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CONTENTS

At the end of this number are the title page, page of general information, and table of contents pages for the entire Volume 8 (1920) of the PROCEEDINGS. These last may be suitably placed at the beginning of the volume for binding.

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MULTIPLEX RADIO TELEGRAPHY AND TELEPHONY*

By

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Various methods have been described for the simultaneous reception with a single antenna of radio signals from more than one transmitter. Similar methods have been recorded for multiplex transmission.† These methods of multiplexing may be classified as follows:

(1) Those employing several radio frequencies and obtaining selectivity by means of radio frequency tuning.

(2) Methods using a single radio frequency and several tone frequencies, selectivity being obtained by audio frequency tuning.

(3) The use of a single radio frequency modulated by several intermediate frequencies each of which has been modulated by an audio frequency. Selectivity is obtained in this case by tuning at the intermediate frequencies.

The principal example of (1) is the divided antenna method employed by Marconi for both transmission and reception and which may be used for either telegraphy or telephony. Method (2) has been used by the Telefunken Company,[1] but has never proven satisfactory owing to difficulties arising from strays. Obviously this method is not applicable to telephony. The third method was originated by Mr. R. A. Heising[2] and is applicable to both telegraphy and telephony. Altho somewhat more complicated than method (1), it is superior in so far as secrecy of transmission is concerned. A consideration of the effect of modulation shows that this method utilizes a considerable range of radio frequencies.

* Received by the Editor, January 9, 1920. Presented before the Seattle Section of the Institute, October 15, 1919; presented before the Institute at New York, February 4, 1920.

† Since the preparation of this paper, it has come to the writer's attention that John Stone Stone patented in 1905 circuits similar to those described for multiplex transmission and reception.

[1] Zenneck, "Wireless Telegraphy," page 325.

[2] Craft and Colpitts, "Radio Telephony," "Proceedings of the American Institute of Electrical Engineers," March, 1919.

Let $\omega_1 = 2\pi \times$ (Carrier Frequency).

$\omega_2 = 2\pi \times$ (Modulation Frequency).

i = Antenna Current.

I_1 = Amplitude of the carrier frequency.

I_2 = Amplitude of the modulation frequency.

k = A constant depending on the degree of modulation.

Then $i = I_1 \sin \omega_1 t$

But I_1 is variable, $I_1 = I_2 \sin \omega_2 t + k$

$i = I_2 \sin \omega_1 t \sin \omega_2 t + k \sin \omega_1 t$

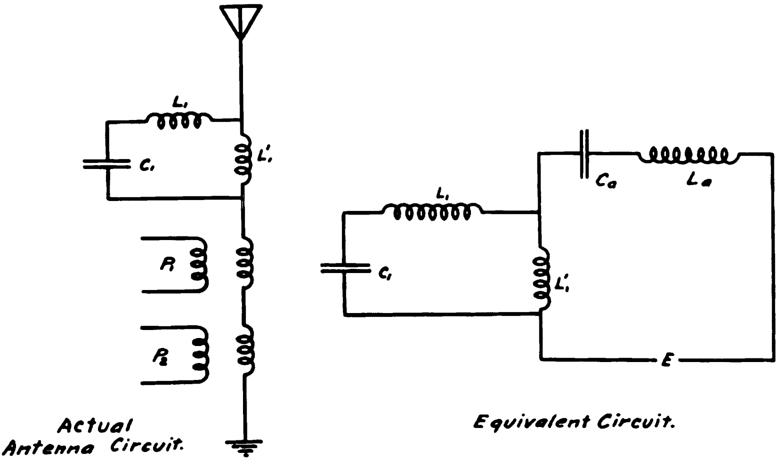
$$i = \frac{I_2}{2} \cos(\omega_1 - \omega_2)t - \frac{I_2}{2} \cos(\omega_1 + \omega_2)t + k \sin \omega_1 t$$

From this last equation, it is apparent that not only the carrier frequency will be present, but also the sum and difference of the carrier and modulation frequencies. Each of these frequencies will be further broadened by voice or other audio frequency modulation. The intermediate frequencies used are of the order of 30 kilo-cycles per second, and therefore the antenna must operate at radio frequencies differing by about 60 kilo-cycles per second. This is not possible at wave lengths exceeding about 400 meters without the use of some special device in the antenna circuit.

From these considerations it would seem that the most simple and practical method of multiplexing a radio system is the use of several radio frequencies, that is, method (1). In order to employ successfully a number of radio frequencies in the same antenna circuit, it is necessary that the antenna be resonant to each of the frequencies, that is, that its reactance be zero for the particular frequencies used. This may be accomplished by the use of certain types of impedance networks in series with the antenna. Figure 1 shows the simplest application of this method. The antenna circuit is given zero reactance for two frequencies by the loop $L_1 C_1$ coupled to the antenna by the common inductance L'_1. Figure 2 shows the equivalent lumped circuit. In this circuit the low frequency antenna capacity is assumed lumped at C_a. L_a is made equal to the sum of the inductances of the two coupling coils, (and load coil if one is used), plus one-third the inductance of the antenna for uniform current.[3] The capacity of the actual antenna is most satisfactorily measured by means of an impedance bridge. If then the natural wave length of the antenna is measured, the induc-

[3] Circular 74 of the Bureau of Standards, page 73.

tance may be calculated. At the transmitter, the two radio frequency sources may be loosely coupled to the antenna by means of the coils P_1 and P_2 as shown in Figure 1. The frequencies of these two sources must, of course, correspond to the two points of zero reactance for the antenna circuit. At the



FIGURES 1 AND 2

receiver P_1 and P_2 would be secondary coils forming parts of two closed circuits each tuned to one of the frequencies to be received. The coupling in both the transmitter and the receiver must be quite loose in order that the points of zero reactance may not be shifted or additional ones introduced.

In a circuit such as shown in Figures 1 and 2, neglecting resistance, the loop reactance is:

$$X_l = \frac{\omega L_1' - \omega^3 C_1 L_1 L_1'}{1 - \omega^2 L_1 C_1 - \omega^2 L_1' C_1}$$

The antenna reactance is:

$$X_a = \frac{\omega^2 L_a C_a - 1}{\omega C_a}$$

The total reactance of the circuit then is:

$$X_t = \frac{\omega^2 L_a C_a - 1}{\omega C_a} + \frac{\omega L_1' - \omega^3 C_1 L_1 L_1'}{1 - \omega^2 L_1 C_1 - \omega^2 L_1' C_1} \qquad (2)$$

The design of such a duplex antenna is much simplified by tuning the loop and antenna circuits independently to the same frequencies. The following familiar relations then hold:

$$K = \frac{L_1'}{\sqrt{(L_1+L_1')(L_a+L_1')}}$$

where K = Coupling coefficient.
L_a+L_1' = Total antenna inductance.
L_1+L_1' = Total loop inductance.

$$f_1 = \frac{f}{\sqrt{1+K}} \qquad (3)$$

$$f_2 = \frac{f}{\sqrt{1-K}} \qquad (4)$$

where f is the frequency to which the antenna and loop have been independently tuned, and f_1 and f_2 are the frequencies for which the reactance is zero

In order to test the practicability of such a method, a duplex radio telephone transmitter was installed in the Electrical Engineering Laboratory of the University of Washington The constants of the antenna circuit used are given in Table 1. The reactance of the equivalent antenna circuit has been calculated for various frequencies and the results are shown graphically in Figure 3. The dotted line shows the loop reactance, the dashed line the antenna reactance, and the full line the reactance of the complete circuit. Two points of zero reactance are to be noted, at 143 and 159 kilo-cycles per second, corresponding to wave lengths of 2,100 and 1,890 meters, respectively

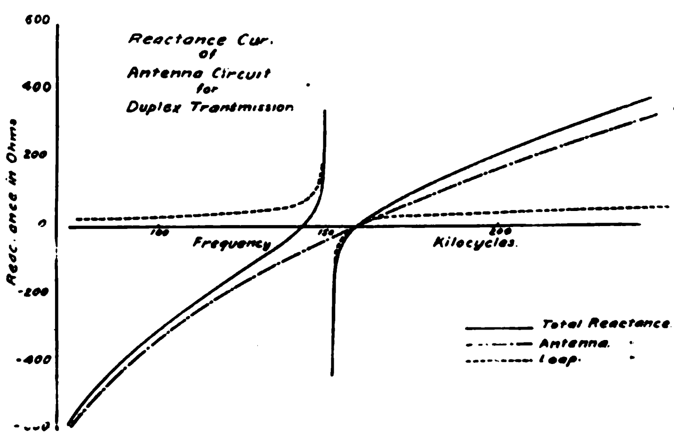


FIGURE 3

TABLE 1

$C_a = 0.00283$ microfarads
$C_1 = 0.00250$ microfarads
$L_1' = 42.1$ microhenrys
$L_a = 356.$ microhenrys
$L_1 = 408.$ microhenrys

The actual circuit employed in the transmitter is shown in Figure 4. Western Electric type "E" vacuum tubes were used,

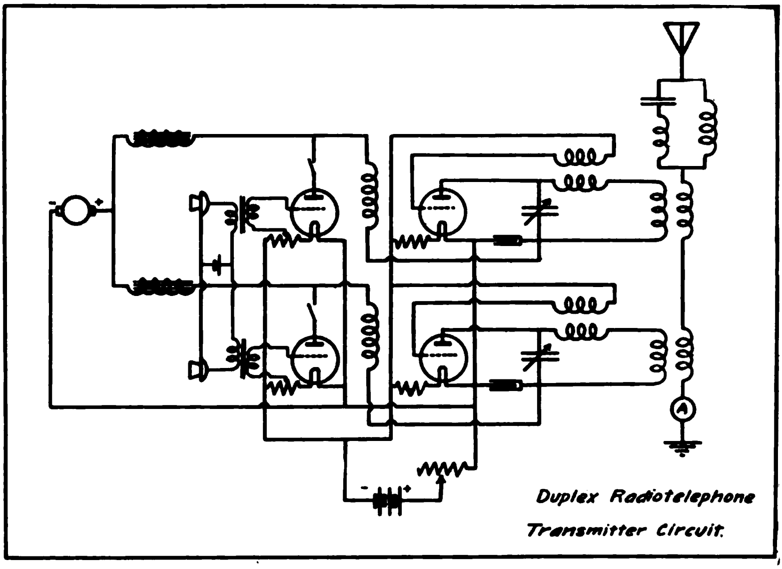


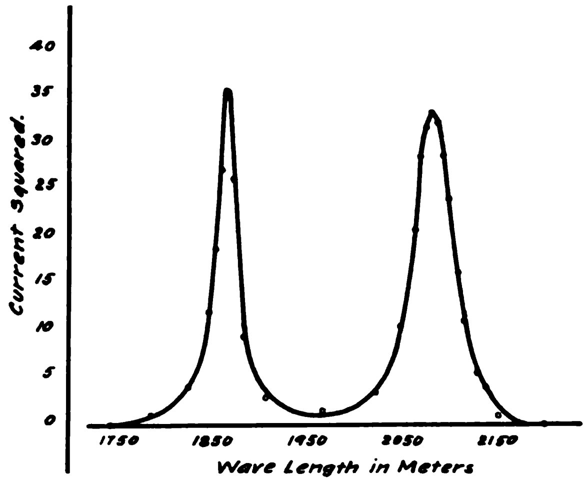
FIGURE 4

both for generation and modulation, the Heising system of modulation being employed.[4] A check of the frequency and wave length of the emitted waves was obtained by taking a resonance curve (Figure 5) with a decremeter placed in inductive relation to the transmitting antenna. The two waves as measured differ less than one per cent from the calculated values. The resonance curve is for the unmodulated waves.

A working test was obtained by installing a duplex receiver at the Seattle Young Men's Christian Association, about five miles (8 km.) distant from the University. The circuits used

[4]Craft and Colpitts, "Radio Telephony," Proceedings of the American Institute of Electrical Engineers," March, 1919, page 360.

are shown schematically in Figure 6. The design of the antenna circuit was similar to that at the transmitter. Two wire telephone lines were available during part of the tests, and were used for talking back to the University as no transmitting apparatus was installed at the Young Men's Christian Association.



Resonance Curve.
of
Duplex Radio Telephone.

FIGURE 5

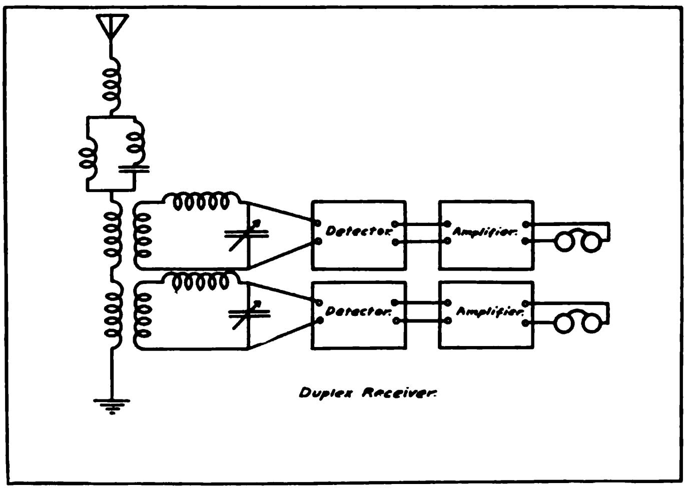


FIGURE 6

With this arrangement, two people at the Young Men's Christian Association could converse with two others at the University with practically no cross talk between the two waves used. Other tests were made allowing a phonograph to play on one of the wave lengths while the other was used for talking. Practically perfect separation was obtained. It was found that a separation of about 15 kilo-cycles between the two carrier waves was the minimum which could be satisfactorily used for telephony, but that for telegraphy the waves could be much closer together.

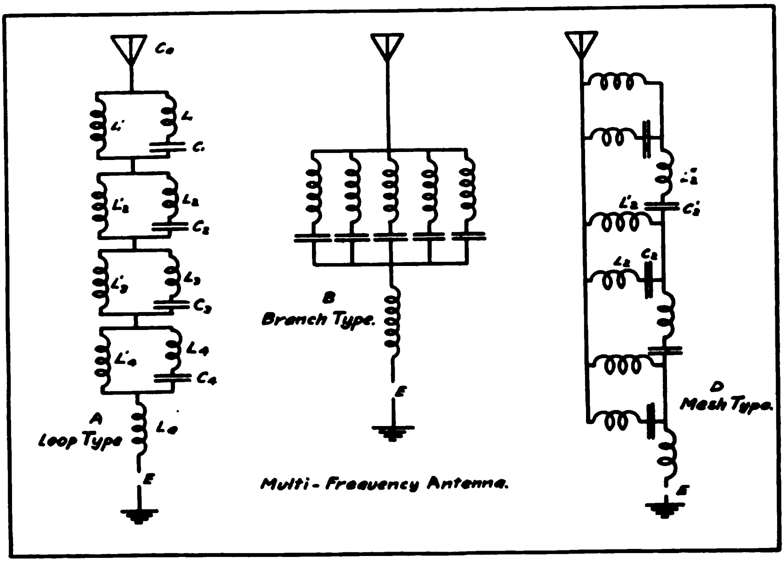


FIGURE 7

The successful completion of these tests suggested that further experiments be conducted with a greater number of frequencies. There are many networks that can be used to give the antenna the desired characteristics. Three typical ones are shown in Figure 7. The *series loop type* shown in Figure 7A gives one more point of zero reactance than there are loops in series with the antenna. The *branch type*, Figure 7B, has as many points of zero reactance as there are branches in the network. The number of points given by the *mesh type*, Figure 7D, is equal to twice the number of units, a unit being defined as consisting of a loop such as as L_2' L_2 C_2 and a series part such as L_2'' C_2' between loops.

The series loop type of network seems to offer the greatest flexibility and ease of design, and was the most thoroly investigated. An experimental network of this type used with an actual antenna to give five points of zero reactance was set up in the University laboratory. The circuit constants used are given in Table 2, the notation referring to Figure 7A. The antenna was excited by five independent vacuum tube generators inductively coupled to the antenna circuits. The coupling coils were inserted in series at the point E in Figure 7A. Figure 8 is a photograph of this "set up." The network is seen on the

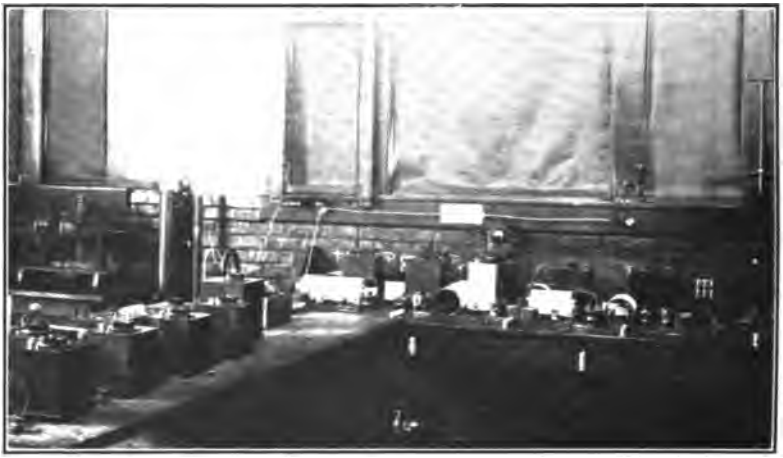


FIGURE 8 -Five-wave Multiplex Radio Transmitter

table to the left and the generator circuits and coupling coils on the table in the background. When using a single tube for each generator the operation was somewhat critical, due to reactions caused by coupling to the antenna circuit. This was overcome by using a small oscillator tube and an amplifier power tube for each of the five waves. The frequencies delivered were then independent of what kind of a system the amplifier tube fed, and depended only on the constants of the circuits of the oscillator tubes.

TABLE 2

C_a = 0.00127 microfarads	L_2' = 213. microhenrys
L_a = 314. microhenrys	L_3' = 179. microhenrys
L_1 = 428. microhenrys	L_4' = 157. microhenrys
L_2 = 522. microhenrys	C_1 = 0.00148 microfarads
L_3 = 922. microhenrys	C_2 = 0.00161 microfarads
L_4 = 980. microhenrys	C_3 = 0.00141 microfarads
L'_1 = 155.5 microhenrys	C_4 = 0.00173 microfarads

The resonance curve shown in Figure 9 was taken by bringing a decremeter in inductive relation to the antenna. The

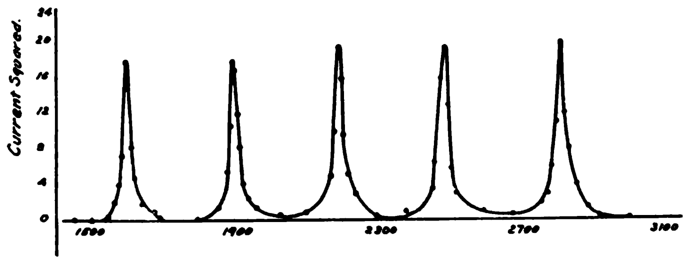


FIGURE 9

reactance of this antenna system has been computed for a number of frequencies and the results plotted in Figures 10 and 11. Figure 10 gives the reactance curves of the separate loops and of the remainder of the circuit, and Figure 11 the reactance of the complete circuit. This last curve is obtained by adding the five curves of Figure 10 algebraically. The discontinuities or points of infinite reactance correspond to the natural frequencies of the loops and the points of zero reactance to the working points or frequencies which are to be radiated by the antenna.

Equation (2) may be extended to apply to a network of this type having four loops by the addition of three more terms to represent the reactance of the additional loops. This equation then becomes:

$$X_t = \frac{\omega^2 L_a C_a - 1}{\omega C_a} + \frac{\omega L_1' - \omega^3 C_1 L_1 L_1'}{1 - \omega^2 C_1 L_1 - \omega^2 C_1 L_1'} + \frac{\omega L_2' - \omega^3 C_2 L_2 L_2'}{1 - \omega^2 C_2 L_2 - \omega^2 C_2 L_2'}$$
$$+ \frac{\omega L_3' - \omega^3 C_3 L_3 L_3'}{1 - \omega^2 C_3 L_3 - \omega^2 C_3 L_3'} + \frac{\omega L_4' - \omega^3 C_4 L_4 L_4'}{1 - \omega^2 C_4 L_4 - \omega^2 C_4 L_4'} \quad (5)$$

The notation refers to Figure 7A.

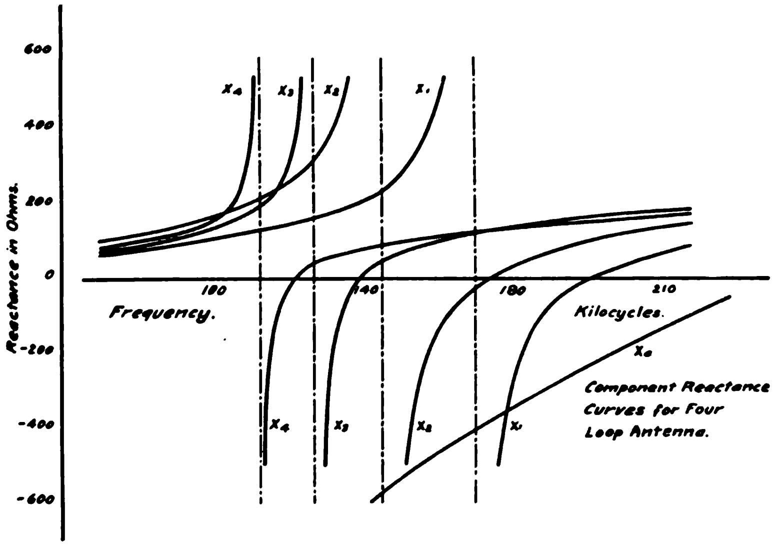


FIGURE 10

In designing a network for use with a given antenna, the problem resolves itself into one of determining the circuit constants necessary to give the points of zero reactance at the desired working frequencies or wave lengths. It will be shown

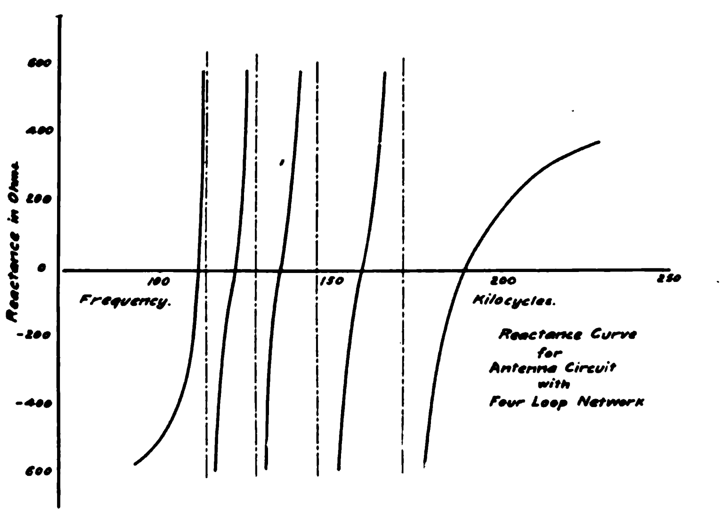


FIGURE 11

how this may be done for a system having five points of zero reactance. Reducing to a common denominator equation (5) becomes:

$$X_t = \frac{A_o\,\omega^{10}+A_2\,\omega^8+A_4\,\omega^6+A_6\,\omega^4+A_8\,\omega^2+A_{10}}{A_1\,\omega^9+A_3\,\omega^7+A_5\,\omega^5+A_7\,\omega^3+A_9\,\omega} \tag{6}$$

where:

$$A_o = C_a\,[L_a\,a_1\,a_2\,a_3\,a_4+L_1'\,\beta_1\,a_2\,a_3\,a_4+L_2'\,a_1\,\beta_2\,a_3\,a_4 + L_3'\,a_1\,a_2\,\beta_3\,a_4+L_4'\,a_1\,a_2\,a_3\,\beta_4]$$

$$A_2 = -C_a\,[L_a\,(a_1\,a_2\,a_3+a_1\,a_2\,a_4+a_1\,a_3\,a_4+a_2\,a_3\,a_4)+L_1'\,(\beta_1\,a_2\,a_3 + \beta_1\,a_3\,a_4+\beta_1\,a_2\,a_4+a_2\,a_3\,a_4)+L_2'\,(\beta_2\,a_1\,a_3+\beta_2\,a_1\,a_4+\beta_2\,a_3\,a_4 + a_1\,a_3\,a_4)+L_3'\,(\beta_3\,a_1\,a_2+\beta_3\,a_2\,a_4+\beta_3\,a_1\,a_4+a_1\,a_2\,a_4) + L_4'\,(\beta_4\,a_1\,a_2+\beta_4\,a_2\,a_3+\beta_4\,a_1\,a_3+a_1\,a_2\,a_3)]+a_1\,a_2\,a_3\,a_4$$

$$A_4 = C_a\,[L_a\,(a_1\,a_2+a_1\,a_3+a_1\,a_4+a_2\,a_3+a_2\,a_4+a_3\,a_4)+L_1'\,(\beta_1\,a_2 + \beta_1\,a_3+\beta_1\,a_4+a_2\,a_3+a_2\,a_4+a_3\,a_4)+L_2'\,(\beta_2\,a_1+\beta_2\,a_3 + \beta_2\,a_4+a_1\,a_3+a_1\,a_4+a_3\,a_4)+L_3'\,(\beta_3\,a_1+\beta_3\,a_2+\beta_3\,a_4+a_1\,a_2 + a_1\,a_4+a_2\,a_4)+L_4'\,(\beta_4\,a_1+\beta_4\,a_2+\beta_4\,a_3+a_1 a_2+a_1\,a_3+a_2\,a_3)] + a_1\,a_2\,a_3+a_1\,a_2\,a_4+a_1\,a_3\,a_4+a_2\,a_3\,a_4$$

$$A_6 = -C_a\,[L_a\,(a_1+a_2+a_3+a_4)+L_1'\,(\beta_1+a_2+a_3+a_4) + L_2'\,(a_1+\beta_2+a_3+a_4)+L_3'\,(a_1+a_2+\beta_3+a_4) + L_4'\,(a_1+a_2+a_3+\beta_4)]+a_1\,a_2+a_1\,a_3+a_1\,a_4+a_2\,a_3 + a_2\,a_4+a_3\,a_4$$

$$A_8 = C_a\,[L_a+L_1'+L_2'+L_3'+L_4']+a_1+a_2+a_3+a_4$$

$$A_{10} = -1$$

$$A_1 = C_a\,a_1\,a_2\,a_3\,a_4$$

$$A_3 = -C_a\,[a_1\,a_2\,a_3+a_1\,a_2\,a_4+a_2\,a_3\,a_4+a_1\,a_3\,a_4]$$

$$A_5 = C_a\,[a_1\,a_2+a_2\,a_3+a_1\,a_3+a_2\,a_4+a_1\,a_4+a_3 a_4]$$

$$A_7 = -C_a\,[a_1+a_2+a_3+a_4]$$

$$A_9 = C_a$$

$$a_1 = C_1\,L_1+C_1\,L_1' \qquad \beta_1 = C_1\,L_1$$

$$a_2 = C_2\,L_2+C_2\,L_2' \qquad \beta_2 = C_2\,L_2$$

$$a_3 = C_3\,L_3+C_3\,L_3' \qquad \beta_3 = C_3\,L_3$$

$$a_4 = C_4\,L_4+C_4\,L_4' \qquad \beta_4 = C_4\,L_4$$

In order that the effective resistance of the antenna system may not be excessive the natural frequencies of the loops must differ considerably from the working frequencies, that is, it is desirable to place the points of infinite reactance about midway between those of zero reactance. The points of infinite reactance are determined by the roots of the denominator of equation (6) equated to zero. If R_1, R_2, R_3, and R_4 are the squares of the respective values of ω for which the reactance is desired infinite, then the following relations satisfy the condition that the denominator equals zero, thereby fixing the infinite points.

$$
\begin{aligned}
a_1 &= \frac{1}{R_1} \\
a_2 &= \frac{1}{R_2} \\
a_3 &= \frac{1}{R_3} \\
a_4 &= \frac{1}{R_4}
\end{aligned}
\tag{7}
$$

This group of equations determines the values of a_1, a_2, a_3, and a_4; and when the frequencies for which the reactance is to be infinite are decided upon, the a's become numerics and their values may be substituted in the numerator.

