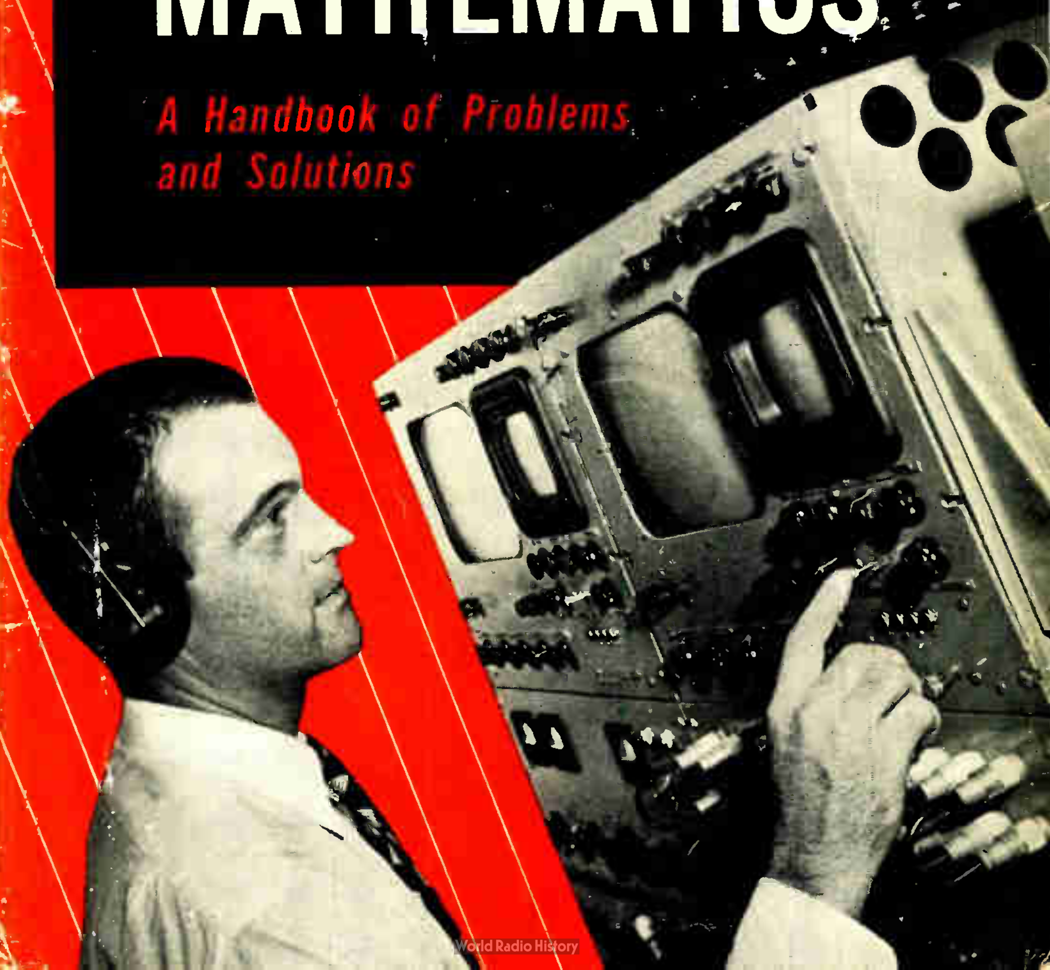
BERNHARD FISCHER, Ph.D.

RADIO and TELEVISION MATHEMATICS

A Handbook of Problems and Solutions



A practical handbook and reference for anyone working in radio, television, or other branches of electronics, this book gives the solutions for nearly 400 problems typical of those encountered in the construction, operation, and servicing of radios, television and other electronic equipment.

The problems, arranged conveniently under electronic headings, include all calculations commonly encountered in electronics, from those involving basic circuit components to those concerned with specialized elements in television and in modern control apparatus.

In each case the author shows how to set up the problem, as well as how to solve it; and all solutions are given in simple, logical steps with no steps omitted. In addition, the book contains a section of extra problems for practice, a complete compilation of formulas used in electronics, mathematical tables, a review of basic mathematics and the use of the slide rule, the J-operator, powers of ten and polar vectors.

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BERNHARD FISCHER

THE author of this book is Vice President in Charge of the Training Division at the American Television Laboratories in Hollywood, California. He has a Ph.D. from the University of Vienna, and taught physics and mathematics in the city school system of Vienna for several years. Coming to this country before the war, he taught a number of special war-training courses at the University of California at Los Angeles until he joined the staff of the Television Laboratories. He is a member of the Institute of Radio Engineers and the Academy of Television Arts and Sciences.

Radio and Television Mathematics

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Radio and Television Mathematics

A HANDBOOK OF PROBLEMS AND SOLUTIONS

Bernhard Fischer, PH.D., VICE PRESIDENT IN CHARGE OF TRAINING, AMERICAN TELEVISION LABORATORIES OF CALIFORNIA

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Preface

This book, as indicated in its subtitle, is a handbook to serve as a guide and reference book for the practical man, as a collection of problems for instructors, and as a review for those who want to acquire a rapid practical skill in solving problems in preparation for radio license examinations given by the United States Federal Communications Commission.

The student of radio will find a number of problems covering the electrical fundamentals of direct and alternating current, as well as representative problems in radio communications including television. The sections on receivers and transmitters include problems to assist in clarifying the principles involved, especially valuable for professional radio and television operators, and problems arising in actual shop work of the technician and the radio amateur. Applicants for United States radiotelephone and radiotelegraph licenses of all classes will find the solutions to all problems of the Study Guide of the Federal Communications Commission requiring mathematical analysis. The radio and television serviceman, though he will find informative problems in all sections, may concentrate his attention on the sections pertaining to receivers, power supplies and measurements. He may also profit by reviewing the problems on control circuits and gas-filled tubes, because of the increased demand for him to service electronic circuits other than those of communication devices in his business. Instructors will find the section *Problems for Further Practice* valuable for home and classroom assignments. Students using the book for home study should devote considerable time to this section for which answers are provided. Since this section contains problems similar to those discussed in detail in previous sections, it is felt that they will add materially to the students' grasp of the subject.

The book is an outgrowth of the material presented in the author's classes in Mathematics of Communications. As a teacher, the author had two purposes in mind when he decided that the following pages should be put into the form of a book.

First, he wanted to demonstrate to the reader engaged in or entering the field of electronics, who is overcome by pessimism as to his mathematical capacities, how it is done. There is no danger of telling the beginner too much. Electronics has become much too universal a field, universal in its all-embracing application and universal in the type of persons working in it. No longer can teachers afford to forget that the obvious may be inconceivable to one unfamiliar with the tools of mathematics.

Second, the author has become increasingly aware of the great educational value of numerical analysis as compared with literal analysis. To most persons except professional mathematicians, figures have a real meaning, whereas letters have not. Also the analysis of a circuit with numerical circuit constants leads to ever simpler results: thus $2 + 3 + 5$ will read 10 in the subsequent step of a calculation, whereas $R_p + R_c + r_p$ will remain $R_p + R_c + r_p$ in the next step, and will accumulate into a tangle of letters which appears monstrous to the beginner. The several literal derivations included in the book aim to clarify in detail the sequence of steps taken, rather than to solve a practical problem in terms of letters.

The intelligent reader knows he will not profit a great deal by merely reading the solutions as presented by the author. He will try to solve the problems for himself and compare his method with the one presented in the text. Only after he has endeavored seriously to solve a problem and has encountered difficulties which he feels he cannot overcome should he resort to the given solution as a last expedient.

A working knowledge of algebra and trigonometry is the mathematical prerequisite to an understanding of this book. The reader who has acquired mathematical knowledge from general textbooks not written especially for the student of electronics, will find supplementary material in the section *Some Important Tools of Radio Mathematics*. To readers who feel that they need a review in mathematics the author recommends standard mathematical textbooks such as "College Algebra" by Paul R. Rider and "Plane and Spherical Trigonometry" by the same author, both published by The Macmillan Company. The electrical prerequisite is a basic knowledge of electricity and radio communications.

The author wishes to extend his thanks for the helpful suggestions he has received from Carter V. Rabinoff, Dean of Instruction of American Television Laboratories of California, and Frederic M. Weil of the Design Engineering Division of Gilfillan Brothers, Inc., of Los Angeles. Thanks are also due to Herbert V. Jacobs, head of the mathematics department of American Television Laboratories of California for checking the proofs and to The Macmillan Company for the editorial assistance rendered by their readers and advisers.

Bernhard Fischer

Hollywood, California

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APPENDIX

SECTION I *Problems and Solutions*

The reader, though equipped with the prerequisites mentioned in the preface, is referred to Section III, "Some important tools of radio mathematics" and Section IV, "Formulas and tables". Many problems of simpler type will require no more than the application of formulas. Others will demand different degrees of thoughtful analysis.

1 *Circuit Components*

Resistance of a Line

1.01 What is the resistance of a one-mile line of copper wire, using No. 10 AWG?

Solution:

The diameter of No. 10 wire is 0.1019 inch = 101.9 circular mils (Table I). Substituting in formula 1.12

$$R = \frac{\rho l}{d^2} \text{ and from Table II, } \rho = 10.4$$

we obtain $$R = \frac{10.4 \times 5280}{101.9^2} = 5.28 \text{ ohms.} \quad \textit{Ans.}$$

Length of a Desired Resistance

1.02 What is the length of No. 27 copper wire necessary to make a shunt of 0.1 ohm?

Solution:

Using $$R = \frac{\rho l}{d^2}$$

we have $$0.1 = \frac{10.4 \times l}{14.2^2} \text{ (Table I),}$$

and $$l = \frac{14.2^2 \times 0.1}{10.4} = 1.94 \text{ feet} = 23.3 \text{ inches.} \quad \textit{Ans.}$$

Constantan Shunt

1.03 What would be the length of the shunt in problem 1.02 if constantan wire were used?

Solution:

Substituting in

$$R = \frac{\rho l}{d^2} \text{ and using Table II}$$

we have

$$0.1 = \frac{295 \times l}{14.2^2}$$

and

$$l = \frac{14.2^2 \times 0.1}{295}$$

$$= 0.0682 \text{ foot} = 0.816 \text{ inch.} \quad \textit{Ans.}$$

A solution can also be obtained by considering the fact that the ratio of the specific resistances is

$$\frac{\rho_{con}}{\rho_{cop}} = \frac{295}{10.4} = 28.4;$$

the constantan shunt will therefore be $\frac{1}{28.4}$ of the copper shunt,

and

$$23.3/28.5 = 0.816 \text{ inch.} \quad \textit{Ans.}$$

Nichrome Shunt

1.04 What would be the length of the shunt in problem 1.02 if nichrome wire were used?

Solution:

The ratio of the specific resistances is (Table II)

$$\frac{\rho_{ni}}{\rho_{cop}} = \frac{600}{10.4} = 57.7.$$

Thus the nichrome shunt will be $\frac{1}{57.7}$ of the copper shunt,

and

$$23.3/57.7 = 0.404 \text{ inch.} \quad \textit{Ans.}$$

Calculating the Wire Gauge

1.05 If the shunt in problem 1.02 is to be 10 inches long, what is the diameter of the wire to be used? What is the approximate AWG number?

Solution:

Using $R = \frac{\rho l}{d^2}$, and expressing the inches in feet

we have $$0.1 = \frac{10.4 \times 10}{d^2 \times 12}$$

and $$d^2 = \frac{10.4 \times 10}{0.1 \times 12}$$

$$d = \sqrt{\frac{104}{1.2}} = \sqrt{86.6}$$

$$= 9.31 \text{ mils} = 0.00931 \text{ inch.}$$

The approximate AWG is No. 31. *Ans.*
For precision work the length should be recalculated for No. 31 wire.

Temperature Considerations

1.06 The field resistance of a dynamic speaker is 445 ohms before operation, at a temperature of 78F. If the operation causes a rise of the temperature to 98F, what is the resistance under operating conditions?

Solution:

Expressing the above temperatures in centigrade (formula 1.411),

$$t_1 = \frac{5}{9}(F - 32)$$

$$= \frac{5}{9}(78 - 32) = 25.56\text{C}$$

$$t_2 = \frac{5}{9}(98 - 32) = 36.7\text{C.}$$

Formula 1.14 can be used to calculate the resistance at 20C with $\alpha_{20} = 0.00393$ (Table II).
The temperature *decrease* is

$$20 - 25.56 = -5.56\text{C.}$$

Substituting in $R_{20} = R_1 [1 + \alpha_{20} (-5.56)]$

we obtain $$R_{20} = 445 [1 - 0.00393 \times 5.56]$$

$$= 445 (1 - 0.0218) = 435 \text{ ohms.}$$

Now calculating the resistance for a temperature *rise*

$$36.7 - 20 = +16.7\text{C}$$

we obtain $R_{36.7} = 435\ (1 + 0.00393 \times 16.7)$

$$= 435 \times 1.0656 = 464 \text{ ohms.} \quad \textit{Ans.}$$

High-Frequency Resistance

1.07 What is the resistance of 2 feet of No. 28 copper wire at 50 megacycles?

Solution:

From Table I we find

$$d = 12.64 \text{ mils}$$
$$= 0.01264 \text{ inch} = 0.0321 \text{ centimeter.}$$

Using the formula No. 1.15

we obtain $R_f = \dfrac{83.2 \times \sqrt{50 \times 10^6}}{0.0321} 10^{-9},$

$$R_f = \frac{83.2 \times 10^3 \times 7.07 \times 10^{-9}}{3.21 \times 10^{-2}}$$
$$= 183.5 \times 10^{-4}$$
$$= 1.835 \times 10^{-2} \text{ ohms per centimeter.}$$

Expressing 24 inches in centimeters

$$24 \times 2.54 = 61 \text{ centimeters;}$$

Thus $R = 1.835 \times 61 \times 10^{-2}$

$$= 112 \times 10^{-2} = 1.12 \text{ ohms.} \quad \textit{Ans.}$$

Comparing High-Frequency and Direct-Current Resistances

1.08 What is the d-c resistance of the conductor in problem 1.07? What is the ratio of the r-f resistance to the d-c resistance?

Solution:

Applying formula 1.12

$$R = \frac{10.4 \times 2}{12.64^2} = \frac{20.8}{160} = 0.13 \text{ ohm.}$$

The ratio is $1.12/0.13 = 8.6$ to 1. *Ans.*

Capacitor Across Alternating-Current Line

1.09 Can a capacitor designed for 150 volts maximum working voltage be used across a 120-volt line?

Solution:

The crest voltage of a 120-volt line is

$$E_{max} = 120 \times \sqrt{2} = 170 \text{ volts.}$$

No, because the rated maximum voltage would be exceeded by 20 volts. *Ans.*

Capacitors in Parallel

1.10 What is the total capacitance of a tuning capacitor of 360 micromicrofarads shunted by a trimmer capacitor of 50 micromicrofarads?

Solution:

$$C = 360 + 50 = 410 \text{ micromicrofarads.} \quad \textit{Ans.}$$

Combining Variable Capacitors

1.11 A 10- to 50-micromicrofarad antenna capacitor and a 30- to 150-micromicrofarad tuning capacitor are available. What ranges of capacitance can be produced?

Solution:

Using C_1 alone: 10 to 50 micromicrofarads. *Ans.*
Using C_2 alone: 30 to 150 micromicrofarads. *Ans.*

Using both in series:

The minimum total series capacitance is

$$c_{ts} = \frac{10 \times 30}{10 + 30} = \frac{300}{40} = 7.5 \text{ micromicrofarads;}$$

the maximum total series capacitance is

$$C_{ts} = \frac{50 \times 150}{50 + 150} = \frac{7500}{200} = 37.5 \text{ micromicrofarads;}$$

range: 7.5 to 37.5 micromicrofarads. *Ans.*

Using both in parallel:

The minimum total parallel capacitance is

$$c_{tp} = 10 + 30 = 40 \text{ micromicrofarads};$$

the maximum total parallel capacitance is

$$C_{tp} = 50 + 150 = 200 \text{ micromicrofarads};$$

range: 40 to 200 micromicrofarads. *Ans.*

Power Factor of Mica Capacitor

1.12 A 0.0004 microfarad mica capacitor is found to have an equivalent series resistance of 0.5 ohm at 500 kilocycles. What is the power factor of the capacitor?

Solution:

Using formula 1.22

$$pf = \frac{R}{X}$$

$$= \frac{R}{\dfrac{1}{2\,\pi f C}} = R \times 2\,\pi f C,$$

we obtain

$$pf = 0.5 \times 6.28 \times 500 \times 10^3 \times 4 \times 10^{-4} \times 10^{-6}$$

$$= 6280 \times 10^{-7} = 0.000628$$

$$= 0.063 \text{ per cent.} \quad \textit{Ans.}$$

Shunt Resistance

1.13 What is the equivalent shunt resistance of the capacitor in problem 1.12?

Solution:

Using formula 1.23

$$R = \frac{1}{2\,\pi f C \times pf},$$

we obtain

$$pf = \frac{1}{6.28 \times 5 \times 10^5 \times 4 \times 10^{-10} \times 6.28 \times 10^{-4}}$$

$$= \frac{1}{39.5 \times 20 \times 10^{-9}}$$

$$= \frac{10^9}{790} = 1{,}270{,}000 \text{ ohms}$$

$$= 1.27 \text{ megohms.} \quad \textit{Ans.}$$

Paper Capacitor

1.14 A paper capacitor consists of a foil-covered paper strip 7 centimeters wide and 3 meters long. The paper is 0.0025 centimeter thick and has a dielectric constant of 2.8. What is the capacitance?

Solution:

The area of the strip is

$$A = 7 \times 300 = 2100 \text{ square centimeters.}$$

Using formula 1.24, we obtain

$$C \text{ (in } \mu\mu\text{f)} = \frac{0.0884 \times 2.8 \times 2100}{0.0025}$$

$$= \frac{8.84 \times 2.8 \times 2.1 \times 10}{2.5 \times 10^{-3}}$$

$$= 20.8 \times 10^4 \ \mu\mu\text{f} = 0.208 \times 10^6 \ \mu\mu\text{f}$$

$$= 0.208 \text{ microfarads.} \quad \textit{Ans.}$$

Capacitors in Parallel

FCC Study Guide Question 2.63*

1.15 If capacitors of 1, 3, and 5 microfarads are connected in parallel, what is the total capacitance?

Solution:

$$C = 1 + 3 + 5 = 9 \text{ microfarads.} \quad \textit{Ans.}$$

* *Study Guide and Reference Material for Commercial Radio Operator Examinations*, issued by the Federal Communications Commission, Washington, D. C.

Three Capacitors in Series

1.16 If capacitors of 5, 3, and 7 microfarads are connected in series, what is the total capacitance?

Solution:

By formula 1.25

$$\frac{1}{C} = \frac{1}{5} + \frac{1}{3} + \frac{1}{7}$$

$$= 0.200 + 0.333 + 0.143 = 0.676$$

$$C = \frac{1}{0.676} = 1.48 \text{ microfarads.} \quad \textit{Ans.}$$

Voltage Rating of Capacitors

FCC Study Guide Question 2.67

1.17 Having available a number of capacitors rated at 400 volts and 2 microfarads each, how many of these capacitors would be necessary to obtain a combination rated at 1600 volts 1.5 microfarads?

Solution:

To increase the rating from 400 to 1600 volts, four capacitors must be connected in series. The capacitance of such a combination would be

$$2/4 = 0.5\ \mu\text{f.}$$

In order to increase the capacitance to 1.5 μf, three series banks must be connected in parallel.

$$0.5 \times 3 = 1.5\ \mu\text{f.}$$

The number of capacitors now is $3 \times 4 = 12$. *Ans.*

Charge of a Capacitor

1.18 What is the charge stored in a 2-microfarad capacitor with a potential difference of 75 volts across the plates?

Solution:

$$Q = E \times C$$
$$= 75 \times 2 \times 10^{-6}$$
$$= 150 \times 10^{-6} = 150 \text{ microcoulombs.} \quad \textit{Ans.}$$

Ten-Plate Capacitor

1.19 What is the capacitance of a 10-plate capacitor with a plate area of $2\frac{1}{2}$ square inches and a distance between the plates of $\frac{1}{8}$ inch?

Solution:

Using formula 1.24 with the constant 0.224 for inches, viz.,

$$C = \frac{0.224 \times K \times A\,(n - 1)}{d},$$

we obtain

$$C = \frac{0.224 \times 2.5 \times 9}{0.125}$$
$$= 40.5 \text{ micromicrofarads.} \quad \textit{Ans.}$$

Removing Plates

1.20 How many plates of the capacitor in problem 1.19 must be removed to reduce the capacitance to 30 micromicrofarads?

Solution:

This problem calls for the solution of formula 1.24 for n.

$$30 = \frac{0.224 \times 2.5\,(n - 1)}{0.125}.$$

Transposing

$$\frac{30 \times 0.125}{0.224 \times 2.5} = n - 1,$$
$$6.7 = n - 1, \text{ and } n = 7.7$$

Since the original capacitor has 10 plates, the number of plates to be removed is

$$10 - 7.7 = 2.3. \quad \textit{Ans.}$$

Two plates may be removed and the desired capacitance is then obtained by bending the outer plates. *Ans.*

Combined Inductors

1.21 Two inductors with an inductance of 15 henries each are connected in

a) series aiding
b) series opposing
c) parallel aiding
d) parallel opposing.

The coupling coefficient $k = 0.6$. What is the total inductance in each case?

Solution:

The mutual inductance is

$$M = k\sqrt{L_1L_2} = 0.6 \times 15 = 9 \text{ henries.}$$

Therefore
$$L_{ta} = L_1 + L_2 + 2M$$
$$= 15 + 15 + 18 = 48 \text{ henries.} \quad \textit{Ans.}$$

$$L_{tb} = L_1 + L_2 - 2M$$
$$= 15 + 15 - 18 = 12 \text{ henries.} \quad \textit{Ans.}$$

$$L_{tc} = \frac{(L_1 + M)(L_2 + M)}{(L_1 + M) + (L_2 + M)}$$
$$= \frac{24 \times 24}{48} = 12 \text{ henries.} \quad \textit{Ans.}$$

$$L_{td} = \frac{(L_1 - M)(L_2 - M)}{(L_1 - M) + (L_2 - M)}$$
$$= \frac{6 \times 6}{12} = 3 \text{ henries.} \quad \textit{Ans.}$$

Coupling Coefficient

FCC Study Guide Question 4.03

1.22 If the mutual inductance between two coils is 0.1 henry, and the coils have inductances of 0.2 and 0.8 henry respectively, what is the coefficient of coupling?

Solution:

Substituting in $M = k \times \sqrt{L_1 L_2}$,

we obtain $0.1 = k\sqrt{0.2 \times 0.8}$,

$0.1 = k \times \sqrt{0.16}$,

and $\dfrac{0.1}{0.4} = k$,

or $k = \frac{1}{4} = 25$ per cent. *Ans.*

Unity Coupling Coefficient

FCC Study Guide Question 4.10

1.23 When two coils of equal inductance are connected in series, with unity coefficient of coupling and their fields in phase, what is the total inductance of the two coils?

Solution:

$$L_t = L_1 + L_2 + 2\,M$$
$$= L + L + 2\,L = 4\,L. \quad \textit{Ans.}$$

Single-Layer Coil

1.24 What is the inductance of a single-layer air core inductor with an inside diameter of 2 inches using 75 turns of No. 25 enamel-covered wire, close-wound? (No. 25 enamel AWG has 51.7 turns per inch.)

Solution:

The length of the coil $l = \dfrac{75}{51.7} = 1.45$ inches, and the inductance is found by using formula 1.32

$$L = \frac{0.2 \times D^2 \times N^2}{3\,D + 9\,l + 10\,C};$$

we obtain
$$L = \frac{0.2 \times 4 \times 75^2}{3 \times 2 + 9 \times 1.45}$$
$$= \frac{0.8 \times 5625}{6 + 13.05}$$
$$= \frac{4500}{19.05} = 236 \text{ microhenries.} \quad \textit{Ans.}$$

Three-Layer Coil

1.25 What is the inductance of a coil having the same specifications as the one in problem 1.24 but having three layers of windings?

Solution:

The thickness of three layers of winding

$$C = \frac{3}{51.7} = 0.058 \text{ inch.}$$

The number of turns

$$N = 75 \times 3 = 225 \text{ turns.}$$

Using $$L = \frac{0.2 \times D^2 \times N^2}{3\,D + 9\,l + 10\,C},$$

we obtain $$L = \frac{0.2 \times 4 \times 50{,}625}{6 + 13.05 + 0.58}$$

$$= \frac{0.8 \times 50{,}625}{19.63} = 2.062 \text{ millihenries.} \quad \textit{Ans.}$$

Diameter of a Coil

1.26 Using the values in problem 1.24, calculate the inside diameter of the coil for an inductance of 144 microhenries.

Solution:

$$144 = \frac{0.2 \times 5625\,d^2}{3\,d + 13.05}$$

$$432\,d + 1880 = 1125\,d^2$$

$$1125\,d^2 - 432\,d - 1880 = 0$$

$$d = \frac{432 \pm \sqrt{186{,}600 + 8{,}450{,}000}}{2250}$$

$$= \frac{432 \pm \sqrt{8{,}636{,}000}}{2250}$$

$$= \frac{3372}{2250} = 1.5 \text{ inches.} \quad \textit{Ans.}$$

Impedance-Matching Transformer

FCC Study Guide Question 6.12

1.27 What turns ratio should a transformer have which is to be used to match a source impedance of 500 ohms to a load of 10 ohms?

Solution:

Since the impedances are proportional to the square of the turns,

we obtain $$\frac{N_p^2}{N_s^2} = \frac{Z_p}{Z_s},$$

and $$\frac{N_p}{N_s} = \sqrt{\frac{Z_p}{Z_s}} = \sqrt{\frac{500}{10}} = 7.1 \text{ to } 1. \quad \textit{Ans.}$$

Shunt Compensation

FCC Study Guide Question 4.19

1.28 In a transformer having a turns ratio of 10 to 1, working into a load impedance of 2000 ohms and out of a circuit having an impedance of 15 ohms, what value of resistance may be connected across the load to effect an impedance match?

Solution:

To find the impedance offered by the transformer secondary we substitute in

$$\frac{Z_2}{Z_1} = \left(\frac{N_2}{N_1}\right)^2,$$

obtaining $$\frac{Z_2}{15} = 100,$$

and $$Z_2 = 100 \times 15 = 1500 \text{ ohms};$$

this value is then substituted in the product-sum formula for parallel resistances:

$$1500 = \frac{2000\,R}{2000 + R},$$

from which $$2000\,R = (2000 + R) \times 1500$$

$$4\,R = (2000 + R) \times 3$$

and $$R = 6000 \text{ ohms.} \quad \textit{Ans.}$$

Effect of Coil Shape

1.29 Two single-layer coils have equal inductances. One coil has a 75-turn winding 3 inches long and is 2 inches in diameter; the second is 2.8 inches long and 1.5 inches in diameter. Calculate the number of turns of the second coil.

Solution:

Using formula 1.32

$$\frac{0.2 \times 2^2 \times 75^2}{3 \times 2 + 9 \times 3} = \frac{0.2 \times 1.5^2 \times N^2}{3 \times 1.5 + 9 \times 2.8}.$$

Dividing by 0.2 and simplifying,

$$\frac{4 \times 5625}{33} = \frac{2.25 \times N^2}{29.7}$$

and

$$N = \sqrt{\frac{4 \times 5625 \times 29.7}{33 \times 2.25}}$$

$$= 95 \text{ turns.} \quad \textit{Ans.}$$

Doubling the Number of Turns

1.30 What would be the inductance of the single-layer coil in problem 1.24 if the number of turns were doubled?

Solution:

From the formula 1.32 for inductance it is obvious that the inductance varies as the SQUARE of the turns.

$$L = 236 \times 2^2$$

$$= 236 \times 4 = 944 \text{ microhenries.} \quad \textit{Ans.}$$

Transformer Efficiency

FCC Study Guide Question 4.11

1.31 If a transformer has a primary voltage of 4400 volts and a secondary voltage of 220 volts, and the transformer has an efficiency of 98 per cent when delivering 23 amperes of secondary current, what is the value of primary current?

Solution:

The secondary power

$$P_2 = 220 \times 23 = 5060 \text{ watts.}$$

The primary power must be greater than this power output since efficiency = 0.98;

$$P_1 = 5060/0.98 = 5160 \text{ watts.}$$

The primary current is thus

$$I_p = 5160/4400 = 1.174 \text{ amperes.} \quad \textit{Ans.}$$

Three-Phase Transformer

FCC Study Guide Question 4.13

1.32 Three single-phase transformers, each with a ratio of 220 to 2200 volts are connected across a 220-volt 3-phase line, primaries in delta. If the secondaries are connected in Y, what is the secondary line voltage?

Solution:

By formula 1.3351

$$E_L = E_\Phi \times \sqrt{3}$$

$$= 2200 \times 1.73 = 3810 \text{ volts.} \quad \textit{Ans.}$$

Step-Down Tansformer

FCC Study Guide Question 5.114

1.33 What is the secondary voltage of a transformer which has a primary voltage of 100, primary turns 200, and secondary turns 40?

Solution:

Since $$\frac{E_2}{E_1} = \frac{N_2}{N_2},$$

we have $$E_2 = \frac{E_1 N_2}{N_2} = \frac{40 \times 100}{200}$$

$$= 20 \text{ volts.} \quad \textit{Ans.}$$

Increasing the Voltage Rating

1.34 The heater winding of a transformer is rated at 7.5 volts and 3 amperes. It consists of 15 turns. How many turns should be added to change the voltage rating to 12.6 volts? What should be the value of the new current rating?

Solution:

$$\frac{N_1}{N_2} = \frac{E_1}{E_2}$$

$$\frac{N_1}{15} = \frac{12.6}{7.5}$$

$$N_1 = \frac{12.6 \times 15}{7.5} = 25.2 \text{ turns.}$$

The turns to be added are

$$25.2 - 15 = 10.2 \text{ turns.} \quad \textit{Ans.}$$

Since the power consumption is to remain unchanged,

$$12.6 \times I = 7.5 \times 3$$

$$I = \frac{7.5 \times 3}{12.6}$$

$$= 1.785 \text{ amperes.} \quad \textit{Ans.}$$

Primary Current

1.35 The type 849 power amplifier tube draws 5 amperes at 11 volts from the secondary of a transformer. What is the primary current: (a) neglecting the losses; (b) at an efficiency of 92 per cent? The primary voltage is 120 volts.

Solution:

(a) Neglecting the losses and assuming $P_1 = P_2$

$$I \times 120 = 11 \times 5$$

$$I = \frac{11 \times 5}{120} = 0.458 \text{ ampere.} \quad \textit{Ans.}$$

(b) At an efficiency of 92 per cent:

$$P_1 \times 0.92 = P_2$$

$$I \times 120 \times 0.92 = 11 \times 5$$

$$I = \frac{11 \times 5}{120 \times 0.92} = 0.498 \text{ ampere.} \quad \textit{Ans.}$$

Transformer Design

1.36 A 300-milliampere power transformer is to be used across a 117-volt 60-cycle line; the secondary voltage is 750 volts, center-tapped. Common grade sheet iron is used ($B = 50{,}000$) for a 2.5 by 4.5-inch core. Calculate the number of turns in the primary, the volts per turn, and the number of turns in the secondary. Neglecting the losses, what number AWG copper wire should be used for the primary and the secondary? Allow 1000 circular mils per ampere.

Solution:

The number of turns of the primary is (formula 1.34)

$$t = \frac{E \times 10^8}{4.44 \times f \times B \times A}$$

$$= \frac{117 \times 10^8}{4.44 \times 60 \times 5 \times 10^4 \times 2.5 \times 4.5}$$

$$= 80 \text{ turns, approximately.} \quad \textit{Ans.}$$

The voltage across one turn will be

$$117/80 = 1.46 \text{ volts per turn.} \quad \textit{Ans.}$$

The number of turns of the secondary is

$$\frac{t}{80} = \frac{750}{117}$$

$$t = \frac{750 \times 80}{117}$$

$$= 512 \text{ turns, tapped at 256 turns.} \quad \textit{Ans.}$$

The current in the secondary is 300 milliamperes; allowing 1000 circular mils per ampere,

$$d = 1000 \times 0.3 = 300 \text{ circular mils.}$$

Use No. 25 AWG wire. *Ans.*

The current in the primary is (disregarding the efficiency)

$$I \times 117 = 0.3 \times 750$$

$$I = \frac{0.3 \times 750}{117} = 1.925 \text{ amperes.}$$

Allowing 1000 circular mils per ampere,

$$d = 1000 \times 1.925 = 1925 \text{ circular mils.}$$

Use No. 17 AWG wire. *Ans.*

Primary Impedance

1.37 The secondary load of a step-down transformer with a turns ratio 1 to 7 is 600 ohms. What is the impedance looking into the primary?

Solution:

Substituting in $\dfrac{Z_1}{Z_2} = \dfrac{N_1^2}{N_2^2}$,

we obtain $\dfrac{Z_1}{600} = \dfrac{7^2}{1}$,

and $Z_1 = 49 \times 600 = 29{,}400$ ohms. *Ans.*

Loud-Speaker Transformer

1.38 What turns ratio will be required to couple a 3.5-ohm voice coil to a plate load of 8500 ohms?

Solution:

Substituting in $\dfrac{Z_1}{Z_2} = \dfrac{N_1^2}{N_2^2}$,

we obtain $\dfrac{8500}{3.5} = \left(\dfrac{N_1}{N_2}\right)^2$,

and $\dfrac{N_1}{N_2} = \sqrt{\dfrac{8500}{3.5}} = 49.3$ to 1. *Ans.*

Voltage Regulation of a Transformer

1.39 The secondary voltage of a transformer dropped from 750 volts no-load voltage to 715 volts under full load. What is the voltage regulation of the transformer?

Solution:

The voltage regulation is the ratio of the voltage decrease to the load voltage.

$$V.R. = \frac{750 - 715}{715}$$

$$= \frac{35}{715} = 0.049 = 4.9 \text{ per cent.} \quad \textit{Ans.}$$

2 Direct-Current Circuits

Filament Resistance

2.01 The transmitting tube type 801 requires a filament voltage of 7.5 volts and a filament current of 1.25 amperes. What is the hot resistance of the filament?

Solution:

Applying Ohm's law, we have

$$R = \frac{E}{I} = \frac{7.5}{1.25} = 6 \text{ ohms.} \quad \textit{Ans.}$$

Current of Commercial Light Bulb

2.02 A light bulb is marked: 110 volts, 50 watts. What current will it draw from the line?

Solution:

$$I = \frac{W}{E} = \frac{50}{110} = 0.455 \text{ ampere.} \quad \textit{Ans.}$$

Voltage Drop Across Line-Cord Resistor

2.03 A resistance cord of 375 ohms is used as a dropping resistor in a 150-milliampere filament string. What is the voltage drop of the cord and what is the combined drop of the filaments?

Solution:

$$E_{cord} = IR = 0.15 \times 375 = 56.2 \text{ volts.} \quad \textit{Ans.}$$

When used across a 120-volt line the filament drop will be

$$E_{filament} = 120 - 56.2 = 63.8 \text{ volts.} \quad \textit{Ans.}$$

Resistance of Light Bulb

2.04 What is the resistance of the bulb in problem 2.02?

Solution:

The hot resistance is

$$R = \frac{E}{I} = \frac{110}{0.455} = 242 \text{ ohms.} \quad \textit{Ans.}$$

Filament Rheostat

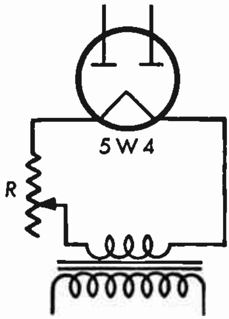

Fig. 2.05 Filament rheostat, used to cause a voltage drop since the voltage across the transformer secondary is too high.

2.05 The rectifier tube type 5W4 has a filament rating of 5 volts and 1.5 amperes. What should be the resistance of a rheostat used to drop the voltage of a 6.3-volt filament transformer to the rated value of the tube, with the sliding contact at mid-position? What should be its minimum wattage rating?

Solution:

The voltage across the rheostat is

$$E_{rh} = 6.3 - 5 = 1.3 \text{ volts.}$$

The resistance from mid-point to the filament terminal is

$$R = \frac{E}{I} = \frac{1.3}{1.5} = 0.867 \text{ ohm.}$$

The whole rheostat has a resistance of

$$R_{rh} = 2 \times 0.867 = 1.74 \text{ ohms.} \quad \textit{Ans.}$$

Its power dissipation will be

$$P = 2 \times E \times I = 2 \times 1.95 = 3.9 \text{ watts.} \quad \textit{Ans.}$$

A rating of approximately 10 to 20 watts should be used for safety.

Heater Element

2.06 A toaster element draws 2.25 amperes from a 117-volt line. What are its resistance and its power consumption?

Solution:

Applying Ohm's law, we have

$$R = \frac{E}{I} = \frac{117}{2.25} = 52 \text{ ohms.} \quad \textit{Ans.}$$

$$P = E \times I = 117 \times 2.25 = 263.5 \text{ watts.} \quad \textit{Ans.}$$

Voltmeter Hookup

2.07 A 0–1 milliampere d'Arsonval meter with an internal resistance of 50 ohms is used to make a voltmeter reading 150 volts full scale. What is the value of the series dropping resistor?

Solution:

The voltage drop across the meter is

$$E_{meter} = I \times R = 0.001 \times 50 = 0.05 \text{ volt.}$$

If the meter is to read 150 volts, the series resistor must cause a voltage drop of

$$E_{resistor} = 150 - 0.05 = 149.95 \text{ volts.}$$

Since the current is to remain 1 milliampere, the dropping resistor is found by Ohm's law

$$R_{series} = \frac{E}{I} = \frac{149.95}{0.001} = 149{,}950 \text{ ohms.} \quad \textit{Ans.}$$

Bleeder Resistor

2.08 A bleeder resistor draws 5 milliamperes from a 275-volt power supply. Find the resistance and the wattage dissipation.

Solution:

Applying Ohm's law, we have

$$R = \frac{E}{I} = \frac{275}{0.005} = 55{,}000 \text{ ohms.} \quad \textit{Ans.}$$

$$P = I^2 R = (5 \times 10^{-3})^2 \times 55 \times 10^3$$
$$= 25 \times 10^{-6} \times 55 \times 10^3$$
$$= 1.375 \text{ watts.} \quad \textit{Ans.}$$

Grid Bias

FCC Study Guide Question 6.135

2.09 In a radio-frequency amplifier stage having a plate voltage of 1250 volts, a plate current of 150 milliamperes, a grid current of 15 milliamperes, and a grid-leak resistance of 4000 ohms, what is the value of the operating grid bias?

Solution:

The only values used in the calculation are the grid current and the grid-leak resistance.

$$E_{bias} = -15 \times 10^{-3} \times 4 \times 10^3$$
$$= -60 \text{ volts.} \quad \textit{Ans.}$$

Resistors in Parallel

2.10 Find the total resistance of 3, 5 and 7 ohms, all connected in parallel.

Solution:

(a) Conductance method:

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{3} + \frac{1}{5} + \frac{1}{7}$$
$$= 0.333 + 0.200 + 0.143$$
$$= 0.676 \text{ mho}$$

$$R_t = \frac{1}{0.676} = 1.48 \text{ ohms.} \quad \textit{Ans.}$$

(b) Product-sum method:

The equivalent resistance of the 3- and 5-ohm resistors is (formula 1.172)

$$R = \frac{\text{product}}{\text{sum}} = \frac{15}{8} = 1.875 \text{ ohms;}$$

this resistance in parallel with 7 ohms yields

$$R = \frac{\text{product}}{\text{sum}} = \frac{1.875 \times 7}{8.875} = 1.48 \text{ ohms.} \quad \textit{Ans.}$$

The second method requires less slide rule work.

(c) Assumed voltage method:

Assuming a voltage of 105 volts, the current through R_1 would be

$$I_1 = \frac{105}{3} = 35 \text{ amperes.}$$

Similarly $$I_2 = \frac{105}{5} = 21 \text{ amperes,}$$

$$I_3 = \frac{105}{7} = 15 \text{ amperes.}$$

$$I_t = I_1 + I_2 + I_3 = 71 \text{ amperes.}$$

Therefore $$R_t = \frac{E_t}{I_t} = \frac{105}{71} = 1.48 \text{ ohms.} \quad \textit{Ans.}$$

This method is more popular for the solution of parallel impedances in a-c circuits.

Series Circuit

2.11 Four resistances of 20, 30, 40, 50 ohms are connected in series across a 4-cell Edison battery. Find the voltage across each resistor. (E of 1 cell = 1.37 volts)

Solution:

$$I_t = \frac{E_t}{R_t} = \frac{4 \times 1.37}{140} = 0.0391 \text{ ampere.}$$

$$E_{20} = 0.0391 \times 20 = 0.782 \text{ volt.} \quad \textit{Ans.}$$

$$E_{30} = 0.0391 \times 30 = 1.173 \text{ volts.} \quad \textit{Ans.}$$

$$E_{40} = 0.0391 \times 40 = 1.564 \text{ volts.} \quad \textit{Ans.}$$

$$E_{50} = 0.0391 \times 50 = 1.96 \text{ volts.} \quad \textit{Ans.}$$

Check: $0.782 + 1.173 + 1.564 + 1.96 = 5.48$ volts.

Also, $4 \times 1.37 = 5.48$ volts.

Panel Lamp

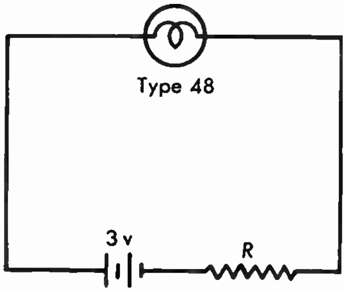

Fig. 2.12 Panel lamp and resistor in series to obtain the rated current and voltage.

2.12 The panel lamp type 48 requires 2 volts and 60 milliamperes. Determine the dropping resistor *R* in Figure 2.12 to operate the lamp from two dry cells in series.

Solution:

The voltage across the resistor is

$$E_r = 3 - 2 = 1 \text{ volt.}$$

Thus $$R = \frac{E}{I} = \frac{1}{0.060} = 16.7 \text{ ohms.} \quad \textit{Ans.}$$

and $$P = \frac{E^2}{R} = \frac{1}{16.7} = 0.06 \text{ watt.} \quad \textit{Ans.}$$

Substituting Odd Resistors

2.13 Since the resistor found in problem 2.12 is not readily available, would it be correct to use a 25- and a 50-ohm resistor in parallel?

Solution:

$$R_t = \frac{\text{product}}{\text{sum}} = \frac{25 \times 50}{75} = \frac{1250}{75} = 16.7 \text{ ohms.}$$

Yes. *Ans.*

Bias Resistor

2.14 The type 6G6 tube has a plate current of 15 milliamperes and a screen current of 2.5 milliamperes. The grid-bias voltage is −9 volts. If cathode bias is used, what should be the resistance of the cathode resistor and what would be its wattage consumption?

Solution:

Since the cathode current consists of the plate current plus the screen current, we have

$$R = \frac{E}{I} = \frac{9}{(15 + 2.5) \times 10^{-3}}$$

$$= \frac{9}{17.5} \times 10^{3} = \frac{9000}{17.5} = 515 \text{ ohms.} \quad \textit{Ans.}$$

$$P = E \times I = 9 \times 17.5 \times 10^{-3} = 0.1575 \text{ watt.} \quad \textit{Ans.}$$

Voltage Divider

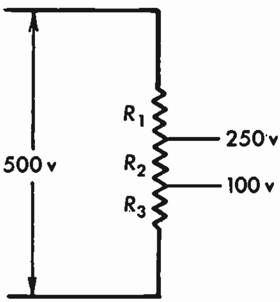

Fig. 2.15 Voltage divider with negligible current drain from taps.

2.15 Find the value of the resistors of the voltage divider in Figure 2.15 for negligible current drain by the load, and a current drain of 7.5 milliamperes by the bleeder.

Solution:

$$R_t = \frac{E_t}{I_t} = \frac{500}{7.5 \times 10^{-3}}$$

$$= \frac{500}{7.5} \times 10^{3}$$

$$= 66.6 \times 10^{3} \text{ ohms}$$

$$= 66,600 \text{ ohms}$$

Since the voltage across R_1 is one-half of the voltage, its value will be

$$R_1 = \frac{66{,}600}{2} = 33{,}300 \text{ ohms.} \quad \textit{Ans.}$$

R_3 will cause a drop of 100 volts; its value will be

$$R_3 = \frac{100}{7.5 \times 10^{-3}} = \frac{100}{7.5} \times 10^3$$
$$= 13.3 \times 10^3 = 13{,}300 \text{ ohms.} \quad \textit{Ans.}$$

R_2 will cause a drop of

$$250 - 100 = 150 \text{ volts}$$
$$R_2 = \frac{150}{7.5} \times 10^3 = 20{,}000 \text{ ohms.} \quad \textit{Ans.}$$

Check: $$R_1 + R_2 + R_3 = 33{,}300 + 20{,}000 + 13{,}300$$
$$= 66{,}600 \text{ ohms.}$$

Voltage Divider with Load Drain

2.16 Under actual working conditions the above voltages (problem 2.15) will rarely be realized. Find the voltages if 3 milliamperes are drained from the 250-volt point and 5 milliamperes from the 100-volt point.

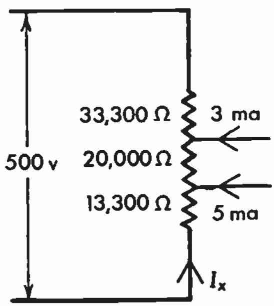

Fig. 2.16 Voltage divider with stated drain from taps.

Solution:

Let the current through the 13,300-ohm resistor be I_x; then $I_x + 0.005$ ampere will flow through the 20,000-ohm resistor, and $I_x + 0.008$ ampere through the 33,000-ohm resistor. By Kirchhoff's second law

$$500 = 33{,}000\,(I_x + 0.008) + 20{,}000\,(I_x + 0.005) + 13{,}000\,I_x$$
$$= 33{,}000\,I_x + 266 + 20{,}000\,I_x + 100 + 13{,}000\,I_x,$$

and $500 = 66{,}000\, I_x + 366,$

$$134 = 66{,}000\, I_x,$$

$$I_x = \frac{134}{66{,}000} = 0.00201 \text{ ampere} = 2.01 \text{ milliamperes.}$$

Using $E = I\,R$, we obtain

$$E_3 = 13.3 \times 10^3 \times 2.01 \times 10^{-3} = 26.7 \text{ volts}$$

$$E_2 = 20 \times 10^3 \times 7.01 \times 10^{-3} = 140.2 \text{ volts}$$

$$E_1 = 33.3 \times 10^3 \times 10.01 \times 10^{-3} = 333.3 \text{ volts.}$$

The voltages at the taps are now:

Voltage at the 100-volt point: 26.7 volts. *Ans.*

Voltage at the 250-volt point: $26.7 + 140.2 = 166.9$ volts. *Ans.*

Check: $26.7 + 140.2 + 333.3 = 500.2$ volts.

Bleeder Design

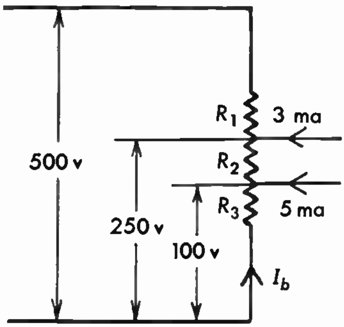

Fig. 2.17 Voltage divider with stated voltages and currents.

2.17 In problem 2.16, 100 and 250 volts are desired at the 5- and 3-milliampere taps, respectively. Calculate R_1, R_2 and R_3, allowing a bleeder current of 10 per cent of the load current.

Solution:

$$I_b = 0.1 \times (3 + 5) = 0.8 \text{ milliampere.}$$

Since $$R_3 = \frac{100}{I_b},$$

$$R_2 = \frac{150}{I_b + 0.005},$$

and $$R_1 = \frac{250}{I_b + 0.008}$$

we obtain, $$R_3 = \frac{100}{0.8 \times 10^{-3}} = 125{,}000 \text{ ohms.} \quad \textit{Ans.}$$

$$R_2 = \frac{150}{5.8 \times 10^{-3}} = 25{,}900 \text{ ohms.} \quad \textit{Ans.}$$

$$R_1 = \frac{250}{8.8 \times 10^{-3}} = 28{,}400 \text{ ohms.} \quad \textit{Ans.}$$

Bleeder Voltage under No Load

2.18 What would be the potential difference at the taps if no load were applied to the circuit in problem 2.17?

Solution:

The voltage across each section can be found from the proportion:

$$\frac{E_1}{E_t} = \frac{R_1}{R_t};$$

$$\frac{E_2}{E_t} = \frac{R_2}{R_t};$$

$$\frac{E_3}{E_t} = \frac{R_3}{R_t}.$$

Now $$R_t = 125{,}000 + 25{,}900 + 28{,}400$$

$$= 179{,}300 \text{ ohms.}$$

Therefore $$E_1 = 500 \times \frac{28{,}400}{179{,}300} = 79.4 \text{ volts.} \quad \textit{Ans.}$$

$$E_2 = 500 \times \frac{25{,}900}{179{,}300} = 72.3 \text{ volts.} \quad \textit{Ans.}$$

$$E_3 = 500 \times \frac{125{,}000}{179{,}300} = 348.3 \text{ volts.} \quad \textit{Ans.}$$

Check: $$79.4 + 72.3 + 348.3 = 500 \text{ volts.}$$

The voltages at the taps will be:

Bottom tap: 0 volts. *Ans.*

Second tap: 79.4 volts. *Ans.*

Third tap: 79.4 + 72.3 = 151.7 volts. *Ans.*

Top tap: 500 volts. *Ans.*

Comparing the load voltages and the no-load voltages:

Tap	Bottom	Second	Third	Top
Load voltage	0	100	250	500
No-load volts	0	79.4	151.7	500

Series-Parallel Circuit

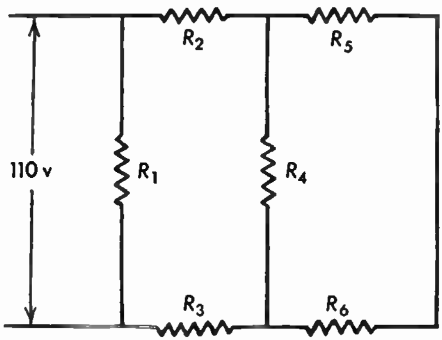

Fig. 2.19a Series-parallel circuit, conventional layout.

2.19 In Figure 2.19a, $R_1 = 80$ ohms, $R_2 = 3$ ohms, $R_3 = 8$ ohms, $R_4 = 6$ ohms, $R_5 = 12$ ohms, $R_6 = 5$ ohms. Find the voltage across R_5.

Solution:

Referring to the equivalent circuit (Figure 2.19b):

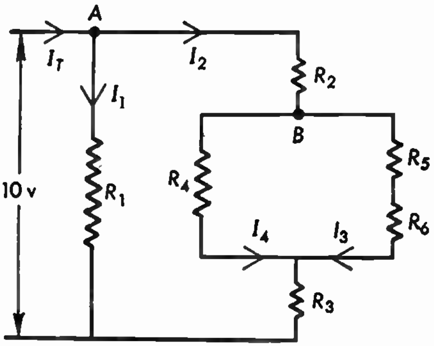

Fig. 2.19b Series-parallel circuit, equivalent to the circuit of Fig. 2.19a.

The total resistance

$$R_{4,5,6,} = \frac{\text{product}}{\text{sum}} = \frac{6 \times (5 + 12)}{6 + (5 + 12)}$$

$$= \frac{102}{23} = 4.44 \text{ ohms.}$$

R_2 and R_3 are in series with the above resistance;

$$4.44 + 3 + 8 = 15.44 \text{ ohms}$$

$$R_t = \frac{80 \times 15.44}{80 + 15.44} = \frac{1233}{95.4} = 13.1 \text{ ohms}$$

$$I_t = \frac{110}{13.1} = 8.42 \text{ amperes.}$$

To find E_5 the current through R_5 must be computed. Using Kirchhoff's first law at A, we have

$$I_t = I_1 + I_2, \text{ or } 8.42 = I_1 + I_2.$$

R_1 and R_{sh} are inversely proportional to their currents:

$$\frac{R_1}{R_{sh}} = \frac{I_{sh}}{I_1}.$$

Now I_{sh} is I_2;

therefore $\dfrac{80}{15.44} = \dfrac{I_2}{I_1}.$

Cross-multiplying

$$80\, I_1 = 15.44\, I_2.$$

We now have the two simultaneous equations:

$$80\, I_1 - 15.44\, I_2 = 0 \qquad (1)$$

$$I_1 + I_2 = 8.42 \qquad (2)$$

Multiplying (2) by 80

$$80\, I_1 + 80\, I_2 = 675 \qquad (3)$$

Subtracting (3) minus (1)

$$95.4\, I_2 = 675,$$

and $I_2 = \dfrac{675}{95.4} = 7.07$ amperes.

Using Kirchhoff's first law at B, we have

$$I_2 = I_3 + I_4,$$

and the shunt law yields

$$\frac{6}{17} = \frac{I_3}{I_4};$$

or

$$I_3 + I_4 = 7.07 \qquad (4)$$

$$17\,I_3 - 6\,I_4 = 0; \qquad (5)$$

multiplying (4) by 6

$$6\,I_3 + 6\,I_4 = 42.5; \qquad (6)$$

adding (5) plus (6)

$$23\,I_3 = 42.5,$$

and

$$I_3 = \frac{42.5}{23} = 1.85 \text{ amperes}$$

$$I_4 = 7.07 - 1.85 = 5.22 \text{ amperes.}$$

Thus

$$E_5 = I_5 \times R_5$$

$$= 1.85 \times 12 = 22.2 \text{ volts.} \quad \textit{Ans.}$$

Check: $E_5 + E_6$ must be equal to E_4

$$E_6 = 1.85 \times 5 = 9.25 \text{ volts}$$

$$E_4 = 5.22 \times 6 = 31.4 \text{ volts}$$

$$E_5 + E_6 = 22.2 + 9.25 = 31.45 \text{ volts}$$

$$E_2 = 7.07 \times 3 = 21.21 \text{ volts.}$$

Now

$$E_3 = 7.07 \times 8 = 56.56 \text{ volts}$$

and

$$21.21 + 56.56 + 31.45 = 109.22 \text{ volts.}$$

An error of less than 1 per cent, viz.,

$$\text{Error} = \frac{110 - 109.22}{110}$$

$$= \frac{0.78}{110} = 0.0071 = 0.71 \text{ per cent}$$

is probably due to slide rule work.

Series-Parallel Circuit

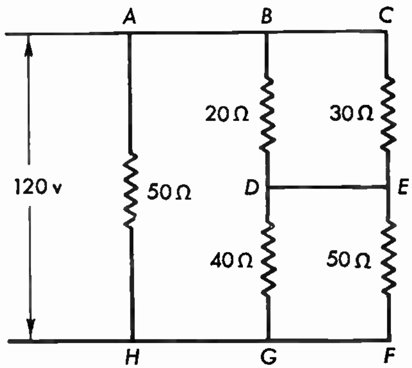

Fig. 2.20a Series-parallel circuit.

2.20 In Figure 2.20a, what is the potential difference between: *AH*, *BD*, *DE*, *DF*?

Solution:

Since *A* and *H* are connected to the terminals of the source there is a potential difference of 120 volts across them. *Ans.*

The equivalent resistance of the 20- and 30-ohm resistors in parallel is

$$R_{20,\ 30} = \frac{20 \times 30}{20 + 30} = \frac{600}{50} = 12 \text{ ohms.}$$

The equivalent resistance of the 40- and 50-ohm resistors in parallel is

$$R_{40,\ 50} = \frac{40 \times 50}{40 + 50} = \frac{2000}{90} = 22.2 \text{ ohms.}$$

We now have an equivalent circuit as indicated in Figure 2.20b:

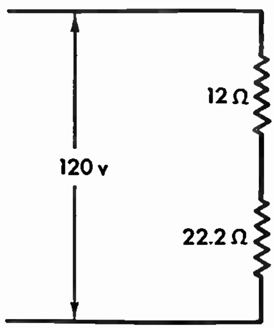

Fig. 2.20b Equivalent circuit. The 12-ohm resistance is equivalent to the resistance of the loop *BCED* of Fig. 2.20a; the 22.2-ohm resistance is equivalent to the resistance of loop *DEFG*.

The current in this circuit is

$$I_e = \frac{120}{34.2} = 3.52$$

By Ohm's law

$$E_{bd} = I_{bd} \times R_{bd}$$
$$= 3.52 \times 12 = 42.2 \text{ volts.} \quad \textit{Ans.}$$
$$E_{df} = 3.52 \times 22.2 = 77.8 \text{ volts.} \quad \textit{Ans.}$$

There is no potential difference between D and E because there is no IR drop. *Ans.*

Batteries in Parallel

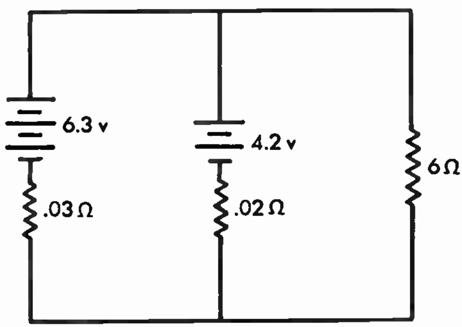

Fig. 2.21a Two sources of e.m.f. connected in parallel across a load of 6 ohms.

2.21 In Figure 2.21a find the load current I_L.

Solution:

(a) By superposition:

This method considers each electromotive force separately. Considering the 6.3-volt source:

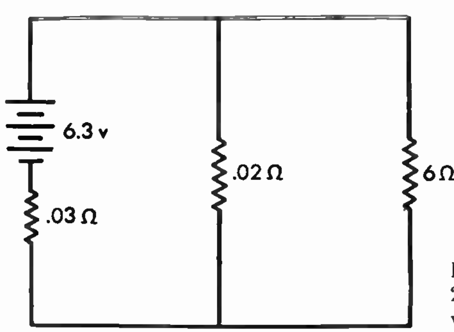

Fig. 2.21b Circuit of Fig. 2.21a, considering the 6.3-volt e.m.f. only.

total resistance

$$R' = \frac{6 \times 0.02}{6 + 0.02} + 0.03 = 0.0499 \text{ ohm},$$

total current

$$I' = \frac{6.3}{0.0499} = 126.18 \text{ amperes},$$

voltage across load

$$E' = 6.3 - (126.18 \times 0.03) = 6.3 - 3.78$$
$$= 2.52 \text{ volts},$$

current through load

$$I'_L = \frac{2.52}{6} = 0.420 \text{ ampere.}$$

Considering the 4.2-volt source:

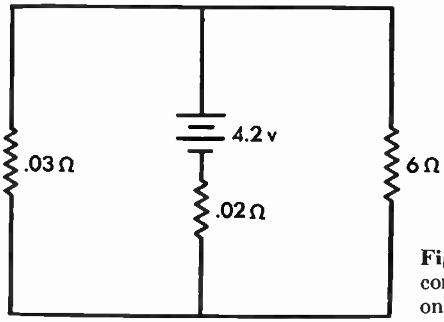

Fig. 2.21c Circuit of Fig. 2.21a, considering the 4.2-volt e.m.f. only.

total resistance

$$R'' = \frac{6 \times 0.03}{6 + 0.03} + 0.02 = 0.0498 \text{ ohm},$$

total current

$$I'' = \frac{4.2}{0.0498} = 84.3 \text{ amperes},$$

voltage across load

$$E'' = 4.2 - (84.3 \times 0.02) = 4.2 - 1.69$$
$$= 2.51 \text{ volts},$$

current through load

$$I''_L = \frac{2.51}{6} = 0.418 \text{ ampere.}$$

The total current due to the simultaneous action of both sources is

$$I_L = 0.418 + 0.420 = 0.838 \text{ ampere.} \quad \textit{Ans.}$$

(b) By Kirchhoff's law, loop method:

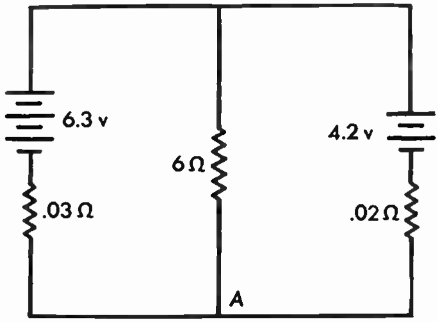

Fig. 2.21d Circuit of Fig. 2.21a, arranged to illustrate a right-hand and a left-hand loop, both containing a source of e.m.f.

