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An Induction Loud Speaker

The Acoustical and Electrical Characteristics of a Loud Speaker Capable of Handling Large Amounts of Energy and which Produces Sounds of Tremendous Volume with Negligible Distortion—The Mathematics of Its Design

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THE loud speaker described in this paper cannot be used for the purposes of the ordinary broadcast listener, but it is an electrical device of extraordinary interest. Because it can handle such large quantities of power and reproduce voice and music with such unusual faithfulness, this device has attracted a great deal of attention. This paper was delivered before a recent meeting of the Radio Club of America, in New York City and is full of the theory and mathematics of design, but it is an interesting and complete presentation of an excellent piece of work.—THE EDITOR

THE problem of reproducing speech and music by electrical means may be arbitrarily divided into four main parts. The first of these concerns the operation, known technically as "pick up." In this operation, the sounds to be reproduced are allowed to produce electrical effects which are usually quite small. The second part of the problem concerns the amplification of the small electrical effects produced by the original sound waves. The third part of the problem concerns the transmission of the electrical signals from one place to another. This usually occurs between the stages of amplification. The fourth part of the problem is that of reproducing sound waves by means of the amplified electrical effects. In case the transmission is accomplished by electrical waves in space, there is still another part of the problem, namely, that of receiving the signals. This may, however, be included in the division of the problem concerning amplification, because many of the considerations involved in radio reception are of a similar nature to those involved in amplification.

This discussion will concern itself mainly with the fourth part of the problem as outlined above; namely the reproduction of speech and music by operating by electrical means upon a particular type of "loud speaker."

The loud speaker, which I shall describe and discuss, is known as the "Induction Loud Speaker," and has already been described in its essential features in previous publications (*Phys. Rev. 17*, p. 257, 1921 and *19*, p. 52, 1922. *Jr. Opt. Soc. Am. 4*, p. 1059, 1922). For the sake of completeness I shall repeat here a brief description of the construction and principle of operation of the instrument.

ESSENTIALS OF THE SPEAKER

THE induction loud speaker consists of two flat circular coils mounted coaxially on either side of a circular sheet of metal such as aluminum. Fig. 1 shows a picture of the parts, and Fig. 2 several models of the assembled instrument. Each coil is made up of sections with annular air spaces between them. These sections are secured to the wooden framework by means of wires which pass around them and through holes in the spider. The sections are connected in series and the terminals of each coil are brought out to two binding posts fixed to the circular frame. The circular diaphragm of aluminum has the same diameter as that of the circular framework. It is lightly held between the two frames by small pieces of felt placed between the diaphragm and each frame at intervals of about 3 inches around its circumference. This method of support leaves the diaphragm quite free to vibrate through such amplitudes as are required of it and allows it to expand when it gets hot. It also allows a certain amount of convection of air to pass upward between the coils and diaphragm and out at the top between the frames and diaphragm. The two coils shown in Fig. 1 are 25 inches in diameter, have an axial width of about $\frac{1}{2}$ inch and contain about 75 pounds of 45 mil wire. The frames and diaphragm are 30 inches in diameter. When mounted the coils are about $\frac{1}{4}$ inch from the aluminum diaphragm, whose thickness is 10 mils.

In operation the instrument is connected as shown in Fig. 3.

The generator sends a direct current through the coils which are connected so that the two magnetic fields due to this current oppose one another. The resultant magnetic field in the space occupied by the diaphragm lies along the radii of the diaphragm. The by-pass condensers C C enable the voice current from the amplifier to pass through the two coils in multiple. From the standpoint of the voice currents, the instrument is an alternating current transformer, the two coils being the primary and the aluminum diaphragm a one-turn secondary. The alternating current in the diaphragm distributes itself throughout the whole diaphragm, and the flow lines are circles concentric with the axis of the diaphragm, and consequently are at right angles to the radius of the diaphragm at all points. The magnetic field, due to the direct current, and the induced voice currents in the diaphragm, are therefore at right angles at all points, and the diaphragm experiences an electrodynamic force of the same character as the wave form of the voice current. This force is distributed fairly uniformly over the whole of the diaphragm, and to a high degree of approximation, the phase of the force is the same at all points, at least for the range of frequencies concerned in the reproduction of speech and music.

CHARACTERISTICS OF THE SPEAKER

THIS instrument reproduces speech and music with remarkable faithfulness, but its sensitiveness is much below that of the more usual types of sound reproducing devices. On account of its size and ruggedness, however, it may be supplied with large amounts of power, so that an enormous volume of sound may be produced. In fact, the device readily lends itself to the field of public address where thousands of people are to be reached in large auditoriums, or even out of doors.