The frequencies for which the reactance is zero are determined by the roots of the numerator of equation (6) equated to zero. If the squares of the values of ω for which the reactance is desired zero are r_1, r_2, r_3, r_4, and r_5, then the following relations, follow from the theory of equations:

$$
\begin{aligned}
\frac{A_2}{A_o} &= -(r_1+r_2+r_3+r_4+r_5) \\
\frac{A_4}{A_o} &= r_1 r_2+r_1 r_3+r_1 r_4+r_1 r_5+r_2 r_3+r_2 r_4+r_2 r_5+r_3 r_4+r_3 r_5+r_4 r_5 \\
\frac{A_6}{A_o} &= -(r_1 r_2 r_3+r_1 r_2 r_4+r_1 r_2 r_5+r_1 r_3 r_4+r_1 r_3 r_5+r_1 r_4 r_5 \\
&\qquad +r_2 r_3 r_4+r_2 r_3 r_5+r_2 r_4 r_5+r_3 r_4 r_5) \\
\frac{A_8}{A_o} &= r_1 r_2 r_3 r_4+r_1 r_3 r_4 r_5+r_1 r_2 r_4 r_5+r_1 r_2 r_3 r_5+r_2 r_3 r_4 r_5 \\
\frac{A_{10}}{A_o} &= -r_1 r_2 r_3 r_4 r_5
\end{aligned}
\tag{8}
$$

This group consists of five simultaneous equations in the unknowns β_1, β_2, β_3, β_4, L_a, L_1', L_2', L_3', L_4', and C_a. Five of these may be given arbitrary values providing care is taken not to choose values which lead to physically impossible solutions. It has been found convenient to give values to β_1, β_2, β_3, and β_4 and to solve for L_a, L_1', L_2', L_3', and L_4' in terms of C_a. The solution may then be carried out by means of determinants.

Equation (5) may be extended to apply to a general case giving $\frac{n}{2}$ points of zero reactance and having $\frac{n}{2}-1$ loops in circuit.

$$X_t = \frac{\omega^2 L_a C_a - 1}{\omega C_a} + \frac{\omega L_1' - \omega^3 L_1 C_1 L_1'}{1 - \omega^2 L_1 C_1 - \omega^2 L_1' C_1} + \cdots + \cdots \frac{\omega L_m' - \omega^3 L_m C_m L_m'}{1 - \omega^2 L_m C_m - \omega^2 L_m' C_m} \qquad (9)$$

Reducing to a common denominator equation (9) becomes of the following general form:

$$X_t = \frac{A_o \omega^n + A_2 \omega^{n-2} + A_4 \omega^{n-4} \cdots + A_{n-2}\omega^2 + A_n}{A_1 \omega^{n-1} + A_3 \omega^{n-3} + A_5 \omega^{n-5} \cdots + A_{n-3}\omega^3 + A_{n-1}\omega} \qquad (10)$$

The coefficients A_o, A_1, A_2, and so on, are functions of the electrical constants of the system as in the case of the four-loop network. The roots of the denominator equated to zero merely show that the points of infinite reactance occur whenever the frequency is equal to the natural frequency of one of the loops. In a manner similar to that shown for the four-loop network, equations may be set up relating the roots and the coefficients of the numerator. There will be $\frac{n}{2}$ of these equations and n variables. In general, this system of equations will be indeterminate, but by reducing the complexity of the network or by imposing certain arbitrary conditions upon it, the system will become determinate. The complexity of the network may be reduced by removing the inductance from the capacity sides of the loops. This is not usually advantageous as it often leads to inconvenient magnitudes for the remaining circuit constants. Arbitrary values may be assigned (within limits) to β_1, β_2, β_3, and so on, as in the four-loop case, and then the values of L_a, L_1', L_2', and so on, may be determined in terms of C_a. The solution is then applicable to any suitable antenna. The values of L_1', L_2', L_3', and so on, can be assigned and solution made for L_a, β_1, β_2, β_3, and so on, if desired.

Equation (10) can be written in the following condensed form.

$$X_t = \frac{\sum_{q=0}^{n/2} A_{2q}\,\omega^{n-2q}}{\sum_{q=2p+1}^{n-1} A_q\,\omega^{n-q}} \qquad (11)$$

The equivalent expansion for the branch type network will be:

$$X_t = \frac{\sum_{q=2p+1}^{n-1} A_q \, \omega^{n-q+2}}{\sum_{q=0}^{n} A_{2q} \, \omega^{n-2q}} \qquad (12)$$

Equation (12) differs from the reciprocal of equation (11) only in the degree of the odd terms. If, however, the series loop type of circuit, Figure 7A, contains capacity in both branches of the loop its reactance is given by equation (12) instead of equation (11). The mesh type of circuit shown in Figure 7D gives a reactance expansion similar to equation (11).

The following numerical solution, of an antenna with a two-loop network is given to illustrate the method of design and calculation of multi-frequency circuits. The results were checked experimentally to show the degree of accuracy that might be expected.

The desired working frequencies were chosen to correspond to wave lengths of 1,600, 1,900, and 2,200 meters. The frequencies for infinite reactance, that is, the frequencies of the respective loops, were taken to correspond to wave lengths of 1,750 and 2,050 meters. β_1 and β_2, that is, the product of inductance and capacity on the condenser side of the loops, were selected for wave lengths of 1,550 and 1,850 meters.

It is first necessary to obtain the equation for the total reactance of the entire network. This is as follows:

$$X_t = \frac{\omega^2 L_a C_a - 1}{\omega C_a} + \frac{\omega L_1' - \omega^3 L_1 C_1 L_1'}{1 - \omega^2 L_1 C_1 - \omega^2 L_1' C_1} + \frac{\omega L_2' - \omega^3 L_2 C_2 L_2'}{1 - \omega^2 L_2 C_2 - \omega^2 L_2' C_2} \qquad (13)$$

In a manner exactly similar to that previously shown for the four-loop network three simultaneous equations are obtained from equation (13).

$$L_a \left[a_1 + a_2 + \frac{A_2}{A_o} a_1 a_2 \right] + L_1' \left[a_2 + \beta_1 + \frac{A_2}{A_o} a_2 \beta_1 \right] + L_2' \left[a_1 + \beta_2 + \frac{A_2}{A_o} a_1 \beta_2 \right] = - \frac{a_1 a_2}{C_a}$$

$$L_a \left[1 - \frac{A_4}{A_o} a_1 a_2 \right] + L_1' \left[1 - \frac{A_4}{A_o} a_2 \beta_1 \right] + L_2' \left[1 - \frac{A_4}{A_o} a_1 \beta_2 \right] = - \frac{a_1 + a_2}{C_a}$$

$$L_a a_1 a_2 \frac{A_6}{A_o} + L_1' a_2 \beta_1 \frac{A_6}{A_o} + L_2' a_1 \beta_2 \frac{A_6}{A_o} = - \frac{1}{C_a}$$

These equations are of the following form:

$$L_a a + L_1' b + L_2' C = \frac{k}{C_a}$$

where

$a_1 = -5.21983 \times 10^{-12}$ $b_1 = -4.35003 \times 10^{-12}$ $c_1 = -4.41122 \times 10^{-12}$
$a_2 = -2.17443$ $b_2 = -1.49029$ $c_2 = -1.58529$
$a_3 = -1.02424 \times 10^{12}$ $b_3 = -0.80364 \times 10^{12}$ $c_3 = -0.8343 \times 10^{12}$

$k_1 = -1.02155 \times 10^{-24}$
$k_2 = -2.04676 \times 10^{-12}$
$k_3 = -1$

Solving these equations by determinant for L_a, L_1', and L_2', we obtain

$$L_a = \frac{0.00884 \times 10^{-12}}{C_a \times 0.01227} \text{ henrys}$$

$$L_1' = \frac{0.002398 \times 10^{-12}}{C_a \times 0.01227} \text{ henrys}$$

$$L_2' = \frac{0.00264 \times 10^{-12}}{C_a \times 0.01227} \text{ henrys}$$

For $C_a = 0.00283$ microfarads

$L_a = 255$ microhenrys

$L_1' = 69$ microhenrys

$L_2' = 76$ microhenrys

The values of L_1, L_2, C_1, and C_2 are then found from the known values of α_1, α_2, β_1, and β_2.

TABLE 3

λ (Assumed)	λ (Experimental)	Per Cent. Error
1,600	1,620	1.2
1,900	1,900	0.0
2,200	2,260	2.7

The discrepancies observed in the experimental check are probably due to the effect of long leads and to the fact that some of the constants were measured at audio frequencies.

The previous equations may be considerably simplified by making $\beta_1 = \beta_2 = L_a C_a$. A network of this type was set up and checked experimentally. The results obtained are as follows:

TABLE 4

λ (Assumed)	λ (Experimental)	Per Cent. Error
1,600	1,605	0.3
1,900	1,905	0.2
2,200	2,208	0.4

It is believed that such systems of multiplexing by means of multi-frequency antennas will be of considerable value for fleet communication in the Navy. It should also prove valuable in marine coast stations designed to connect ship radio telephone stations with the existing land wire telephone system. Another possible application is that of trans-oceanic communication at long wave lengths.

In high power stations, the output is often limited by the corona voltage of the antenna. It is to be noted that in using a multi-frequency antenna for multiplex transmission, the maximum instantaneous voltage will be the arithmetic sum of the maximums of the several frequencies impressed. This will occur at a relatively low frequency, but demands consideration. Another limitation of the methods proposed is the lack of flexibility of adjustment. The effective resistance of such networks becomes excessive when it is attempted to place the working waves very close together. Some additional work should be done studying the effective resistance and efficiency of multi-frequency antennas. In spite of these limitations the method will probably find considerable application in special instances.

The writers are deeply indebted to Dr. C. E. Magnusson and Prof. L. F. Curtis of the Electrical Engineering Department of the University of Washington for the excellent facilities provided for the experiments described, and for their interest in the work. Thanks is also due to Mr. V. I. Kraft, of the Seattle Young Men's Christian Association for his generosity in allowing the use of their antenna and premises. It should also be mentioned that the Navy Department, thru Lieutenant-Commander Frank Luckel, District Communication Superintendent at the Navy Yard, Puget Sound, very kindly extended the privilege of using the transmitter. The Office of the Chief Signal Officer of the Army co-operated by furnishing the necessary vacuum tubes without which the experiments would have been impossible.

University of Washington,
Seattle, Washington,
December 16, 1919

SUMMARY: The authors classify the chief methods of multiplex radio communication as multi-radio frequency, mono-radio-frequency with multi-audio-frequency, and mono-radio frequency modulated at several super-audio-frequencies each of which is itself modulated at audio frequency.

The authors classify antenna systems for the first method as of the series loop type, the branch type, and the mesh type. Each of these systems has zero reactance at a number of radio frequencies. The series type is investigated mathematically for several cases and checked experimentally with good accuracy.

Duplex and quintuplex radio telephony and telegraphy over 5 miles (8 km.) at wave lengths around 2,000 meters were experimentally accomplished, and are described.

A CONTRIBUTION TO THE THEORY OF MAGNETIC FREQUENCY CHANGERS *

By

J. Zenneck

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I. ARRANGEMENT OF CIRCUITS

The arrangement on which the following theoretical considerations are based, is well known.† It consists of two iron cores, A and B (Figure 1), on each of which are wound three coils; a primary coil (N_1), a direct current coil (N_0) and a secondary coil (N_2).

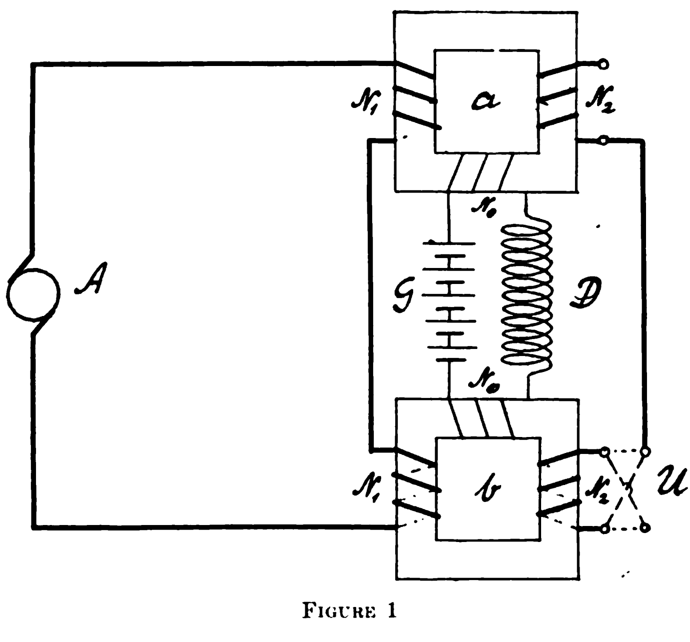

Figure 1

* Presented before The Institute of Radio Engineers, New York, September 1, 1915.

† See "Radio Frequency Changers," by A. N. Goldsmith, Proceedings of The Institute of Radio Engineers, Volume 3, page 55.

The two primary coils N_1 are connected in series, and inserted in the circuit of the radio-frequency alternator A. The direct current coils N_0 are fed by a direct current source G, the circuit of which contains a choke coil D. These coils are wound on the two iron cores in such a way that if, at a given moment the direct current in the iron core A assists the magnetic field of the primary current, in the iron core B the magnetic fields of the direct current and that of the primary current are opposed to each other. The secondary coils N_2 can either be connected in series (assisting) or in opposition by a switch U.

II. ASSUMPTIONS

1. The inductance of the coil D is assumed to be so large that no radio-frequency current of any appreciable amplitude is allowed to flow in the direct current circuit.

2. The size and material of the two iron cores are alike and the corresponding coils have the same number of turns (N_1, N_o, N_2).

3. The magnetic field intensity in the iron cores being H, the magnetic induction B cannot be assumed as being proportional to H. In fact, the operation of the magnetic frequency changers is due precisely to the fact that such proportionality does not exist. The relation between the magnetic induction and the magnetic field intensity may therefore be expressed by an equation:

$$B = sH - s'H^3 \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where s and s' are constants of the iron used.

The magnetic flux in the iron cores (Φ_a and Φ_b) is proportional to B, and the magnetic field intensity H is proportional to the number of ampere turns (F_a, F_b). We may therefore write

$$\left.\begin{aligned} \Phi_a &= pF_a - qF_a^3 \\ \Phi_b &= pF_b - qF_b^3 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

p and q being constants.

The curve corresponding to this equation has a maximum for

$$\left.\begin{matrix} F_a \\ F_b \end{matrix}\right\} = \sqrt{\frac{p}{3q}} = S$$

Evidently we are not entitled to use equation (2) beyond this point, as then by a further increase of the ampere turns, a decrease of the magnetic flux would be produced. This implies that always

$$\left.\begin{matrix} F_a \\ F_b \end{matrix}\right\} \leqq S \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Substituting the maximum value S in equation (2), we get

$$\left.\begin{aligned}\Phi_a &= pF_a\left[1-\frac{1}{3}\left(\frac{F_a}{S}\right)^2\right]\\ \Phi_b &= pF_b\left[1-\frac{1}{3}\left(\frac{F_b}{S}\right)^2\right]\end{aligned}\right\} \quad \ldots\ldots \quad (4)$$

The curve corresponding to this equation is shown in Figure 2.

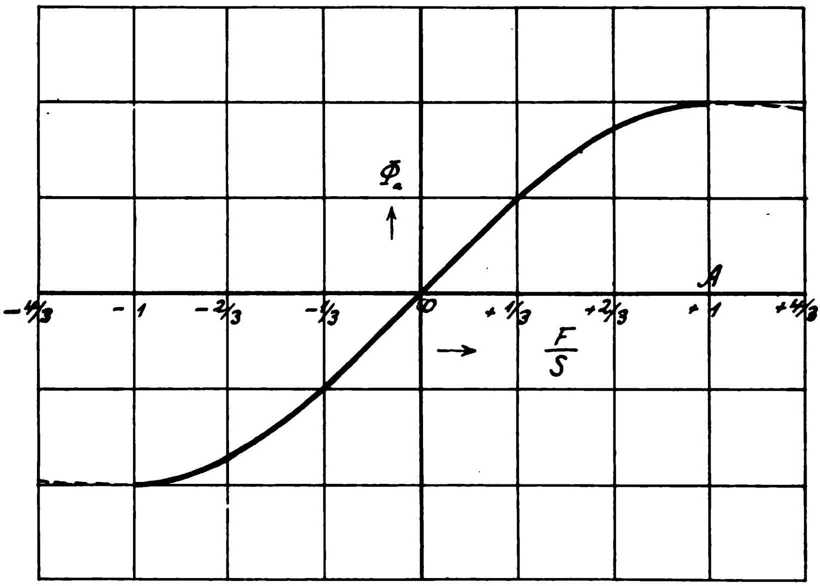

FIGURE 2

III. EMF. IN THE SECONDARY COILS

As the secondary coils in Figure 1 are not inserted in a closed circuit, no secondary current i_2 of any appreciable amplitude is present. The ampere turns F_a and F_b are therefore only due to the primary current i_1 and the direct current I_o, that is

$$\left.\begin{aligned}F_a &= N_1 i_1 + N_o I_o\\ F_b &= N_1 i_1 - N_o I_o.\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (5)$$

As to the primary current, the assumption may be made that it is of substantially sinusoidal form; that is

$$i_1 = I_1 \sin \omega t \;.\;\;.\;\;.\;\;.\;\;. \qquad (6)$$

This assumption seems to be harmless, but is not so under the conditions of equation (4). It is easy to show that under these conditions even when the voltage of the alternator is exactly sinusoidal, in the emf. of the primary circuit a third harmonic is produced, the amplitude of which is of the same order of magnitude as that of the voltage of the alternator. There is indeed but one means for substantially realising the condition of equation (6): namely, inserting into the primary circuit a condenser C_1 and an inductance L_1, thereby substituting the arrangement of Figure 3 for that of Figure 1, and adjusting the capacity C_1 and the inductance L_1 so that the primary circuit is at least approximately in resonance with the frequency of the alternator. The use of equation (6) is therefore restricted to this case.*

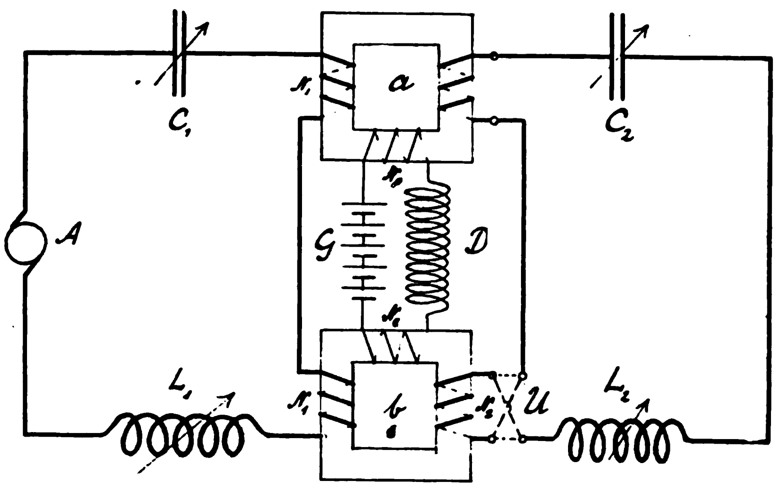

FIGURE 3

a. Connecting first the secondary coils *in series, and additively*, the emf. e_2 induced in them, is determined by the equation

$$e_2 = -N_2 \frac{d}{dt}(\Phi_a + \Phi_b)$$

Now by equations (4), (5) and (6)

$$\Phi_a = p\,(N_1 i_1 + N_o I_o)\left\{1 - \frac{1}{3}\left(\frac{N_1 i_1 + N_o I_o}{S}\right)^2\right\}$$

$$\Phi_b = p\,(N_1 i_1 - N_o I_o)\left\{1 - \frac{1}{3}\left(\frac{N_1 i_1 - N_o I_o}{S}\right)^2\right\}$$

* I am well aware that the mathematical method of this paper is substantially correct only as long as the amplitudes of the current harmonics are small compared to that of the fundamental.

and therefore

$$\Phi_a+\Phi_b=2\,p\,N_1\,i_1\left\{1-\left(\frac{N_o\,I_o}{S}\right)^2-\frac{1}{3}\left(\frac{N_1\,i_1}{S}\right)^2\right\}$$

$$=2\,p\,N_1\,I_1\,sin\,\omega\,t\left\{1-\left(\frac{N_o\,I_o}{S}\right)^2-\frac{1}{3}\left(\frac{N_1\,I_1\,sin\,\omega\,t}{S}\right)^2\right\}$$

Since

$$sin^3\,\omega\,t=\frac{3}{4}\,sin\,\omega\,t-\frac{1}{4}\,sin\,3\,\omega\,t,$$

we get

$$\Phi_a+\Phi_b=2\,p\,N_1\,I_1\left\{1-\left(\frac{N_o\,I_o}{S}\right)^2-\frac{1}{4}\left(\frac{N_1\,I_1}{S}\right)^2\right\}sin\,\omega\,t$$

$$-\frac{p}{6}S\left(\frac{N_1\,I_1}{S}\right)^3 sin\,3\,\omega\,t$$

and therefore

$$e_2=-2\,p\,\omega\,N_1\,N_2\,I_1\left\{1-\left(\frac{N_o\,I_o}{S}\right)^2-\frac{1}{4}\left(\frac{N_1\,I_1}{S}\right)^2\right\}cos\,\omega\,t$$

$$+\frac{1}{2}\,p\,\omega\,N_2\,S\left(\frac{N_1\,I_1}{S}\right)^3 cos\,3\,\omega\,t.$$

This means that in addition to an emf. of the fundamental frequency of the alternator an emf. of a frequency three times greater than that of the alternator is produced. The arrangement discussed is therefore a device for *tripling* the frequency.