Kirchhoff's second law for the left loop

$$6.3 - 0.03\, I_1 - 6\,(I_1 + I_2) = 0$$

Kirchhoff's second law for the right loop

$$4.2 - 0.02\, I_2 - 6\,(I_1 + I_2) = 0.$$

Simplifying

$$6.03\, I_1 + 6\, I_2 = 6.3$$
$$6\, I_1 + 6.02\, I_2 = 4.2.$$

Multiplying by 6 and 6.03 respectively

$$36.18\, I_1 + 36\, I_2 = 37.8$$
$$36.18\, I_1 + 36.3006\, I_2 = 25.326$$
$$-\ 0.3006\, I_2 = 12.474$$
$$I_2 = \frac{12.474}{-0.3006}$$
$$= -41.497 \text{ amperes.}$$

Substituting

$$6\, I_1 + 6.02\,(-41.497) = 4.2$$
$$6\, I_1 - 249.812 = 4.2$$
$$6\, I_1 = 254.012$$
$$I_1 = 42.335$$
$$I_1 + I_2 = 42.335 - 41.497$$
$$= 0.838 \text{ ampere.} \quad \textit{Ans.}$$

(c) By Kirchhoff's law, node method:

As in writing loop equations, currents are expressed in terms of I, I_1, I_2 etc., to satisfy Kirchhoff's current law, so in writing node equations voltages are first expressed in terms of E, E_1, E_2, etc., to satisfy Kirchhoff's voltage law. In this problem, let the voltage across the 6-ohm resistor be E, then the voltage across the 0.03-ohm resistance must be $6.3 - E$, the voltage across the 0.02-ohm resistance $E - 4.2$, with the polarities as indicated in Figure 2.21d.

Check: $6.3 - (6.3 - E) - E = 0$ (left loop)

$6.3 - (6.3 - E) - (E - 4.2) - 4.2 = 0$ (outline)

Kirchhoff's first law for node A yields the following algebraic sum of three currents, one flowing toward A and two flowing away from A:

$$\frac{6.3 - E}{0.03} + \frac{-(E - 4.2)}{0.02} + \frac{-E}{6} = 0,$$

or

$$\frac{630 - 100\,E}{3} - \frac{100\,E - 420}{2} - \frac{E}{6} = 0.$$

Multiplying by 6

$$1260 - 200\,E - 300\,E + 1260 - E = 0$$

$$2520 = 501\,E$$

$$E = \frac{2520}{501}.$$

Thus the current through the 6-ohm resistor is

$$I = \frac{E}{6} = \frac{2520}{501 \times 6} = 0.838 \text{ ampere.} \quad \textit{Ans.}$$

Unbalanced Bridge—Solution by Kirchhoff's Law

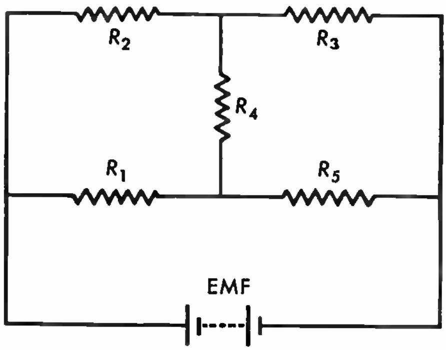

Fig. 2.22a Unbalanced bridge.

2.22 In the unbalanced-bridge circuit of Figure 2.22a, $R_1 = 1$ ohm, $R_2 = 2$ ohms, $R_3 = 3$ ohms, $R_4 = 4$ ohms, $R_5 = 5$ ohms. Assuming an electromotive force of 100 volts, find the total resistance, the total current, and the current through R_4.

Solution:

The total resistance is of course independent of the applied voltage. Assuming a voltage of 100 volts, we obtain the diagram (Figure 2.22b) for the currents and voltages.

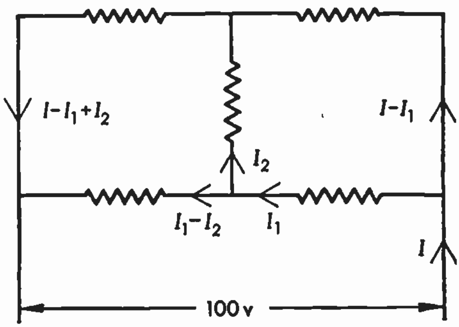

Fig. 2.22b Current layout on the unbalanced bridge.

I_1 will flow through the 5-ohm resistor,

$I - I_1$ will flow through the 3-ohm resistor,

I_2 will flow through the 4-ohm resistor,

the sum of the latter two currents through the 2-ohm resistor,

and $I - I_2$ will flow through the 1-ohm resistor.

We now apply Kirchhoff's second law:

For the lower loop

(1) $$100 = 5\,I_1 + (I_1 - I_2),$$

for the outline

(2) $$100 = 3\,(I - I_1) + 2\,(I - I_1 + I_2),$$

for the right loop

(3) $$3\,(I - I_1) = 5\,I_1 + 4\,I_2.$$

Many computers hesitate to assume a direction of the current flow through the shunt resistor (R_4 = 4 ohms in this case). There is no reason for this whatsoever. Should the assumed direction be opposite to the actual direction, the current through the shunt resistor I_2 will come out negative. Simplifying equations (1), (2), and (3)

(1) $$6\,I_1 - I_2 = 100$$

(2) $$5\,I - 5\,I_1 + 2\,I_2 = 100$$

(3) $$-3\,I + 8\,I_1 + 4\,I_2 = 0.$$

Multiplying equation (2) by 2

$$10\,I - 10\,I_1 + 4\,I_2 = 200$$

(3) $$-3\,I + 8\,I_1 + 4\,I_2 = 0$$

Subtracting $$13\,I - 18\,I_1 = 200 \qquad (4)$$

Multiplying equation (1) by 2

$$12\,I_1 - 2\,I_2 = 200$$

(2) $$5\,I - 5\,I_1 + 2\,I_2 = 100$$

Adding $$5\,I + 7\,I_1 = 300 \qquad (5)$$

Multiplying equation (4) by 7 and equation (5) by 18

$$91\,I - 126\,I_1 = 1400$$

$$90\,I + 126\,I_1 = 5400$$

Adding $$181\,I = 6800$$

and $$I = 6800/181$$

$$I = 37.57 \text{ amperes.} \quad \textit{Ans.}$$

The total resistance is

$$R = 100/37.57 = 2.662 \text{ ohms.} \quad \textit{Ans.}$$

From equation (5) we find

$$I_1 = (300 - 187.9)/7 = 16.02 \text{ amperes.}$$

From equation (1) we finally compute the shunt current of the unbalanced bridge through R_4:

$$I_2 = -100 + 96.14$$

$$= -3.86 \text{ amperes.} \quad \textit{Ans.}$$

Delta-Star Transformation

2.23 In problem 2.22 find the total current with the aid of a delta-star transformation. Also find the current through the shunt resistor R_4.

Solution:

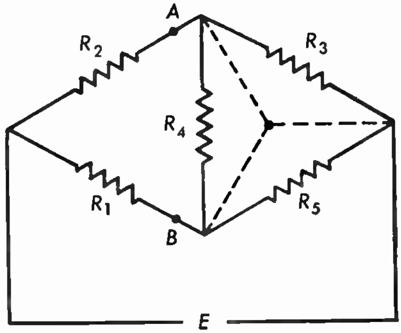

Fig. 2.23a Unbalanced bridge of Fig. 2.22a represented by two delta loops.

In Figure 2.23a we shall remove the Δ-circuit to the right of $A\ B$ and transform it into a Y-circuit to obtain the equivalent circuit of Figure 2.23b.

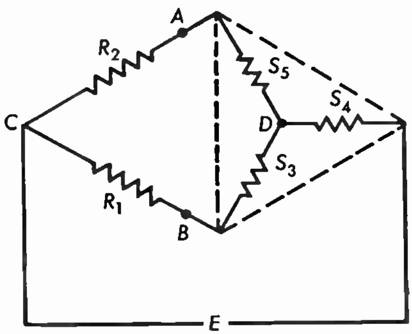

Fig. 2.23b The equivalent circuit of Fig. 2.23a, the right loop transformed to a star circuit.

The star resistances S_4, S_5, S_3 are:

Opposite R_4 is

$$S_4 = (3 \times 5)/(3 + 4 + 5) = 15/12 = 1.25 \text{ ohms.}$$

Opposite R_5 is

$$S_5 = (3 \times 4)/12 = 12/12 = 1 \text{ ohm.}$$

Opposite R_3 is

$$S_3 = (4 \times 5)/12 = 20/12 = 1.667 \text{ ohms.}$$

The parallel combination R_p consists of the branches $2 + 1 = 3$ ohms, and $1 + 1.667 = 2.667$ ohms

$$R_p = (3 \times 2.667)/(3 + 2.667) = 1.412 \text{ ohms,}$$

and the total resistance

$$R_t = R_p + S_4 = 1.412 + 1.25 = 2.662 \text{ ohms.}$$

The total current is

$$I_t = 100/(2.662) = 37.6 \text{ amperes.} \quad \textit{Ans.}$$

The voltage drop across S_4 is

$$E_4 = 37.6 \times 1.25 = 47 \text{ volts.}$$

The voltage drop across $C\,D$ is $100 - 47 = 53$ volts. The drops across S_5 and S_3 are:

$$E_5 = 53 \times \frac{1}{1 + 2} = 17.65 \text{ volts}$$

$$E_3 = 53 \times \frac{1.667}{1.667 + 1} = 33.1 \text{ volts}$$

The potential difference between A and B is

$$33.1 - 17.65 = 15.45 \text{ volts,}$$

and $$I_2 = 15.45/4 = 3.86 \text{ amperes.} \quad \textit{Ans.}$$

Thévenin's Theorem

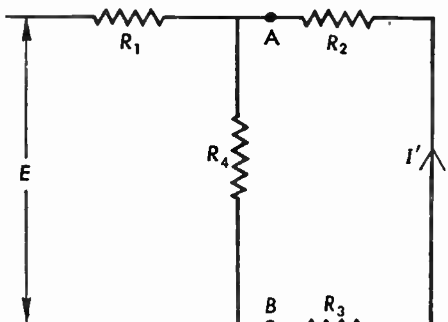

Fig. 2.24a Series-parallel circuit.

2.24 In the circuit of Figure 2.24a, $E = 100$ volts, $R_1 = R_2 = R_3 = R_4 = 5$ ohms. Find I'.

Solution:

(a) By calculating the total resistance.

The series-parallel resistance consists of R_1 in series with a parallel combination. One branch of the parallel combination is R_4, the other branch is $(R_2 + R_3)$. The total resistance is

$$R_t = 5 + \frac{5 \times 10}{5 + 10} = 8.33 \text{ ohms.}$$

The total current $I_t = \dfrac{100}{8.33} = 12$ amperes.

The voltage drop across R_1 is

$$E_1 = 12 \times 5 = 60 \text{ volts.}$$

The voltage drop across R_4 is $100 - 60 = 40$ volts; the same voltage exists across $R_2 + R_3$.

$$I' = 40/(5 + 5) = 4 \text{ amperes.} \quad \textit{Ans.}$$

(b) By Thévenin's theorem.

Let AB be the network terminals and $R_2 + R_3$ the load.

The equivalent circuit is represented by Figure 2.24b

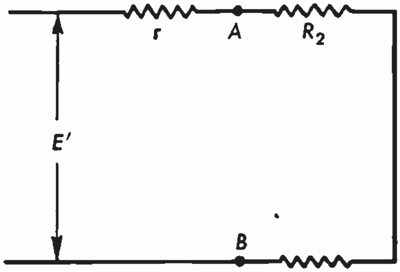

Fig. 2.24b Equivalent circuit. The network to the left of AB is represented by a "Thévenin generator" with an e.m.f. of E' volts and an internal resistance of r ohms.

To find the network resistance r:

With the voltage E short-circuited, the network impedance, looking from AB to the left, consists of two 5-ohm resistors in parallel.

$$r = \frac{5 \times 5}{5 + 5} = 2.5 \text{ ohms.}$$

To find the equivalent voltage E':

Under no-load condition E' is determined by the voltage divider $(R_1 + R_4)$.

$$E' = 100 \times \frac{5}{5 + 5} = 50 \text{ volts}$$

$$I' = \frac{E'}{R_2 + R_3 + r} = \frac{50}{12.5} = 4 \text{ amperes.} \quad \textit{Ans.}$$

Thévenin's Theorem Applied to Bridge Network

2.25 Find the current through R_4 in problem 2.22 with the aid of Thévenin's theorem.

Solution:

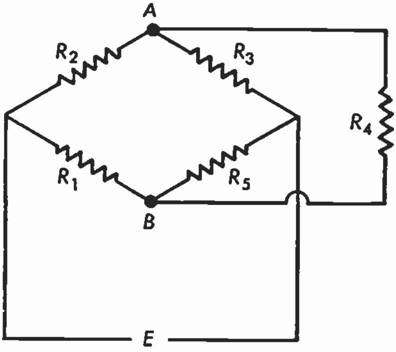

Fig. 2.25a Unbalanced bridge. R_4 to be disconnected and the remainder of the network to be simplified to a source of e.m.f. with an internal resistance.

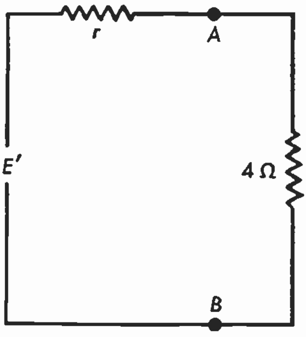

Fig. 2.25b Equivalent circuit. The circuit to the left of *AB* is "Thévenins generator."

Let the 4-ohm resistor be the load, and *A B* the terminals of the network. Figure 2.25b will then be the equivalent circuit.

To find the internal resistance of the network r:

With the 100-volt source short-circuited by line *SC*, the network impedance as seen from *A B* consists of the series-parallel resistances as indicated in Figure 2.25c.

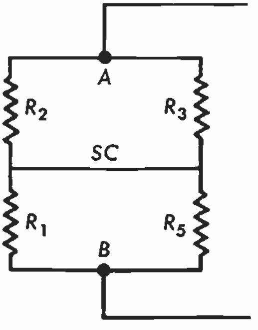

Fig. 2.25c The resistances making up the internal resistance r of Fig. 2.25b.

$$\frac{2 \times 3}{2 + 3} = 1.2 \text{ ohms} \quad \text{and} \quad \frac{1 \times 5}{1 + 5} = 0.83 \text{ ohms}$$

$$r = 1.2 + 0.83 = 2.03 \text{ ohms}$$

To find the equivalent voltage E':

The potential difference between A and B is determined by the voltage drops across the 3- and 5-ohm resistors under no-load conditions, i.e., with the resistor R_4 not in the circuit.

$$E_3 = 100 \times \frac{3}{3+2} = 60 \text{ volts,}$$

and

$$E_5 = 100 \times \frac{5}{5+1} = 83.3 \text{ volts}$$

$$E_{ab} = 83.3 - 60 = 23.3 \text{ volts,}$$

and the current I_2 through the shunt resistor is

$$I_2 = \frac{23.3}{2.03 + 4} = 3.86 \text{ amperes.} \quad \textit{Ans.}$$

Thévenin's Theorem for Network Containing Two Sources of Electromotive Force

2.26 Solve problem 2.21 with the aid of Thévenin's theorem.

Solution:

With the 6-ohm load removed, the internal resistance of the network consists of the 0.03- and the 0.02-ohm resistances in parallel.

$$R_i = \frac{0.03 \times 0.02}{0.03 + 0.02} = \frac{0.0006}{0.05} = 0.012 \text{ ohm.}$$

The equivalent voltage under no-load is

$$E_{eq} = 6.3 - I \times 0.03.$$

But the current in the battery loop is determined by the electromotive force (6.3 − 4.2) volts, and by the resistance (0.03 + 0.02) ohm.

Therefore $$I = \frac{2.1}{0.05} = 42 \text{ amperes.}$$

Substituting

$$E_{eq} = 6.3 - 42 \times 0.03$$
$$= 6.3 - 1.26 = 5.04 \text{ volts.}$$

The load current I_L is determined by the equivalent electromotive force, the equivalent internal resistance, and the load resistor, which is in series with the internal resistance.

$$I_L = \frac{5.04}{6 + 0.012} = \frac{5.04}{6.012} = 0.837 \text{ ampere.} \quad \textit{Ans.}$$

3 *Alternating-Current Circuits*

Wavelength

3.01 What is the wavelength of a 550-kilocycle signal? Of a 1.35-megacycle signal?

Solution:

$$\lambda_1 = \frac{3 \times 10^8}{f} = \frac{3 \times 10^8}{550 \times 10^3} = 545 \text{ meters.} \quad \textit{Ans.}$$

$$\lambda_2 = \frac{3 \times 10^8}{1350 \times 10^3} = 222 \text{ meters.} \quad \textit{Ans.}$$

Frequency

3.02 What is the frequency of a 42-centimeter wave?

Solution:

$$f = \frac{3 \times 10^8}{0.42} = 7.14 \times 10^8 = 714 \text{ megacycles.} \quad \textit{Ans.}$$

Period

3.03 What is the period of a 42-centimeter wave?

Solution:

The period is the duration of one cycle. Since there are 714 million cycles in one second, this duration will be

$$P = \frac{1}{714 \times 10^6}$$

$$= \frac{1}{7.14 \times 10^8} = 0.14 \times 10^{-8}$$

$$= 0.0014 \times 10^{-6} = 0.0014 \text{ microsecond.} \quad \textit{Ans.}$$

Instantaneous Current

3.04 What is the instantaneous value of an alternating current at 212 degrees of its cycle? The peak value is 7 amperes.

Solution:

Using $i = I \sin \theta$,

we have $7 \sin 212° = 7\,(-\sin 32°)$

$= -7 \times 0.53 = -3.71$ amperes. *Ans.*

Instantaneous Voltage

3.05 What is the instantaneous value of a 60-cycle alternating voltage of 75 volts peak value, 5000 microseconds after the beginning of the cycle?

Solution:

$$e = E \sin 2\,\pi\,f\,t$$
$$= 75 \times \sin 377 \times 5000 \times 10^{-6}$$
$$= 75 \times \sin 1.885.$$

Since 1.885 radians $= 1.885 \times 57.3$ degrees, we have

$$e = 75 \times \sin 108°$$
$$= 75 \sin 72° = 75 \times 0.951$$
$$= 71.4 \text{ volts.} \quad \textit{Ans.}$$

Pulsating Direct Current

3.06 A full-wave rectified pulsating direct voltage has a peak value of 157 volts. Calculate its d-c component.

Solution:

The d-c component is the average value of 157 volts peak.

$$E_{av} = 0.637 \times 157 = 100 \text{ volts.} \quad \textit{Ans.}$$

Alternating and Direct Currents Combined

3.07 What is the effective voltage resulting from a 90-volt d-c source and a 120-volt 60-cycle alternating voltage in series?

Solution:

Since the effective value is the square root of the sum of the squares, we have

$$E_{eff} = \sqrt{E_{dc}^2 + E_{ac}^2}$$
$$= \sqrt{120^2 + 90^2} = \sqrt{14{,}400 + 8100}$$
$$= \sqrt{22{,}500} = 150 \text{ volts.} \quad \textit{Ans.}$$

Phase Considerations

3.08 In a capacitive-resistive circuit the current leads the voltage by 37 degrees. The frequency is 60 cycles, and the peak values of voltage and current are 100 volts and 2 amperes respectively. Write the equations of the current and the voltage indicating the phase difference. At the time when the voltage is 50 volts, what is the current?

Solution:

Since $\theta = 37 \text{ degrees} = 0.646 \text{ radian}$

and $i = I \sin (2 \pi f t + \theta)$

we have $i = 2 \sin (377 t + 0.646)$. *Ans.*

$$e = E \sin 2 \pi f t = 100 \sin 377 t. \quad \textit{Ans.}$$

In order to find the value of the current when $e = 50$ volts, we must find the value of t.

Substituting in the voltage equation:

$$50 = 100 \sin 377 t$$
$$0.5 = \sin 377 t$$
$$0.5 = \sin 30°,$$

which, expressed in radians, becomes

$$0.5 = \sin 0.524$$
$$377 \text{ t} = 0.524$$
$$t = \frac{0.524}{377} = 1.39 \times 10^{-3} \text{ second.}$$

The equation of the current then yields

$$\begin{aligned} i_{50} &= 2 \sin (377 \times 1.39 \times 10^{-3} + 37°) \\ &= 2 \sin (0.524 + 0.646) \\ &= 2 \sin 1.170 = 2 \sin 67° \\ &= 2 \times 0.92 = 1.84 \text{ amperes.} \quad \textit{Ans.} \end{aligned}$$

The value of the current may be found more quickly by remembering that the voltage, having an instantaneous value of half its peak value, is at 30 degrees of its cycle, since sin 30° = 1/2. The current, leading the voltage by 37 degrees, is at 67 degrees of its cycle (30° + 37°); thus

$$i = 2 \sin 67° = 2 \times 0.92 = 1.84 \text{ amperes.}$$

Motor Input Current

FCC Study Guide Question 4.135

3.09 If a 15-horsepower 220-volt single-phase alternating-current motor is 92 per cent efficient when delivering its full rated output, what is the input current at a power factor of 0.85?

Solution:

The efficiency is

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}.$$

The output in watts is

$$15 \times 746 = 11{,}190 \text{ watts.}$$

Substituting $$0.92 = \frac{11{,}190}{P_i}$$

Transposing $$P_i = \frac{11{,}190}{0.92} = 12{,}160 \text{ watts}$$

Using the power formula for alternating current

$$P_i = E_i \times I_i \times \cos\theta,$$

we obtain $$12{,}160 = 220 \times I_i \times 0.85,$$

and $$I_i = \frac{12{,}160}{220 \times 0.85} = 65 \text{ amperes.} \quad \textit{Ans.}$$

Resistance and Capacitance in Series

3.10 A voltage of 50 volts, 120 cycles exists across a series circuit, consisting of a capacitor of 6 microfarads and a resistor of 250 ohms. What is the power dissipated by the resistor?

Solution:

$$X_c = \frac{1}{2\,\pi f C}$$

$$= \frac{1}{6.28 \times 120 \times 6 \times 10^{-6}}$$

$$= \frac{1{,}000{,}000}{753 \times 6} = 220 \text{ ohms.}$$

The circuit impedance is

$$|Z| = \sqrt{250^2 + 220^2} = 332 \text{ ohms;}$$

the current is

$$I = \frac{50}{332} = 0.1505 \text{ ampere.}$$

The voltage across the resistor is

$$E_r = I\,R = 0.1505 \times 250 = 37.6 \text{ volts.}$$

The power dissipated by the resistor is

$$P_r = 37.6 \times 0.1505 = 5.65 \text{ watts.} \quad \textit{Ans.}$$

Alternate computation using the power factor:

$$P = E \times I \times \cos\theta$$

$$= \frac{50 \times 0.15 \times 250}{332} = 5.65 \text{ watts.}$$

Resistance and Inductance in Series

3.11 A current of 500 milliamperes is flowing in a circuit of 521 ohms resistance and an inductance in series. The voltage across the circuit is 280 volts. What is the value of the inductance, if the frequency is 120 cycles?

Solution:

The circuit impedance is

$$Z = \frac{E}{I} = \frac{280}{0.5} = 560 \text{ ohms}$$

X_l is a side of the impedance triangle; the other side is 521, and the hypotenuse is 560.

Therefore, $X_l = \sqrt{560^2 - 521^2} = 205$ ohms.

Also $X_l = 2\pi f L$.

Substituting $205 = 753\,L$,

and $L = \frac{205}{753} = 0.272$ henry. *Ans.*

Resistance, Capacitance, and Inductance, in Series

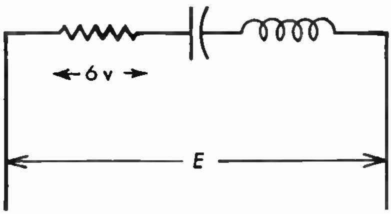

Fig. 3.12 Resistance, capacitance and inductance in series.

3.12 A series circuit consists of a 0.5-megohm resistor, a 3-micro-microfarad capacitor and a 200-millihenry inductor. A radio-frequency signal of 600 kilocycles is applied across the circuit, causing the voltage of 6 volts across the resistor. What is the voltage of the applied signal?

Solution:

$$X_l = 2\pi f L$$

$$= 6.28 \times 600 \times 10^3 \times 200 \times 10^{-3}$$

$$= 3767 \times 200 = 754{,}000 \text{ ohms}$$

$$X_c = \frac{1}{2\pi f C}$$

$$= \frac{1}{6.28 \times 6 \times 10^2 \times 10^3 \times 3 \times 10^{-12}}$$

$$= \frac{1}{3.767 \times 10^3 \times 10^3 \times 3 \times 10^{-12}}$$

$$= \frac{10^6}{3.767 \times 3} = 88{,}600 \text{ ohms}$$

$$X_l - X_c = 754{,}000 - 88{,}600 = 665{,}400 \text{ ohms}$$

$$Z = \sqrt{500{,}000^2 + 665{,}400^2} = 833{,}000 \text{ ohms}$$

$$I_r = \frac{E_r}{R} = \frac{6}{5 \times 10^5} = 1.2 \times 10^{-5} \text{ ampere.}$$

Being a series circuit, I_r is the circuit current and the applied signal voltage is

$$E = I\,Z = 1.2 \times 10^{-5} \times 8.33 \times 10^5$$

$$= 10 \text{ volts.} \quad \textit{Ans.}$$

j-Notation in a Series Circuit

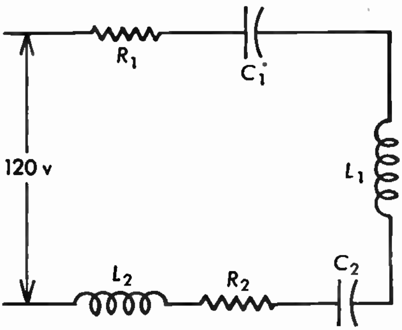

Fig. 3.13 Impedance elements in series.

3.13 In Figure 3.13, $R_1 = 300$ ohms, $R_2 = 210$ ohms; the reactances of the other circuit components are:

$$X_{c_1} = 23 \text{ ohms,} \qquad X_{l_1} = 1200 \text{ ohms,}$$

$$X_{c_2} = 810 \text{ ohms} \quad \text{and} \quad X_{l_2} = 50 \text{ ohms.}$$

Find the voltage across the 810-ohm capacitive reactance.

Solution:

The vector value of the impedance is

$$\mathring{Z} = 300 - j\,23 + j\,1200 - j\,810 + 210 + j\,50$$

$$= 510 + j\,1250 - j\,833$$

$$= 510 + j\,417 \text{ vector ohms.}$$

The absolute value of the impedance is

$$|Z| = \sqrt{510^2 + 417^2} = 658 \text{ ohms.}$$

Using Ohm's law, we obtain

$$I = \frac{120}{658} = 0.183 \text{ ampere,}$$

and $$E_c = I\,X_c = 0.183 \times 810 = 148 \text{ volts.} \quad \textit{Ans.}$$

Phase Difference

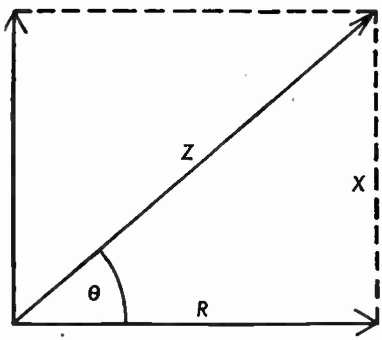

Fig. 3.14 Vector diagram for the total impedance of Fig. 3.13.

3.14 In problem 3.13 find the phase difference between the voltage and the current, and the power factor of the circuit.

Solution:

$$\mathring{Z} = 510 + j\,417,$$

and $$\tan\theta = \frac{X}{R}$$

$$\theta = \tan^{-1}\frac{417}{510} = \tan^{-1} 0.817$$

$$\tan^{-1} 0.817 = 39.2°$$

The phase difference is 39.2°, the voltage is leading. *Ans.*

$$\text{pf} = \cos 39.2°$$
$$= 0.775 = 77.5 \text{ per cent}$$

or $$\frac{R}{Z} = \frac{510}{658} = 0.775 = 77.5 \text{ per cent.} \quad \textit{Ans.}$$

Resistance and Capacitance in Parallel

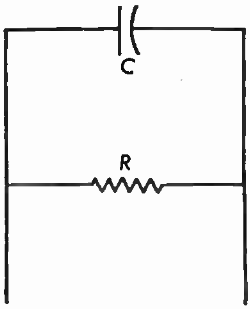

Fig. 3.15 Resistance and capacitance in parallel.

3.15 In Figure 3.15, $C = 5$ microfarads, $R = 500$ ohms, $f = 50$ cycles. Find the equivalent series circuit and the power factor.

Solution:

$$X_c = \frac{1}{2\pi f C}$$

$$= \frac{1}{6.28 \times 50 \times 5 \times 10^{-6}} = 636 \text{ ohms.}$$

Substituting in

$$\frac{1}{\mathring{Z}_t} = \frac{1}{\mathring{Z}_1} + \frac{1}{\mathring{Z}_2}$$

we obtain

$$\frac{1}{\mathring{Z}_t} = \frac{1}{500} + \frac{1}{-j\,636},$$

and since

$$\frac{1}{-j} = +j$$

$$\frac{1}{\mathring{Z}_t} = 0.002 + j\,0.001571.$$

The reciprocal

$$\mathring{Z}_t = \frac{1}{0.002 + j\,0.001571}.$$

Rationalizing by using the conjugate number

$$\mathring{Z}_t = \frac{0.002 - j\,0.001571}{0.002^2 + 0.001571^2}$$

$$= \frac{0.002 - j\,0.001571}{6.47 \times 10^{-6}}$$

$$= \frac{2000}{6.47} - \frac{j\,1571}{6.47}$$

The equivalent series circuit is

$$\mathring{Z}_t = 309 - j\,243. \quad \textit{Ans.}$$

To find the power factor:

$$|Z_t| = \sqrt{309^2 + 243^2} = 393 \text{ ohms},$$

and

$$pf = \frac{R}{Z} = \frac{309}{393}$$

$$= 0.785 = 78.5 \text{ per cent.} \quad \textit{Ans.}$$

Alternate solution using polar vectors:

$$\mathring{Z}_t = \frac{\mathring{Z}_1 \mathring{Z}_2}{\mathring{Z}_1 + \mathring{Z}_2} = \frac{(500\underline{/0^\circ})(636\underline{/-90^\circ})}{500 - j\,636}$$

$$= \frac{500 \times 636\underline{/-90^\circ}}{810\underline{/-51.8^\circ}}$$

$$= 392.8\underline{/-38.2^\circ}$$

$$= 309 - j\,243. \quad \textit{Ans.}$$

Capacitance and Inductance in Parallel

3.16 In Figure 3.16, $X_c = 5200$ ohms, $X_l = 5100$ ohms, $I = 5$ microamperes. Find the total impedance and the voltage E.

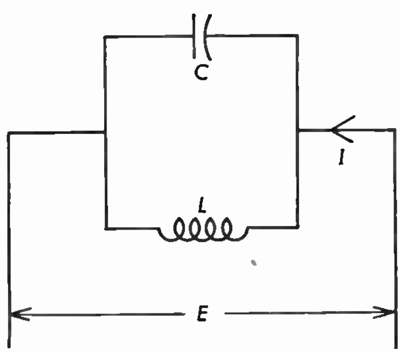

Fig. 3.16 Capacitance and inductance in parallel.

Solution:

$$\mathring{Z}_t = \frac{\mathring{Z}_1 \times \mathring{Z}_2}{\mathring{Z}_1 + \mathring{Z}_2} = \frac{j\,5100 \times (-j\,5200)}{j\,5100 - j\,5200}$$

$$= \frac{26{,}500{,}000}{-j\,100}$$

$$= +j\,265{,}000$$

$$|Z_t| = 265{,}000 \text{ ohms.} \quad \textit{Ans.}$$

$$E_t = I\,Z_t = 5 \times 10^{-6} \times 2.65 \times 10^5$$

$$= 1.325 \text{ volts.} \quad \textit{Ans.}$$

Resistance and Inductance in Parallel

3.17 In Figure 3.17, L = 15 millihenries, R = 75,000 ohms; the frequency f = 550 kilocycles. Find the equivalent series circuit and the power factor.

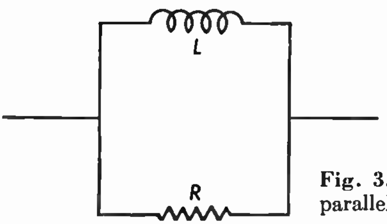

Fig. 3.17 Resistance and inductance in parallel.

Solution:

$$X_l = 2\,\pi\,f\,L = 51{,}800 \text{ ohms}$$

$$\mathring{Z}_t \text{ (in kilohms)} = \frac{\mathring{Z}_1\,\mathring{Z}_2}{\mathring{Z}_1 + \mathring{Z}_2}$$

$$= \frac{j\,51.8 \times 75}{75 + j\,51.8}$$

$$= \frac{j\,3{,}890\,(75 - j\,51.8)}{5600 + 2680}$$

$$= \frac{j\,291{,}000 + 201{,}000}{8280}$$

$$= 24.3 + j\,35.1 \text{ vector kilohms}$$

$$= 24{,}300 + j\,35{,}100 \text{ vector ohms}$$

$$|Z_t| = \sqrt{24.3^2 + 35.1^2}$$
$$= 42.7 \text{ kilohms} = 42{,}700 \text{ ohms.}$$
$$pf = \frac{R}{Z} = \frac{24.3}{42.7}$$
$$= 0.568 = 56.8 \text{ per cent.} \quad \textit{Ans.}$$

Resistance, Capacitance, and Inductance in Parallel

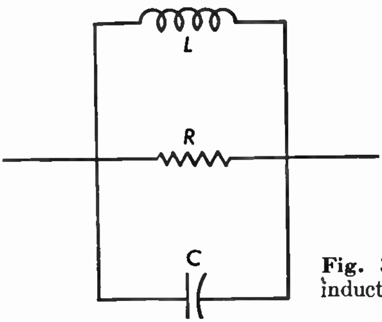

Fig. 3.18 Resistance, capacitance and inductance in parallel.

3.18 In Figure 3.18, $L = 5$ henries, $R = 5000$ ohms, $C = 1$ microfarad and $f = 250$ cycles. Find the equivalent series circuit, the absolute value of the impedance and the phase angle.

Solution:

$$X_L = 2\,\pi f L = 6.28 \times 250 \times 5 = 7850 \text{ ohms}$$
$$X_c = \frac{1}{2\,\pi f C} = \frac{1}{6.28 \times 250 \times 1 \times 10^{-6}}$$
$$= \frac{10^{-6}}{6.28 \times 250} = \frac{1{,}000{,}000}{1571} = 637 \text{ ohms.}$$

We can now write the following equation, the reactances in kilohms:

$$\frac{1}{\mathring{Z}_t} = \frac{1}{5} + \frac{1}{j\,7.85} + \frac{1}{-j\,0.637}$$
$$= 0.2 - j\,0.1273 + j\,1.571$$
$$= 0.2 + j\,1.444 \text{ vector mhos,}$$

and the reciprocal

$$\mathring{Z}_t = \frac{1}{0.2 + j\,1.444} = \frac{0.2 - j\,1.444}{0.04 + 2.07}$$

$$= \frac{0.2}{2.11} - \frac{j\,1.44}{2.11}$$

$$= 0.0948 - j\,0.683 \text{ vector kilohms.}$$

The equivalent circuit is

$$\mathring{Z}_t = 94.8 - j\,683 \text{ vector ohms.} \quad \textit{Ans.}$$

The phase angle is

$$\theta = \tan^{-1}\frac{683}{94.8} = \tan^{-1} 7.22$$

$$= 82.1°\text{, voltage lagging.} \quad \textit{Ans.}$$

The absolute value of the impedance is

$$|Z| = \frac{683}{\sin 82.1°} = \frac{683}{0.9905} = 689 \text{ ohms.} \quad \textit{Ans.}$$

Methods of Solving Parallel Circuits

3.19 Depending upon previous training, engineers use different methods to compute parallel circuits. The following problem illustrates the four popular methods:

In Figure 3.19, C = 500 micromicrofarads, L = 0.1 millihenry, R = 50 ohms and f = 750 kilocycles. Find the equivalent series circuit, the phase angle, and the absolute value of the impedance.

Solution:

(a) Parallel impedances method:

$$\mathring{Z}_t = \frac{\mathring{Z}_1 \times \mathring{Z}_2}{\mathring{Z}_1 + \mathring{Z}_2}$$

The branch impedances $\mathring{Z}_1$ and $\mathring{Z}_2$ can be expressed in j-notation after X_l and X_c are found.

$$X_l = 2\,\pi f L$$

$$= 6.28 \times 750 \times 0.1 \times 10^3 \times 10^{-3}$$

$$= 471 \text{ ohms}$$

$$X_c = \frac{1}{2\,\pi f\,C}$$

$$= \frac{1}{6.28 \times 750 \times 10^3 \times 500 \times 10^{-12}}$$

$$= 425 \text{ ohms}$$

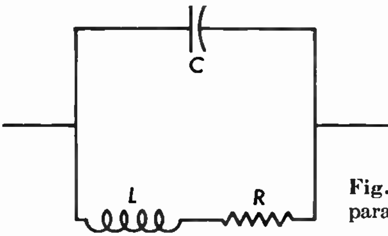

Fig. 3.19 Coil and Capacitor in parallel.

$$\mathring{Z}_t = \frac{(-j\,425)(50 + j\,471)}{(-j\,425) + (50 + j\,471)}$$

$$= \frac{200{,}000 - j\,21{,}250}{50 + j\,46}$$

$$= \frac{10^4\,(20 - j\,2.125)}{10\,(5 + j\,4.6)};$$

after multiplying numerator and denominator by $(5 - j\,4.6)$ we obtain

$$\frac{10^3\,(90.2 - j\,102.6)}{5^2 + 4.6^2}$$

$$= \frac{90{,}200 - j\,102{,}600}{46.16}$$

$$= 1960 - j\,2220. \quad \textit{Ans.}$$

$$|Z_t| = \sqrt{1960^2 + 2220^2}$$

$$= 2970 \text{ ohms.} \quad \textit{Ans.}$$

$$\theta = \tan^{-1}\frac{222}{196} = \tan^{-1} 1.13$$

$$= 48.5°\text{, voltage lagging.} \quad \textit{Ans.}$$

(b) Assumed voltage method:

Let us assume a voltage of 1000 volts across the circuit. The branch currents, expressed in vector amperes, will then be

$$\mathring{I}_1 = \frac{1000}{-j\,425} = j\,2.35$$

$$\mathring{I}_2 = \frac{1000}{50 + j\,471}$$

$$= \frac{1000\,(50 - j\,471)}{50^2 + 471^2}$$

$$= \frac{1000\,(50 - j\,471)}{224{,}300}$$

$$= 0.223 - j\,2.1$$

The total current is the sum of the branch currents

$$\mathring{I}_t = \mathring{I}_1 + \mathring{I}_2 = 0.223 + j\,0.25,$$

and by Ohm's law

$$\mathring{Z}_t = \frac{1000}{\mathring{I}_t}$$

$$= \frac{1000}{0.223 + j\,0.25}$$

$$= \frac{1000\,(0.223 - j\,0.25)}{0.223^2 + 0.25^2}$$

$$= \frac{1000\,(0.223 - j\,0.25)}{0.0497 + 0.0625}$$

$$= 1000\left(\frac{0.223}{0.112} - j\,\frac{0.250}{0.112}\right)$$

$$= 1000\,(1.97 - j\,2.23)$$

$$= 1970 - j\,2230. \quad \textit{Ans.}$$

The answers agree within the limits of slide rule accuracy.

(c) Polar vector method:

The capacitive branch containing pure capacitance has a polar vector

$$\mathring{Z}_1 = 425\underline{/-90°}$$

The magnitude of Z_2 is $\sqrt{50^2 + 471^2} = 473$, and the phase is

$$\tan^{-1}(471/50) = \tan^{-1}(9.72) = 83.9°.$$

The polar vector of the inductive branch is therefore

$$\mathring{Z}_2 = 473\underline{/83.9°}.$$

The total impedance is

$$\mathring{Z}_t = \frac{\mathring{Z}_1 \times \mathring{Z}_2}{\mathring{Z}_1 + \mathring{Z}_2} = \frac{(425\underline{/-90°})\,(473\underline{/83.9°})}{50 + j\,46}.$$

Expressing the denominator in polar form

$$\mathring{Z}_t = \frac{(425\underline{/-90°})\,(473\underline{/83.9°})}{67.8\underline{/42.6°}}$$

$$= 2975\underline{/48.7°} \text{ vector ohms.} \quad \textit{Ans.}$$

This agrees with the magnitude and phase found by the method of parallel impedances within the limit of slide-rule accuracy.

(d) Admittance method:

$$\mathring{Y}_1 = G_1 + j\,B_1$$

Since the capacitive branch has no resistance, we obtain

$$\mathring{Y}_1 = 0 + j\,(425/425^2) = j\,0.00236$$

$$\mathring{Y}_2 = G_2 - j\,B_2$$

$$= \frac{50}{50^2 + 471^2} - j\,\frac{471}{50^2 + 471^2}$$

$$= \frac{50}{224{,}341} - j\,\frac{471}{224{,}341}$$

$$= 0.000223 - j\,0.00211.$$

Adding the conductances and the susceptances algebraically

$$\mathring{Y}_t = 0.000223 + j\,0.00025$$

$$= 10^{-4}\,(2.23 + j\,2.50) \text{ vector mho.}$$

The absolute value of the admittance is

$$|Y_t| = 10^{-4}\sqrt{2.23^2 + 2.5^2}$$
$$= 10^{-4}\sqrt{4.97 + 6.25}$$
$$= 10^{-4}\sqrt{11.22}$$
$$= 3.35 \times 10^{-4} \text{ mho}$$

The absolute value of the impedance is the reciprocal of the admittance value

$$|Z_t| = 10^4 \times (1/3.35) = 2975 \text{ ohms.} \quad \textit{Ans.}$$

To find the phase angle, the values of the vector notation of the admittance can be used:

$$\theta = \tan^{-1}(2.5/2.23)$$
$$= 48.5°, \text{ voltage lagging.} \quad \textit{Ans.}$$

Resistance in Both Branches

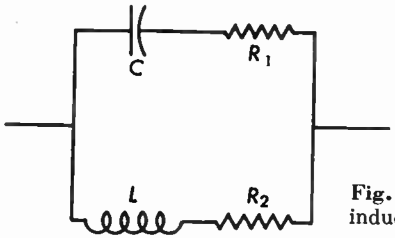

Fig. 3.20 Capacitive-resistive and inductive-resistive branches in parallel.

3.20 In Figure 3.20, C = 506 micromicrofarads, R_1 = 70 ohms, L = 87 microhenries, R_2 = 50 ohms and f = 1.5 megacycles. Find the equivalent series circuit, the absolute value of the impedance, and the phase angle.

Solution:

(a) By rectangular vectors:

$$X_L = 2\,\pi\,f\,L$$
$$= 6.28 \times 1.5 \times 10^6 \times 87 \times 10^{-6}$$
$$= 820 \text{ ohms.}$$

$$X_c = 1/(2\,\pi\,f\,C)$$
$$= 1/(6.28 \times 1.5 \times 10^6 \times 506 \times 10^{-12})$$
$$= 210 \text{ ohms.}$$

$$1/\mathring{Z}_t = 1/\mathring{Z}_1 + 1/\mathring{Z}_2$$
$$= \frac{1}{70 - j\,210} + \frac{1}{50 + j\,820}$$
$$= \frac{70 + j\,210}{70^2 + 210^2} + \frac{50 - j\,820}{50^2 + 820^2}$$
$$= \frac{70 + j\,210}{4900 + 44{,}100} + \frac{50 - j\,820}{2500 + 670{,}000}$$
$$= \frac{70 + j\,210}{49{,}000} + \frac{50 - j\,820}{672{,}500}$$
$$= \frac{1}{10^4}\left(\frac{70}{4.9} + j\frac{210}{4.9} + \frac{50}{67} - j\frac{820}{67}\right)$$
$$= \frac{1}{10^4}(14.28 + j\,42.9 + 0.746 - j\,12.21)$$
$$= \frac{1}{10^4}(15.03 + j\,30.69) \text{ vector mhos.}$$

$$\mathring{Z}_t = 10^4 \frac{1}{15.03 + j\,30.69}$$
$$= 10^4 \frac{15.03 - j\,30.69}{15.03^2 + 30.7^2}$$
$$= 10^4 \frac{15.03 - j\,30.7}{1167}$$
$$= 128.7 - j\,262.5 \text{ vector ohms.} \quad \textit{Ans.}$$

The absolute value of the impedance is

$$|Z| = \sqrt{128.7^2 + 262.5^2}$$
$$= \sqrt{16{,}800 + 68{,}800}$$
$$= \sqrt{85{,}600} = 292.5 \text{ ohms.} \quad \textit{Ans.}$$

The phase angle is

$$\theta = \tan^{-1}\frac{262.5}{128.7} = \tan^{-1} 2.05$$
$$= 64°, \text{ voltage lagging.} \quad \textit{Ans.}$$

(b) Alternate solution by polar vectors:

$$\mathring{Z}_t = \frac{\mathring{Z}_1 \times \mathring{Z}_2}{\mathring{Z}_1 + \mathring{Z}_2}$$

$$= \frac{(70 - j\,210)\,(50 + j\,820)}{70 - j\,210 + 50 + j\,820}$$

$$= \frac{(70 - j\,210)\,(50 + j\,820)}{120 + j\,610}$$

$$= \frac{\sqrt{70^2 + 210^2}\underline{/\tan^{-1}(-210/70)}\sqrt{50^2 + 820^2}\underline{/\tan^{-1}(820/50)}}{\sqrt{120^2 + 610^2}\underline{/\tan^{-1}(610/120)}}$$

$$= \frac{221\underline{/-71.6^\circ}\ 821\underline{/86.5^\circ}}{621\underline{/78.9^\circ}}$$

$$= 292.5\underline{/-64^\circ}. \quad \textit{Ans.}$$

This answer is identical with the one obtained by rectangular vectors.

Series-Parallel Network

3.21 In Figure 3.21, $L_1 = 0.005$ henry, $L_2 = 0.002$ henry, $L_3 = 0.001$ henry, $R_1 = 400$ ohms, $R_2 = 200$ ohms, $C_4 = 0.1$ microfarad, $C_5 = 0.05$ microfarad, $E = 60$ volts, 6000 cycles. Find the total current I and the power factor.

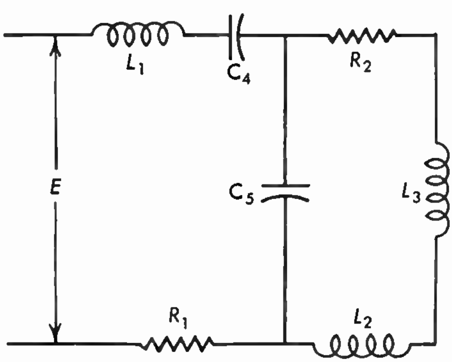

Fig. 3.21 Series-parallel network of impedance elements.

Solution:

$$
\begin{aligned}
X_1 &= 2\pi f L_1 \\
&= 6.28 \times 6 \times 10^3 \times 0.005 = 188.5 \text{ ohms} \\
X_2 &= 2\pi f L_2 \\
&= 6.28 \times 6 \times 10^3 \times 0.002 = 75.4 \text{ ohms} \\
X_3 &= 2\pi f L_3 \\
&= 6.28 \times 6 \times 10^3 \times 0.001 = 37.7 \text{ ohms} \\
X_4 &= 1/(2\pi f C_4) \\
&= 1/(6.28 \times 6 \times 10^3 \times 0.1 \times 10^{-6}) = 265 \text{ ohms} \\
X_5 &= 1/(2\pi f C_5) \\
&= 1/(6.28 \times 6 \times 10^3 \times 0.05 \times 10^{-6}) = 531 \text{ ohms.}
\end{aligned}
$$

In Figure 3.21, R_1, X_1, and X_4 are in series with the parallel branches Z_1, Z_2. The branch Z_1 is X_5, and the branch Z_2 consists of R_2, X_2, X_3 in series. To find the impedance of the parallel combination we first find the rectangular and the polar notations of Z_1 and Z_2.

Step 1.

$$
\begin{aligned}
\mathring{Z}_1 &= -j\,531 = 531\underline{/-90^\circ} \\
\mathring{Z}_2 &= 200 + j\,37.7 + j\,75.4 \\
&= 200 + j\,113.1 = 230\underline{/29.6^\circ} \\
\mathring{Z}_1 + \mathring{Z}_2 &= -j\,531 + j\,113.1 + 200 \\
&= 200 - j\,417.9 = 464\underline{/-64.4^\circ}
\end{aligned}
$$

Step 2.

$$
\begin{aligned}
\mathring{Z}\text{ (parallel)} &= \frac{\mathring{Z}_1\,\mathring{Z}_1}{\mathring{Z}_1 + \mathring{Z}_2} \\
&= \frac{531\underline{/-90^\circ}\ 230\underline{/29.6^\circ}}{464\underline{/-64.4^\circ}} \\
&= 263\underline{/4^\circ} \\
&= 263\,(\cos 4^\circ + j \sin 4^\circ) \\
&= 263\,(0.998 + j\,0.0697) \\
&= 262 + j\,18.35
\end{aligned}
$$

Step 3.

The total impedance is

$$
\begin{aligned}
\mathring{Z}_t &= 400 + j\,188.5 - j\,265 + 262 + j\,18.4 \\
&= 662 - j\,58.1 = 665\underline{/5^\circ} \text{ ohms.}
\end{aligned}
$$

$$|I_t| = \frac{60}{665} = 0.0902 = 90.2 \text{ milliamperes.} \quad \textit{Ans.}$$

$$\text{power factor} = \cos 5° = 99.6 \text{ per cent.} \quad \textit{Ans.}$$

Step 2 can also be calculated by rectangular vectors:

$$\frac{1}{\mathring{Z}_{parallel}} = \frac{1}{-j\,531} + \frac{1}{200 + j\,113.1}$$

$$= j\frac{1}{531} + \frac{200 - j\,113.1}{40{,}000 + 12{,}800}$$

$$= j\,0.00188 + 0.00378 - j\,0.00214$$

$$= 10^{-3}\,(3.78 - j\,0.26)$$

$$\mathring{Z}_{parallel} = 10^3\,\frac{3.78 + j\,0.26}{14.3 + 0.067}$$

$$= 10^3\,(0.262 + j\,0.0181)$$

$$= 262 + j\,18.1 \text{ etc.}$$

The total impedance is then found by adding X_1, X_4 and R_1 to this parallel combination as was done above in Step 3.

Resonant Frequency

3.22 A series resonant circuit consists of 30 millihenries inductance and 0.005 microfarad capacitance. What is the resonant frequency?

Solution:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{6.28\sqrt{30 \times 10^{-3} \times 0.005 \times 10^{-6}}}$$

$$= \frac{1}{6.28\sqrt{3 \times 10 \times 10^{-3} \times 5 \times 10^{-3} \times 10^{-6}}}$$

$$= \frac{1}{6.28 \times 10^{-6} \times \sqrt{150}}$$

$$= \frac{10^6}{6.28 \times 12.25}$$

$$= \frac{1{,}000{,}000}{77} = 13{,}000 \text{ cycles} = 13 \text{ kilocycles.} \quad \textit{Ans.}$$

Voltage at Resonance

3.23 A series resonant circuit tuned to 1500 kilocycles has a resistance of 50 ohms and an inductance of 50 microhenries. A voltage of 10 volts is applied across the circuit. Calculate the voltage across the resistor and the capacitor.

Solution:

Using
$$C = \frac{1}{4\,\pi^2 L f^2},$$

we obtain
$$C = \frac{1}{39.5 \times 50 \times 10^{-6} \times (1.5 \times 10^6)^2}$$

$$= \frac{1}{39.5 \times 50 \times 1.5^2 \times 10^6} = 225\ \mu\mu\text{f}.$$

$$Z = R = 50 \text{ ohms.}$$

$$I = \frac{10}{50}$$

$$= 0.2 \text{ amperes} = 200 \text{ milliamperes.}$$

And at resonance,

$$E_r = I \times R = 10 \text{ volts.} \quad \textit{Ans.}$$

and
$$E_c = I \times X_c = 0.2 \times \frac{1}{2\,\pi f C}$$

$$= \frac{0.2}{6.28 \times 1.5 \times 10^6 \times 225 \times 10^{-12}}$$

$$= 0.2 \times 472 = 94.4 \text{ volts.} \quad \textit{Ans.}$$

Resonant Capacitance

3.24 What capacitance will tune to resonance with a 15-henry choke at 60 cycles?

Solution:

Using $$C = \frac{1}{4\,\pi^2\,L\,f^2},$$

we obtain $$C = \frac{1}{39.4 \times 15 \times 3600}$$

$$= \frac{1}{3.94 \times 10 \times 1.5 \times 10 \times 3.6 \times 10^3}$$

$$= \frac{1}{21.3} \times 10^{-5}$$

$$= 0.047 \times 10^{-5} = 0.47 \text{ microfarad.} \quad \textit{Ans.}$$

Effective Resistance

3.25 An inductor of 50 microhenries has a figure of merit of $Q = 22.5$ at 4 megacycles. Find the effective resistance of the coil at this frequency.

Solution:

$$X_l = 2\,\pi\,f\,L$$

$$= 6.28 \times 4 \times 10^6 \times 50 \times 10^{-6}$$

$$= 1256 \text{ ohms.}$$

Since $$Q = \frac{X}{R},$$

we obtain $$22.5 = \frac{1256}{R},$$

and the resistance at this frequency is

$$R = \frac{1256}{22.5} = 55.8 \text{ ohms.} \quad \textit{Ans.}$$

Parallel Resonance

3.26 A tank circuit consists of an inductance of 20 microhenries, a capacitance of 50 micromicrofarads, and a coil resistance of 25 ohms.

What is the resonant frequency, the impedance at resonance, and the Q of the circuit?

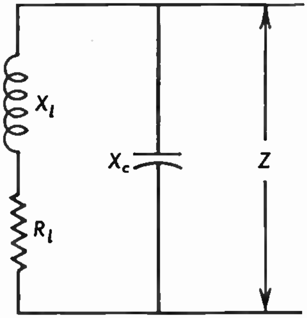

Fig. 3.26 Parallel-resonant circuit.

Solution:

General considerations:

The impedance of the parallel (non-resonant) circuit Fig. 3.26, where $\mathring{Z}_1 = -jX_c$, $\mathring{Z}_2 = R + jX_l$ and $\mathring{Z}_1 + \mathring{Z}_2 = R + j(X_l - X_c)$ is

$$|Z| = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{X_c \sqrt{R^2 + X_l^2}}{\sqrt{R^2 + (X_l - X_c)^2}}$$

$$= X_c \sqrt{\frac{R^2 + X_l^2}{R^2 + (X_l - X_c)^2}}$$

If the merit of the coil is at all good, then R^2 in the numerator is so small compared with X_l^2 that it can be neglected. At resonance X_l and X_c will be practically equal, and the quantity $(X_l - X_c)^2$ in the denominator can be neglected compared with R^2. We then have at resonance,

$$Z = X_c \sqrt{\frac{X_l^2}{R^2}} = \frac{X_l X_c}{R} = \frac{X^2}{R} = XQ$$

For this and the following problems we shall use the formulas:

$$Z = \frac{X^2}{R}, \text{ and } Z = XQ$$

In this problem:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{6.28 \times \sqrt{20 \times 10^{-6} \times 50 \times 10^{-12}}}$$

$$= \frac{1}{6.28 \times 10^{-9}\sqrt{100 \times 10}}$$

$$= \frac{1}{6.28 \times 10^{-8} \times 3.16}$$

$$= \frac{10^8}{6.28 \times 3.16} = 5.03 \text{ megacycles.}$$

$$X_l = 2\pi f L$$

$$= 6.28 \times 5.03 \times 10^6 \times 20 \times 10^{-6}$$

$$= 632 \text{ ohms}$$

$$Z = \frac{X^2}{R} = \frac{632^2}{25} = 16{,}000 \text{ ohms.} \quad \textit{Ans.}$$

$$Q = \frac{Z}{X} = \frac{16{,}000}{632} = 25.3. \quad \textit{Ans.}$$

Q of Tank Circuit

3.27 A radio-frequency voltage of 50 volts exists across the capacitor plates of a tank circuit and a current of 0.001 ampere is flowing from and to the tank; the 50 micromicrofarad capacitor is tuned to 2.5 megacycles. What is the Q of the circuit?

Solution:

$$X_c = X_l = \frac{1}{2\pi f C}$$

$$= \frac{1}{6.28 \times 2.5 \times 10^6 \times 50 \times 10^{-12}}$$

$$= 1273 \text{ ohms}$$

$$Z = \frac{E}{I} = \frac{50}{10^{-3}} = 50{,}000 \text{ ohms}$$

$$Q = \frac{Z}{X} = \frac{50{,}000}{1273} = 39.3. \quad \textit{Ans.}$$

Series Radio-Frequency Resistance

3.28 A tank circuit tuned to 7.5 megacycles has an impedance of 75,000 ohms. The capacitance is 12 micromicrofarads. What is the effective resistance of the inductor and the Q of the circuit?

Solution:

$$X_c = \frac{1}{2\,\pi\,f\,C}$$

$$= \frac{1}{6.28 \times 7.5 \times 10^6 \times 12 \times 10^{-12}}$$

$$= \frac{10^6}{6.28 \times 7.5 \times 12} = 1770 \text{ ohms}$$

$$Q = \frac{Z}{X} = \frac{75{,}000}{1770} = 42.3. \quad \textit{Ans.}$$

$$R = \frac{X}{Q} = \frac{1770}{42.3} = 41.9 \text{ ohms.} \quad \textit{Ans.}$$

Tank Capacitor

3.29 An inductance of 50 microhenries with an effective resistance of 27.2 ohms is to be tuned to a frequency of 2600 kilocycles. What capacitor must be used? Calculate the circuit Q and the total impedance of the circuit.

Solution:

Using $$C = \frac{1}{4\,\pi^2 f^2\,L}$$

we obtain $$C = \frac{1}{39.5 \times (2.6 \times 10^6)^2 \times 50 \times 10^{-6}}$$

$$= \frac{10^{-6}}{39.5 \times 6.75 \times 50} = \frac{10^{-6}}{13{,}300}$$

$$= 75 \times 10^{-12} = 75 \text{ micromicrofarads.} \quad \textit{Ans.}$$

$$X_l = 2\,\pi\,f\,L = 6.28 \times 2.6 \times 10^6 \times 50 \times 10^{-6}$$

$$= 817 \text{ ohms}$$

$$Q = \frac{817}{27.2} = 30. \quad \textit{Ans.}$$

$$Z = X\,Q = 817 \times 30 = 24{,}510 \text{ ohms.} \quad \textit{Ans.}$$

Loaded Tank Circuit

3.30 An inductance of 100 microhenries is used with a 30-micro-microfarad capacitance in a parallel-resonant circuit. The resistance of the coil is 30 ohms. The tank circuit is loaded with 6000 ohms resistance. Find the total impedance and the Q of the circuit with and without load.

Solution:

(a) Without load:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{6.28\sqrt{100 \times 10^{-6} \times 30 \times 10^{-12}}}$$

$$= \frac{1}{6.28\sqrt{3000 \times 10^{-18}}}$$

$$= \frac{1}{6.28\sqrt{30 \times 10^{-16}}}$$

$$= \frac{1}{6.28 \times 10^{-8} \times 5.48}$$

$$= \frac{10^6}{34.4 \times 10^{-2}}$$

$$= \frac{100}{34.4} \times 10^6 = 2.9 \text{ megacycles}$$

$$X_l = 2\pi f L = 6.28 \times 2.9 \times 10^6 \times 100 \times 10^{-6}$$

$$= 1820 \text{ ohms}$$

$$Q = \frac{X}{R} = \frac{1820}{30} = 60.6 \quad \textit{Ans.}$$

$$Z = XQ = 1820 \times 60.6 = 110{,}000 \text{ ohms.} \quad \textit{Ans.}$$

(b) With load:

The 110,000 ohms can be considered as purely resistive at resonance. The total impedance of the circuit is

$$Z' = \frac{110{,}000 \times 6000}{110{,}000 + 6000} = 5690 \text{ ohms}$$

$$Q' = \frac{Z'}{X} = \frac{5690}{1820} = 3.13. \quad \textit{Ans.}$$

Inductance and Capacitance

3.31 A loaded parallel-resonant circuit, tuned to 7 megacycles, has a total impedance of 10,000 ohms and an over-all $Q = 17$. Calculate the coil resistance and the value of the inductance and the capacitance.