This instrument embodies several features which are obviously of great importance for the faithful reproduction of speech and music. In the first place, the diaphragm is aperiodic which, while contributing to the instrument's lack of sensitiveness, eliminates all distortion due to resonance. In the second place, the force moving the diaphragm is distributed fairly uniformly over its whole surface so that the diaphragm moves as a whole, there being no tendency for it to vibrate in segments, which might result in resonance at frequencies corresponding to its partial vibrations. Thirdly, the large area of the diaphragm results in relatively efficient radiation over the lower range of frequencies, without the use of a horn. In speech and many forms of music most of the sound energy is carried by the lower frequency components, while the naturalness of speech is lost if these lower frequencies are not present in sufficient quantity. In the fourth place, the instrument is simple and rugged in construction and does not require any fine adjustments. When once put into operating condition it will remain so indefinitely.

OPERATION OF A LARGE DIAPHRAGM

IN ORDER to make some calculations of what we should expect in the performance of a large area diaphragm, we shall make certain simplifying

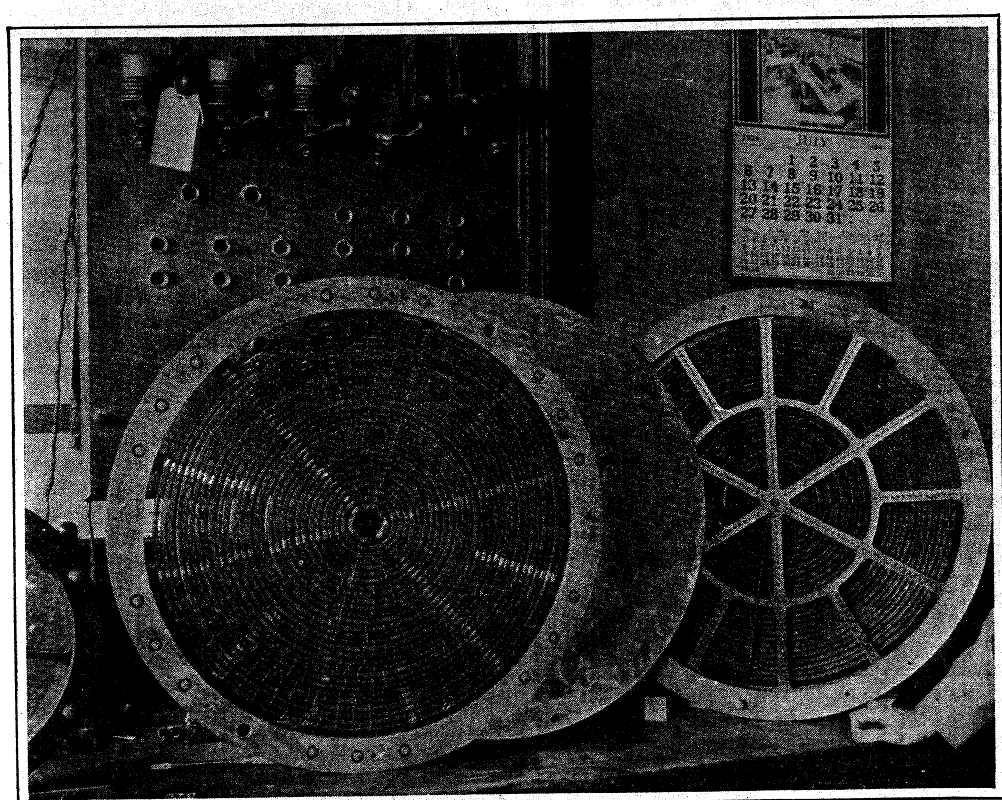

FIG. I
Several of the Hewlitt Induction loud speakers in a corner of the research laboratory of the General Electric Company at Schenectady

assumptions in regard to the boundary conditions in the surrounding medium, and in regard to the driving forces acting on the diaphragm.

In the first place, we shall assume that the diaphragm moves as a whole when vibrating. The degree to which this is realized in practice depends upon the distribution and phases of the electrodynamic forces over the diaphragm; upon the natural periods of vibration in which the diaphragm may vibrate owing to its elastic properties; and upon the manner in which it is supported. The polarizing field, which is radial, is weak near the center, but fairly uniform over the major portion of the diaphragm. The magnitude of this component of the field in gauss is very roughly given by the total ampere-turns on both sides of the diaphragm divided by the diameter of the diaphragm. The induced current in the diaphragm should be most densely distributed in the central portion of the diaphragm where the radial field is the weakest. Since the electrodynamic force acting on the diaphragm is proportional to the product of the radial field and the current induced in the diaphragm, it would seem that to a first degree of approximation we would be justified in assuming that the force is uniformly distributed over the diaphragm. This assumption neglects whatever phase difference exist between the induced currents in the different parts of the diaphragm.