The amplitude of the emf. of triple frequency is proportional to the third power of the primary amplitude, but does not depend on the direct current. If therefore no direct current at all were used, the same amplitude of the emf. of triple frequency would result. The only function of the direct current is to diminish the amplitude of the unwelcome emf. of the fundamental frequency; but as long as the restriction expressed by equation (3) holds, this amplitude cannot be reduced to zero.

b. Connecting the secondary coils *in opposition*, the emf. e_2 induced in them is determined by

$$e_2=-N_2\,\frac{d}{d\,t}\,(\Phi_a-\Phi_b).$$

As

$$\Phi_a - \Phi_b = 2\,p\,N_o I_o \left\{1 - \left(\frac{N_1 I_1}{S}\right)^2 - \frac{1}{3}\left(\frac{N_o I_o}{S}\right)^2\right\}$$

$$= 2\,p\,N_o I_o \left\{1 - \left(\frac{N_1 I_1 \sin \omega t}{S}\right)^2 - \frac{1}{3}\left(\frac{N_o I_o}{S}\right)^2\right\}$$

$$= 2\,p\,N_o I_o \left\{1 - \frac{1}{2}\left(\frac{N_1 I_1}{S}\right)^2 + \frac{1}{2}\left(\frac{N_1 I_1}{S}\right)^2 \cos 2\,\omega t - \frac{1}{3}\left(\frac{N_o I_o}{S}\right)^2\right\},$$

we get

$$e_2 = 2\,p\,\omega\,N_2 S\left(\frac{N_o I_o}{S}\right)\left(\frac{N_1 I_1}{S}\right)^2 \sin 2\,\omega t \quad \cdots \quad (7)$$

This equation shows first, that as long as the primary current is purely sinusoidal, the emf. induced in the secondary coils is purely sinusoidal and of twice the frequency of the primary current. The arrangement represented is therefore a device for *doubling* the frequency and an ideal one; as neither the fundamental frequency nor harmonics higher than the second are present in the emf. induced in the secondary coils.

Further, it is evident from equation (7) that for the production of this emf., the direct current is absolutely necessary. For by making the direct current zero, the amplitude of e_2 also would be reduced to zero.

Finally, the amplitude of e_2 being proportional to I_o and to I_1^2, and I_o and I_1 being limited by the condition (3), that is

$$N_1 I_1 + N_o I_o \leq S \qquad \text{or}$$

$$\frac{N_1 I_1}{S} + \frac{N_o I_o}{S} \leq 1,$$

The question arises, which ratio of $\frac{N_1 I_1}{S}$ and $\frac{N_o I_o}{S}$ produces the maximum amplitude of e_2. This question is answered by Figure 4, in which the abscissas are proportional to $\frac{N_1 I_1}{S}$, the ordinates proportional to E_2, the amplitude of e_2, and the assumption has been made, that the value of $\frac{N_1 I_1}{S} + \frac{N_o I_o}{S}$ is the highest possible, namely 1. The curve in Figure 4 shows that the amplitude E_2 reaches a marked maximum for $\frac{N_1 I_1}{S} = \frac{2}{3}$, that is, (compare Figure 2), when the magnetic flux Φ_a due to the alternating primary current varies from the value corresponding to $F_a = +S$ to the value corresponding to $F_a = -\frac{1}{3}S$ and Φ_b from $+\frac{1}{3}S$ to $-S$.

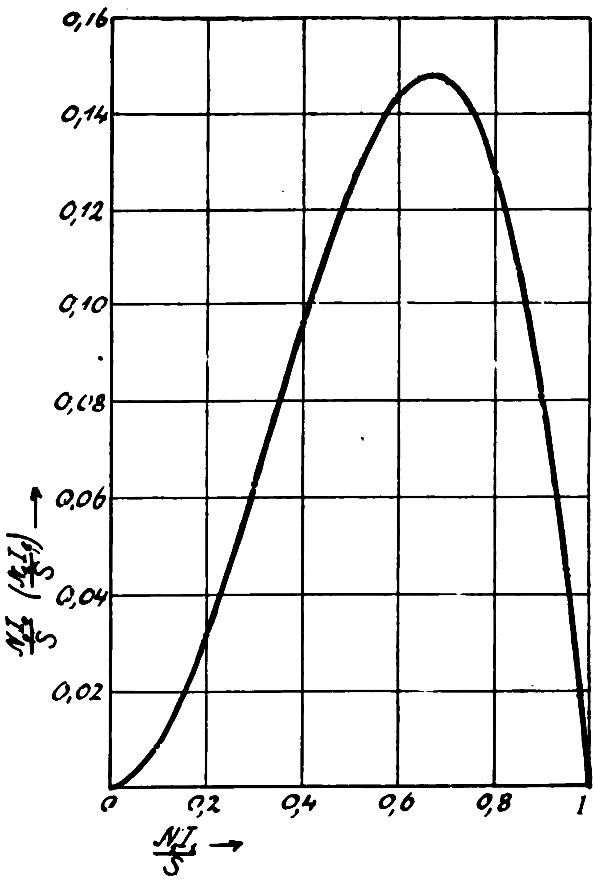

FIGURE 4

IV. THE PRIMARY CIRCUIT

In calculating the primary current i_1 as a function of the emf. e_1 of the alternator, the secondary circuit being disconnected, we restrict ourselves to the case, where the primary circuit is at least approximately in resonance with the frequency of the alternator. In this case we may neglect all harmonics of the current. We retain the equation

$$i_1 = I_1 \sin \omega t$$

and assume $e_1 = E_1 \sin(\omega t + \phi)$.

Then the amplitude I_1 and the phase difference ϕ are the terms to be calculated.

a. The differential equation for the condenser circuit of Figure 3, acted upon by an emf. e_1 is

$$R_1 \frac{d i_1}{d t} + L_1 \frac{d^2 i_1}{d t^2} + N_1 \frac{d^2}{d t^2}(\Phi_a + \Phi_b) + \frac{i_1}{C_1} = \frac{d e_1}{d t} \quad \cdot \cdot \cdot \cdot \quad (8)^*$$

Under the assumptions just mentioned, and using for $(\Phi_a + \Phi_b)$ the term calculated in *IIIa*, its solution is

$$I_1 = \frac{E_1}{\sqrt{R_1^2 + \left[\omega\left(L_1 + L_1'\left\{1 - \left(\frac{N_o I_o}{S}\right)^2 - \frac{1}{4}\left(\frac{N_1 I_1}{S}\right)^2\right\}\right) - \frac{1}{\omega C_1}\right]^2}}$$

$$\tan\phi = \frac{\omega\left(L_1 + L_1'\left\{1 - \left(\frac{N_o I_o}{S}\right)^2 - \frac{1}{4}\left(\frac{N_1 I_1}{S}\right)^2\right\}\right) - \frac{1}{\omega C_1}}{R_1} \qquad (9)$$

where $L_1' = 2p\, N_1^2$. The term

$$L_1'\left\{1 - \left(\frac{N_o I_o}{S}\right)^2 - \frac{1}{4}\left(\frac{N_1 I_1}{S}\right)^2\right\}$$

is to be considered as being the inductance due to the iron cores.

If no direct current were used, and if the amplitude of the primary current were so small that $\frac{N_1 I_1}{S} \ll 1$, we would have obtained the ordinary equations for a condenser circuit with the constant inductance $L_1 + L_1'$, i.e.:

$$I_1 = \frac{E_1}{\sqrt{R_1^2 + \left[\omega(L_1 + L_1') - \frac{1}{\omega C_1}\right]^2}}$$

$$\tan\phi = \frac{\omega(L_1 + L_1') - \frac{1}{\omega C_1}}{R_1} \qquad (10)$$

Comparing equations (9) and (10), it is easily seen that the direct current has not altered conditions greatly; it only diminishes the inductance due to the iron cores by the constant term $-\left(\frac{N_o I_o}{S}\right)^2$. But the term caused by the primary current, $-\frac{1}{4}\left(\frac{N_1 I_1}{S}\right)^2$, changes the whole situation as soon as an appre-

* L_1 being the inductance of the primary circuit outside of the frequency changer.

ciable primary current is present and therefore $\frac{1}{4}\left(\frac{N_1 I_1}{S}\right)^2$ is no longer $\ll 1$.

b. One difference is immediately seen, when we specify the resonance condition; that is, the condition which makes the ratio $I_1 : E_1$ a maximum.

In the ordinary case of constant inductance (equation 10) this condition is:

$$\omega(L_1+L_1') - \frac{1}{\omega C_1} = 0 \quad \cdot \cdot \cdot \cdot \qquad (11)$$

but in our case it is represented by

$$\omega\left(L_1+L_1'\left\{1-\left(\frac{N_o I_o}{S}\right)^2 - \frac{1}{4}\left(\frac{N_1 I_1}{S}\right)^2\right\}\right) - \frac{1}{\omega C_1} = 0 \cdot \cdot \cdot \qquad (12)$$

While in the ordinary case of constant inductance, there is one definite resonance frequency independent of the E.M.F. of the alternator, in our case, there is no general resonance condition at all. The resonance frequency, as might have been expected before hand, depends on the amplitude of the primary current and therefore on the excitation of the alternator. When for one voltage of the alternator the circuit has been adjusted so as to be in resonance to the frequency of the alternator, it is no longer in resonance as soon as the excitation, and therefore the voltage of the alternator, is changed.

In one respect, our case and the ordinary one are alike. When the resonance condition is fulfilled, the amplitude of the current is determined only by the effective resistance R_1 and the equations (9) as well as (10) become

$$I_1 = \frac{E_1}{R_1}; \quad \tan\phi = 0.$$

c. Nevertheless, owing to the fact that because of the term $-\frac{1}{4}\left(\frac{N_1 I_1}{S}\right)^2$, the inductance is no longer constant, the whole situation is changed in a very fundamental way. This can be best shown by drawing for different frequencies the characteristics of the circuit; that is, by plotting curves having as abscissas the amplitudes (or R.M.S. values) I_1 of the current and as ordinates the amplitudes (or R.M.S. values) E_1 of the E.M.F. of the alternator.

In the case of constant inductance, according to equation (10), all these curves would be straight lines, and all points fulfilling the resonance condition (11) would be situated on one of them; namely on the straight line represented by the equation $I_1 = \frac{E_1}{R_1}$.

In our case, corresponding to equation (9) the state of affairs is quite different, as is seen from Figure 5, which has been calculated under the following assumptions:

$$R_1 = 5\,\text{ohms};\ C_1 = 0.1\ \mu\text{f. (mfd.)};\ \frac{L_1'}{L_1} = 10;$$

$$\frac{N_o I_o}{S} = \frac{1}{3};\quad \frac{N_1 I_{1(max)}}{S} = \frac{2}{3}\ \text{for}\ I_{1(max)} = 100\ \text{amperes};$$

$$\omega\left(L_1 + L_1'\left\{1 - \left(\frac{N_o I_o}{S}\right)^2\right\}\right) - \frac{1}{\omega C_1} = 0$$

for $\omega = 2\pi \times 10{,}000$ cycles per sec.

This means that the primary current being so small that $\frac{1}{4}\left(\frac{N_1 I_1}{S}\right)^2 \ll 1$, the circuit is in resonance to the frequency 10,000 cycles per second.

All the characteristic curves of Figure 5 are curved lines, many of them with a falling portion. A falling characteristic of a circuit means unstable conditions.* All points, therefore, which are situated on a falling part of such a curve, altho they are actually given by equation (9), cannot be realized in practice, at least so far as stationary conditions are concerned.

It will be noted that the resonance points defined by equation (12) are nothing more than the points, where the straight line corresponding to the equation

$$I_1 = \frac{E_1}{R_1} = \frac{E_1}{5}$$

(that is, the dashed line OA in Figure 5) is a tangent to the characteristic curves. In those curves in which these points are very near to the boundary between the rising and the falling portions, the resonance is theoretically stable, as well as in the curves which are entirely rising. Yet from a practical point of view, such resonance may be considered as unstable, since a very small variation of the current or the voltage may cause instability, and therefore no operator would be willing to work the set under such delicate conditions of adjustment.

The curves of Figure 5 show another feature, which may be interesting from a practical point of view. Suppose the normal voltage of the alternator to be about 500 volts. Then the most favorable operating conditions would be those represented by

* In the following discussion, for the sake of simplicity, the characteristic of the alternator is assumed to be a straight line parallel to the I axis.

the point A of the curve $n = 10{,}600$; as at this point the maximum current of 100 amperes would be produced by the normal voltage. But this point is so near to the falling, and therefore unstable part of the curve, that no operator would take the risk of working at this point. Therefore, from a practical point of

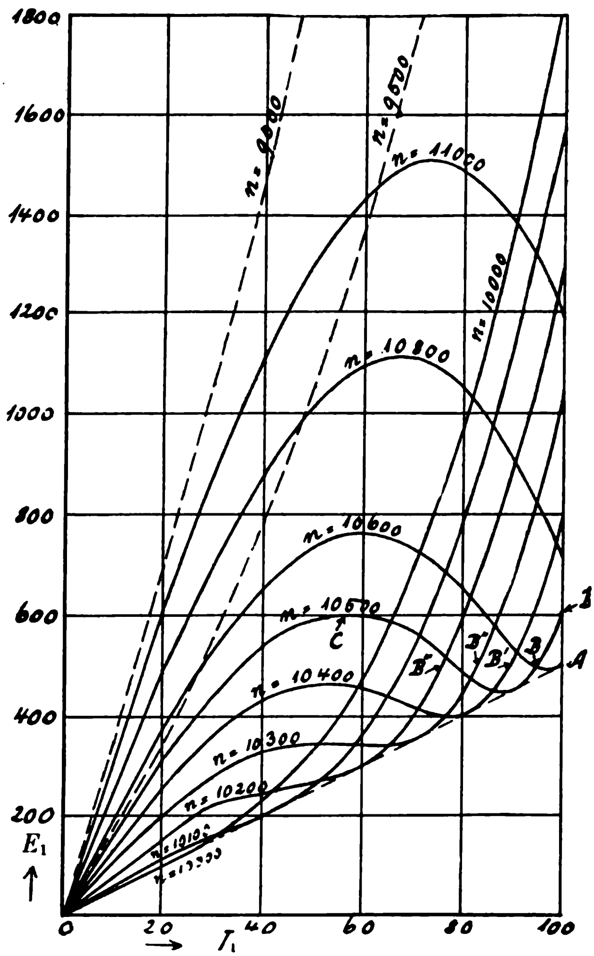

FIGURE 5

view, the best point on the curves is the point B on the curve $n = 10{,}500$. To reach this point, we may either over-excite the alternator up to 600 volts while keeping its frequency constant at 10,500 cycles per second and then proceed along the curve

$n_1 = 10{,}500$ up to the point C. Then, as soon as the voltage is further increased slightly, the current will jump from the value of about 56 amperes at the point C to the value of about 100 amperes at the point D. By then decreasing the excitation of the alternator to the normal value required for 500 volts, we reach the desired point B. If we desire to avoid this troublesome sudden jump in the current, we may proceed in the following way. The alternator is so excited as to give about the normal voltage of 500 volts at a frequency of 10,200 cycles per second. That is, the set is being worked at the point B'''. Then, by keeping the voltage constant, but gradually increasing the frequency to 10,500, we pass to the point B thru the points B'' and B'.

d. The interesting phenomena represented by the curves of Figure 5 appear only if that part of the inductance which is due to the primary current (that is, the term $-L_1' \frac{1}{4}\left(\frac{N_1 I_1}{S}\right)^2$), is allowed to become a rather large percentage of the whole inductance. If the assumptions in **c** are changed merely in such a way that for $I_{1(max)} = 100$ amperes, $\frac{N_1 I_{1(max)}}{S}$ is only $\frac{1}{3}$, the characteristic curves are those of Figure 6. They have no falling parts at all, and therefore the conditions are stable everywhere.

e. How far the characteristics are affected by the value of the resistance R_1, is easily deduced from equation (9), and is demonstrated by Figure 7. The curves of this figure have been calculated under the same assumptions as those of Figure 6, but the resistance R_1, instead of being 5 ohms, as in Figure 6, was assumed to be 1 ohm.

V. THE SECONDARY CURRENT OF A FREQUENCY DOUBLER

In considering the secondary circuit of the arrangement shown in Figure 3, we shall restrict ourselves to the case where the secondary coils are connected in opposition and therefore, according to *IIIb*, a doubling of the frequency is effected.

As to the primary circuit the assumption made in IV may still hold. The assumptions to be made for the secondary circuit correspond to those made for the primary. The secondary circuit may be a condenser circuit (Figure 3) and so adjusted that it is at least approximately in resonance to twice the frequency of the alternator. These assumptions being made, we may neglect all higher harmonics in the secondary circuit and may express the secondary current i_2 by the equation

$$i_2 = I_2 \sin (2 \omega t + a).$$

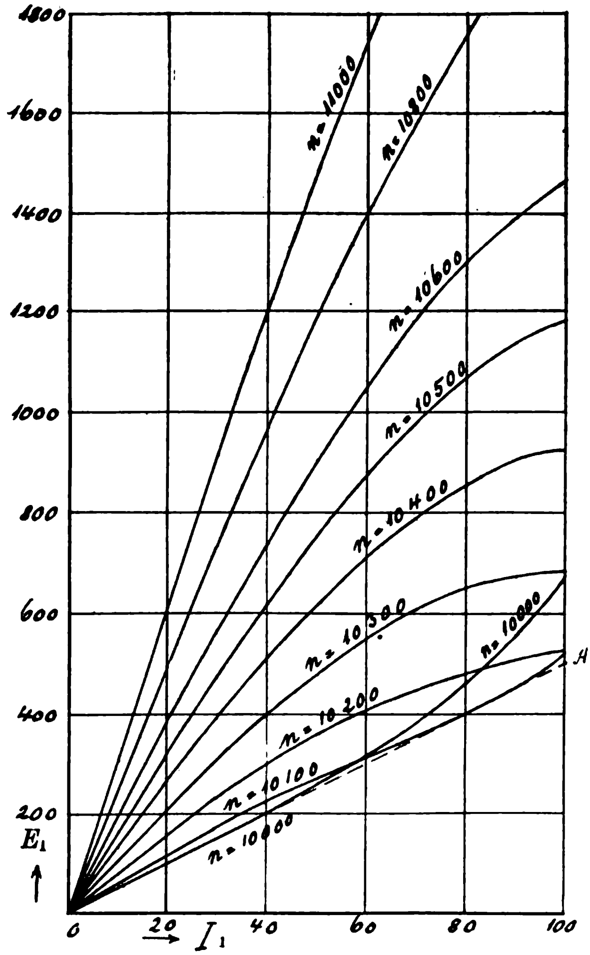

FIGURE 6

The differential equation for the secondary circuit is

$$R_2 \frac{d\, i_2}{d\, t} + L_2 \frac{d^2\, i_2}{d\, t^2} + N_2 \frac{d^2}{d\, t^2} (\Phi_a - \Phi_b) + \frac{i_2}{C_2} = 0 \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \; (13)^*$$

* L_2 being the inductance of the secondary circuit outside of the frequency changer.

The terms Φ_a and Φ_b have to be calculated from equation (4), where

$$\left.\begin{aligned} F_a &= N_1 i_1 + N_o I_o + N_2 i_2 \\ F_b &= N_1 i_1 - N_o I_o - N_2 i_2, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (14)$$

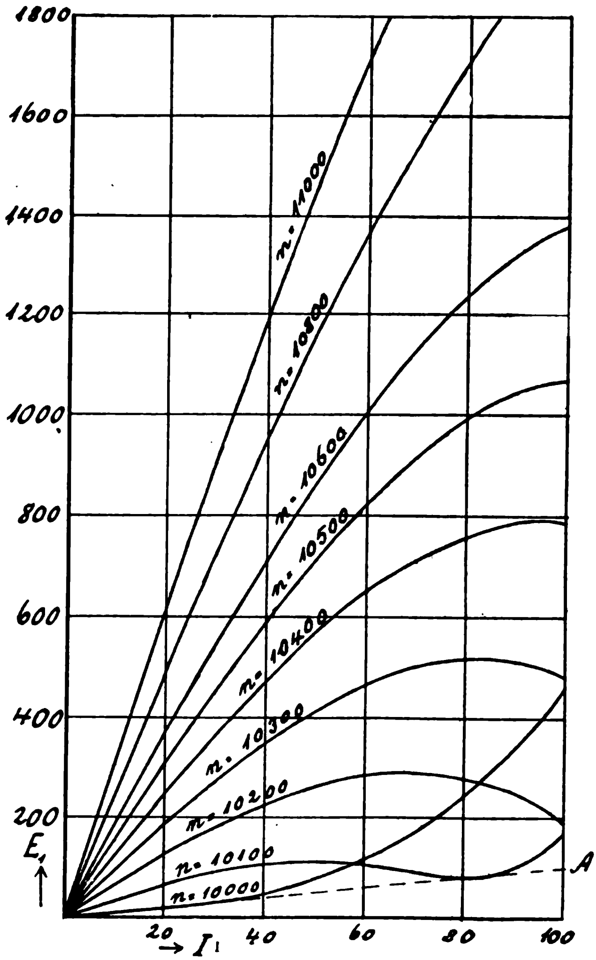

FIGURE 7

since an appreciable secondary current may now be flowing. Substituting these terms in equation (4), we get

$$\Phi_a - \Phi_b = 2p\ N_o I_o \left\{1 - \left(\frac{N_1 i_1}{S}\right)^2 - \left(\frac{N_2 i_2}{S}\right)^2 - \frac{1}{3}\left(\frac{N_o I_o}{S}\right)^2\right\}$$

$$+ 2p\ N_2 i_2 \left\{1 - \left(\frac{N_o I_o}{S}\right)^2 - \left(\frac{N_1 i_1}{S}\right)^2 - \frac{1}{3}\left(\frac{N_2 i_2}{S}\right)^2\right\}$$

$$= 2p\ N_o I_o \left\{1 - \left(\frac{N_1 I_1 \sin \omega t}{S}\right)^2 - \left(\frac{N_2 I_2 \sin(2\omega t + a)}{S}\right)^2 - \frac{1}{3}\left(\frac{N_o I_o}{S}\right)^2\right\}$$

$$+ 2p\ N_2 I_2 \sin(2\omega t + a) \left\{1 - \left(\frac{N_o I_o}{S}\right)^2 - \left(\frac{N_1 I_1 \sin \omega t}{S}\right)^2 - \frac{1}{3}\left(\frac{N_2 I_2 \sin(2\omega t + a)}{S}\right)^2\right\}$$

$$= 2p\ N_o I_o \left\{1 - \frac{1}{2}\left(\frac{N_1 I_1}{S}\right)^2 + \frac{1}{2}\left(\frac{N_1 I_1}{S}\right)^2 \cos 2\omega t - \frac{1}{2}\left(\frac{N_2 I_2}{S}\right)^2 - \frac{1}{3}\left(\frac{N_o I_o}{S}\right)^2\right\}$$

$$+ 2p\ N_2 I_2 \sin(2\omega t + a) \left\{1 - \left(\frac{N_o I_o}{S}\right)^2 - \frac{1}{2}\left(\frac{N_1 I_1}{S}\right)^2 - \frac{1}{4}\left(\frac{N_2 I_2}{S}\right)^2\right\}$$

$$+ \frac{p}{2} \cdot N_2 I_2 \cdot \left(\frac{N_1 I_1}{S}\right)^2 \sin a$$

This gives

$$N_2 \frac{d^2}{dt^2}(\Phi_a - \Phi_b) = -4\omega^2 p N_2 \,.\, N_o I_o \cdot \left(\frac{N_1 I_1}{S}\right)^2 \cos 2\omega t$$

$$- 8\omega^2 p N_2^2 I_2 \left\{1 - \left(\frac{N_o I_o}{S}\right)^2 - \frac{1}{2}\left(\frac{N_1 I_1}{S}\right)^2 - \frac{1}{4}\left(\frac{N_2 I_2}{S}\right)^2\right\} \sin(2\omega t + a)$$

and by substituting this expression in equation (13), we easily get

$$R_2 I_2 = 2\omega p N_2 \,.\, N_o I_o \,.\, \left(\frac{N_1 I_1}{S}\right)^2 \cdot \cos a,$$

$$\left[2\omega\left(L_2 + L_2'\left\{1 - \left(\frac{N_o I_o}{S}\right)^2 - \frac{1}{2}\left(\frac{N_1 I_1}{S}\right)^2 - \frac{1}{4}\left(\frac{N_2 I_2}{S}\right)^2\right\}\right) - \frac{1}{2\omega C_2}\right] I_2$$

$$= -2\omega p N_2 \,.\, N_o I_o \,.\, \left(\frac{N_1 I_1}{S}\right)^2 \sin a \qquad (14a)$$

or

$$I_2=\frac{E_2}{\sqrt{R_2^2+\left[2\omega\left(L_2+L_2'\left\{1-\left(\frac{N_oI_o}{S}\right)^2-\frac{1}{2}\left(\frac{N_1I_1}{S}\right)^2-\frac{1}{4}\left(\frac{N_2I_2}{S}\right)^2\right\}\right)-\frac{1}{2\omega C_2}\right]^2}} \tag{15}$$

$$\tan\alpha=-\frac{2\,\omega\left(L_2+L_2'\left\{1-\left(\frac{N_oI_o}{S}\right)^2-\frac{1}{2}\left(\frac{N_1I_1}{S}\right)^2-\frac{1}{4}\left(\frac{N_2I_2}{S}\right)^2\right\}\right)-\frac{1}{2\,\omega C_2}}{R_2} \tag{15a}$$

where E_2 has the value given by equation (7) and $L_2'=2\,p\,N_2^2$.