Solution:

$$X = \frac{Z}{Q} = \frac{10,000}{17} = 588 \text{ ohms},$$

$$R = \frac{X}{Q} = \frac{588}{17} = 34.6 \text{ ohms.} \quad \textit{Ans.}$$

To find L, we use

$$X_l = 2\,\pi\,f\,L,$$

and obtain

$$588 = 6.28 \times 7 \times 10^6 \times L,$$

$$\frac{588}{6.28 \times 7} \times 10^{-6} = L,$$

and

$$L = 13.4 \text{ microhenries.} \quad \textit{Ans.}$$

Likewise, at resonance

$$588 = \frac{1}{2\,\pi\,f\,C},$$

and

$$C = \frac{1}{6.28 \times 7 \times 10^6 \times 588}$$

$$= 38.6 \text{ micromicrofarads.} \quad \textit{Ans.}$$

Audio-Frequency High-Pass Filter

3.32 Design a filter to reject all frequencies lower than 200 cycles, to work into a 600-ohm terminal impedance.

Solution:

The filter will be a high-pass filter.

Using formula 3.611 and formula 3.612,

we obtain

$$L = \frac{600}{12.57 \times 200} = 0.239 \text{ henries.} \quad \textit{Ans.}$$

$$C = \frac{1}{12.57 \times 200 \times 600}$$

$$= \frac{1}{1.257 \times 10 \times 2 \times 10^2 \times 6 \times 10^2}$$

$$= \frac{1}{15.1 \times 10^5} = \frac{1}{1.51} \times 10^{-6}$$

$$= 0.662 \text{ microfarad.} \quad \textit{Ans.}$$

Radio-Frequency High-Pass Filter

3.33 Design a filter with a terminal impedance of 2000 ohms to reject all frequencies lower than 4.7 megacycles.

Solution:

A high-pass filter is needed. The cutoff frequency is 4.7 megacycles. Using formula 3.611 and formula 3.612,

we obtain
$$L = \frac{2000}{12.57 \times 4.7 \times 10^6}$$

$$= 33.8 \times 10^{-6} = 33.8 \text{ microhenries.} \quad \textit{Ans.}$$

$$C = \frac{1}{12.57 \times 4.7 \times 10^6 \times 2 \times 10^3}$$

$$= \frac{1}{1.18 \times 10^9}$$

$$= \frac{1}{0.118} \times 10^{-12}$$

$$= 8.45 \text{ micromicrofarads.} \quad \textit{Ans.}$$

Low-Pass Filter

3.34 Design a filter to attenuate all frequencies higher than 10,000 cycles, working into an 8000-ohm impedance.

Solution:

The filter is a low-pass filter (formula 3.62).

Using
$$L = \frac{R}{\pi f}$$

we obtain
$$L = \frac{8000}{3.14 \times 10{,}000} = 0.255 \text{ henry.} \quad \textit{Ans.}$$

$$C = \frac{1}{\pi f R}$$

$$= \frac{1}{3.14 \times 10{,}000 \times 8000}$$

$$= \frac{1}{3.14 \times 80} \times 10^{-6}$$

$$= 0.00398 \times 10^{-6}$$

$$\cong 0.004 \text{ microfarad.} \quad \textit{Ans.}$$

Wave Filter

3.35 A wave filter is to pass only 480 to 520 kilocycles, working into a resistance of 1000 ohms. Calculate the series and the shunt components.

Solution:

Using the formulas for band-pass filters 3.631 to 3.634 we obtain

$$L_1 = \frac{R}{\pi (f_2 - f_1)}$$

$$= \frac{1000}{3.14 \times 40{,}000}$$

$$= 7.96 \text{ millihenries.} \quad \textit{Ans.}$$

$$L_2 = \frac{(f_2 - f_1) R}{4 \pi f_1 f_2}$$

$$= \frac{4 \times 10^4 \times 10^3}{12.57 \times 480 \times 520 \times 10^6}$$

$$= 12.7 \text{ microhenries.} \quad \textit{Ans.}$$

$$C_1 = \frac{f_2 - f_1}{4 \pi f_1 f_2 R}$$

$$= \frac{4 \times 10^4}{12.57 \times 4.8 \times 5.2 \times 10^{10} \times 10^3}$$

$$= 12.7 \text{ micromicrofarads.} \quad \textit{Ans.}$$

$$C_2 = \frac{1}{\pi (f_2 - f_1) R}$$

$$= \frac{1}{3.14 \times 4 \times 10^4 \times 10^3}$$

$$= 0.00796 \text{ microfarad.} \quad \textit{Ans.}$$

Wave Rejector

3.36 A local station operating at 750 to 760 kilocycles is to be rejected by a filter working into a resistance of 1000 ohms. Find the filter components.

Solution:

To reject a stated frequency band use the formulas for band elimination, No. 3.641 ff.,

$$L_1 = \frac{(f_2 - f_1)\,R}{\pi f_1 f_2}$$

$$= \frac{10^4 \times 10^3}{3.14 \times 7.5 \times 7.6 \times 10^{10}}$$

$$= \frac{10^{-3}}{3.14 \times 7.5 \times 7.6}$$

$$= 0.00557 \times 10^{-3}$$

$= 5.57$ microhenries. *Ans.*

$$C_1 = \frac{1}{4\,\pi\,(f_2 - f_1)\,R}$$

$$= \frac{1}{12.57 \times 10^4 \times 10^3}$$

$$= 0.0796 \times 10^{-7}$$

$= 0.00796$ microfarad. *Ans.*

$$L_2 = \frac{R}{4\,\pi\,(f_2 - f_1)}$$

$$= \frac{10^3}{12.57 \times 10^4}$$

$$= 0.0796 \times 10^{-1}$$

$= 7.94$ millihenries. *Ans.*

$$C_2 = \frac{(f_2 - f_1)}{\pi f_1 f_2 R}$$

$$= \frac{10^4}{3.14 \times 7.5 \times 7.6 \times 10^{10} \times 10^3}$$

$$= 0.00557 \times 10^{-9}$$

$= 5.57$ micromicrofarads. *Ans.*

Kirchhoff's Law for Alternating Currents

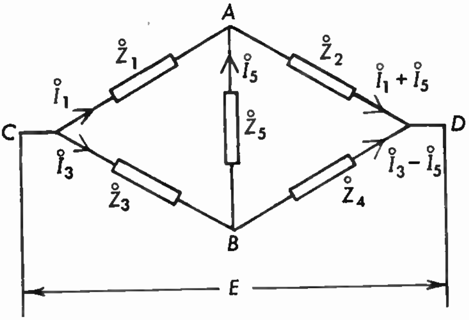

Fig. 3.37 Unbalanced impedance bridge.

3.37 In the circuit of Figure 3.37

$$\mathring{Z}_1 = 15 + j\,20$$
$$\mathring{Z}_2 = 0 + j\,25$$
$$\mathring{Z}_3 = 0 - j\,10$$
$$\mathring{Z}_4 = 40 - j\,0$$
$$\mathring{Z}_5 = 100 - j\,100$$

The applied voltage $E = 10$ volts. Using Kirchhoff's laws find the current flowing through $\mathring{Z}_5$.

Solution:

Let the current through $\mathring{Z}_1$ be $\mathring{I}_1$. the current through $\mathring{Z}_3$ be $\mathring{I}_3$, and the current through $\mathring{Z}_5$ be $\mathring{I}_5$; the direction of $\mathring{I}_5$ is assumed arbitrarily, a convenient procedure which is discussed in problem 2.22. In the circuit of Figure 3.37 the arrow is pointing upward, therefore a current of $\mathring{I}_1 + \mathring{I}_5$ will flow through $\mathring{Z}_2$, and a current of $\mathring{I}_3 - \mathring{I}_5$ will flow through $\mathring{Z}_4$.

Applying Kirchhoff's second law to the lower path yields

$$10 - \mathring{I}_3\,(-j\,10) - (\mathring{I}_3 - \mathring{I}_5)\,(40) = 0; \tag{1}$$

to the left loop

$$\mathring{I}_1\,(15 + j\,20) - \mathring{I}_5\,(100 - j\,100) - \mathring{I}_3\,(-j\,10) = 0; \tag{2}$$

to the right loop

$$\mathring{I}_5\,(100 - j\,100) + (\mathring{I}_1 + \mathring{I}_5)\,(j\,25) - (\mathring{I}_3 - \mathring{I}_5)\,(40) = 0. \tag{3}$$

Rearranging

$$\mathring{I}_3(-40 + j\,10) + \mathring{I}_5(40) + 10 = 0 \qquad (1)$$

$$\mathring{I}_1(15 + j\,20) + \mathring{I}_3(j\,10) - \mathring{I}_5(100 - j\,100) = 0 \qquad (2)$$

$$\mathring{I}_1(j\,25) + \mathring{I}_3(-40) + \mathring{I}_5(100 - j\,100 + j\,25 + 40) = 0. \qquad (3)$$

Simplifying

$$\mathring{I}_3(-40 + j\,10) + \mathring{I}_5(40) + 10 = 0 \qquad (1)$$

$$\mathring{I}_1(15 + j\,20) + \mathring{I}_3(j\,10) - \mathring{I}_5(100 - j\,100) = 0 \qquad (2)$$

$$\mathring{I}_1(j\,25) + \mathring{I}_3(-40) + \mathring{I}_5(140 - j\,75) = 0. \qquad (3)$$

These simultaneous equations are solved by the method of substitution as follows:

Expressing $\mathring{I}_1$ in terms of $\mathring{I}_3$ and $\mathring{I}_5$ in (3)

$$\mathring{I}_1 = \frac{\mathring{I}_3(-40) + \mathring{I}_5(140 - j\,75)}{-j\,25}$$

$$= \mathring{I}_3(-j\,1.6) + \mathring{I}_5(3 + j\,5.6).$$

Substituting in (2)

$$\mathring{I}_3(-j\,1.6)(15 + j\,20) + \mathring{I}_5(3 + j\,5.6)(15 + j\,20) + \mathring{I}_3(+j\,10) + \mathring{I}_5(-100 + j\,100) = 0$$

Multiplying

$$\mathring{I}_3(-j\,24 + 32) + \mathring{I}_5(45 + j\,84 + j\,60 - 112) + \mathring{I}_3(+j\,10) + \mathring{I}_5(-100 + j\,100) = 0$$

Simplifying

$$\mathring{I}_3(32 - j\,14) + \mathring{I}_5(-167 + j\,244) = 0$$

Expressing $\mathring{I}_3$ in terms of $\mathring{I}_5$

$$\mathring{I}_3 = \mathring{I}_5\,\frac{167 - j\,244}{32 - j\,14} = \mathring{I}_5\,\frac{296\underline{/-55.6^\circ}}{34.9\underline{/-23.6^\circ}}$$

$$= \mathring{I}_5\,8.48\underline{/-32^\circ} = \mathring{I}_5(7.19 - j\,4.5).$$

Substituting in (1)

$$\mathring{I}_5 (7.19 - j\,4.5)(-40 + j\,10) + 40\,\mathring{I}_5 + 10 = 0$$

$$\mathring{I}_5 (-288 + j\,180 + j\,71.9 + 45 + 40) + 10 = 0$$

$$\mathring{I}_5 (-203 + j\,251.9) = -10$$

$$\mathring{I}_5 = \frac{10}{203 - j\,251.9} = \frac{10}{323\underline{/-50.9^\circ}}$$

$$= 0.0308\underline{/50.9^\circ} \text{ vector ampere.} \quad \textit{Ans.}$$

Impedance of Bridge Network

3.38 Calculate the equivalent impedance of the bridge network of problem 3.37.

Solution:

Using the value found for $\mathring{I}_5$, viz.,

$$\mathring{I}_5 = 0.0308\underline{/50.9^\circ}$$

$$= 0.0194 + j\,0.0239,$$

we can write Kirchhoff's law for the upper path

$$10 = \mathring{I}_1 \mathring{Z}_1 + (\mathring{I}_1 + \mathring{I}_5)\mathring{Z}_2$$

$$10 = \mathring{I}_1 (15 + j\,20) + (\mathring{I}_1 + 0.0194 + j\,0.0239)(j\,25)$$

$$10 = \mathring{I}_1 (15 + j\,45) + j\,0.486 - 0.598$$

$$\mathring{I}_1 = \frac{10.598 - j\,0.486}{15 + j\,45}$$

$$= \frac{10.6\underline{/-2.6^\circ}}{47.5\underline{/71.6^\circ}} = 0.223\underline{/-74.2^\circ}$$

$$= 0.0603 - j\,0.215.$$

Similarly, the equation of the lower path is

$$10 = \mathring{I}_3 \mathring{Z}_3 + (\mathring{I}_3 - \mathring{I}_5)\mathring{Z}_4$$

$$10 = \mathring{I}_3 (-j\,10) + (\mathring{I}_3 - 0.0194 - j\,0.0239)\,40$$

$$10 = \mathring{I}_3 (40 - j\,10) - 0.777 - j\,0.957$$

$$\mathring{I}_3 (40 - j\,10) = 10.777 + j\,0.957$$

$$\mathring{I}_3 = \frac{10.78 + j\,0.957}{40 - j\,10}$$

$$= \frac{10.8\underline{/5.1°}}{41.2\underline{/-14°}} = 0.262\underline{/19.1°}$$

$$= 0.248 + j\,0.086$$

The total current is

$$\mathring{I}_t = \mathring{I}_1 + \mathring{I}_3 = 0.0603 - j\,0.215 + 0.248 + j\,0.086$$

$$= 0.3083 - j\,0.129 = 0.334\underline{/-22.6°}.$$

The total impedance is then found by Ohm's law

$$Z_t = \frac{10}{0.334\underline{/-22.6°}} = 29.9\underline{/22.6°}$$

$$= 27.6 + j\,11.5 \text{ vector ohms.} \quad \textit{Ans.}$$

General Solution of the Delta-Star Transformation

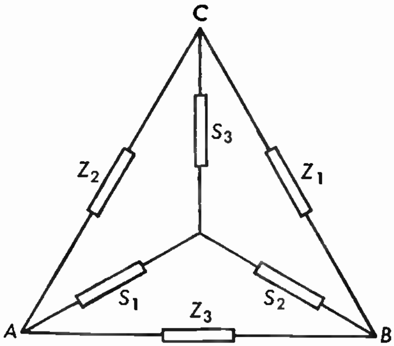

Fig. 3.39 Equivalent delta and star impedances.

3.39 Calculate the equivalent star circuit of Figure 3.39.

Solution:

Viewed from $A\ B$ the impedance of the delta circuit is Z_3 in parallel with $Z_1 + Z_2$.

By the product-sum rule

$$Z_{ab} = \frac{(Z_1 + Z_2)\,Z_3}{Z_1 + Z_2 + Z_3}$$

Viewed from A B the impedance of the star circuit is

$$Z_{ab} = S_1 + S_2$$

We therefore have the identity

$$\frac{(Z_1 + Z_2)\, Z_3}{Z_1 + Z_2 + Z_3} = S_1 + S_2. \qquad (1)$$

Similarly

$$\frac{(Z_2 + Z_3)\, Z_1}{Z_1 + Z_2 + Z_3} = S_2 + S_3, \qquad (2)$$

and

$$\frac{(Z_3 + Z_1)\, Z_2}{Z_1 + Z_2 + Z_3} = S_3 + S_1. \qquad (3)$$

Subtracting (2) − (3) we obtain

$$\frac{(Z_2 + Z_3)\, Z_1 - (Z_3 + Z_1)\, Z_2}{Z_1 + Z_2 + Z_3} = S_2 - S_1$$

$$\frac{Z_1 Z_2 + Z_1 Z_3 - Z_2 Z_3 - Z_1 Z_2}{Z_1 + Z_2 + Z_3} = S_2 - S_1$$

$$\frac{Z_1 Z_3 - Z_2 Z_3}{Z_1 + Z_2 + Z_3} = S_2 - S_1 \qquad (4)$$

Rewriting (1)

$$\frac{Z_1 Z_3 + Z_2 Z_3}{Z_1 + Z_2 + Z_3} = S_2 + S_1 \qquad (1)$$

Adding (1) + (4)

$$\frac{2\, Z_1 Z_3}{Z_1 + Z_2 + Z_3} = 2\, S_2$$

and

$$S_2 = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3};$$

likewise

$$S_1 = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3},$$

and

$$S_3 = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}.$$

Rule: Each star impedance is equal to the product of the two including delta impedances, divided by the sum of all delta impedances.

Delta-Star Transformation for Alternating Currents

3.40 Find the equivalent impedance and the total current of the bridge network of problem 3.37 with the aid of a delta-star transformation.

Solution:

The star impedance opposite $\mathring{Z}_2$ is

$$\mathring{S}_2 = \frac{(40 - j\,0)\,(100 - j\,100)}{j\,25 + 40 + 100 - j\,100}$$

$$= \frac{(40\underline{/0^\circ})\,(141\underline{/-45^\circ})}{158\underline{/-28.3^\circ}} = 35.7\underline{/-16.7^\circ}$$

$$= 34.1 - j\,10.3$$

Similarly $$\mathring{S}_4 = \frac{(25\underline{/90^\circ})\,(141\underline{/-45^\circ}}{158\underline{/-28.3^\circ}} = 22.3\underline{/73.3^\circ}$$

$$= 6.41 + j\,21.4$$

and $$\mathring{S}_5 = \frac{(25\underline{/90^\circ})\,(40\underline{/0^\circ}}{158\underline{/-28.3^\circ}} = 6.32\underline{/118.3^\circ}$$

$$= -3 + j\,5.54$$

$\mathring{S}_5$ is in series with the parallel combination consisting of the two branches $\mathring{Z}_a$ and $\mathring{Z}_b$, where

$$\mathring{Z}_a = \mathring{Z}_1 + \mathring{S}_4 = 21.4 + j\,41.4 = 46.6\underline{/62.6^\circ}$$

and $$\mathring{Z}_b = \mathring{Z}_3 + \mathring{S}_2 = 34.1 - j\,20.3 = 40\underline{/-30.7^\circ}$$

$$\mathring{Z}_a + \mathring{Z}_b = 55.5 + j\,21.1$$

$$= 59.5\underline{/20.8^\circ}$$

the parallel combination is

$$\mathring{Z}_p = \frac{\mathring{Z}_a\,\mathring{Z}_b}{\mathring{Z}_a + \mathring{Z}_b}$$

$$= \frac{(46.6\underline{/62.6^\circ})\,(40\underline{/-30.7^\circ})}{59.5\underline{/20.8^\circ}}$$

$$= 31.4\underline{/11.1^\circ} = 30.8 + j\,6.05.$$

The total impedance is

$$\mathring{Z}_t = \mathring{Z}_p + \mathring{S}_5$$
$$= 27.8 + j\,11.59$$
$$= 29.9\underline{/22.6°} \text{ vector ohms.} \quad \textit{Ans.}$$

$$\mathring{I}_t = \frac{10}{29.9\underline{/22.6°}}$$
$$= 0.334\underline{/-22.6°} \text{ vector amperes.} \quad \textit{Ans.}$$

Thévenin's Theorem for Alternating Currents

3.41 Solve problem 3.37 with the aid of Thévenin's theorem.

Solution:

Let $\mathring{Z}_5$ be the load, and A and B the terminals of the Thévenin generator. The computation of the equivalent internal impedance $\mathring{Z}_i$ and the equivalent electromotive force $\mathring{E}_{ab}$ follows closely the one outlined in problem 2.26, except that the resistances are replaced by complex impedances.

With the electromotive force short-circuited, $\mathring{Z}_i$, as seen from $A\,B$, consists of the two parallel combinations of $\mathring{Z}_1$ and $\mathring{Z}_2$, plus $\mathring{Z}_3$ and $\mathring{Z}_4$.

$$\mathring{Z}_{1,2} = \frac{\mathring{Z}_1\mathring{Z}_2}{\mathring{Z}_1 + \mathring{Z}_2} = \frac{(15 + j\,20)\,(0 + j\,25)}{15 + j\,45}$$
$$= \frac{(25\underline{/53.1°})\,(25\underline{/90°})}{47.5\underline{/71.5°}} = 13.13\underline{/71.6°}$$
$$= 4.15 + j\,12.5$$
$$\mathring{Z}_{3,4} = \frac{(0 - j\,10)\,(40 + j\,0)}{40 - j\,10} = \frac{(10\underline{/-90°})\,(40\underline{/0°})}{41.4\underline{/-14°}}$$
$$= 9.67\underline{/-76°} = 2.34 - j\,9.33$$
$$\mathring{Z}_i = \mathring{Z}_{1,2} + \mathring{Z}_{3,4} = 4.15 + j\,12.5 + 2.34 - j\,9.33$$
$$= 6.49 + j\,3.17$$

This internal generator impedance is in series with the load $\mathring{Z}_5$, which adds up to a total impedance of

$$\mathring{Z}_t = \mathring{Z}_i + \mathring{Z}_5 = 6.49 + j\,3.17 + 100 - j\,100$$
$$= 106.5 + j\,96.8 = 144\underline{/-42.2°}$$

To find the equivalent electromotive force $\mathring{E}_{ab}$, the voltage drop across $\mathring{Z}_4$ and $\mathring{Z}_2$ must be calculated.

By the potentiometer rule

$$\mathring{E}_4 = 10 \frac{\mathring{Z}_4}{\mathring{Z}_3 + \mathring{Z}_4} = 10 \frac{40}{40 - j\,10}$$

$$= \frac{400}{41.4\underline{/-14.05^\circ}} = 9.67\underline{/14.05^\circ}$$

$$= 9.33 + j\,2.34$$

$$\mathring{E}_2 = 10 \frac{\mathring{Z}_2}{\mathring{Z}_1 + \mathring{Z}_2} = 10 \frac{j\,25}{15 + j\,45}$$

$$= \frac{250\underline{/90^\circ}}{47.5\underline{/71.5^\circ}} = 5.27\underline{/18.5^\circ}$$

$$= 5 + j\,1.67.$$

The potential difference is

$$\mathring{E}_{ab} = \mathring{E}_4 - \mathring{E}_2 = 4.33 + j\,0.67$$

$$= 4.43\underline{/8.7^\circ}$$

The current through $\mathring{Z}_5$ is therefore

$$\mathring{I}_5 = \frac{\mathring{E}_{ab}}{\mathring{Z}_t} = \frac{4.43\underline{/8.7^\circ}}{144\underline{/-42.2^\circ}}$$

$$= 0.0308\underline{/50.9^\circ} \text{ vector ampere.} \quad \textit{Ans.}$$

M-Derived Filter

3.42 To obtain sharper attenuation in the cutoff region of the filter of problem 3.34 an inductance L_2 is inserted in series with the shunt-arm capacitance which is $(1 - m^2)/4\,m$ times the inductance L of problem 3.34. The other filter components change as follows: the series-arm inductance becomes $L_1' = m\,L$, the shunt-arm capacitance becomes $C' = m\,C$ where $m = \sqrt{1 - (f_c/f_\infty)^2}$, f_c is the cutoff frequency of the prototype filter (of problem 3.34), f_∞ is the frequency of very high attenuation of the m-derived filter. If $f_\infty = 11{,}000$ cycles, find the components of the m-derived filter.

Solution:

The value of m is

$$m = \sqrt{1 - (f_c/f_\infty)^2}$$
$$= \sqrt{1 - (0.909)^2} = \sqrt{1 - 0.826}$$
$$= \sqrt{0.174} = 0.417$$

The new series inductance is

$$L_1' = m\,L = 0.417 \times 0.255 = 0.106 \text{ henry.} \quad \textit{Ans.}$$

The new shunt capacitance is

$$C' = m\,C = 0.417 \times 0.00398$$
$$= 0.00165 \text{ microfarad} = 1650 \text{ micromicrofarads.} \quad \textit{Ans.}$$

The shunt-arm inductance is

$$L_2' = \frac{1 - m^2}{4\,m} L$$
$$= \frac{1 - 0.174}{4 \times 0.417} \times 0.255$$
$$= \frac{0.826}{1.668} \times 0.255 = 0.1261 \text{ henry.} \quad \textit{Ans.}$$

4 Vacuum-Tube Fundamentals

Electron Emission

4.01 Calculate the thermionic emission of an oxide-coated cathode for a temperature of 1200K, with material constants $A = 0.005$ and $b = 11{,}750$.

Solution:

Using Dushman's equation

$$I = A\,T^2\,\epsilon^{-\frac{b}{T}},$$

we obtain
$$I = 0.005 \times (1200)^2 \times \epsilon^{-\frac{11750}{1200}}$$
$$= 5 \times 10^{-3} \times 1.44 \times 10^6 \times \epsilon^{-9.79}$$

The last factor, unless special means are available, is found with the aid of logarithm tables or the slide rule *L*-scale.

$$\log \epsilon^{-9.79} = -9.79 \times \log \epsilon$$
$$= -9.79 \times 0.4343$$
$$= -4.252$$
$$= 0.748 - 5$$
$$\epsilon^{-9.79} = \text{antilog } (0.748 - 5)$$
$$= 5.6 \times 10^{-5}.$$

Substituting:

$$I = 5 \times 10^{-3} \times 1.44 \times 10^6 \times 5.6 \times 10^{-5}$$
$$= 40.4 \times 10^{-2} = 404 \times 10^{-3}$$
$$= 404 \text{ milliamperes per square centimeter.} \quad \textit{Ans.}$$

Two-Electrode Tube Emission

4.02 The following readings were observed in a space-charge-limited 2-electrode tube:

$$I = 70 \text{ milliamperes}, \; E_p = 82 \text{ volts.}$$

What current can be expected when a plate voltage of 100 volts is applied?

Solution:

Substituting in the emission equation (formula 4.12)

$$I = K E_p^{\frac{3}{2}}$$

we obtain $70 \times 10^{-3} = K \times 82^{\frac{3}{2}}$, which solved for K, yields

$$K = \frac{70 \times 10^{-3}}{\sqrt[2]{82^3}}$$

$$= \frac{70 \times 10^{-3}}{\sqrt[2]{551{,}000}} = \frac{70 \times 10^{-3}}{742}$$

$$= 0.0943 \times 10^{-3} = 9.43 \times 10^{-5}.$$

At a plate voltage of 100 volts the current to be expected is

$$I = 9.43 \times 10^{-5} \times \sqrt[2]{100^3}$$

$$= 9.43 \times 10^{-5} \times 10^3 = 9.43 \times 10^{-2}$$

$$= 94.3 \times 10^{-3} = 94.3 \text{ milliamperes.} \quad \textit{Ans.}$$

Triode Plate Current

4.03 The triode type 6P5 has the following typical operation data obtained from the information of the manufacturer:

plate voltage	100 volts
plate current	2.5 milliamperes
grid voltage	−5 volts
amplification factor	13.8

If the plate voltage were increased to 150 volts, what plate current could be expected?

Solution:

We shall use the equation for space current in triodes (formula 4.13)

$I_p + I_g = K\left(E_g + \frac{E_p}{\mu}\right)^{\frac{3}{2}}$ to find K. Since E_g is negative, making $I_g = 0$

we obtain $$2.5 \times 10^{-3} = K\left(-5 + \frac{100}{13.8}\right)^{\frac{3}{2}}$$

and $$K = \frac{2.5 \times 10^{-3}}{\sqrt[2]{(7.24 - 5)^3}}$$

$$= \frac{2.5 \times 10^{-3}}{\sqrt[2]{2.24^3}} = \frac{2.5 \times 10^{-3}}{\sqrt{11.3}}$$

$$= \frac{2.5 \times 10^{-3}}{3.36} = 0.744 \times 10^{-3}$$

$$= 7.44 \times 10^{-4}$$

Using this value of K and the formula 4.13 we obtain

$$I_p = 7.44 \times 10^{-4}\left(-5 + \frac{150}{13.8}\right)^{\frac{3}{2}}$$

$$= 7.44 \times 10^{-4}\,(-5 + 10.85)^{\frac{3}{2}}$$

$$= 7.44 \times 10^{-4}\,\sqrt{5.85^3}$$

$$= 7.44 \times 10^{-4}\,\sqrt{200}$$

$$= 7.44 \times 10^{-4} \times 14.1 = 105 \times 10^{-4}$$

$= 10.5$ milliamperes. *Ans.*

(A value little different from this can be read from an experimentally determined family of curves for the 6P5 tube.)

Amplification Factor

4.04 The circuit of Figure 4.04 is used to examine the tube T. It is found that with a grid voltage of -6 volts and a plate voltage of 211 volts, the milliammeter reads 8 milliamperes; an increase of the plate voltage to 250 volts causes a rise of the plate current to 12.3 milliamperes. The plate voltage is then decreased to its original value and the grid bias is changed to −4 volts. The milliammeter again reads 12.3 milliamperes. What is the amplification factor of the tube?

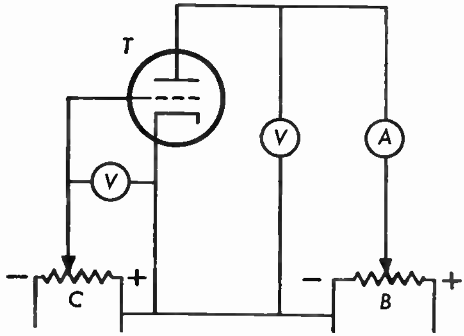

Fig. 4.04 Experimental circuit for determining triode curves.

Solution:

$$\mu = \frac{\Delta E_p}{\Delta E_g} = \frac{250 - 211}{6 - 4} = \frac{39}{2} = 19.5. \quad \textit{Ans.}$$

Plate-Resistance

4.05 What is the plate resistance of the tube in problem 4.04?

Solution:

$$r_p = \frac{\Delta E_p}{\Delta I_p} = \frac{250 - 211}{(12.3 - 8)\ 10^{-3}} = \frac{39}{4.3} \times 10^3 = 9100 \text{ ohms.} \quad \textit{Ans.}$$

Transconductance

4.06 In problem 4.04 what mutual conductance would be indicated by the readings taken?

Solution:

$$g_m = \frac{\Delta I_p}{\Delta E_g}$$

$$= \frac{(12.3 - 8) \times 10^{-3}}{6 - 4}$$

$$= \frac{4.3 \times 10^{-3}}{2} = 2.15 \times 10^{-3} \text{ mho}$$

$$= 2150 \text{ micromhos.} \quad \textit{Ans.}$$

Also, from the values calculated in problem 4.04 and 4.05 and using formula 4.23

$$g_m = \frac{\mu}{r_p} = \frac{19.5}{9100} = 0.00215 \text{ mho}$$

$$= 2150 \text{ micromhos}$$

Determining Tube Constants

4.07 The following data were observed in a laboratory test:

E_p volts	E_g volts	I_p milliamperes
250	−8	8
215	−6	8
250	−7	10

Find the approximate value of μ, r_p, and g_m at the point where $E_p = 250$ volts and $E_g = -8$ volts.

Solution:

$$\mu = \frac{\Delta E_p}{\Delta E_g} = \frac{250 - 215}{8 - 6} = \frac{35}{2} = 17.5. \quad \textit{Ans.}$$

$$g_m = \frac{\Delta I_p}{\Delta E_g} = \frac{2 \times 10^{-3}}{1} = 2000 \text{ micromhos.} \quad \textit{Ans.}$$

$$r_p = \frac{\mu}{g_m} = \frac{17.5}{2 \times 10^{-3}} = 8750 \text{ ohms.} \quad \textit{Ans.}$$

Output Voltage

4.08 An input voltage of 2 volts peak is impressed on the grid of the tube in problem 4.04. If a plate resistor of 5000 ohms is used in the plate circuit, what is the output voltage?

Solution:

The equivalent circuit is shown in Figure 4.08.

The output voltage is that part of the voltage μE_g which exists across R_p in the voltage divider $r_p + R_p$.

$$E_o = \mu E_g \frac{R_p}{r_p + R_p}$$

$$= \frac{19.5 \times 2 \times 5000}{9100 + 5000} = 13.8 \text{ volts.} \quad \textit{Ans.}$$

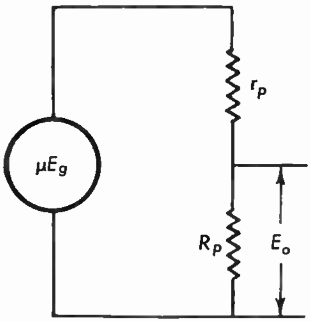

Fig. 4.08 Equivalent circuit of triode with resistive load.

Voltage Amplification

4.09 What is the voltage amplification in problem 4.08?

Solution:

The voltage amplification is the ratio of the output voltage E_o to the input voltage E_i.

$$G = \frac{E_o}{E_i} = \frac{13.8}{2} = 6.9. \quad \textit{Ans.}$$

Increasing the Load Resistance

4.10 What would be the voltage amplification in problem 4.08 if the plate resistor were 50,000 ohms?

Solution:

$$G = \frac{\mu R_p}{r_p + R_p} = \frac{19.5 \times 5 \times 10^4}{(9.1 \times 10^3) + (50 \times 10^3)}$$

$$= \frac{19.5 \times 5 \times 10}{59.1} = 16.5. \quad \textit{Ans.}$$

Load Line

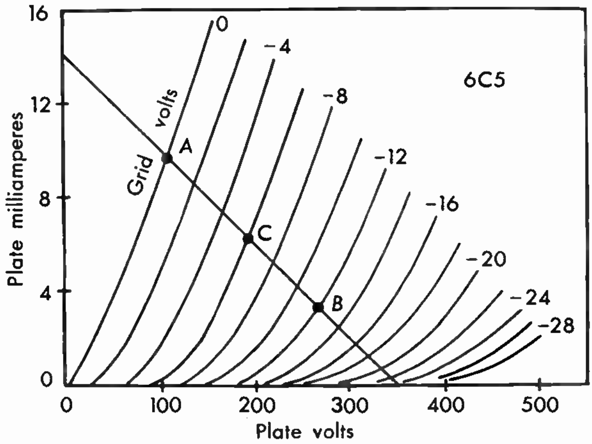

Fig. 4.11 Plate voltage-plate current characteristics of the triode type 6C5.

4.11 Using the $E_p - I_p$ characteristics of the type 6C5 (Figure 4.11), find the x and y intercepts of the load line for a plate load of $R = 25{,}000$ ohms and plot the load line by joining these points. The supply voltage is 350 volts.

Solution:

The load line is the graph of the straight-line equation

$$e = E_o - iR,$$

which is the mathematical statement of the fact that the plate voltage e is smaller than the $B+$ voltage E_o by the voltage drop iR across the plate resistor.

Now at $i = 0$,

$$e = E_o = 350 \text{ volts } (x\text{-intercept}). \quad \textit{Ans.}$$

At $e = 0$,

$$i = \frac{E_o}{R} = \frac{350}{25{,}000} = 0.014$$

$$= 14 \text{ milliamperes } (y\text{-intercept}). \quad \textit{Ans.}$$

No-Signal Plate Voltage and Plate Current

4.12 In problem 4.11 we decide to choose −6-volt bias as the operating point; what is the plate voltage at zero signal? What is the plate current at zero signal?

Solution:

The co-ordinates of the operating point, i.e., the point of intersection of the load line with the curve $E_g = -6$ is P (195, 6.4).

Therefore

$$\text{the plate voltage} = 195 \text{ volts}$$
$$\text{and the plate current} = 6.4 \text{ milliamperes.} \quad \textit{Ans.}$$

Maximum and Minimum Plate Voltage

4.13 Assuming a signal of 2 volts peak at the grid in problem 4.11, what would be the maximum and minimum values of the plate voltage?

Solution:

The projection of the respective points of intersection of the load line with the curves $E_g = -8$ and $E_g = -4$ upon the x-axis yields

$$E_{min} = 170 \text{ volts}, \quad E_{max} = 220 \text{ volts.} \quad \textit{Ans.}$$

Plate Voltage

FCC Study Guide Question 6.133

4.14 What is the d-c plate voltage of a resistance-coupled amplifier stage which has a plate-supply voltage of 260 volts, a plate current of 1 milliampere, and a plate-load resistance of 100,000 ohms?

Solution:

$$\begin{aligned} E_p &= E_B - I\,R \\ &= 260 - 10^{-3} \times 10^5 \\ &= 260 - 100 = 160 \text{ volts.} \quad \textit{Ans.} \end{aligned}$$

Grid Bias

FCC Study Guide Question 6.135

4.15 In a radio-frequency amplifier stage having a plate voltage of 1250 volts, a plate current of 150 milliamperes, a grid current of

15 milliamperes, and a grid leak resistance of 4000 ohms, what is the value of the operating grid bias?

Solution:

The only values used are the grid current and the grid leak resistance.

$$E = I\,R = 15 \times 10^{-3} \times 4 \times 10^{3} = 60 \text{ volts.} \quad \textit{Ans.}$$

Conversion Calculations for Beam-Power Tube

4.16 From the tube manual the following data are obtained for the typical operation of the beam power tube type 6L6: plate voltage 350 volts, screen voltage 250 volts, grid voltage −18 volts, plate current 54 milliamperes, screen current 2.5 milliamperes, maximum signal power output 10.8 watts, plate resistance 33,000 ohms, transconductance 5200 micromhos, load resistance 4200 ohms; if only 245 volts plate voltage are available, what should be the other operating data?

Solution:

The conversion factor of the plate voltage is

$$F_e = \frac{245}{350} = 0.7$$

The new bias and the new screen voltage will be proportional to the new plate voltage

$$E'_{sc} = 0.7 \times 250 = 175 \text{ volts.} \quad \textit{Ans.}$$

$$E'_g = -18 \times 0.7 = -12.6 \text{ volts.} \quad \textit{Ans.}$$

The current conversion factor can be found by using the fundamental law of vacuum tubes that $I_p \propto E_p^{\frac{3}{2}}$; therefore the current conversion factor is

$$F_i = F_e^{\frac{3}{2}} = F_e\,F_e^{\frac{1}{2}} = 0.7\sqrt{0.7} = 0.585$$

$$I'_p = 54 \times 0.585 = 31.6 \text{ milliamperes.} \quad \textit{Ans.}$$

$$I'_{sc} = 2.5 \times 0.585 = 1.46 \text{ milliamperes.} \quad \textit{Ans.}$$

The resistor conversion factor is found by (Ohm's law)

$$F_r = \frac{F_e}{F_i} = \frac{0.7}{0.585} = 1.2$$

Thus $r_p = 33{,}000 \times 1.2 = 39{,}500$ ohms

$R_p = 4200 \times 1.2 = 5000$ ohms

The mutual conductance conversion factor

$$F_{gm} = \frac{F_i}{F_e} = \frac{1}{F_r} = 0.835$$

$$g'_m = 5200 \times 0.835 = 4350 \text{ micromhos.} \quad \textit{Ans.}$$

The power conversion factor is found by

$$F_p = F_e \times F_i = 0.7 \times 0.585 = 0.41$$

$$P' = 10.8 \times 0.41 = 4.4 \text{ watts.} \quad \textit{Ans.}$$

Note: The RCA tube manual contains a conversion chart upon which the preceding discussion is based.

Conversion Calculations for Triode

4.17 From the tube manual the following data are taken for the triode type 6C5: plate voltage = 250 volts, grid voltage = −8 volts, plate current = 8 milliamperes, plate resistance = 10,000 ohms, amplification factor = 20, transconductance = 2000 micromhos. Calculate the operating data for a plate voltage (E'_p) = 150 volts.

Solution:

$$F_e = \frac{150}{250} = 0.6$$

$$E'_g = -8 \times 0.6 = -4.8 \text{ volts.} \quad \textit{Ans.}$$

$$F_i = 0.6 \times \sqrt{0.6} = 0.465$$

$$I'_p = 8 \times 0.465 = 3.72 \text{ milliamperes.} \quad \textit{Ans.}$$

$$F_r = \frac{0.6}{0.465} = 1.29$$

$$r'_p = 10{,}000 \times 1.29 = 12{,}900 \text{ ohms.} \quad \textit{Ans.}$$

$$F_{gm} = \frac{1}{1.29} = 0.775$$

$$g'_m = 2000 \times 0.775 = 1550 \text{ micromhos.} \quad \textit{Ans.}$$

Check: $\mu' = r'_p \times g'_m = 12{,}900 \times 1550 \times 10^{-6} = 20$

Triode Power Output

4.18 The power amplifier tube type 801 has a transconductance of 1600 micromhos, a plate resistance of 5000 ohms, and a load resistance of 12,000 ohms. What is the power output at a peak signal input of 35 volts?

Solution:

The power output is

$$P = I^2 R_p$$

$$= \left[\frac{\mu E_g}{r + R_p}\right]^2 R_p,$$

and

$$\mu = g_m \times r = 1600 \times 10^{-6} \times 5000$$

$$= 1.6 \times 5 = 8.$$

The rms signal input is

$$E_g = \frac{E_g\, peak}{\sqrt{2}} = \frac{35}{\sqrt{2}}.$$

Substituting

$$P = \frac{8^2 \times 35^2 \times 12{,}000}{17{,}000^2 \times 2}$$

$$= \frac{64 \times 1220 \times 12{,}000}{289{,}000{,}000 \times 2} = 1.63 \text{ watts.} \quad \textit{Ans.}$$

Pentode Power Output

4.19 The power pentode type 6K6 operates at 250 volts plate voltage, and 30 milliamperes plate current. What is the approximate power output?

Solution:

Using the fact that the plate efficiency of a pentode is approximately 33 per cent when operated as a class A amplifier, we obtain

$$P_{in} = 250 \times 30 \times 10^{-3} = 7.5 \text{ watts},$$

and

$$P_{out} = 7.5 \times 0.33 = 2.5 \text{ watts, approximately.} \quad \textit{Ans.}$$

Maximum Power Output

4.20 In problem 4.18, what should be the value of the plate load resistor for maximum power output and what would be the maximum power output in watts?

Solution:

The value of the plate load resistor should be

$$R_p = r = 5000 \text{ ohms.} \quad \textit{Ans.}$$

The maximum power output will be

$$P_{max} = \frac{(\mu E_g)^2}{(r + r)^2} \times r$$

$$= \frac{(\mu E_g)^2}{4r}$$

$$= \frac{(8 \times 35)^2}{20{,}000 \times 2} = 1.87 \text{ watts.} \quad \textit{Ans.}$$

Undistorted Power Output

4.21 In problem 4.18, what should be the value of the plate load resistor for maximum undistorted power and what will be the power output in this case?

Solution:

The value of the plate load resistor should be

$$R_p = 2r = 10{,}000 \text{ ohms}$$

$$P_{undist.} = \frac{(\mu E_g)^2}{(2r + r)^2} \times 2r$$

$$= \frac{(\mu E_g)^2}{9r^2} \times 2r = \frac{2(\mu E_g)^2}{9r}$$

$$= \frac{2(8 \times 35)^2}{45{,}000 \times 2} = 1.74 \text{ watts.} \quad \textit{Ans.}$$

Plate Efficiency

4.22 The beam power tube type 6V6 has an a-c power output of 4.5 watts while the plate current is 47 milliamperes and the plate voltage 250 volts. What is the plate efficiency?

Solution:

The plate efficiency of a power amplifier tube is the ratio of the a-c power output to the product of the plate direct current times the plate direct voltage at full signal.

Therefore
$$\eta = \frac{4.5}{250 \times 47 \times 10^{-3}}$$
$$= \frac{4.5}{2.5 \times 4.7}$$
$$= \frac{4.5}{11.75} = 0.383 = 38.3 \text{ per cent.} \quad \textit{Ans.}$$

Approximate Load Resistance

4.23 The plate voltage of the power pentode tube 6F6, operated as single-tube class A amplifier is 285 volts while the average plate current is 38 milliamperes. How could the approximate value of the load resistance be found for the purpose of replacing the output transformer?

Solution:

The approximate value of the load resistor can be found from the formula

$$R_L = \frac{E_p}{I_o}$$
$$= \frac{285}{38 \times 10^{-3}} = 7500 \text{ ohms.}$$

The critical reader may wonder how an approximation such as this can be justified. The correct method of calculating the load resistance, if enough information were available, would be to use

$$R = \frac{E_{max} - E_{min}}{I_{max} - I_{min}} \quad \text{(Figure 4.23)}$$

However, in the case of a pentode operated with minimum distortion, the triangle ABC will be nearly similar to the triangle $I_o\, E_p\, O$.

Therefore
$$\frac{E_{max} - E_{min}}{I_{max} - I_{min}} \cong \frac{E_p}{I_o},$$

making the load line l nearly parallel to l'. This would not be true for the highly distorted line d. The line of minimum distortion is found by rotating a ruler marked with equal distances about the operating point P until the distances PA and PB are as nearly equal as possible.

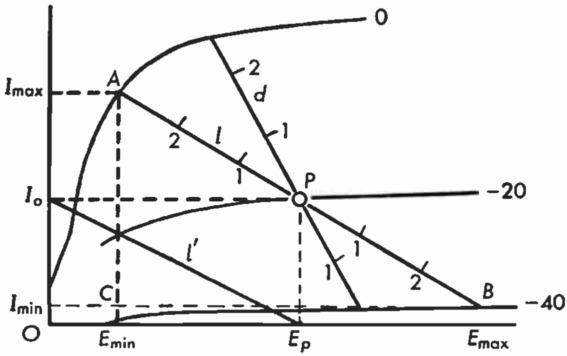

Fig. 4.23 Plate voltage-plate current characteristics of the pentode type 6F6. A scale intersecting the −20 volts grid bias curve will have a nearly equal swing of approximately 3 units between the 0 and the −40 curve, indicating a small amount of distortion.

Plate Efficiency

FCC Study Guide Question 4.93

4.24 The direct-current input power to the final amplifier stage is exactly 1500 volts and 700 milliamperes. The antenna resistance is 8.2 ohms and the antenna current is 9 amperes. What is the plate efficiency of the final amplifier?

Solution:

The efficiency is the ratio of the output power to the input power expressed in per cent

$$\eta = \frac{P_o}{P_i}.$$

The output power is determined by the antenna resistance and the antenna current,

$$P_o = I^2 R = 9^2 \times 8.2$$
$$= 81 \times 8.2 = 664 \text{ watts.}$$

The input power is

$$P_i = E_{d\text{-}c} \times I_{d\text{-}c}$$
$$= 1500 \times 700 \times 10^{-3} = 1.5 \times 700$$
$$= 1050 \text{ watts.}$$

Thus the efficiency is

$$\eta = \frac{P_o}{P_i} = \frac{664}{1050} = 0.633 = 63.3 \text{ per cent.} \quad \textit{Ans.}$$

Stage Amplification

FCC Study Guide Question 4.173

4.25 What is the stage amplification obtained with a single triode operating with the following constants:

plate voltage	250	volts
plate current	20	milliamperes
plate impedance	5000	ohms
load impedance	10,000	ohms
grid bias	5.4	volts
amplification factor	24	

Solution:

Figure 4.25 represents the equivalent circuit, where r_p is the plate impedance, Z_p the load impedance.

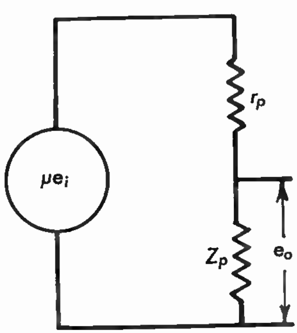

Fig. 4.25 Equivalent triode circuit.

The output voltage e_o as it appears across Z_p is (formula 2.26)

$$e_o = \mu \, e_i \frac{Z_p}{Z_p + r_p},$$

and the stage amplification or voltage amplification is

$$G = \frac{e_o}{e_i} = \frac{\mu Z_p}{Z_p + r_p}$$

$$= \frac{24 \times 10{,}000}{10{,}000 + 5000} = 16. \quad \textit{Ans.}$$

5 *Amplifiers*

Medium-Frequency Response

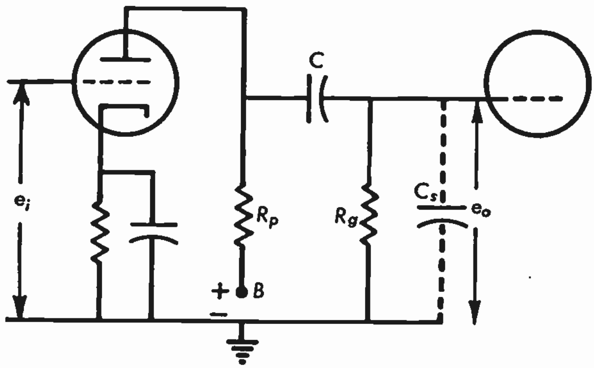

Fig. 5.01a Resistance-capacitance coupled amplifier stage.

5.01 In the resistance-coupled amplifier stage of Figure 5.01a, $R_p = 250{,}000$ ohms, $C = 0.01$ microfarad, $R_g = 1$ megohm, the effective shunt capacitance $C_s = 75$ micromicrofarads, the plate resistance $r_p = 240{,}000$ ohms, and the amplification factor $\mu = 65$. Calculate the voltage amplification at a frequency of 1000 cycles. Do not use formulas. Neglect the cathode resistor and capacitor.

Solution:

At this frequency the reactance of the coupling capacitor is

$$X_c = 1/(2\pi f C) = 15{,}900 \text{ ohms}$$

and the reactance of the shunt capacitance is

$$X_s = 1/(2\pi f C_s) = 2{,}120{,}000 \text{ ohms.}$$

The reactance of the coupling capacitor is so small compared with the value of R_g that the voltages appearing across R_p and R_g are practically equal. The load thus consists of a 250,000-ohm and a 1-megohm resistor in parallel. Compared with this parallel combination the reactance of C_s is too big to have any shunt effect. The equivalent circuit therefore is that shown in Figure 5.01b:

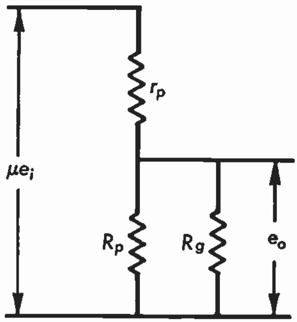

Fig. 5.01b Equivalent circuit of Fig. 5.06a for medium frequencies (approx. 1000 cycles).

The grid signal input e_i appears as a generator voltage μe_i across $r_p + R_p$. The series-parallel combination acts as a voltage divider. The output voltage e_o is that part of the voltage μe_i which appears across the parallel resistors.

$$e_o = \mu e_i \times \frac{\text{parallel resistance}}{\text{total resistance}}$$

The value of the parallel resistance is

$$\frac{250{,}000 \times 1{,}000{,}000}{250{,}000 + 1{,}000{,}000} = 200{,}000 \text{ ohms.}$$

The value of the total resistance is

$$200{,}000 + 240{,}000 = 440{,}000 \text{ ohms.}$$

Substituting

$$e_o = \mu e_i \times \frac{200{,}000}{440{,}000} = \mu e_i \times 0.455,$$

and the voltage amplification is

$$G = \frac{e_o}{e_i} = \mu \times 0.455$$

$$= 65 \times 0.455 = 29.6. \quad \textit{Ans.}$$

High-Frequency Response

5.02 Calculate the voltage amplification of the resistance-coupled amplifier stage of Figure 5.01a, problem 5.01, at a frequency of 11,800 cycles.

Solution:

At this frequency the reactance of the coupling capacitor has become so small that it can be considered a short circuit. However, the shunt capacitance has a reactance of

$$X_s = \frac{1}{2\,\pi\, f\, C_s} = 180{,}000 \text{ ohms}$$

which is comparable to the plate resistor. Figure 5.02 is the equivalent circuit.

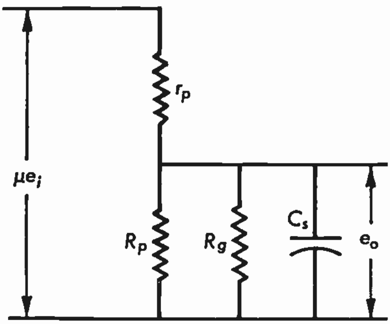

Fig. 5.02 Equivalent circuit of Fig. 5.01a for high frequencies (approx. 10,000 cycles).

$$e_o = \mu\, e_i \times \frac{Z\ parallel}{Z\ total}$$

Z_p consists of two parallel resistors with an equivalent resistance of 200,000 ohms shunted by the reactance

$$\mathring{X} = -j\,\frac{1}{2\,\pi\, f\, C_s}$$

$$\mathring{Z}_p = \frac{200{,}000 \times \mathring{X}}{200{,}000 + \mathring{X}}$$

$$\mathring{Z}_t = 240{,}000 + \frac{200{,}000\,\mathring{X}}{200{,}000 + \mathring{X}}$$

$$= \frac{4.8 \times 10^{10} + 240{,}000\,\mathring{X} + 200{,}000\,\mathring{X}}{200{,}000 + \mathring{X}}$$

$$= \frac{4.8 \times 10^{10} + 440{,}000\,\mathring{X}}{200{,}000 + \mathring{X}}$$

$$\frac{\mathring{Z}_p}{\mathring{Z}_t} = \frac{200{,}000\,\mathring{X}}{200{,}000 + \mathring{X}} \div \frac{4.8 \times 10^{10} + 440{,}000\,\mathring{X}}{200{,}000 + \mathring{X}}$$

$$= \frac{200{,}000\,\mathring{X}}{4.8 \times 10^{10} + 440{,}000\,\mathring{X}}$$

$$= \frac{2\,\mathring{X}}{4.8 \times 10^{5} + 4.4\,\mathring{X}}$$

This fraction is a complex quantity. The absolute value is

$$\left|\frac{Z_p}{Z_t}\right| = \frac{2 \times 180{,}000}{\sqrt{(4.8 \times 10^5)^2 + (4.4 \times 180{,}000)^2}}$$

$$= \frac{360{,}000}{\sqrt{10^{10}\,(23 + 19.4 \times 3.25)}}$$

$$= \frac{3.6 \times 10^5}{10^5 \sqrt{86}}$$

$$= \frac{3.6}{9.26} = 0.388$$

$$e_o = \mu\, e_i \times 0.388,$$

and the voltage amplification is

$$G = \frac{e_o}{e_i} = 65 \times 0.388 = 25.2. \quad \textit{Ans.}$$

Low-Frequency Response

5.03 Calculate the voltage amplification of the circuit of Figure 5.01, problem 5.01, at 30 cycles.

Solution:

At this frequency the reactance of the shunt capacitance is too big to be considered. However, the reactance of the coupling capacitor $X = 1/(2\pi f C) = 531{,}000$ ohms, which is comparable to the resistance of the circuit. The equivalent circuit is represented by Figure 5.03.

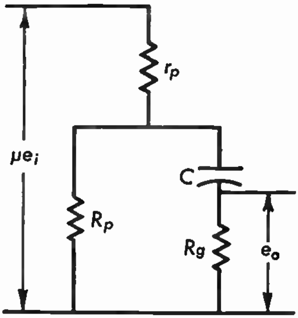

Fig. 5.03 Equivalent circuit of Fig. 5.01a for low frequencies (approx. 50 cycles).

The voltage across the parallel combination is

$$e_p = \mu e_i \frac{Z\ parallel}{Z\ total}$$

The impedance of the parallel combination, in kilohms, is

$$\mathring{Z}_p = \frac{250 \times (1000 - j\,531)}{250 + (1000 - j\,531)}$$

$$= 250 \times \frac{1000 - j\,531}{1250 - j\,531}$$

$$= 250 \times \frac{1 - j\,0.531}{1.25 - j\,0.531}$$

$$= 250 \times \frac{(1 - j\,0.531)\,(1.25 + j\,0.531)}{1.25^2 + 0.531^2}$$

$$= 250 \times \frac{1.25 - j\,0.664 + j\,0.531 + 0.282)}{1.57 + 0.282}$$

$$= 250 \times \frac{1.53 - j\,0.133}{1.85}$$

$$= 135 \times (1.53 - j\,0.133) = 207 - j\,18 \text{ kilohms.}$$

This impedance is practically a pure resistance of 207 kilohms. The total impedance is

$$Z_t = 207 + 240 = 447 \text{ kilohms.}$$

Substituting in the first equation

$$e_p = \mu\, e_i \times \frac{207}{447} = 65 \times e_i \times \frac{207}{447} = 30.1\, e_i$$

The output voltage across the 1-megohm resistor is (using megohms)

$$\begin{aligned} e_o &= e_p \times \frac{R_g}{\sqrt{R_g^2 + X^2}} \\ &= e_p \times \frac{1}{\sqrt{1^2 + 0.531^2}} \\ &= e_p \times \frac{1}{1.132} = 0.882\, e_p. \end{aligned}$$

Substituting the value found for e_p

$$e_o = 30.1\, e_i \times 0.882 = 26.6\, e_i\,,$$

and the voltage amplification is

$$G = \frac{e_o}{e_i} = 26.6. \quad \textit{Ans.}$$

General Solution of the Medium-Frequency Response

5.04 Derive a formula for the frequency response of a resistance-coupled amplifier at medium frequencies.

Solution:

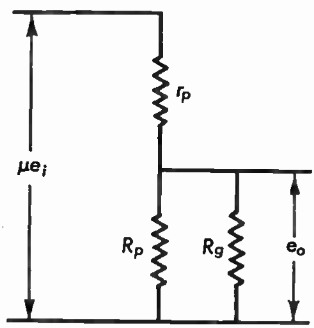

Fig. 5.04 Resistance-capacitance coupled amplifier at medium frequencies.

As stated in problem 5.01, the reactance of the coupling capacitor and of the shunt capacitance can be neglected. Figure 5.04 represents the equivalent circuit for medium frequencies.

Let R_{pg} be the value of the plate resistor and the grid resistor in parallel, then the output voltage as it appears across R_{pg} is

$$e_o = \mu\, e_i \times \frac{R_{pg}}{R_{pg} + r_p}.$$

The voltage amplification is obtained by transposing e_i

$$G_m = \frac{e_o}{e_i} = \frac{\mu\, R_{pg}}{R_{pg} + r_p}$$

$$= \frac{\mu\, R_{pg}}{R_{pg} + r_p} \times \frac{r_p}{r_p}\text{; since } \frac{\mu}{r_p} = g_m$$

$$= g_m \frac{R_{pg}\, r_p}{R_{pg} + r_p}.$$

The fraction in the above expression, being the product divided by the sum of two resistors, is the value of R_p, R_g and r_p, all connected in parallel. Let this term be R', then the frequency response at medium frequencies is

$$G_m = g_m\, R',$$

where G_m is the ratio of the output voltage to the input voltage,

g_m = the transconductance of the tube, and

R' = the plate resistance, the load resistor and the grid resistor in parallel. *Ans.*

Medium-Frequency Response Using the Formula for Voltage Amplification at Medium Frequencies

5.05 Using the formula derived in problem 5.04 calculate the medium frequency response of the resistance coupled amplifier stage of Figure 5.01a, problem 5.01.

Solution:

$$G_m = g_m\, R'$$

$$g_m = \frac{\mu}{r_p} = \frac{65}{2.4 \times 10^5}$$

$$= 27 \times 10^{-6} = 270 \text{ micromhos.}$$

By formula 1.173

$$R' = \frac{1,000,000}{1 + (1,000,000/240,000) + (1,000,000/250,000)}$$

$$= \frac{1,000,000}{1 + 4.17 + 4}$$

$$= 109,100 \text{ ohms.}$$

Substituting

$$G_m = 270 \times 10^{-6} \times 0.1091 \times 10^6 = 29.5. \quad \textit{Ans.}$$

General Solution of the High-Frequency Response

5.06 Derive a formula for the frequency response of a resistance-coupled amplifier at high frequencies.

Solution:

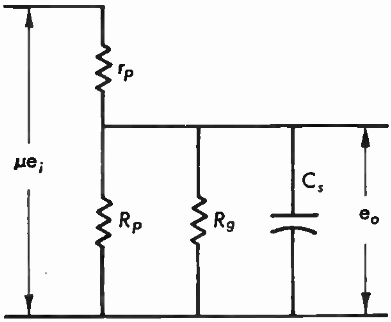

Fig. 5.06 Resistance-capacitance coupled amplifier at high frequencies.