In regard to resonant periods the diaphragm is so large, and so loosely held between the edges of the supporting framework, that the fundamental period would be only a few cycles per second. Moreover the restoring force is so small, and the dissipation so great on account of the looseness with which the diaphragm is held, that the partial vibrations would not arise with appreciable intensity. The fact that the diaphragm is held around the edges should not affect its motion very far from the edge, for the maximum amplitudes of motion under ordinary conditions of use would not exceed 1 mm. for frequencies as low as 30 cycles, and for higher frequencies, the amplitude falls off almost as fast as the inverse square of the frequency. Actual listening tests have shown that the quality of speech or music produced by a large diaphragm, say 2 feet in diameter, suspended by two strings cannot be distinguished from that produced by one clamped around the edges.

INTENSITY OF SOUND WAVES FROM A LARGE DIAPHRAGM

WE SHALL also assume that the diaphragm is bounded by an infinite plane which is at rest, and that the medium extends indefinitely in all directions on both sides of the plane. In actual practice, the instrument is not bounded by a large plane. This assumption introduces very little error into the calculations we shall make for waves short compared to the circumference of the diaphragm, but when the length of the waves becomes comparable to the circumference of the diaphragm, the calculation will give too great radiation, and the error will be greater, the longer the waves.

The problem of calculating the intensity of the sound waves given off from a vibrating diaphragm under the conditions as we have limited them has been solved by Lord Rayleigh. (*Theory of Sound*, Vol. II, p. 162–169).

The equation of motion for a simple harmonic application of force is

$$m\frac{d^2x}{dt^2} + k\frac{dx}{dt} + n^2x = F\cos\omega t$$

where x is the displacement of the diaphragm from its position of equilibrium, F is the maximum value of the harmonic force impressed on the diaphragm, $\omega = 2\pi$ times the frequency, n^2 is the elastic force opposing displacement for unit displacement,

$$m = m_o + \frac{\pi\rho}{2\,\alpha^3}\,K,\ (2\,\alpha\,R)$$

$$\text{Where } K_1\,(z) = \frac{Z}{\pi}\left(\frac{Z^3}{1^2.3} - \frac{Z^5}{1^2.3^2.5} + \frac{Z^7}{1^2.3^2 5^2.7} - \text{etc.}\right)$$

and m_o is the mass of the diaphragm, ρ is the density of the air, $\alpha = \frac{2\pi}{\lambda}$. λ is the wave length of the air vibration set up by the diaphragm, R is the radius of the diaphragm.

$$k = v\,\rho\,\pi\,R^2\left(1 - \frac{J_1\,(2\,\alpha\,R)}{\alpha\,R}\right)$$

$J_1\,(z)$ is the Bessell function of the 1st order of z, and v is the velocity of sound.

In the case under discussion, the diaphragm vibrates across a radial magnetic field, so that there is a magnetic damping force acting on the diaphragm in addition to that due to the emission of sound waves. The approximate calculation of this effect is given in appendix I and is shown to consist of two force terms, one multiplying the displacement, and the other the velocity. Both terms are shown to be negligible compared to the other terms present.

The force driving the diaphragm arises from the interaction of the radial magnetic field and the currents induced in the diaphragm by those in the coils. In appendix II, the approximate magnitude of this force is calculated and shown to be

$$F = H\sqrt{\frac{2\,W_o A}{r}}$$

where H is the strength of the radial magnetic field, W_o is the audio power transferred from the coils to the diaphragm, r is the superficial resistivity of the diaphragm and A is its area. The square of the force acting on the diaphragm is thus proportional to its area for definite values of H, W_o, and r.