Taking into consideration that, for a given primary current, the term $\frac{1}{2}\left(\frac{N_1I_1}{S}\right)^2$ in the denominator of equation (15) is constant as well as $\left(\frac{N_oI_o}{S}\right)^2$, and comparing equation (15) with the corresponding equation (9) for the primary circuit, it is easy to see that both of these equations are of quite the same form. Therefore all that has been said in discussing equation (9) under heading (IV), holds good for this equation also.

VI. THE LOADED FREQUENCY DOUBLER

In IV, the primary current had been calculated for an unloaded frequency changer; that is, on the assumption that $i_2=0$. In V the secondary circuit of a frequency doubler has been considered under the condition that the primary current be given. The loaded frequency doubler which is now to be considered is the case in which the secondary circuit is closed, and the only quantity given for the primary circuit is the emf. e_1 of the alternator. In this case the differential equations (8) and (13) hold at the same time, and for both circuits the number of ampere-turns is determined by equation (14).

Substituting the terms of equation (14) in equation (4), we get

$$\Phi_a+\Phi_b=2\,p\,N_1\,i_1\left\{1-\frac{1}{3}\left(\frac{N_1\,i_1}{S}\right)^2-\left(\frac{N_o\,I_o}{S}\right)^2-\left(\frac{N_2\,i_2}{S}\right)^2-2\left(\frac{N_o\,I_o}{S}\right)\left(\frac{N_2\,i_2}{S}\right)\right\}.$$

Under the same assumptions which had been made above for the primary and secondary circuits, this expression is easily transformed into

$$\Phi_a+\Phi_b=2\,p\,N_1\,I_1\left\{1-\left(\frac{N_o I_o}{S}\right)^2-\frac{1}{4}\left(\frac{N_1 I_1}{S}\right)^2 -\frac{1}{2}\left(\frac{N_2 I_2}{S}\right)^2\right\} sin\,\omega t - 2\,p\,N_1 I_1 \cdot \frac{N_o I_o}{S} \cdot \frac{N_2 I_2}{S} \cdot cos\,(\omega t+a)$$

Introducing this term into the differential equation (8) for the primary circuit, its solution becomes

$$\left.\begin{aligned} I_1 &= \frac{E_1}{\sqrt{(R_1+R_1')^2+\left[\omega\left(L_1+L_1''\right)-\frac{1}{\omega C_1}\right]^2}} \\ tan\,\phi &= \frac{\omega\,(L_1+L_1'')-\frac{1}{\omega C_1}}{R_1+R_1'} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

where

$$R_1'=\omega L_1' \cdot \left(\frac{N_o I_o}{S}\right) \cdot \left(\frac{N_2 I_2}{S}\right) \cdot cos\,a \cdot \quad . \quad . \quad . \quad . \qquad (17)$$

$$L_1=2\,p\,N_1^2$$

$$L_1''=L_1'\left\{1-\left(\frac{N_o I_o}{S}\right)^2-\frac{1}{4}\left(\frac{N_1 I_1}{S}\right)^2-\frac{1}{2}\left(\frac{N_2 I_2}{S}\right)^2 +\left(\frac{N_o I_o}{S}\right)\left(\frac{N_2 I_2}{S}\right) sin\,a\right\} \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

The additional resistance term R_1' is caused by the energy absorption in the secondary circuit. It is easy to show that $R_1' I_1^2$ is identical with $R_2 I_2^2$. Owing to this term the apparent total resistance of the primary circuit R_1+R_1' is no longer constant, but depends on the value of I_2 and a, and therefore on the load of the frequency changer. The terms in L_1'', namely:

$$-\frac{1}{2}\left(\frac{N_2 I_2}{S}\right)^2+\left(\frac{N_o I_o}{S}\right)\left(\frac{N_2 I_2}{S}\right) sin\,a,$$

are due to the reaction of the secondary circuit on the magnetic field of the frequency changer.

As for the secondary circuit, the solution of the differential equation (13) is the same as that given in V, namely equation (15). But since the value of I_1 in this equation is not given in this case, but has to be calculated from equation (16), and since on the other hand, equation (16) contains I_2 which is to be derived from equation (15), the situation is far more complicated than in §§ IV or V. It is indeed scarcely possible from the two equations (15) and (16) to get in a general way a clear idea of what really happens. It may be well therefore to restrict ourselves to the discussion of two special cases.

a. The first case may be defined by the condition that the secondary circuit is always adjusted in resonance to twice the frequency of the alternator.

According to this assumption, and to equations (15) and (15a), we get

$$I_2 = \frac{E_2}{R_2}; \quad \alpha = 0,$$

and from equations (17) and (18),

$$R_1' = \omega L_1' \cdot \left(\frac{N_o I_o}{S}\right)\left(\frac{N_2 I_2}{S}\right) \quad . \quad . \quad . \quad . \quad . \qquad (17a)$$

$$L_1'' = L_1'\left\{1 - \left(\frac{N_o I_o}{S}\right)^2 - \frac{1}{4}\left(\frac{N_1 I_1}{S}\right)^2 - \frac{1}{2}\left(\frac{N_2 I_2}{S}\right)^2\right\} \quad . \quad . \quad . \qquad (18a)$$

According to equation (7) E_2 being proportional to I_1^2, the term $\left(\frac{N_2 I_2}{S}\right)$ becomes proportional to I_1^2 and $\left(\frac{N_2 I_2}{S}\right)^2$ proportional to I_1^4.

I have calculated the characteristics of the primary circuit with the same values as those on which the calculation of the curves in Figure 7 was based; and assuming further that

$$\frac{N_2 I_{2(max)}}{S} = \frac{1}{3}, \quad R_1'_{(max)} = 4 R_1,$$

$I_{2(max)}$ being the secondary current produced by a primary current $I_{1(max)} = 100$ amperes at a frequency of 11,000 cycles per second and $R_1'_{(max)}$ being the value of R_1' under the same conditions.

The curves are reproduced in Figure 8. They show, that by the reaction of the secondary current, conditions are created which resemble very much those discussed in § IV. I therefore need not go into any details.

Supposing the normal voltage E_1 of the alternator to be about 500 volts, the most favorable conditions are represented by the part *AB* of the curve, corresponding to the frequency 10,300 cycles per second; the equilibrium is stable, and for the normal voltage, the primary, and therefore the secondary current, and the output of the frequency changer are remarkably large.

The efficiency η of the frequency doubler is equal to the output divided by the total input. Therefore

$$\eta = \frac{R_1'}{R_1 + R_1'},$$

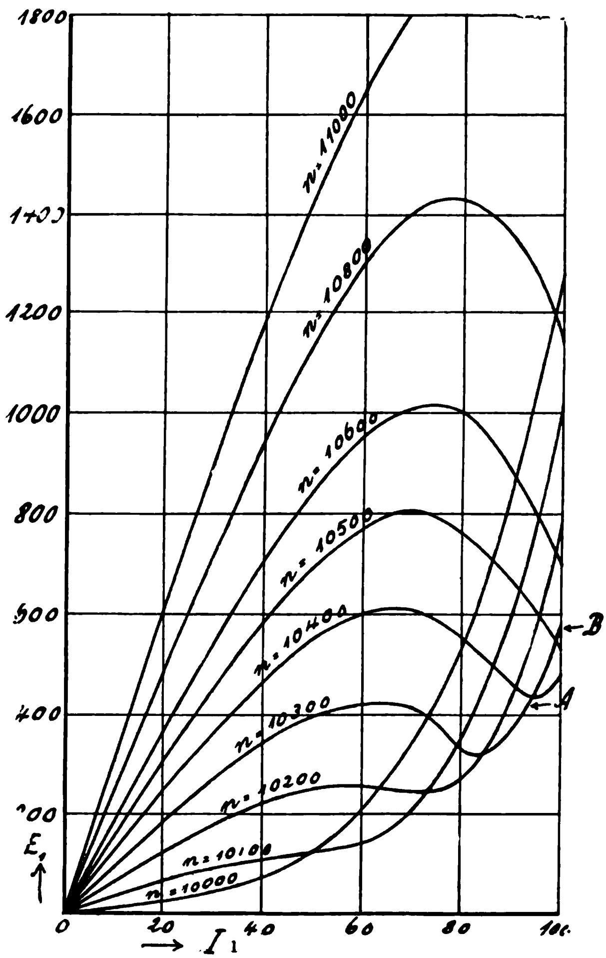

FIGURE 8

and according to equation (17a) it depends largely on the value of I_2 and therefore of I_1. Supposing that for a given frequency and for the current I_2 corresponding to a primary current $I_{1(max)} = 100$ amperes, $R_1' = k \cdot R_1$, we get

$$\eta = \frac{k \cdot \left(\frac{I_1}{100}\right)^2}{1 + k\left(\frac{I_1}{100}\right)^2}.$$

The values of η which correspond to different primary currents and to values of k from 1 to 5, are represented in Figure 9.

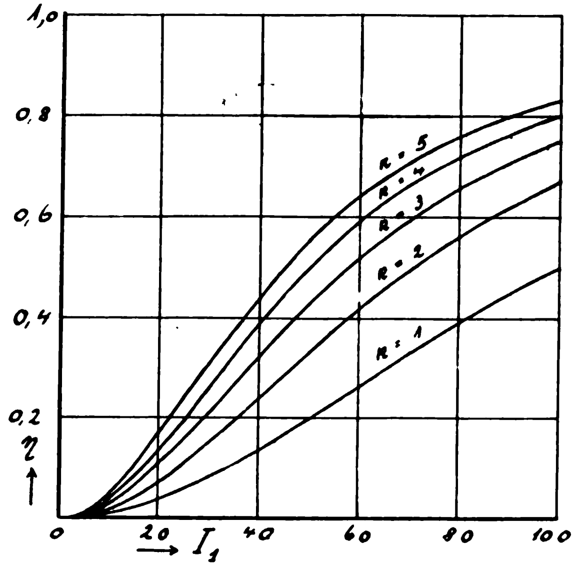

FIGURE 9

b. The second case is based on the assumption that the secondary circuit is adjusted for resonance to one frequency, and then is not varied again. The question to be considered is to what extent the characteristics are changed when the frequency of the alternator is varied.

As soon as the secondary circuit is out of resonance to twice the frequency of the primary current, in equations (17) and (18) a is no longer equal to zero, and in the expression for L_1'' (equation (18)), the term $+\left(\frac{N_0 I_0}{S}\right) \cdot \left(\frac{N_2 I_2}{S}\right) \sin a$ no longer disappears, but may have a considerable influence on the value of L_1''. In order to diminish the mathematical difficulties of the problem, the inductance L_2 of the secondary circuit may be assumed to be so large that in the expression for I_2, (equation (5)), the terms containing the primary and secondary current compared with L_2 are too small to change the whole inductance of the secondary circuit materially. Then the conditions are substantially those of a condenser circuit with a constant inductance and therefore, as is well known, the equations (15) and (15a) can be replaced by

$$I_2 = \frac{E_2}{R_2\sqrt{1+\frac{x}{(d/2\pi)^2}}}; \quad \tan a = -\frac{x}{d/2\pi},$$

where d is the logarithmic decrement of the secondary circuit, and

$$x = \frac{n-n_r}{n},$$

n_r being the resonance frequency.

The decrement d of the secondary circuit may be 0.05, and the frequency n_r (to which the secondary circuit is in resonance) may be $=2\times 10{,}300$ cycles per second. The assumptions made above regarding the primary and secondary circuit may still hold. Then we get the curves of Figure 10 as the char-

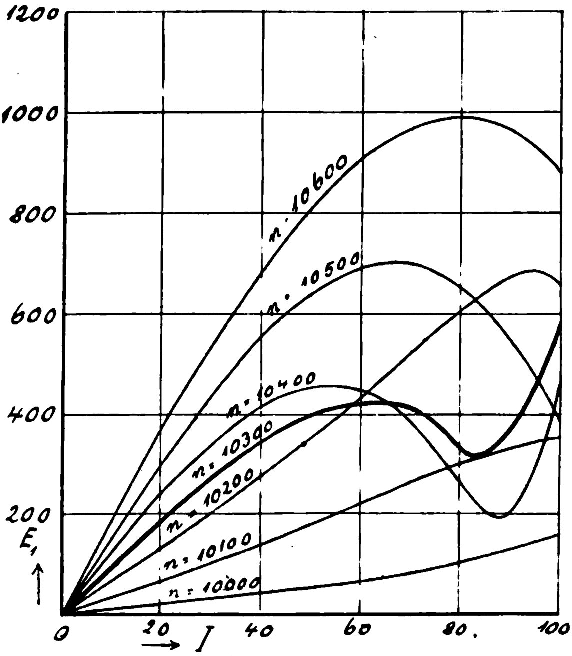

FIGURE 10

acteristics of the primary circuit, according to equations(16), (17) and (18).

These curves show one remarkable feature. In the expression for L_1'' in equation (18) the term $\left(\frac{N_0 I_0}{S}\right) \cdot \left(\frac{N_2 I_2}{S}\right) \sin \alpha$ is positive for frequencies below the resonance frequency and negative for frequencies above it, and under the assumptions made above, this term has a marked influence on the values of L_1''. Owing to this fact the curves for frequencies above the resonance frequency and those below it have quite a different character. Therefore an increase of the frequency of the alternator by 1 per cent. affects the operating conditions in a quite different manner from a decrease of the frequency by the same amount.

VII. INFLUENCE OF THE HYSTERESIS EFFECT

All previous considerations have been based upon the magnetisation curve Figure 2.

a. There is an obvious objection to the use of this curve; namely, that the real magnetisation curve ought to show the hysteresis effect. According to the experiments of Fassbender and Hupka * on the behaviour of iron, we might expect a magnetisation curve like Figure 11 rather than one like Figure 2.

As far as the primary circuit of the unloaded frequency doubler is concerned, this magnetisation curve can easily be represented by an analytical expression which still allows the integration of the differential equation. The magnetic fluxes Φ_a and Φ_b corresponding to Figure 2 are of the form:

$$\Phi_a = U + V \sin \omega t$$
$$\Phi_b = U' + V' \sin \omega t,$$

which may be derived from equations (4) and (5) by neglecting all harmonic terms. In order to get the magnetisation curve for Φ_a of Figure 11 we have only to add the term

$$-r V \cos^3 \omega t \, (1 - \sin \omega t),$$

where in the case of Figure 11, $r = 0{,}45$. We therefore get

$$\Phi_a = U + V \left[\sin \omega t - r \cos^3 \omega t \, (1 - \sin \omega t)\right].$$

The corresponding equation for Φ_b is

$$\Phi_b = U' + V' \left[\sin \omega t - r \cos^3 \omega t (1 + \sin \omega t)\right].$$

* H. Fassbender and E. Hupka, "Jahrbuch der drahtlosen Telegraphie," 6, page 133, 1912.

In consequence of these, we have

$$N_1(\Phi_a+\Phi_b)=A+B\left[\sin\omega t-\frac{3}{4}r\cos\omega t\right], \quad (19)$$

again neglecting all harmonic terms. Comparing this expression with that calculated in III*a*, we find

$$B=L_1'\left\{1-\left(\frac{N_oI_o}{S}\right)^2-\frac{1}{4}\left(\frac{N_1I_1}{S}\right)^2\right\}I_1.$$

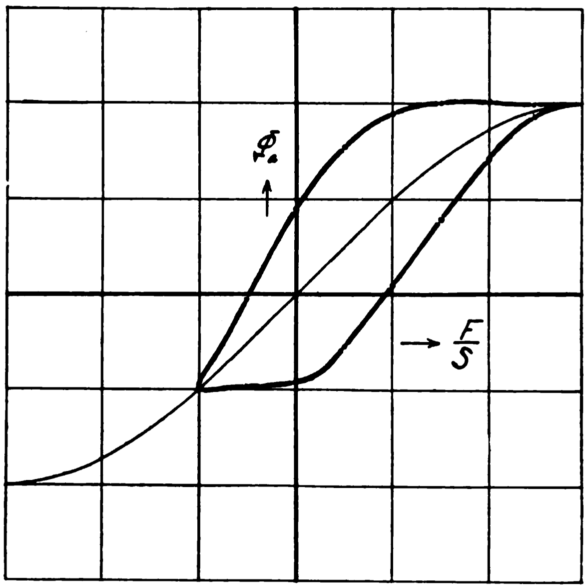

FIGURE 11

Substituting this in equation (8) for the primary circuit of the unloaded frequency doubler, we obtain the result that the conditions in the primary circuit are changed by the additional term $-\frac{3}{4}Br\cos\omega t$ only in so far as there is added to the resistance R_1 a term R_1'' where

$$R_1''=+\frac{3}{4}\omega rL_1'\left\{1-\left(\frac{N_oI_o}{S}\right)^2-\frac{1}{4}\left(\frac{N_1I_1}{S}\right)^2\right\}$$

This term, expressing the iron losses, depends on the primary current. But even assuming $\frac{N_1I_{1(max)}}{S}$ to be $=\frac{2}{3}$ (compare IV**c**) for $I_{1(max)}=100$ amperes, the variation of the value of this term is not more than about 11 per cent., when I_1 is changed from 0 to 100 amperes.

The conditions for the loaded frequency doubler become somewhat more complicated, but it is easy to show that the results obtained in VI are not changed materially.

b. A by far more serious objection may be raised against the harmless looking equation (3)

$$\left.\begin{matrix} F_a \\ F_b \end{matrix}\right\} \leqq S$$

This equation, which is absolutely necessary under the assumption of equation (2), does not allow an increase of the primary and secondary currents such as would make the number of ampere turns greater than that corresponding to the point A in Figure 1. Now, a real magnetisation curve—not taking into account the hysteresis effect—looks rather like the curve Figure 12. There are good reasons to believe that in many respects the results obtained would be very interesting if we were able actually to increase the ampere turns so as to reach the part AB of the magnetisation curve Figure 12. We are therefore seriously handicapped by equation (3). I shall try to surmount this limitation, and I hope to be able, in a later paper, to report on the results of this later investigation.

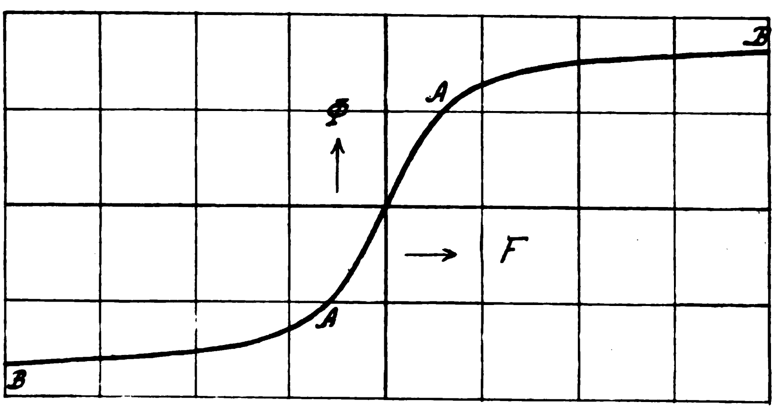

FIGURE 12

SUMMARY: Using iron-core one-to-one transformers, with (or without) supplementary d.c. excitation, the well-known magnetic frequency doublers and triplers can be arranged. The theory of these is developed, assuming a cubic relation between flux and field intensity.

The emf. in the secondary, the effect of supplementary d.c. excita-

tion, the primary current and phase, and the secondary current for the unloaded doubler are derived. The results are extended to the loaded doubler, the efficiency and operating characteristics of which are calculated.

The modification of the derived formulas due to iron hysteresis is obtained.

The theory is clearly illustrated by a number of calculated curves; of which the primary current-voltage curves show an interesting combination of stable (rising) and unstable (falling) portions. The resulting desirable methods of practical operation are fully considered.

The theory is to be further developed to cover phenomena dependent on iron saturation above the knee-point of the magnetization curve.

SOME CHARACTERISTICS OF THE FREQUENCY DOUBLER AS APPLIED TO RADIO TRANSMISSION*

By

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1. General Theory and Value of Sinusoidal EMF. Induced in the Secondary

In the well known Joly-Vallauri system, as shown in Figure 1, we have a pair of transformers T_a T_b excited with direct current A_D, where e_1, e_2 denote the voltages in primary and secondary; e_a, e_b denote the voltages of each transformer T_a, T_b, a_1,a_2 denote the primary currents and secondary currents respectively, and for simplicity, the number of windings are assumed to be all the same and unity.

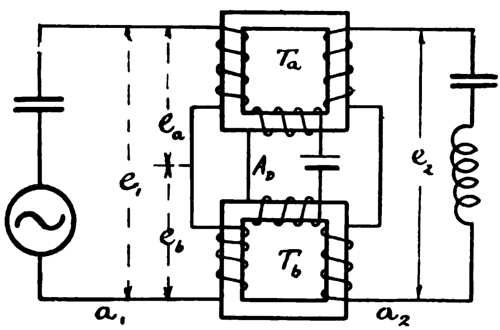


Figure 1

The resonance effect being generally utilized in radio engineering to get the maximum output, the wave forms of oscillating current a_1,a_2 can be assumed to be nearly sine curves, or $a_1 = A_1 \sin \omega t$, $a_2 = A_2 \sin 2 \omega t$. This assumption makes it easy to solve all problems dealing with the frequency transformer.

* Received by the Editor, September 28, 1919.

The Lissajous figure, shown in Figure 2, is taken with a Braun tube, the light spot being deflected horizontally by the primary current a_1 of 50,000 cycles and vertically by the secondary current a_2 of 100,000 cycles in a circuit of the type used in radio practice. This figure clearly shows that the wave forms of a_1 and a_2 both are nearly sinusoidal.

FIGURE 2

The general principle of the frequency doubler is generally assumed to be as follows:

When certain relations exist between A_D and a_1 in the magnetizing current $A_D + a_1$, the emf. e_a induced in the secondary of transformer T_a would take the form

$$e_a = P_1 \sin \omega t + P_2 \sin 2 \omega t$$

Similarly the emf. e_b induced in the secondary of the other transformer T_b, excited by the current $A_D - a_1$ would be

$$e_b = P_1 \sin (\omega t - \pi) + P_2 \sin (2 \omega t - 2 \pi).$$

If e_a and e_b are superposed, we get

$$e_2 = e_a + e_b = 2 P_2 \sin \omega t$$

which is a pure sinusoidal wave of double frequency, the fundamental wave being entirely cancelled out.

The necessary relation of A_D and a_1 required can be easily determined experimentally, but we can also find it graphically by considering the magnetization curve of the iron cores.

Taking the core the magnetization curve of which is shown in Figure 3, if we try to plot the curves of flux and induced emf., assuming arbitrary ampere-turns by trial so as to obtain a curve of secondary induced emf. e_2 of pure sine wave form, we shall find it desirable to take the primary current a_1 so that $A B = 1.6$ to $1.8\ C D$. For example, in Figure 3, taking a_1 so that $A B = 1.8\ C D$, we get the curve ϕ_a and $\frac{d \phi_a}{d t}$ or e_a as in Figure 4.

Adding e_a and e_b, which have the same wave form with 180° phase, difference we obtain the nearly sinusoidal wave of double frequency, as shown in Figure 5.

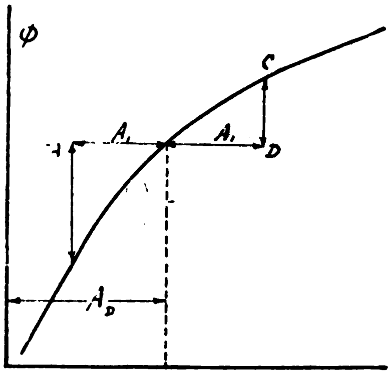


FIGURE 3

From my calculations given below, the relation $AB = 1.667\ CD$ was deduced for the same condition.