As stated in problem 5.02 the reactance of the coupling capacitor can be neglected, but the reactance of the shunt capacitance must be included. Figure 5.06, therefore represents the equivalent circuit for high frequencies. Let R_{pg} be the value of the plate resistor and the grid resistor in parallel, $\mathring{Z}_p$ the impedance of R_{pg} and X_s in parallel, then the output voltage, as it appears across $\mathring{Z}_p$, is

$$e_o = \mu\, e_i \frac{Z_p}{Z_t}.$$

Now
$$\mathring{Z}_p = \frac{-j\, X_s\, R_{pg}}{-j\, X_s + R_{pg}}$$

and
$$\mathring{Z}_t = r_p + \mathring{Z}_p$$
$$= \frac{r_p(-j X_s + R_{pg}) - j X_s R_{pg}}{-j X_s + R_{pg}}$$
$$= \frac{-j X_s (r_p + R_{pg}) + r_p R_{pg}}{-j X_s + R_{pg}}.$$

The common denominator $-j X_s + R_{pg}$ will be canceled in the fraction $\mathring{Z}_p/\mathring{Z}_t$, therefore

$$\frac{\mathring{Z}_p}{\mathring{Z}_t} = \frac{-j X_s R_{pg}}{-j X_s (r_p + R_{pg}) + r_p R_{pg}}.$$

Dividing numerator and denominator by $-j X_s$ and multiplying them by r_p

$$\frac{\mathring{Z}_p}{\mathring{Z}_t} = \frac{R_{pg}}{r_p + R_{pg} + (r_p R_{pg})/(-j X_s)} \times \frac{r_p}{r_p}.$$

The sum $r_p + R_{pg}$ in the denominator can be rewritten as follows: Using the formula for parallel resistances and the symbol R' with the same meaning as in problem 5.05, we obtain

$$\frac{r_p R_{pg}}{r_p + R_{pg}} = R'$$

and
$$\frac{r_p R_{pg}}{R'} = r_p + R_{pg}.$$

Substituting
$$\frac{\mathring{Z}_p}{\mathring{Z}_t} = \frac{r_p R_{pg}}{(r_p R_{pg})/R' + r_p R_{pg}/(-j X_s)} \times \frac{1}{r_p}$$

and canceling $r_p R_{pg}$
$$= \frac{1}{1/R' + 1/(-j X_s)} \times \frac{1}{r_p}$$

Multiplying by $\frac{R'}{R'}$
$$= \frac{R'}{1 + R'/(-j X_s)} \times \frac{1}{r_p},$$

the absolute value of which is

$$\left|\frac{Z_p}{Z_t}\right| = \frac{R'}{\sqrt{1 + (R'^2/X^2)}} \times \frac{1}{r_p}.$$

Substituting in the equation for voltage output

$$e_o = e_i \times \frac{\mu}{r_p} \times \frac{R'}{\sqrt{1 + (R'/X_s)^2}}.$$

Since $\mu/r_p = g_m$ and since $g_m R'$ is the amplification at medium frequencies (G_m), the voltage amplification for high frequencies is

$$G_h = \frac{e_o}{e_i} = \frac{G_m}{\sqrt{1 + (R'/X_s)^2}}. \quad \textit{Ans.}$$

High-Frequency Response Using the Formula for Voltage Amplification at High Frequencies

5.07 Using the formula derived in problem 5.06 calculate the high-frequency response of the resistance-coupled amplifier stage of Figure 5.01a, problem 5.01.

Solution:

The values of the response for voltage amplification at medium frequencies and of the equivalent resistance R' were found in problem 5.05:

$$G_m = 29.5 \text{ and } R' = 109{,}100 \text{ ohms.}$$

Using

$$G_h = \frac{G_m}{\sqrt{1 + (R'/X_s)^2}}$$

with $X_s = 1/(2\,\pi\,f\,C_s) = 180{,}000$ ohms, we obtain

$$G_h = \frac{29.5}{\sqrt{1 + (0.607^2)}}$$

$$= \frac{29.5}{\sqrt{1 + 0.368}}$$

$$= 29.5/1.17 = 25.2. \quad \textit{Ans.}$$

General Solution of the Low-Frequency Response

5.08 Derive a formula for the frequency response of a resistance-coupled amplifier at low frequencies.

Solution:

As stated in problem 5.03, the reactance of the shunt capacitor can be neglected, but the reactance of the coupling capacitor must be included. Figure 5.08 therefore represents the equivalent circuit for low frequencies.

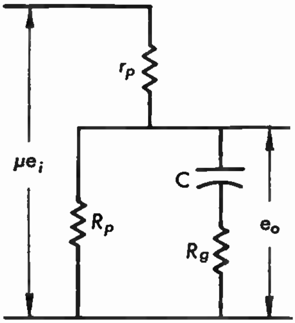

Fig. 5.08 Resistance-capacitance coupled amplifier at low frequencies.

We shall again attempt to express the response in terms of the medium-frequency response. It will be helpful to remember that

$$G_m = g_m R'$$

$$= \frac{g_m r_p R_p R_g}{r_p R_p + r_p R_g + R_p R_g}$$

Let e_p be the voltage across R_p, then the output voltage as it appears across R_g is

$$e_o = e_p \times \frac{R_g}{R_g + \mathring{X}} \text{ where } \mathring{X} = -jX.$$

It is therefore necessary to find e_p first; e_p is that part of the generator voltage μe_i which appears across the parallel combination Z_p.

$$e_p = \mu e_i \frac{Z_p}{Z_p + r_p}.$$

Using the product-sum formula we obtain Z_p:

$$\mathring{Z}_p = \frac{R_p \times (R_g + \mathring{X})}{R_p + (R_g + \mathring{X})}.$$

Substituting this value for Z_p in the equation of the voltage across the plate resistor

$$e_p = \mu e_i \frac{\dfrac{R_p(R_g + \mathring{X})}{R_p + R_g + \mathring{X}}}{r_p + \dfrac{R_p(R_g + \mathring{X})}{R_p + R_g + \mathring{X}}}$$

$$= \mu e_i \frac{R_p(R_g + \mathring{X})}{r_p R_p + r_p R_g + r_p \mathring{X} + R_p R_g + R_p \mathring{X}}$$

$$= \mu e_i \frac{R_p(R_g + \mathring{X})}{r_p R_p + r_p R_g + R_p R_g + \mathring{X}(r_p + R_p)}.$$

Substituting this value for e_p in the equation of the output voltage

$$e_o = \mu e_i \frac{R_p(R_g + \mathring{X})}{r_p R_p + r_p R_g + R_p R_g + \mathring{X}(r_p + R_p)} \times \frac{R_g}{R_g + \mathring{X}} \times \frac{r_p}{r_p}$$

Canceling $R_g + \mathring{X}$

$$= e_i \frac{\mu}{r_p} \frac{R_g r_p R_p}{r_p R_p + r_p R_g + R_p R_g + \mathring{X}(r_p + R_p)}$$

and regrouping

$$= e_i \times \frac{g_m (r_p R_p R_g)/(r_p + R_p)}{\dfrac{r_p R_p + r_p R_g + R_p R_g}{r_p + R_p} + \mathring{X}}.$$

For the sake of clarity we shall examine the numerator and denominator separately.

The real part of the denominator becomes, after factoring,

$$\frac{r_p R_p + R_g(r_p + R_p)}{r_p + R_p},$$

and after performing the division

$$\frac{r_p R_p}{r_p + R_p} + R_g = R''$$

It is obvious that the real part of the denominator constitutes a resistance R'', formed by the parallel combination of plate load resistor and plate resistance in series with the grid resistor. Thus the denominator may be written:

$$R'' + \mathring{X}.$$

Expanding the fractional numerator

$$\frac{g_m r_p R_p R_g}{r_p + R_p} \times \frac{r_p R_p + r_p R_g + R_p R_g}{r_p R_p + r_p R_g + R_p R_g}$$

it becomes $$G_m \times R''.$$

Using this simplified numerator and denominator

$$e_o = e_i \frac{G_m R''}{R'' + \mathring{X}}$$

$$= e_i \frac{G_m}{1 + \mathring{X}/R''}.$$

The absolute value of the low-frequency response is

$$G_l = \frac{e_o}{e_i} = \frac{G_m}{\sqrt{1 + (X/R'')^2}}. \quad Ans.$$

Low-Frequency Response Using the Formula for Voltage Amplification at Low Frequencies

5.09 Using the formula derived in problem 5.08 calculate the low frequency response of the resistance-coupled amplifier stage of Figure 5.01a, problem 5.01.

Solution:

$$G_l = \frac{G_m}{\sqrt{1 + (X/R'')^2}}$$

The value of the response at medium frequencies G_m was found in problem 5.04:

$$G_m = 29.5$$

$$X = 1/(2\pi f C) = 531{,}000 \text{ ohms}$$

$$R'' = 1{,}000{,}000 + \frac{240{,}000 \times 250{,}000}{240{,}000 + 250{,}000}$$

$$= 1{,}122{,}500 \text{ ohms}$$

$$G_l = \frac{29.5}{\sqrt{1 + 0.472^2}}$$

$$= \frac{29.5}{\sqrt{1 + 0.223}}$$

$$= \frac{29.5}{1.108} = 26.6. \quad \textit{Ans.}$$

Replacement of the Coupling Capacitor

5.10 A radio receiver is inoperative because of an open coupling capacitor, the rating of which cannot be read or measured. If the ohmmeter registers 750,000 ohms grid-leak resistance, what would be an adequate value of coupling capacitance?

Solution:

For good bass response a large value of capacitance would be desirable, but too large a capacitor would not allow the charge of the capacitor to leak off.

A time constant of 1/250 of a second is a practical value for the capacitance-resistance product.

$$C \times R = \frac{1}{250}$$

$$C = \frac{1}{250 \times R}$$

$$= \frac{1}{250 \times 750 \times 10^3}$$

$$= \frac{1}{2.5 \times 7.5 \times 10^7}$$

$$= 0.053 \times 10^{-7} = 0.005 \times 10^{-6}$$

$$= 0.005 \text{ microfarad, approximately.} \quad \textit{Ans.}$$

Leaky Coupling Capacitor

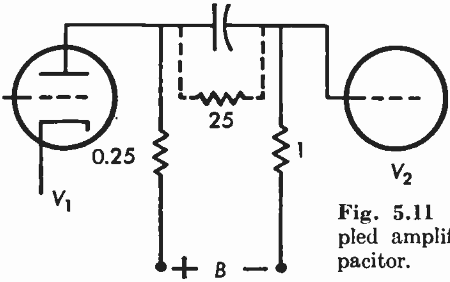

Fig. 5.11 Resistance-capacitance coupled amplifier with leaky coupling capacitor.

5.11 While looking for the reason for distortion in a radio receiver it is found that the audio-frequency stage has a leaky coupling capacitor with a leakage resistance of 25 megohms. The plate resistor is 0.25 megohm, and the grid-leak resistor is 1 megohm. What is the value of the positive voltage on the grid of the tube V_2 in Figure 5.11 caused by this leakage, if the plate supply is 125 volts?

Solution:

$$R_t = 0.25 + 25 + 1 = 26.25 \text{ megohms}$$

$$I = \frac{125}{26.25 \times 10^6} \quad 4.76 \text{ microamperes}$$

$$E_g = I R_g = 4.76 \times 10^{-6} \times 10^6 = 4.76 \text{ volts}$$

A positive potential of 4.76 volts is opposing the normal grid bias, hence the distortion. *Ans.*

Grid-Leak Resistor

5.12 The transmitting triode type 802 requires a grid bias of −90 volts. If this voltage is obtained by the grid-leak resistor and the grid direct current is 4.5 milliamperes, what will be the value of the resistance and what power rating will be adequatc?

Solution:

$$R = \frac{E}{I} = \frac{90}{4.5 \times 10^{-3}} = 20 \times 10^3 = 20{,}000 \text{ ohms.} \quad \textit{Ans.}$$

$$P = E \times I = 90 \times 4.5 \times 10^{-3} = 0.405 \text{ watts.}$$

A 2-watt resistor will work with a safety factor of 4, approximately. *Ans.*

Cathode Resistor and Capacitor

5.13 The power amplifier triode type 45 requires a grid voltage of −56 volts, obtained by means of the voltage drop through a resistor in the plate return lead. The plate current is 36 milliamperes. Find the value of the bias resistor, the by-pass capacitor, and determine practical power and voltage ratings, respectively.

Solution:

$$R = \frac{56}{36 \times 10^{-3}} = 1560 \text{ ohms.} \quad \textit{Ans.}$$

$$P = E \times I = 56 \times 36 \times 10^{-3} = 2 \text{ watts.}$$

A 5-watt wire-wound resistor will be adequate. *Ans.*

Let the opposition of X_c be 1/10 of R for the lowest frequency to be by-passed, then $X_c = 1560/10 = 156$ ohms;

the capacitance having this reactance at 50 cycles is

$$C = \frac{1}{2\pi f X_c} = \frac{1}{6.28 \times 50 \times 156}$$

$$= 20.4 \text{ microfarads.}$$

A 25-microfarad 100-volt electrolytic capacitor will be adequate. *Ans.*

Line Amplifier

FCC Study Guide Question 4.65

5.14 If a preamplifier having a 600-ohm output is connected to a microphone so that the power output is −40 decibels, and assuming the mixer system to have a loss of 10 decibels, what must be the voltage amplification necessary in the line amplifier in order to feed +10 decibels into the transmitter line?

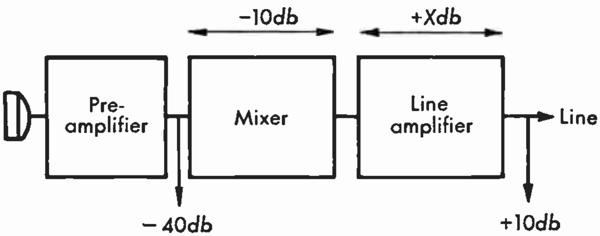

Fig. 5.14 Block diagram of audio equipment with line amplifier.

Solution:

Figure 5.14 represents the block diagram for this problem. The gain in decibels can be added algebraically

$$(-40) + (-10) + (+x) = +10$$

$$x = 10 + 50 = 60 \text{ decibels}$$

The number of decibels calculated by the voltage ratio is

$$N_{db} = 20 \log \frac{E_i}{E_2}$$

$$60 = 20 \log \frac{E_1}{E_2}$$

$$3 = \log \frac{E_1}{E_2}$$

$$\frac{E_1}{E_2} = \text{antilog } 3 = 1000$$

The ratio of the input to the output voltage is 1000 to 1. *Ans.*

Decibel Loss

FCC Study Guide Question 4.66

5.15 If the power output of a modulator is decreased from 1000 watts to 10 watts, how is the power loss expressed in decibels?

Solution:

$$N_{db} = 10 \log \frac{P_1}{P_2}$$

$$N_{db} = 10 \log \frac{1000}{10}$$

$$N_{db} = 10 \log 100 = 10 \times 2$$

$$N_{db} = 20 \text{ db loss} \quad \text{or} \quad -20 \text{ decibels.}$$ *Ans.*

Percentage of Modulation at Stated Decibel Input

FCC Study Guide Question 4.70

5.16 If 100 per cent modulation is obtained with an input level of 60 decibels, what percentage of modulation will be obtained with an input level of 45 decibels?

Solution:

The percentage modulation is not proportional to the decibel level but to the voltage level. A loss of 60 − 45 = 15 decibels of the modulation voltage means a voltage ratio of E_1/E_2 which is found as follows:

$$15 = 20 \log \frac{E_1}{E_2}$$

$$\frac{15}{20} = \log \frac{E_1}{E_2}$$

$$\log \frac{E_1}{E_2} = 0.75$$

$$\frac{E_1}{E_2} = \text{antilog } 0.75$$

$$= 5.63,$$

i.e., the ratio of the larger voltage to the smaller is 5.63 to 1.

$$\frac{x}{1} = \frac{100}{563} = 0.178$$

$$x = 17.8 \text{ per cent.} \quad \textit{Ans.}$$

Input for Stated Gain

FCC Study Guide Question 4.81

5.17 If a certain audio-frequency amplifier has an over-all gain of 40 decibels and the output is 6 watts, what is the input?

Solution:

$$N_{db} = 10 \log \frac{P_1}{P_2}$$

$$40 = 10 \log \frac{6}{P_2}$$

$$4 = \log \frac{6}{P_2}$$

$$\text{antilog } 4 = \frac{6}{P_2}$$

$$10{,}000 = \frac{6}{P_2},$$

and the input is

$$P_2 = \frac{6}{10{,}000} = 0.0006 \text{ watt.} \quad \textit{Ans.}$$

Triode Power Sensitivity

5.18 Which of the two operating conditions of the triode type 45 works with a greater power sensitivity?

1) Plate voltage 180 volts, grid signal rms 28 volts, output 0.825 watts.
2) Plate voltage 275 volts, grid signal rms 50 volts, output 2 watts.

Solution:

The power sensitivity is the ratio of the power output to the square of the rms signal input voltage.

1) $$\text{power sensitivity}_1 = \frac{P}{E_g^2} = \frac{0.825}{28^2} = \frac{0.825}{784}$$
$$= 0.00105 \text{ mho} = 1050 \text{ micromhos}$$

2) $$\text{power sensitivity}_2 = \frac{2}{50^2} = \frac{2}{2{,}500} = 0.0008 \text{ mhos}$$
$$= 800 \text{ micromhos}$$

The first. *Ans.*

Pentode Power Sensitivity

5.19 Which of the two operating conditions of the pentode type 6F6 works with a greater power sensitivity?

1) $E_p = 250$ volts, $E_g = 15$ volts, $P = 3.2$ watts
2) $E'_p = 285$ volts, $E'_g = 20$ volts, $P' = 4.5$ watts

Solution:

1) $$\text{power sensitivity}_1 = \frac{3.2}{15^2} = \frac{3.2}{225}$$
$$= 0.0142 \text{ mho} = 14{,}200 \text{ micromhos}$$

2) $$\text{power sensitivity}_2 = \frac{4.5}{20^2} = \frac{4.5}{400}$$
$$= 0.01125 \text{ mho} = 11{,}250 \text{ micromhos}$$

The first. *Ans.*

It is interesting to compare the power sensitivity of the triode with the one of the pentode; the pentode is about ten times more sensitive.

Transmitting Triode Power Sensitivity

5.20 What is the power sensitivity of the triode tube type 845, operating with a grid voltage input of 52 volts rms and a power output of 24 watts?

Solution:

$$\text{power sensitivity} = \frac{24}{52^2} = \frac{24}{2700}$$
$$= 0.009 \text{ mho}$$
$$= 9000 \text{ micromhos.} \quad \textit{Ans.}$$

Class A. Graphical Determination of the Power Output

5.21 From the load line drawn in problem 4.10 calculate the power output of the amplifier working with a zero-signal bias of −6 volts.

Solution:

The peak signal input is 6 volts, since the grid is not allowed to go positive in class A. The intersection of the load line with the $E_g = 0$ curve is

A (110 volts, 9.5 milliamperes).

The intersection of the load line with the curve $E_g = -12$ is

B (270 volts, 3.5 milliamperes)

The power output is

$$P = \frac{(I_{max} - I_{min})(E_{max} - E_{min})}{8}$$
$$= \frac{(9.5 - 3.5) \times 10^{-3} \times (270 - 110)}{8}$$
$$= \frac{6 \times 160 \times 10^{-3}}{8}$$
$$= 0.12 \text{ watt.} \quad \textit{Ans.}$$

Graphical Determination of the Second Harmonic Distortion

5.22 What is the second-harmonic distortion of the class A amplifier working under the conditions of problem 5.21?

Solution:

The intersection of the load line with the zero-signal grid-voltage curve $E_g = -6$ has an ordinate

$$I_o = 6.3 \text{ milliamperes}$$

The second-harmonic distortion is

$$D_2 = \frac{0.5\,(I_{max} + I_{min}) - I_0}{I_{max} - I_{min}}$$

$$= \frac{0.5\,(9.5 + 3.5) - 6.3}{9.5 - 3.5}$$

$$= \frac{6.5 - 6.3}{6}$$

$$= \frac{0.2}{6} = 0.033$$

$$= 3.3 \text{ per cent.} \quad \textit{Ans.}$$

Response of Transformer-Coupled Amplifier

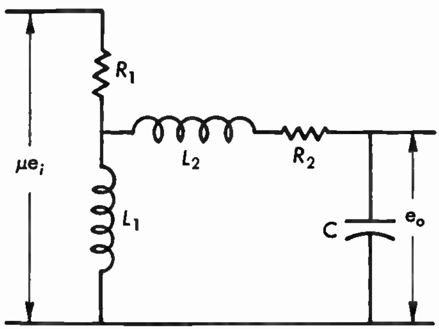

Fig. 5.23 Equivalent circuit of transformer-coupled audio amplifier.

5.23 The circuit of Figure 5.23 represents the equivalent circuit of a transformer-coupled audio amplifier, where R_1 is the plate resistance plus primary resistance, L_1 is the primary inductance, L_2 is the secondary leakage inductance, R_2 is the reflected secondary resistance, C is the total shunt capacitance. If $R_1 = 15{,}000$ ohms, $L_1 = 40$ henries, $L_2 = 0.5$ henries, $R_2 = 1000$ ohms, $C = 1500$ micromicrofarads, calculate the voltage amplification at 1000 cycles, assuming a 1 to 1 transformer ratio, and an amplification factor $\mu = 9$.

Solution:

$$X_{L_1} = 2\pi f L_1 = 251{,}000 \text{ ohms}$$

$$X_{L_2} = 2\pi f L_2 = 3140 \text{ ohms}$$

$$X_c = \frac{1}{2\pi f C} = 106{,}000 \text{ ohms.}$$

It is evident that the oppositions of R_2 and L_2 are not comparable with the one of C, and the parallel circuit will simply be L_1 and C in parallel.

$$e_o = \mu e_i \frac{Z\ parallel}{Z\ total}$$

$$\mathring{Z}_p = \frac{\mathring{Z}_1 \mathring{Z}_2}{\mathring{Z}_1 + \mathring{Z}_2}$$

$$\mathring{Z}_p = \frac{(j X_{L_1})(-j X_c)}{j X_{L_1} - j X_c} = \frac{X_{L_1} X_c}{j(X_{L_1} - X_c)}$$

$$= -j \frac{251 \times 106 \times 10^6}{(251 - 106)\, 10^3}$$

$$= -j \frac{26{,}800 \times 10^3}{145} = -j\, 185{,}000.$$

Note: $\frac{1}{j} = -j.$

$$\mathring{Z}_t = 15{,}000 - j\, 185{,}000$$

$$|Z_t| = \sqrt{15{,}000^2 + 185{,}000^2} = 185{,}600 \text{ ohms}$$

$$\frac{Z_p}{Z_t} = \frac{185}{185.6} = 0.996$$

$$e_o = \mu e_i \times 0.996$$

The voltage amplification at 1000 cycles is

$$G = \frac{e_o}{e_i} = \mu \times 0.996$$

$$= 9 \times 0.996 = 8.96. \quad \textit{Ans.}$$

At 1000 cycles the impedance of the parallel circuit is so big compared with R that practically no voltage drop occurs in R_1, and the voltage amplification is most approximately equal to the amplification factor of the tube.

Transformer Coupling. Low-Frequency Response

5.24 In Figure 5.23, problem 5.23, calculate the voltage amplification at 50 cycles.

Solution:

$$X_{L_1} = 2\,\pi f\,L_1 = 12{,}600 \text{ ohms}$$

$$X_{L_2} = 2\,\pi f\,L_2 = 157 \text{ ohms}$$

$$X_c = \frac{1}{2\,\pi f\,C} = 2{,}120{,}000 \text{ ohms}$$

$$e_o = \mu\, e_i\, \frac{Z_p}{Z_t}$$

Z_p practically consists of X_{L_1} only, since X_{L_2} is negligible and X_c is practically an open circuit.

$$\mathring{Z}_t = 15{,}000 - j\,12{,}600$$

$$|Z_t| = 10^3\sqrt{15^2 + 12.6^2} = 19{,}500 \text{ ohms}$$

$$G = \frac{e_o}{e_i} = \mu\,\frac{12{,}600}{19{,}500} = 9 \times 0.646$$

$$= 5.82. \quad \textit{Ans.}$$

Transformer Coupling. High-Frequency Response

5.25 Calculate the voltage amplification of the transformer-coupled amplifier of Figure 5.23, problem 5.23, (a) at a frequency of 10,000 cycles, (b) at the resonant frequency of C and L_2.

Solution:

(a) At 10,000 cycles

$$X_{l_1} = 2\,\pi f\,L_1 = 2{,}510{,}000 \text{ ohms,}$$

$$X_{l_2} = 2\,\pi f\,L_2 = 31{,}400 \text{ ohms,}$$

$$X_c = \frac{1}{2\,\pi f\,C} = 10{,}600 \text{ ohms.}$$

Clearly X_{l_1} will have a negligible shunting effect. The equivalent circuit then becomes a series circuit and the output voltage is the one appearing across X_c.

$$e_o = \mu\, e_i \frac{X_c}{Z_t}.$$

Neglecting L_1

$$\mathring{Z}_p = 1000 + j\,31{,}400 - j\,10{,}600$$
$$= 1000 + j\,20{,}800.$$
$$\mathring{Z}_t = 15{,}000 + \mathring{Z}_p = 16{,}000 + j\,20{,}400$$
$$|Z_t| = \sqrt{16{,}000^2 + 20{,}800^2} = 26{,}200 \text{ ohms}$$
$$e_o = \mu\, e_i \times \frac{10{,}600}{26{,}200}.$$

The high-frequency voltage amplification is

$$G = \frac{e_o}{e_i} = 9 \times 0.404 = 3.64. \quad \textit{Ans.}$$

(b) The resonant frequency is

$$f = \frac{1}{6.28\sqrt{L_2\, C}}$$
$$= \frac{10^5}{6.28\sqrt{0.5 \times 15 \times 10^2 \times 10^{-12}}}$$
$$= 5820 \text{ cycles.}$$

At this frequency

$$X_{l_1} = 2\,\pi f L_1 = 1{,}460{,}000 \text{ ohms}$$
$$X_{l_2} = 2\,\pi f L_2 = 18{,}400 \text{ ohms}$$
$$X_c = \frac{1}{2\,\pi f C} = 18{,}400 \text{ ohms}$$

L_1 can be neglected, and the circuit consists of a series connection, viz.,

$$Z_t = 15{,}000 + j\,18{,}400 + 1000 - j\,18{,}400$$
$$= 16{,}000 \text{ ohms.}$$

The voltage as it appears across C is

$$e_o = \mu\, e_i \times \frac{X_c}{Z_t}$$

$$= 9\, e_i \times \frac{18{,}400}{16{,}000}$$

$$= 9 \times 1.15 \times e_i$$

$$e_o = 10.35\, e_i$$

and $$G_{res} = \frac{e_o}{e_i} = 10.35. \quad Ans.$$

There will be an outspoken peak in the frequency response curve at this frequency.

Impedance Coupling

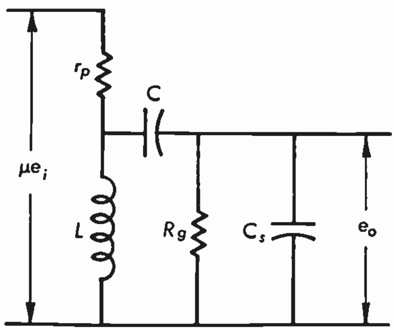

Fig. 5.26 Equivalent circuit of an impedance-coupled audio amplifier.

5.26 The circuit of Figure 5.26 represents the equivalent circuit of an impedance-coupled audio amplifier, where r_p is the plate resistance, L the coupling inductance, C the coupling capacitance, R_g the grid resistance, C_s the shunt capacitance (combined output capacitance of the first tube, input capacitance of the second tube and distributed capacitance of the winding). If r_p = 100,000 ohms, L = 500 henries, C = 0.05 microfarad, R_g = 0.5 megohm, and C_s = 250 micromicrofarads, calculate the voltage amplification at 1000 cycles and an amplification factor μ = 40.

Solution:

At 1000 cycles

$$X_c = 1/(2\pi f C) = 1/(6.28 \times 1000 \times 0.05 \times 10^{-6})$$
$$= 3180 \text{ ohms}$$
$$X_s = 1/(2\pi f C_s) = 1/(6.28 \times 1000 \times 250 \times 10^{-12})$$
$$= 637{,}000 \text{ ohms.}$$
$$X_L = 2\pi f L = 6.28 \times 1000 \times 500 = 3{,}140{,}000 \text{ ohms.}$$

The parallel combination, for all practical considerations, consists of X_s and R_g in parallel

$$\mathring{Z}_p = \frac{-j X_s \times R_g}{-j X_s + R_g}$$
$$= \frac{-j\,637{,}000 \times 500{,}000}{-j\,637{,}000 + 500{,}000}$$
$$= \frac{10^{10}(-j\,31.8)}{10^5(5 - j\,6.37)}$$
$$= 10^5 \frac{(-j\,31.8)(5 + j\,6.37)}{5^2 + 6.37^2}$$
$$= 10^5 \frac{-j\,159 + 203}{25 + 40.5}$$
$$= 10^5 (3.1 - j\,2.43)$$
$$= 310{,}000 - j\,243{,}000.$$

The total impedance of the circuit is the above expression plus the plate resistance

$$\mathring{Z}_t = 310{,}000 - j\,243{,}000 + 100{,}000$$
$$= 410{,}000 - j\,243{,}000.$$

The output voltage as it appears across Z_p is

$$e_o = \mu\, e_i \times \frac{Z_p}{Z_t}.$$

The absolute value of the impedance ratio is

$$\left|\frac{Z_p}{Z_t}\right| = \frac{\sqrt{310^2 + 243^2}}{\sqrt{410^2 + 243^2}} = 0.825.$$

Substituting $e_o = 40\, e_i \times 0.825,$

and the voltage amplification is

$$G = \frac{e_o}{e_i} = 40 \times 0.825 = 33. \quad \textit{Ans.}$$

Decibel Gain

5.27 Express the voltage amplification of problem 5.26 in decibels.

Solution:

$$N_{db} = 20 \log \frac{E_1}{E_2}$$

$$N_{db} = 20 \log 32.3 = 20 \times 1.5092$$

$$= 30.2 \text{ decibels.} \quad \textit{Ans.}$$

Power Gain

5.28 An amplifier has an over-all voltage amplification of 5000. The input voltage is obtained across a 0.5-megohm resistor, the output voltage is fed into a 600-ohm transmission line. What is the power gain in decibels?

Solution:

Assuming an input voltage of 1 volt

$$\text{Input Power} = \frac{1^2}{500{,}000} = \frac{1}{5} \times 10^{-5} = 2 \times 10^{-6} \text{ watts}$$

$$\text{Output Power} = \frac{5000^2}{600} = \frac{25{,}000{,}000}{600} = 41{,}600 \text{ watts}$$

$$N_{db} = 10 \log \frac{P_1}{P_2} = 10 \log \frac{4.16}{2} \times 10^{10}$$

$$= 10 \log 2.08 \times 10^{10}$$

$$= 10\,(0.3181 + 10)$$

$$= 10 \times 10.32 = 103.2 \text{ decibels.} \quad \textit{Ans.}$$

Audio-Frequency and Radio-Frequency By-pass Capacitors

5.29 Find the value of the cathode by-pass capacitor to shunt a 1500-ohm resistor

1) for 550 kilocycle broadcast frequencies
2) for 400 cycle audio frequencies

Solution:

1) The reactance of the capacitor should be

$$1500 \div 10 = 150 \text{ ohms or less}$$

$$C = \frac{1}{2\,\pi\,f\,X_c}$$

$$= \frac{1}{6.28 \times 550 \times 10^3 \times 1.5 \times 10^2}$$

$$= 0.00194 \text{ microfarad.} \quad \textit{Ans.}$$

A 0.002-microfarad capacitor would work.

2)
$$C = \frac{1}{2\,\pi\,f\,X_c}$$

$$= \frac{1}{6.28 \times 400 \times 150}$$

$$= 2.65 \text{ microfarads.} \quad \textit{Ans.}$$

A 2.5-microfarad (electrolytic) by-pass capacitor will be used, with a voltage rating equal to or higher than the voltage drop across the resistor. *Ans.*

Radio-Frequency Choke

5.30 A plate tank circuit, tuned to 1500 kilocycles is shunt-fed from a power supply through a radio-frequency choke coil. The impedance of the tank circuit is 5000 ohms. Disregarding the distributed capacitance of the choke coil, what should be its inductance?

Solution:

The value of the inductive reactance should be 10 times the impedance of the circuit or more.

$$X_l = 5000 \times 10 = 50{,}000 \text{ ohms.}$$

From

$$X_l = 2\pi f L$$

we obtain

$$L = \frac{X}{2\pi f}$$

$$= \frac{5 \times 10^4}{6.28 \times 1500 \times 10^3}$$

$$= 0.0053 = 5.3 \text{ millihenries}$$

A radio-frequency choke of an inductance of 5 millihenries or higher will be adequate. *Ans.*

Pentode Bias Resistor

5.31 The 25B6 power amplifier pentode works with a grid bias of −15 volts while a screen current of 5 milliamperes and a plate current of 45 milliamperes are flowing. What is the value of the cathode resistor and what is its rating?

Solution:

The cathode current is the sum of the plate current and screen current.

$$I_c = (45 + 5) \times 10^{-3} = 50 \times 10^{-3} \text{ amperes}$$

$$R = \frac{15}{50 \times 10^{-3}} = 300 \text{ ohms.} \quad \textit{Ans.}$$

$$P = E \times I = 15 \times 50 \times 10^{-3} = 0.75 \text{ watts}$$

A 2-watt carbon, or better, a 5-watt wire-wound resistor should be used. *Ans.*

Push-Pull Bias Resistor

5.32 What should be the value of the cathode resistor in problem 4.55 if two 25B6 tubes are used in push-pull?

Solution:

Since twice the current will flow through the cathode resistor, half the resistance will be required.

$$R = 300 \div 2 = 150 \text{ ohms.} \quad \textit{Ans.}$$

The power will be

$$15 \times 100 \times 10^{-3} = 1.5 \text{ watts.}$$

A 10-watt wire-wound resistor would work with a safety factor of 6, approximately. *Ans.*

Screen Dropping Resistor

5.33 The pentode amplifier type 6SK7 is operated at a plate voltage of 250 volts, a screen voltage of 100 volts, a plate current of 9.2 milliamperes, and a screen current of 2.4 milliamperes. What are the resistance and the power rating of the screen dropping resistor if the screen and the plate obtain their voltages from the same supply?

Solution:

The screen dropping resistor will drop a voltage of

$$E_r = 250 - 100 = 150 \text{ volts.}$$

Applying Ohm's law we obtain for the screen dropping resistor,

$$R_{sc} = \frac{E_r}{I_{sc}} = \frac{150}{2.4 \times 10^{-3}} = 62{,}500 \text{ ohms.} \quad \textit{Ans.}$$

The power dissipation of the screen resistor is

$$P_{sc} = E_r \times I_{sc} = 150 \times 2.4 \times 10^{-3} = 0.36 \text{ watts.}$$

A 1-watt resistor will work with a safety factor of 3, approximately. *Ans.*

Plate Efficiencies of Classes A, B, and C Amplifiers

5.34 The transmitting triode type 801 when used as a class A audio-frequency amplifier has an undistorted power output of 3.8 watts when the direct current is 30 milliamperes and the peak audio-frequency grid voltage is 50 volts. When used as a class B push-pull amplifier the power output is 45 watts, while the direct plate current is 130 milliamperes and the peak radio-frequency grid voltage is 320 volts. When operated as a class C radio-frequency amplifier the power output is 25 watts, the direct plate current is 65 milliamperes and the peak radio-frequency grid voltage is 260 volts. The applied plate voltage is 600 volts throughout. Calculate the plate efficiencies.

Solution:

The grid voltage, while serving to calculate the plate sensitivity, is not necessary to calculate the plate efficiency.

$$\text{Class A:}\quad \eta = \frac{3.8}{600 \times 0.03} = 0.211 = 21.1 \text{ per cent.} \quad \textit{Ans.}$$

$$\text{Class B:}\quad \eta = \frac{45}{600 \times 0.13} = 0.577 = 57.7 \text{ per cent.} \quad \textit{Ans.}$$

$$\text{Class C:}\quad \eta = \frac{25}{600 \times 0.065} = 0.64 = 64 \text{ per cent.} \quad \textit{Ans.}$$

Class A. Maximum Root-Mean-Square Signal Input

FCC Study Guide Question 6.131

5.35 What is the maximum permissible rms value of audio voltage which can be applied to the grid of a class A audio amplifier which has a grid bias of 10 volts?

Solution:

Since the grid is not allowed to become positive in a class A amplifier, the grid voltage will be zero when the signal voltage is at its peak or 10 volts. The rms signal voltage is

$$E = 10 \times 0.707 = 7.07 \text{ volts.} \quad \textit{Ans.}$$

Class AB_2. Coupling of the Driver Tube

5.36 Two type 6L6 beam power tubes are operated as a push-pull, class AB_2 amplifier. The manufacturer recommends a plate voltage of 360 volts, a screen voltage of 270 volts, a peak signal input of 72 volts. A 6F6 tube is used for the driver stage, triode-connected. It operates at 250 volts plate voltage with a recommended plate resistance of 4000 ohms and a power output of 0.85 watts. What is the turns ratio of the input transformer?

Solution:

To find the voltage across the primary we use

$$P = E^2/R$$

$$0.85 = E^2/4000$$

$$E^2 = 4000 \times 0.85 = 3400$$

$$E = \sqrt{3400} = 58.5 \text{ volts.}$$

The peak signal output voltage of the driver tube is

$$E_{peak} = 58.5 \times 1.414 = 82.5 \text{ volts.}$$

The turns ratio is proportional to the voltage ratio

$$\frac{N_1}{N_2} = \frac{82.5}{72} = 1.145 \text{ to } 1. \quad \textit{Ans.}$$

Phase Inverter

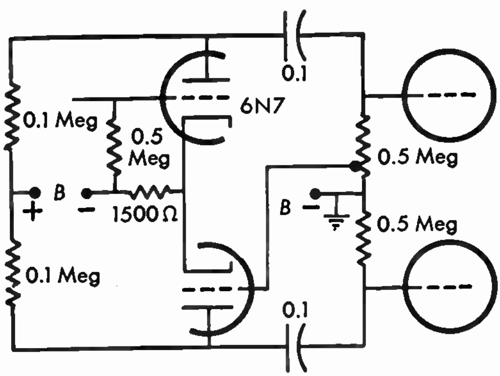

Fig. 5.37a Push-pull amplifier with phase inverter.

5.37 The amplification factor of the duo-triode tube used in the circuit of Figure 5.37a is 35, the plate resistance 11,000 ohms. The tube is used as a phase inverter. Where should the 0.5-megohm resistor be tapped to provide a balanced signal input to the push-pull circuit?

Solution:

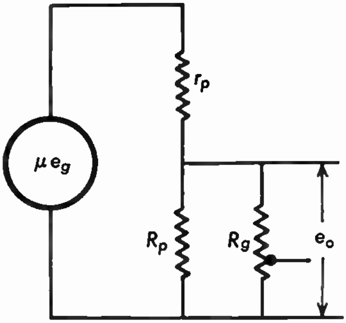

Fig. 5.37b Equivalent circuit of the output of the upper half of the 6N7 triode of Fig. 5.37a.

The lower triode has the same voltage amplification as the upper. If the voltage amplification is 30, for example, then 1/30 of the voltage across the 0.5-megohm resistor must be fed into the grid of the lower tube. To obtain the actual voltage amplification in this circuit we use the formula for the voltage output of a resistance-coupled amplifier at medium frequencies (Figure 5.37b),

$$e_o = \mu\, e_g \frac{Z_{parallel}}{Z_{total}}$$

$$Z_p = \frac{0.1 \times 0.5}{0.1 + 0.5} = \frac{0.05}{0.6} = 0.0832 \text{ megohms} = 83{,}200 \text{ ohms}$$

$$Z_t = 83{,}200 + 11{,}000 = 94{,}200 \text{ ohms}$$

$$G = e_o/e_g = \mu \times \frac{83{,}200}{94{,}200} = 35 \times 0.883 = 30.9$$

The same voltage gain will be realized in the lower triode, therefore only 1/30.9 of the output voltage of the upper triode will be the input voltage to the lower grid.

To obtain the resistance from the tap to ground, we divide

$$500{,}000 \div 30.9 = 16{,}200 \text{ ohms.} \quad \textit{Ans.}$$

The other part of the resistor is

$$500{,}000 - 16{,}200 = 483{,}800 \text{ ohms.} \quad \textit{Ans.}$$

Class B Tube Selection

5.38 The plate dissipation of the power amplifier type 6K6 is 8.5 watts. Can two tubes be used for a class B audio-frequency amplifier with a power output of 40 watts? What is the maximum power obtainable if an efficiency of 45 per cent is realized?

Solution:

The plate efficiency of a class B amplifier is 50 per cent, approximately.

The plate dissipation of both tubes is

$$2 \times 8.5 = 17 \text{ watts}$$

$$\eta = \frac{40}{40 + 17} = 0.702 = 70.2 \text{ per cent.}$$

This efficiency cannot be realized in class B amplifiers. *Ans.*

To calculate the maximum power output obtainable we substitute in the equation for the efficiency

$$0.45 = \frac{P}{P + 17}.$$

$$0.45\,P + 7.65 = P$$

$$7.65 = 0.55\,P$$

$$P = \frac{7.65}{0.55} = 13.9 \text{ watts.} \quad \textit{Ans.}$$

Class B Grid Bias

FCC Study Guide Question 5.200

5.39 What is the correct value of negative grid bias for operation as a class B amplifier, for a vacuum tube of the following characteristics: plate voltage 1000, plate current 127 milliamperes, filament voltage 4 volts, filament current 5.4 amperes, mutual conductance 8000 micromhos, and amplification factor 25?

Solution:

A class B amplifier is biased to cutoff. The equation for the plate current of a triode is

$$I_p = K\left(E_g + \frac{E_p}{\mu}\right)^{\frac{3}{2}}$$

where I_p is the plate current, K the perveance, E_g the grid voltage, E_p the plate voltage, μ the amplification factor. The tube is biased to cutoff when $I_p = 0$; we then have

$$K\left(E_g + \frac{E_p}{\mu}\right)^{\frac{3}{2}} = 0$$

and dividing by K and raising both sides to the 2/3 power we obtain

$$E_g + \frac{E_p}{\mu} = 0$$

and

$$E_g\,(\text{cutoff}) = -\frac{E_p}{\mu} = -\frac{1000}{25}$$

$$= -40 \text{ volts.} \quad \textit{Ans.}$$

Class C Grid Bias

FCC Study Guide Question 5.65

5.40 A triode transmitting tube, operating with a plate voltage of 1250 volts, has a filament voltage of 10 volts, a filament current of 3.25 amperes, and a plate current of 150 milliamperes. The amplification factor is 25. What value of control-grid bias must be used for operation as a class C amplier?

Solution:

Referring to problem 5.39 we first obtain cutoff bias for the given plate voltage

$$E_g\,(\text{cutoff}) = -\frac{E_p}{\mu} = -\frac{1250}{25} = -50 \text{ volts}$$

If the tube is biased $2\frac{1}{2}$ times cutoff (an average value for class C), then

$$E_g\,(\text{bias}) = (-50) \times 2.5$$

$$= -125 \text{ volts, approximately.} \quad \textit{Ans.}$$

Class C Total Space Current

5.41 One set of operating conditions of a radio-frequency amplifier pentode is as follows:

direct plate voltage	1000 volts
direct grid voltage	−100 volts
radio-frequency grid voltage	140 volts peak
direct plate current	45 milliamperes
direct screen current	35 milliamperes
direct grid current	5.5 milliamperes
power output	16 watts
cutoff bias	−30 volts

What is the total space current flowing away from the filament?

Solution:

The total space current is the sum of grid current, screen current and plate current

$$I_t = 35.5 + 35 + 45 = 85.5 \text{ milliamperes.} \quad \textit{Ans.}$$

Class C Grid Driving Power

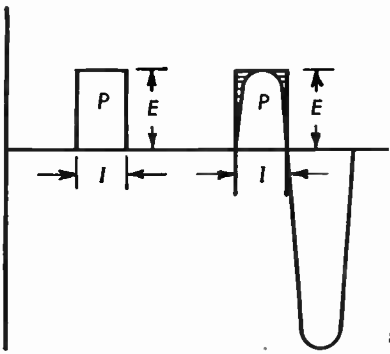

Fig. 5.42 Diagram illustrating the driving power of a class C amplifier.

5.42 In problem 5.41 what is the average grid driving power for the given operating conditions?

Solution:

The peak grid voltage is the determining voltage for calculation of power, since the current flows only during that small part of the cycle when the grid voltage is near its peak. The actual driving power as bounded by the sine wave in Figure 5.42 is about 10 per cent less than the one represented by the area of the rectangle $E \times I$.

$$P = 0.9 \times E_{peak} \times I$$
$$= 0.9 \times 140 \times 5.5 \times 10^{-3}$$
$$= 0.7 \text{ watts, approximately.} \quad \textit{Ans.}$$

Class C Root-Mean-Square Signal Output

5.43 In problem 5.41 what is the minimum permissible value of the direct plate voltage during the active part of the cycle and what is the output rms signal?

Solution:

The plate voltage will sink to its minimum value while the grid voltage is at its maximum; to prevent the flow of excessive grid current the plate voltage should not be less than the peak grid voltage

$$E_{p\ min} = E_{g\ max} = 140 \text{ volts.}$$

The plate swing from 1000 to 140 volts is

$$1000 - 140 = 860 \text{ volts.}$$

The plate signal rms voltage is

$$860 \times 0.707 = 608 \text{ volts.} \quad \textit{Ans.}$$

Class C Active Part of the Cycle

5.44 In problem 5.41 calculate the angle in degrees during which the tube is conducting under the given operating conditions.

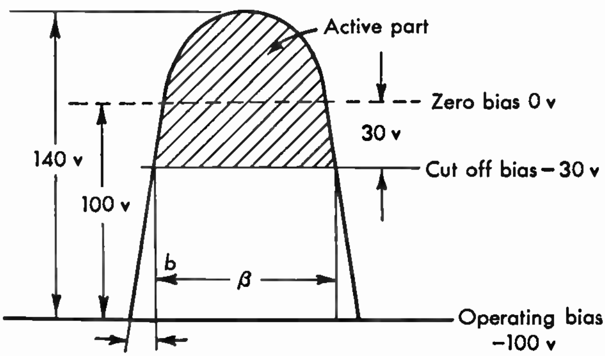

Fig. 5.44 Diagram illustrating the bias conditions of a class C amplifier.

Solution:

From Figure 5.44, we read

$$b = 140 \times \sin\theta,$$

where b is the difference between the operating bias and the cutoff bias:

$$b = 100 - 30 = 70 \text{ volts.}$$

Substituting

$$70 = 140 \sin\theta$$

$$\sin\theta = 0.5,\ \theta = \sin^{-1} 0.5 = 30°$$

The active angle is

$$\beta = 180 - (2 \times 30)$$

$$= 120 \text{ degrees.} \quad \textit{Ans.}$$

Class C Operating Bias

5.45 From Figure 5.44, problem 5.44, calculate the operating bias which would change the active angle to 150 degrees.

Solution:

$$\theta = \frac{180 - \beta}{2} = \frac{180 - 150}{2} = 15°$$

$$b = 140 \times \sin 15° = 140 \times 0.259 = 36.3 \text{ volts.}$$

The new operating bias is

$$-36.3 - 30 = -66.3 \text{ volts, approximately.} \quad \textit{Ans.}$$

Class C Grid Impedance

5.46 In problem 5.41 what is the average grid impedance exhibited by the tube under the given operating conditions?

Solution:

Using the formula $P = \dfrac{E^2}{Z}$, where P is the value of the grid driving power found in problem 5.42 and E the rms value of the grid voltage

$$E_{rms} = 140 \times 0.707 = 99 \text{ volts.}$$

Substituting $$0.7 = \frac{99^2}{Z},$$

and $$Z = \frac{9800}{0.7} = 14{,}000 \text{ ohms.} \quad \textit{Ans.}$$

Class C Tank Circuit Design

5.47 The following typical operation is given in the tube manual for the radio-frequency amplifier type 808:

direct plate voltage	1000 volts
direct grid voltage	−210 volts
radio-frequency grid voltage	360 volts peak
direct plate current	120 milliamperes
direct grid current	35 milliamperes
grid resistor	6000 ohms
driving power	11.5 watts
power output	85 watts

Assuming that the circuit $Q = 10$, calculate the inductance and capacitance necessary to tune the circuit to 4.5 megacycles.

Solution:

The peak signal voltage at the plate is about 90 per cent of the plate voltage.

$$E_m = 0.9 \times 1000 = 900 \text{ volts.}$$

Since the tank circuit acts like a pure resistance at resonance, the tank circuit power is

$$P = \frac{E^2}{Z}$$

where E is the rms value of the voltage or

$$P = \frac{(E_m/\sqrt{2})^2}{Z} = \frac{E_m^2}{2\,Z},$$

where E_m is the peak value of the voltage. From the data we find that the power delivered to the tank circuit is

$$P = 85 \text{ watts.}$$

Since at resonance $Z = X\,Q$,

we have $$P = \frac{E_m^2}{2\,X\,Q}$$

Substituting $$85 = \frac{900^2}{2 \times X \times 10}$$

and $$X = \frac{810{,}000}{85 \times 2 \times 10} = 477 \text{ ohms.}$$

At 4.5 megacycles we can substitute in the formula

$$X_L = 2\,\pi\,f\,L$$

$$477 = 6.28 \times 4.5 \times 10^6 \times L$$

$$L = \frac{477}{6.28 \times 4.5} \times 10^{-6}$$

$$= 16.9 \text{ microhenries.} \quad \textit{Ans.}$$

To obtain the capacitance we solve the formula

$$f = \frac{1}{2\,\pi\,\sqrt{L\,C}} \text{ for } C,$$

and obtain after squaring and transposing

$$C = \frac{1}{4\,\pi^2\,f^2\,L}$$

$$= \frac{1}{39.5 \times 4.5^2 \times 10^{12} \times 16.9 \times 10^{-6}}$$

$$= \frac{1}{39.5 \times 20.3 \times 16.9} \times 10^{-6}$$

$$= 0.000074 \times 10^{-6}$$

$$= 74 \text{ micromicrofarads.} \quad \textit{Ans.}$$

Matching Class A to Class C

FCC Study Guide Question 4.164

5.48 Given a class C amplifier with a plate voltage of 1000 volts and a plate current of 150 milliamperes, which is to be modulated by a class A amplifier with a plate voltage of 2000 volts, plate current of 200 milliamperes, and a plate resistance of 15,000 ohms. What is the proper turns ratio for the coupling transformer?

Solution:

The plate load resistance of the class *A* amplifier must be twice its plate resistance for distortionless operation.

$$R_L = 2 \times 15{,}000 = 30{,}000 \text{ ohms.}$$

The modulation impedance of the class C tube is

$$Z_m = \frac{E}{I} = \frac{1000}{0.15} = 6667 \text{ ohms}$$

The turns ratio of the matching transformer is

$$\frac{N_p}{N_s} = \sqrt{\frac{30{,}000}{6667}} = 2.12 \text{ to } 1. \quad \textit{Ans.}$$

6 *Oscillators*

Hartley Oscillator

6.01 An inductance of 28.3 microhenries is used for a Hartley-type vacuum tube oscillator, tuned to a frequency of 7500 kilocycles. Find the value of the capacitor.

Solution:

$$f = \frac{1}{2\,\pi\,\sqrt{L\,C}}$$

$$f^2 = \frac{1}{4\,\pi^2\,L\,C}$$

$$C = \frac{1}{4\,\pi^2\,f^2\,L}$$

$$= \frac{1}{39.5 \times 56 \times 10^{12} \times 28.3 \times 10^{-6}}$$

$= 15.96$ micromicrofarads. *Ans.*

Colpitts Oscillator

6.02 An inductor of 28.3 microhenries is used for a Colpitts oscillator. A fixed capacitor of 25 micromicrofarads is available. What is the value of the other capacitor required to tune the circuit to 7500 kilocycles?

Solution:

Since the capacitors are in series and C_t is equal to the capacitor of the Hartley oscillator in the preceding problem, viz., 15.96 micromicrofarads, we have

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_2} = \frac{1}{C_t} - \frac{1}{C_1} = \frac{C_1 - C_t}{C_1 C_t}$$

$$C_2 = \frac{C_1 C_t}{C_1 - C_t} = \frac{25 \times 15.96}{25 - 15.96}$$

$$= 44.2 \text{ micromicrofarads.} \quad \textit{Ans.}$$

Tuned-Plate Oscillator

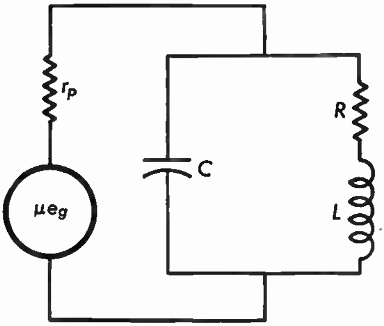

Fig. 6.03 Equivalent circuit of a tuned-plate oscillator.

6.03 The circuit of Figure 6.03 is the equivalent circuit of a tuned-plate oscillator. The inductance is 100 microhenries with an effective resistance of 50 ohms. The capacitance is 100 micromicrofarads and the plate resistance is 9500 ohms. Find the value of the resonant frequency

(a) neglecting the plate resistance and the coil resistance

(b) including the plate resistance and the coil resistance.

Solution:

In this problem slide rule work will not show the different frequencies for (a) and (b). Full arithmetical computation or good logarithm tables should be employed.

(a) Neglecting the plate resistance and the coil resistance

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.2831853 \times \sqrt{100 \times 100 \times 10^{-18}}}$$

$$= \frac{10^7}{6.2831853} = 1{,}591{,}549 \text{ cycles}$$

$$= 1591.5 \text{ kilocycles, approximately.} \quad \textit{Ans.}$$

(b) Including the plate resistance and the coil resistance

$$f' = \frac{1}{2\pi\sqrt{LC}} \times \sqrt{\frac{R + r_p}{r_p}}.$$

The value of the second factor is

$$\sqrt{\frac{9550}{9500}} = 1.0026281,$$

and $f' = 1{,}591{,}549 \times 1.0026281 = 1{,}595{,}641$
$= 1595.6$ kilocycles. *Ans.*

This demonstrates the small degree of usefulness of the oscillator for transmitting purposes, since variations in the tube characteristics will cause the frequency to vary far beyond the limit of the legal frequency tolerances.

Frequency Tolerance

6.04 In problem 6.03, by what percentage does the frequency calculated by including the plate resistance and the coil resistance exceed the frequency found by the simpler formula?

Solution:

The factor $\sqrt{\frac{R + r}{r}} = 1.00263$

i.e., f' will exceed f by 0.00263 of the value of f.

$0.00263 = 0.263$ per cent. *Ans.*

(International Broadcast tolerance = 0.005 per cent!)

Armstrong Oscillator

6.05 The equation for the resonant frequency of the oscillator of Figure 6.05 is

$$f = \frac{1}{2\pi\sqrt{\frac{C\,L\,r + L'\,R}{r}}}.$$

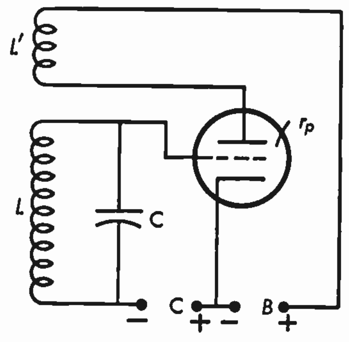

Fig. 6.05 Armstrong oscillator.

If the circuit were used as a local oscillator in a superheterodyne receiver where the frequency tolerance is comparatively great, how could this formula be simplified?

Solution:

Any typical value of plate resistance and radio-frequency resistance of the tank coil, e.g., 10,000 ohms and 50 ohms, respectively, will make the second term of the numerator under the radical sign ($L' R$) much smaller than the first term ($L r$). If the inductance of the tickler coil is $L/2$, then

$$L' R = (L/2) \times 50 = 25L.$$

The term $$L r = 100{,}000\, L.$$

The ratio of the first to the second term is

$$\frac{100{,}000\, L}{25\, L} = 400 \text{ to } 1.$$

Neglecting $L' R$, the formula simplifies to

$$f = \frac{1}{2\,\pi \sqrt{C \dfrac{L\, r}{r}}}$$

$$= \frac{1}{2\,\pi \sqrt{L\, C}}. \quad \textit{Ans.}$$

Crystal-Controlled Oscillator

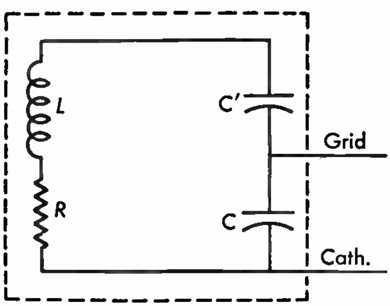

Fig. 6.06 Equivalent circuit of an oscillator crystal.

6.06 Figure 6.06 represents the equivalent circuit of a crystal-controlled oscillator. If $L = 3.5$ henries, $C = 5$ micromicrofarads, $R = 6000$ ohms and $C' = 0.05$ micromicrofarad, what change in frequency could be accomplished by connecting a 10-micromicrofarad capacitor from the grid terminal to the cathode terminal of the crystal?

Solution:

The capacitance of the crystal is

$$C_t = \frac{C\,C'}{C + C'} = \frac{5 \times 0.05}{5 + 0.05} = \frac{0.25}{5.05}$$
$$= 0.0495 \text{ micromicrofarad.}$$

Neglecting R the resonant frequency is

$$f = \frac{1}{2\,\pi\,\sqrt{L\,C_t}}$$
$$= \frac{1}{6.28\,\sqrt{3.5 \times 0.0495 \times 10^{-12}}}$$
$$= 382.5 \text{ kilocycles.}$$

With the 10-micromicrofarad shunt, C becomes 15 micromicrofarads,

and $$C_t' = \frac{0.05 \times 15}{0.05 + 15}$$
$$= 0.0498 \text{ micromicrofarads,}$$

$$f' = \frac{1}{6.28\,\sqrt{3.5 \times 0.0498 \times 10^{-12}}}$$
$$= 381.4 \text{ kilocycles.}$$

The change in frequency would be 1 kilocycle, approximately. *Ans.*

Q of a Crystal

6.07 Calculate the approximate Q of the crystal described in problem 6.06.

Solution:

The resonant frequency of the crystal is 383 kilocycles.

$$Q = \frac{X}{R}$$
$$= \frac{6.28 \times 383 \times 10^3 \times 3.5}{6 \times 10^3} \cong 1400. \quad \textit{Ans.}$$

Frequency Stability of a Crystal

6.08 If the shunt capacitor in problem 6.06 were increased to any desired value of capacitance, what would be the theoretical limit to which the rated frequency could be changed by this external application?

Solution:

Using the equation $\frac{1}{C_t} = \frac{1}{C} + \frac{1}{C'}$, it is evident that $\frac{1}{C}$ approaches the value of zero as C is increased to any desired value.

Then $$\frac{1}{C_t} \cong \frac{1}{C'}$$

and $$C_t = C' = 0.05 \text{ micromicrofarad.}$$

The resonant frequency would then be

$$f_{min} = \frac{1}{6.28\sqrt{3.5 \times 0.05 \times 10^{-12}}}$$

$$= 380.5 \text{ kilocycles, approximately.} \quad \textit{Ans.}$$

Conclusion: The frequency of a crystal is variable only within small limits by means of a shunt capacitor.

Temperature Coefficient

6.09 An X-cut crystal having a temperature coefficient of −19 parts per million (ppm) is rated 318.27 kilocycles at 18 degrees centigrade. What will be the change in cycles if the temperature falls to 16 degrees?

Solution:

Since the change is 19 parts per million per degree, a change of 2 degrees centigrade will cause a frequency change of

$$2 \times 318{,}270 \times 19 \times 10^{-6} = 12.1 \text{ cycles.}$$

Since the temperature coefficient is negative an increase of the frequency by 12.1 cycles, approximately, is to be expected. *Ans.*

Effect of Temperature on Frequency Multipliers

FCC Study Guide Question 5.192

6.10 A transmitter is operating on 5000 kilocycles, using a 1000-kilocycle crystal with a temperature coefficient of −4 cycles per megacycle per degree centigrade. If the crystal temperature increases 6 degrees centigrade, what is the change in the output frequency of the transmitter?

Solution:

The crystal frequency is 1 million cycles. The frequency of the oscillator stage will decrease

$$6 \times (-4) = -24 \text{ cycles.}$$

This decrease will be multiplied by 5 because the transmitter operates on 5000 kilocycles;

$$(-24) \times 5 = -120 \text{ cycles.} \quad \textit{Ans.}$$

Capacitance Coupling of Radio-Frequency Amplifier

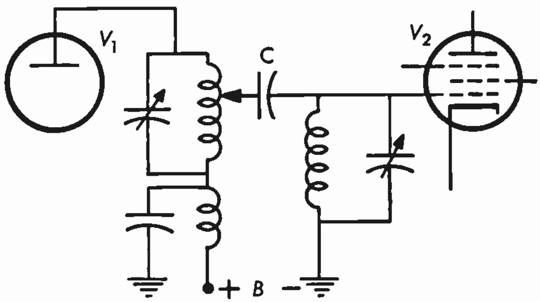

Fig. 6.11 Capacitance-coupled r-f amplifier.