Returning to the equation of motion of the diaphragm we may calculate the power expended by the driving force F cos ωt.

$$\text{This is } W = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}}\left[m\frac{d^2x}{dt^2} + k\frac{dx}{dt} + n^2x\right]\frac{dx}{dt}\,dt$$

$$= \frac{k\,F^2}{2\left(k^2 + \left[\frac{n^2 - \omega^2 m}{\omega}\right]^2\right)}$$

Estimation of n^2 for the diaphragm under consideration shows it to be entirely negligible compared to

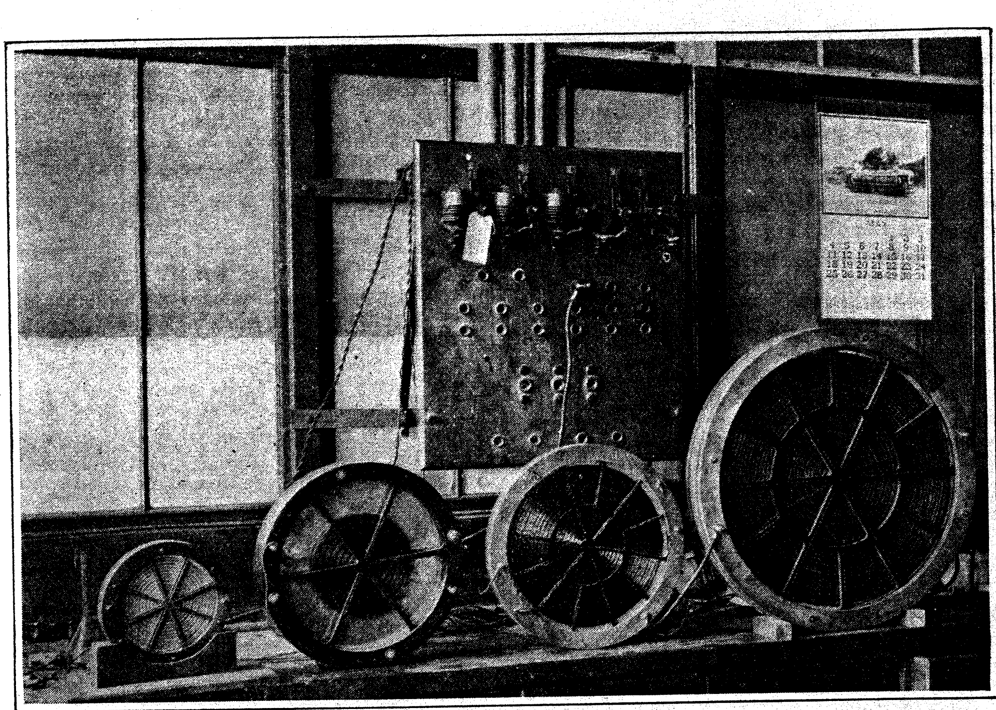

FIG. 2
Various size models of the induction loud speaker

$\omega^2 m$ for all frequencies with which we are concerned. The sound energy radiated each second then becomes

$$W = \frac{k F^2}{2 (K^2 + \omega^2 m^2)}$$

For very short waves this is radiated almost as a beam of plane waves of cross section equal to that of the diaphragm. As the waves get longer, the beam spreads out, and when the length of the waves is comparable to the circumference of the diaphragm the radiation passes out in all directions, and at the same time the above expression for W gives too large a value for the total radiation, because the diaphragm is not bounded by an infinite plane at rest. If a sound measuring device were placed in front of the vibrating diaphragm, and its indications taken for a wide range of frequencies, these indications would be proportional to W, calculated from the above expression, only for wavelengths short compared to the circumference of the diaphragm. For increasing wavelengths comparable to and larger than the circumference, the indications of the measuring instrument would increase less rapidly than W calculated from the above expression for two reasons. First, because on account of the greater spreading for long wavelengths a less proportion of the energy radiated would enter the measuring instrument, and second, because the expression gives too great a value for the radiation at long wavelengths. This consideration should be borne in mind when examining the tables and curves to follow showing the sound energy radiated from the diaphragms as a function of the frequency.

CALCULATION OF SOUND RADIATION

THE sound radiation will be calculated for several different sizes of diaphragms. In order to make the results comparable, we shall assume that the radial magnetic field has the same strength for all sizes of diaphragm. As is shown in appendix III, this corresponds approximately to dissipating an amount of direct current power proportional to the square of the diameter of the instrument. We shall also assume that the same amount of voice-current power will be supplied to all sizes of instrument, that is, we shall employ the full output of a given audio amplifier to drive all instruments. As has been shown, this means that the force actuating the diaphragm is proportional to its radius. A comparison of the results so obtained will favor the smaller instruments from the standpoint of *total sound output*, for the radial field may be made stronger at a constant temperature of operation, and more audio power may be safely supplied to the larger than to the smaller instruments. With the same limiting temperature of operation the field of the largest instrument discussed might be from one to two times as great as that of the smallest, while the audio power input might be from ten to twenty times as great, so that the total sound energy output might be twenty to forty times as great in the case of the largest instrument. For any one instrument, however, these considerations would not affect the relative amount of sound energy output at different frequencies. It might be remarked at this point that as the sound energy output is proportional to the product of the strength of the polarizing field, and the audio current in the diaphragm, and as the total power supplied is limited by the allowable temperature rise, the sound energy output is a maximum when the two powers are equal (see appendix IV). But owing to the great disparity in the cost of polarizing and audio power it is advisable to use polarizing power to within a small percentage of the allowable dissipation. For example, using the 25-inch instrument with 800 watts of polarizing power, and 30 watts of audio power, the sound pressure output is about 15 per cent. of what it would be using 415 watts of each kind of power.