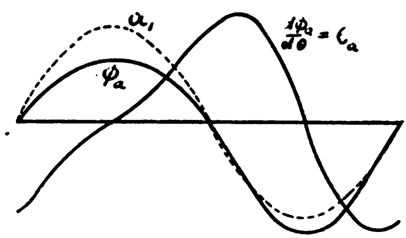


FIGURE 4

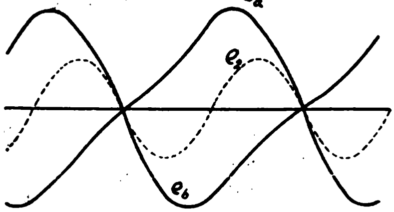


FIGURE 5

In the ideal case, where no useless higher harmonics are considered, the flux in each transformer core would be

$$\phi_a = -\int e_a\, dt = -\int (P_1 \sin \omega t + P_2 \sin 2\,\omega t)\, dt$$

$$= \frac{P_1}{\omega} \cos \omega t + \frac{P_2}{2\,\omega} \cos 2\,\omega t$$

and

$$\phi_{\omega t=0} = \frac{P_1}{\omega} + \frac{P_2}{2\,\omega}.$$

$$\phi_{\omega t=\pi} = -\frac{P_1}{\omega} + \frac{P_2}{2\,\omega}.$$

Taking

$$\frac{P_1}{P_2} = k,$$

we get the relation

$$\frac{\phi_{\omega t=0}}{-\phi_{\omega t=\pi}} = \frac{2\,k+1}{2\,k-1} = X.$$

We now plot the curves e_a', e_a'', e_a''' in Figure 6, assuming

$$e_a' = P_1 \sin \omega t + P_2' \sin 2\,\omega t$$

$$e_a'' = P_1 \sin \omega t + P_2'' \sin 2\,\omega t$$

$$e_a''' = P_1 \sin \omega t + P_2''' \sin 2\,\omega t$$

where $$P_2''' < P_2'' < P_2'$$

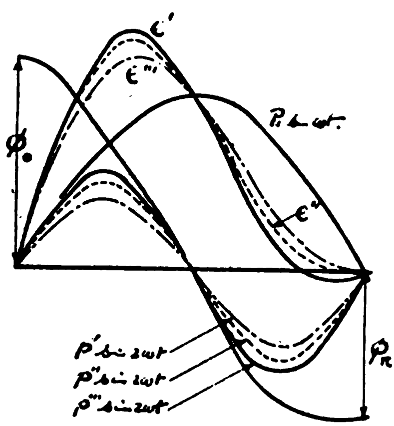


FIGURE 6

In order that the amplitude of the second harmonic should be as great as possible, the curve e_a' is the best of the three given. But it is impossible that the curve e_a pass thru the zero line during the interval from $\omega t = C$ to π, as e_a' does, since $\frac{d\phi_a}{dt}$ can never be zero during that time, as seen from Figure 6. Therefore, in the curve e_a, which would give the maximum realisable second harmonic, $P_2 \sin 2\,\omega t$, the value $\frac{d\phi}{dt}$ should be equal to zero at $\omega t = \pi$.

or

$$\left(\frac{de}{dt}\right)_{\omega t = \pi} = 0,$$

$$\left[\omega P_1 \cos \omega t + 2\,\omega P_2 \cos 2\,\omega t\right]_{\omega t = \pi} = 0.$$

Then we have $\quad P_1 = 2\,P_2, \quad$ or $\; k = 2.$

Substituting this value in the condition X above, we get

$$X = \frac{\phi_{\omega t = 0}}{-\phi_{\omega t = \pi}} = \frac{2\,k+1}{2\,k-1} = 1.66.$$

This means that $A\,B = 1.667\,C\,D$.

Let us call the case of the secondary induced emf. of pure sinusoidal wave form, a "sine wave system."

2. Impulsive Emf. Induced in the Secondary.

In the sine wave system a_1 and e_2 can not be great enough to get the sufficient secondary power, owing to the limitation $AB = (X)\,(CD)$, mentioned above.

In practice, it is usually required to get as much energy as possible, the induced emf. e_2 being used for impulse excitation of the secondary oscillation, and the second harmonic only being effectively resonated by proper choice of the secondary circuit frequency. Therefore it is not essential that the secondary induced emf. be a pure sinusoidal wave of double frequency, but it is usually necessary to obtain the maximum impulsive secondary emf. of double frequency. Thus the flux density change due to a_1 should be large in order to get a large secondary induced emf. for a given size of core. That is, a larger a_1 is required as compared with A_D than for the case of the sine wave system.

Taking $A_1 > A_D$ in the example shown in Figure 7, the effective value of a_1 to be 10 amperes (or $\sqrt{2} \times 10$ as the maximum amplitude A_1), and $A_D = 5$, then the corresponding flux curve can be plotted as shown in the curve ϕ_a (assuming that its saturation curve be as shown in Figure 8) and we can get the curve e_a from the inclination of ϕ_a or $\frac{d\phi_a}{dt}$. Similarly the curve e_b is obtained, which has the same wave form, but with a phase difference of 180° from e_a. The secondary voltage of double

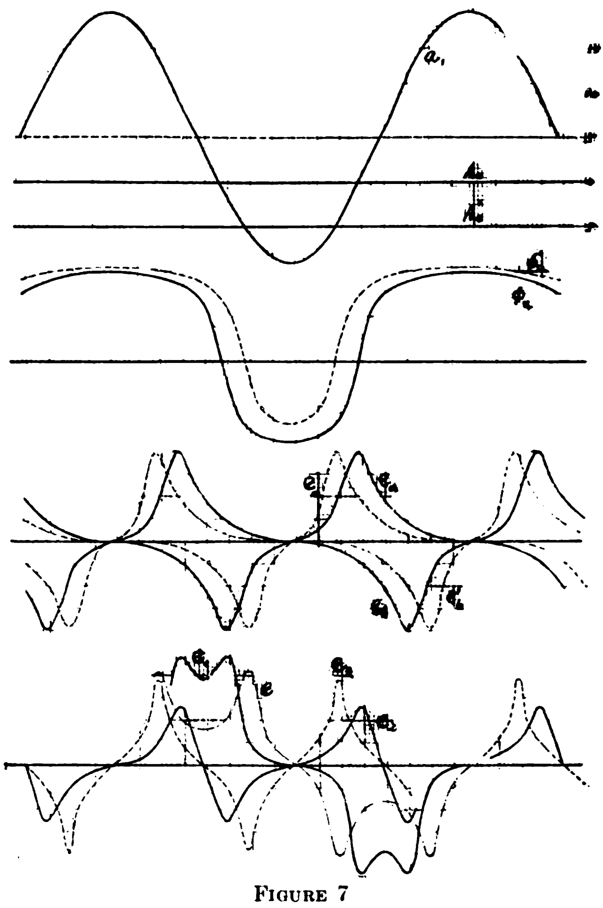


FIGURE 7

frequency will be the sum of e_a and e_b, and the primary voltage e_1 is the difference of e_a and e_b, as shown in Figure 7. But if the excitation $A_D' = 10$ (or effective value of a_1), the flux curve will be ϕ_a', then e_a', e_b' are as plotted, and $e_a' + e_b' = e_2'$ and $e_a' - e_b' = e_1'$ will be as shown in the figure.

As seen in the figure, the wave forms e_a and e_b' both are not sinusoidal but sharp-peaked or impulsive waves, containing many higher harmonics. The maximum amplitude of e_2 does not come at equal intervals while that of e_2' does come with a regular period of double frequency. The amplitude of e_2' is much greater than that of e_2, therefore we can say that A_D should be equal to the effective value of a_1 in order to get the maximum impulsive emf. of double frequency.

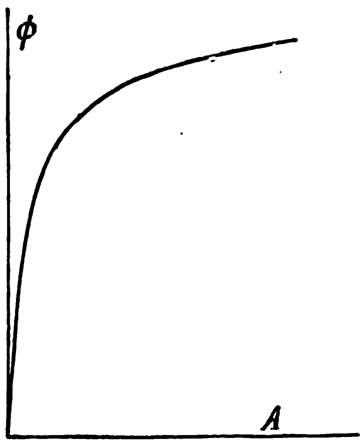


FIGURE 8

The fact is explained as follows:

The amplitude of e_2 will be the maximum at the same phase as the maximum amplitude of e_a and e_b or $\frac{d\phi_a}{dt}$. But $\frac{d\phi_a}{dt}$ should be the maximum when the excitation current $A_D \pm A_1 \sin \omega t$ in the cores becomes zero, if we neglect the effect of the hysterisis loop. Hence $A_D \pm A_1 \sin \omega t$ should be zero at the phase of $\frac{\pi}{4}$ or $\frac{3}{4}\pi$ in order to produce the maximum amplitude of e_a at intervals of $\frac{\pi}{2}$ (or π of double frequency) as seen in Figure 7.

We have

$$A_D - A_1 \sin \frac{\pi}{4} = 0$$

or

$$A_D = \frac{A_1}{\sqrt{2}}.$$

This means that d.c. ampere-turns are equal to the effective value of primary ampere-turns.

If A_D be increased to the point of magnetic saturation, always maintaining the relation $A_1 = \sqrt{2} A_D$, the curve e_a becomes gradually a peaked wave, and the fundamental wave tends to disappear, the position of the maximum amplitude being unchanged.

As an example ϕ', ϕ'', ϕ''' in Figure 9 show the flux curve taking $A_D = \frac{A_1}{\sqrt{2}} = 5$, $A_D = \frac{A_1}{\sqrt{2}} = 10$, $A_D = \frac{A_1}{\sqrt{2}} = 15$ respectively

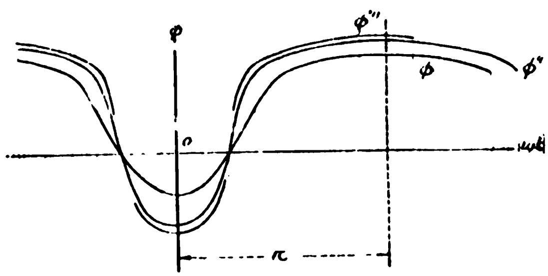


FIGURE 9

in the saturation curve shown in Figure 8. Furthermore, the corresponding induced emf. e_a', e_a'', e_a''' become as shown in Figure 10. Thus if A_D excite the core near to magnetic saturation, the curve e_a become a somewhat peaked wave which consists mainly of the second harmonic, and there is almost no fundamental wave

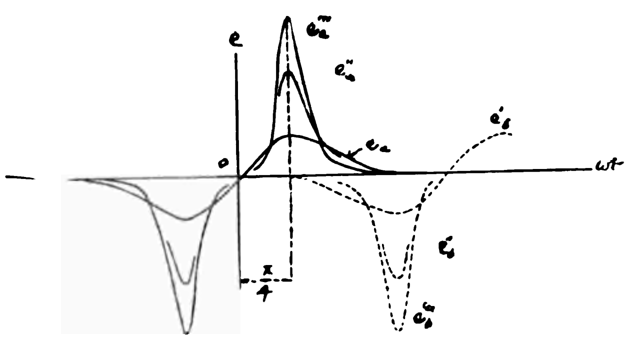


FIGURE 10

component which will have to be cancelled out when e_a is superposed on e_b to get e_2 (see e_a'', e_b'' or e_a''', e_b'''). If A_D excite the core above the knee of the saturation curve, the curve e_a becomes too peaked and narrow, (see the curve e_a''') and the second higher harmonic also tends to disappear.

Therefore it is preferable that the difference of flux change due to $+A_1$ and $-A_1$ shall be as great as possible, taking A_D so that the excitation is at the knee of the saturation curve, (see the curve e_a'' where $A_D=10$; that is A_D is the excitation corresponding to the knee of the saturation curve).

Thus the two conditions $A_D=\frac{A_1}{\sqrt{2}}$ and A_D equal to the excitation corresponding to the knee of the saturation curve are necessary to get the maximum impulsive wave of double frequency.

I will call this system the "impulsive wave system." It is important in practice at the lower radio-frequencies and is discussed in the following sections.

Let us consider a special imaginary case where the magnetic flux of the core increases linearly with excitation up to the knee of the saturation curve (for example, with an excitation of 4 amperes), and past this point, the flux maintains a constant value, as shown in Figure 11.

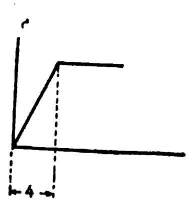


FIGURE 11

Taking $A_D=4$, and the effective value of the primary current (the wave form of which is a pure sine) as 4 amperes, if we plot the flux curve corresponding to the excitation a_1+A_D and a_1-A_D, we get the curves of ϕ_a and ϕ_b as shown in Figure 12. If, then we plot the curves e_a and e_b as calculated from the slope of ϕ_a and ϕ_b, we can see from these curves that the effective value of e_2 in this case is equal to the sum of effective values of e_a and e_b, that the effective value of e_2 is equal to the effective value of e_1, that the wave form of e_1 is sinusoidal, and that the form factor of e_a is equal to that of a sine wave.

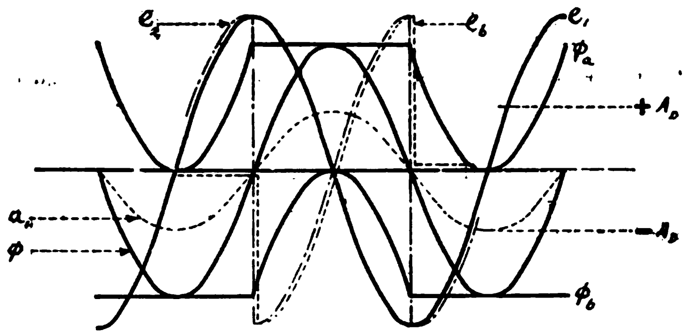


FIGURE 12

3. Distortion of Voltage Wave Due to the Load Current

We will explain the effect of loading the frequency changer, using as an example the core the characteristic of which is shown in Figure 8.

The secondary voltage at no load can be plotted as shown in Figure 13. If the primary current is a_{1_0} and exciting current A_D, then the flux curve will be ϕ_o in Figure 14, and the terminal voltage of each transformer will be e_{a_0}. Therefore we can easily obtain the secondary voltage by adding the ordinates of e_b which have the same wave form but with 180° phase difference.

(a). Inductive Load

On loading the changer inductively, the secondary current lags behind the emf. Taking the curve of the secondary inductive current a_{2l} in Figure 13, which lags 90 degree behind the emf. of no load, the maximum amplitude of which should be at the phase of X in the figure, we can plot the corresponding flux curve of a transformer the exciting current of which is $a_{2l}+a_{1_0}+A_D$, or $L+A_D$, as shown in Figure 14. Thus we get quite an irregular wave form e_{al} by calculating $\frac{d\phi_l}{dt}$ from the curve ϕ_l as shown Figure 15. Therefore, if we try to plot the wave form of e_2 by adding e_{a_0} and e_{b_0}, the wave forms of which are the same but with 180° phase difference, we can easily see that an irregular wave form is obtained, and its amplitude is comparatively low, hence the output can not be great in that case.

(b). Capacity Load

Assuming secondary capacity load current a_{2_c}, with phase 90° ahead of the secondary emf. at no load, the sum of a_1 and a_{2_0}

will be C, as shown in Figure 13. Then, if we plot the flux curve of a transformer corresponding to the exciting current a_1+C or $a_1+a_{2_b}+A_D$, we get the curve ϕ_c as shown in Figure 14.

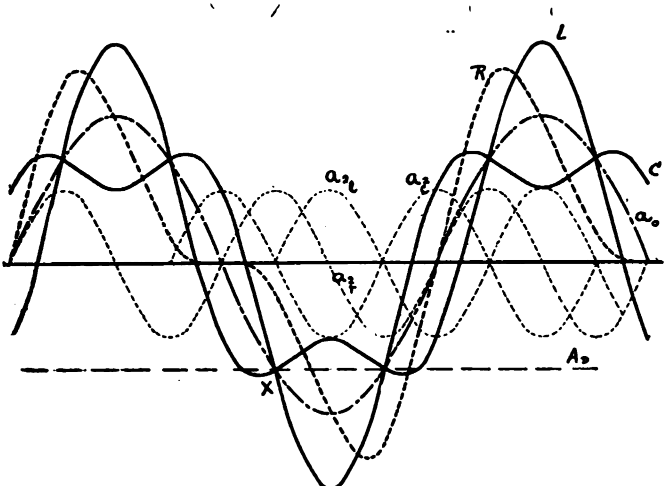


fig. 13.

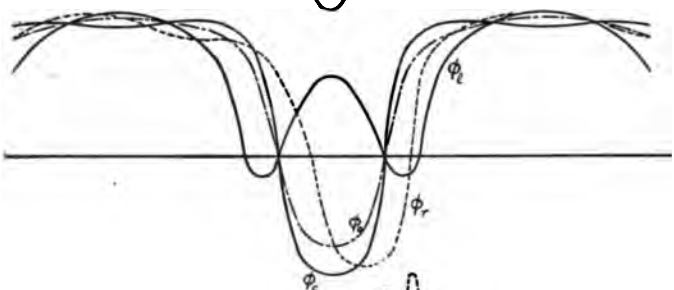


fig. 14.

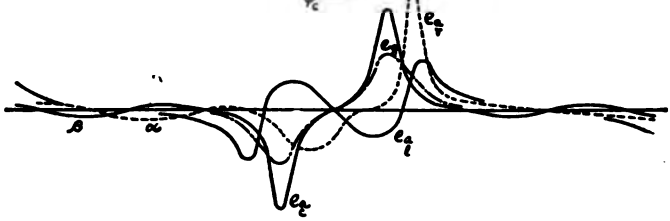


fig. 15.

FIGURES 13 to 15

Similarly we can get e_{a_c} as shownin Figure 15 with maximum amplitude far greater than e_{a_c}. So if we plot e_2 by adding e_{a_e} and e_{b_e}, we find that the amplitude of e_2 is great and the wave has quite a regular form of double frequency where the part β of the curve is obtained when e_{a_e} and e_{b_e} are superposed to get e_2.

These capacity or inductive loads dissipate practically no power since the phase difference of a_2 and e_2 is nearly equal to 90°. Practical operation should always lie between these two, the resonance effect being employed.

(c) Actual Case of Loading

If the maximum amplitude of secondary induced emf. were always at the phase of the point X in Figure 13, the secondary current a_2 should also be in the same phase when the secondary circuit is just at resonance condition. But in the actual case, the phase of the induced emf. e_2 will shift by a certain amount as soon as the secondary current a_2 flows since the wave form of the flux is distorted by the secondary current. For example, assume a_{2_r} the secondary current as shown in Figure 13. Then the curve $a_1+a_{2_r}$ is represented by R, and ϕ_r will be the flux curve for this case. The voltage curve of the transformer will be e_{a_r}, which has an unsymmetrical form as shown, and in this example the secondary current a_{2_r} is advanced about 60° before the induced emf. e_2.

(d) Maximum Output

As just mentioned, the induced secondary emf. e_2 is high when the output is supplied to a capacity load. On the other hand, it is evident that the output at the resonance condition is a maximum if the secondary induced emf. is kept constant. Therefore we can say that the maximum output is obtainable when the secondary current leads the emf. by a certain small amount. This amount depends on the relative values of a_1, a_2, A_D, and on the saturation curve of the core. In practice, a variometer in the secondary circuit is adjusted so as to obtain maximum current or maximum output, since the effective resistance of the secondary circuit is constant. Therefore it should be remembered that the current and induced emf. in the secondary, in practical cases are not exactly in phase but that the current somewhat leads the emf. e_2.

We have next to find how many secondary turns are required to get the maximum secondary output. As we can see from Figure 14, the curve of ϕ_r deviates from ϕ_o because of the secondary current, and the greater the secondary current a_{2_r} the more is the flux curve distorted. At the same time the part a of the curve shown in Figure 15 tends to increase in amplitude which is very unfavorable as regards obtaining large values of e_2 since this part of e_{a_r} cancels the other part of e_{b_r} in the secondary and accordingly the amplitude of e_{2_r} can not be sufficiently large.

From these considerations, we can conclude that:

The secondary ampere-turns should lie within such limits that the secondary ampere-turns at full load will not produce any considerable amplitude corresponding to a in Figure 15, because of distortion of the flux on load based on the saturation characteristic curve of the core used.

4. Oscillograms Obtained in Experiments

As the oscillograph will not correctly trace waves at high frequency, I took many oscillograms during the experiments at 50 cycles, using the core the characteristic of which is shown in Figure 8 and the connection diagram as shown in Figure 1.

Case 1

At no load, the wave form of e_a, e_b, e_1, and e_2 gave the oscillograms shown in Figure 16 (a_1, e_1). From Figure 17 (e_a, e_b), and Figure 18 (e_1, e_2), we can see:

(1) The primary voltage e_1 has a V-topped wave form.

(2) The secondary voltage of double frequency has a uniform period if the exciting d.c. ampere-turns are equal to the effective value of primary ampere-turns.

(3) The phase difference between e_1 and a_1 is about 90°.

Case 2

On resistance load, the wave forms of e_1, e_a, e_b, a_1, e_2, and a_2 were taken in the oscillograms as shown in Figure 19 (a_1, e_1), Figure 20 (e_a, e_b), and Figure 21 (a_2,e_2) whence we can see that:

(1) The primary voltage of the transformer is distorted a little and the upper and lower parts have become asymmetrical.

(2) The primary voltage has a flat asymmetrical V-topped wave form.

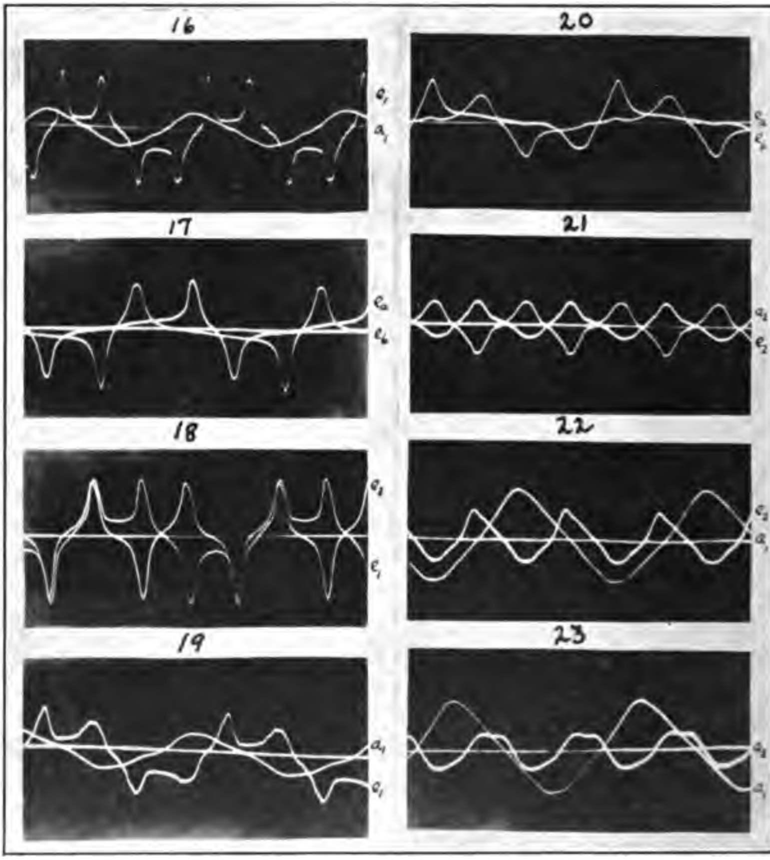
(3) The resultant secondary terminal voltage is somewhat low and asymmetrical.

Case 3

On capacity, inductance and resistance load, where the variable inductance was adjusted so as to obtain the maximum current, the wave forms a_1, a_2 and the secondary terminal voltage e_2 were taken in oscillograms, as shown in Figure 22 (a_1,e_2), and Figure 23 (a_1,a_2).

This case actually occurs in radio transmitters as already mentioned, and from these oscillograms we can see that the secondary current somewhat leads before secondary terminal voltage e_2.

These results are all clearly explained by the diagrams shown in Figure 13, 14, and 15.



