2^2 + a^2)]^2}} \qquad (28)$$

which may also be written

$$V = \frac{E}{\omega_1 C\sqrt{\left(r + \frac{L}{C}\cdot g\right)^2 + \left(\omega_1 L - \frac{1 + g\,r}{\omega_1 C}\right)^2}}. \qquad (29)$$

Substitute (25) in (14)

$$i = -V\sqrt{(\omega_1 C)^2 + g^2}\,\cos\left[\omega_1 t + \phi + \tan^{-1}\left(\frac{-g}{\omega_1 C}\right)\right] \qquad (30)$$

Let the maximum value of $i = I$, then

$$I = V\sqrt{(\omega_1 C)^2 + g^2} \qquad (31)$$

There are two ways of securing maximum potential or current in an oscillatory circuit. Either the inductance may be varied while the capacitance remains constant or the capacitance may be varied while the inductance remains constant. If ω_1 and C remain constant, and L is varied, an inspection of (31) shows that both I and V are simultaneously a maximum for all values of r and g.

For maximum V by varying L, let $\frac{dV}{dL} = 0$; this gives

$$\omega_1 = \sqrt{\frac{1}{LC} - \left(\frac{g}{C}\right)^2} \qquad (32)$$

For maximum V by varying C, let $\frac{dV}{dC} = 0$; this gives

$$\omega_1 = \sqrt{\frac{1}{LC} - \left(\frac{r}{L}\right)^2}. \qquad (33)$$

It is seen from (32) and (33) that with large conductance and small resistance in the oscillatory circuit, it is necessary to vary the capacitance to obtain maximum V at resonance; and, with large resistance and small conductance the inductance must be varied to get maximum V at resonance.

The potential energy, $\frac{1}{2}CV^2$, is always a maximum at resonance unless the conductance is large, then the maximum is obtained by varying the capacitance when

$$\omega_1 = \sqrt{\frac{1+gr}{LC - \frac{gL(rC+gL)}{1+gr}}} \tag{34}$$

and is obtained by varying the inductance when

$$\omega_1 = \sqrt{\frac{1}{LC} - \left(\frac{g}{C}\right)^2}. \tag{35}$$

Substituting (29) in (31), the maximum current amplitude

$$I = \frac{E\sqrt{\omega_1^2 C^2 + g^2}}{\omega_1 C \sqrt{\left(r+\frac{L}{C}g\right)^2 + \left(\omega_1 L - \frac{1+gr}{\omega_1 C}\right)^2}}. \tag{36}$$

The condition for resonance is that $\omega_1 = \sqrt{\frac{1+gr}{LC}}$. It will be noted that the reactance depends to a small extent upon the resistance and conductance when both are considered present. In (36) g is generally negligible compared to $\omega_1 C$ and the product $g\,r$ is negligible compared to unity, therefore (36) may be written

$$I = \frac{E}{\sqrt{\left(r+\frac{L}{C}g\right)^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}} \tag{37}$$

When $g=0$ this reduces to the familiar expression

$$I = \frac{E}{\sqrt{r^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}} \tag{38}$$

And when $r=0$ equation (37) becomes

$$I = \frac{E}{\frac{L}{C}\sqrt{g^2 + \left(\omega_1 C - \frac{1}{\omega_1 L}\right)^2}} \tag{39}$$

The product of $\frac{L}{C}$ times a conductance or susceptance has the dimensions of a resistance or reactance.

To determine the maximum I of equation (36) by varying C, let $\frac{dI}{dC}=0$; this gives

$$\omega_1 = \sqrt{\frac{1+2gr}{LC} + \left(\frac{g}{C}\right)^2} \tag{40}$$

Equation (32) applies to both potential and current; but equations (33) and (40) show that by varying the capacitance, the potential and current do not attain a maximum value simultaneously; the potential reaches a maximum at a frequency less than resonance and the current at a frequency greater than resonance frequency. When the conductance is negligible, all maximum values are coincident with resonance except maximum potential when the capacitance is varied for a maximum.

The radiation resistance is not large enough to change appreciably the frequency for maximum values of current and potential, otherwise equations (32), (33), and (40) could be used to determine experimentally whether radiation causes resistance or conductance damping; furthermore, maximum radiation probably takes place when the product of instantaneous potential and current, vi, is a maximum; that is, when the transfer of energy in the oscillatory circuit is a maximum. Maximum vi is displaced 45° from both maximum potential and maximum current and occurs at double the frequency.

The expressions derived for maximum potential and current are based upon a constant frequency of the impressed e. m. f. A slight fluctuation in the frequency introduces a reactance into the oscillatory circuit. The detrimental effect of this reactance increases as the ratio $\frac{L}{C}$ increases and as the effective resistance, $r+\frac{L}{C}g$, decreases.