VALUES OF DIFFERENT DIAPHRAGMS

THE calculation has been carried out for five different sizes of diaphragm assuming a uniform field strength H=300 gauss, and that the audio power input is 1 watt in each case.

The diaphragms are all of aluminum .025 cm. thick. The following table gives the results of the calculations and these are represented graphically in Fig. 4.

TABLE I

Radius	$\frac{30}{\pi}$ cm.	$\frac{60}{\pi}$ cm.	$\frac{100}{\pi}$ cm.	$\frac{150}{\pi}$ cm.	$\frac{300}{\pi}$ cm.
Frequency cycles/sec.	W in Kiloergs per sec.				
30		8.81	16.5	26.9	53.4
60	2.85	8.05	16.6	27.4	55.0
100	2.55	7.91	16.5	27.1	57.1
150	2.46	8.40	16.2	26.2	50.6
200	2.51	7.63	15.1	25.1	31.0
300	2.48	7.22	13.5	17.2	16.9
400	2.28	6.23	9.61	8.69	9.3
600	2.18	4.65	4.23	4.69	6.9
750	1.96	3.13	2.83		
1000	1.61	1.56	1.68		
1500	0.80	0.78			
2000	0.39				
3000	0.18				

As already stated, it should be borne in mind that the actual frequency characteristic as perceived by one standing in front of the instrument would not be so pronounced as indicated by the table and curves, because the calculation gives too great a value for the radiation at low frequencies, and also the lower the frequency the more the spreading of the sound. Moreover the response of the ear mechanism is proportional to the sound wave pressure rather than to the energy flux. At any given frequency the sound wave pressure is proportional to the square root of the energy flux. Still another consideration is the relation between the impedance of the amplifier and that of the loud speaker. Fig. 5 shows the impedance-frequency curve for the 25-inch or $R = \frac{100}{\pi}$ cm. instrument, provided with an aluminum diaphragm .025 cm. thick. The by-pass condensers shown in Fig. 3 were 3 mfd. each. It is apparent that if the power amplifier has an impedance of 1000 ohms, then the power delivered to the loud speaker is going to fall off rapidly below a frequency of 600 cycles, which will prevent the excessive radiation of low frequencies. In fact, quite noticeable changes in the general pitch level of the reproduced speech of music can be accomplished by adjusting the impedance of the loud speaker by means of transformers. The difference in the directivity of the loud speaker for short and long waves is shown by the progressive loss in articulation, particularly with the larger instruments, as the angle between the axis of the instrument and a line drawn from the instrument to the observer is increased. When using the larger instruments out of doors and in auditoriums it is well to use at least two in order to so direct them as to minimize the effect just mentioned.

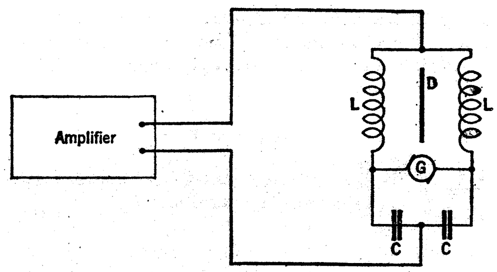

FIG. 3

The circuit diagram of the Hewlitt induction loud speaker. L L are the two flat coils; D, the aluminum diaphragm; G, a direct current generator; and C C, by-pass condensers

QUALITY OF THE SPEAKER ON LOW FREQUENCIES

IN ORDER to arrive at some idea as to how great an error is made in assuming for the purposes of calculation that the diaphragm is bounded by an infinite plane at rest, a large board, 6 feet square, was prepared with a circular hole of variable diameter in the center into which various size instruments could be placed. It was found with the smallest instrument, $R = \frac{30}{\pi}$ cm., the general pitch level of speech and music was noticeably lowered when placed in the hole in the board, while with the instrument $R = \frac{100}{\pi}$ cm. this lowering of the general pitch level was barely noticeable. This means that, for the range of frequencies with which we are ordinarily concerned in the reproduction of speech and music, the failure of the expression for W at low frequencies is of little importance. Of course, there is still left the effect of the greater spreading of the lower frequencies.