FIGURES 16 to 23

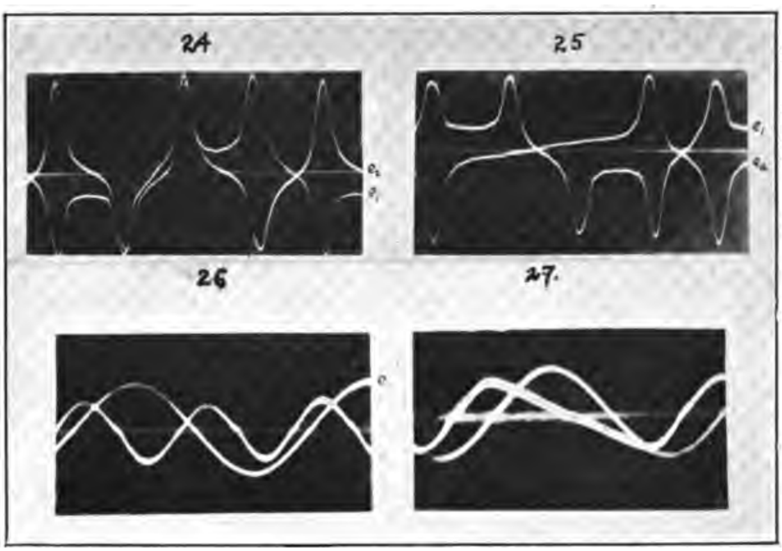
5. COMPARISON OF THE SINE WAVE SYSTEM AND THE IMPULSIVE WAVE SYSTEM

As already mentioned, the secondary emf. of the pure sinusoidal wave obtained by the sine wave system is applied efficiently in producing the secondary output but the flux density change being limited by the condition $AB+X.CD$, the induced emf. or output can not be great as desirable for a given size of core.

Ample space is required for the winding of A_D but the iron loss per unit volume of the core may be small. On the other hand, in the case of the maximum impulsive wave system which has been treated in sections 2 to 4, the flux density change can be made great enough to induce a high secondary emf. because of the necessary conditions that $A_D = A_1/\sqrt{2}$ and A_D = the excitation required to cause the flux just to reach the knee of the saturation curve. Thus the maximum secondary induced emf. wherein many higher harmonics are included, is utilized for impulse excitation in the production of the secondary oscillations wherein the second harmonic predominates because of the resonance effect. Consequently, a satisfactory amount of energy can be drawn from the source. The iron loss per unit volume is great in this case but the total volume of the core may be much less than in the former case.

These two systems correspond closely to the cases of the oscillations of the first type and the second type in Poulsen arcs.

I took oscillograms of e_1, e_a, e_2 for each system adjusting the conditions to the best values; e_1, e_a, e_2 for the impulsive wave system are shown in Figures 24 and 25, and e_1, e_a, e_2 for the sine wave system are shown in Figures 26 and 27. From these curves, we see that the curve e_2 is approximately sinusoidal and the curve e_1 has a flat top in the case of sine wave; while the curve



FIGURES 24 TO 27

e_2 is a peak-topped wave and the curve e_1 has a sharp V-shaped top in the case of impulsive wave.

In practice, it is important to cause the oscillations to have as much energy as possible by using the method of impulsive wave excitation, even tho many higher harmonics are produced, provided the iron loss in the core is of allowable value for the given frequencies. These considerations are similar to those governing the choice of oscillations of the second type in the Poulsen arc. But the sine wave system should be used, if the frequency is too high to enable reasonable losses for a given material of the core.

SUMMARY: A general analysis is given of the theory of the ferromagnetic frequency doubler. The production of secondary circuit energy is secured either by the induction of a sinusoidal secondary emf. or by the production of a sharply peaked, "impulsive" secondary emf. In each case, secondary resonance is employed to emphasize the desired double frequency.

The sinusoidal wave and impulsive wave systems are oscillographically studied and compared for different types of secondary load. The impulsive system is recommended as having higher efficiency wherever the frequency is sufficiently low to avoid excessive iron core losses with that form of wave.

THE STATUS OF THE STATIC FREQUENCY DOUBLER IN RADIO ENGINEERING PRACTICE*

By

FREDERICK C. RYAN

(RADIO ENGINEER)

Further advancement in the art of radio communication calls for the co-ordination of development centralized on the physical phenomena producing the most nearly ideal results so far attained. It is obvious that the method of signal reception based on the interaction of two radio frequency currents properly related to one another, their resultant giving a note of audible frequency, represents our nearest approach to perfection. If both of these currents are of sustained wave form and the receiver is of the vacuum relay type of proper characteristics, the signal note will be purely musical and the pitch subject to variation at will. There is also a marked gain in audio frequency energy by reason of the employment of the received energy in conjunction with that supplied by the locally produced oscillations.

Based on these premises, a consummation of our technical ideals will be fulfilled thru concentration of effort on the development of devices for the most economical generation and radiation of sustained wave radio frequency energy. Ultimate practical perfection requires resourcefulness in reducing energy losses in apparatus and circuits; flexibility of frequency variation; stability of operating frequency; elimination of the undesirable effects of harmonics including those inherent in the generator or induced by reason of the necessarily distributed constants of a radiating circuit; production of sufficient power; and the required speeds of signaling.

Present practice presents as a basis for development the theories applied in the design of radio frequency generators of the direct current arc oscillator, power electron tube, and rotating mechanical types. The arc method involves the passage of an electric current from a copper to a carbon electrode, surrounded by an hydrogenous atmosphere and acted upon by a tranverse magnetic field. The gas serves as a vehicle of de-

* Received by the Editor November 26, 1917. Bibliography is at end of paper.

ionization of the space between the electrodes by reason of its high coefficient of diffusion and is also beneficial because of its high heat conductivity thus preventing local temperature rises. The magnetic field is an adjunct for ionic removal. By these means rapid growth of the ignition voltage is assured and the apparatus acts as a converter of continuous current energy into sustained radio frequency energy. The application of these hypotheses has afforded us means, at the present time, of producing ample energy for practical wave propagation over normal transmission distances. Rapid manipulation of frequency is obtained by the simple process of varying the antenna system inductance thru appropriate external variable reactance, because the frequency is determined wholly by the circuit constants. Signaling is performed by a small percentage change in radiated frequency, thru the change of inductance by electrical or mechanical methods. It may also be accomplished by shifting of the radio frequency energy from the radiating to a dissipating system. Practical operation of the arc oscillator is only successful when accomplished without disturbance of the factors governing the constancy of the frequency and oscillation amplitude. Therefore the supply circuit must be fixed and signaling done by one of the methods discussed. The inherently unstable characteristics of the arc make it difficult to design absorbing systems and to devise appropriate manipulation thereof to obtain signal control with a minimum effect on the constancy of energy and frequency. Prevalence of harmonics generated and the disturbances caused by the control apparatus detract much from an otherwise desirable generator.

The arc systems losses are somewhat larger than usual, and adding these to those produced by the continuous energy flow at full power causes the operating efficiency to become quite low.

By making use of the properties of free electronic emission from incandescent bodies in vacua, and causing thermionic currents to flow thence to anodes strategically situated we are able to produce radio frequency alternating currents by the employment of a proper control. If a third electrode be brought into the space between such a cathode and anode, the number of electrons returning to the cathode will increase if this body be negatively charged; and, on the other hand, if it be positively charged it will tend to neutralize the negative charges on the electrons in transit. By these artifices the thermionic current is decreased to a minimum or increased to saturation. By introducing into the electrode circuits appropriate inductances and capacities and causing interaction by mutual inductance between them, the regenerative action brought about by the transfer of energy back and forth will cause the device to act as a source of radio frequency alternating current or sustained wave energy. The work already done along these lines promises early fulfillment of the realization of means for accomplishing radio communication with simplicity of signal control, both telegraphic and telephonic, ease of manipulation of energy and high generating efficiency. At this time the high first cost and prohibitive operating expenses exclude their consideration as an economical commercial proposition.

Numerous investigators have engaged in the design of rotating radio frequency alternators. The two types attaining some degree of success are represented by the Alexanderson and the Goldschmidt types of machines. These machines generate and deliver directly to the radiating circuit energy of its fundamental frequency. The Alexanderson alternator[1] has been built and operated in units up to 75 kilowatts and to frequencies of 100,000 cycles. The inductor type of construction is followed in this machine, using a continuous wave armature winding. In order to obtain radio frequencies, it is necessary to have a large number of slots and rotating field projections, together with a high peripheral speed. The output is a function of the air gap clearance because the generated voltage is nearly inversely proportional to the air gap length. Carefully built flexible shafts are necessary because of the high speeds involved, in order that the rotating member may revolve about its exact center of mass. The mechanical design, selection of materials and assembly of a machine of this type involves considerable expense if successful practical operation is to be expected. The windage and friction losses are quite high thereby giving relatively low efficiencies. The question of speed regulation is all important in order to secure stability at the operating frequency. Flexibility of frequency change brings up intricate speed design questions. For medium energy output at not too high a frequency, and where wave length changes are not often required, this apparatus will doubtless find a field of application. The control of such a generator for either telephonic or telegraphic use has been beautifully worked out by Mr. Alexanderson.*

The Goldschmidt type of alternator[2] may be described as a combined generator and frequency multiplying device. By

*See PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, Volume 4, number 2, page 101, April, 1916.

appropriate choice of circuit constants, various tuned circuits are shunted across the stator and rotor windings, serving to intensify and increase the higher harmonics of the generated frequency. That higher harmonic desired for use appears in a resonant radiating circuit. Each of these reflection steps involve losses in machine and circuits, hence it is desirable to reach the requisite frequency with as few steps as possible. Therefore, the machine is designed for a high initial frequency and speed. The mechanical design is exceedingly complex, very small air gaps being necessary. The flux carrying structure is of very thin steel laminations, separated by paper for insulation against eddy currents. As the rotor carries conductors, great care is necessary to secure proper mechanical strengths for successful operation at high speeds. Abundant power is made available by this design, signaling being facilitated thru controlling the direct current exciting field. Wave length change is obviously somewhat difficult to carry out, necessitating cumbersome mechanism.

The construction and operation of alternators giving frequencies up to 10,000 cycles in large power units and as great as 20,000 cycles in small sizes is a practical commercial proposition. By using the inductor type of machine the rotor of which carries no windings, the mechanical structure becomes very rugged. Operating speeds will not be high enough to cause excessive windage and friction losses. By the exercise of an ordinary amount of ingenuity, questions of slot size for windings and dimensions of pole projections on the rotating field are soluble If now, means are available for raising the frequency so generated to a value suitable for efficient radiation, we obtain another method for attacking the problem.

The static frequency doubler serves such a purpose. By its use the generated frequency may be stepped up as many times as desired. Since each step doubles the frequency, the required value is obtainable without undue complication.

Before taking up the operation of this scheme of frequency multiplication the theory involved will be discussed. If two windings be placed upon a core of ferromagnetic material, one excited by a direct current and the other carrying an alternating current, the value of the reactance presented the alternating current will vary with the degree of magnetization produced by the direct current winding because of the changing permeability of the magnetic core at different flux densities.

The magnetizing force produced by a coil in air is numerically equal to the field intensity.

$$H = \frac{0.4 \pi N I}{l} K$$

wherein N = turns,
I = current in amperes,
l = length of magnetic circuit,
K = constant, depending on shape of coil.

and the magnetic induction in a medium of permeability μ assuming the coil shape giving $K = 1$ will be

$$B = \frac{0.4 \pi N I}{l} \mu = H \mu.$$

Since $\phi = B A$,

and $L = N \frac{\phi}{10^8 i}$,

$$L = \frac{0.4 \pi N^2 A}{10^8 l} \mu$$

where A = cross sectional area of magnetic circuit,
ϕ = total flux,
L = inductance,

it is obvious that the inductance, L, varies as the total flux, which in turn is a function of the permeability μ. Hence the control of the value of reactance thru the direct current magnetization is had, because the same flux threads both coils.

The use of ferromagnetic material for carrying the lines of induction caused by currents of radio frequency brings up the question as to the character of the laws governing the flux density, depth of penetration of the flux, the iron losses, and whether the permeability varies with change of frequency. If theory confirms the practical aspect, it is to be presumed that the effective permeability and flux density under given conditions will vary inversely as the frequency. It has been shown that the depth of penetration of the flux in iron is

$$S = \frac{3570}{\sqrt{\lambda \mu f}}$$

wherein λ = conductivity,
f = frequency.

Experiments carried out along lines to confirm these assumptions have been performed by Mr. Alexanderson, and confirmation of the theory was fulfilled.

The losses in iron comprise those due to hysteresis and eddy currents, the values of which are given by the expressions,

$$P_h = N f V B^{1.6} 10^{-7} \text{ watts}$$
$$P_e = 1.645\, V d^2 f^2 B^2 10^{-11} \text{ watts}$$

wherein N and d are constants,
V = volume of iron.

As the first increases directly and the latter as the square of the frequency it can be seen that the eddy current losses predominate at the higher frequencies. A decrease in hysteresis loss (Ewing) is suggested by reason of the rapid agitation of the molecules at the higher speeds of magnetic reversal. The total core losses in practice will be found to become less as the frequency is increased because the flux density for a given duty becomes less as the frequency is increased. Hence the losses are well within reason when using iron at radio frequencies. Combined with the fact that the effective permeability is still an appreciable quantity at these frequencies it is noted that use of ferromagnetic materials is perfectly feasible and is proving successful in practice.

The static frequency doubler consists essentially of two closed iron cores. Each of these carries a primary and a secondary alternating current winding and also a direct current magnetizing coil. The manner of connection is shown in Figure 1. The

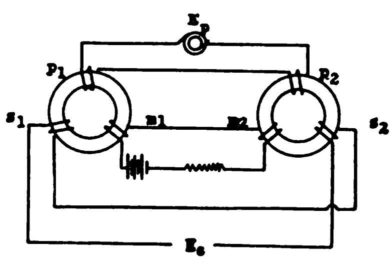


FIGURE 1

two primary windings p_1 and p_2 are wound in the same direction. The secondary windings s_1 and s_2 are reversed with respect to one another, as are also the direct current magnetizing windings m_1 and m_2. If an alternating current of potential E is applied to the primaries in series and at the same time the direct current coils are excited, a small flux change during the first half cycle will be produced in the transformer in which the alternating and direct current fluxes add. During the same half cycle in the other transformer the two fluxes oppose one another and there will be a larger flux change. Since the secondary windings are reversed relative to one another, two opposing e.m.f.'s of differing amplitude will be induced in the circuit during this half cycle. Their resultant will be an e.m.f. wave having an amplitude the difference of the generated amplitudes. During the second half cycle, the phenomena will be reversed, but the resultant induced e.m.f. in the secondary circuit will be of a similar character but of course displaced in phase relative to the induced e. m. f. of the first half cycle. This results in an alternating e.m.f. at the terminals of the secondaries in series of twice the frequency of the primary e. m. f. These voltage curves are shown in Figure 2. As may be seen from this figure, the constant magnetization caused by the direct current i produces a dissymmetry in the two loops of magnetic induction, which have opposite senses with regard to the base magnetization.

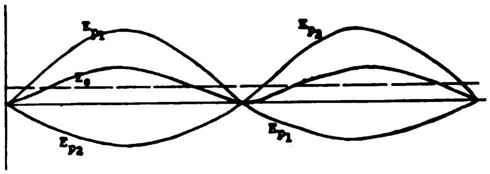


FIGURE 2

The primary voltage, e can contain only odd harmonics because of the well-known inherent characteristics of rotating generating machinery. The magnetizing current must, therefore contain even harmonics, in order to produce a flux having even harmonics, since these are not supplied by the primary e.m.f.

The action of these ferromagnetic frequency changers of the polarized core type has been given in some detail by Goldsmith.[3]

In order to bring out the theoretical treatment of these phenomena the mathematical discussions of Messrs. Kuhn[4] and Joly are here reproduced, with the notation somewhat abridged. At any time t, the resultant magnetization in core A will be

$$M_a = N I_a = N_o i_o + N i_t,$$

wherein

$N i_o$ = direct current ampere turns.

$N_o i_t$ = a. c. ampere-turns at the time t.

and the total flux in the core is

$$\Phi_a = \Phi_o + \Phi_1 \sin(\omega t - \phi_1) + \Phi_2 \sin(2\omega t - \phi_2) + \cdots$$

In core B, the resultant magnetization at any time t will be

$$M_b = N I_b = -N_o I_o + N i_t,$$

but at time t

$$= -\left\{ i_{\left(t+\frac{T}{2}\right)} N_1 - N_o I_o \right\}$$

From this we obtain the series expressing the flux in the second core

$$\Phi_b = -\Phi_o + \Phi_1 \sin(\omega t - \phi_1) - \Phi_2 \sin(2\omega t - \phi_2) + \cdots .$$

The phase angle in the second transformer is shifted one-half period with respect to the first transformer. The total primary flux Φ_p is

$$\Phi_p = \Phi_a + \Phi_b = 2\left\{\Phi_1 \sin(\omega t - \phi_1) + \Phi_3 \sin(3\omega t - \phi_3) + \cdots\right.$$

while the secondary flux is

$$\Phi_s = (\Phi_a - \Phi_b) = 2\left\{\Phi_o + \Phi_2 \sin(2\omega t - \phi_2) + \Phi_4 \sin(4\omega t - \phi_4) + \cdots .\right.$$

From this it is obvious that the secondary voltage e_2, which is generated by the flux Φ_s, contains only even harmonics.

To complete the mathematical discussion of the problem it is necessary to be able to express the $B-H$ curves of iron by an empirical formula. The necessary conditions can be embodied in an empirical expression of the following form:

$$B = A \tan^{-1} a x + C x$$

where x = ampere-turns per cm. length of magnetic circuit,
B = induction.

In order to make this function satisfy, for values of an experimentally determined curve, the three constants A, a, and C must be obtained.

From Figure 3, the three points p_1, p_2, and p_3 are chosen on the curve. These values are substituted in the above equation. A and C are eliminated as follows:—

$$B_1 = A \tan^{-1} a x_1 + C x_1$$

$$B_2 = A \tan^{-1} a x_2 + C x_2$$

$$B_3 = A \tan^{-1} a x_3 + C x_3$$

$$\frac{B_1 x_2 - B_2 x_1}{B_3 x_2 - B_2 x_3} = \frac{x_2 \tan^{-1} a x_1 - x_1 \tan^{-1} a x_2}{x_2 \tan^{-1} a x_3 - x_3 \tan^{-1} a x_2}$$

If a is obtained by trial, and substituted in two of the original equations, two linear equations in A and C result, which are easily solved. The agreement between the actual and theoretical curves is found to be very close, tho for more accurate work an equation having four arbitrary constants could be used.

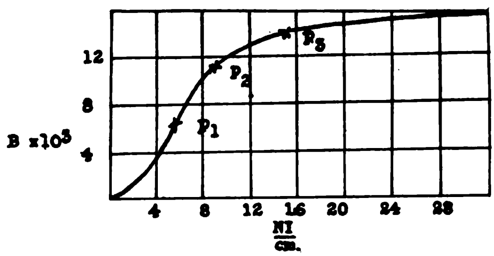


FIGURE 3

Suppose a potential e_1 is impressed on the primaries of the device. The instantaneous value of the alternating current is

$$i_1 = I \cos \omega t$$

The magnetizing ampere-turns per unit of magnetic circuit length can be written as follows:—

$$\frac{N i_1}{\text{cm.}} = x_1 \cos \omega t$$

If sufficient sized inductances are put into the direct current circuit, its reactance may be made large enough to prevent the flow of induced alternating current, and the exciting current i will contain practically no alternating current component, and therefore may be expressed as

$$i_o = I_o = \text{constant},$$

and

$$\frac{N i_o}{\text{cm.}} = x_o = \text{constant}.$$

The question now arises as to what are the forms of the secondary voltages e_a and e_b. The iron losses are neglected for the sake of simplicity. Using the relation.

$$B = A \tan^{-1} a x + C x_1 \qquad \text{where } x = \frac{N i}{\text{cm.}},$$

we can write for the flux density in the two transformers A and B:—

$$B_a = A \tan^{-1} a\,(x_1 \cos \omega t + x_o) + C\,(x_1 \cos \omega t + x_o).$$
$$B_b = A \tan^{-1} a\,(x_1 \cos \omega t - x_o) + C\,(x_1 \cos \omega t - x_o).$$

The induced emf. for an iron cross section of F sq. cm., is, for the two cases:

$$\frac{e_a}{N} = -F \frac{d B_a}{d t} 10^{-8} \text{ (volts per turn)}$$

$$\frac{e_b}{N} = -F \frac{d B_b}{d t} 10^{-8} \text{ (volts per turn)}$$

Substituting B_a and B_b from the previous equations we get:—

$$\frac{e_a}{N} = (F \omega 10^{-8})\, x_1 \sin \omega t \left(\frac{A a}{1 + a^2 (x_1 \cos \omega t + x_o)^2} + C \right)$$

$$= (F \omega 10^{-8}) \sin \omega t$$

$$\left\{ \frac{A}{a x_1} \cdot \frac{2}{\cos 2 \omega t + 4 \frac{x_o}{x_1} \cos \omega t + 2 \left(\frac{x_o}{x_1} \right)^2 + 2 \left(\frac{1}{a x_1} \right)^2 + 1} + C x_1 \right\}$$

$$\frac{e_b}{N} = (F \omega 10^{-8})\, x_1 \sin \omega t \left(\frac{A a}{1 + a^2 (x_1 \cos \omega t + x_o)^2} + C \right)$$

$$= (F \omega 10^{-8}) \sin \omega t$$

$$\left\{ \frac{A}{a x_1} \cdot \frac{2}{\cos 2 \omega t + 4 \frac{x_o}{x_1} \cos \omega t + 2 \left(\frac{x_o}{x_1} \right)^2 + 2 \left(\frac{1}{a x_1} \right)^2 + 1} + C x_1 \right\}$$

These two equations differ only in a phase angle of one-half period. Hence we may make the following substitutions

$$\sin \omega t \text{ for } \sin \omega \left(t + \frac{T}{2} \right)$$

$$\cos \omega t \text{ for } \cos \omega \left(t + \frac{T}{2} \right)$$

$$\cos 2 \omega t \text{ for } \cos 2 \omega \left(t + \frac{T}{2} \right)$$

and the former equations may be rewritten as follows:—

$$e_b = -(F \omega 10^{-8}) \sin \omega \left(t + \frac{T}{2} \right)$$

$$\left\{ \frac{A}{a x_1} \cdot \frac{2}{\cos 2\omega \left(t + \frac{T}{2} \right) + 4 \frac{x_o}{x_1} \cos \omega \left(t + \frac{T}{2} \right) + 2 \left(\frac{x_o}{x_1} \right)^2 + 2 \left(\frac{1}{a x_1} \right)^2 + 1} + C x_1 \right\}$$

also $$e_{b(t)} = -e_{a\left(t + \frac{T}{2}\right)}$$

On the primary side the voltages add:

$$e_1 = e_a + e_b.$$

The induced total voltage can therefore only contain odd harmonics of twice the amplitude of each voltage and in phase, since on the secondary side, where the windings are connected in opposition, all the odd harmonics disappear.

A numerical case illustrating these relations is cited. Use is made of the saturation curve of Figure 4 which was found to be represented by the equation

$$B = 9800\ tan^{-1}\left(0.41 \frac{N\,i}{\text{cm.}}\right) + 21.0 \frac{N\,i}{\text{cm.}}$$

where $A = 9{,}800$,
$a = 0.41,$
$C = 21.0$.

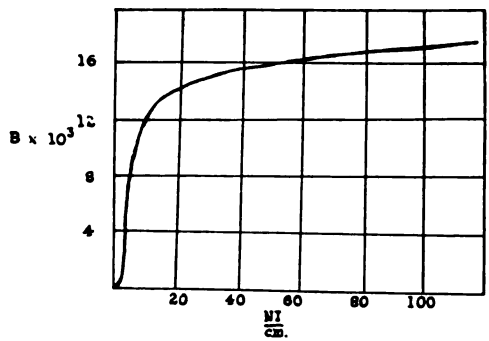


FIGURE 4

The two transformers are assumed to be saturated to the knee of the curve by means of alternating current i_1

$$x_1 = 12.2 \frac{N\,i}{\text{cm.}} . \qquad a\,x_1 = 5.0$$

Three cases are worked out with the direct current excitation having the following values

$$\frac{x_o}{x_1} = 0.25,\ 0.50, \text{ and } 0.75.$$

For these values of $\frac{x_o}{x_1}$ and x_1, the curves are shown in Figures 5, 6, and 7. Upon substitution the values of e_a becomes

$\frac{x_o}{x_1} = 0.25$ . . . . $e_a{}' = \kappa\, e_a{}'$

$= 0.50$ . . . . $e_a{}'' = \kappa\, e_a{}''$

$= 0.75$ . . . . $e_a{}''' = \kappa\, e_a{}'''$

The character of each voltage curve for one core is the same for all cases, consisting of a pointed wave which is symmetrical to the right and left of the zero point, and which differs with various magnetizing excitation currents x_0 only in height and phase position of the maximum ordinate.