6.11 The radio-frequency amplifier of Figure 6.11 has a grid input impedance of 5000 ohms to V_2, and a load of 4000 ohms for V_1. What is a suitable value for the coupling capacitor if the circuit is tuned to 8 megacycles?

Solution:

The reactance of the capacitor should be 1/10 of the input impedance of V_2 or less.

$$\frac{Z_i}{10} = \frac{1}{2\pi f C}$$

$$C = \frac{1}{6.28 \times 8 \times 10^6 \times 500}$$

$$= 39.9 \text{ micromicrofarads or more.} \quad \textit{Ans.}$$

Grid and Plate Tank Circuit

6.12 Disregarding the method of coupling, what is the value of the inductance and capacitance in both the grid and the plate circuit, if a Q of 10 is provided for each circuit of problem 6.11?

Solution:

Grid circuit:

$$Z = QX$$

$$5000 = 10 \times X$$

$$X = 500 \text{ ohms}$$

$$L = \frac{X}{2\pi f} = \frac{500}{6.28 \times 8 \times 10^6}$$

$$= 9.95 \text{ microhenries.} \quad \textit{Ans.}$$

$$C = \frac{1}{2\pi f X} = \frac{1}{6.28 \times 8 \times 10^6 \times 500}$$

$$= 39.8 \text{ micromicrofarads.} \quad \textit{Ans.}$$

Plate circuit:

$$4000 = 10 \times X$$

$$X = 400$$

$$L' = \frac{X}{2\pi f} = \frac{400}{6.28 \times 8 \times 10^6}$$

$$= 7.95 \text{ microhenries.} \quad \textit{Ans.}$$

$$C' = \frac{1}{2\pi f X} = \frac{1}{6.28 \times 8 \times 10^6 \times 400}$$

$$= 49.7 \text{ micromicrofarads.} \quad \textit{Ans.}$$

Oscillator Tank Circuit Design

6.13 From the tube manual we obtain the following typical operating conditions for the radio-frequency power amplifier and oscillator triode type 801:

direct plate voltage	600	volts
direct grid voltage	−150	volts
peak radio-frequency grid voltage	260	volts
direct plate current	65	milliamperes
direct grid current	15	milliamperes
grid resistor	10,000	ohms
driving power	4	watts
power output	25	watts

Calculate the values of the capacitance and inductance of a Hartley oscillator tuned to a frequency of 2000 kilocycles with a circuit $Q = 35$, working as close as possible under the typical operating conditions.

Solution:

The total radio-frequency voltage across the tank circuit is equal to the sum of the radio-frequency plate voltage plus the radio-frequency grid voltage. The peak radio-frequency plate voltage is about 90 per cent of the direct plate voltage

$$\text{peak radio-frequency plate voltage} = 0.9 \times 600 = 540 \text{ volts.}$$

The peak voltage across the tank circuit is

$$E_t = 540 + 260 = 800 \text{ volts.}$$

The tank circuit acts like a pure resistance at resonance, and the tank circuit power is

$$P = \frac{E_{rms}^2}{Z} = \frac{(E_t/\sqrt{2})^2}{Z} = \frac{E_t^2}{2\,Z}$$

but Z at resonance is equal to $X\,Q$.

Substituting

$$25 = \frac{800^2}{2 \times X \times 35},$$

and $$X = \frac{640{,}000}{2 \times 25 \times 35} = 366 \text{ ohms}$$

$$L = \frac{X}{2\,\pi f} = \frac{366}{6.28 \times 2 \times 10^6}$$

$$= 28.7 \text{ microhenries.} \quad \textit{Ans.}$$

$$C = \frac{1}{4\,\pi^2 f^2 L} = \frac{1}{39.5 \times 4 \times 10^{12} \times 28.7 \times 10^{-6}}$$

$$= 221 \text{ micromicrofarads.} \quad \textit{Ans.}$$

7 *Transmitters*

Continuous-Wave Telegraphy. Grid Keying

7.01 The cutoff voltage of the tube used in the grid keying circuit of Figure 7.01 is −75 volts. Show that no current will flow when the key is in the up-position.

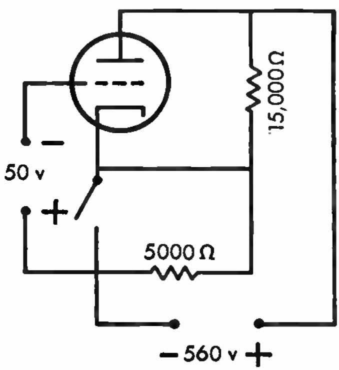

Fig. 7.01 Grid keying circuit.

Solution:

The voltage across the 5000-ohm resistor is

$$E_5 = 560 \times \frac{5}{15 + 5} = 140 \text{ volts.}$$

The grid is connected to the negative side of this potential, the cathode to the positive; E_5 is in series with the voltage of the C supply.

$$E_g = -50 + (-140) = -190 \text{ volts}$$

This is far beyond the cutoff of the tube. *Ans.*

Grid Keying with Click Filter

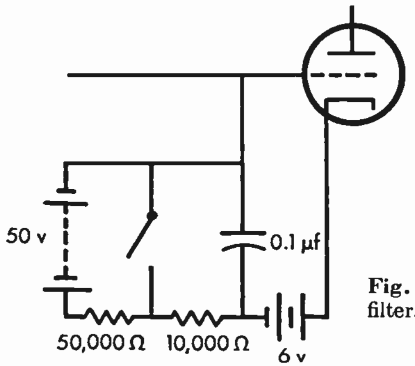

Fig. 7.02 Grid keying with click filter.

7.02 In the keying circuit of Figure 7.02, what is the approximate bias voltage for *make* and for *break*? What is the time constant for *make* and for *break*?

Solution:

The *make* bias is approximately −6 volts if the grid current is negligible. The *make* time constant is determined by the capacitor and the 50,000-ohm resistor.

$$CR = 0.1 \times 10^{-6} \times 10 \times 10^{3}$$
$$= 1 \times 10^{-3} = 1 \text{ millisecond.}$$

The *break* bias is approximately $(-50) + (-6) = -56$ volts. The *break* time constant is determined by the capacitor and the 50,000-ohm and 10,000-ohm resistors.

$$CR = 0.1 \times 10^{-6} \times (50 + 10) \times 10^{3}$$
$$= 6 \text{ milliseconds.} \quad \textit{Ans.}$$

Current Increase through Modulation

FCC Study Guide Question 4.97

7.03 If a transmitter is modulated 100 per cent by a sinusoidal tone, what percentage increase in antenna current will occur?

Solution:

The formula for the effective total current is

$$I_{mod} = I_c \sqrt{1 + (m^2/2)},$$

where I_{mod} = the current during modulation,

I_c = the carrier current,

m = the degree of modulation.

At m = 100 per cent = 1.00, the current during modulation is

$$I_{mod} = I_c \sqrt{1 + 0.5} = I_c \times 1.225,$$

or the antenna current will increase by 22.5 per cent. *Ans.*

The derivation is as follows:

The increase in antenna current is due to the addition of the side-band currents, the amplitude of which is 1/2 the amplitude of the carrier multiplied by the degree of modulation, i.e. $(m\, I_c)/2$ for each carrier.

The average power dissipated by the total current after modulation across a radiation resistance R is $(I^2_{mod}R)/2$, the power of the carrier is $(I_c^2\, R)/2$ and the power of each sideband is

$$\left(\frac{m\, I_c}{2}\right)^2 \frac{R}{2}.$$

Since $$P_{total} = P_{lower\ sideband} + P_{carrier} + P_{upper\ sideband}$$

we have $$\frac{I^2_{mod}\, R}{2} = \left(\frac{m\, I_c}{2}\right)^2 \frac{R}{2} + \frac{I_c^2\, R}{2} + \left(\frac{m\, I_c}{2}\right)^2 \frac{R}{2}$$

Canceling $\frac{R}{2}$, $$I^2_{mod} = \frac{m^2\, I_c^2}{4} + I_c^2 + \frac{m^2\, I_c^2}{4}$$

and $$I^2_{mod} = I_c^2 + I_c^2 \frac{m^2}{2} = I_c^2\left(1 + \frac{m^2}{2}\right).$$

Hence $$I_{mod} = \sqrt{I_c^2\left(1 + \frac{m^2}{2}\right)}$$

$$= I_c \sqrt{1 + \frac{m^2}{2}}.$$

At 100 per cent modulation m = 1.00,

and $$I_{mod} = I_c \sqrt{1 + 0.5} = 1.225\, I_c. \quad \textit{Ans.}$$

Current During Modulation

FCC Study Guide Question 6.25

7.04 A ship's transmitter has an antenna current of 8 amperes using A1 emission. What would be the antenna current when this transmitter is 100 per cent modulated by sinusoidal modulation?

Solution:

Type A1 is continuous-wave emission, unmodulated. Referring to problem 7.03 the modulation will increase the current by 22.5 per cent.

$$I = 8 + (8 \times 0.225) = 9.8 \text{ amperes.} \quad \textit{Ans.}$$

Power for Modulation

FCC Study Guide Question 6.26

7.05 The direct-current plate input to a modulated class C amplifier, with an efficiency of 60 per cent, is 200 watts. What value of sinusoidal audio-frequency power is required in order to insure 100 per cent modulation; 50 per cent modulation?

Solution:

(a) Using the formula

$$P_{sb} = 0.5\, m^2\, P_c\,,$$

where P_{sb} is sideband power

m the degree of modulation

P_c the carrier power,

we obtain: at 50 per cent modulation

$$P_{sb} = 0.5 \times (0.50)^2 \times 200 = 25 \text{ watts;} \quad \textit{Ans.}$$

at 100 per cent modulation

$$P_{sb} = 0.5\,(1.00)^2 \times 200 = 100 \text{ watts.} \quad \textit{Ans.}$$

(b) By analysis.

The value of the sinusoidal audio power is equal to the sideband power introduced by the modulation. The amplitude of the sideband waves is 1/2 the carrier × the degree of modulation; the power being proportional to the square of the amplitude of voltage or current, will be

$$(1/2 \times \text{degree of modulation})^2 \times P_{carrier}$$

for one sideband.

For 100 per cent modulation

$$1/4 \times (1.00)^2 \times 200 = 50 \text{ watts for one sideband,}$$
$$100 \text{ watts for two sidebands.} \quad \textit{Ans.}$$

For 50 per cent modulation

$$(1/2 \times 0.5)^2 \times 200 = \frac{0.25 \times 200}{4}$$
$$= 12.5 \text{ watts for one sideband,}$$
$$25 \text{ watts for two sidebands.} \quad \textit{Ans.}$$

Comparing Different Degrees of Modulation

7.06 Which of the two signals is more effective:

(a) a carrier amplitude of 150 volts 50 per cent modulated

or (b) a carrier amplitude of 100 volts 100 per cent modulated?

Solution:

The amplitude of the sound-frequency voltage after detection is proportional to both the carrier amplitude and the degree of modulation; we write

$$E_o = k \times (\text{percentage modulation}) \times E_c$$

where k depends on the circuit constants and tube characteristics of the detector.

$$\text{(a)} \quad E_o = k \times 0.50 \times 150 = 75\,k \text{ peak volts,}$$
$$\text{(b)} \quad E_o = k \times 1.00 \times 100 = 100\,k \text{ peak volts.}$$

The signal (b) is more effective. *Ans.*

Bandwidth During Modulation

FCC Study Guide Question 6.28

7.07 What is the total bandwidth of a transmitter using A2 emission with a modulating frequency of 800 cycles and a carrier frequency of 500 kilocycles? What are the upper and the lower frequencies?

Solution:

A2 is modulated telegraphy.

The bandwidth is $800 \times 2 = 1600$ cycles.

The upper frequency

$$f_u = 500 + 1.6 = 501.6 \text{ kilocycles.}$$

The lower frequency

$$f_l = 500 - 1.6 = 498.4 \text{ kilocycles.} \quad \textit{Ans.}$$

Ratio of Peak Currents

FCC Study Guide Question 6.44

7.08 In 100 per cent amplitude modulation, what is the ratio of peak antenna current to unmodulated antenna current?

Solution:

Since the peak current of each sideband is (see problem 7.03)

$$0.5 \times 1.00 \times I_c = 0.5 \times I_c\,,$$

the total peak current is

$$0.5\,I_c + I_c + 0.5\,I_c = 2\,I_c.$$

The ratio of the peak current to the unmodulated current is 2 to 1. *Ans.*

Ratio of Peak Powers

FCC Study Guide Question 6.45

7.09 In 100 per cent modulation, what is the ratio of instantaneous peak antenna power to unmodulated antenna power?

Solution:

Referring to problem 7.08, and realizing that the power is proportional to the square of the current ($I^2 R$),
we have

$$P_m : P_c = 2^2 R : 1^2 R = 4 \text{ to } 1. \quad \textit{Ans.}$$

Sideband Power

FCC Study Guide Question 4.57

7.10 At 100 per cent modulation, what percentage of the total output power is in the sidebands?

Solution:

Since the sideband power is equal to 1/2 the carrier power, the sideband power is

$$P_m = 0.5\, P_c,$$

and
$$\frac{\text{sideband power}}{\text{total power}} = \frac{P_m}{P_m + P_c}$$
$$= \frac{0.5\, P_c}{0.5\, P_c + P_c} = \frac{0.5}{1.5}$$
$$= \frac{1}{3} = 0.333 = 33\tfrac{1}{3} \text{ per cent.} \quad \textit{Ans.}$$

Power Reduction in Sidebands

FCC Study Guide Question 4.80

7.11 If you decrease the percentage of modulation from 100 per cent to 50 per cent, by what percentage have you decreased the power in the sidebands?

Solution:

Referring to the discussion in problem 7.05, the power of the sidebands is reduced from 100 watts to 25 watts.

$$\frac{100 - 25}{100} = 0.75 = 75 \text{ per cent.} \quad \textit{Ans.}$$

Peak Voltage During Modulation

FCC Study Guide Question 4.104

7.12 If a vertical antenna has a resistance of 500 ohms and a reactance of zero at its base, and an antenna power input of 10 kilowatts, what is the peak voltage to ground under 100 per cent modulation conditions?

Solution:

The peak power at 100 per cent modulation being proportional to the square of the current amplitude will be 4 times the unmodulated power since the current is 2 times the unmodulated current.

$$4 \times 10 = 40 \text{ kw} = 40{,}000 \text{ watts}$$

Using the formula (all peak values)

$$P = \frac{E^2}{R},$$

we obtain
$$40{,}000 = \frac{E^2}{500},$$

and
$$E = \sqrt{500 \times 40{,}000}$$
$$= 1000\sqrt{20} = 4473 \text{ volts.} \quad \textit{Ans.}$$

Modulator Input

FCC Study Guide Question 4.115

7.13 A certain transmitter has an output of 100 watts. The efficiency of the final modulated amplifier stage is 50 per cent. Assuming that the modulator has an efficiency of 66 per cent, what plate input to the modulator is necessary for 100 per cent modulation of this transmitter?

Solution:

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}$$

$$0.50 = \frac{100}{\text{input}};$$

$$\text{input} = 100 \div 0.5 = 200 \text{ watts.}$$

At 100 per cent modulation the modulator must supply half of the power

$$200/2 = 100 \text{ watts.}$$

The efficiency of the modulator = 66 per cent

$$0.66 = \frac{100}{\text{input}}.$$

The input to the modulator is

$$100/(0.66) = 151.5 \text{ watts.} \quad \textit{Ans.}$$

Modulation Reading from Ammeter

7.14 The output current of a transmitter is 8 amperes when the unmodulated carrier is radiated. With modulation applied, the current increases to 15.2 amperes. What is the percentage modulation?

Solution:

Since the increase due to both sidebands is equal to the carrier current times the modulation percentage, we have

$$\text{Increase} = 15.2 - 8 = 7.2 \text{ amperes}$$

$$7.2 = 8 \times m;$$

$$m = \frac{7.2}{8} = 90 \text{ per cent.} \quad \textit{Ans.}$$

Channel Widths

7.15 What channel width is required for a radio signal emitted as

(a) modulated telegraphy (800-cycle note)

(b) radio telephone (200- to 2000-cycle speech)

(c) broadcast (100- to 5000-cycle music)

Solution:

Regardless of the carrier frequency the sideband frequency of (a) will be 800 cycles above and below the assigned frequency:

Width for (a) $800 + 800 = 1600 = 1.6$ kilocycles. *Ans.*,

for (b) $2000 + 2000 = 4$ kilocycles. *Ans.*,

for (c) $5000 + 5000 = 10$ kilocycles. *Ans.*

Resistance of Cooling System

FCC Study Guide Question 4.122

7.16 The 50-kilowatt output stage of a broadcast transmitter, having a final amplifier efficiency of 33 per cent, has a plate current of 10 amperes. If the water-cooling system leakage-current meter reads 11 milliamperes, what is the resistance of the water system from plate to ground?

Solution:

The efficiency is

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{P_o}{E \times I}$$

$$0.33 = \frac{50 \times 10^3}{E \times 10}.$$

Transposing

$$E = \frac{50 \times 10^3}{10 \times 0.33} = 15{,}150 \text{ volts.}$$

This voltage also exists across the cooling system. Its resistance therefore is

$$R = \frac{E}{I} = \frac{15{,}150}{11 \times 10^{-3}}$$

$$= 1{,}377{,}000 \text{ ohms.} \quad \textit{Ans.}$$

Transmitter Power Loss

FCC Study Guide Question 4.126

7.17 A 50-kilowatt transmitter employs 6 tubes in push-pull parallel in the final class B linear stage, operating with a 50-kilowatt output and an efficiency of 33 per cent. Assuming that all of the heat radiation is to the water-cooling system, what amount of power must be dissipated from each tube?

Solution:

The efficiency is

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}$$

$$0.33 = \frac{50 \times 10^3}{P_i}$$

$$P_i = \frac{50 \times 10^3}{0.33} = 151{,}500 \text{ watts.}$$

The total loss is the input minus the output

$$Loss_t = 151{,}500 - 50{,}000 = 101{,}500 \text{ watts.}$$

The loss per tube is

$$Loss_1 = \frac{101{,}500}{6} = 16{,}917 \text{ watts.} \quad \textit{Ans.}$$

Interference Caused by Modulation

7.18 A 500-kilocycle carrier is modulated by a 1000-cycle note. If the note contains harmonics up to the 6th, what are the frequency ranges of interference?

Solution:

The channel width of the modulated signal is

$$2 \times 1000 = 2000 = 2 \text{ kilocycles};$$

it extends from 499 to 501 kilocycles.

The channel width of the harmonics is

$$2 \times 6000 = 12{,}000 = 12 \text{ kilocycles};$$

it extends from 494 to 506 kilocycles.

The zones of interference will be from 494 to 499 kilocycles and from 501 to 506 kilocycles. *Ans.*

Modulation Power for Stated Tube

7.19 The transmitting tube type 860 is operated at a plate voltage of 2000 volts and a plate current of 85 milliamperes. Calculate the power which must be supplied by the modulator for a 100 per cent sinusoidal plate modulation.

Solution:

Since the efficiency of a class C amplifier is almost constant whether it is modulated or unmodulated, the modulator power will be subject to the same proportion of losses as the carrier input.

The carrier input is

$$2000 \times 85 \times 10^{-3} = 170 \text{ watts}.$$

The modulator power must be $\frac{1}{2}$ of this power:

$$170/2 = 85 \text{ watts.} \quad \textit{Ans.}$$

Modulation Impedance

7.20 The power amplifier type 805 operates as class C with a plate voltage of 1250 volts, a direct plate current of 160 milliamperes and a d-c grid bias of -160 volts. What is the modulation impedance for these conditions?

Solution:

Only the plate voltage and the plate current are used for the computation of the modulation impedance.

$$Z_m = \frac{E_p}{I_p} = \frac{1250}{160 \times 10^{-3}} = 7800 \text{ ohms.} \quad \textit{Ans.}$$

Impedance Matching

7.21 The class C amplifier type 833 operating at a plate voltage of 2000 volts is to be modulated with a class B modulator using two type 805 tubes with a plate supply of 1250 volts, a rated power output of 300 watts, and a rated plate to plate load of 6700 ohms. If the coupling is accomplished by an output transformer, calculate its proper turns ratio for 100 per cent modulation.

Solution:

Assuming a transformer efficiency of 90 per cent, a power of

$$300 \times 0.9 = 270 \text{ watts}$$

will be available for modulation. The class C amplifier will need an input of

$$2 \times 270 = 540 \text{ watts}$$

for 100 per cent modulation, and its plate current will be

$$\frac{540}{2000} = 270 \text{ milliamperes;}$$

the modulation impedance will be

$$Z_m = \frac{2000}{270 \times 10^{-3}} = 9100 \text{ ohms.}$$

The turns ratio is obtained from the formula

$$\left(\frac{N_2}{N_1}\right)^2 = \frac{Z_2}{Z_1},$$

where Z_2 is the modulation impedance.

Therefore $$\frac{N_2}{N_1} = \sqrt{\frac{9100}{6700}} = 1.17 \text{ to } 1. \quad \textit{Ans.}$$

Oscillator Frequency Tolerance

FCC Study Guide Question 5.196

7.22 A station has an assigned frequency of 8000 kilocycles and a frequency tolerance of plus or minus 0.04 per cent. The oscillator operates at 1/8 the output frequency. What is the maximum permitted deviation of the oscillator frequency, in cycles, which will not exceed the tolerance?

Solution:

The frequency tolerance at the antenna is 0.04 per cent of the assigned frequency. Expressing 0.04 per cent in powers of 10

$$0.04 \text{ per cent} = 0.04 \times 10^{-2} = 4 \times 10^{-4},$$

The permissible frequency deviation at the antenna is

$$f_a = 8 \times 10^3 \times 4 \times 10^{-4} = 3200 \text{ cycles.}$$

The permissible frequency deviation in the oscillator stage is

$$f_o = 3200/8 = 400 \text{ cycles.} \quad \textit{Ans.}$$

Grid Modulation. Carrier Power

7.23 A radio-frequency amplifier tube has a rated plate dissipation of 60 watts. If used for grid modulation what will be the approximate unmodulated radio-frequency output for the tube?

Solution:

Since the plate-voltage swing under carrier conditions can utilize only about half of the d-c supply, in order to be doubled at the crest of an audio cycle, the efficiency is about 1/2 of an unmodulated class C amplifier.

$$\eta = 70/2 = 35 \text{ per cent, approximately.}$$

The plate dissipation will be

$$100 - 35 = 65 \text{ per cent, approximately.}$$

We have the proportion

$$\frac{P}{35} = \frac{60}{65},$$

and

$$P = \frac{35 \times 60}{65}$$

$$= 32.3 \text{ watts, approximately.} \quad \textit{Ans.}$$

Grid Modulation Versus Plate Modulation

7.24 Using the data in problem 7.23, what would be the approximate carrier power that can be expected for plate modulation?

Solution:

The plate-voltage swing in this case can utilize the whole d-c supply; the current will be multiplied by 2 and the power ($I^2 R$) by 4. The power output will be

$$P = 32.3 \times 4$$

$$= 129.2 \text{ watts at 100 per cent modulation.}$$

One-third of this power will be audio-frequency power, and the carrier power will be

$$P_c = 129.2 \times 0.66 = 85 \text{ watts, approximately.} \quad \textit{Ans.}$$

Frequency Modulation. Deviation from Carrier

7.25 A frequency-modulated carrier operates between 36,985 and 37,015 kilocycles. What is the carrier frequency and what is the frequency deviation?

Solution:

The band-width is

$$37{,}015 - 36{,}985 = 30 \text{ kilocycles}$$

The frequency deviation from the mid-frequency is

$$30/2 = 15 \text{ kilocycles.} \quad \textit{Ans.}$$

and the carrier frequency is

$$36{,}985 + 15 = 37{,}000 \text{ kilocycles} = 37 \text{ megacycles.} \quad \textit{Ans.}$$

Frequency-Modulation Index

7.26 A 37-megacycle carrier is frequency-modulated, so as to have a channel width of 100-kilocycles. What will be the modulation index (a) for an 800-cycle note, (b) for 3000-cycle telephone service and (c) for 15,000-cycle high-fidelity music?

Solution:

The modulation index is the ratio of the frequency deviation to the modulating frequency.

The frequency deviation is

$$100/2 = 50 \text{ kilocycles}$$

The modulation indices are

(a) for code $50{,}000/800 = 62.5.$ *Ans.*

(b) for speech $50{,}000/3{,}000 = 16.7.$ *Ans.*

(c) for music $50{,}000/15{,}000 = 3.33.$ *Ans.*

Phase Splitting in Reactance-Tube Circuit

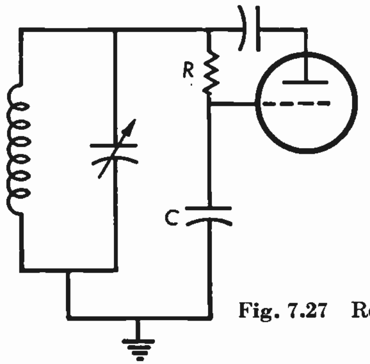

Fig. 7.27 Reactance tube phase-splitting circuit.

7.27 The phase-splitting circuit *R-C* of the reactance tube modulator of Figure 7.27 is to be designed so as to make the circuit practically resistive and so that the current through *C* leads the voltage by no more than 0.5 degree. If $C = 20$ micromicrofarads and the oscillator operates at a frequency of 7 megacycles what should be the value of *R*?

Solution:

$$\tan 0.5° = \frac{X}{R}$$

$$0.0087 = \frac{X}{R};$$

now $$X_c = 1/(2\pi f C) = 1135 \text{ ohms.}$$

$$0.0087 = \frac{1135}{R}$$

$$R = \frac{1135}{8.7 \times 10^{-3}} = 130{,}000 \text{ ohms.} \quad \textit{Ans.}$$

Frequency Division

7.28 A frequency-modulation transmitter is to operate at a frequency of 85.5 megacycles with a band-width of 100 kilocycles. If the output frequency is the result of a frequency multiplication by 9, what frequency deviation should the reactance tube oscillator be able to produce, and to which frequency will its tank circuit be tuned?

Solution:

The output deviation is

$$100/2 = 50 \text{ kilocycles.}$$

Since the frequency deviation increases directly with the frequency multiplication, the fundamental deviation is

$$50/9 = 5.55 \text{ kilocycles.} \quad \textit{Ans.}$$

The tank circuit is tuned to

$$85.5/9 = 9.5 \text{ megacycles.} \quad \textit{Ans.}$$

Audio Frequency in Frequency Modulation

7.29 In problem 7.28 find the highest audio frequency for which the reactance tube modulator exhibits a modulation index of $m = 5$?

Solution:

Using the formula $$m = \frac{F_d}{f_m},$$

we obtain $$5 = \frac{50{,}000}{f_m},$$

and $$f_m = 50{,}000 \div 5$$
$$= 10{,}000 \text{ cycles.} \quad \textit{Ans.}$$

Channel Width for High-Fidelity Music

7.30 What should be the frequency limits of a 82.5 megacycle frequency-modulation broadcast station if it is desired to obtain a minimum modulation index of $m = 5$?

Solution:

Substituting in $$m = \frac{F_d}{f_m}$$

and recognizing the fact that 15,000 cycles is the upper audio-frequency limit,

we obtain $$5 = \frac{F_d}{15{,}000}$$

and $$F_d = 5 \times 15{,}000 = 75{,}000 \text{ cycles}$$
$$= 0.075 \text{ megacycles.}$$

the frequency limits would then be

$$85.5 + 0.075 = 85.575 \text{ megacycles.} \quad \textit{Ans.}$$

and $$85.5 - 0.0075 = 85.425 \text{ megacycles.} \quad \textit{Ans.}$$

Frequency Modulation in the Broadcast Band

7.31 Show why it is impractical to operate a frequency-modulation station in the regular broadcast band, illustrating the problem for the case of a station with an assigned frequency of 1000 kilocycles.

Solution:

If an audio-frequency range of 10,000 cycles is to be reproduced with a modulation index of $m = 5$, the deviation frequency would be

$$m = \frac{F_d}{f_m}$$

$$5 = \frac{F_d}{10{,}000},$$

and $$F_d = 10{,}000 \times 5$$
$$= 50{,}000 \text{ cycles or } 50 \text{ kilocycles.}$$

The channel width would be

$$50 \times 2 = 100 \text{ kilocycles.} \quad \textit{Ans.}$$

This would occupy a frequency range from

$$1000 - 50 = 950 \text{ kilocycles}$$

to

$$1000 + 50 = 1050 \text{ kilocycles;}$$

this width is occupied by 10 amplitude-modulated stations.

Note: According to the United States Standards which allow a deviation of $\pm$ 75 kilocycles the occupied band would be

$$75 \times 2 = 150 \text{ kilocycles}$$

a width which could be used by

$$150 \div 10 = 15 \text{ amplitude-modulated stations.} \quad \textit{Ans.}$$

8 *Receivers*

Bandspread Capacitors

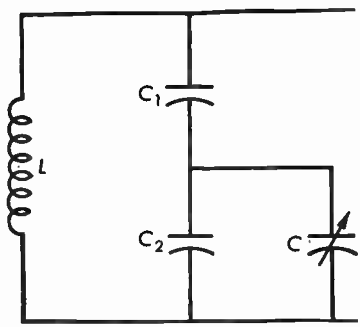

Fig. 8.01 Tank circuit with bandspread capacitor.

8.01 In the tuning circuit of Figure 8.01, $C_1 = 50$ micromicrofarads, $C_2 = 25$ micromicrofarads, $C = 10$ to 100 micromicrofarads, $L = 50$ microhenries.

What is the frequency range of the tuning circuit?

(a) with the capacitance C only

(b) with C and C_2, C_1 short-circuited

(c) with all three in the circuit

Which of the three arrangements will have the greatest bandspread effect?

Solution:

(a) The upper frequency limit

$$F_a = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.28\sqrt{50 \times 10^{-6} \times 10 \times 10^{-12}}}$$

$$= 7150 \text{ kilocycles.} \quad \textit{Ans.}$$

The lower frequency limit

$$f_a = \frac{1}{6.28\sqrt{50 \times 10^{-6} \times 100 \times 10^{-12}}} = 2260 \text{ kilocycles. } \quad \textit{Ans.}$$

(b) The extreme values of the total capacitance are

$$c_b = 10 + 25 = 35 \text{ micromicrofarads.}$$

$$C_b = 100 + 25 = 125 \text{ micromicrofarads.}$$

$$F_b = \frac{1}{6.28\sqrt{50 \times 10^{-6} \times 35 \times 10^{-12}}}$$

$$= 3810 \text{ kilocycles. } \quad \textit{Ans.}$$

$$f_c = \frac{1}{6.28\sqrt{50 \times 10^{-6} \times 125 \times 10^{-12}}}$$

$$= 2010 \text{ kilocycles. } \quad \textit{Ans.}$$

(c) $$c_c = \frac{50 \times (10 + 25)}{50 + (10 + 25)} = 20.6 \text{ micromicrofarads.}$$

$$C_c = \frac{50 \times (100 + 25)}{50 + (100 + 25)} = 35.7 \text{ micromicrofarads.}$$

$$F_c = \frac{1}{6.28\sqrt{50 \times 10^{-6} \times 20.6 \times 10^{-12}}}$$

$$= 4960 \text{ kilocycles. } \quad \textit{Ans.}$$

$$f_c = \frac{1}{6.28\sqrt{50 \times 10^{-6} \times 35.7 \times 10^{-12}}}$$

$$= 3680 \text{ kilocycles. } \quad \textit{Ans.}$$

The tuning ranges are:

(a) 7150 − 2260 = 4890 kilocycles.

(b) 3810 − 2010 = 1800 kilocycles.

(c) 4960 − 3680 = 1280 kilocycles.

(c) will spread the narrowest band over the whole dial. *Ans.*

Band Spreading with Permeability Tuning

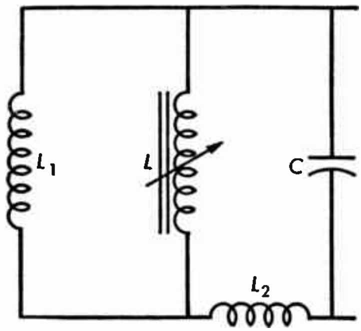

Fig. 8.02 Tank circuit with permeability tuning.

8.02 Write the equation of the resonant frequency with the following parts in the circuit:

(a) C and L
(b) C, L and L_1, aiding
(c) C, L and L_1, opposing
(d) C, L, L_1, L_2, aiding
with no mutual inductance in any case.

Solution:

(a) With one inductance in the circuit the usual equation for resonant frequency is written:

$$f_a = \frac{1}{2\pi\sqrt{LC}}. \quad Ans.$$

(b) The total inductance is found by means of a formula corresponding to the product-sum formula used for resistances in parallel.

$$L_t = \frac{L L_1}{L + L_1}.$$

Therefore $$f_b = \frac{1}{2\pi\sqrt{\dfrac{L L_1}{L + L_1} C}}. \quad Ans.$$

(c) $f_c = f_b$, since there is no mutual inductance.

(d) The total inductance exceeds the value found under (b) by L_2 henries

$$L_T = L_2 + \frac{L L_1}{L + L_1}$$

The resonant frequency is therefore

$$f_d = \frac{1}{2\pi\sqrt{\left(L_2 + \frac{L L_1}{L + L_1}\right) C}}. \quad Ans.$$

Possible Frequency Combinations

8.03 A superheterodyne receiver works with intermediate-frequency stages tuned to 456 kilocycles. If a signal of 1750 kilocycles is to be received, what could be the frequency of the oscillator, and what would be the image frequency of the lower of the two possible oscillator frequencies?

Solution:

The upper oscillator frequency is

$$F = 1750 + 456 = 2206 \text{ kilocycles.} \quad Ans.$$

The lower oscillator frequency is

$$f = 1750 - 456 = 1294 \text{ kilocycles.} \quad Ans.$$

In the latter case a signal frequency of

$$f_s = 1294 - 456 = 838 \text{ kilocycles}$$

would also mix to the intermediate frequency. 838 kilocycles would therefore be the image frequency. *Ans.*

Tracking Range

8.04 A broadcast superheterodyne receiver has 456-kilocycle intermediate-frequency coils. What is the necessary tuning range of the oscillator to cover the broadcast band?

Solution:

The broadcast band is from 550 to 1500 kilocycles. The lowest oscillator frequency must therefore be

$$f = 550 + 456 = 1006 \text{ kilocycles.} \quad Ans.$$

The highest oscillator frequency is

$$F = 1500 + 456 = 1956 \text{ kilocycles.} \quad Ans.$$

Image Frequency

FCC Study Guide Question 3.71

8.05 If a superheterodyne receiver is tuned to a desired signal at 1000 kilocycles and its conversion oscillator is operating at 1300 kilocycles, what would be the frequency of an incoming signal which would possibly cause image reception?

Solution:

If the intermediate-frequency stage is tuned to the difference frequency

$$1300 - 1000 = 300 \text{ kilocycles intermediate frequency,}$$

then an image signal of

$$1300 + 300 = 1600 \text{ kilocycles}$$

will also give a beat frequency of 300 kilocycles.
1600 kilocycles is the image frequency. *Ans.*
If the intermediate-frequency stage were tuned to the sum frequency

$$1300 + 1000 = 2300 \text{ kilocycles intermediate frequency,}$$

an image signal of

$$2300 + 1300 = 3600 \text{ kilocycles}$$

would produce a beat note of 2300 kilocycles intermediate frequency. In this case 1000 and 3600 kilocycles would be the image frequencies. The first arrangement is the more common.

Interference

FCC Study Guide Question 5.72

8.06 A superheterodyne receiver, having an intermediate frequency of 465 kilocycles and tuned to a broadcast station on 1450 kilocycles, is receiving severe interference from an image signal. What is the frequency of the interfering station?

Solution:

The oscillator frequency is

$$1450 + 465 = 1915 \text{ kilocycles,}$$

and since

$$1915 + 465 = 2380 \text{ kilocycles,}$$

a signal of 2380 kilocycles comes from the interfering station. *Ans.*

Mixer-Oscillator Frequency

FCC Study Guide Question 5.73

8.07 A superheterodyne receiver is tuned to 1712 kilocycles and the intermediate frequency is 456 kilocycles. What is the frequency of the mixer oscillator?

Solution:

It is the usual practice to tune the mixer-oscillator to a frequency higher than the incoming signal. In this case the mixer-oscillator frequency is

$$F_{mo} = 1712 + 456 = 2168 \text{ kilocycles.} \quad \textit{Ans.}$$

Intermediate Frequency

FCC Study Guide Question 6.57

8.08 If a superheterodyne receiver is receiving a signal of 1000 kilocycles and the mixing oscillator is tuned to 1500 kilocycles, what is the intermediate frequency?

Solution:

The intermediate frequency is the difference between the mixing oscillator frequency and the signal frequency.

Intermediate frequency $= 1500 - 1000 = 500$ kilocycles. *Ans.*

Oscillator Coil and Capacitor

8.09 The oscillator circuit of a broadcast receiver operating at an intermediate frequency of 456 kilocycles is capacitance-tuned, and the maximum capacitance is 365 micromicrofarads. What is the inductance of the oscillator coil, and what is the lowest capacitance of the variable capacitor?

Solution:

The oscillator frequencies are

$$550 + 456 = 1006 \text{ kilocycles,}$$

$$1500 + 456 = 1956 \text{ kilocycles.}$$

Solving the formula

$$f = \frac{1}{2\pi\sqrt{LC}}$$

for L, and remembering that the lowest frequency will require the highest capacitance, we obtain

$$L = \frac{1}{4\pi^2 f^2 C}$$

$$= \frac{1}{39.5 \times 1006^2 \times 10^6 \times 365 \times 10^{-12}}$$

$$= 68.5 \text{ microhenries.} \quad \textit{Ans.}$$

The lowest capacitance will cause oscillations at the highest frequency, therefore

$$C = \frac{1}{4\pi^2 f^2 L}$$

$$= \frac{1}{39.5 \times 1956^2 \times 10^6 \times 68.5 \times 10^{-6}}$$

$$= 99.5 \text{ micromicrofarads.} \quad \textit{Ans.}$$

Detector Efficiency

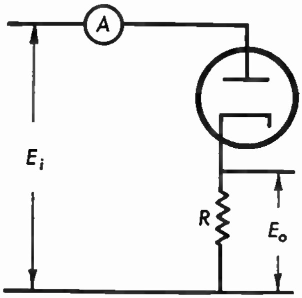

Fig. 8.10 Diode detector.

8.10 The diode detector of Figure 8.10 has a plate resistance of 5000 ohms. The peak input signal $E_i = 30$ volts. When a load resistor of $R = 0.5$ megohms is used, the ammeter reads 40 microamperes. What is the efficiency of the detector?

Solution:

The efficiency of a detector is

$$\eta = \frac{\text{d-c output}}{\text{peak signal input}}.$$

Under the given operating conditions the output voltage is

$$E_o = I\,R = 40 \times 10^{-6} \times 0.5 \times 10^{6} = 20 \text{ volts},$$

causing an efficiency of

$$\eta = \frac{E_o}{E_i} = \frac{20}{30} = 0.66 = 66 \text{ per cent.} \quad \textit{Ans.}$$

Power Consumed by Detector

8.11 In problem 8.10 what power does the diode draw from the input circuit?

Solution:

Since the diode current flows only when the signal voltage is near its peak value, the power drawn from the input circuit is approximately

$$P = E_i \times I = 30 \times 40 \times 10^{-6}$$
$$= 1200 \text{ microwatts.} \quad \textit{Ans.}$$

Loading Effect

8.12 What is the equivalent input resistance of the diode in problem 8.10?

Solution:

Since $\quad P = \dfrac{E_{eff}^2}{R_{eff}} \quad$ and $\quad E_{eff} = E_{peak}/\sqrt{2},$

we obtain $\quad P = \dfrac{(E_i/\sqrt{2})^2}{R_{eff}},$

and

$$R_{eff} = \frac{E_i^2}{2\,P}$$
$$= \frac{900}{2 \times 1200 \times 10^{-6}}$$
$$= 375{,}000 \text{ ohms.} \quad \textit{Ans.}$$

Note: A similarly derived formula

$$R_{eff} = \frac{R}{2\,\eta}$$

yields

$$R_{eff} = \frac{500{,}000}{2 \times 0.66}$$
$$= 375{,}000 \text{ ohms.} \quad \textit{Ans.}$$

Television Detector Efficiency

8.13 Because of the band-pass characteristic of the input circuit, the equivalent input resistance of a television detector R_{eff} should be in the order of about 10 kilohms. If the plate resistance of the detector tube is 5000 ohms, what value of load resistance should be applied, and what will be the efficiency of the detector?

Solution:

Since the plate resistance is 5000 ohms, the load resistor, being in series with it, must also be 5000 ohms to add up to 10 kilohms.
Using the formula given in problem 8.12, viz.,

$$R_{eff} = \frac{R}{2\eta}$$

we obtain $$10{,}000 = \frac{5000}{2\eta},$$

and $$\eta = \frac{5000}{20{,}000} = 0.25 = 25 \text{ per cent.} \quad \textit{Ans.}$$

Diode Versus Plate Detection

8.14

(a) $$E_d = \eta \times m \times E_i$$

(b) $$E_p = \eta \times \mu \times m \times E_i$$

are formulas for computing the audio-frequency voltage E_d or E_p as they appear across the load of a diode or plate detector, respectively. If both operate with an efficiency of $\eta = 80$ per cent receiving a 95 per cent modulated signal and both are to deliver a 15-volt audio-frequency signal, what input voltage E_i must the radio-frequency stages deliver to (a) the diode; (b) the plate detector, which has a $\mu = 35$?

Solution:

Substituting in (a)

$$15 = 0.8 \times 0.95 \times E_i.$$

The signal voltage necessary for diode detection is

$$E_i = \frac{15}{0.8 \times 0.95} = 19.75 \text{ volts.} \quad \textit{Ans.}$$

Substituting in (b)

$$15 = 0.8 \times 35 \times 0.95 \times E_i$$

The signal voltage necessary for plate detection is

$$E_i = \frac{15}{0.8 \times 35 \times 0.95}$$

$$= 0.564 \text{ volts.} \quad \textit{Ans.}$$

Volume Control

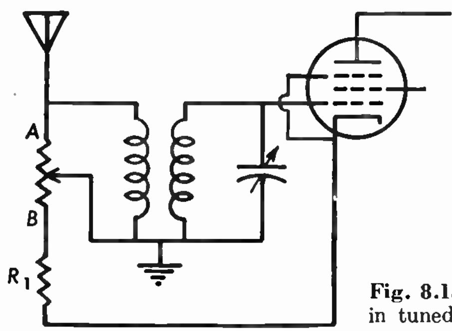

Fig. 8.15 Volume control, common in tuned radio frequency receivers.

8.15 The antenna-bias type volume control of Figure 8.15 has a value of 10,000 ohms and is variable only from A to B. The 6K7 tube is operated under the following conditions: $E_p = 90$ volts, $E_s = 90$ volts, $I_p = 5.4$ milliamperes, $I_s = 1.3$ milliamperes, and $E_g = -3$ volts. Calculate the value of the variable range of the tapped volume control. If this type volume control were not available for replacement, what would be a fair value for R_1 and R_{ab} employing two separate parts.

Solution:

The cathode current

$$I_c = I_p + I_s = 6.7 \text{ milliamperes.}$$

Since the bias at full volume is −3 volts, we have

$$R = \frac{3}{6.7 \times 10^{-3}} = 450 \text{ ohms, approximately;}$$

$$R_{ab} = 10{,}000 - 450$$
$$9550 \text{ ohms, approximately.} \quad \textit{Ans.}$$

A fair value for R_{ab} would be 10,000 ohms, and for R_1 500 ohms, making a total resistance of 10,500 ohms. The percentage of error would then be

$$\text{error} = \frac{\text{difference}}{\text{original value}}$$

$$= \frac{500}{10{,}000} = 0.05 = 5 \text{ per cent.}$$

Automatic Volume Control

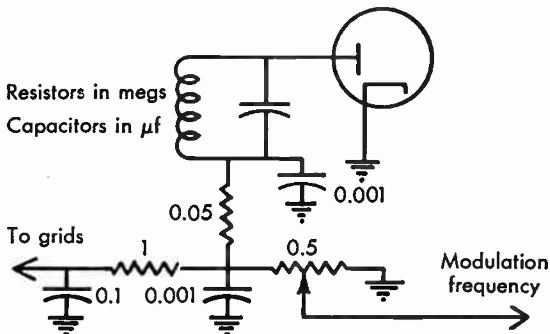

Fig. 8.16 Automatic volume control.

8.16 In the automatic volume control circuit of Figure 8.16 what is the approximate time constant during the charge period and during the discharge period?

Solution:

During the charge period i.e. when the diode is conducting the resistance-capacitance circuit consists of the 1-megohm and the 0.05-megohm resistor in series with the 0.1 microfarad capacitor.

$$t_c = (1 + 0.05)\, 10^6 \times 0.1 \times 10^{-6} = 1.05 \times 0.1$$
$$= 0.105 \text{ second.} \quad \textit{Ans.}$$

During the discharge period the 0.1 microfarad capacitor will discharge through the 1-megohm resistor in series with the 0.5-megohm volume control.

$$t_d = (1 + 0.5) \times 10^6 \times 0.1 \times 10^{-6}$$
$$= 0.15 \text{ second.} \quad \textit{Ans.}$$

Tuning Eye

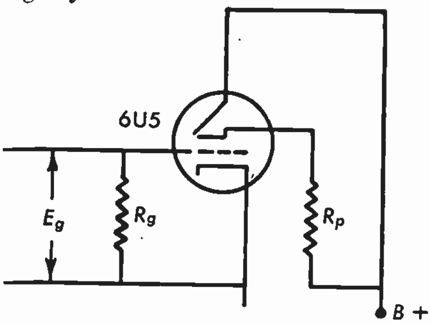

Fig. 8.17 Electron-ray tube indicator.

8.17 In the tuning-eye circuit of Figure 8.17, $R_g = 2$ megohms, $I_p = 190$ microamperes, $R_p = 0.5$ megohm, $B+ = 100$ volts, and $E_g = 0.1$ volt. What is the potential difference between the ray-control electrode and the target?

Solution:

The potential difference between the ray-control electrode and the target is determined by the current flow through R_p.

$$E_p = I_p R_p$$
$$= 190 \times 10^{-6} \times 0.5 \times 10^6$$
$$= 95 \text{ volts.} \quad \textit{Ans.}$$

The potential difference is 95 volts, the target being positive and the ray-control electrode negative.

Negative Feedback

8.18 An amplifier has an input signal of 4.5 volts and an output of 45 volts, of which 1/45 is fed back to the input in opposite phase to the input. What is the new gain, and what is the gain reduction factor?

Solution:

To obtain the same output it will require a 5.5-volt signal, since 1/45 of the output (= 1 volt) in opposite phase will cancel 1 volt. The new gain is

$$G_1 = \frac{E_o}{E_g'} = \frac{45}{5.5} = 8.17$$

To find the gain reduction factor we first calculate the no-feedback gain:

$$G = \frac{E_o}{E_g} = \frac{45}{4.5} = 10,$$

and the gain reduction factor is

$$\frac{G_1}{G} = \frac{8.17}{10} = 0.817. \quad \textit{Ans.}$$

Reduction of Distortion

8.19 In problem 8.18, if 9 per cent harmonic distortion was present without negative feedback, how much harmonic distortion is present with feedback?

Solution:

Since the harmonic distortion is reduced by a factor approximately equal to the gain reduction factor,

$$\text{Percentage of distortion} = 9 \times 0.817 = 7.36 \text{ per cent.} \quad \textit{Ans.}$$

Frequency Distortion

8.20 An amplifier has a voltage input of 10 volts peak and a voltage output of 160 volts for the audio-frequency band, except in the region of 3500 cycles where the output is 290 volts for the same signal input. If 5 per cent of the output voltage is applied to the input in opposite phase, what is the frequency distortion with and without feedback?

Solution:

(a) Without feedback

$$G = \frac{160}{10} = 16.$$

The voltage amplification at 3500 cycles is

$$G = \frac{290}{10} = 29.$$

The frequency distortion at 3500 cycles is

$$\frac{29 - 16}{16} = 0.813 = 81.3 \text{ per cent.} \quad \textit{Ans.}$$

There is a hump of 81 per cent excess gain at 3500 cycles.

(b) With feedback

In order to have a comparison basis for the computation, let us refer the input voltages to the 160-volt output, 5 per cent of which will be the feedback voltage:

$$160 \times 0.05 = 8 \text{ volts.}$$

It will now require an input signal of

$$8 + 10 = 18 \text{ volts}$$

in order to obtain 160 volts output, since 8 volts will be canceled out by the feedback. The new gain is

$$G = \frac{160}{18} = 8.88.$$

At 3500 cycles, however, where the voltage amplification is 29, a smaller net grid input would cause the same output of 160 volts, viz.,

$$160/29 = 5.5 \text{ volts.}$$

This net grid voltage will require an incoming signal of

$$8 + 5.5 = 13.5 \text{ volts}$$

to overcome the negative feedback. The gain with feedback at 3500 cycles is

$$G_1 = \frac{160}{13.5} = 11.85$$

The frequency distortion at 3500 cycles is

$$\frac{11.85 - 8.88}{8.88} = 0.335$$

There is only 33.5 per cent excess gain at 3500 cycles when negative feedback is applied. *Ans.*

Negative Feedback for Transformer Coupling

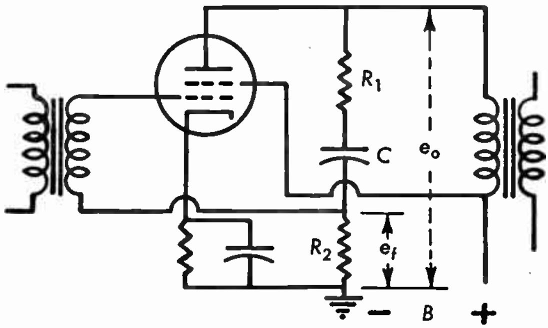

Fig. 8.21 Transformer-coupled amplifier with negative feedback.

8.21 In the circuit of Figure 8.21, $R_1 = 50{,}000$ ohms, $R_2 = 2500$ ohms. Calculate the feedback factor β.

Solution:

The feedback factor is that part of the output voltage which is applied in opposition to the input signal.

$$\beta = \frac{e_f}{e_o} = \frac{2500}{50{,}000 + 2500}$$

$$= \frac{2.5}{52.5} = 0.0475$$

$$= 4.75 \text{ per cent, approximately.} \quad \textit{Ans.}$$

Note: The blocking capacitor C must be big enough to offer little reactance to the frequencies involved.

Negative Feedback for Resistance Coupling

8.22 In the circuit of Figure 8.21a C_1 and C_2 are large enough to offer negligible reactance at audio frequencies; $r_p = 100{,}000$ ohms, $R_L = 0.5$ megohm, $R_g = 0.75$ megohm. What value of the feedback resistor R_f will provide 10 per cent negative feedback?

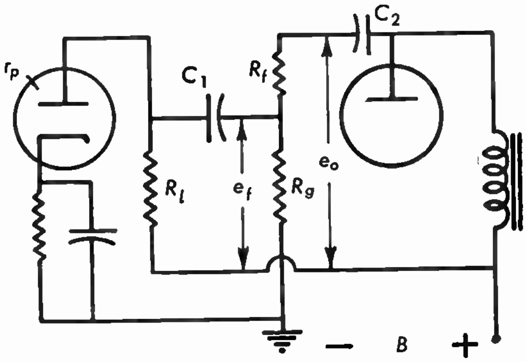

Fig. 8.22a Resistance-coupled amplifier with negative feedback.

Solution:

The feedback factor $\beta = \dfrac{e_f}{e_o}$. Figure 8.22b represents the equivalent circuit.

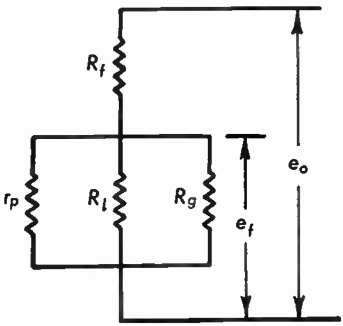

Fig. 8.22b Equivalent circuit at Fig. 8.22a.

The parallel resistor combination is (in kilohms)

$$R = \frac{750}{1 + 750/500 + 750/100} \quad \text{(formula 1.173)}$$

$$= 750/10 = 75 \text{ kilohms.}$$

The feedback factor is

$$\beta = \frac{R}{R + R_f}.$$

Substituting $$0.1 = \frac{75{,}000}{R_f + 75{,}000}$$

$$0.1\,R_f + 7500 = 75{,}000$$

$$0.1\,R_f = 75{,}000 - 7500$$

$$0.1\,R_f = 67{,}500,$$

and $$R_f = 675{,}000 \text{ ohms.} \quad \textit{Ans.}$$

Negative Feedback for Push-Pull Circuit

8.23 In the circuit of Figure 8.23, $R_1 = 50{,}000$ ohms, $R_f = 5000$ ohms; the coupling capacitor C is large enough to offer only negligible reactance at audio frequencies. Calculate the feedback factor β.

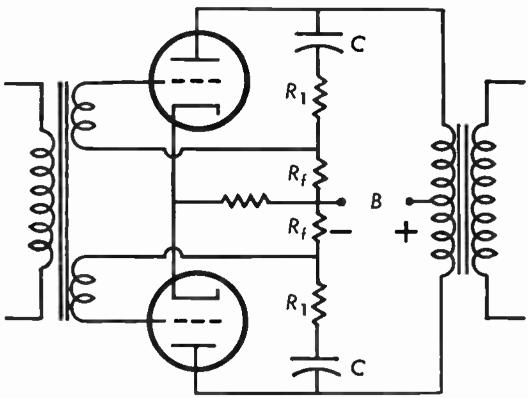

Fig. 8.23 Push-pull circuit with negative feedback.

Solution:

The effective voltage divider is connected from the capacitor to B minus.

$$\beta = \frac{R_f}{R_f + R_1}$$

$$= \frac{5000}{55{,}000} = 0.909 = 9.09 \text{ per cent.} \quad \textit{Ans.}$$

Infinite Attenuation

8.24 In the circuit of Figure 8.21, problem 8.21, the input signal is 10 volts, the output is 100 volts. What value of R_2 will kill the signal?

Solution:

The input signal will be canceled out when the feedback voltage is equal and opposite to the input signal. We then have the proportion

$$\frac{100}{10} = \frac{50{,}000 + R_2}{R_2}$$

$$10\,R_2 = 50{,}000 + R_2,$$

and

$$R_2 = 50{,}000/9 = 5555 \text{ ohms.} \quad \textit{Ans.}$$

Tone Control. Bass Attenuation

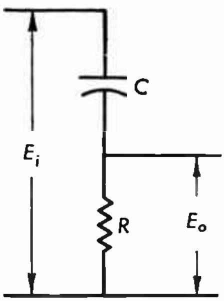

Fig. 8.25 Resistance-capacitance circuit for bass attenuation.

8.25 Write the equation of the voltage output as a function of the input for the bass attenuation circuit of Figure 8.25.

Solution:

$$E_o = E_i \frac{R}{R + X_c},$$

where

$$X_c = -j\frac{1}{2\,\pi f C},$$

or

$$|E_o| = E_i \frac{R}{\sqrt{R^2 + X_c^2}} \text{ volts.} \quad \textit{Ans.}$$

Tone Control. Treble Attenuation

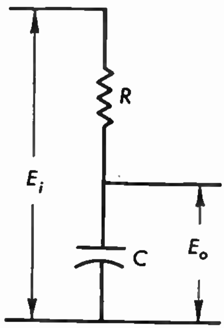

Fig. 8.26 Resistance-capacitance circuit for treble attenuation.

8.26 Write the equation of the voltage output E_o as a function of the voltage input E_i of the treble-attenuation circuit of Figure 8.26. Discuss the extreme cases.

Solution:

$$\mathring{E}_o = E_i \frac{X_c}{R + X_c},$$

where

$$X_c = -j \frac{1}{2 \pi f C}$$

$$|E_o| = E_i \frac{X_c}{\sqrt{R^2 + X_c^2}} \text{ volts.} \quad \textit{Ans.}$$

Discussion: When the frequency approaches zero, X_c will approach infinity, R will be very small compared with X_c, and negligible. Therefore, at bass notes

$$E_o \rightarrow E_i \frac{X_c}{X_c} \rightarrow = E_i.$$

The full bass response will appear across C. When the frequency approaches infinity, X_c approaches zero, and the fraction $\dfrac{X_c}{R + X_c}$ will be

$$0/R = 0,$$

therefore at treble notes

$$E_o \rightarrow E_i \times 0 \rightarrow 0.$$

The treble notes will be infinitely attenuated.

Loudspeaker Power

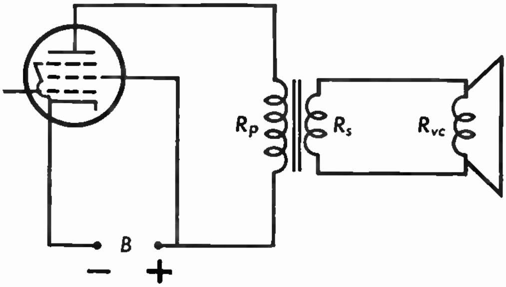

Fig. 8.27a Audio output circuit.

8.27 In the circuit of Figure 8.27a $R_p = 250$ ohms, $R_s = 0.8$ ohms, $R_{vc} = 4.5$ ohms, the step-down ratio is 20 to 1. What part of the total power output of 4.5 watts will be transferred to the loudspeaker?

Solution:

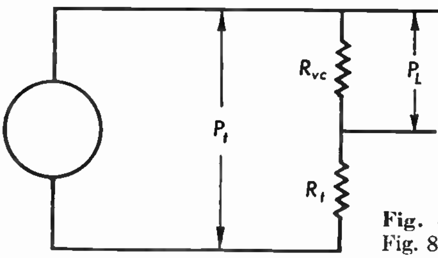

Fig. 8.27b Equivalent circuit of Fig. 8.27a.

Since the inductance need not be considered for power output, the equivalent circuit is approximately represented in Figure 8.27b. The part transferred to the loudspeaker is

$$P_l = 4.5 \frac{R_{vc}}{R_t + R_{vc}}.$$

The transformer resistance R_t is the resistance of the primary winding of the power transformer plus the reflected resistance of the secondary

$$R_t = R_p + n^2 R_s$$

$$= 250 + 400 \times 0.8 = 580 \text{ ohms.}$$

R_{vc} is the reflected resistance of the voice coil,

$$R_{vc} = 400 \times 4.5 = 1800 \text{ ohms.}$$

The power transferred to the speaker is

$$P_L = 4.5 \times \frac{1800}{1800 + 580}$$

$$= 4.5 \times \frac{1800}{2380} = 3.4 \text{ watts.} \quad \textit{Ans.}$$

Output Transformer Efficiency

8.28 What is the efficiency of the output transformer of problem 8.27?

Solution:

The efficiency is the ratio of the output power to the input power. In the case of problem 8.27 it is the ratio of the power delivered to the voice coil to the power delivered to the primary of the transformer. The power input to the transformer is 4.5 watts, and as was found in problem 8.27, the power delivered to the voice coil is 3.4 watts. Therefore the efficiency is

$$\eta = \frac{P_{vc}}{P_t}$$

$$= \frac{3.4}{4.5} = 0.756 = 75.6 \text{ per cent.} \quad \textit{Ans.}$$

Miller Effect

8.29 Due to the Miller effect the radio-frequency stage of a receiver experiences a noticeable degree of detuning in a circuit normally operating with a gain of 10, a grid-cathode capacitance of 1.8 micromicrofarads, and a grid-plate capacitance of 1.4 micromicrofarads. If a strong signal has caused the automatic volume control to drop the gain to 8, what change in input capacitance will detune the circuit?