On the whole, after taking everything into consideration, it appears that the instrument ought to reproduce well the lower frequencies which is necessary for naturalness in the reproduction of the human voice and for richness of quality in music. It is seen that the smaller diaphragms should give a fairly flat frequency characteristic over a greater range than do the larger ones. That is, the higher tones should be relatively more important in the smaller than in the larger instruments. These conclusions are borne out by experience.

APPENDIX I

WE SHALL only attempt to get a rough estimate of the order of magnitude of the magnetic damping force acting on the diaphragm owing to its vibration across the radial magnetic field. For this purpose let us suppose that the metal composing the diaphragm is concentrated into a single circular turn of wire of circular cross section whose diameter is one half that of the diaphragm. Let this ring vibrate parallel to its axis with displacement x, velocity v and amplitude A. Then $x = A \sin \omega t$ and $v = \omega A \cos \omega t$. The induced electromotive force is $e = v c H$, where c is the circumference of the circular wire and H is the strength of the radial magnetic field. Then

$$e = \omega A c H \cos \omega t$$
$$= E \cos \omega t, \text{ where } E = \omega A c H$$

Applying Kirchoff's law, letting i be the induced current

$$i r + L \frac{di}{dt} = E \cos \omega t$$

where r and L are the resistance and inductance of the wire. From this follows

$$i = \frac{E}{\sqrt{r^2 + (\omega L)^2}} \cos (\omega t - \Theta) \text{ where } \tan \Theta = \frac{\omega L}{r}$$

The reaction of the field on this current is

$$f = i c H = \frac{\omega A}{Z} (C H)^2 \cos (\omega t - \Theta)$$
$$\text{where } Z = \sqrt{r^2 + (\omega L)^2}$$

or

$$f = \frac{\omega A (C H)^2}{Z^2} \left[r \cos \omega t + \omega L \sin \omega t \right]$$
$$= r \left(\frac{C H}{Z} \right)^2 \cdot \frac{dx}{dt} + \omega^2 L \left(\frac{C H}{Z} \right)^2 \cdot x$$

In order to take account of this force, we may assume that this is the magnetic drag that would act on the diaphragm represented by the ring, and we may then add the above coefficients of $\frac{dx}{dt}$ and X to the corresponding coefficients in the original equation of motion. To carry this out for a particular case, the instrument $R = \frac{100}{\pi}$ cm. with an aluminum diaphragm .025 cm. thick was chosen. It is assumed that H = 300 gauss; calculation of the other quantities concerned give

$$r = 3.52 \times 10^5 \text{ e.m.u.}$$
$$L = 7.41 \times 10^2 \text{ ''}$$
$$C H = 2.95 \times 10^4 \text{ ''}$$

If we let $a = r \left(\frac{C H}{Z} \right)^2$

$$\frac{b}{\omega} = \omega L \left(\frac{C H}{Z} \right)^2$$

then the expression for the sound energy radiated is

$$W = \frac{k F^2}{2 \left[(k + a)^2 + \left(\frac{b}{\omega} - \omega m \right)^2 \right]}$$

The radiation in kiloergs /sec. and the amplitude in cm. calculated for this instrument for an input of 1 watt of audio power is given in table II.

TABLE II

FREQUENCY CYCLES /SEC.	W KILOERGS /SEC.	A CM.
30	16.9	1960×10^{-5}
60	16.9	500 ''
150	16.7	85 ''
300	13.5	24 ''
600	4.2	7 ''

From a comparison of the values of W in Table II with those for the same instrument in Table I it is apparent that the damping of the magnetic field has no appreciable effect on the frequency-radiation characteristic of the loud speaker.

APPENDIX II

IN ORDER to get an approximate idea of the periodic force driving the diaphragm, let us assume that the audio power is transferred quantitatively to the diaphragm, and is there dissipated in heat. The audio impedance with diaphragm is only a few per cent. of that without diaphragm, and it is seen from Table I that with a field strength of 300 gauss somewhat less than 0.2 per cent. of the audio power is converted into sound radiation. The above assumption is, therefore, justified for a first approximation. We shall also assume that the induced current in the diaphragm is uniformly distributed. Let I be the maximum value of a sine wave audio current through an annulus of the diaphragm, 1 cm. wide, and let *r* be the superficial resistivity of the diaphragm. Let A be the area of the diaphragm, and W_o the audio power supplied. Then $I^2 r A = 2 W_o$, and the maximum value of the force on the diaphragm is $H I A = H \sqrt{\frac{2 W_o A}{r}}$ where H is the strength of the radial magnetic field. For a given thickness of diaphragm of a given material, a given field strength, and a definite supply of audio power, the square of the force driving the diaphragm is proportional to the area of thc diaphragm.