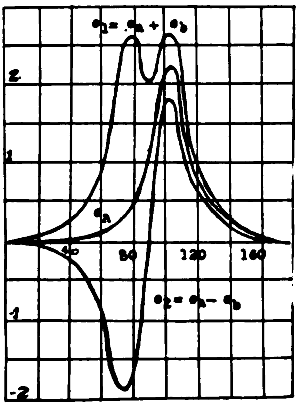


FIGURE 5

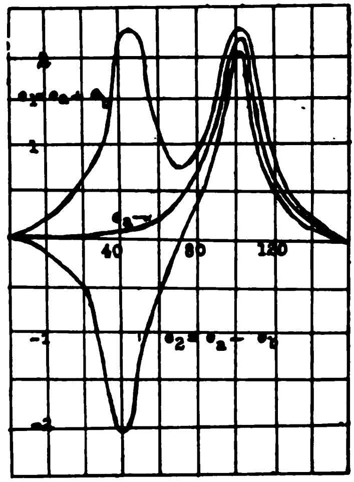


FIGURE 6

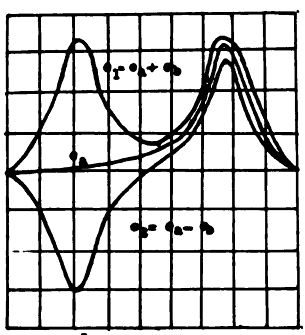


FIGURE 7

The position of the maximum ordinate can be found with sufficient accuracy by

$$\phi_{max} = -\cos^{-1}\frac{x_0}{x_1}$$

and the following values pertain to the cases given

where $\frac{x_o}{x_1} = \begin{cases} 0.25 \\ 0.50 \\ 0.75 \end{cases} \qquad \phi = \begin{cases} 104^\circ \\ 120^\circ \\ 138^\circ \end{cases}$

The even harmonics are related as follows:

$$e_2 = e_a - e_b = \kappa\, e_2$$

Odd harmonics are eliminated by the method of connection, that is

$$e_1 = e_a + e_b = \kappa\, e_1$$

The curves of the even harmonics are shown in Figures 5, 6, and 7.

Analysis of the total voltage to the sixth harmonic indicate that not only a strong second but also other higher harmonics exist under the condition that the magnetizing current is of sinusoidal shape.

Returning to the practical aspect concerning the actual operating conditions. A medium frequency alternator connected to a first doubling stage will have a rather high synchronous impedance and hence the transformer would only furnish a small magnetizing current. To overcome this, use is made of neutralizing capacity in this circuit and the advantages of resonance obtained. It has been demonstrated that it is possible to operate alternators of the required type in parallel, requirements for synchronizing being the same as for ordinary generators, the relation between reactance and resistance indicating the existence of certain limits within which stable operation is obtained. Therefore, if any difficulty is experienced in obtaining very large outputs from a single unit, more may be used.

Taking the hypothetical case of a required communication circuit over a distance of 1,800 miles (3,000 kilometers), the wave length for best operating efficiency is found in practice to be about 8,000 meters corresponding to approximately 38,000 cycles. If an initial generated frequency of 9,500 cycles is used, the desired operating frequency can be reached thru two doubling stages. This is indeed a simple process and with so few stages of transformation the losses will not become large and a transmitter of relatively high efficiency is availab'e. Small desired frequency changes are obtainable thru variation of alternator speed and larger steps are attained thru fewer or more doubling stages.

Energy control with this system is facilitated by the permeability—inductance relation already discussed. Some of the possible methods have already been mentioned, and are further discussed for the sake of completeness.

If resonance is obtained by the use of series capacity in each doubler circuit, the conditions for maximum energy in the circuits will be very critical. If, now, any of the direct current magnetising values are varied, a change of reactance will take place in the alternating current circuits and hence a small variation in this circuit will produce large energy changes in the radio frequency circuits thru detuning.

Figure 8[5] illustrates a method for transmitting telegraphic signals. A key, *k*, is arranged in shunt to a resistance *R* in the

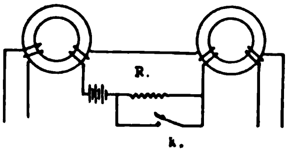


FIGURE 8

auxiliary circuit, the electrical constants being so arranged that short-circuiting of this resistance will produce the necessary change of inductance in the energy circuit to disturb markedly the resonant condition.

Figure 9 depicts an arrangement whereby a musical note of audio frequency may be superimposed on the radio frequency current. This is accomplished by additional coils on the cores,

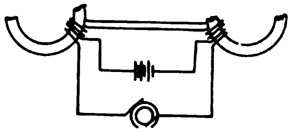


FIGURE 9

which coils are supplied with the required audio frequency current, thus serving to produce periodical variations in the magnetic flux.

In Figure 10 is shown a method of telephonic control by means of a microphone in the auxiliary circuit.

These few illustrations of control methods show the ease with which energy manipulation is obtained in using such a system.

In order to obtain a maximum of efficiency in the antenna, the last coil of the doubling system may be connected, if desired, to an ordinary transformer, of proper dimensions, for the purpose of raising or lowering the voltage to the proper value to fit the constants of the radiating system.

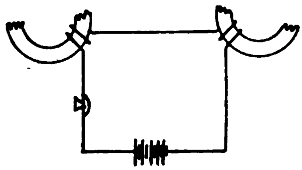


FIGURE 10

A schematic arrangement of a complete transmitter is shown in Figure 11.

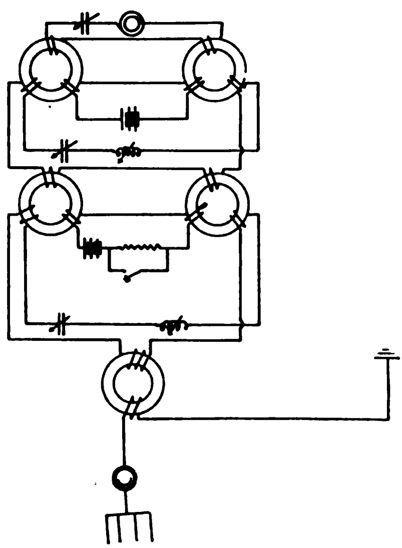


FIGURE 11

Sufficient information is now available at radio frequencies to place the design of alternators and transformers of the type required for the operation of such a system on a practical basis.

Other obstacles such as obtaining laminated core material of proper thinness, of the order of 1 to 2 mils (0.001 to 0.002 inch or 0.02 to 0.04 mm.), and the close speed regulation of prime movers, can be met to any necessary degree.

References:

1. "Magnetic Properties of Iron at Frequencies Up to 200,000 Cycles," Alexanderson, "Proceedings A. I. E. E.," 1911.
2. "The Goldschmidt System of Radio Telegraphy," Emil E. Mayer, "Proceedings I. R. E.," Volume 2, Number 1, Page 69. 1914.
3. "Radio Frequency Changers," Alfred N. Goldsmith, "Proceedings I. R. E.," Volume 3, Number 1, page 55, 1915. See also Goldsmith, "Radio Telephony" (Wireless Press, New York, 1918), pages 182 to 203.
4. "Jahrb. der Draht. Tel.," Epstein, Kuhn, and Joly, 1915, and 1916.
5. U. S. Patent number 1,181,556, Von Arco and Meissner.

SUMMARY: The system of sustained wave generation based on the use of a radio or high audio frequency alternator and frequency multipliers is considered.

The mathematical expressions for the secondary voltages of the unloaded ferromagnetic doubler of the polarized core type are derived and explained.

THEORY OF ANTENNA RADIATION*

By

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The problem of antenna radiation has been attacked mathematically, notably by Hertz, Macdonald, Pearson and Lee, Love, Abraham, Bennett, and Hack. In no case, however, has the mathematical investigation included the condition of the substantially sinusoidal distribution of voltage and current along the antenna. In a series of papers published in the PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, I have shown how the variable distribution of inductivity L (self induction coefficient), and capacity C affects such distribution along the antenna. In the subjoined paper it will be shown that Eccles' formula for radiation involving as it does an equation of the following type

$$I = \frac{a}{\sqrt{r}} \cdot \varepsilon^{-\beta r}$$

is theoretically deducible, and shown to be dependent on the substantially sinusoidal voltage distribution of the antenna. This will be done without in any manner invoking the principle of the so-called Heaviside Layer. Again it will be shown that there is theoretical ground for the Cohen "dispersion factor" $(1+Ax)$, for which latter see "The Electrician," London, February 25, 1916. "Fading" will also be explained.

The present writing, it will be found, is limited to the case of a finite vertical grounded antenna with a flat earth. Cylindrical co-ordinates will, therefore, have to be resorted to. The Heaviside-Hertz equations for the field will be set up and solved by the symbolic methods of Oliver Heaviside. However, in order more easily to investigate the problem in hand it will be found desirable to split up the investigation of the radiation phenomena into two parts. These are called respectively, "voltaic effect" and "galvanic effect." The reason for this is as follows:

* Received by the Editor, July 9, 1919.

In Maxwell's theory of the electromagnetic medium two circuital laws are made fundamental. It is necessary to consider the radiation system as really constituting an electric circuit. Current passes up, along the antenna, thru the conductive medium of the antenna wire to flow off afterwards into the dielectric medium in much the same manner as current flows across the insulating medium from the opposite sides of a condenser. From this point of view, the antenna wire and the current in it, by setting up a variable external magnetic field (and consequent dielectric stresses), produce radiation phenomena which, altho contributing nothing from an energy standpoint according to the application made of Poynting's Theorem, yet sets up distant field components that ought to be taken into account when considering a complete radiation theory. The above type of action limited as it is by the finite type of antenna is referred to as "galvanic effect." It represents the leaky transformer effect.

The second type, having reference to the rest of the electric circuit, is designated the voltaic effect, because it is found to be dependent upon the distribution of electrical potential along the antenna conductor with respect to earth. The direct effect corresponds to the so-called electrostatic induction. It also sets its magnetic field components by its variation.

Insofar as the inductivity L and capacity C per unit of length of antenna are substantially constant except for 10 per cent. of the antenna height near the bottom of the antenna and 5 per cent. near the top, the theoretical work becomes much simplified in consequence. In any case the subjoined investigation should be of interest in view of the work of Messrs. Weagant and Taylor, mentioned in the PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, of June, 1919.

DIFFERENTIAL EQUATIONS FOR VERTICAL GROUNDED ANTENNA. In order to take into account the radiation in all directions from a vertical grounded antenna on a flat earth, we shall resort to the use of cylindrical co-ordinates. The antenna will be assumed as extending upward in the z direction perpendicular to the earth's surface in which the co-ordinates x and r are taken. In this case, the only component of magnetic force H_θ possible will be that having circular symmetry about the z or antenna axis. On the other hand, the displacement current components will be D_r and D_r but with no D_z component.

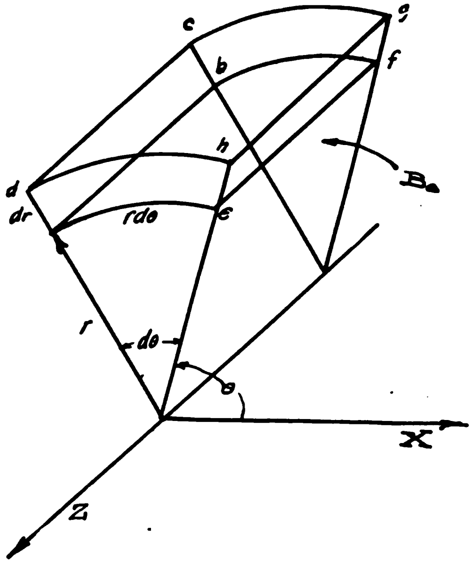

FIGURE 1

To arrive at the necessary differential equations, consider the flux passing thru the face hf. It will be $B\,dz\,dr$. Then

$$-\frac{dB}{dt}dz\,dr = \mathscr{E}_z dz + \left(\mathscr{E}_z + \frac{d\mathscr{E}_z}{dr}dr\right)(-dz) + \mathscr{E}_r(-dr) + \left(\mathscr{E}_r + \frac{d\mathscr{E}_r dz}{dz}\right)dr$$

$$\frac{dB}{dt} = \frac{d\mathscr{E}_z}{dr} - \frac{d\mathscr{E}_r}{dz}.$$

Considering now the face cf, the displacement is $D_z \cdot dr \cdot rd\theta$. We will have, therefore

$$\frac{dD_z}{dt} \cdot dr\ \ rd\theta = H(-rd\theta) + \left(H + \frac{dH}{dr}dr\right)(r+dr)\,d\theta$$

$$\frac{dD_z}{dt} = \frac{1}{r}H + \frac{dH}{dr}.$$

Taking now the face af, the displacement is $D_r \cdot dz \cdot rd\theta$ Thus

$$\frac{dD_r}{dt} \cdot dz \cdot rd\theta = H \cdot rd\theta + \left(H + \frac{dH}{dz}dz\right)(-rd\theta)$$

$$\frac{dD_r}{dt} = -\frac{dH}{dz}$$

Introducing the relations

$$B=\mu H$$

$$D=\kappa \mathcal{E}$$

$$\mu \kappa \frac{d^2 D_r}{d t^2}=\frac{d^2 D_r}{d z^2}+\frac{d^2 D_r}{d r^2}+\frac{1}{r}\frac{d D_r}{d r}-\frac{1}{r^2}D_r.$$

Symbolizing the t and z operators, the last equation takes the form:

$$\frac{d^2 D}{d r^2}+\frac{1}{r}\frac{d D}{d r}-\left(\frac{1}{r^2}+\Delta^2\right)D=0$$

where

$$\Delta^2=q^2-z_1^2;\ q^2=-\frac{1}{V}\cdot\frac{d^2}{d t^2};\ z_1^2=\frac{d^2}{d z^2}$$

and V is the velocity of light.

Voltaic Effect from Vertical Grounded Antenna. We have seen that the differential equations of condition lead to the form

$$\frac{d^2 D_r}{d r^2}+\frac{1}{r}\frac{d D_r}{d r}-\left(\frac{1}{r^2}+\Delta^2\right)D_r=0$$

A suitable solution will be

$$D_r=r_1\cdot K_o\,(\Delta\, r)\cdot A$$

where K_o is a zeroth Bessel of the second kind with $r_1=\frac{d}{d r}$ and A any desired function independent of r, but that may involve the time t or height z. For a proper solution, it will be necessary to have at

$$r=a \text{ that } D_r=D_o$$

with a as the radius of the antenna wire and where D_o is the known value of the displacement due to the distributed voltage v_z along the vertical aerial.

In any case, the elemental charge dQ for any height $d z$ of the aerial will be given in electrostatic units by

$$dQ=\kappa\left(\frac{d C}{d z}\cdot d z\right)v$$

if C is the capacity per unit of length. Dividing, therefore, by the elemental cylindrical area involved, $2\pi a\cdot d z$, then

$$D_o=\frac{1}{2\pi a}\cdot\frac{dQ}{d z}=\frac{\kappa}{2\pi a}\cdot\frac{d C}{d z}\cdot v.$$

As stated above for a vertical aerial, except within ten per cent of the ground, we can take over the range $0<z<h$, where h is the antenna height

$$\frac{dC}{dz}=a$$

Moreover, for, $0<z<h$

$$v=v_z\cdot sin\, pt$$

$$v_z=v_o\cdot sin\frac{\pi z}{2h}.$$

Substituting in the above, but employing the shift operator S (see, for example, Heaviside's "Electromagnetic Theory," volume 2, page 413; volume 3, page 236), such that

$$S\cdot z^o=(1-\varepsilon^{hz_1})\cdot z^o$$

$$D_r=\frac{r_1\cdot K_o(\Delta r)}{\{r_1 K_o(\Delta r)\}_{r=a}}\cdot\frac{a\kappa}{2\pi a}\cdot v_o\cdot sin\frac{\pi z}{2h}\cdot S z^o\cdot sin\, pt\ .$$

from which to obtain H_θ and D_z, and so on. In reality, as will be pointed out elsewhere, we ought to take

$$v=v_z\cdot sin\,(pt+\psi)$$

to allow for radiation resistance of the antenna. Radiation is assumed, in other words, to bring the voltage and current distributions into phase with one another. However, the above is sufficient to arrive at the requisite formulas.

It will be well to point out the nature of the graph belonging to

$$y=v_z\cdot S z_o=v_o\cdot sin\frac{\pi z}{2h}\cdot S z^o.$$

Clearly the function

$$y=S z^o$$

implies a flat top curve only over the range $0<z<h$. Therefore the function

$$y=v_o S z^o\cdot sin\frac{\pi z}{2h}$$

will mean that the sine function extends only over the range $o<z<h$, and that beyond, the function represents the axis of z where $y=0$. This will mean that over the range h, differentiations and integrations are to be performed on the function $v_o\cdot sin\frac{\pi z}{2h}$ as operand; but beyond this range, the operand is zero. This distinction is very impoıtant, for integrations of zero lead to finite quantities.

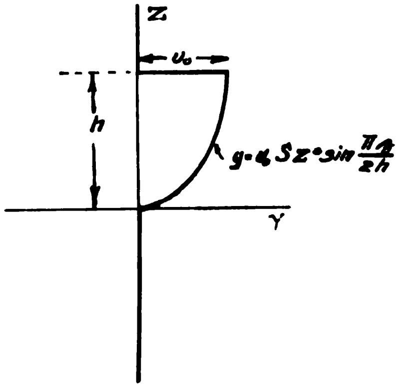

FIGURE 2

It is usual to write

$$r_1 \cdot K_o(\Delta r) = -\Delta K_1(\Delta r)$$

so that

$$D_r = \frac{K_1(\Delta r)}{K_1(\Delta a)} \cdot \frac{a \kappa v}{2\pi a} \cdot S \cdot z^o$$

For values in which $8\Delta r > 1$ the first term only need be taken into account in the divergent series development for $K_1(\Delta r)$ which from the generalized series

$$K_m(\Delta r) = \left(\frac{2}{\pi \Delta r}\right)^{\frac{1}{2}} \cdot \varepsilon^{-\Delta r} \left\{ 1 - \frac{1^2 - 4m^2}{1! \cdot 8\Delta r} + \frac{(1^2 - 4m^2)(3^2 - 4m^2)}{2! \cdot (8\Delta r)^2} - \right\}$$

is obtained by putting $m = 1$

$$\therefore \frac{K_1(\Delta r)}{K_1(\Delta a)} \equiv \left(\frac{a}{r}\right)^{\frac{1}{2}} \cdot \varepsilon^{-\Delta (r-a)}$$

and

$$D_r \equiv \left(\frac{a}{r}\right)^{\frac{1}{2}} \cdot \frac{a \quad v}{2\pi a} \cdot \varepsilon^{-\Delta (r-a)} \cdot S \cdot z^o$$

Over the range $0 < z < h$, the function v occurring in D_r is a sinoidal function of z, so that within the above limits the operator involving Δ will reduce to a pure exponential. We have

$$v_z = v_o \cdot sin \frac{\pi z}{2h}; \quad z_1{}^2 = -\left(\frac{\pi}{2h}\right)^2$$

moreover,

$$q^2 = -\beta^2 = -\left(\frac{2\pi}{\lambda}\right)^2$$

so that on introduction in Δ it follows

$$\Delta^2 = q^2 - z_1{}^2 = \left(\frac{\pi}{2h}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2$$

However, it is known that for a simple vertical antenna we can write, seemingly due to radiation resistance effects,

$$\lambda = M \cdot 4h \text{ with } M > 1$$

That is, the wave length λ is always slightly larger than four times the vertical height h. Making the substitution for h,

$$\Delta^2 = \left(\frac{2\pi}{\lambda}\right)^2 \{M^2 - 1\} = \left(\frac{2\pi}{\lambda}\right)^2 \cdot N^2$$

$$\Delta = \frac{2\pi}{\lambda} \cdot N$$

For the range $z > h$, that is, above the antenna height, a different type of treatment will have to be resorted to.

The functions considered with $z < h$ will be primed to distinguish them from the same functions in the region $z > h$, in which latter case they will be double primed. By the formula for D_r

$$D_r' = \frac{\alpha \kappa}{2\pi\sqrt{a}} \cdot \sin\frac{\pi z}{2h} \cdot \sin pt \cdot \frac{v_0}{\sqrt{r}} \cdot \varepsilon^{-\frac{2\pi N}{\lambda}(r-a)}$$

$$i_r' = \frac{\alpha \kappa}{\sqrt{a}} \cdot V \cdot \sin\frac{\pi z}{2h} \cdot \cos pt \cdot \frac{v_0}{\lambda\sqrt{r}} \cdot \varepsilon^{-\frac{2\pi N}{\lambda}(r-a)}$$

This latter suggests very strongly the radiation formula of Eccles ("Wireless Telegraphy and Telephony," 2nd edition, page 153, London: Benn Brothers). The above formula for the radial component of current is in greater conformity with the Eccles form than with that due to Austin-Cohen.

Working backwards, since Eccles gives the coefficient

$$2\pi N = 2 \times 10^{-4}$$

$$\Delta = \frac{2 \times 10^{-4}}{\lambda}$$

It will be remembered that the condition obtaining for taking the first term of the series expansion $K_0(\Delta r)$ was that for great accuracy

$$8\Delta r > 1$$

or

$$r > \frac{1}{8} \cdot \frac{\lambda}{2 \times 10^{-4}} = \frac{10.000}{16} \lambda \doteq 600\lambda.$$

If, then, λ is in kilometers, the distance r would be in kilometers. Again from the above, we have

$$N^2 = \frac{10^{-8}}{\pi^2}; \quad M^2 = 1 + 10^{-9}$$

$$\therefore M - 1 + 5 \times 10^{-10}$$

In other words, according to the above theory, coupled with experiment for a vertical antenna, the wave length λ is very closely 4 times the antenna height.

From the componental equations, it now follows

$$H_\theta' = \frac{2a\kappa}{\pi\sqrt{a}} \cdot V \cdot \cos\frac{\pi z}{2h} \cdot \cos pt \cdot \frac{v_0 h}{\lambda\sqrt{r}} \cdot \varepsilon^{\frac{-2\pi N}{\lambda}(r-a)}$$

$$-\frac{dH_\theta'}{dt} = \frac{4a}{\mu\sqrt{a}} \cdot \cos\frac{\pi z}{2h} \cdot \sin pt \cdot \frac{v_0 h}{\lambda^2\sqrt{r}} \cdot \varepsilon^{\frac{-2\pi N}{\lambda}(r-a)}.$$

To find i_z we have

$$i_z = \left(\frac{1}{r} + r_1\right) H,$$

and therefore the vertical component of current density at the receiving end is

$$i_z' = \left(\frac{1}{2r} - \frac{2\pi N}{\lambda}\right) \cdot \frac{2a}{\pi V\mu\sqrt{a}} \cdot \cos\frac{\pi z}{2h} \cdot \frac{v_0 h}{\lambda\sqrt{r}} \cdot \varepsilon^{\frac{-2\pi N}{\lambda}(r-a)} \cos pt$$

Thus, so far as the law of radiation goes, it is seen that a great deal depends on whether the reception of signals is made to depend more on i_z than on $-\frac{dH}{dt}$. In the case of i_z, the law as to distances is according to $\frac{1}{r^{\frac{3}{2}}}$; whereas as for $-\frac{dH}{dt}$, it is according to $\frac{1}{\sqrt{r}}$. So far as the vertical antenna is concerned for the voltaic effect, the greater the height h the better. The term $\left(\frac{1}{2r} - \frac{2\pi N}{\lambda}\right)$ is suggestive of the Cohen factor $(1+Ar)$, which was suggested to account for dispersion and diffraction.

The fact that λ appears in the exponent as a denominator, as well as in the body of the equations for i_z and $-\frac{dH_\theta}{dt}$, for a vertical antenna, bespeaks an optimum wave length as Dr. Cohen first suggested.