Solution:

The normal input capacitance to the tube is

$$C_i = C_{gk} + C_{gp}\,(1 + G)$$

which in the above circuit is

$$\begin{aligned} C_i &= 1.8 + 1.4\,(1 + 10) \\ &= 17.2 \text{ micromicrofarads.} \end{aligned}$$

The bias from the automatic volume control will cause an input capacitance

$$\begin{aligned} C_i' &= 1.8 + 1.4\,(1 + 8) \\ &= 14.4 \text{ micromicrofarads.} \end{aligned}$$

The change in input capacitance is

$$\begin{aligned} \Delta C_i &= 17.2 - 14.4 \\ &= 2.8 \text{ micromicrofarads.} \quad \textit{Ans.} \end{aligned}$$

or

$$\frac{2.8}{17.2} = 0.163 = 16.3 \text{ per cent.} \quad \textit{Ans.}$$

9 *Power Supplies*

Half-Wave Rectifier. Capacitor Input

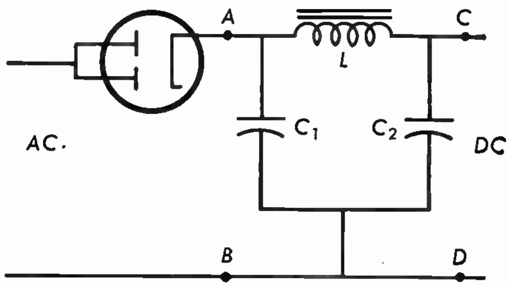

Fig. 9.01a Half-wave rectifier with filter circuit.

9.01 The rectifier type 25Z5 is used in the rectifier circuit of Figure 9.01a. The line voltage is 117 volts, 60 cycles, $L = 15$ henries, $C_1 = C_2 = 16$ microfarads. The resistance of the choke is 30 ohms. The d-c load draws 50 milliamperes from the rectifier. What is the *approximate* ripple voltage at AB? At CD?

Solution:

By formula 9.11,

$$E_{ab} = \frac{I\sqrt{2}}{2\pi f C}$$

$$= \frac{50 \times 10^{-3} \times 1.414}{6.28 \times 60 \times 16 \times 10^{-6}}$$

$$= 11.75 \text{ volts, approximately.} \quad \textit{Ans.}$$

To find the ripple voltage at CD it is necessary to calculate the reactance of the choke

$$X_L = 2\pi f L = 6.28 \times 60 \times 15$$

$$= 5650 \text{ ohms}$$

Compared with the reactance the d-c resistance of 300 ohms can be neglected.

$$X_c = \frac{1}{2\,\pi\, f\, C} = \frac{1}{6.28 \times 60 \times 16 \times 10^{-6}}$$

$$= 166 \text{ ohms}$$

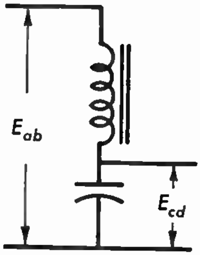

Fig. 9.01b Voltage-divider circuit of the filter output of Fig. 9.01a.

$$E_{cd} = E_{ab} \times \frac{X_c}{Z},$$

but Z is practically X_L, therefore the ripple voltage at CD is

$$E_{cd} = 11.75 \times \frac{166}{5650}$$

$$= 0.344 \text{ volts, approximately.} \quad \textit{Ans.}$$

Per Cent Ripple of Half-Wave Rectifier

9.02 From the manufacturer's operation characteristics ($I_{dc} - E_{dc}$ curve) of the 25Z5 it is found that the rectifier of problem 9.01 will operate with 129 direct volts at input to filter when the d-c drain is 50 milliamperes. Calculate the per cent ripple of the filter in problem 9.01 at AB and CD.

Solution:

The per cent ripple is the ratio of the ripple voltage to the direct voltage. At AC it is

$$\text{Per cent ripple at } AB = \frac{E_{a\text{-}c}}{E_{d\text{-}c}} = \frac{11.75}{129} = 0.0912$$

$$= 9.12 \text{ per cent, approximately.} \quad \textit{Ans.}$$

The direct voltage at CD is determined by the drop across the choke

$$E_{ch} = IR = 50 \times 10^{-3} \times 300 = 15 \text{ volts.}$$

The direct voltage at CD is

$$E'_{d\text{-}c} = 129 - 15 = 114 \text{ volts}$$

The per cent ripple at CD is

$$\text{Per cent ripple at } CD = \frac{0.344}{114} = 0.00301$$

$$= 0.301 \text{ per cent, approximately.} \quad \textit{Ans.}$$

Full-Wave Rectifier. Capacitor Input

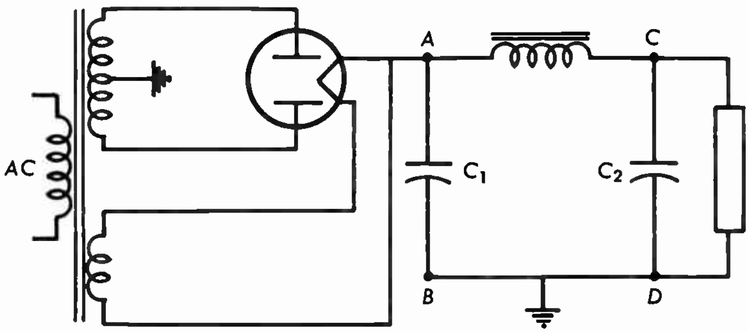

Fig. 9.03a Full-wave rectifier with filter circuit.

9.03 The rectifier type 80 is used in the rectifier circuit of Figure 9.03a. The line voltage is 117 volts, 60 cycles, the secondary high voltage is 800 volts from plate to plate, $L = 25$ henries, 2000 ohms, $C_1 = 4$ microfarads, $C_2 = 8$ microfarads. The d-c load draws 80 milliamperes from the rectifier. What is the *approximate* ripple voltage at AB? At CD?

Solution:

Since this is a full-wave rectifier, the ripple frequency $f = 120$ cycles.

By formula 9.11,

$$E_{ab} = \frac{I\sqrt{2}}{2\,\pi f C}$$

$$= \frac{80 \times 10^{-3} \times \sqrt{2}}{6.28 \times 120 \times 4 \times 10^{-6}}$$

$$= 37.6 \text{ volts.} \quad \textit{Ans.}$$

$$X_l = 2\,\pi f L$$

$$= 6.28 \times 120 \times 25 = 18{,}800 \text{ ohms.}$$

The impedance of the choke

$$Z_l = \sqrt{18{,}800^2 + 2000^2} = 18{,}900 \text{ ohms,}$$

$$X_c = \frac{1}{2\,\pi f C} = \frac{1}{754 \times 8 \times 10^{-6}}$$

$$= 166 \text{ ohms.}$$

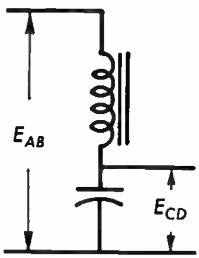

Fig. 9.03b Voltage-divider circuit of the filter output of Fig. 9.03a.

Figure 9.03b represents the equivalent voltage-divider circuit.

$$E_{cd} = E_{ab}\frac{X_c}{Z_t};$$

now Z_t is practically Z_l, since X_c is very small compared with Z_l.

Therefore $$E_{cd} = 37.6 \times \frac{166}{18{,}900} = 0.331 \text{ volts.} \quad \textit{Ans.}$$

Per Cent Ripple. Full-Wave Rectifier

9.04 From the manufacturer's operation characteristic ($I_{d\text{-}c} - E_{d\text{-}c}$ curve) of the type 80 tube it is found that the rectifier of problem 9.03 will operate with 450 direct volts at input to filter when the rms input voltage from plate to plate is 800 volts and the d-c drain is 80 milliamperes. Calculate the per cent ripple of the filter in problem 9.03 at AB and at CD.

Solution:

$$\text{Per cent ripple at } AB = \frac{E_{a\text{-}c}}{E_{d\text{-}c}} = \frac{37.6}{375} = 0.1004$$

$$= 10.04 \text{ per cent, approximately.} \quad \textit{Ans.}$$

The voltage drop across the choke is

$$E_{ch} = IR = 80 \times 10^{-3} \times 2000 = 160 \text{ volts.}$$

The voltage at the filter output is therefore

$$E_{cd} = 450 - 160 = 290 \text{ volts,}$$

$$\text{Per cent ripple at } CD = \frac{0.331}{290} = 0.00114$$

$$= 0.114 \text{ per cent, approximately.} \quad \textit{Ans.}$$

Full-Wave Rectifier. Fourier Analysis

9.05 The harmonic composition of the output of a full-wave rectifier is

$$y = \frac{2}{\pi} E \left(1 + \frac{2}{2^2 - 1} \cos 2x - \frac{2}{4^2 - 1} \cos 4x + \frac{2}{6^2 - 1} \cos 6x \cdots \right)$$

Where E is the peak of the alternating voltage applied to the rectifier tube.

Calculate: (a) the d-c component,
(b) the amplitude of the second harmonic
(c) the amplitude of the 4th harmonic
(d) the amplitude of the 10th harmonic

in terms of the a-c input.

Solution:

The d-c component is constant and is found by multiplying $2E/\pi$ times the constant number in the parentheses.

$$E_{d\text{-}c} = 2E/\pi \times 1 = +0.637 E \text{ volts.} \quad \textit{Ans.}$$

The harmonic amplitudes (peaks) are found by multiplying $2\,E/\pi$ times the coefficients of $\cos nx$, where n indicates the nth harmonic.

$$E_2 = \frac{2}{\pi} E \times \frac{2}{2^2 - 1} = +E\frac{2 \times 2}{3\,\pi}$$
$$= +0.425\,E \text{ volts.} \quad Ans.$$

$$E_4 = \frac{2}{\pi} E \times \frac{-2}{4^2 - 1} = -E\frac{4}{15\,\pi}$$
$$= -0.0849\,E \text{ volts.} \quad Ans.$$

$$E_{10} = \frac{2}{\pi} E \times \frac{2}{10^2 - 1} = E\frac{4}{99\,\pi}$$
$$+0.0129\,E \text{ volts.} \quad Ans.$$

Choke Input. Ripple Calculations

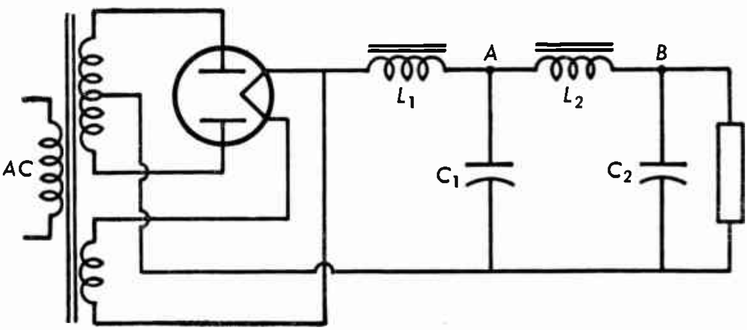

Fig. 9.06a Full-wave rectifier with two-section filter.

9.06 The rectifier circuit of Figure 9.06a is connected to a 60-cycle a-c line. The secondary voltage is 550 volts peak per plate. $L_1 = 25$ henries, 1000 ohms; $L_2 = 15$ henries, 300 ohms; $C_1 = C_2 = 8$ microfarads. The equivalent load resistance across the output capacitor is 6000 ohms. Calculate the approximate per cent ripple across the input and the output capacitor. Do not use formulas.

Solution

(a) *Input capacitor*

The main alternating voltage is the one of the second harmonic; in terms of the a-c peak input it was found in problem 9.05 to be

$$E_2 = E_p \times 0.425 \text{ peak volts.}$$

The rms value of this ripple voltage is

$$E_{a\text{-}c} = \frac{E_p \times 0.425}{\sqrt{2}}$$

$$= \frac{550 \times 0.425}{\sqrt{2}} = 165 \text{ volts.}$$

The per cent ripple is the ratio of the average voltage to the rms ripple voltage. The average voltage (direct voltage) at the input to the filter is

$$E_{d\text{-}c} = 0.637 \times 550 = 350 \text{ volts.}$$

The direct voltage across the input capacitor is determined by the voltage drop across the first choke. We have a direct voltage divider circuit from the filament through the chokes and the load to ground.

$$E'_{ch} = E_{d\text{-}c} \times \frac{1000}{1000 + 300 + 6000}$$

$$= 350 \times 0.137 = 48 \text{ volts.}$$

The direct voltage across the input capacitor is

$$E_i = E_{d\text{-}c} - E_{ch}$$

$$= 350 - 48 = 302 \text{ volts.}$$

The alternating voltage across the input capacitor $E'_{a\text{-}c}$ is found from the alternating voltage divider of Figure 9.06b.

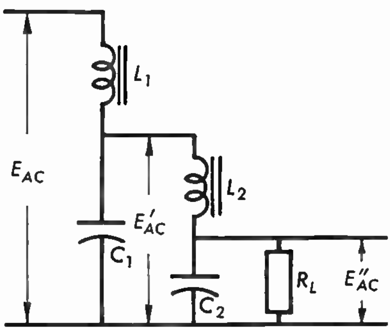

Fig. 9.06b Equivalent circuit of the filter sections of Fig. 9.06a.

The reactance of the 8-microfarad capacitor at the ripple frequency of 120 cycles is

$$X_c = \frac{1}{2\pi f C}$$

$$= \frac{1}{754 \times 8 \times 10^{-6}} = 166 \text{ ohms.}$$

The reactance of the 25-henry choke is

$$X_l' = 2\,\pi\,f\,L_1$$

$$= 754 \times 25 = 18{,}850 \text{ ohms.}$$

The reactance of the 15-henry choke is

$$X_l'' = 2\,\pi\,f\,L_2$$

$$= 754 \times 15 = 11{,}300 \text{ ohms.}$$

It is clear that the shunt effect of the second choke in series with C_2 will be negligible compared with the small reactance of C_1 (approximately 11,000 ohms shunting 160 ohms!). A further simplification is brought in by the fact that the first choke is determined by its reactance only, since

$$Z_{ch} = \sqrt{18{,}850^2 + 1000^2}$$

$$\cong 18{,}850 \text{ ohms, approximately.}$$

The voltage divider thus consists of X_l' and X_c only.

$$E'_{a\text{-}c} = E_{a\text{-}c} \times \frac{-j\,166}{+j\,18{,}850 + (-j\,166)}$$

$$= 165 \times 0.00888 = 1.467 \text{ volts}$$

The per cent ripple at the input capacitor is therefore

$$\text{Per cent ripple (input)} = \frac{E'_{a\text{-}c}}{E_i} = \frac{1.467}{302} = 0.00486$$

$$= 0.486 \text{ per cent, approximately.} \quad \textit{Ans.}$$

(b) *Output capacitor*

The direct voltage across the output capacitor is determined by the voltage drop across the second choke. The drop across the second choke is

$$E''_{ch} = 350 \times \frac{300}{1000 + 300 + 6000}$$

$$= 350 \times 0.0411 = 14.4 \text{ volts.}$$

The direct voltage across the input capacitor is therefore

$$E_o = E_i - E''_{ch} = 302 - 14.4$$

$$= 287.6 \text{ volts.}$$

The alternating voltage across the input capacitor $E''_{a\text{-}c}$ is found to be that part of $E'_{a\text{-}c}$ which appears across C_2; the shunt effect of the 6000-ohm load is negligible compared with the 166-ohm reactance.

Thus
$$E''_{a\text{-}c} = E'_{a\text{-}c} \times \frac{-j\,166}{j\,11{,}300 + (-j\,166)}$$
$$= 1.467 \times 0.0148 = 0.0218 \text{ volts.}$$

The per cent ripple at the output is therefore

$$\text{Per cent ripple (output)} = \frac{E''_{a\text{-}c}}{E_o} = \frac{0.0218}{287.6} = 0.000076$$
$$= 0.0076 \text{ per cent, approximately.} \quad \textit{Ans.}$$

Filter Formulas

9.07 Calculate the per cent ripple across the input and the output capacitors in problem 9.06 with the aid of the ripple-filter formulas.

Solution:

(a) Input capacitor

The direct voltage across the input capacitor is found as in problem 9.06

$$E_i = 302 \text{ volts.}$$

The alternating voltage across the input capacitor is

$$E'_{a\text{-}c} = E_{a\text{-}c} \times \frac{1}{(2\,\pi\,f)^2\,L_1\,C_1}.$$

$E_{a\text{-}c}$ is found as in problem 9.06

$$E_{a\text{-}c} = 165 \text{ volts rms;}$$

substituting
$$E'_{a\text{-}c} = 165 \times \frac{1}{754^2 \times 25 \times 8 \times 10^{-6}}$$
$$= 165 \times \frac{10^6}{569{,}000 \times 200}$$
$$= 165 \times 0.0088 = 1.453 \text{ volts;}$$

the per cent ripple at the input capacitor is therefore

$$\text{Per cent ripple (input)} = \frac{E''_{a\text{-}c}}{E_i} = \frac{1.453}{302} = 0.00482$$
$$= 0.482 \text{ per cent, approximately.} \quad \textit{Ans.}$$

(b) Output capacitor

The direct voltage across the input capacitor is found as in problem 9.06

$$E_o = 287.6 \text{ volts;}$$

the alternating voltage across the output capacitor is

$$E''_{a\text{-}c} = E'_{a\text{-}c} \frac{1}{(2\,\pi\,f)^2 \times L_2\,C_2}$$

$$= 1.453 \frac{1}{754^2 \times 15 \times 8 \times 10^{-6}}$$

$$= \frac{1.453 \times 10^6}{569{,}000 \times 120} = 0.0213 \text{ volts;}$$

the per cent ripple at the output capacitor is therefore

$$\text{Per cent ripple (output)} = \frac{E''_{a\text{-}c}}{E_o} = \frac{0.0213}{287.6} = 0.0000744$$

$$= 0.00744 \text{ per cent, approximately.} \quad \textit{Ans.}$$

Critical Inductance

9.08 A full-wave choke-input rectifier, working from a 60-cycle line is to deliver a direct output voltage of 350 volts and a current of 75 milliamperes. What is the critical input inductance?

Solution:

By formula 9.24,

$$L = \frac{R}{1130} = \frac{350/(75 \times 10^{-3})}{1130} = \frac{350 \times 10^3}{75 \times 1130}$$

$$= 4.13 \text{ henries or more.} \quad \textit{Ans.}$$

Regulation By Bleeder

9.09 An audio-frequency amplifier has a plate supply of 450 volts and a current drain from the power supply of 120 milliamperes at full signal, and 25 milliamperes at no signal. Calculate the value of a bleeder resistor which will prevent the current from falling below 25 per cent of the maximum drain.

Solution:

Let the bleeder current be x milliamperes. The maximum current drain is

$$I_{max} = (120 + x) \text{ milliamperes.}$$

The minimum current drain is

$$I_{min} = (25 + x).$$

Since the minimum current is 25 per cent of the maximum current, we can write

$$0.25\,(120 + x) = 25 + x$$
$$30 + 0.25\,x = 25 + x$$
$$5 = 0.75\,x$$
$$x = \frac{5}{0.75} = 6.65 \text{ milliamperes}$$

Neglecting the voltage drop due to increased drain, the bleeder resistor is

$$R_b = \frac{E}{I} = \frac{450}{6.65 \times 10^{-3}} = 67{,}700 \text{ ohms.} \quad \textit{Ans.}$$

Its power dissipation will be

$$P = I\,E$$
$$= 6.65 \times 10^{-3} \times 450 = 2.99 \text{ watts.} \quad \textit{Ans.}$$

A 10-watt wire-wound resistor will provide a safety factor of 4, approximately.

Improving Voltage Regulation by Bleeder

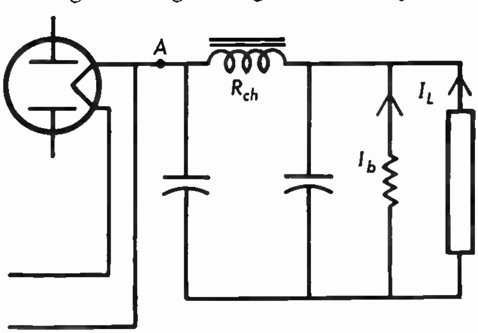

Fig. 9.10 Filter with bleeder resistor.

9.10 The rectifier circuit of Figure 9.10 has a choke resistance $R_{ch} = 250$ ohms and a load demand of $I_L = 90$ milliamperes. A type

5Y4 rectifier tube is used with the following data obtained from the information of the manufacturer:

D-c load in milliamperes	Direct volts at A
0	495
30	435
90	370
120	345

Calculate the voltage regulation without and with a bleeder current of 30 milliamperes. Also find the resistance and the dissipation of the bleeder resistor.

Solution:

(a) Without bleeder

The no-load voltage is

$$V = 495 \text{ volts.}$$

The load voltage is the voltage at 90 milliamperes minus the voltage drop,

$$\begin{aligned} v &= 370 - I_{ch} R_{ch} \\ &= 370 - 90 \times 10^{-3} \times 250 \\ &= 370 - 22.5 = 347.5 \text{ volts.} \end{aligned}$$

The voltage regulation then is

$$\begin{aligned} \frac{V - v}{v} &= \frac{495 - 347.5}{347.5} = \frac{147.5}{347.5} \\ &= 0.425 = 42.5 \text{ per cent.} \quad \textit{Ans.} \end{aligned}$$

(b) With bleeder

The no-load voltage is

$$\begin{aligned} V_b &= 435 - 30 \times 10^{-3} \times 250 \\ &= 435 - 7.5 = 427.5 \text{ volts.} \end{aligned}$$

The load voltage is the voltage at 90 plus 30 milliamperes, minus the voltage across R_{ch}

$$\begin{aligned} v_b &= 345 - 120 \times 10^{-3} \times 250 \\ &= 345 - 30 = 315 \text{ volts.} \end{aligned}$$

The voltage regulation then is

$$\frac{V_b - v_b}{v_b} = \frac{427.5 - 315}{315} = \frac{112.5}{315} = 0.357$$

$$= 35.7 \text{ per cent.} \quad \textit{Ans.}$$

The bleeder resistor is

$$R_b = \frac{V_b}{I_b} = \frac{427.5}{30 \times 10^{-3}}$$

$$= 14{,}200 \text{ ohms, approximately.} \quad \textit{Ans.}$$

The power dissipation of the bleeder resistor is

$$P_b = 427.5 \times 30 \times 10^{-3} = 12.8 \text{ watts.} \quad \textit{Ans.}$$

Swinging Choke

9.11 In problem 9.09, between what critical values should the swinging choke vary between the no-signal current and the full-signal current?

Solution:

The no-signal current is the load current plus the bleeder current

$$I_{min} = 25 + 6.65 = 31.65 \text{ milliamperes.}$$

The effective load resistance is

$$R' = \frac{E}{I_{min}} = \frac{450}{31.65 \times 10^{-3}} = 14{,}200 \text{ ohms,}$$

and the corresponding critical inductance is

$$L' = \frac{R'}{1130} = \frac{14{,}200}{1130} = 12.6 \text{ henries.} \quad \textit{Ans.}$$

Likewise $\quad I_{max} = 120 + 6.65 = 126.65$ milliamperes,

$$R'' = \frac{450}{126.7 \times 10^{3}} = 3560 \text{ ohms,}$$

$$L'' = \frac{R''}{1130} = \frac{3560}{1130} = 3.14 \text{ henries.} \quad \textit{Ans.}$$

Filter Design for Stated Per Cent Ripple

9.12 A choke-input 2-section, full-wave rectifier works into an effective load of 6000 ohms. Neglecting the d-c drops, determine the first filter so as to provide a per cent ripple of 1 per cent. What should be the inductance of the second choke if both capacitors are equal and a final ripple of 0.01 per cent is required?

Solution:

The critical input inductance is

$$L = \frac{R}{1130} = \frac{6000}{1130} = 5.2 \text{ henries}$$

Assuming an applied peak voltage of E volts, the input to the filter will be (problem 9.05)

$$\begin{aligned} E_{a\text{-}c} &= 0.425\,E \text{ volts peak} \\ &= 0.425\,E \times 0.707 \\ &= 0.301\,E \text{ volt rms.} \end{aligned}$$

The direct voltage is approximately

$$E_{d\text{-}c} = 0.637\,E \text{ volts.}$$

$$\text{Per cent ripple} = \frac{E_{a\text{-}c}}{E_{d\text{-}c}}$$

or

$$0.01 = \frac{0.301\,E \times A}{0.637\,E},$$

where A is the attenuation factor by which the alternating voltage 0.301 E must be multiplied in order to obtain the alternating voltage across the first capacitor.

Upon cancellation of E and transposition we obtain

$$A = \frac{0.01 \times 0.637}{0.301} = 0.0211,$$

and by formula 9.22

$$A = \frac{1}{(2\,\pi\,f)^2\,L\,C}$$

yielding

$$0.0211 = \frac{1}{754^2 \times 5.2 \times C},$$

and
$$C = \frac{1}{754^2 \times 5.2 \times 0.0211}$$
$$= \frac{1}{569{,}000 \times 5.2 \times 0.0211}$$
$$= \frac{10^{-3}}{5.69 \times 5.2 \times 2.11}$$
$$= 0.016 \times 10^{-3} = 16 \times 10^{-6}$$
$$= 16 \text{ microfarads.} \quad Ans.$$

Likewise, since the same attenuation is expected from the second section (1/100 of the first) and the direct voltage is assumed to be the same

$$L_2 = L_1 \quad \text{if} \quad C_2 = C_1.$$

Therefore $$L_2 = 5.2 \text{ henries.} \quad Ans.$$

Equivalent Load Resistance

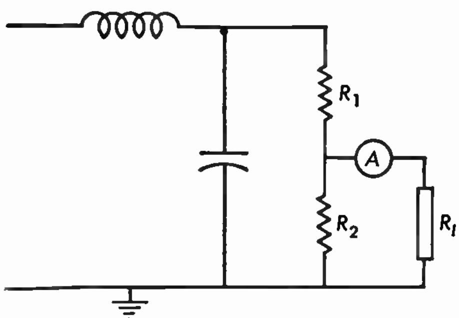

Fig. 9.13 Filter with two-section bleeder and load resistance.

9.13 The output voltage of the power supply of Figure 9.13, $E_{d\text{-}c} = 200$ volts. The two sections of the bleeder are: $R_1 = 3000$ ohms and $R_2 = 2000$ ohms. The reading of the ammeter is 50 milliamperes. What is the equivalent resistance of the load R_L?

Solution:

By Ohm's law the voltage across R_L is

$$E_L = I\,R_L = 0.05\,R_L. \tag{1}$$

By the potentiometer rule the same voltage is

$$E_L = 200 \times \frac{R_{parallel}}{R_{total}}$$

or

$$E_L = 200 \times \frac{\dfrac{2000 \times R_L}{2000 + R_L}}{\dfrac{2000 \times R_L}{2000 + R_L} + 3000}. \tag{2}$$

Equating (1) and (2)

we obtain

$$0.05\, R_L = 200 \times \frac{\dfrac{2000\, R_L}{2000 + R_L}}{\dfrac{2000\, R_L}{2000 + R_L} + 3000}.$$

Multiplying numerator and denominator of the right term by $(2000 + R_L)$ we obtain

$$0.05\, R_L = 200\, \frac{2000\, R_L}{2000\, R_L + 6{,}000{,}000 + 3000\, R_L}.$$

Canceling and simplifying

$$0.05\, R_L = 200\, \frac{2\, R_L}{5\, R_L + 6000}.$$

Dividing both sides by R_L

$$0.05 = \frac{200 \times 2}{5\, R_L + 6000}.$$

Transposing

$$0.25\, R_L + 300 = 400,$$

$$0.25\, R_L = 100,$$

and

$$R_L = \frac{100}{0.25} = 400 \text{ ohms.} \quad \textit{Ans.}$$

Resistance-Capacitance Decoupling Filter

9.14 The plate voltage of the input tube of a high-gain public-address amplifier is obtained through a decoupling filter. The ripple frequency is 120 cycles. A resistor of 5000 ohms is used in the decoupling filter. Determine the value of the capacitor which will reduce the ripple voltage to 1 per cent of the value at the output of the power supply.

Solution:

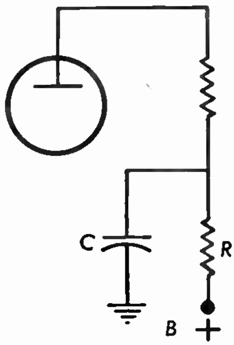

Fig. 9.14a Resistance-capacitance decoupling filter.

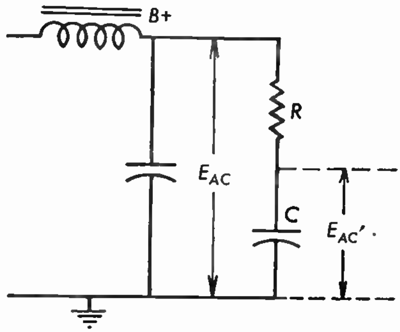

Fig. 9.14b Decoupling filter as connected across the B-supply.

The decoupling filter *R-C* of Figure 9.14a is connected across the power supply essentially as indicated in Figure 9.14b. The ratio of the ripple voltages is

$$\frac{E'_{a\text{-}c}}{E_{a\text{-}c}} = \frac{1}{100}; \text{ but it is also}$$

$$\frac{E'_{a\text{-}c}}{E_{a\text{-}c}} = \frac{X_c}{\sqrt{X_c^2 + R^2}}.$$

Therefore $$\frac{X_c}{\sqrt{X_c^2 + R^2}} = \frac{1}{100},$$

and $$\frac{X_c^2}{X_c^2 + R^2} = 0.0001,$$

$$0.0001\, X_c^2 + 0.0001\, R^2 = X_c^2.$$

Neglecting $0.0001\, X_c^2$ and substituting the given value of R we have

$$0.0001 \times 5000^2 = X_c^2,$$

and $$X_c = \sqrt{10^{-4} \times 5000^2} = 50 \text{ ohms}.$$

Substituting in $$X_c = \frac{1}{2\,\pi\, f\, C}$$

we obtain
$$50 = \frac{1}{754 \times C},$$

and
$$C = \frac{1}{754 \times 50}$$
$$= 26.8 \text{ microfarads.} \quad \textit{Ans.}$$

A 25-microfarad electrolytic capacitor will be suitable.

Mercury-Vapor Rectifier

9.15 A power supply using mercury-vapor rectifier tubes operates with a peak voltage of 2200 volts from plate to plate, a bleeder current of 25 milliamperes and a load current of 250 milliamperes. A choke-input, 2-section filter is used, with choke resistances of 350 and 300 ohms. Calculate the direct output voltage.

Solution:

The direct input voltage to the filter is

$$E_i = 0.637 \times \frac{2200}{2} = 700 \text{ volts.}$$

The voltage drop across the tube is

$$E_{tu} = 15 \text{ volts (constant for mercury-vapor rectifiers).}$$

The voltage drop across the chokes is

$$E_{ch} = I_{ch} \times R_{ch}$$
$$= (25 + 250) \times 10^{-3} \times (350 + 300)$$
$$= 275 \times 10^{-3} \times 650 = 178.5 \text{ volts.}$$

The output voltage is equal to the input voltage minus the drops:

$$E_o = 700 - 15 - 178.5 = 506.5 \text{ volts.} \quad \textit{Ans.}$$

Plate-to-Plate Voltage

9.16 A power supply using mercury-vapor rectifier tubes works into a load of 200 milliamperes. What is the transformer voltage necessary to produce a direct output voltage of 500 volts, if a choke-input 2-section filter is used with choke resistances of 350 and 300 ohms?

Solution:

The transformer voltage is determined by the direct filter-input voltage. This voltage is 500 volts plus the voltage drops across the chokes and the (active) tube. Now the tube drop is

$$E_{tu} = 15 \text{ volts},$$

and the choke drops are

$$E_{ch} = 200 \times 10^{-3} \times 650 = 130 \text{ volts}.$$

The input voltage is therefore

$$E_i = 500 + 15 + 130 = 645 \text{ volts}.$$

Since this is approximately the average of the peak input voltage we have the equation

$$645 = 0.637 \times E_{peak},$$

and $$E_{peak} = \frac{645}{0.637} = 1000 \text{ volts, approximately.} \quad \textit{Ans.}$$

The plate-to-plate voltage will be

$$E_{pp} = 2 \times 1000 = 2000 \text{ volts peak}$$

or $$= \frac{2000}{\sqrt{2}} = 1414 \text{ volts rms.} \quad \textit{Ans.}$$

Voltage Divider Design

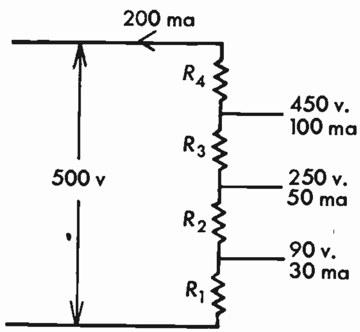

Fig. 9.17 Voltage divider.

9.17 A voltage divider is to be installed in the power supply of problem 9.16, with voltage taps and current drains as indicated in Figure 9.16. Calculate the resistance and the power dissipation of the resistors R_1, R_2, R_3, and R_4.

Solution:

The total current drain is

$$I_t = 100 + 50 + 30 = 180 \text{ milliamperes.}$$

The bleeder current is

$$I_b = 200 - 180 = 20 \text{ milliamperes.}$$

This current will flow through R_1. The value of this resistor is therefore

$$R_1 = \frac{90}{20 \times 10^{-3}} = 4500 \text{ ohms.} \quad \textit{Ans.};$$

$$P_1 = 90 \times 20 \times 10^{-3} = 1.8 \text{ watts.} \quad \textit{Ans.}$$

The current through R_2 is

$$I_2 = 20 + 30 = 50 \text{ milliamperes,}$$

and the voltage across R_2 is

$$E_2 = 250 - 90 = 160 \text{ volts.}$$

These values of current and voltage require a resistor of

$$R_2 = \frac{160}{50 \times 10^{-3}} = 3200 \text{ ohms.} \quad \textit{Ans.},$$

which will dissipate a power of

$$P_2 = 160 \times 50 \times 10^{-3} = 8 \text{ watts.} \quad \textit{Ans.}$$

The current through R_3 is

$$I_3 = 20 + 30 + 50 = 100 \text{ milliamperes,}$$

and the voltage across R_3 is

$$E_3 = 450 - 250 = 200 \text{ volts.}$$

The necessary resistance is therefore

$$R_3 = \frac{200}{100 \times 10^{-3}} = 2000 \text{ ohms.} \quad \textit{Ans.},$$

and the dissipated power

$$P_3 = 200 \times 100 \times 10^{-3} = 20 \text{ watts.} \quad \textit{Ans.}$$

The current through R_4 is

$$I_4 = 20 + 30 + 50 + 100 = 200 \text{ milliamperes,}$$

and the voltage across R_4 is

$$E_4 = 500 - 450 = 50 \text{ volts,}$$

therefore

$$R_4 = \frac{50}{200 \times 10^{-3}} = 250 \text{ ohms.} \quad \textit{Ans.},$$

and $$P_4 = 50 \times 200 \times 10^{-3} = 10 \text{ watts.} \quad \textit{Ans.}$$

Vibrator Power Supply

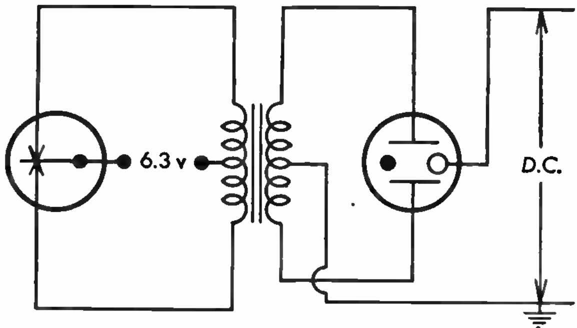

Fig. 9.18 Vibrator-power supply with gas-filled rectifier tube.

9.18 The starting voltage of the gas-filled 0Z4 tube of the vibrator power supply of Figure 9.18 is 300 volts peak, according to the information provided by the manufacturer. When connected to a 6.3-volt automobile battery, the plate-to-plate voltage of the transformer secondary is 460 volts rms. Assuming the same step-up ratio from the dual primary to the full secondary, show if the power supply can be expected to operate on a run-down battery with a terminal voltage of 5.6 volts. Also calculate the approximate minimum battery-voltage demand.

Solution:

The effective step-up ratio of the vibrating direct voltage to the secondary peak voltage is

$$\frac{E_p}{E_s} = \frac{6.3}{460 \times \sqrt{2}} = \frac{1}{103.2}.$$

The secondary voltage at 5.6 volts vibrator input may be

$$E_s' = 5.6 \times 103.2 = 579 \text{ volts peak},$$

with a corresponding peak voltage per plate of

$$579/2 = 289.5 \text{ volts.}$$

The power supply will be inoperative. *Ans.*

Disregarding the voltage drops in the secondary circuit, the plate-to-plate secondary voltage should be

$$E_s'' = 2 \times 300 = 600 \text{ volts peak,}$$

which would require a d-c input of more than

$$E_p'' = 600/103.2 = 5.81 \text{ volts.} \quad \textit{Ans.}$$

Vibrator Time Efficiency

9.19 A vibrator operates with a vibrating frequency of 115 cycles per second. If the time required by the reed to move from one contact to the other is 890 microseconds, what is the time efficiency of the vibrator?

Solution:

The time efficiency is the ratio of the contact time to the total time of a cycle. The total duration of 1 cycle is

$$T_t = \frac{1}{f} = \frac{1}{115} = 0.008696 \text{ second}$$

$$= 8696 \text{ microseconds.}$$

The contact time is

$$T_c = 8696 - 890 = 7806 \text{ microseconds.}$$

The time efficiency is therefore

$$\eta_T = \frac{7806}{8696} = 0.899 = 89.9 \text{ per cent.} \quad \textit{Ans.}$$

Glow-Discharge Tube. Voltage Regulation

9.20 An unregulated d-c supply of 200 volts is to be used as a voltage-regulated source of 150 volts and 25 milliamperes. Which VR tube will be used, and what is the resistance and the power dissipation of the dropping resistor?

Solution:

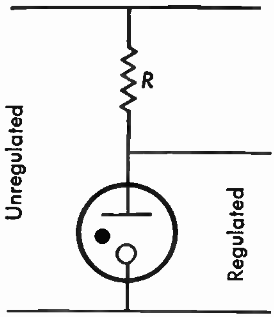

Fig. 9.20 Voltage regulation by glow-discharge tube.

A glow-discharge tube type VR-150/30 or the equivalent type OD3 will be used, because this type maintains a constant potential difference of 150 volts over a varying range of current drains up to 30 milliamperes.

Allowing a glow-tube current of 20 milliamperes, the current through R is

$$I_r = 20 + 25 = 45 \text{ milliamperes.}$$

The voltage drop across R is

$$E_r = 200 - 150 = 50 \text{ volts.}$$

The required resistance is therefore

$$R = \frac{50}{45 \times 10^{-3}} = 1110 \text{ ohms.} \quad \textit{Ans.}$$

$$P = E \times I$$

$$= 50 \times 45 \times 10^{-3} = 2.25 \text{ watts.} \quad \textit{Ans.}$$

Filament Supply for a-c/d-c Superheterodyne

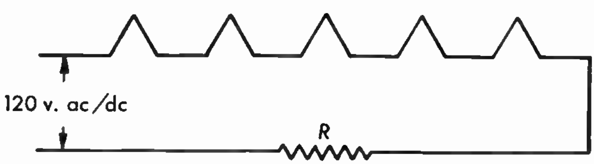

Fig. 9.21 A-c/d-c filament hookup with lamp-cord resistor.

9.21 A universal-type superheterodyne receiver employs the following tubes: 6SA7, 6SK7, 6SQ7, 25L6, and 25Z5. The line voltage is

120 volts. Calculate the necessary value of the lamp-cord resistor R in series with the filament string, and the power lost in the cord resistor.

Solution:

There are three 6.3-volt and two 25-volt tubes in the circuit. The voltage drop across the filament string is

$$E_f = (3 \times 6.3) + (2 \times 25) = 68.9 \text{ volts.}$$

This leaves a voltage across R of

$$E_r = 120 - 68.9 = 51.1 \text{ volts,}$$

and the heater current being 0.3 ampere, the resistance of the cord is

$$R = \frac{51.1}{0.3} = 170 \text{ ohms, approximately.} \quad \textit{Ans.}$$

The power lost in the cord is

$$P = E \times I$$
$$= 51.1 \times 0.3 = 15.33 \text{ watts.} \quad \textit{Ans.}$$

A-Supply for Tuned-Radio-Frequency Receiver with Pilot Lamp

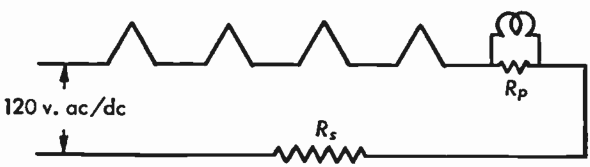

Fig. 9.22 A-c/d-c filament hookup with ballast resistor and pilot lamp.

9.22 The following tubes are used in a tuned-radio-frequency receiver of the transformerless type: 6K7, 6J7, 25L6, and 25Z5. Provisions are made for a type 46 pilot light (6.3 volts, 0.25 ampere). Calculate the resistance of the series dropping resistor R_s and of the pilot shunt resistor R_p, and their power dissipation.

Solution:

The tubes have a heater current of 0.3 ampere. The total voltage across the tubes and the pilot is

$$E_f = (3 \times 6.3) + (2 \times 25) = 68.9 \text{ volts.}$$

The series dropping resistor is therefore

$$R_s = \frac{120 - 68.9}{0.3} = \frac{51.1}{0.3}$$

$$= 170 \text{ ohms, approximately.} \quad \textit{Ans.}$$

The power dissipation of the cord is

$$P_c = 51.1 \times 0.3 = 15.3 \text{ watts.} \quad \textit{Ans.}$$

The resistance of the pilot lamp shunt can be calculated by recognizing that a current of 0.05 ampere will flow through it if 0.25 ampere is to flow through the lamp. Therefore

$$R_p = \frac{6.3}{0.05} = 126 \text{ ohms.} \quad \textit{Ans.};$$

its power dissipation is

$$P_p = 6.3 \times 0.05 = 0.315 \text{ watt.} \quad \textit{Ans.}$$

Sound-Truck Power Supply

9.23 A dynamotor-type power supply of a sound-truck amplifier consumes 500 watts from two storage batteries connected in series, 6.3 volts each. The internal resistance of each battery is 0.015 ohm. What is the voltage at the input to the power supply while the amplifier is "on"?

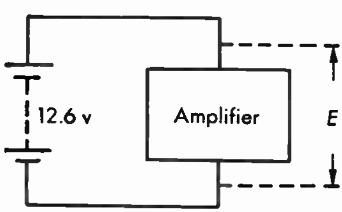

Fig. 9.23a Storage battery driving an amplifier with stated power consumption.

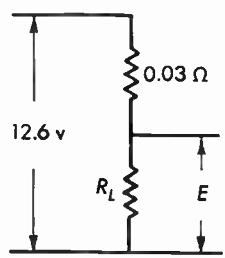

Fig. 9.23b Equivalent circuit of Fig. 9.23a.

Solution:

The total internal resistance is

$$R_i = 2 \times 0.015 = 0.03 \text{ ohm}.$$

In the equivalent voltage-divider circuit of Figure 9.23b, the terminal voltage is

$$E = 12.6 \times \frac{R_L}{R_L + 0.03}. \tag{1}$$

Also, from the given power consumption we have

$$500 = \frac{E^2}{R_L}. \tag{2}$$

These two simultaneous equations are most conveniently solved by substitution. From (2) we obtain

$$R_L = E^2/500,$$

which substituted in (1), yields

$$E = 12.6 \times \frac{E^2/500}{E^2/500 + 0.03}.$$

Multiplying numerator and denominator by 500 we obtain

$$E = \frac{12.6\,E^2}{E^2 + 15},$$

which, after dividing both terms by E, becomes

$$1 = \frac{12.6\,E}{E^2 + 15}$$

and $$E^2 + 15 = 12.6\,E,$$

or $$E^2 - 12.6\,E + 15 = 0.$$

Using the quadratic formula

$$E = \frac{-(-12.6) \pm \sqrt{159 - 60}}{2}$$

$$= \frac{12.6 \pm 9.95}{2}$$

$$= 11.28 \text{ volts or } 1.32 \text{ volts}.$$

Obviously the voltage at the input to the dynamotor is 11.28 volts, and 1.32 volts is the voltage drop in the storage battery. *Ans.*

Repair of "Burnt-Out" 35Z5

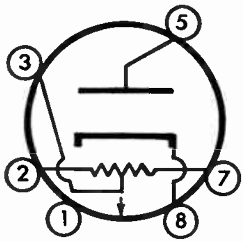

Fig. 9.24 Base connections of the 35Z5 rectifier tube. Prongs 2 and 7 are the heater terminals, prong **3** is the pilot-light tap.

9.24 A continuity test of the heater prongs of a 35Z5 rectifier tube (35 volts, 0.15 ampere) shows continuity between prongs 3 and 7 and an open circuit between prongs 2 and 3. How could this trouble be remedied?

Solution:

By connecting a resistor between prongs 2 and 3. A voltage drop of 6 to 8 volts usually exists between prongs 2 and 3 (6–8 volt pilot light). If the pilot is omitted

$$R_{2,3} = \frac{6}{0.15} = 40 \text{ ohms approximately} \quad \textit{Ans.}$$

and $$P_{2,3} = 6 \times 0.15 = 0.9 \text{ watts};$$

a resistor of 5-watt rating would be safe. *Ans.*

Open Pilot Lamp

9.25 In the filament circuit of problem 9.22, show how a burnt-out pilot lamp will affect the A-supply of the receiver.

Solution:

With the pilot-lamp filament open, the total resistance of the circuit will be changed. Assuming that the hot resistance of the tube filaments will not change substantially, it will be

$$R_{tubes} = \frac{62.6}{0.3} = 209 \text{ ohms, approximately.}$$

The total resistance

$$R_t' = 209 + 126 + 170 = 496 \text{ ohms,}$$

whereas with the pilot lamp burning it was

$$R_t = \frac{120}{0.3} = 400 \text{ ohms.}$$

The current now will be

$$I' = \frac{120}{496} = 0.241 \text{ ampere.}$$

The voltage drop across the tubes will be smaller; the voltage drop across the pilot lamp shunt

$$E_p' = I_p R_p = 0.241 \times 126 = 30.4 \text{ volts}$$

instead of 6.3 volts; its power dissipation will be

$$P_p' = 30.4 \times 0.241 = 7.3 \text{ watts, approximately.}$$

The set would probably play for a short time with distortion; resistor R_p will be charred and damaged. *Ans.*

Three-Way Portable Receiver

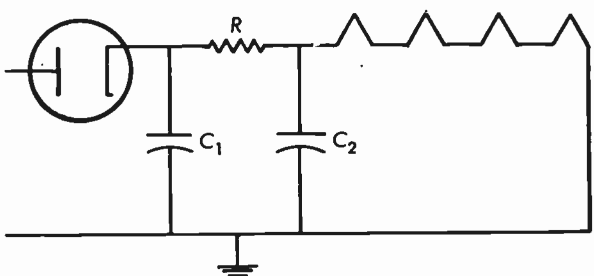

Fig. 9.26 Power supply suitable for a-c, d-c, and battery supplies.

9.26 A three-way portable receiver (a-c, d-c, battery) uses a series filament hookup of the following tubes: 1A7, 1N5, 1H5, 1A5. The tubes are d-c heated from a 117Z6 rectifier tube. The current drain of the tubes is 0.05 ampere and the rectifier output voltage 120 volts. What is the value of the series dropping resistor and what is the wasted power?

Solution:

Figure 9.26 represents the filament circuit when the switch is on a-c/d-c position. The filament drop is

$$R_f = 4 \times 1.5 = 6 \text{ volts.}$$

The drop across R is

$$E_r = 120 - 6 = 114 \text{ volts,}$$

and
$$R = \frac{114}{0.05} = 2280 \text{ ohms.} \quad \textit{Ans.}$$

Thus
$$P = 114 \times 0.05 = 5.7 \text{ watts.} \quad \textit{Ans.}$$

The rating should be 20 watts for safety.

Burn-Out in Turned-Off Receiver

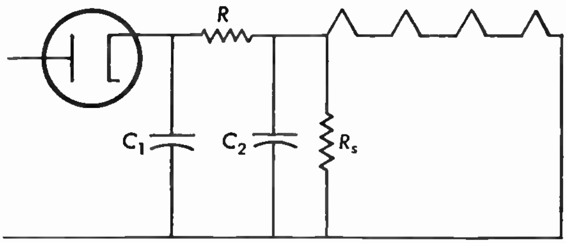

Fig. 9.27 Filament supply for portable receivers with safety discharge resistor R_s.

9.27 It was found that tubes of 3-way portable receivers which have been removed from the socket will burn out immediately when put back in, even with the switch in "off" position. A safety bleeder-resistor R_s, added to later models and connected as in Figure 9.27, will prevent this if certain precautions are taken. If R_s = 500,000 ohms and C_2 = 10 microfarads, what time must elapse after turning off the switch before the tube can be plugged back in with safety?

Solution:

The capacitor C_2, having been charged to 120 volts and thus causing the burn-out, must discharge to 6 volts (4 × 1.5) to make the plug-in safe.

Using $E_t = E_o \epsilon^{-\frac{t}{RC}}$

and $RC = 5 \times 10^5 \times 10 \times 10^{-6} = 5,$

we have

$$6 = 120 \epsilon^{-\frac{t}{5}},$$

$$6 = 120 \epsilon^{-0.2t},$$

$$0.05 = \epsilon^{-0.2t},$$

$$\log 0.05 = \log \epsilon^{-0.2t},$$

$$\log 0.05 = -0.2t \log \epsilon,$$

$$0.699 - 2 = -0.2t \times 0.4343,$$

$$-1.301 = -0.08686t$$

$$t = \frac{1.301}{0.08686} = 14.98 \text{ seconds.} \quad \textit{Ans.}$$

The tube can be put back after 15 seconds, approximately, but under no circumstances while the switch is in "on" position.

A-Supply for Phonograph Amplifier

9.28 An a-c/d-c phonograph amplifier uses a 70L7 rectifier power-amplifier tube in series with a 12SJ7 driver. The 70L7 has a heater current of 0.3 ampere, the 12SJ7 a heater current of 0.15 ampere. Design a filament circuit for a line voltage of 120 volts.

Solution:

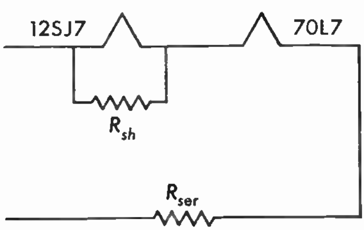

Fig. 9.28 Series filament hookup for phonograph amplifier.

The circuit will be connected as indicated in Figure 9.28. The filament voltage is

$$E_f = 70 + 12.6 = 82.6 \text{ volts}$$

The voltage drop across R_{ser} is

$$E_s = 120 - 82.6 = 37.4 \text{ volts,}$$

and $$R_{ser} = \frac{37.4}{0.3} = 125 \text{ ohms, approximately.} \quad \textit{Ans.}$$

The current through the shunt resistor is

$$I_{sh} = 0.3 - 0.15 = 0.15 \text{ ampere.}$$

and $$R_{sh} = \frac{6.3}{0.15} = 42 \text{ ohms.} \quad \textit{Ans.}$$

The power dissipation of R_{ser} is

$$P_{ser} = I^2 R = 0.09 \times 125 = 11.25 \text{ watts.}$$

The power dissipation of R_{sh} is

$$P_{sh} = E \times I = 6.3 \times 0.15$$
$$= 0.95 \text{ watt, approximately.}$$

A power rating of about 3 to 4 times these values should be used for safety. *Ans.*

Substitution of a 14C5 Tube for a 25L6 Tube

9.29 Calculate the resistance necessary to replace a 25L6 (25 volts, 0.3 ampere) beam power tube by using a 14C5 (12.6 volts, 0.225 ampere), and draw a diagram of the circuit.

Solution:

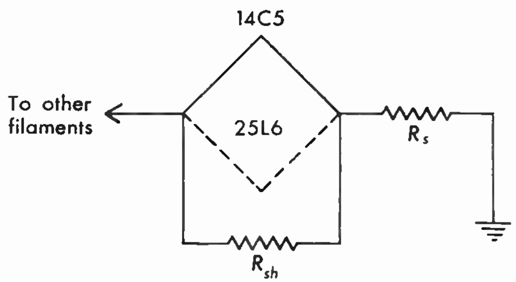

Fig. 9.29 Diagram illustrating the substitution of a 25L6 beam-power tube by using the equivalent type 14C5.

The circuit will be connected as indicated in Figure 9.29. The series resistor will absorb a drop of

$$E_s = 25 - 12.6 = 12.4 \text{ volts.}$$

The current through R_s is 0.3 ampere.

Thus $$R_s = \frac{12.4}{0.3} = 41 \text{ ohms.} \quad \textit{Ans.}$$

Its power rating will be determined by its power dissipation.

$$P_s = 12.4 \times 0.3 = 3.72 \text{ watts.}$$

Perhaps a resistor rated 10 watts would work satisfactorily. The shunt resistor will carry a current of

$$I_{sh} = 0.3 - 0.225 = 0.075 \text{ ampere.}$$

Its value will be

$$R_{sh} = \frac{12.6}{0.075} = 168 \text{ ohms, approximately.} \quad \textit{Ans.},$$

and its dissipation

$$P_{sh} = 12.6 \times 0.075 = 0.945 \text{ watt.} \quad \textit{Ans.}$$

Substituting 6-volt Tubes for 12-volt Tubes

9.30 The tubes of a midget superheterodyne receiver using 12SA7, 12SK7, 12SQ7, 50L6, and 35Z5 are to be substituted by using 6SA7, 6SK7, 6SQ7, 25L6, and 12SN7, all 0.3-ampere heater tubes. The 12SN7 is used as a rectifier with all grids and plates tied together to make the anode. If no provisions are made for the pilot lamp, what is the resistance of the lamp-cord resistor to be used in series with the filament? The line voltage is 120 volts.

Solution:

The potential difference across the filament string is

$$E_f = 3 \times 6.3 + 12.6 + 25 = 56.5 \text{ volts.}$$

The voltage across the cord resistor will be

$$E_c = 120 - 56.5 = 63.5 \text{ volts.}$$

Since the 6-volt tubes have a heater current of 0.3 ampere, the resistance of the cord is

$$R_c = \frac{63.5}{0.3} = 212 \text{ ohms, approximately.} \quad \textit{Ans.}$$

Using a Selenium Rectifier for Substitution

9.31 A rectifier-beam-power amplifier type 70L7 is to be substituted by using a selenium rectifier type NC-5 and a simple beam power amplifier. Find the resistance and the wattage dissipation of the series filament dropping resistor.

Solution:

The tube manual gives a current rating of 0.15 ampere for the heater of the 70L7. If a type 35L6 which has the same heater current is used as an output tube the resistor to be installed must cause a voltage drop of

$$E_R = 70 - 35 = 35 \text{ volts,}$$

and

$$R = \frac{35}{0.15} = 233 \text{ ohms.} \quad \textit{Ans.}$$

A 230-ohm resistor will work.

The wattage dissipation is

$$P_R = E_R \times I_R$$

$$= 35 \times 0.15 = 5.25 \text{ watts.} \quad \textit{Ans.}$$

A resistor rated 10 to 25 watts may be used for safety.
If a type 50L6 output tube is used the resistor to be installed must cause a voltage drop of

$$E_R = 70 - 50 = 20 \text{ volts}$$

$$R = \frac{20}{0.15} = 133 \text{ ohms.} \quad \textit{Ans.}$$

A 130- or 135-ohm resistor may be used. The wattage dissipation will be

$$P_R = 20 \times 0.15 = 3 \text{ watts.} \quad \textit{Ans.}$$

10 *Antennas and Transmission Lines*

Hertz Antenna in Inches

10.01 Derive a formula for the length of a Hertz antenna in inches and megacycles, disregarding the "end effect."

Solution:

The length is approximately equal to 1/2 the wavelength.

Using $$l = \frac{1}{2}\lambda = \frac{1}{2} \times \frac{3 \times 10^8}{f_{mc} \times 10^6} \text{ meters,}$$

where f_{mc} is the frequency in megacycles, we have

$$l = \frac{1}{2} \times \frac{3 \times 10^8}{f_{mc} \times 10^6} \times 39.37 \text{ inches}$$

$$= \frac{3 \times 39.37 \times 10^2}{2} \times \frac{1}{f_{mc}} = \frac{5906}{f_{mc}} \text{ inches.} \quad \textit{Ans.}$$

"End Effect" in Antennas

10.02 If the "end effect" reduces the length of the antenna by 5 per cent, calculate the length of a Hertz antenna for 5 and for 25 megacycles.

Solution:

Using the formula derived in problem 10.01, we obtain for 5 megacycles:

$$l = \frac{5906}{5} \times 0.95 = 1122 \text{ inches}$$

$$= 93.5 \text{ feet.} \quad \textit{Ans.}$$

For 25 megacycles:

the antenna will be 1/5 the length of 93.5 feet

$$\frac{1}{5} \times 93.5 = 18.7 \text{ feet.} \quad \textit{Ans.}$$

Resonant Frequency

10.03 What is the resonant frequency of a Hertz antenna 27.5 feet long?

Solution:

The actual length is about 95 per cent of the electrical length. We have

$$l \times 0.95 = 27.5$$

and

$$l = \frac{27.5}{0.95} = 28.9 \text{ feet} = 348 \text{ inches.}$$

Using the formula derived in problem 10.01

$$l = \frac{5906}{f_{mc}}$$

we obtain

$$348 = \frac{5906}{f_{mc}},$$

and

$$f_{mc} = \frac{5906}{348} = 16.9 \text{ megacycles.} \quad \textit{Ans.}$$

Height in Wavelengths

FCC Study Guide Question 4.105

10.04 If a vertical antenna is 405 feet high and is operated at 1250 kilocycles, what is its physical height expressed in wavelengths? (1 meter = 3.28 feet.)

Solution:

The wavelength corresponding to 1250 kilocycles

$$\lambda = \frac{3 \times 10^8}{1.25 \times 10^6} = 240 \text{ meters.}$$

But

$$405 \text{ feet} = 405 \times 0.3048 = 124 \text{ meters.}$$

The given height then, is not a full wavelength, but

$$\frac{124}{240} = 0.516 \text{ wavelength.} \quad \textit{Ans.}$$

Height in Feet

FCC Study Guide Question 4.106

10.05 What must be the height of a vertical radiator 1/2 wavelength high if the operating frequency is 1100 kilocycles?

Solution:

Since it is 1/2 a wavelength long we can use the formula of problem 10.01.

$$h = \frac{5906}{1.1} \text{ inches}$$

$$= \frac{5906}{1.1 \times 12} \text{ feet}$$

$$= 448 \text{ feet.} \quad \textit{Ans.}$$

Antenna Current for Reduced Power

FCC Study Guide Question 4.90

10.06 If the antenna current is 9.7 amperes for 5 kilowatts, what is the current necessary for a power of 1 kilowatt?

Solution:

Using $P = I^2 R,$

we can calculate the antenna resistance:

$$5000 = 9.7^2 R$$

$$R = \frac{5000}{9.7^2} = 53.2 \text{ ohms.}$$

Then $1000 = I^2 \times 53.2,$

$$I^2 = \frac{1000}{53.2} = 18.8,$$

and $I = \sqrt{18.8} = 4.33$ amperes. *Ans.*

Antenna Current for Stated Power

FCC Study Guide Question 4.91

10.07 What is the antenna current when a transmitter is delivering 900 watts into an antenna having a resistance of 16 ohms?

Solution:

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{900}{16}} = 7.5 \text{ amperes.} \quad \textit{Ans.}$$

Effect of Current Increase

FCC Study Guide Question 4.95

10.08 If the reading of the ammeter connected at the base of a Marconi antenna is increased to 2.77 times its original value, what is the increase in output power?

Solution:

The resistance being equal in both cases, we obtain

$$P_1 = I^2 R$$

$$P_2 = (2.77\, I)^2 R = 7.67 \times I^2 R.$$

The ratio of the increased power to the original power is

$$\frac{P_2}{P_1} = \frac{7.67 \times I^2 R}{I^2 R} = 7.67;$$

the power is increased to 7.67 times its original value. *Ans.*

Antenna Power

FCC Study Guide Question 4.92

10.09 If the day input power to a broadcast station antenna having a resistance of 20 ohms is 2000 watts, what would be the night input power if the antenna current were cut in half?

Solution:

Day current: $I = \sqrt{\frac{P}{R}} = \sqrt{\frac{2000}{20}} = 10$ amperes.

The night current is:

$$10 \div 2 = 5 \text{ amperes;}$$

therefore $P_n = I^2 R = 5^2 \times 20 = 500$ watts. *Ans.*

Alternate solution:

The power is proportional to the square of the current. A reduction of the current by 1/2 will cause a reduction of the power by $(1/2)^2 = 1/4$. Hence the night power is

$$P_n = 2000/4 = 500 \text{ watts.} \quad \textit{Ans.}$$

Two-Wire Transmission Line

10.10 A 2-wire transmission line consists of No. 12 wire AWG; the distance between the centers is 10 inches. What is the characteristic impedance of the line?