APPENDIX III

THE power dissipated in the instrument has to be eliminated through the faces of the coils, and in the absence of forced ventilation, the amount of power that can be dissipated from instruments of various size with a given mean temperature rise of the coils will be proportional to the area of the coils. The induction loud speakers have been designed to operate at a temperature of 100° C. The power to be dissipated is practically the polarizing power, since the audio power under actual conditions of operation is only a few per cent. of the polarizing power. The following brief analysis will show the relation between the polarizing voltage, the number of turns, and the linear dimensions of the coils:

Let R = radius of one pancake coil
r = resistance one pancake coil
t = axial depth of winding
n = number of turns in one pancake
E = voltage on one pancake
S = space factor of windings
ρ = specific resistance of the wire.

Let us assume a constant space factor for coil windings when using wire of various sizes, and for various size coils. This factor may vary from 0.40 to 0.50, and takes account of the thickness of insulation, the circular section of the wire, and the space between sections for the passage of sound waves.

$$\text{Then } r = \frac{\pi R n \rho}{\frac{R t s}{n}} = \frac{\pi \rho}{S} \cdot \frac{n^2}{t} = K_1 \frac{n^2}{t} \qquad (1)$$

For the 25-inch instrument described in this paper, K_1 has a value of 1.31×10^{-5} at 20° C with r measured in ohms, and t in cm. This is a good representative value for K_1 and corresponds to a space factor of 0.426 and a specific resistivity of 1.78×10^{-6} ohms per cm³.

Equation (1) shows that the resistance of a pancake can be expressed in terms of the number of turns of wire and the axial thickness of the winding, independent of the diameter of the instrument.

The power dissipated in one pancake coil is

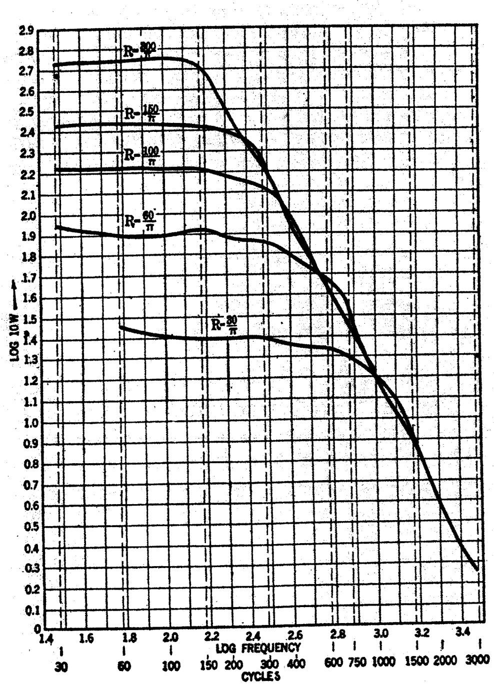

FIG. 4

$\frac{E^2}{r} = \frac{E^2 t}{K_1 n^2}$. Let us suppose this proportional to the exposed area of the pancake coil. Then

$$\frac{E^2 t}{K_1 n^2} = K_2 R^2$$

When both pancakes are mounted together as they are in the assembled instrument, it is found that a temperature rise of 80° C corresponds to a value of $K_2 = 0.50$ when R is measured in cm., and the power in watts. $K_2 R^2$ then gives the power dissipated as heat in each pancake coil.

Solving the last equation for n we have

$$n = \frac{E}{R} \sqrt{\frac{t}{K_1 K_2}} \qquad (2)$$

that is for a given operating voltage, the number of turns is proportional to $\frac{\sqrt{t}}{R}$. The current is $i = \frac{E}{r} = \frac{Et}{K_1 n^2}$

The ampere-turns for one pancake coil are $ni = \frac{Et}{K_1 n}$

and by Ampere's law, the strength of the radial component of the magnetic field contributed by one pancake is approximately proportional to $\frac{ni}{R} = \frac{Et.}{K_1 n R}$

Let us see what condition must be fulfilled in order that we may have the same strength of magnetic field for instruments of all sizes, that is

$$\frac{Et}{K_1 n R} = K_3 \qquad (3)$$

If E is expressed in volts, and the other quantities expressed as previously specified, then K3 is approximately 1.6 times the strength of the radial component of the magnetic field due to one pancake, or 0.8 times the total radial component when both pancakes are present, the strength of the magnetic field being expressed in gauss.