Considering now the case where $z > h$, that is heights above the ground greater than the antenna height, the function v is zero over this range so that in D_r it is necessary to interpret the operator function

$$\varepsilon^{-\Delta(r-a)} \cdot S^1 z^0 = S \cdot \varepsilon^{-\Delta(r-a)} z^0$$

with $$\Delta^2 = q^2 - z_1^2;\ q = jp = \frac{2\pi}{\lambda}\cdot j.$$

It can be shown that the above operator function reduces to the following form:

$$\varepsilon^{-\Delta(r-a)}\cdot z^o = \cos\frac{2\pi}{\lambda}.(r-a)\cdot z^o - \sin\frac{2\pi(r-a)}{\lambda}\cdot J_s\left(\frac{2\pi z}{\lambda}\right)\cdot j$$

where $$J_s(\beta y) = \beta\int J_o(\beta y)\cdot dy.$$

Applying now the shift operator S,

$$S\cdot\varepsilon^{-\Delta(r-a)}\cdot z^o = -\sin\frac{2\pi}{\lambda}(r-a)\cdot SJ_s\left(\frac{2\pi z}{\lambda}\right)\cdot j$$

since Sz^o is zero for $z>h$. On the other hand, SJ_s is given by

$$S\cdot J_{s\ z>h} = \left[J_s\left(\frac{2\pi z}{\lambda}\right) - J_s\left(\frac{2\pi(z-h)}{\lambda}\right)\right]_{z>h}$$

The formula for displacement D_r for $z>h$ now becomes instead

$$D_r'' \equiv \frac{a\kappa}{2\pi\sqrt{a}}\cdot\frac{v_o}{\sqrt{r}}\cdot\sin\frac{2\pi(r-a)}{\lambda}\cdot\cos pt\cdot\left[J_s\left\{\frac{2\pi(z-h)}{\lambda}\right\} - J_s\left(\frac{2\pi z}{\lambda}\right)\right]_{z>h}$$

The attenuation factor $[\ \ ]_{z>h}$ approaches zero for large values of h. *Thus a stationary wave is indicated so far as the voltaic effect alone is concerned, with nodal points spaced half a wave length λ apart,* that is, for the radial component of current as against i_r',

$$i_r'' = -\frac{aV\kappa}{\sqrt{a}}\cdot\frac{v_o}{\lambda\sqrt{r}}\cdot\sin\frac{2\pi(r-a)}{\lambda}\cdot\sin pt\cdot\left[J_s\left\{\frac{2\pi(z-h)}{\lambda}\right\} - J_s\left(\frac{2\pi z}{\lambda}\right)\right]_{z>h}$$

Again, from the auxiliary differential equations of condition—

$$H_\theta'' = -\frac{aV\kappa}{\sqrt{a}}\cdot\frac{v_o}{\lambda\sqrt{r}}\cdot\sin\frac{2\pi(r-a)}{\lambda}\cdot\sin pt\cdot\left[J_{s2}\left\{\frac{2\pi(z-h)}{\lambda}\right\} - J_{s2}\left(\frac{2\pi z}{\lambda}\right)\right]_{z>h}$$

if by J_{s2} is meant the integration of J_s with respect to z.

Thus, integrating—

$$\int J_s(\beta y)\cdot dy = \frac{(\beta y)^2}{2} - \frac{(\beta y)^4}{4\times 3.2^2} + \frac{(\beta y)^6}{6\times 5.2^2.4^2} - \frac{(\beta y)^8}{8\times 7.2^2.4^2.6^2} + \cdots = J_{s2}$$

Operating, now, according to

$$i_z = \left(\frac{1}{r} + r_1\right) H_\theta$$

$$i_z'' = -\left[\frac{1}{2r} + \frac{2\pi}{\lambda} \cdot cot\left\{\frac{2\pi(r-a)}{\lambda}\right\}\right] \cdot \frac{aV\kappa}{\sqrt{a}} \cdot \frac{v_0}{\lambda\sqrt{r}} \cdot sin\frac{2\pi(r-a)}{\lambda} \cdot sin\, pt\, [J_{s2}]_{z>h}$$

The above formulas must throw considerable light on the problem of the horizontal antenna, for the latter can be considered as made up of two abutting vertical antennas combined with an image of the whole below the surface of the earth. *In fact, "fading" as a function of the wave length changes is explained by the J_s nodal spacings.* It is therefore particularly interesting that the Eccles formula conforms to those derived for a vertical antenna proper. Moreover, so far as reception is concerned, it is seen to be important whether the reception loop be near the ground or not.

Galvanic Effect from a Vertical Grounded Antenna. If the fundamental differential equations of condition are examined, it will be seen that they must obtain for galvanic effect as well. However, the conditions to be met are different in the two cases. For the present case the limiting condition is the conduction current distribution in the vertical antenna wire itself setting up a value of H_θ at the surface to correspond. It will, therefore, be necessary first to obtain the required differential equation in H_θ and then solve the same subject to the conditions just mentioned. From this latter, the values of D_r and D_z can be derived.

Insofar as

$$t_1 \cdot D_r = -z_1 H$$

$$H = -\frac{t_1}{z_1} \cdot D_r.$$

Then, substituting, we have

$$\frac{d^2 H_\theta}{d r^2} + \frac{d^2 H_\theta}{d z^2} + \frac{1}{r} \cdot \frac{d H_\theta}{d r} - \left(\frac{1}{r^2} + \frac{1}{V^2} \cdot \frac{d^2}{d t^2}\right) H_\theta = 0.$$

Rewriting in symbolical notation (dropping subscripts for the time being)

$$\frac{d^2 H}{d r^2} + \frac{1}{r}\frac{d H}{d r} - \left(\frac{1}{r^2} + \Delta^2\right) H = 0$$

where $\Delta^2 = q^2 - z_1^2$

We have seen that a solution will be of the form

$$H = \frac{K_1(\Delta r)}{K_1(\Delta a)} \cdot A$$

where A is again any desired function, but independent of r, but that may involve the time t and the height z. Evidently where $r = a$

$$H_{r=a} = H_o = A$$

and must accord with the distribution of H at the surface of the antenna wire. Because, in electromagnetic units,

$$H_o \times 2\pi a = I$$

$$A = \frac{I}{2\pi a}$$

However, the current I in the conductor being distributed, will conform to a function of the following type:

$$I = I_z \cdot \cos\frac{\pi z}{2h} \cdot S z^o$$

with $$I_z = I_o \cdot \sin pt,$$

and the shift operator function $S z_o$, implying that no current I occurs where $z > h$. Hence the formula for H_θ is given by

$$I_\theta = \frac{K_1(\Delta r)}{K_1(\Delta a)} \cdot \frac{I_o}{2\pi a} \cdot \cos\frac{\pi z}{2h} \cdot \sin pt \cdot S z^o$$

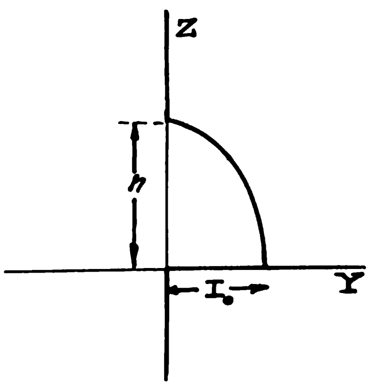

Figure 3

Again here it is necessary to remember that the sinoid $\cos\frac{\pi z}{2h}$

has two zero branches beyond the range $0<z<h$, so that H_θ will have two sets of solutions. In the range h, the solution will be primed as before, and in the region $z>h$, double primes will be employed. Considering the h region first, then, as before,

$$\Delta = \frac{2\pi N}{\lambda}$$

$$H_\theta' = \frac{1}{2\pi\sqrt{a}} \cdot \cos\frac{\pi z}{2h} \cdot \sin pt \cdot \frac{I_o}{\sqrt{r}} \cdot \varepsilon^{\frac{-2\pi N}{\lambda}(r-a)} \cdot Sz^o$$

$$-\frac{d H_\theta'}{dt} = \frac{V}{\sqrt{a}} \cdot \cos\frac{\pi z}{2h} \cdot \sin pt \cdot \frac{I_o}{\lambda\sqrt{r}} \cdot \varepsilon^{-\frac{2\pi N}{\lambda}(r-a)} \cdot S \cdot z^o$$

To obtain D_r', we note first

$$i_r = -z_1 H$$

$$i_r = \frac{1}{4} \sin\frac{\pi z}{2h} \cdot \sin pt \cdot \frac{I_o}{\sqrt{a} \cdot h \cdot \sqrt{r}} \cdot \varepsilon^{-\frac{2\pi N}{\lambda}(r-a)}$$

$$D_r' = \frac{1}{8\pi V} \cdot \sin\frac{\pi z}{2h} \cdot \cos pt \cdot \frac{I_o \lambda}{\sqrt{a} \cdot h\sqrt{r}} \cdot \varepsilon^{-\frac{2\pi N}{\Delta}(r-a)}$$

Again, because

$$i_z = \left(\frac{1}{r} + r_1\right) H,$$

we have

$$i_z' = \left(\frac{1}{2r} - \frac{2\pi N}{\lambda}\right) \frac{1}{2\pi} \cdot \frac{1}{\sqrt{a}} \cdot \cos\frac{\pi z}{2h} \cdot \sin pt \cdot \frac{I_o}{\sqrt{r}} \cdot \varepsilon^{-\frac{2\pi N}{\lambda}(r-a)}$$

$$D_z' = -\left(\frac{1}{2r} - \frac{2\pi N}{\lambda}\right) \cdot \frac{1}{4\pi^2 V\sqrt{a}} \cdot \cos\frac{\pi z}{2h} \cdot \cos pt \cdot \frac{I_o \lambda}{\sqrt{r}} \cdot \varepsilon^{-\frac{2\pi N}{\lambda}(r-a)}$$

The double prime components occurring in the region $z>h$ can be obtained in a similar manner to the procedure for the voltaic radiation. The formula i_z' for the galvanic component again shows the Cohen factor tho of somewhat different form from the corresponding voltaic component.

ANTENNA RADIATION RESISTANCE OF VERTICAL GROUNDED ANTENNA. If we attempt to apply the Poynting radiation theorem to a vertical grounded antenna, it will be sufficient to know the values of $\mathscr{E}$ and H over the surface of the aerial for finding the instantaneous watts radiated. Taking the current on the antenna as given by

$$I_z = I_o \cdot \sin pt$$

it will be necessary to take

$$v_z = v_0 \cdot \sin(pt + \psi)$$

as against

$$v_z = v_0 \cos pt$$

which would be the case of a non-radiating transmission line. (In reality, ψ is a slowly varying function of z). Since

$$v_z = v_0 \{\cos\psi \cdot \sin pt + \sin\psi \cdot \cos pt\}$$

It will be necessary in the formula developed involving $v = v_0 \sin pt$ to replace pt by $(pt+\psi)$ in order to allow for the wattage loss by radiation. At the surface of the aerial wire $(r=a)$, the voltaic displacement D_r is given, and therefore

$$\mathscr{E}_r = \frac{1}{\kappa} D_r = \frac{a}{2\pi a} \cdot \sin\frac{\pi z}{2h} \cdot v_0 \cdot \sin(pt+\psi) \cdots \text{(voltaic, } r=a) \quad (1)$$

The value of H_θ therefore reduces to

$$H_\theta = \frac{2a\kappa}{\pi a} \cdot V \cdot \cos\frac{\pi z}{2h} \cos(pt+\psi) \cdot \frac{v_0 h}{\lambda} \cdots \text{(voltaic, } r=a) \quad (2)$$

Again

$$E_z = \frac{a h v_0}{\pi^2 a}\left(\frac{1}{2a} - \frac{2\pi N}{\lambda}\right) \cdot \cos\frac{\pi z}{2h} \cdot \sin(pt+\psi) \cdots \text{(voltaic, } r=a) \quad (3)$$

Taking now the galvanic components by

$$H_\theta = \frac{I_0}{2\pi a} \cdot \cos\frac{\pi z}{2h} \cdot \sin pt \cdots \text{(galvanic, } r=a) \quad (4)$$

$$\mathscr{E}_r = \frac{1}{\kappa} D_r = -\frac{I_0}{8\pi a V \kappa} \cdot \frac{\lambda}{h} \cdot \sin\frac{\pi z}{2h} \cdot \cos pt \quad (5)$$

$$E_z = \frac{1}{\kappa} D_z = -\frac{I_0 \lambda}{4\pi^2 V \kappa a} \cdot \left(\frac{1}{2a} - \frac{2\pi N}{\lambda}\right) \cdot \cos\frac{\pi z}{2h} \cos pt \quad (6)$$

Considering the time elements separately in the radiation formula with vector notation $div\, \mathbf{W} = div\, V\mathbf{E} \cdot \mathbf{H}$, it is necessary to note that any time components such as $\sin pt \cdot \cos pt$. must reduce to zero over a cycle period $T = \frac{1}{f}$. The true wattage components in the formula necessitates that $\frac{d\mathscr{E}}{dt}$ give an average resultant value when multiplied by $\frac{dH}{dt}$. Combining therefore (4) with $\sin pt.$ · elements in (1), (2), and (3), let

$$\frac{I_0}{2\pi a}=A;\quad \frac{2a\kappa}{\pi a}\cdot V\cdot\frac{r_0 h}{\lambda}=M$$

$$\frac{I_0}{8\pi a V\kappa}\cdot\frac{\lambda}{h}=B;\quad \frac{I_0}{4\pi^2 V\kappa}\cdot\frac{\lambda}{a}\left(\frac{1}{2a}-\frac{2\pi N}{\lambda}\right)=C$$

Then for the $sin\,pt$ terms only

$$H_s\cdot sin\,pt=\left\{A\cdot cos\frac{\pi z}{2h}-M\,sin\psi\cdot cos\frac{\pi z}{2h}\right\}sin\,pt=$$

$$(A-M\,sin\psi)\cdot cos\frac{\pi z}{2h}\cdot sin\,pt.$$

Again, let

$$\frac{r_0 a}{2\pi a}=N;\quad \frac{a h r_0}{\pi^2 a}\left(\frac{1}{2a}-\frac{2\pi N}{\lambda}\right)=P$$

For the $sin\,pt$ electric components together

$$\mathcal{E}_s\cdot sin\,pt.=\left\{N\cdot sin\frac{\pi z}{2h}+P\cdot cos\frac{\pi z}{2h}\right\}cos\psi\cdot sin\,pt.$$

For the $cos\,pt$ terms in H,

$$H_c\cdot cos\,pt=M\cdot cos\left(\frac{\pi z}{2h}\right)\cdot cos\psi\cdot cos\,pt$$

On the other hand, for the electric terms combined:

$$\mathcal{E}_c\cdot cos\,pt=\left\{N\cdot sin\psi\cdot sin\frac{\pi z}{2h}-B\cdot sin\frac{\pi z}{2h}-C\,cos\frac{\pi z}{2h}\right\}\cdot cos\,pt$$

$$=\left\{(N\,sin\psi-B)\,sin\frac{\pi z}{2h}-C\,cos\frac{\pi z}{2h}\right\}cos\,pt$$

Over an element of area $2\pi a\cdot dz=dS$ with forces $\mathcal{E}$ and H, the average wattage will be

$$dW=2\pi a\cdot dz\,\frac{1}{T}\int_0^T(\mathcal{E}_c\,cos\,pt+E_s\,sin\,pt)\,(H_c\,cos\,pt+H_s\,sin\,pt)\,dt$$

$$=2\pi a\cdot dz\cdot\frac{1}{T}\int_0^T(\mathcal{E}_c H_c\,cos^2 pt+E_s H_s\,sin^2 pt)\,dt.$$

Since over a period $T=\frac{1}{f}$, terms involving $sin\,pt\cdot cos\,pt$ reduce to zero. We can therefore investigate the terms separately

$$dW_c=\pi a\cdot dz\cdot\mathcal{E}_c\cdot H_c;\quad dW_s=\pi a\cdot dz\cdot\mathcal{E}_s H_s.$$

However,

$$\mathcal{E}_c H_c=\left\{(N\cdot sin\psi-B)\,sin\frac{\pi z}{2h}-C\,cos\frac{\pi z}{2h}\right\}M\,cos\frac{\pi z}{2h}\cdot cos\psi$$

$$=MN\cdot sin2\psi\cdot sin\frac{\pi z}{2h}-\frac{MB}{2}cos\psi\cdot sin\frac{\pi z}{2h}-MC\cdot cos\psi\cdot cos^2\frac{\pi z}{2h}.$$

Similarly, therefore

$$\mathfrak{E}_s H_s = \frac{A\,N}{2} \cdot sin\frac{\pi z}{2h} \cdot cos\psi - \frac{MN}{4} \cdot sin\,2\psi \cdot sin\frac{\pi z}{2h} +$$
$$A\,P\,cos\psi \cdot cos^2\frac{\pi z}{2h} - \frac{MP}{2} sin\,2\psi \cdot cos^2\frac{\pi z}{2h}.$$

Adding the above terms together we note

$$A\,P = M\,C; \quad M\,B = A\,N$$

$$M\,P = \frac{2\,a^2\,V\,\kappa}{\pi^3} \cdot \frac{h^2}{\lambda\,a^2} \cdot v_o^2\left(\frac{1}{2a} - \frac{2\pi N}{\lambda}\right)$$

$$\therefore d\,W = d\,W_s + d\,W_c = \frac{a^2\,V\,\kappa}{\pi^2} \cdot \frac{h^2}{a\,\lambda} \cdot v_o^2\left\{\frac{1}{2\,a} - \frac{2\pi N}{\lambda}\right\} sin\,2\psi \cdot cos^2\frac{\pi z}{2h} \cdot d\,z\,.$$

Integrating over the whole length of the aerial, then, because

$$\int_o^h cos^2\frac{\pi z}{2h} \cdot d\,z = \frac{h}{2}$$

$$W_h = \frac{a^2\,V\,\kappa}{\pi^2} \cdot \frac{h^3}{2\,\lambda\,a} \cdot v_o^2\left\{\frac{1}{2\,a} - \frac{2\pi N}{\lambda}\right\} sin\,2\psi - \frac{a^2\,V\,\kappa}{2\,\pi^2} \cdot \frac{h^3}{\lambda\,a^2} \cdot v_o^2 \cdot sin\,\psi \cdot cos\,\psi.$$

since N is small. The above represents the radiation from the body of the wire only and does not include in any manner the radiation by means and thru the earth's surface.

To evaluate ψ, the wattage can be derived from a consideration of the current and voltage in the aerial. The average wattage consumed in the antenna at any distance z and over the interval $d\,z$ is given in terms of maximum voltage and current by

$$dW_h = \frac{d\,z}{T}\int_o^T\left\{I_o \cdot cos\frac{\pi z}{2h} \cdot sin\,pt\right\}\frac{d}{dz}\left\{v_o \cdot sin\frac{\pi z}{2h} \cdot sin(pt+\psi)\right\} \cdot dt$$
$$= \frac{d\,z}{T}\int_o^T I_o v_o \cdot \frac{\pi}{2\,h} \cdot \left(cos\frac{\pi z}{2h}\right)^2 \cdot (sin\,pt)^2 \cdot cos\,\psi \cdot dt,$$

because we can neglect the $sin\,pt \cdot cos\,pt$ term over an entire interval T. The above reduces to

$$d\,W_h = \frac{\pi}{4\,h} \cdot I_o v_o \cdot cos\psi \cdot \left(cos\frac{\pi z}{2h}\right)^2 \cdot d\,z$$

Integrating, therefore, over the whole length h of the aerial:

$$W_h = \frac{\pi}{8} \cdot I_o v_o \cdot cos\,\psi$$

This latter must equate to the value given previously. Thus

$$\frac{\pi}{8} I_o v_o \cos\psi = \frac{a^2}{2\pi^2 V\mu} \cdot \frac{h^3}{\lambda a^2} \cdot v_o^2 \cdot \sin\psi \cos\psi$$

$$\sin\psi = \frac{\pi^3}{4a^2} \cdot \frac{\lambda a^2}{h^3} \cdot V\mu \frac{I_o}{v_o}$$

To express the above in terms of the radiation resistance, let this latter be defined by

$$\frac{W_h}{(I_o/\sqrt{2})^2} = \frac{2W}{I_o^2} = R_h = \frac{\pi}{4} \cdot \frac{I_o v_o \cdot \cos\psi}{I_o^2} = \frac{\pi}{4} \cdot \cos\psi \cdot \frac{v_o}{I_o}.$$

For any vertical type of aerial the ratio $\frac{v_o}{I_o}$ is a known constant, and depends merely on the ratio of the inductance to the permittance per unit of length. Defining this ratio by

$$\Omega = \frac{v_o}{I_o}; \quad R_h = \frac{\pi}{4} \cdot \Omega \cdot \cos\psi$$

For convenience, let

$$A = \frac{\pi^3}{4a^2} \cdot \frac{\lambda a^2}{h^3},$$

then

$$\sin\psi = A\frac{V\mu}{\Omega}; \quad \cos\psi = \sqrt{1 - \left(\frac{AV\mu}{\Omega}\right)^2}$$

$$\therefore R_h = \frac{\pi}{4}\Omega\sqrt{1 - \left(\frac{AV\mu}{\Omega}\right)^2} = \frac{\pi}{4}\sqrt{\Omega^2 - \left\{\frac{\pi^3}{4a^2} \cdot \frac{\lambda a^2}{h^3} \cdot V\mu\right\}^2}$$

Thus, for radiation from the antenna wire itself in the case of a vertical aerial, the height should be as great as possible; for since the wave length is proportional to the height, an increase in height will more than compensate for the increase in wave length. Observe that the above takes no note of radiation from the surface of the earth. The constant a is defined by the relation $\frac{dC}{dz}$ above. For its evaluation, see the paper by the author on "The Vertical Grounded Antenna as a Generalized Bessels Antenna," PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, December, 1918, volume 6, number 6.

SUMMARY: After considering the previous investigations of antenna radiation, the author attacks this problem taking into account the sinusoidal distribution of current and voltage along the antenna. The vertical grounded antenna only is considered.

A detailed mathematical investigation of radiation and field strengths at remote points follows. Formulas resembling the Eccles radiation formula, and the Cohen dispersion and diffraction factor formula are obtained and interpreted.

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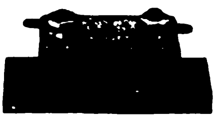

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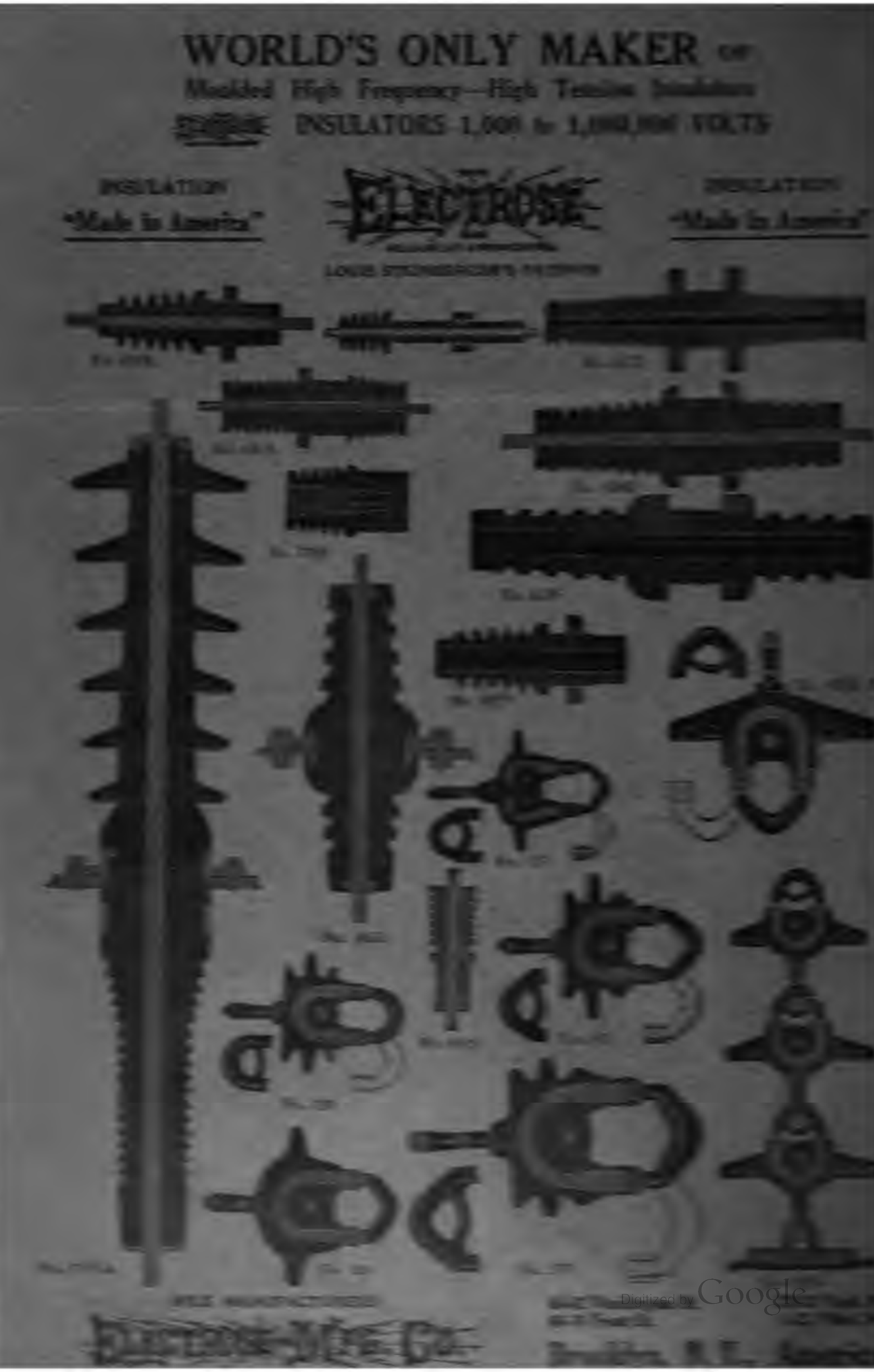
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