Solution:

The diameter of No. 12 wire is 81 mils = 0.081 inch,

the radius $r = \frac{0.081}{2} = 0.0405$ inch,

the distance $s = 10$ inches;

by formula 10.21 $Z = 276 \times \log \frac{s}{r}$

$$= 276 \log \frac{10}{0.0405}$$

$$= 276 \log 247$$

$$= 276 \times 2.3927 = 660 \text{ ohms.} \quad \textit{Ans.}$$

Concentric Line

10.11 The outer conductor of a coaxial transmission line consists of copper tubing 0.05 inch thick, with an outside diameter of 1.8 inches. The diameter of the inner conductor is 0.55 inch. What is the characteristic impedance of the line?

Solution:

Using $Z = 138 \log \frac{D}{d}$, (formula 10.22)

and $D = 1.8 - 0.1 = 1.7$ inches,

we obtain $Z = 138 \log \frac{1.7}{0.55} = 138 \log 3.09$

$$= 138 \times 0.49 = 67.6 \text{ ohms.} \quad \textit{Ans.}$$

Power to Transmission Line

FCC Study Guide Question 4.88

10.12 An antenna is fed by a properly terminated 2-wire transmission line. The current in the line at the output end is 3 amperes. The surge impedance of the line is 500 ohms. How much power is supplied to the line?

Solution:

The antenna resistance is equal to the surge impedance, since the line is properly terminated.

The antenna power is therefore:

$$P = I^2 R = 3^2 \times 500 = 4500 \text{ watts} = 4.5 \text{ kilowatts.} \quad \textit{Ans.}$$

The power supplied to the line is also 4.5 kilowatts, since losses are negligible with proper termination.

Transmission Line Current

FCC Study Guide Question 4.89

10.13 If the daytime transmission-line current of a 10-kilowatt transmitter is 12 amperes, and the transmitter is required to reduce to 5 kilowatts at sunset, what is the new value of transmission-line current?

Solution:

We first calculate the resistance.

Using $$P = I_d^2 R,$$

we obtain $$10{,}000 = 12^2 R,$$

and $$R = \frac{10{,}000}{144} = 69.5 \text{ ohms.}$$

Again substituting in

$$P = I_n^2 R,$$

we obtain $$5000 = I_n^2 \times 69.5,$$

$$I_n^2 = \frac{5000}{69.5} = 72,$$

and $$I_n = \sqrt{72} = 8.5 \text{ amperes.} \quad \textit{Ans.}$$

Alternate solution:

Since the power is proportional to the square of the current, we have

$$\frac{I_n^2}{I_d^2} = \frac{P_n}{P_d}$$

and
$$I_n = \sqrt{\frac{I_n^2 P_n}{P_d}}$$

$$= I_d \sqrt{\frac{5}{10}} = 12 \times \sqrt{0.5}$$

$$= 8.5 \text{ amperes.} \quad \textit{Ans.}$$

Concentric Transmission Line Peak Voltage

FCC Study Guide Question 4.99

10.14 The power input to a 72-ohm concentric transmission line is 5000 watts. What is the peak voltage between the inner conductor and the sheath?

Solution:

Using the formula

$$P = \frac{E^2}{R},$$

we obtain
$$5000 = \frac{E^2}{72}$$

and
$$E^2 = 5000 \times 72 = 360{,}000.$$

$$E = \sqrt{360{,}000} = 600 \text{ volts.}$$

The peak voltage between the inner conductor and the sheath is

$$E_{peak} = E \times \sqrt{2}$$

$$= 600 \times 1.414 = 848.4 \text{ volts.} \quad \textit{Ans.}$$

Loss in Long Transmission Line

FCC Study Guide Question 4.100

10.15 A long transmission line delivers 10 kilowatts into an antenna; at the transmitter end the line current is 5 amperes, and at the coupling house it is 4.8 amperes. Assuming the line current to be properly terminated and the losses in the coupling system negligible, what is the power lost in the line?

Solution:

When the line is properly terminated its resistance is equal to the antenna resistance.

Using the formula

$$P = I^2 R$$

we obtain

$$10{,}000 = 4.8^2 \times R,$$

$$R = \frac{10{,}000}{23.04} = 434 \text{ ohms.}$$

The power input to the line is determined by the above resistance and the line current,

$$P_i = I^2 R$$

$$= 5^2 \times 434 = 10{,}850 \text{ watts.}$$

Since the transmission line delivers only 10,000 watts, the loss is

$$P_{loss} = 10{,}850 - 10{,}000$$

$$= 850 \text{ watts.} \quad \textit{Ans.}$$

Attenuation in Decibels

10.16 What is the attenuation in decibels of the transmission line in problem 10.10 for a frequency of 9 megacycles?

Solution:

The radio-frequency resistance of the line is, by formula 10.24

$$R = \frac{\sqrt{F}}{5\,d} = \frac{3}{5 \times 0.081}$$

$$= \frac{3}{0.405} = 7.4 \text{ ohms per 100 feet.}$$

By formula 10.25

$$A = \frac{4.35\,R}{Z_0} = \frac{4.35 \times 7.4}{660}$$

$$= 0.0488 \text{ decibel per 100 feet.} \quad \textit{Ans.}$$

Efficiency of Transmission

10.17 If the line in problem 10.16 were 1000 feet long, what would be the efficiency of the transmission?

Solution:

$$\eta = \frac{P_o}{P_i};$$

now at 1000 feet the attenuation will be

$$0.0488 \times 10 = 0.488 \text{ decibels};$$

from
$$N_{db} = 10 \log \frac{P_i}{P_o}$$

and
$$0.0488 = \log \frac{P_i}{P_o}$$

we find
$$\frac{P_i}{P_o} = \text{antilog } 0.0488 = 1.119,$$

and
$$\frac{P_o}{P_i} = \frac{1}{1.119} = 0.895$$

$$= 89.5 \text{ per cent.} \quad \textit{Ans.}$$

Attenuation of Coaxial Line

10.18 What is the attenuation of the line in problem 10.11 at 16 megacycles?

Solution:

The resistance of the line per 100 feet is, by formula 10.23

$$R_f = 0.1 \left(\frac{D \times d}{D + d}\right) \sqrt{F}$$

$$= \frac{0.1 \times 1.7 \times 0.55 \times 4}{2.2}$$

$$= 0.17 \text{ ohm per 100 feet.}$$

Thus
$$A = \frac{4.35\, R_f}{Z_o} = \frac{4.35 \times 0.17}{67.6}$$

$$= 0.0109 \text{ decibel per 100 feet.} \quad \textit{Ans.}$$

Line Loss

10.19 What is the line loss in decibels if the line in problem 10.18 is 500 feet long?

Solution:

$$\text{Loss} = 0.0109 \times 5 = 0.0545 \text{ decibel.} \quad \textit{Ans.}$$

Length of Line

10.20 How long can be the line in problem 10.18 at the most, if an efficiency of not less than 80 per cent is to be realized?

Solution:

$$\eta = \frac{P_o}{P_i} = 80 \text{ per cent} = 0.8$$

$$\frac{P_i}{P_o} = \frac{1}{0.8} = 1.25.$$

The corresponding number of decibels is

$$N_{db} = 10 \log 1.25$$

$$= 10 \times 0.0969 = 0.969 \text{ decibel.}$$

Since 0.0109 decibel will be lost per each 100 feet of distance, we have

$$0.969/0.0109 = 88.9 \qquad \text{100-foot distances}$$

or 8890 feet. *Ans.*

Characteristic Impedance

10.21 Find the series equivalent of the characteristic impedance of a line, the circuit constants of which at 1000 cycles were found to be: $R = 10$ ohms, $L = 3.5$ millihenries, $C = 0.01$ microfarad, $G = 0.25$ micromho.

Solution:

The characteristic impedance is

$$Z = \sqrt{\frac{R + j\,2\,\pi f L}{G + j\,2\,\pi f C}}.$$

Now $2\pi f L = 6.28 \times 10^3 \times 3.5 \times 10^{-3} = 22,$

and $2\pi f C = 6.28 \times 10^3 \times 0.01 \times 10^{-6} = 62.8 \times 10^{-6}.$

The polar expressions are:

$$R + j2\pi f L = 10 + j22 = 24.2\underline{/65.6^\circ}$$

$$G + j2\pi f C = 0.25 \times 10^{-6} + j62.8 \times 10^{-6}$$

$$= 10^{-6}(0.25 + j\,62.8).$$

The real part of the vector, 0.25, can be neglected with considerably less than 1 per cent error.

Thus $G + j2\pi f C = 10^{-6} \times j62.8 = 62.8 \times 10^{-6}\underline{/+90^\circ}.$

Substituting

$$Z = \sqrt{\frac{24.2\underline{/65.6^\circ}}{10^{-6} \times 62.8\underline{/+90^\circ}}}$$

$$= \sqrt{10^6\,(24.2 \div 62.8)\,\underline{/65.6 - (+90^\circ)}}$$

$$= 10^3\sqrt{0.384\underline{/-24.4^\circ}}$$

$$= 10^3\,(0.620\underline{/-12.2^\circ}) = 620\underline{/-12.2^\circ}.$$

The equivalent j-notation is

$$Z = 620\cos(-12.2^\circ) + j\,620\sin(-12.2^\circ)$$

$$= 606 - j\,131.$$

The series equivalent circuit consists of

606 ohms resistance,

131 ohms capacitive reactance. *Ans.*

Determining Characteristic Impedance by Opening and Short-Circuiting the Load End

10.22 With the aid of a bridge instrument it is found that a resistance of 675 ohms and an inductance of 0.025 henry will neutralize the line when the load end is open-circuited; when the load end is short-circuited the readings are 700 ohms and 0.015 henries; the frequency is 1000 cycles. What is the series equivalent circuit for the characteristic impedance of the line?

Solution:

We shall use formula 10.262

$$Z_o = \sqrt{Z_{Ls} Z_{Lo}}.$$

$$X_{Lo} = 2\pi f L_o = 6280 \times 0.025 = 157 \text{ ohms};$$

this reactance will neutralize $-j$ 157 ohms in the line.

$$X_{Ls} = 2\pi f L_s = 6280 \times 0.015 = 94 \text{ ohms};$$

this reactance will neutralize $-j$ 94 ohms in the line.

$$Z_{Ls} = 675 - j\,157 = 695\underline{/-13.1^\circ}$$

$$Z_{Lo} = 700 - j\,94 = 707\underline{/-7.6^\circ}.$$

Therefore

$$\begin{aligned} Z_o &= \sqrt{Z_{Ls} Z_{Lo}} \\ &= \sqrt{694\underline{/-13.1^\circ} \times 707\underline{/-7.6^\circ}} \\ &= \sqrt{491{,}000\underline{/-20.7^\circ}} \\ &= 701\underline{/-10.35^\circ} \text{ vector ohms.} \end{aligned}$$

The equivalent j-notation is

$$\begin{aligned} Z_o &= 701 \cos(-10.35^\circ) + j\,701 \sin(-10.35^\circ) \\ &= 688 - j\,126. \end{aligned}$$

The series equivalent circuit consists of

688 ohms resistance

126 ohms capacitive reactance. *Ans.*

Field Strength of Non-Resonant Wire

10.23 Using the formula

$$\epsilon = \frac{60\,\pi}{d\,\lambda} I\,(\delta l) \cos\theta,$$

where ϵ = field strength in volts per meter,

d = distance from antenna in meters,

θ = angle of elevation with respect to a plane perpendicular to the antenna wire,

λ = wavelength of radiated wave in meters,

δl = length of elementary antenna in meters,

I = current in antenna;

calculate the field strength of a vertical wire 25 feet long, with a radio-frequency current of 4 amperes at a frequency of 1000 kilocycles, in a horizontal plane through the center of the wire, at distances of 1/2, 1, 5, 10, 50, and 100 miles, assuming the wire to be remote from ground.

Solution:

The wavelength in meters is

$$\lambda = \frac{3 \times 10^8}{1000 \times 10^3} = 300 \text{ meters},$$

and since 1 foot = 0.3048 meter, 1 mile = 1.609×10^3 meters; and $\theta = 0°$ because the direction is within the plane defined above, for θ, we can obtain a simplified formula by substituting in

$$\epsilon = \frac{60\,\pi\, I\,(\delta l)\cos\theta}{\lambda} \times \frac{1}{d}$$

$$= \frac{188 \times 4 \times 25 \times 0.3048 \times 1}{300} \times \frac{1}{d \times 1.609 \times 10^3}$$

$$= \frac{0.0119}{d}, \text{ where } d \text{ is in miles.}$$

The field strengths at the required distances are

$$\epsilon_{0.5} = \frac{0.0119}{d} = 0.0238$$

$$= 23.8 \text{ millivolts per meter.} \quad \textit{Ans.}$$

Recognizing that the field strength is inversely proportional to the distance, we obtain

$$\epsilon_1 = \frac{1}{2} \times \epsilon_{0.5} = 11.9 \text{ millivolts per meter.} \quad \textit{Ans.}$$

$$\epsilon_5 = \frac{1}{10} \times \epsilon_{0.5} = 2.38 \text{ millivolts per meter.} \quad \textit{Ans.}$$

$$\epsilon_{10} = \frac{1}{10} \times \epsilon_1 = 1.19 \text{ millivolts per meter.} \quad \textit{Ans.}$$

$$\epsilon_{50} = \frac{1}{10} \times \epsilon_5 = 0.238 \text{ millivolt per meter.} \quad \textit{Ans.}$$

$$\epsilon_{100} = \frac{1}{100} \times \epsilon_1 = 0.119 \text{ millivolt per meter}$$

$$= 119 \text{ microvolts per meter.} \quad \textit{Ans.}$$

Pattern of Resonant Wire

10.24 Using the formula

$$\epsilon = \frac{60\, I}{d} \quad \frac{\cos\left(\pi \frac{L}{\lambda} \cos\theta\right)}{\sin\theta},$$

valid for a wire the length of which is an odd number of half wavelengths, where

ϵ = field strength in volts per meter,
d = distance to antenna in meters,
I = current in amperes at a current loop,
L = length of antenna in meters,
λ = wavelength in meters,
θ = angle of elevation measured with respect to the wire axis;

calculate and plot the directional characteristics of an antenna $1\frac{1}{2}$ wavelengths long in a plane containing the wire.

Solution:

The first fraction in the above formula is a constant determined by the distance and the current. The relative field strength is determined by the second fraction. Calculating the relative field strength in steps of 10 degrees with $L = (3/2)\,\lambda$, we obtain for $\theta = 0°$,

$$\epsilon_0 = \frac{\cos\left(\frac{3\,\pi}{2} \cos 0°\right)}{\sin 0°}$$

$$= \frac{\cos\,(270° \times 1)}{\sin 0°} = \frac{0}{0}.$$

This is an *indeterminate* form, the value of which is not readily obtainable by elementary mathematics. However, for a very small angle, say $\theta = 0.1°$, it is found from the tables that

$$\sin 0.1° = 0.00174$$

$$\cos 0.1° = 1.0000,$$

therefore $$\epsilon_{0.1} = \frac{\cos(1.5\,\pi\cos 0.1°)}{\sin 0.1°}$$

$$= \frac{\cos(270° \times 1)}{0.00174} = \frac{0}{0.00174}$$

$$= 0. \quad \textit{Ans.} \quad \text{(Point 1)}$$

$$\epsilon_{10} = \frac{\cos(1.5\,\pi\cos 10°)}{\sin 10°}$$

$$= \frac{\cos(270° \times 0.985)}{0.174} = \frac{\cos 266°}{0.174}$$

$$= -\frac{\cos(266° - 180°)}{0.174} = -\frac{\cos 86°}{0.174}$$

$$= -\frac{0.0698}{0.174} = -0.401. \quad \textit{Ans.} \quad \text{(Point 2)}$$

$$\epsilon_{20} = \frac{\cos(1.5\,\pi \times \cos 20°)}{\sin 20°}$$

$$= \frac{\cos(270° \times 0.94)}{0.342} = \frac{\cos 253.8°}{0.342} = -\frac{\cos 73.8°}{0.342}$$

$$= -\frac{0.279}{0.342} = -0.816. \quad \textit{Ans.} \quad \text{(Point 3)}$$

$$\epsilon_{30} = \frac{\cos(270° \times \cos 30°)}{\sin 30°}$$

$$= \frac{\cos(270° \times 0.866)}{0.5} = \frac{\cos 233.8°}{0.5}$$

$$= -\frac{53.8°}{0.5} = -\frac{0.59}{0.5}$$

$$= -1.18. \quad \textit{Ans.} \quad \text{(Point 4)}$$

$$\epsilon_{40} = \frac{\cos(1.5\,\pi\cos 40°)}{\sin 40°}$$

$$= \frac{\cos(270° \times 0.7604)}{0.6428} = \frac{\cos 206.9°}{0.6428}$$

$$= -\frac{26.9°}{0.6428} = -\frac{0.8918}{0.6428}$$

$$= -1.39. \quad \textit{Ans.} \quad \text{(Point 5)}$$

$$\epsilon_{50} = \frac{\cos(1.5\,\pi \cos 50°)}{\sin 50°}$$

$$= \frac{\cos(270° \times 0.6428)}{0.766} = \frac{\cos 173.4°}{0.766}$$

$$= -\frac{(\cos 180 - 173.4°)}{0.766} = -\frac{\cos 6.6°}{0.766}$$

$$= -\frac{0.9934}{0.766} = -1.297. \quad \textit{Ans.} \quad \text{(Point 6)}$$

From the last two answers it is seen that the peak of the lobe is between 40 and 50 degrees. It will therefore be useful to calculate the field strength at 45°.

$$\epsilon_{45} = \frac{\cos(1.5\,\pi \times \cos 45°)}{\sin 45°}$$

$$= \frac{\cos(270° \times 0.707)}{0.707} = \frac{\cos 191°}{0.707} = -\frac{\cos 11°}{0.707}$$

$$= -\frac{0.9816}{0.707} = -1.389. \quad \textit{Ans.} \quad \text{(Point 7)}$$

The field strength at 40° is nearly equal to the field strength at 45°. The peak is to be expected somewhere near 42.5°.

$$\epsilon_{42.5} = \frac{\cos(1.5\,\pi \cos 42.5°)}{\sin 42.5°}$$

$$= \frac{\cos(270° \times 0.7373)}{0.6756} = \frac{\cos 190.7°}{0.6756} = \frac{\cos 199.1°}{0.6756}$$

$$= -\frac{\cos 19.1°}{0.6756} = -\frac{0.9455}{0.6756}$$

$$= -1.4. \quad \textit{Ans.} \quad \text{(Point 8)}$$

$$\epsilon_{60} = \frac{\cos(270° \times \cos 60°)}{\sin 60°}$$

$$= \frac{\cos(270° \times 0.5)}{0.866} = \frac{\cos 135°}{0.886} = -\frac{\cos 45°}{0.866}$$

$$= -\frac{0.707}{0.866} = -0.815. \quad \textit{Ans.} \quad \text{(Point 9)}$$

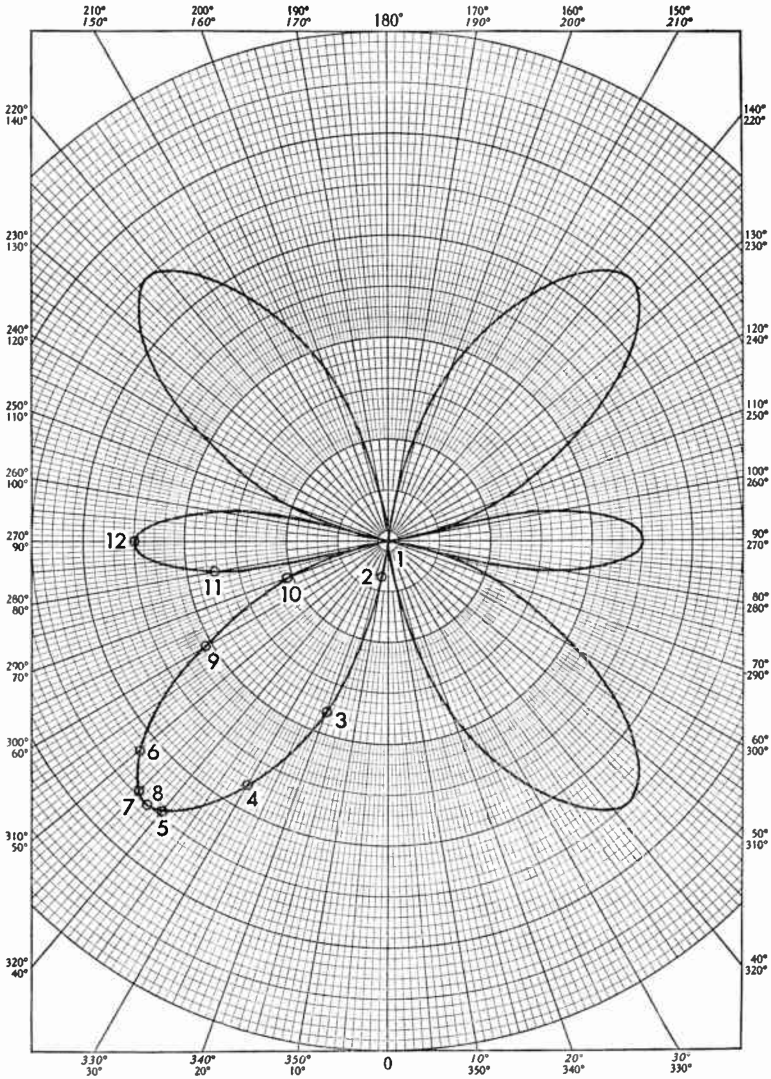

Fig. 10.24 Field strength pattern of an antenna $1\frac{1}{2}$ wavelengths long in a plane containing the wire.

$$\epsilon_{70} = \frac{\cos(270° \times \cos 70°)}{\sin 70°}$$

$$= \frac{\cos(270° \times 0.342)}{0.9397} = \frac{\cos 92.34°}{0.9397} = \frac{\cos 87.66°}{0.9397}$$

$$= -\frac{0.0407}{0.9397} = -0.0432. \quad \textit{Ans.} \quad \text{(Point 10)}$$

$$\epsilon_{80} = \frac{\cos(270° \times \cos 80°)}{\sin 80°}$$

$$= \frac{\cos(270° \times 0.1736)}{0.9848} = \frac{\cos 46.9°}{0.9848} = \frac{0.6833}{0.9848}$$

$$= 0.694. \quad \textit{Ans.} \quad \text{(Point 11)}$$

$$\epsilon_{90} = \frac{\cos(270° \times \cos 90°)}{\sin 90°}$$

$$= \frac{\cos(270° \times 0)}{1} = \frac{\cos 0°}{1}$$

$$= 1. \quad \textit{Ans.} \quad \text{(Point 12)}$$

Using the absolute values of the field strengths obtained for the above directions and the fact that the field strength distribution is symmetrical about the center of the antenna* we obtain the pattern plotted in Figure 10.24.

Directional Array

10.25 Using the formula

$$R.F.S. = \frac{\sin n\,\pi\,(a \cos\theta + b)}{n \sin \pi\,(a \cos\theta + b)},$$

where $R.F.S.$ = relative field strength of n radiators, radiating uniformly in all directions remote from ground,

n = number of radiators,

a = spacing of adjacent radiators in wavelengths,

b = phase difference between adjacent radiators in cycles,

* F. E. Terman, *Radio Engineering* (2d ed.; New York: McGraw-Hill Book Co., 1937), p. 656 f.

θ = angle with respect to a line joining the radiators, in a plane perpendicular to the radiators, through the radiators;

calculate and plot the directional characteristics in a horizontal plane obtained by two vertical wires spaced 1/4-wavelength apart, with the current through one wire leading the current through the other wire by 90 degrees, assuming the wires to be remote from ground.

Solution:

We shall first calculate the relative field strength at 0°, 45°, 90°, 135° and 180°, and shall provide other values between these directions if they are necessary to clarify the pattern.

In all computations

$$a = 0.25 \text{ wavelength},$$
$$b = 0.25 \text{ cycle},$$
$$n = 2 \text{ radiators}.$$

The field strength in a direction of 0° is

$$R.F.S. = \frac{\sin n\,\pi\,(a \cos\theta + b)}{n \sin \pi\,(a \cos\theta + b)}$$

$$= \frac{\sin 2\,\pi\,(0.25 \cos 0^\circ + 0.25)}{2 \sin \pi\,(0.25 \cos 0^\circ + 0.25)}$$

$$= \frac{\sin (360^\circ \times 0.5)}{2 \sin (180^\circ \times 0.5)}$$

$$= \frac{0}{2} = 0. \quad \textit{Ans.} \quad \text{(Point A)}$$

In a direction of 45°

$$R.F.S. = \frac{\sin 360^\circ\,(0.25 \cos 45^\circ + 0.25)}{2 \sin 180^\circ\,(0.25 \cos 45^\circ + 0.25)}$$

$$= \frac{\sin 360^\circ\,(0.25 \times 0.707 + 0.25)}{2 \sin 180^\circ\,(0.25 + \cos 45^\circ + 0.25)}$$

$$= \frac{\sin (360^\circ \times 0.427)}{2 \sin (180^\circ \times 0.427)}$$

$$= \frac{\sin 153.8^\circ}{2 \sin 76.9^\circ} = \frac{0.442}{2 \times 974} = \frac{0.442}{1.948}$$

$$= 0.227. \quad \textit{Ans.} \quad \text{(Point B)}$$

In a direction of 90°

$$R.F.S. = \frac{\sin 360° (0.25 \cos 90° + 0.25)}{2 \sin 180° (0.25 \cos 90° + 0.25)}$$

$$= \frac{\sin (360° \times 0.25)}{2 \sin (180° \times 0.25)}$$

$$= \frac{\sin 90°}{2 \sin 45°} = \frac{1}{2 \times 0.707} = \frac{1}{\sqrt{2}}$$

$= 0.707.$ *Ans.* (Point C)

In a direction of 135°

$$R.F.S. = \frac{\sin 360° (0.25 \cos 135° + 0.25)}{2 \sin 180° (0.25 \cos 135° + 0.25)}$$

$$= \frac{\sin 360° (-0.707 \times 0.25 + 0.25)}{2 \sin 180° (-0.707 \times 0.25 + 0.25)}$$

$$= \frac{\sin 360° (-0.177 + 0.25)}{2 \sin 180° (-0.117 + 0.25)}$$

$$= \frac{\sin (360° \times 0.073)}{2 \sin (180° \times 0.073)}$$

$$= \frac{\sin 26.3°}{2 \sin 13.15°} = \frac{0.443}{2 \times 0.227} = \frac{0.443}{0.454}$$

$= 0.976.$ *Ans.* (Point D)

In a direction of 180°

$$R.F.S. = \frac{\sin 360° (0.25 \cos 180° + 0.25)}{2 \sin 180° (0.25 \cos 180° + 0.25)}$$

$$= \frac{\sin 360° (-0.25 + 0.25)}{2 \sin 180° (-0.25 + 0.25)} = \frac{0}{0}$$

This indeterminate form can be found by employing an angle slightly smaller than 180°, say 179.5°.

Then $$R.F.S. = \frac{\sin 360° (0.25 \cos 179.5° + 0.25)}{2 \sin 180° (0.25 \cos 179.5° + 0.25)}$$

$$= \frac{\sin 360° (-0.99996 \times 0.25 + 0.25)}{2 \sin 180° (-0.99996 \times 0.25 + 0.25)}$$

$$= \frac{\sin 360° (-0.24999 + 0.25)}{2 \sin 180° (-0.24999 + 0.25)}$$

$$= \frac{\sin (360° \times 10^{-5})}{2 \sin (180° \times 10^{-5})}$$

$$= \frac{\sin 0.0036°}{2 \sin 0.0018°} = \frac{\sin 12.96'}{2 \sin 6.48'}$$

$$= \frac{0.000063}{2 \times 0.0000315} = 1. \quad \textit{Ans.} \quad \text{(Point E)}$$

This answer can also be established by recognizing the fact that the sine of a small angle is proportional to the angle, so that

$$\frac{\sin 2x}{2 \sin x} = \frac{2 \sin x}{2 \sin x} = 1.$$
$$\lim x \to 0$$

After plotting the above 5 points on polar co-ordinates, viz.,

$$A = 0\underline{/0°}$$

$$B = 0.227\underline{/45°}$$

$$C = 0.707\underline{/90°}$$

$$D = 0.976\underline{/135°}$$

$$E = 1\underline{/180°}$$

we notice that perhaps two additional points, say $\theta = 60°$, and $\theta = 120°$, will make the pattern more complete.

In a direction of 60°

$$R.F.S. = \frac{\sin 360° (0.25 \cos 60° + 0.25)}{2 \sin 180° (0.25 \cos 60° + 0.25)}$$

$$= \frac{\sin 360° (0.25 \times 0.5 + 0.25)}{2 \sin 180° (0.25 \times 0.5 + 0.25)}$$

$$= \frac{\sin (360° \times 0.375)}{2 \sin (180° \times 0.375)} = \frac{\sin 135°}{2 \sin 67.5°}$$

$$= \frac{0.707}{2 \times 0.923} = \frac{0.707}{1.846}$$

$$= 0.383. \quad \textit{Ans.} \quad \text{(Point F)}$$

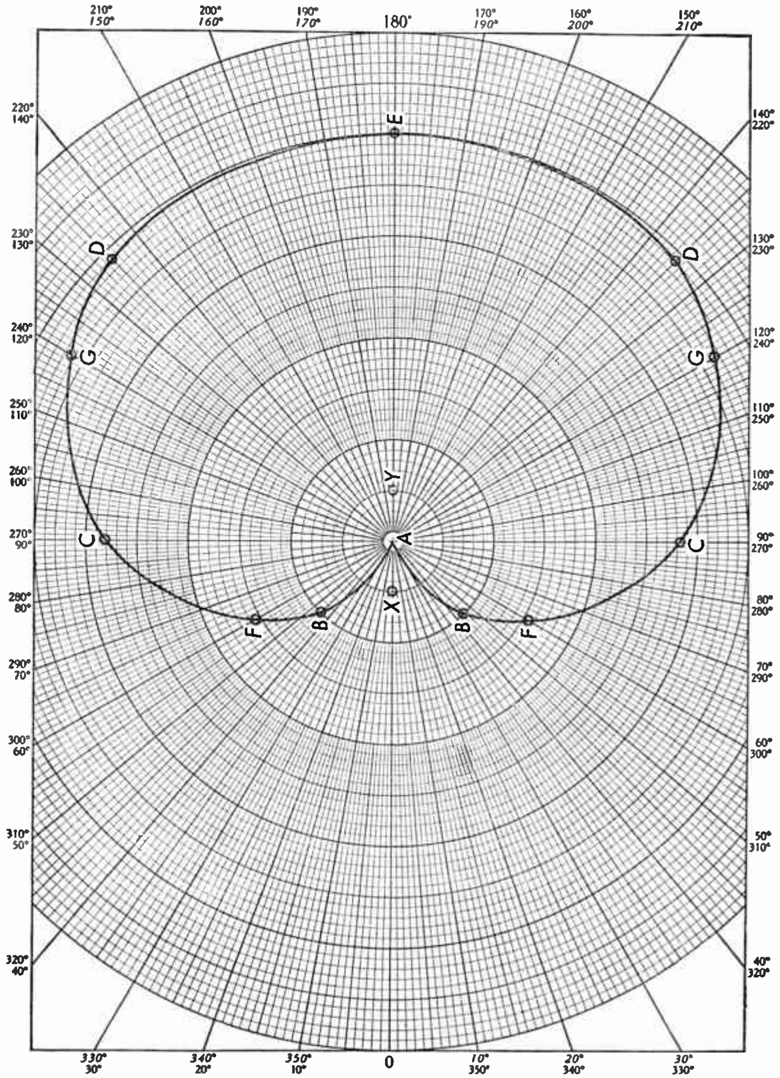

Fig. 10.25 Field strength pattern in a horizontal plane obtained by two vertical wires spaced $\frac{1}{4}$ wavelength apart and fed 90 degrees out of phase.

In a direction of 120°

$$R.F.S. = \frac{\sin 360° (0.25 \cos 120° + 0.25)}{2 \sin 180° (0.25 \cos 120° + 0.25)}$$

$$= \frac{\sin 360° (-0.5 \times 0.25 + 0.25)}{2 \sin 180° (-0.5 \times 0.25 + 0.25)}$$

$$= \frac{\sin (360° \times 0.125)}{2 \sin (180° \times 0.125)} = \frac{\sin 45°}{2 \sin 22.5°}$$

$$= \frac{0.707}{2\ (0.383)} = \frac{0.707}{0.766}$$

$= 0.924.$ *Ans.* (Point G)

We now add the points

$$F = 0.383\underline{/60°}$$

and

$$G = 0.924\underline{/120°},$$

and since the field strength is symmetrical about the line joining the radiators X and Y, we obtain the pattern of Figure 10.25.

Note: The unidirectional action of this array can also be deduced from the following simple considerations:

1. When the radiator Y is at the beginning of the cycle, X is, according to the given data, at +90° of its cycle. The radiation leaving Y at the time $t = 0$, arrives at X a quarter of a cycle later, because the distance between the two radiators is $\lambda/4$; at this instant the signal at X is at 180° of its cycle, i.e., in opposition to the radiation from Y. The algebraic sum of the two signals is zero and there is no signal in the direction from Y to X.

2. When X is at +90° of its cycle, Y is at 0°. The radiation of R_1 arrives at Y, one-quarter of a cycle later, at which time the signal at Y has also advanced to +90°. Both signals, being in phase, add up to twice the signal of one radiator, in the direction from A to B.

Quarter-Wave Line Matching

10.26 A transmitting dipole exhibiting an impedance of 72 ohms is to be connected to a 2-wire line with a characteristic impedance of 600 ohms by using a quarter-wave line as a matching "transformer."

What is the length of the dipole and of the matching line, and what is the characteristic impedance of the quarter-wave line if the frequency is 8 megacycles?

Solution:

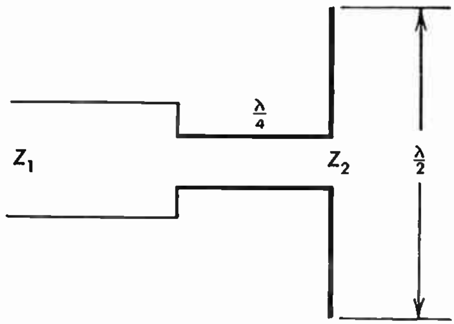

Fig. 10.26 Quarter-wave matching stub, coupling a two-wire line into a dipole.

The arrangement is shown in Figure 10.26.

The wavelength is

$$\lambda = \frac{3 \times 10^8}{8 \times 10^6} = 37.5 \text{ meters.}$$

The dipole length is (disregarding the "end effect")

$$\frac{\lambda}{2} = \frac{37.5}{2} = 18.75 \text{ meters}$$

$$= 18.75 \times 3.28 = 61.6 \text{ feet.} \quad \textit{Ans.}$$

The length of the matching line is

$$\frac{\lambda}{4} = \frac{61.6}{2} = 30.8 \text{ feet.} \quad \textit{Ans.}$$

The impedance of the matching line is the geometric mean of the "primary" and "secondary" impedances (formula 10.263).

$$Z_0 = \sqrt{Z_1 Z_2}$$

$$= \sqrt{72 \times 600} = 208 \text{ ohms.} \quad \textit{Ans.}$$

11 *Television*

Speed of Scanning Beam

11.01 What is the speed of the scanning beam at the end of the beam if the picture frame is 10 inches wide and a 525-line system is used with 60 fields per second and interlaced scanning? Disregard the flyback time in this problem.

Solution:

The 525 lines will be swept out in

$$t_{525} = \frac{1}{60} \times 2 = \frac{1}{30} \text{ second,}$$

each line being 10 inches long, with

$$l_{sec} = 30 \times 525 \text{ lines per second to be scanned,}$$

the distance traveled by the beam is

$$S_{sec} = 525 \times 30 \times 10 \text{ inches}$$

$$= 157{,}500 \text{ inches per second.}$$

$$S_{hour} = 157{,}500 \times 3600$$

$$= 567{,}000{,}000 \text{ inches per hour}$$

$$= 47{,}200{,}000 \text{ feet per hour}$$

$$= 8950 \text{ miles per hour, approximately.}$$

or 14,400 kilometers per hour. *Ans.*

Rate of Transmission of Picture Elements

11.02 What is the number of picture elements scanned per second in problem 11.01, if each line contains 485 picture elements and 40 lines are inactive?

Solution:

The number of the active lines is

$$525 - 40 = 485;$$

the active lines per second are

$$485 \times 30 = 14{,}550.$$

Since each line contains 485 picture elements, we have

$$p_e = 14{,}550 \times 485$$
$$= 7{,}050{,}000 \text{ picture elements per second.} \quad \textit{Ans.}$$

Horizontal Scanning and Retrace

11.03 A sync system requires a horizontal synchronizing signal the base of which is 0.18 H, the front porch of the pedestal 0.0225 H, the following part of the pedestal including the horizontal synchronizing pulse and the back porch 0.14 H, of which 0.08 H is taken by the synchronizing pulse; H is the time from the start of one line to the start of the next line. Find the duration of scanning and retrace time, the ratio k_h of the retrace to the scanning velocity and the actual trace and retrace velocities of the beam when scanning the mosaic of the iconoscope ($3\frac{9}{16}'' \times 4\frac{3}{4}''$) and the screen of a $9'' \times 12''$ kinescope.

Solution:

The number of lines per second scanned in a 525-line, 30-frame system is

$$n = 525 \times 30 = 15{,}750 \text{ lines per second.}$$

The duration of one line is

$$H = \frac{1}{15{,}750} = 63.5 \times 10^{-6} \text{ second}$$
$$= 63.5 \text{ microseconds.}$$

Neglecting the front porch and assuming that the retrace time is identical with the pedestal width, viz.,

$$t_r = P_w = 0.14 + 0.0225 = 0.1625\ H$$
$$= 0.1625 \times 63.5 = 10.3 \text{ microseconds.} \quad \textit{Ans.}$$

Accordingly the scanning time is

$$t_{sc} = 63.5 - 10.3 = 53.2 \text{ microseconds.} \quad \textit{Ans.}$$

The ratio of the velocities, or the number expressing how many times faster the scanning spot travels when flying back compared to its scanning speed is

$$k_h = \frac{1.00 - 0.1625}{0.1625} = \frac{0.8375}{0.1625} = 5.15. \quad \textit{Ans.}$$

Also $$k_h = \frac{53.2}{10.3} = 5.15$$

The horizontal scanning velocity at the mosaic of the iconoscope is

$$v_{hsc} = \frac{4.75}{53.5 \times 10^{-6}} = 0.0888 \times 10^6 \text{ inches}$$

$$= 88{,}800 \text{ inches} = 7400 \text{ feet}$$

$$= 2260 \text{ meters per second.} \quad \textit{Ans.}$$

The retrace velocity being 5.15 times as fast is

$$v_{hr} = 2260 \times 5.15 = 11{,}600 \text{ meters per second.} \quad \textit{Ans.}$$

The kinescope scanning and retrace velocities can be found by multiplying the above values by the factor

$$\frac{12}{4.75} = 2.53$$

which takes the larger width of the reproducing tube into account.

Hence $$v'_{hsc} = 2260 \times 2.53 = 5720 \text{ meters per second.} \quad \textit{Ans.}$$

$$v'_{hr} = 11{,}600 \times 2.53 = 29{,}300 \text{ meters per second.} \quad \textit{Ans.}$$

Vertical Scanning and Retrace

11.04 The R.M.A. standards require a vertical blanking time of 0.05 V with a positive tolerance of 0.03 V and a negative tolerance of 0.0 V where V is the time from the start of one field to the start of next. Find the duration of the scanning and retrace time and the ratio k_v of the flyback velocity to the scanning velocity.

Solution:

The number of fields per second scanned in a 525-line, 30-frame system is $30 \times 2 = 60$ fields.

The duration of one field is

$$V = \frac{1}{60} = 16.7 \text{ milliseconds.} \quad \textit{Ans.}$$

Neglecting the equalizing time and assuming that the retrace time is identical with the vertical blanking time

$$t_r' = t_{bl} = 0.05 \times 16.7$$
$$= 0.836 \text{ milliseconds.} \quad \textit{Ans.}$$

The scanning time is $16.7 - 0.836 = 15.864$ milliseconds.

The ratio of the velocities is

$$k_v = \frac{1.00 - 0.05}{0.05} = 19. \quad \textit{Ans.}$$

Also
$$k_v = \frac{15.864}{0.836} = 19.$$

Number of Inactive Lines

11.05 Using the standards stated in problem 11.04 find the maximum and the minimum number of inactive lines per frame and per field in a 525-line system employing interlaced scanning.

Solution:

The smallest number of inactive lines during which the picture is blanked out is

$$n_i = 525 \times 0.05 = 26.25 \text{ lines per frame}$$
$$\text{or } 13.125 \text{ lines per field.} \quad \textit{Ans.}$$

The maximum number of inactive lines is

$$N_i = 525 \times (0.05 + 0.03) = 42 \text{ lines per frame}$$
$$\text{or } 21 \text{ lines per field.} \quad \textit{Ans.}$$

Number of Picture Elements per Frame

11.06 A 16-millimeter motion-picture frame contains about 125,000 effective picture elements. Compare this figure with the present FCC standard television employing 525 lines per frame with 5 per cent vertical retrace time using 60 fields interlaced scanning, a transmission system with a maximum video frequency of 4 megacycles, a ratio of width to height of 4 to 3 and operating with a utilization ratio of 75 per cent.

Solution:

(a) Limitation by beam diameter.

The number of active lines is

$$n_a = 525 \times 0.95 = 498 \text{ lines}$$

Assuming that the beam diameter is equal to the width of a line it will fit into the frame 498 times vertically. The number of beam diameters contained in each line is determined by the aspect ratio

$$n_{dl} = \frac{4}{3} \times 498 = 665$$

The maximum number of picture elements per pattern as limited by the beam diameter is

$$N_{pe} = 665 \times 498 = 332{,}000 \text{ picture elements.}$$

As far as the beam diameter is concerned the television frame could therefore contain

$$\frac{332{,}000}{125{,}000} = 2.66 \text{ times}$$

as many picture elements as does a 16-millimeter motion-picture frame. *Ans.*

(b) Limitation by highest video frequency.

At the highest video frequency given, and assuming a scene consisting of alternating black and white spots of the size of picture elements, the total number of picture elements transmitted in one second is

$$N_{sec} = 4{,}000{,}000 \times 2 = 8{,}000{,}000$$

since the positive half of the cycle may transmit the white spot and the negative half of the cycle the black spot. With 30 frames transmitted in one second the number of picture elements per frame as limited by the video frequency is

$$N_{vf} = \frac{8{,}000{,}000}{30} = 267.000.$$

As far as the given video frequency is concerned the television frame could therefore contain

$$\frac{267{,}000}{125{,}000} = 2.22 \text{ times}$$

as many picture elements as does a 16-millimeter motion-picture frame. *Ans.*

(c) Limitation by resolution.

At a utilization ratio of 75 per cent the number of picture elements resolved in a vertical direction is

$$N_v = 0.75 \times 498 = 373 \text{ picture elements.}$$

The number of picture elements resolved in each line is determined by the highest video frequency.

$$f_{max} = \frac{1}{2} \times 373 \times N_L \times 30$$

where the factor $\frac{1}{2}$ is due to the fact that each cycle can contain 2 picture elements as was explained under (b), and 30 is the number of frames per second.

Thus
$$n_L = \frac{2 \times 4{,}000{,}000}{373 \times 30}$$
$$= 714 \text{ picture elements per line.}$$

Since the flyback time is about 18 per cent of total line time the number of visible picture elements per line may be

$$n_L = 714 \times 0.82$$
$$= 585 \text{ visible picture elements.}$$

Hence the total number of picture elements per frame as limited by the resolution is

$$N_r = 373 \times 585 = 218{,}000.$$

As far as the actual resolution of picture elements is concerned a television frame could contain

$$\frac{218{,}000}{125{,}000} = 1.74 \text{ times}$$

as many picture elements as does a 16-millimeter motion-picture frame. *Ans.*

Television Broadcast Band

11.07 The Federal Communications Commission defines television broadcast band as follows: "The term *television broadcast band* means those frequencies in the band extending from 44 to 216 megacycles which are assignable to television broadcast stations. These frequencies are 44 to 50 megacycles, 54 to 72 megacycles, 76 to 88 megacycles and 174 to 216 megacycles." How many stations could operate in each "television horizon"?

Solution:

Since a bandwidth of 6 megacycles is occupied by each television channel the available number of channels is:

$$
\begin{aligned}
50 - 44 &= 6 \text{ megacycles} = 1 \text{ station (unassigned)} \\
72 - 54 &= 18 \text{ megacycles} = 3 \text{ stations} \\
88 - 76 &= 12 \text{ megacycles} = 2 \text{ stations} \\
216 - 174 &= 42 \text{ megacycles} = 7 \text{ stations} \\
&\qquad \text{Sum} \quad 13 \text{ stations.} \quad \textit{Ans.}
\end{aligned}
$$

Television Horizon

11.08 Prove the formula $r = 1.23\sqrt{h}$, where r is the radius of the horizon in miles, h the height in feet.

Solution:

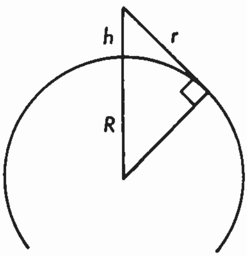

Fig. 11.08 Distance of the horizon r as seen from an altitude h.

Let R be the radius of the earth, r the radius of the horizon (more accurately the distance from the top of the antenna to the horizon), then we have a right triangle, where

$$(R + h)^2 = R^2 + r^2,$$

$$R^2 + 2\,Rh + h^2 = R^2 + r^2.$$

Subtracting R^2 $$2Rh + h^2 = r^2,$$

and $$r = \sqrt{2Rh + h^2}.$$

Since the radius of the earth is 3960 miles, approximately, and the height of the antenna is, for example, 1 mile if located on top of a mountain of 5280 feet absolute altitude (not above sea level), the value of h^2 will be negligibly small compared with $2Rh$; we can write

$$r = \sqrt{2Rh}$$

all values in the same unit, miles or kilometers. If h is in feet then h feet will be $\frac{h}{5280}$ of a mile; with $R = 3960$ miles, we obtain

$$r = \sqrt{2 \times 3960 \times \left(\frac{h}{5280}\right)}$$
$$= \sqrt{1.5\,h} = 1.23\sqrt{h}. \quad \textit{Ans.}$$

Television Area

11.09 What area will be covered by a television antenna located on a hill with an effective height of 568 feet?

Solution:

$$r = 1.23\sqrt{568} = 29.3 \text{ miles}$$
$$A = \pi r^2 = 3.14 \times 29.3^2 = 2690 \text{ square miles.} \quad \textit{Ans.}$$

Distance between Relay Stations

11.10 What is the maximum distance between two television relay stations, if one antenna has an effective height of 420 feet and the other 375 feet?

Solution:

If the line joining the two stations is allowed to be a tangent of the earth, the distance is

$$D = r_1 + r_2$$
$$= 1.23(\sqrt{h_1} + \sqrt{h_2})$$
$$= 1.23(20.5 + 19.4)$$
$$= 1.23 \times 39.9 = 49.1 \text{ miles.} \quad \textit{Ans.}$$

Kinescope Electrostatic Deflection

11.11 A cathode-ray tube has a beam length $L = 21.5$ centimeters, measured from the center of the deflecting plates to the screen, and a voltage $e_d = 720$ volts between the deflecting plates which are separated by a distance $h = 0.5$ centimeter and have a length of $l = 3$ centimeters. Find the deflection D at the fluorescent screen if the applied second anode voltage is $E = 10{,}000$ volts. Also find the deflection sensitivity for the stated operating conditions.

Solution:

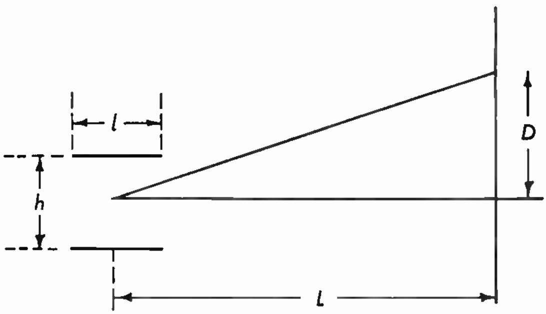

Fig. 11.11 Diagram illustrating the deflection of an electron beam by electrostatic deflection.

Using the formula

$$D = \frac{L\,l\,e_d}{2\,E\,h}$$

we have

$$D = \frac{21.5 \times 3 \times 720}{2 \times 10{,}000 \times 0.5}$$

$$= 21.5 \times 0.216 = 4.64 \text{ centimeters.} \quad \textit{Ans.}$$

The electrostatic deflection sensitivity is

$$S_e = \frac{720}{4.64} = 155.2 \text{ volts per centimeter.} \quad \textit{Ans.}$$

Magnetic Deflection

11.12 A cathode-ray tube has a beam length of $L = 21.5$ centimeters, measured from the center of the deflecting field to the screen, an axial length $l = 3$ centimeters of the region of the uniform magnetic field. Calculate the flux density B that exists in the magnetic field deflecting the beam a distance of $D = 12.5$ centimeters when a second-anode voltage of $E = 10{,}000$ volts is applied.

Solution:

Using the formula

$$D = 0.297 \frac{LlB}{\sqrt{E}}$$

we have

$$12.5 = \frac{0.297 \times 21.5 \times 3\,B}{\sqrt{10{,}000}}$$

making

$$1250 = 19.15\,B,$$

and

$$B = \frac{1250}{19.15}$$

$$= 65 \text{ lines per square centimeter.} \quad \textit{Ans.}$$

Illumination of Mosaic

11.13 A source of 900 candles is mounted in a reflector that reflects 85 per cent of the light in a beam of 30° onto a perfectly diffusing white surface 20 feet away from the source. The area of the surface is 9 square feet and the reflection coefficient is 80 per cent. Using the simplified formula $E_p = B/(2f^2)$ where E_p is the illumination of the plate in foot candles, f the ratio of the focal length to the diameter of the lens opening, and B the brightness in candles per square foot, find the illumination of the camera plate by the given source if an $f:6$ lens is used.

Solution:

The light flux radiated from a source of 900 candles in all directions

$$F = 4\pi I$$

$$= 12.56 \times 900 = 11{,}300 \text{ lumens}$$

of which 85 per cent, i.e.

$$11{,}300 \times 0.85 = 9{,}600 \text{ lumens}$$

are reflected by the reflector.

The area subtended by a solid angle of 30° at a distance of 20 feet is found as follows:

Consider the cross section of the conical beam which is an isosceles triangle. The angle at the vertex of the triangle is 30° and the altitude of the triangle is 20 feet.

Let half the base be r,

then $$\frac{r}{20} = \tan 15°$$

and $$r = 20 \times \tan 15° = 5.36 \text{ feet.}$$

The intercepted area is

$$A = \pi r^2 = 90 \text{ square feet.}$$

The illuminated area intercepts

$$F_A = 9600 \times \frac{9}{90} = 960 \text{ lumens.}$$

Since the area is 9 square feet the illumination of the area

$$E_A = \frac{960}{9} = 107 \text{ lumens per square foot.}$$

The brightness of the perfectly diffusing area with a reflection coefficient of 80 per cent is

$$B = \frac{E_A R}{\pi} = \frac{107 \times 0.8}{3.14}$$

$$= 27.2 \text{ candles per square foot.}$$

Thus the average illumination of the camera plate is

$$E_p = \frac{B}{2f^2} = \frac{27.2}{2 \times 36}$$

$$= 0.378 \text{ foot-candles.} \quad \textit{Ans.}$$

Image-Orthicon Electron Multiplier

11.14 The total gain obtainable from a five-stage multiplier such as is used in an image orthicon is 200 to 500. Calculate the secondary emission ratio of each stage corresponding to the two stated values of gain.

Solution:

(a) for 200.

The secondary emission ratio for each of the 5 stages is

$$S = \sqrt[5]{200}$$

$$\log S = \frac{1}{5} \log 200 = 0.2 \times 2.301$$

$$\log S = 0.4602$$

$$S = \text{antilog } 0.4602 \cong 2.9 \text{ to } 1. \quad \textit{Ans.}$$

(b) for 500

$$S = \sqrt[5]{500}$$

$$\log S = \frac{1}{5} \log 500 = 0.2 \times 2.699$$

$$\log S = 0.5398$$

$$S = \text{antilog } 0.5398 \cong 3.5 \text{ to } 1. \quad \textit{Ans.}$$

Note that the small change in the secondary emission ratio from approximately 3 to 3.5 will change the gain from 200 to 500.

Orthicon Sensitivity

11.15 The discharge current I_d of the mosaic target of an orthicon-pickup tube, i.e. the current which constitutes the video signal, is 1 microampere for 28 foot-candles of illumination, 0.3 microampere for 7 foot-candles of illumination and 0.043 microampere for 1 foot-candle of illumination. Calculate the sensitivity of the orthicon for high, medium, and low illuminations, expressed in microvolts per foot-candle when a coupling resistor of 10,000 ohms is used.

Solution:

(a) For 28 foot-candles.

The voltage due to the discharge current is

$$E_1 = I_1 R = 10^{-6} \times 10^{4} = 10^{-2}.$$

The sensitivity expressed in microvolts per foot-candle is

$$S_1 = \frac{1}{28} \times 10^{-2} = 0.0358 \times 10^{-2}$$

$$= 3.58 \times 10^{-4}$$

$$= 358 \times 10^{-6} \text{ volts per foot-candle.} \quad \textit{Ans.}$$

(b) For 7 foot-candles.

$$E_2 = I_2 R = 0.3 \times 10^{-6} \times 10^4 = 3 \times 10^{-3}$$

$$S_2 = \frac{1}{7} \times 3 \times 10^{-3} = 0.428 \times 10^{-3} = 428 \times 10^{-6}$$

$= 428$ microvolts per foot-candle. *Ans.*

(c) For 1 foot-candle.

$$E_3 = I_3 R = 0.043 \times 10^{-6} \times 10^4 = 4.3 \times 10^{-4}$$

$$S_3 = 4.3 \times 10^{-4} = 430 \times 10^{-6}$$

$= 430$ microvolts per foot-candle. *Ans.*

Object Size at Stated Distance from Screen

11.16 A person was tested and found to have a minimum viewing angle of 1.5 minutes. What is the smallest distance d the person will be able to distinguish on a screen, 25 feet away from his eye?

Solution:

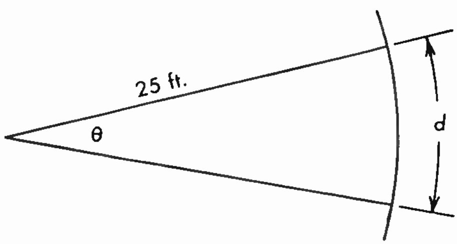

Fig. 11.16 Distance d distinguishable when the viewing angle is θ.

The distance d, though a curved arc, can be considered as practically a straight line.

The distance is $$d = 2\pi r \times \frac{\theta}{360}, \ \theta \text{ in degrees};$$

or $$d = 2\pi r \times \frac{\theta}{360 \times 60}, \ \theta \text{ in minutes};$$

substituting $$d = \frac{6.28 \times 25 \times 1.5}{360 \times 60} = 0.0109 \text{ foot}$$

$= 0.131$ inch. *Ans.*

(A little more than 1/8 inch.)

Critical Viewing Distance for Picture Elements

11.17 The test pattern of a television station indicates a vertical resolution of 390 picture elements. What is the critical viewing distance for a person whose minimum viewing angle is one minute, if the screen is 9 × 12 inches?

Solution:

At a resolution of 390 picture elements the average diameter of a picture element is the height of the screen divided by the vertical resolution:

$$d_{pc} = \frac{9}{390} = 0.0231 \text{ inch.}$$

Assuming two black picture elements separated by a white picture element the object distance

$$s_o = 2 \times 0.0231 = 0.0462 \text{ inch.}$$

An angle of one minute is 0.000291 radian, i.e. at a distance of 1 unit an angle of 1 minute would intercept an arc of 0.000291 unit.
We have the proportion

$$\frac{x}{1} = \frac{0.0462}{0.000291} = \frac{462}{2.91}$$

and

$$x = 158.8 \text{ inches}$$

$$= 13.22 \text{ feet.} \quad \textit{Ans.}$$

From a greater distance the two picture elements would merge into one with a consequent loss of detail. From a smaller distance no more detail could be observed and only the undesirable structure of the picture would become apparent.

Critical Viewing Distance for Scanning Lines

11.18 A television raster contains 525 lines with a vertical blanking time of 0.05 V, with a tolerance of +0.03 and −0.0, where V is the time from the start of one field to the start of the next field. What is the critical distance for seeing the screen as a homogeneous rectangle of light if the minimum viewing angle is 1 minute and the screen is 9 × 12 inches?

Solution:

The number of retrace lines (invisible lines) is

$$n_r = 525 \times 0.05$$
$$= 26.25 \cong 27 \text{ lines}$$

(Note: There is no negative tolerance, therefore 26 lines would not be correct.)

The number of active (visible) lines is

$$n_a = 525 - 27 = 498 \text{ lines.}$$

The distance between two lines is

$$d_2 = \frac{9}{498} = 0.018 \text{ inch.}$$

Since 1 minute is 0.000291 radian we have the proportion

$$\frac{x}{1} = \frac{0.018}{0.00029} = \frac{180}{2.9}$$

$$x = 62.2 \text{ inches} = 5.2 \text{ feet.} \quad \textit{Ans.}$$

Closer viewing will reveal the line structure of the raster.

Differentiating Circuit

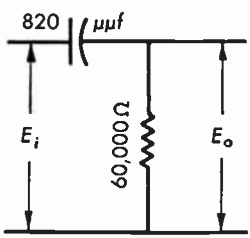

Fig. 11.19 One-stage differentiating circuit.

11.19 The amplified synchronizing pulses of a television signal are fed in composite form to the differentiating circuit of Figure 11.19. If a system of 525 lines and 60 fields employing interlaced scanning is used, what will be the percentage output of the line-scanning signal, if the fundamental frequency only is considered?

Solution:

The differentiating circuit is a voltage-divider circuit, the output voltage of which is

$$E_o = E_i \times \frac{R}{Z},$$

where Z is the impedance formed by the 60-kilohm resistor and the 820-micromicrofarad capacitor in series.

The line-scanning frequency is determined by the fact that 525 lines are scanned 30 times per second. The horizontal frequency therefore is

$$f_h = 30 \times 525 = 15{,}750 \text{ cycles}$$

$$X_h = \frac{1}{2\pi f_h C} = \frac{1}{6.28 \times 15{,}750 \times 820 \times 10^{-12}}$$

$$= 12{,}350 \text{ ohms.}$$

The impedance is therefore

$$Z = \sqrt{12.35^2 + 60^2} = 61.2 \text{ kilohms.}$$

Substituting in the voltage-divider formula, we have

$$E_o = E_i \frac{60}{61.2}$$

and $$\frac{E_o}{E_i} \cong 0.98 \text{ or } 98 \text{ per cent.} \quad \textit{Ans.}$$

Integrating Circuit

11.20 The amplified synchronizing pulses of a television signal are fed in composite form to an integrating circuit (Figure 11.20). In a system employing 525 lines per frame and 60 fields per second, what will be the percentage output of the line-scanning signal, if the fundamental frequency only is considered?

Solution:

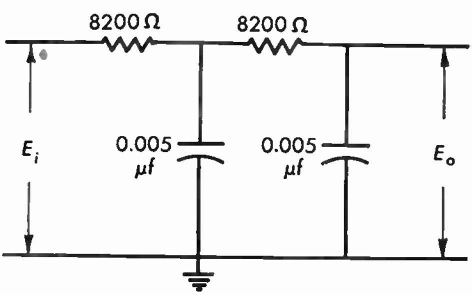

Fig. 11.20 Two-stage integrating circuit.

The integrating circuit is a voltage-divider circuit, the output voltage of which is

$$E_o = E_i \times \frac{X}{Z},$$

where X is the reactance of the shunt capacitance, and Z is the impedance formed by the resistor and capacitor in series. The line-scanning frequency is

$$f_h = 30 \times 525 = 15{,}750 \text{ cycles.}$$

Thus $$X_h = \frac{1}{6.28 \times 15{,}750 \times 0.005 \times 10^{-6}} = 2020 \text{ ohms,}$$

and $$Z = \sqrt{8200^2 + 2020^2} = 8450 \text{ ohms.}$$

Substituting

$$\frac{E_o'}{E_i} = \frac{2020}{8450} = 0.239 = 23.9 \text{ per cent.}$$

Approximately 23.9 per cent of the input to the first section will appear across the first capacitor.