Eliminating $\frac{E}{Rn}$ from (2) and (3) we have $t = \frac{K_1 K_3^2}{K_2}$

or the axial thickness of all the coils must be the same. Actually, the demands on the instrument from the standpoint of an audio transformer are such that the thickness of the coils may be made larger in the larger instruments than in the smaller ones. Therefore, with the larger instruments we may have a stronger radial magnetic field than with the smaller ones when operating at the same temperature.

In order to design the windings for an induction loud speaker, the following procedure will be found to be as direct as any. Suppose the approximate radius, R, of the pancake coils ,and E, half the polarizing voltage, are given. First choose t, the axial thickness of the pancake, which may be 0.5 inches for coils as small as 4.5 in. radius to 1.0 inch for coils as large as 18 in. radius. Choose next, a space factor between 0.40 and 0.50. Calculate K. and take $K_2 = 0.50$. Then $H = \frac{K_3}{0.8} = 1.25\sqrt{\frac{K_2 t}{K_1}}$

which should be at least as large as 280 e.m.u. If H is not this large, t or s should be adjusted accordingly. The resistance of the pancake coil is given by $\frac{E^2}{K_2R^2}$. If d is the density of copper $\pi R^2 t Sd$ gives the mass of copper in the pancake, and from this and the resistance, the size of wire to be used may be read off from a wire table. The number of turns may be calculated from equation (1). Using the above figures as approximations, the coil may now be accurately designed by obvious procedure. If the polarizing voltage is very low, say less than 100 volts, the instrument may require excessive current, and be of very low impedance, while if the voltage is high, the reverse conditions may be encountered. 250 to 500 volts for polarizing have been found to give very satisfactory results, both from the standpoint of polarizing current and audio impedance for all sizes of instruments so far built.

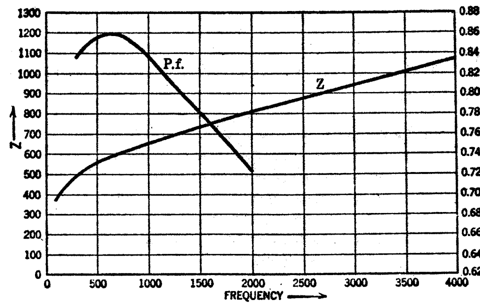

FIG. 5

APPENDIX IV

LET us determine the condition for the maximum amount of sound radiation output for a given total amount of polarizing and audio power input. The force driving the diaphragm is directly proportional to the strength of the radial magnetic field, which in turn is directly proportional to the polarizing current i_1. The force is also proportional to the audio current in the diaphragm, which in turn is proportional to the audio current in the coils i_2. The sound pressure output p is proportional to the force acting on the diaphragm and, therefore, we may write

$$p = K_1 i_1 i_2 \qquad (1)$$

If r_1 and r_2 are the d-c and audio resistances, respectively, of the instrument, then the condition that the total power supplied shall be independent of the relative proportions of polarizing and audio powers, is

$$i_1{}^2r_1 + i_2{}^2r_2 = K_2 \qquad (2)$$

differentiating (1) and (2) with respect to i_1, we have

$$\frac{dp}{di_1} = K_1i_2 + K_1i_1\frac{di_2}{di_1} \qquad (3)$$

$$2i_2\frac{di_2}{di_1}r_2 + 2i_1r_1 = 0 \qquad (4)$$

From (4) we have $\frac{di_2}{di_1} = -\frac{i_1}{i_2} \cdot \frac{r_1}{r_2}$ (5)

Substituting (5) in (3) and equating to 0, we have $i_2{}^2r_2 = i_1{}^2r_1$, which is the condition that p shall be a maximum. It therefore follows that the audio and polarizing powers should be equal in order that the maximum sound radiation should be produced.

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NEUTRALIZING AND TUNED RADIO FREQUENCY

ANOTHER paper presented before the Radio Club of America will appear in RADIO BROADCAST for September. It is by C. L. Farrand and deals with his further findings in the field of tuned radio frequency amplification, especially in the important matter of neutralization. The progress of Mr. Farrand's experiments is traced and diagrams and photographs show clearly his research in this very important subject.—THE EDITOR.