In equation (37), let $\omega_1 L=\frac{1}{\omega_1 C}$ and let ρ represent $r+\frac{gL}{C}$, then the resonance current

$$I_r=\frac{E}{\rho}. \tag{41}$$

Let the fluctuation in frequency be equivalent to changing the frequency by a factor a. Substituting $a\,\omega_1$ for ω_1 in (37) then (41) will become

$$I_a=\frac{E}{\sqrt{\rho^2+\left[\frac{1}{\omega_1 C}\left(\frac{a^2-1}{a}\right)\right]^2}} \tag{42}$$

where I_a is the measured resonance current.

The term "frequency factor" may be applied to the ratio

$$\frac{I_a}{I_r}=\frac{\rho}{\sqrt{\rho^2+\left(\frac{a^2-1}{a\,\omega_1 C}\right)^2}} \tag{43}$$

which is the power factor of a circuit in which all of the reactance is due to frequency fluctuations of the impressed e. m. f.

In a receiving antenna circuit, damping is due to the losses in the circuit and to the energy withdrawn for useful work. The former consists of re-radiation, resistance, and conductance losses; and the latter consists of the energy withdrawn by the detector circuit or its equivalent. To obtain maximum energy in the detector circuit, the familiar principle applies, viz., that the damping due to useful energy withdrawn must equal the damping due to energy loss.

Applications and Numerical Examples

Some of the equations will be stated in a form required for the substitution of practical units. Numerical examples will refer, unless otherwise noted, to a standard antenna of 0.002 microfarad capacitance, 10^5 cycles frequency (that is, of wave length 3,000 meters), and 5 ohms effective resistance. Let C_m = capacitance in microfarads, L = inductance in cm., r_o = the resistance in ohms, ρ_o = the effective resistance in ohms, and g_m = conductance in mhos.

To estimate the effect of conductance upon damping, the conductance may be expressed as an equivalent resistance by the relation $\frac{r}{2L}=\frac{g}{2C}$, or, $r=\frac{L}{C}g=\frac{g}{\omega^2 C^2}$, which shows that an equivalent resistance is directly proportional to the conductance. In practical units

$$r_o=\frac{10^{12}}{4\pi^2}\cdot\frac{g_m}{f^2 C_m^2} \tag{46}$$

When $g_m=10^{-6}$ mhos, $r_o=0.62$ ohms; i. e., one megohm antenna insulation is equivalent to ⅝ ohm antenna resistance in its damping effect.

The ratio of resonance frequency to the frequency of a free oscillation is expressed by

$$\frac{\sqrt{\frac{1+gr}{LC}}}{\sqrt{\frac{1+gr}{LC}-\frac{\rho^2}{4L^2}}}=\sqrt{\frac{1}{1-\left(\frac{\delta}{2\pi}\right)^2}} \tag{47}$$

where the logarithmic decrement, $\delta=\frac{2\pi^2}{10^6}\rho_o C_m f$. When $\delta=0.2$, the ratio $(47)=1.0005$, or a variation of 50 cycles at a frequency of 10^5 cycles. The decrement of the standard antenna is 0.02 for which the ratio (47) is practically unity.

The maximum potential in practical units neglecting g and in (32) and (33) is

$$V=\frac{10^6}{2\pi}\cdot\frac{E}{\rho_o f C_m}. \tag{48}$$

Let $E=100$ volts, then $V=16,000$ volts.

The frequency factor may be expressed, with sufficient approximation, in practical units by

$$\frac{\rho_o}{\sqrt{\rho_o^2+\frac{1}{10}\left(\frac{10^4 h}{f C_m}\right)^2}} \tag{49}$$

where $h=100\ (a-1)=$ the per cent. frequency variation.

The per cent. frequency variation (h) and the corresponding frequency factor are tabulated below for a standard antenna, and also for the same antenna with 10 ohms effective resistance.

TABLE I

Frequency Variation (%)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Impedance (ohms)	5	5.25	5.9	6.9	8.1	9.4	10.7	12.1	13.6	15.0	16.6
Frequency Factor (%)	100	95	85	72.5	62	53	46	41	37	33	30
Impedance (ohms)	10	10.1	10.5	11.1	11.8	12.7	13.8	14.9	16.1	17.4	18.7
Frequency Factor (%)	100	98.8	95	90	85	79	73	67	62	57	53

The ratio $\frac{L}{C}$ is naturally high in a receiving antenna circuit while the efficiency is greatly increased by reducing the effective resistance. The re-radiation resistance, which may be considered equal to the radiation resistance, cannot be reduced, but all joulean loss should be made as small as practicable.

The effect of resistance and conductance upon the frequency is generally only of theoretical interest, but becomes appreciable in a sustained oscillation receiving or measuring circuit which is highly damped or sharply tuned.

The expressions derived are based upon Ohm's law; conductances, such as detector current and corona current, do not follow this law but approach more nearly to it than does an equivalent resistance.

SUMMARY: The free oscillations produced on a circuit having capacity, inductance, resistance, and conductance (leakance) are studied. The transient and permanent conditions with sustained oscillations are similarly treated. The resonance and energy relations of such circuits are carefully considered, together with the influence of conductance and resistance on decrement of the circuit and period thereof.