Total percentage output:

$$\frac{E_o}{E_i} = 0.239 \times 0.239 = 0.0571$$

$$= 5.71 \text{ per cent, approximately.} \quad \textit{Ans.}$$

Two-Section Integrator

11.21 Assuming that the integrated field scanning signal across the first capacitor Figure 11.20 is e_f volts and that the line scanning signal is e_l volts what is the percentage output of e_f and e_l across the second capacitor if the fundamental frequency only is considered?

Solution:

The reactance of the capacitor at the field-repetition frequency is

$$X_V = \frac{1}{6.28 \times 60 \times 0.005 \times 10^{-6}} = 531{,}000 \text{ ohms.}$$

We have the reactive-resistive voltage divider ($X_V - R$).

Since $$Z_V = \sqrt{8.2^2 + 531^2} \cong 531 \text{ kilohms}$$

the percentage output of the field scanning signal, for all practical considerations is

$$\frac{E_o}{e_f} = \frac{X_V}{Z} = \frac{531}{531} = 1 = 100 \text{ per cent.} \quad \textit{Ans.}$$

The percentage output of the line scanning signal, or of whatever is left of it after passing the first section is 23.9 per cent of e_l or 5.71 per cent of the original input. *Ans.* (See problem 11.20.)

Universal Time-Constant Chart

11.22 Using the formulas 11.1 and 11.12 on charge and discharge of a capacitor in a resistance-capacitance circuit calculate the charge and discharge voltage across the capacitor e_c in terms of the applied voltage E after a time of 1/2 RC, 1 RC, 2 RC, 3 RC, 4 RC and 5 RC seconds. Plot a *universal time-constant chart.*

Solution:

(a) Charge.

Using $$e_c = E\left(1 - \epsilon^{-\frac{t}{RC}}\right)$$

and $$t = 0.5\,RC \text{ second}$$

we have
$$e_{c_{1/2}} = E\left(1 - \epsilon^{-\frac{0.5\,RC}{RC}}\right)$$
$$= E\,(1 - \epsilon^{-0.5}) = E\left(1 - \frac{1}{\sqrt{\epsilon}}\right)$$
$$= E\left(1 - \frac{1}{1.649}\right) = E\,(1 - 0.607)$$
$$= E\,(0.393) \cong 39 \text{ per cent of } E. \quad \textit{Ans.}$$

When $$t = 1\,RC$$
$$e_{c_1} = E\left(1 - \epsilon^{-\frac{RC}{RC}}\right)$$
$$= E\,(1 - \epsilon^{-1}) = E\left(1 - \frac{1}{\epsilon}\right) = E\,(1 - 0.368)$$
$$= E\,(0.632) \cong 63 \text{ per cent of } E. \quad \textit{Ans.}$$

When $$t = 2\,RC$$
$$e_{c_2} = E\,(1 - \epsilon^{-\frac{2\,RC}{RC}})$$
$$= E\,(1 - \epsilon^{-2}) = E\left(1 - \frac{1}{\epsilon^2}\right) = E\left(1 - \frac{1}{7.39}\right)$$
$$= E\,(1 - 0.135) = E\,(0.865)$$
$$\cong 86 \text{ per cent of } E. \quad \textit{Ans.}$$

When $t = 3\ RC$

$$e_{c_3} = E\,(1 - \epsilon^{-\frac{3\ RC}{RC}})$$

$$= E\,(1 - \epsilon^{-3}) = E\left(1 - \frac{1}{\epsilon^3}\right) = E\left(1 - \frac{1}{20.1}\right)$$

$$= E\,(1 - 0.049) = E\,(0.951)$$

$\cong$ 95 per cent of E. *Ans.*

When $t = 4\ RC$

$$\epsilon_{c_4} = E\left(1 - \frac{1}{\epsilon^{-4}}\right)$$

$$= E\left(1 - \frac{1}{54.6}\right) = E\,(1 - 0.0183) = E\,(0.9817)$$

= 98 per cent of E

When $t = 5\ RC$

$$e_{c_5} = E\left(1 - \frac{1}{\epsilon^5}\right)$$

$$= E\left(1 - \frac{1}{148}\right) = E\,(1 - 0.0067) = E\,(0.9933)$$

$\cong$ 99 per cent of E. *Ans.*

(b) Discharge.

Using the formula

$$e_c = E\,C^{-\frac{t}{RC}}$$

the values of the discharge voltages can readily be found by using the second-term values of the expression $(1 - \epsilon^{-\frac{t}{RC}})$ in the preceding calculations. Thus, for $t = 0.5\ RC$ this value is 0.607 or approximately 61 per cent of E. *Ans.*

When $t = 1\ RC$ seconds

$$e_{c_1} = 0.368\ E$$

$\cong$ 37 per cent of E. *Ans.*

Likewise $e_{c_2} = 14$ per cent of E,

$e_{c_3} = 5$ per cent of E,

$e_{c_4} = 2$ per cent of E,

and $e_{c_5} = 1$ per cent of E. *Ans.*

Table for plotting the universal time-constant chart

Time in seconds	Per cent of applied voltage while charging	Per cent of full voltage while discharging
0.5 *RC*	39	61
1 *RC*	63	37
2 *RC*	86	14
3 *RC*	95	5
4 *RC*	98	2
5 *RC*	99	1

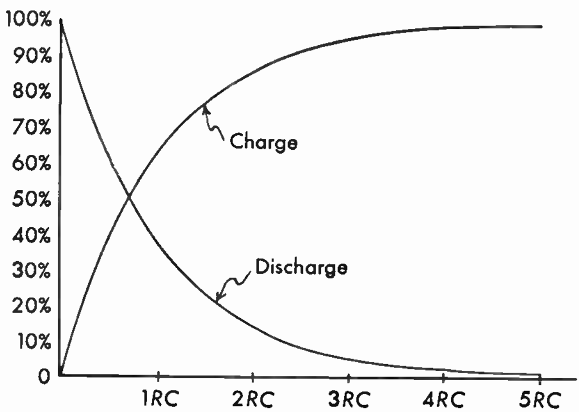

Fig. 11.22 Universal charge and discharge curves (see text).

Integrated Output

11.23 Using the universal time-constant chart find the voltage of the integrated output Figure 11.23b at the peaks A, B, C, D, E, and F and at the troughs between these points. The input wave has the form of Figure 11.23a, the time intervals being 0.5 RC and 2 RC throughout.

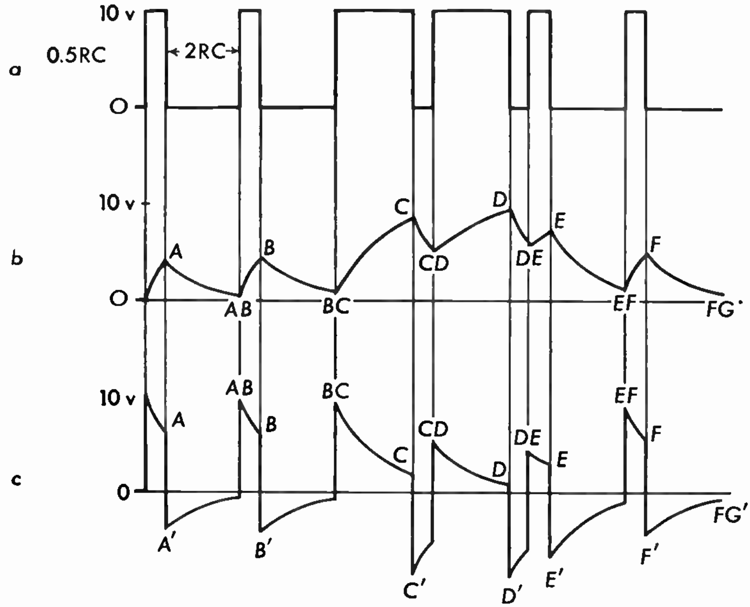

Fig. 11.23 *a*. Rectangular pulses. The short intervals on the time axis are all $\frac{1}{2}$ RC second, the long intervals all 2 RC seconds.

b. Integrated output of the waveform *a*.

c. Differentiated output of the waveform *a*. To avoid crowding, the lettering of the points referred to in the text as AB', BC', CD', DE', and EF' is omitted. AB' is 10 volts below AB, BC' 10 volts below BC etc.

Solution:

The leading edges of the curve (i.e. the left-hand edges which occur sooner) are due to the charge of the capacitor, and the trailing edges are due to discharge.

When the voltage of 10 volts is applied across the integrating circuit the capacitor will charge to 39 per cent of the applied voltage.

Therefore $E_A = 10 \times 0.39 = 3.9$ volts. *Ans.*

During the following time of 2 RC seconds when no voltage is applied, the condenser will discharge to 14 per cent of voltage at the beginning of the discharge period. The voltage across the capacitor will reach a trough of

$$E_{AB} = 3.9 \times 0.14 = 0.546 \text{ volt.}$$

When the second narrow pulse is applied the capacitor voltage will oppose the applied voltage, since its negative plate is tied to the negative plate of the applied voltage. The net voltage across the integrating circuit is

$$10 - 0.546 = 9.454 \text{ volts.}$$

During the following charge time of 0.5 RC seconds the capacitor voltage will increase by the amount of

$$9.454 \times 0.39 = 3.69 \text{ volts,}$$

and will reach the peak B.

$$E_B = 0.546 + 3.69 = 4.236 \text{ volts.} \quad \textit{Ans.}$$

During the following discharge period of 2 RC seconds this voltage will be reduced to

$$E_{BC} = 4.236 \times 0.14 = 0.593 \text{ volt.}$$

Similarly

$$10 - 0.593 = 9.407 \text{ volts}$$

$$9.407 \times 0.86 = 8.1 \text{ volts}$$

and

$$E_C = 8.1 + 0.593 = 8.693 \text{ volts.} \quad \textit{Ans.}$$

$$E_{CD} = 8.693 \times 0.61 = 5.3 \text{ volts.} \quad \textit{Ans.}$$

$$10 - 5.3 = 4.7 \text{ volts}$$

$$4.7 \times 0.86 = 4.04 \text{ volts}$$

and

$$E_D = 5.3 + 4.04 = 9.34 \text{ volts.} \quad \textit{Ans.}$$

$$E_{DE} = 9.34 \times 0.61 = 5.7 \text{ volts}$$

$$10 - 5.7 = 4.3 \text{ volts}$$

$$4.3 \times 0.39 = 1.68 \text{ volts}$$

$$E_E = 5.7 + 1.68 = 7.38 \text{ volts.} \quad \textit{Ans.}$$

$$E_{EF} = 7.38 \times 0.14 = 1.03 \text{ volts.} \quad \textit{Ans.}$$

$$10 - 1.03 = 8.97 \text{ volts}$$

$$8.97 \times 0.39 = 3.5 \text{ volts}$$

$$E_F = 1.03 + 3.5 = 4.53 \text{ volts.} \quad \textit{Ans.}$$

$$E_{FG} = 4.53 \times 0.14 = 0.635 \text{ volts.} \quad \textit{Ans.}$$

Differentiated Output

11.24 Find the voltages of the differentiated output Figure 11.23c at the points A, A', etc. to FG'. The input wave has the form of Figure 11.23.

Solution:

The voltages across the resistor could be found by using the charge and discharge formulas or by using the universal time-constant chart. A simpler way is the following:

The required voltage values can be found by remembering that

(1) *at any time the voltage across the resistor plus the voltage across the capacitor must be equal to the applied voltage of 10 volts or zero volts (Kirchhoff's second law)*;

(2) *whenever the applied voltage changes the change appears across the resistor instantaneously.*

At the time $t = 0$ the full pulse voltage appears across the resistor

$$E_0 = 10 \text{ volts.} \quad Ans.$$

At the time A, 1/2 RC seconds later, when the capacitor voltage is 3.9 volts the resistor voltage is

$$E_A = 10 - 3.9 = 6.1 \text{ volts.} \quad Ans.$$

When the pulse voltage ceases or equals zero the voltage across the capacitor cannot change instantaneously, but the resistor voltage can. The full change will appear across the resistor

$$E'_A = 6.1 - 10 = -3.9 \text{ volts.} \quad Ans.$$

From the time A' to the time AB the applied voltage remains zero, and the voltage across the resistor is equal and opposite to the voltage across the capacitor.

At AB' $\quad E'_{AB} = -0.54$ volt. *Ans.*

From this point the applied voltage is again 10 volts. The voltage across the resistor immediately increases by 10 volts.

$$E_{AB} = -0.54 + 10 = 9.46 \text{ volts.} \quad Ans.$$

At B the voltage is only

$$E_B = 10 - 4.24 = 5.76 \text{ volts.} \quad Ans.$$

At B' it drops by 10 volts

$$E'_B = 5.76 - 10 = -4.24 \text{ volts.} \quad Ans.$$

At BC' the voltage is equal and opposite to the voltage across the capacitor

$$E'_{BC} = 0.59 \text{ volt.} \quad Ans.$$

Similarly

$$E_{BC} = -0.59 + 10 = 9.41 \text{ volts.} \quad Ans.$$

$$E_C = 10 - 8.69 = 1.31 \text{ volts.} \quad Ans.$$

$$E'_C = 1.31 - 10 = -8.69 \text{ volts.} \quad Ans.$$

$$E'_{CD} = -5.3 \text{ volts.} \quad Ans.$$

$$E_{CD} = 10 - 5.3 = 4.7 \text{ volts.} \quad Ans.$$

$$E_D = 10 - 9.34 = 0.66 \text{ volt.} \quad Ans.$$

$$E'_D = 0.66 - 10 = -9.34 \text{ volts.} \quad Ans.$$

$$E'_{DE} = -5.7 \text{ volts.} \quad Ans.$$

$$E_{DE} = 10 - 5.7 = 4.3 \text{ volts.} \quad Ans.$$

$$E_E = 10 - 7.38 = 2.62 \text{ volts.} \quad Ans.$$

$$E'_E = 2.62 - 10 = -7.38 \text{ volts.} \quad Ans.$$

$$E'_{EF} = -3.5 \text{ volts.} \quad Ans.$$

$$E_{EF} = 10 - 3.5 = 6.5 \text{ volts.} \quad Ans.$$

$$E_F = 10 - 4.53 = 5.47 \text{ volts.} \quad Ans.$$

$$E'_F = 5.47 - 10 = -4.53 \text{ volts.} \quad Ans.$$

$$E'_{FG} = -0.635 \text{ volts.} \quad Ans.$$

Voltage Distribution across Capacitors

11.25 In Figure 11.25 $E_s = 120$ volts, $C_1 = 4$ microfarads, $C_2 = 8$ microfarads. Find the voltage E_2 across the plates of the capacitor C_2. Also find a general formula for E_2 in terms of E_s, C_1, and C_2.

Solution:

(a) The voltage being inversely proportional to the capacitance we have

$$\frac{E_1}{E_2} = \frac{8}{4} \tag{1}$$

Also $$E_1 + E_2 = 120 \tag{2}$$

Solving for E_1 in (1)

$$E_1 = \frac{8\,E_2}{4} = 2\,E_2$$

Substituting in (2)

$$2\,E_2 + E_2 = 120,$$

$$3\,E_2 = 120,$$

and $$E_2 = 40 \text{ volts.} \quad \textit{Ans.}$$

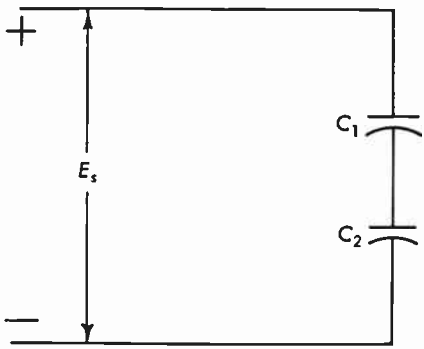

Fig. 11.25 Direct voltage E_s applied across two capacitors in series.

(b) Since the charge across a capacitor is directly proportional to both the capacitance and the voltage across the plates we have

$$Q_1 = E_1 \times C_1 \text{ and } Q_2 = E_2 \times C_2. \qquad (1, 2)$$

When the source voltage E_s is applied, a certain number of electrons will rush to the lower plate of C_2 and build up the charge Q_2, but exactly the same number of electrons will be repelled from the upper plate of C_2 and will rush to the lower plate of C_1. It is evident that

$$Q_1 = Q_2. \qquad (3)$$

The equations (1) and (2) can therefore be rewritten

$$E_1\,C_1 = E_2\,C_2$$

and, after transposing

$$\frac{E_1}{E_2} = \frac{C_2}{C_1} \qquad (4)$$

Also, by Kirchhoff's second law

$$E_s = E_1 + E_2 \qquad (5)$$

The simultaneous equations (4) and (5) can be solved for E_2 by the method of substitution:

From (4) $$E_1 = \frac{E_2 C_2}{C_1}.$$

Substituting in (5)

$$E_s = \frac{E_2 C_2}{C_1} + E_2,$$

$$E_s = E_2\left(\frac{C_2}{C_1} + 1\right),$$

$$E_s = E_2 \frac{C_1 + C_2}{C_1},$$

and $$E_2 = E_s \frac{C_1}{C_1 + C_2}. \quad \textit{Ans.}$$

In this problem

$$E_2 = 120 \times \frac{4}{4 + 8} = 40 \text{ volts.}$$

Counter Circuit

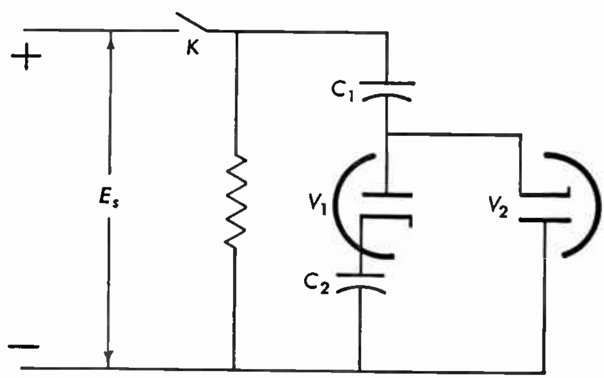

Fig. 11.26 Electronic counter circuit, based on the principle of the voltage distribution of two capacitors in series. The voltage across the capacitor C_2 is usually employed to unblock the grid of the tube of a discharge circuit.

11.26 In the electronic counter circuit Figure 11.26 the signal voltage $E_s = 100$ volts, $C_1 = 0.05$ microfarads, and $C_2 = 2$ microfarads. Find the voltage that will be built up across C_2 after the source voltage E_s is applied twice by closing the key K.

Solution:

When the key is closed V_1 will conduct and the voltage distribution across the capacitors C_1 and C_2 is found by the formula derived in problem 11.25

$$E_2 = 100 \times \frac{0.05}{2.05} = 2.44 \text{ volts.}$$

When the key is released C_1, the lower plate of which has accumulated a negative charge, will cut off V_1 and fire V_2, through which it will discharge. The charge on C_2 will remain. When the key is closed again the voltage across C_2 will oppose the applied voltage, since the negative plate is tied to the negative terminal.

The net signal voltage is now

$$E_s' = 100 - 2.44 = 97.56 \text{ volts}$$

The new voltage built up across C_2

$$E_2' = 97.56 \times \frac{0.05}{2.05} = 2.38 \text{ volts}$$

The total voltage across C_2 after two pulses is

$$E_T = 2.44 + 2.38 = 4.82 \text{ volts.} \quad \textit{Ans.}$$

Counting Voltage after a Stated Number of Pulses

11.27 Derive a formula for the voltage accumulated across the capacitor C_2 of Figure 11.26 of problem 11.26 after a given number of pulses of the signal voltage E_s has been applied to the counter circuit.

Solution:

Let E_1 be the voltage across C_2 after the first pulse, E_2 the voltage after the second pulse and E_n the voltage after the nth pulse then, as was shown in problems 11.25 and 11.26

$$E_1 = E_s \frac{C_1}{C_1 + C_2},$$

and if the ratio $\frac{C_1}{C_1 + C_2}$ is denoted by r

$$E_1 = rE_s$$

Also

$$E_2 = \mathrm{E}_1 + r\,(E_s - E_1)$$

since the applied voltage E_s will be opposed by the voltage E_1, as was shown in the previous problem.

Thus $E_2 = rE_s + rE_s - r^2E_s$

$$E_2 \quad rE_s [1 + (1 - r)]$$

$$E_3 = E_2 + r (E_s - E_2)$$

$$= rE_s [1 + (1 - r)] + r \{E_s - rE_s [1 + (1 - r)]\}.$$

$$= rE_s [1 + (1 - r)] + rE_s \{1 - r [1 + (1 - r)]\}$$

$$= rE_s \{1 + (1 - r) + 1 - r - r + r^2\}$$

$$= rE_s \{1 + (1 - r) + (1 - 2r + r^2)\}$$

$$= rE_s [1 + (1 - r) + (1 - r)^2].$$

By inductive conclusion

$$E_n = rE_s [1 + (1 - r) + (1 - r)^2 + (1 - r)^3 + \dots (1 - r)^{n-1}]$$

The expression in brackets is a geometric series with n terms and 1 as the first term.

The sum of this series is

$$S = \frac{1 - (1 - r)^n}{1 - (1 - r)} = \frac{1 - (1 - r)^n}{r}$$

Hence $E_n = rE_s \dfrac{1 - (1 - r)^n}{r} = E_s [1 - (1 - r)^n]$. *Ans.*

Counting Voltage after an Infinite Number of Pulses

11.28 Using the formula derived in the preceding problem, what is the final voltage across C_2 after a great number of pulses have been applied?

Solution:

Using $E_n = E_s [1 - (1 - r)^n]$

it is readily seen that since $r = \dfrac{C_1}{C_1 + C_2}$ is smaller than 1, the quantity $(1 - r)$ will also be smaller than 1. A proper fraction raised to an infinite power becomes infinitesimally small and the value $(1 - r)^n$ will approach zero when $n = \infty$.

Hence $$E_n = E_s(1 - 0) = E_s. \quad Ans.$$

After a great number of pulses the final voltage across C_2 approximates the value of the applied pulse voltage.

Using the Counting-Voltage Formula

11.29 Calculate the voltage built up across C_2 in problem 11.26 after 2 pulses and after 5 pulses.

Solution:

(a) After 2 pulses.

Using $$E_n = E_s[1 - (1 - r)^n],$$

where $$n = 2 \text{ and } E_s = 100 \text{ volts},$$

we have $$E_2 = 100\left[1 - \left(1 - \frac{0.05}{2.05}\right)^2\right]$$

$$= 100\,[1 - (1 - 0.0244)^2]$$

$$= 100\,[1 - 0.9756^2] = 100\,(1 - 0.952)$$

$$= 100 \times 0.048 = 4.8 \text{ volts.} \quad Ans.$$

(b) After 5 pulses.

$$E_5 = 100\,(1 - 0.9756^5)$$

$$= 100\,(1 - 0.884) = 11.6 \text{ volts.} \quad Ans.$$

Incremental Voltage Caused by a Given Pulse

11.30 Find the voltage increase across C_2 in problem 11.26 caused by the 5th pulse applied to the counting circuit.

Solution:

Using r for the ratio $\dfrac{C_1}{C_1 + C_2}$ the voltage increase due to the first pulse is

$$e_1 = E_s r$$

Due to the 2nd pulse

$$e_2 = (E_s - rE_s)\,r = E_s r\,(1 - r)$$

Due to the 3rd pulse

$$\begin{aligned} e_3 &= (E_s - e_2 - e_1)\, r \\ &= (E_s - E_s r + r^2 E_s - r E_s)\, r \\ &= E_s r\,(1 - 2\,r + r^2) \\ &= E_s r\,(1 - r)^2 \end{aligned}$$

Due to nth pulse

$$e_n = E_s r\,(1 - r)^{n-1}\,.$$

Hence the 5th pulse will cause an incremental voltage of

$$\begin{aligned} e_5 &= 100 \times 0.0244\,(1 - 0.0244)^4 \\ &= 2.44 \times 0.9756^4 = 2.22 \text{ volts.} \quad \textit{Ans.} \end{aligned}$$

Maximum Number of Pulses

11.31 A pulse counter of the type of Figure 11.26, problem 11.26 is to be designed to operate at a range where the incremental voltage is more than 2 per cent of applied pulse voltage. What is the permissible number of pulses?

Solution:

Using the formula derived in the preceding problem

$$e_n = E_s r\,(1 - r)^{n-1}$$

and since 2 per cent of 100 volts is 2 volts

we obtain

$$\begin{aligned} 2 &= 100 \times 0.0244 \times 0.9756^{n-1} \\ \frac{2}{2.44} &= 0.9756^{n-1} \\ 0.82 &= 0.9756^{n-1} \\ \log 0.82 &= \log 0.9756^{n-1} \\ \log 0.82 &= (n - 1) \log 0.9756 \\ (0.914 - 1) &= (n - 1)\,(0.989 - 1) \\ n - 1 &= \frac{0.086}{0.011} = 7.8 \\ n &= 8.8 \end{aligned}$$

The pulse counter should be designed to operate with 8 pulses or less. *Ans.*

Number of Pulses for Stated Voltage

11.32 Calculate the number of pulses necessary to build up 25 volts or slightly more across the capacitor C_2 in problem 11.26.

Solution:

Using $$E_n = E_s[1 - (1 - r)^n]$$

we have $$25 = 100[1 - (1 - r)^n]$$

$$0.25 = 1 - (1 - r)^n$$

$$(1 - r)^n = 0.75$$

But $$1 - r = 0.9756.$$

Therefore $$0.9756^n = 0.75$$

$$n \log 0.9756 = \log 0.75$$

$$n = \frac{\log 0.75}{\log 0.9756} = \frac{0.875 - 1}{0.989 - 1}$$

$$= \frac{-0.125}{-0.011} = 11.35$$

12 pulses will be required. *Ans.*

Filter-Time Constant of Video Power Supply

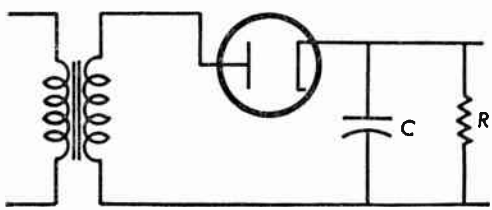

Fig. 11.33a High-voltage rectifier with resistance-capacitance filter.

11.33 A 60-cycle alternating voltage is applied to the half-wave rectifier circuit of Figure 11.10a. What is the value of the time constant RC if a ripple voltage of 1 per cent is admitted?

Solution:

Let E_p be the peak voltage and E_t the voltage after the discharge time t, then

$$E_t = E_p \epsilon^{-\frac{t}{RC}}.$$

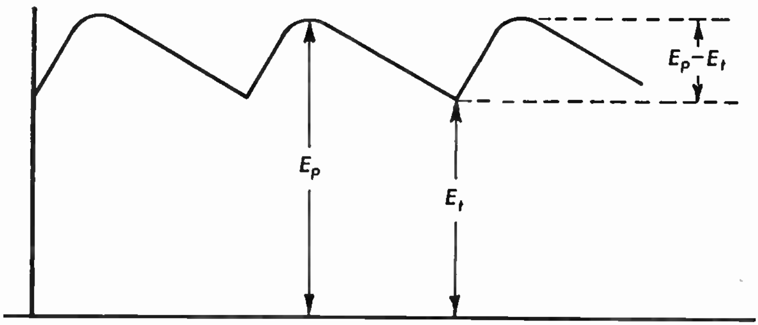

Fig. 11.33b Fluctuating direct voltage as obtained from the rectifier of Fig. 11.33a.

The per cent ripple is approximately the ratio of the voltage difference $E_p - E_t$ to the peak voltage E_p (Figure 11.10b)

$$\text{Per cent ripple} = \frac{E_p - E_t}{E_p}$$

$$= 1 - \frac{E_t}{E_p};$$

now

$$\frac{E_t}{E_p} = \epsilon^{-\frac{t}{RC}},$$

therefore $\quad$ Per cent ripple $= 1 - \epsilon^{-\frac{t}{RC}}.$

This quantity is made equal to 1 per cent or 0.01.

$$1 - \epsilon^{-\frac{t}{RC}} = 0.01$$

$$0.99 = \epsilon^{-\frac{t}{RC}};$$

but $\quad t = \frac{1}{60},$

therefore
$$0.99 = \epsilon^{-\frac{1}{60\,R\,C}}$$

$$\log 0.99 = \log \epsilon^{-\frac{1}{60\,R\,C}}$$

$$\log 0.99 = -\frac{1}{60\,R\,C}\log \epsilon$$

$$0.9956 - 1 = -\frac{1}{60\,R\,C} \times 0.4343$$

$$-0.0044 = -\frac{1}{60\,R\,C} \times 0.4343$$

$$R\,C = \frac{0.4343}{60 \times 0.0044}$$

$$= 1.64 \text{ seconds.} \quad \textit{Ans.}$$

Focus and Centering

11.34 A television receiver employs a high-voltage bleeder circuit of the type of Figure 11.34, where $R_1 = R_2 = 500{,}000$ ohms, $R_3 = 3$ megohms, $R_4 = 1$ megohm, $R_5 = 2$ megohms and $R_6 = R_7 = 1$ megohm. The voltage at the cathode of the rectifier is +2700 volts. Find the variable ranges of voltage of the focus control and of the horizontal and vertical centering controls. Also find the potential difference of the focus control voltage and of the centering voltage with respect to ground when the controls are at midposition.

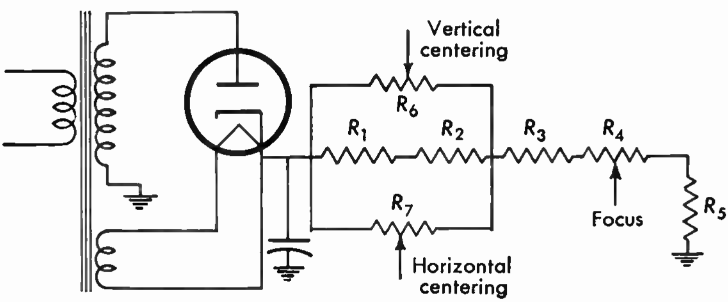

Fig. 11.34 High-voltage rectifier with provisions for focusing and centering.

Solution:

The total resistance of all bleeder resistors is

$$R_T = 0.333 + 3 + 1 + 2 = 6.33 \text{ megohms}$$

of which the 0.333 megohm is obtained as the equivalent resistance of three 1-megohm resistors connected in parallel. The voltage across the focus control is found by using the potentiometer rule, viz.,

$$E_f = 2700 \times \frac{1}{6.33} = 428 \text{ volts.} \quad \textit{Ans.}$$

The voltage across the resistor closest to ground is twice that amount, i.e. 856 volts, which is also the potential difference of the lower end of the focus potentiometer with respect to ground. The upper end of the focus potentiometer is

$$856 + 428 = 1284 \text{ volts}$$

above ground. The midpoint is therefore

$$E_{fm} = \frac{1284 + 856}{2} = 1070 \text{ volts}$$

above ground. *Ans.*

The voltage drop across the centering controls is

$$E_c = 2700 \times \frac{0.333}{6.33} = 142 \text{ volts.} \quad \textit{Ans.}$$

This voltage exists across both the horizontal and the vertical centering controls. The midpoint voltage is

$$E_{cm} = 2700 - \frac{142}{2} = 2629 \text{ volts.} \quad \textit{Ans.}$$

Video Amplifier Plate-Load Resistor

11.35 The amplifier of Figure 11.35 has a total shunt capacitance of 40 micromicrofarads. Calculate the output voltage for 1 kilocycle and 1 megacycle,

(a) using a plate resistor of 100 kilohms,
(b) using a plate resistor of 1200 ohms.

Which of the two resistors will provide a more level frequency response?

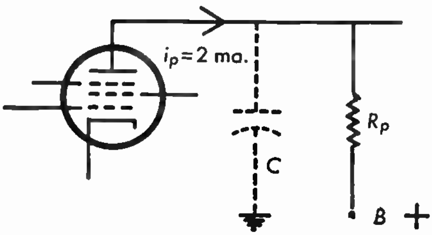

Fig. 11.35 Constant-current tube with resistive-capacitive load.

Solution:

(a) Using a plate resistor of 100 kilohms

The amplitude of the output voltage is

$$E_o = i_p \times Z_p.$$

Now $$Z_p = \frac{R \times (-j\,X)}{R + (-j\,X)};$$

for 1 kilocycle

$$X_{kc} = \frac{1}{2\,\pi\,f\,C} = \frac{1}{6.28 \times 10^3 \times 40 \times 10^{-12}}$$

$$= 3{,}980{,}000 \text{ ohms};$$

for 1 megacycle

$$X_{mc} \text{ will be } \frac{1}{1000} \text{ of the above value:}$$

$$X_{mc} = \frac{3{,}980{,}000}{1000} = 3980 \text{ ohms.}$$

The corresponding value of Z will be

$$|Z_{kc}| = \frac{10^5 \times 3.98 \times 10^6}{\sqrt{(10^5)^2 + (3.98 \times 10^6)^2}}$$

$$\cong \frac{3.98 \times 10^{11}}{3.98 \times 10^6} \cong 100{,}000 \text{ ohms.}$$

Note that $(10^5)^2$ of the denominator is negligible compared with $(3.98 \times 10^6)^2$.

$$Z_{mc} = \frac{10^5 \times 3.98 \times 10^3}{\sqrt{(10^5)^2 + (3.98 \times 10^3)^2}}$$

$$\cong \frac{3.98 \times 10^8}{10^5} \cong 3980 \text{ ohms.}$$

Since the first term under the radical sign is 10^{10}, the other about 10^7, the term $(3.98 \times 10^3)^2$ can be neglected.

The corresponding voltage outputs will be:

for 1 kilocycle

$$E_o = i\,Z_{kc}$$
$$= 0.002 \times 10^5 = 200 \text{ volts.} \quad \textit{Ans.};$$

for 1 megacycle

$$E_o = i\,Z_{mc}$$
$$= 0.002 \times 3980 = 7.96 \text{ volts.} \quad \textit{Ans.}$$

(b) Using a plate resistor of 1200 ohms.

$$Z_{kc} = \frac{1200 \times 3.98 \times 10^6}{\sqrt{1200^2 + (3.98 \times 10^6)^2}}$$

$$\cong \frac{1200 \times 3.98 \times 10^6}{3.98 \times 10^6} \cong 1200 \text{ ohms.}$$

$$Z_{mc} = \frac{1200 \times 3.98 \times 10^3}{\sqrt{1200^2 + (3.98 \times 10^3)^2}}$$

$$= \frac{1200 \times 3.98 \times 10^3}{4.16 \times 10^3} \cong 1140 \text{ ohms.}$$

The corresponding output voltages are:

for 1 kilocycle

$$E_o = i\,Z = 0.002 \times 1200 = 2.4 \text{ volts.} \quad \textit{Ans.};$$

for 1 megacycle

$$E_o = i\,Z = 0.002 \times 1140 = 2.28 \text{ volts.} \quad \textit{Ans.}$$

The 1200-ohm resistor will provide a more level frequency response. *Ans.*

High-Frequency Compensation by Shunt Peaking

11.36 In the video amplifier of Figure 11.36a, $R_p = 2000$ ohms, $r_p = 100{,}000$ ohms, $L_p = 50$ microhenries, $C_c = 0.25$ microfarad, $R_g = 2$ megohms, $C_i = 12.5$ micromicrofarads, $C_o = 7.5$ micromicrofarads, the transconductance of V_1, $g_m = 7700$ micromhos. Calculate the voltage gain at 4 megacycles (a) with the shunt peaking coil L_p short-circuited, (b) with the shunt peaking coil L_p in the circuit.

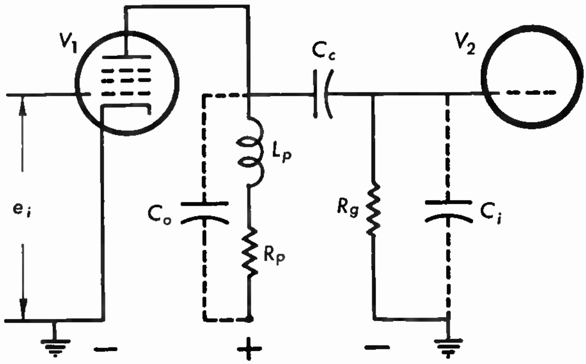

Fig. 11.36a Video amplifier with high-frequency compensation by using a shunt-peaking coil.

Solution:

At a frequency of 4 megacycles

$$X_p = 2\,\pi f\,L_p = 6.28 \times 4 \times 10^6 \times 50 \times 10^{-6}$$
$$= 1256 \text{ ohms},$$
$$X_c = \frac{1}{2\,\pi f\,C_c} = \frac{1}{6.28 \times 4 \times 10^6 \times 0.25 \times 10^{-6}}$$
$$= 0.159 \text{ ohm},$$
$$X_i = \frac{1}{6.28 \times 4 \times 10^6 \times 12.5 \times 10^{-12}}$$
$$= 0.00318 \times 10^6 = 3180 \text{ ohms};$$

and by proportion

$$X_o = 3180 \times \frac{12.5}{7.5} = 5300 \text{ ohms}.$$

(a) In the equivalent circuit of Figure 11.36b, X_c is replaced by a short circuit and R_g across C_i by an open circuit.

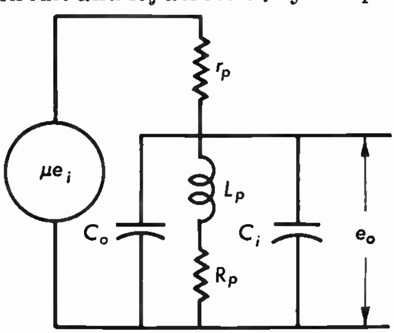

Fig. 11.36b Equivalent circuit of Fig. 11.36a.

Using formula 5.12 for the output voltage across the plate-load impedance Z we obtain

$$e_o = \mu\, e_i \frac{Z}{Z + r_p} \text{ or}$$

$$G = \frac{e_o}{e_i} = \frac{\mu\, Z}{Z + r_p}\,.$$

From the values computed above it follows that Z is in the order of thousands, whereas r_p = 100,000 ohms. Hence the term Z in the denominator can be neglected, yielding the simple equation for the gain of a video amplifier employing pentodes (i.e., high plate resistance)

$$G = \frac{\mu\, Z}{r_p} = g_m\, Z \text{ (formula 4.23).}$$

Calculating Z:

A further simplification is introduced by the fact that R_g is too great a resistance to have any shunt effect. Thus Z consists of a parallel circuit of which R_p is one branch, and $C_T = C_o + C_i = 20$ micro-microfarads the other branch.

At 4 megacycles $\quad X_T = \dfrac{1}{2\,\pi\, f\, C_T} = 1990$ ohms.

The absolute value of Z, with the shunt peaking inductor short-circuited, is

$$|Z| = \frac{R\, X_T}{\sqrt{R^2 + X_T^2}} = \frac{2000 \times 1990}{2830} = 1410 \text{ ohms,}$$

and the uncompensated gain is

$$G = g_m\, Z = 7700 \times 10^{-6} \times 1410 = 10.87. \quad \textit{Ans.}$$

(b) The value of $\mathring{Z}$, when the shunt peaking inductor is applied, is (product-sum formula)

$$\mathring{Z} = \frac{(-j\, X_T) \times (R_p + j\, X_p)}{R_p + j\,(X_p - X_T)}$$

$$= \frac{(-j\, 1990)\,(2000 + j\, 1256)}{2000 - j\, 634}.$$

$$|Z| = \frac{1990\sqrt{2000^2 + 1256^2}}{\sqrt{2000^2 + 634^2}}$$

$$= \frac{1990 \times 2360}{2090} = 2245 \text{ ohms,}$$

and the compensated gain is

$$G = g_m Z = 7700 \times 10^{-6} \times 2245 = 17.3. \quad \textit{Ans.}$$

High-Frequency Compensation by Series Peaking

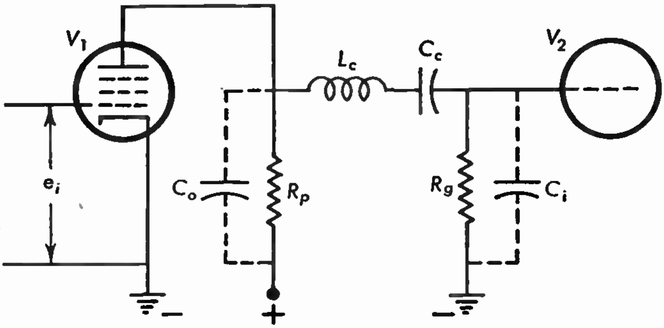

Fig. 11.37a Video amplifier with series-peaking coil for high-frequency compensation.

11.37 In the video amplifier of Figure 11.37a, R_p = 2000 ohms, r_p = 100,000 ohms, L_c = 100 microhenries, C_c = 0.25 microfarad, R_g = 2 megohms, C_i = 12.5 micromicrofarads, C_o = 7.5 micromicrofarads, the transconductance of V_1, g_m = 7700 micromhos. Calculate the gain at 4 megacycles (a) with the series peaking coil L_c short-circuited, (b) with the series peaking coil L_c in the circuit.

Solution:

(a) The voltage gain with the series peaking coil short-circuited, is equal to the gain found in problem 11.36, since the circuit constants are identical. Thus the uncompensated gain at 4 megacycles is

$$G = 10.87. \quad \textit{Ans.}$$

(b) Figure 11.37b represents the equivalent circuit when the series peaking inductor is applied. Its reactance is

$$X_L = 2\,\pi\, f\, L = 6.28 \times 4 \times 10^6 \times 100 \times 10^{-6}$$
$$= 2520 \text{ ohms.}$$

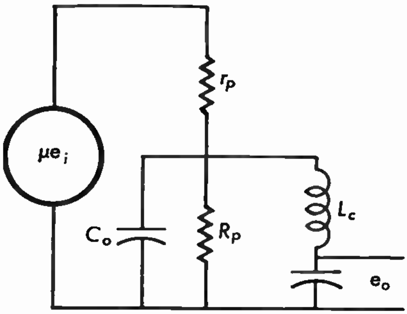

Fig. 11.37b Equivalent circuit of Fig. 11.37a.

Omitting the coupling capacitor and the grid resistor for the reasons outlined in problem 11.36, the gain, as it appears across R_p, is

$$G_R = g_m\, Z,$$

where Z is the parallel combination of the branch X_o, the branch R_p, and the branch X_i and X_L. The first and the third, being purely reactive, can be computed by using the value for X_o and X_i of problem 11.36.

$$Z_x = \frac{(-j\,X_o) \times j\,(X_L - X_i)}{-j\,X_o + j\,(X_L - X_i)}$$

$$= \frac{(-j\,5300) \times j\,(2520 - 3180)}{-j\,5300 + j\,(2520 - 3180)}$$

$$= \frac{j^2\,5300 \times 660}{-j\,5960} = -j\,587.$$

This reactance is shunted by R_p; therefore

$$Z = \frac{-j\,587 \times 2000}{-j\,587 + 2000}$$

and

$$|Z| = \frac{2000 \times 587}{\sqrt{2000^2 + 587^2}}$$

$$= \frac{2000 \times 587}{2070} = 567 \text{ ohms.}$$

Hence

$$G_R = 7700 \times 10^{-6} \times 567 = 4.37$$

The voltage passed on to the next stage is determined by the *reactive* voltage divider LC_i. The series compensated gain is therefore

$$G = 4.37 \times \frac{-j\,X_i}{-j\,X_i + j\,X_L}$$

$$= 4.37 \times \frac{3180}{660} = 21. \quad \textit{Ans.}$$

Series-Shunt Compensation

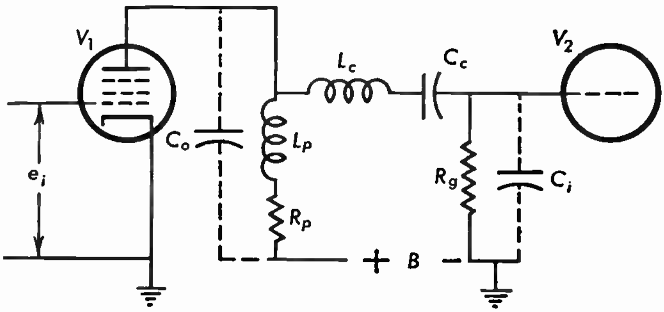

Fig. 11.38a Video amplifier with series and shunt-peaking coils for high-frequency compensation.

11.38 In the video amplifier of Figure 11.38a, R_p = 3600 ohms, r_p = 100,000 ohms, C_c = 0.25 microfarad, R_g = 2 megohms, C_i = 12.5 micromicrofarads, C_o = 7.5 micromicrofarads, L_p = 30 microhenries, L_c = 130 microhenries, the tube V_1 has a g_m = 7700 micromhos. Calculate the voltage gain at 4 megacycles (a) without the application of the series-shunt compensation, (b) with the application of the series and shunt compensating coils.

Solution:

At a frequency of 4 megacycles

$$X_c = 0.159 \text{ ohm}, X_i = 3180 \text{ ohms}, X_o = 5300 \text{ ohms}$$

(problem 11.36),

$$X_p = 2\,\pi\,f\,L_p = 6.28 \times 4 \times 10^6 \times 30 \times 10^{-6}$$

$$= 755 \text{ ohms},$$

$$X_c = 2\,\pi\,f\,L_c = 3260 \text{ ohms.}$$

(a) Without the series-shunt compensation the voltage amplification is

$$G = g_m Z, \text{ (problem 11.36)}$$

where Z is the impedance of R_p and the $C_i + C_o$ in parallel. Since

$$X_T = 1990 \text{ ohms (problem 11.36)}$$

$$|Z| = \frac{R_p \times X_T}{\sqrt{R_p^2 + X_T^2}} = \frac{3600 \times 1990}{\sqrt{3600^2 + 1990^2}}$$

$$= \frac{3600 \times 1990}{4110} = 1740 \text{ ohms.}$$

Hence the uncompensated gain is

$$G = 7700 \times 10^{-6} \times 1740 = 13.4. \quad \textit{Ans.}$$

(b) When the compensating inductors are applied, the equivalent circuit is represented by Figure 11.38b.

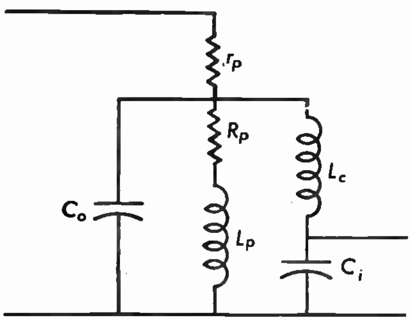

Fig. 11.38b Equivalent circuit of Fig. 11.38a.

The impedance Z is made up by three parallel branches, two of which are reactive, the third being inductive-resistive. The coupling capacitor is replaced by a short circuit, and the grid resistor by an open circuit across C_i. The j-representation of both reactive branches, in kilohms is

$$X_{ct} = \frac{-j\,5.3\,(j\,3.26 - j\,3.18)}{-j\,5.3 + j\,3.26 - j\,3.18}$$

$$= \frac{-j^2\,0.424}{-j\,5.22} = j\,0.0812 \text{ kilohms.}$$

Hence $$Z = \frac{(j\,0.0812)\,(3.6 + j\,0.755)}{j\,0.0812 + 3.6 + j\,0.755}$$

$$= \frac{-0.0594 + j\,0.283}{3.6 + j\,0.836},$$

and $$|Z| = \frac{\sqrt{0.0594^2 + 0.283^2}}{\sqrt{3.6^2 + 0.836^2}} = \frac{0.288}{3.7}$$

$$= 0.078 \text{ kilohms} = 78 \text{ ohms.}$$

The voltage amplification across Z is

$$G_z = 7700 \times 10^{-6} \times 78 = 0.6006.$$

The voltage passed on to the next stage is determined by the *reactive* voltage divider L_c/C_i.

Therefore $$G = 0.6006 \times \frac{-j\,3.18}{-j\,3.18 + j\,3.26}$$

$$= 0.6006 \times \frac{3.18}{0.08} = 24.02. \quad \textit{Ans.}$$

Low-Frequency Compensation

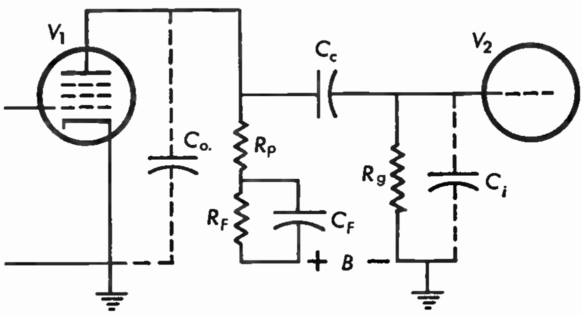

Fig. 11.39a Resistance-capacitance coupled amplifier with the low-frequency compensation circuit R_FC_F.

11.39 In the video amplifier of Figure 11.39a, $R_p = 2000$ ohms, $R_F = 5000$ ohms, $C_F = 8$ microfarads, $R_g = 1$ megohm, $C_c = 0.04$ microfarad, $C_o = 5.2$ micromicrofarads, $C_i = 10.4$ micromicrofarads, V_1 has a $g_m = 7700$ micromhos, $r_p = 100{,}000$ ohms. Calculate the voltage gain at 10 cycles (a) without, (b) with the compensating circuit R_FC_F.

Solution:

(a) At a frequency of 10 cycles

$$X_c = \frac{1}{2\,\pi\, f\, C} = 398{,}000 \text{ ohms},$$

$$X_i = \frac{1}{2\,\pi\, f\, C_i} = 1530 \text{ megohms},$$

$$X_o = 2 \times X_i = 3060 \text{ megohms}.$$

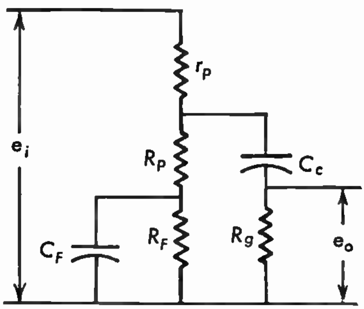

Fig. 11.39b Equivalent circuit of Fig. 11.39a.

The equivalent circuit is represented by Figure 11.39b. Both X_i and X_o are omitted since they act like an open circuit at this frequency. With the decoupling filter R_FC_F short-circuited, the impedance Z of the parallel combination R_p-C_c-R_g is determined by R_p only, since R_g and X_c will have no shunt effect.

The voltage gain as it appears across Z, is

$$G_z = g_m\, Z = 7700 \times 10^{-6} \times 2000 = 15.4$$

But only the voltage across R_g is passed on to the next tube. The voltage gain without compensation is therefore

$$G = 15.4 \frac{R_g}{\sqrt{R_g^2 + X_c^2}}$$

$$= 15.4 \times 0.935 = 14.3. \quad \textit{Ans.}$$

(b) When the compensating circuit R_FC_F is applied. Z is determined by the series-parallel circuit $R_pR_FC_F$.

Since $$X_F = \frac{1}{6.28 \times 10 \times 8 \times 10^{-6}} = 1990 \text{ ohms}$$

the series-equivalent circuit of R_FC_F is (all values in kilohms)

$$\mathring{Z}_p = \frac{-j\,1.99 \times 5}{5 - j\,1.99} = \frac{-j\,9.95\,(5 + j\,1.99)}{25 + 3.95}$$

$$= \frac{19.8}{28.95} - j\,\frac{49.75}{28.95}$$

$$= 0.685 - j\,1.72 \text{ vector kilohms}$$

C_c and R_g having negligible shunt effect, the impedance

$$\mathring{Z} = 2000 + 685 - j\,1720 = 2685 - j\,1720$$

and $$|Z| = \sqrt{2685^2 + 1720^2} = 3180 \text{ ohms.}$$

Hence $$G_2 = g_m Z = 7700 \times 10^{-6} \times 3180 = 24.5$$

and the low-frequency compensated gain is

$$G = 24.5 \times 0.935 = 22.9. \quad \textit{Ans.}$$

Cathode Follower

11.40 In the cathode follower circuit of Figure 11.40a, $R_k = 1500$ ohms, $r_p = 10{,}000$ ohms, $\mu = 25$. Find the stage gain of the circuit.

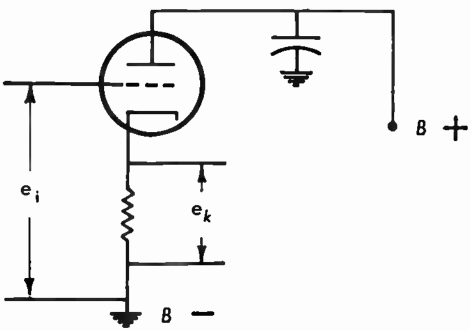

Fig. 11.40a Cathode follower, using triode tube.

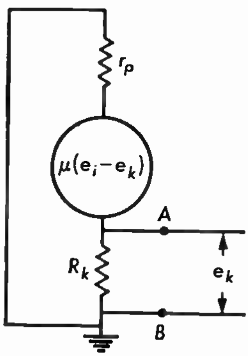

Fig. 11.40b Equivalent circuit of cathode follower.

Solution:

With the cathode follower unby-passed, the signal as it appears from grid to cathode is $(e_i - e_k)$ volts. The equivalent circuit of Figure 11.40b therefore contains a generator voltage of $\mu\,(e_i - e_k)$

in series with r_p and R_k. The output voltage as it appears across R_k, is by the potentiometer rule,

$$e_k = \mu\,(e_i - e_k)\,\frac{R_k}{R_k + r_p}$$

or

$$e_k = 25\,(e_i - e_k)\,\frac{1500}{11{,}500},$$

$$e_k = 3.26\,e_i - 3.26\,e_k,$$

$$4.26\,e_k = 3.26\,e_i.$$

The stage gain is

$$G = \frac{e_k}{e_i} = \frac{3.26}{4.26} = 0.765. \quad \textit{Ans.}$$

General Solution of the Cathode Follower

11.41 Derive the general solution for the gain and the output impedance of the cathode follower circuit of Figure 11.40a.

Solution:

The output voltage is by the potentiometer rule

$$e_k = \mu\,(e_i - e_k)\,\frac{R_k}{r_p + R_k}.$$

This equation is solved for e_k as follows:

Multiplying

$$e_k = \frac{\mu\, e_i\, R_k}{r_p + R_k} - \frac{\mu\, e_k\, R_k}{r_p + R_k};$$

transposing the last term and factoring yields

$$e_k\left(1 + \frac{\mu\, R_k}{r_p + R_k}\right) = \frac{\mu\, e_i\, R_k}{r_p + R_k},$$

and

$$e_k\,\frac{r_p + R_k + \mu\, R_k}{r_p + R_k} = \frac{\mu\, e_i\, R_k}{r_p + R_k},$$

which after transposing and canceling yields

$$e_k = \frac{\mu\, e_i\, R_k}{r_p + R_k\,(\mu + 1)}.$$

The gain is the ratio e_k/e_i,

$$G = \frac{e_k}{e_i} = \frac{\mu\, R_k}{r_p + R_k\,(\mu + 1)}. \quad \textit{Ans.}$$

After dividing numerator and denominator by $(\mu + 1)$, this equation will read

$$G = \frac{\dfrac{\mu}{\mu + 1} R_k}{\dfrac{r_p}{\mu + 1} + R_k},$$

which corresponds to the formula

$$G = \frac{\mu R}{r + R}.$$

The equivalent plate resistance is $r' = \dfrac{r_p}{\mu + 1}$, the equivalent amplification factor is $\mu' = \dfrac{\mu}{\mu + 1}$. Figure 11.41 is the equivalent circuit.

The output impedance Z_o is R_k in parallel with r'.

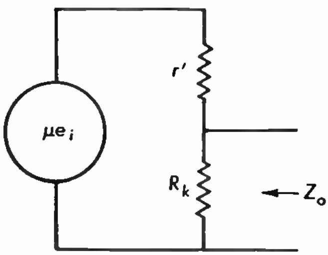

Fig. 11.41 Equivalent circuit of cathode follower for calculating the output impedance. The equivalent plate resistance r' and the equivalent generator μe_i make the circuit analogous to a plate coupled triode amplifier.

$$Z_o = \frac{r' R_k}{r' + R_k}$$

$$= \frac{\dfrac{r_p}{\mu + 1} R_k}{\dfrac{r_p}{\mu + 1} + R_k}$$

$$= \frac{r_p R_k}{r_p + R_k (\mu + 1)}. \quad \textit{Ans.}$$

Cathode Follower Output Impedance

11.42 With the aid of the formulas derived in problem 11.41, find the gain and the output impedance of the cathode follower circuit of problem 11.16.

Solution:

$$G = \frac{\mu R_k}{r_p + R_k(\mu + 1)}$$

$$= \frac{25 \times 1500}{10^4 + 1500\,(26)}$$

$$= \frac{37{,}500}{49{,}000} = 0.765. \quad \textit{Ans.}$$

$$Z_o = \frac{r_p R_k}{r_p + R_k(\mu + 1)}$$

$$= \frac{10^4 \times 1500}{10^4 + 1500\,(26)}$$

$$= \frac{1.5 \times 10^6}{4.9 \times 10^3} = 306 \text{ ohms.} \quad \textit{Ans.}$$

Fourier Analysis of a Saw-Tooth Wave

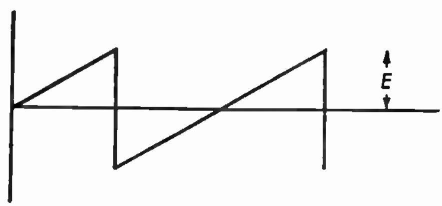

Fig. 11.43 Saw-tooth wave.

11.43 The Fourier analysis of the saw-tooth wave of Figure 11.43 yields the following harmonic components:

$$y = \frac{2E}{\pi}\left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x \cdots\right)$$

Find the amplitudes of the first five components assuming $E = 1$ volt.

Solution:

$$e_1 = \frac{2E}{\pi}\sin x$$

The coefficient of $\sin x$ is the amplitude of e_1.

$$E_1 = \frac{2 \times 1}{\pi} = 0.637 \text{ volt.} \quad \textit{Ans.}$$

$$e_2 = \frac{2E}{\pi}\left(-\frac{1}{2}\sin 2x\right) = -\frac{E}{\pi}\sin 2x$$

The amplitudes are:

$$E_2 = -\frac{1}{\pi} = -0.319 \text{ volt.} \quad Ans.$$

$$E_3 = \frac{2E}{3\pi} = 0.212 \text{ volt.} \quad Ans.$$

$$E_4 = -\frac{2E}{4\pi} = -\frac{1}{2\pi} = -0.159 \text{ volt.} \quad Ans.$$

$$E_5 = \frac{2E}{5\pi} = 0.127 \text{ volt.} \quad Ans.$$

Using the Sigma Notation of Fourier Series

11.44 A short rectangular pulse is given by the equation

$$y = kE + \frac{2E}{\pi}\sum_{n=1}^{n=\infty}\frac{1}{n}\sin nk\pi \cos nx$$

where k is a proper fraction expressing the duration of the pulse as a fraction of a cycle,

n = the order of the harmonic considered
E = the amplitude of the pulse.

Find the amplitudes of the fundamental and second harmonic and write their equations for a pulse voltage $E = 10$ volts, and a pulse duration of 36 degrees.

Solution:

Using $$A_n = \frac{2E}{\pi} \times \frac{1}{n}\sin(nk\pi)$$

and $$k = \frac{36}{360} = 0.1 \text{ cycle}$$

yields an amplitude of the fundamental wave of

$$A_1 = \frac{2 \times 10}{3.14} \times \frac{1}{1}\sin(1 \times 0.1 \times \pi)$$
$$= 6.37 \sin 0.1\pi,$$

where the angle 0.1π is in radians or, since π radians = 180 degrees

$$A_1 = 6.37 \times \sin 18° = 6.37 \times 0.309$$
$$= 1.97 \text{ volts.} \quad Ans.$$

The equation of the fundamental wave is therefore

$$e_1 = 1.97 \cos x. \quad Ans.$$

Likewise
$$A_2 = \frac{2 \times 10}{3.14} \times \frac{1}{2} (\sin 2 \times 0.1 \times \pi)$$
$$= 6.37 \times 0.5 \sin (0.2\,\pi)$$
$$= 6.37 \times 0.5 \sin 36°$$
$$= 6.37 \times 0.5 \times 0.588 = 1.89 \text{ volts.} \quad \textit{Ans.}$$

and
$$e_2 = 1.89 \cos 2x. \quad \textit{Ans.}$$

Reflective Projection System

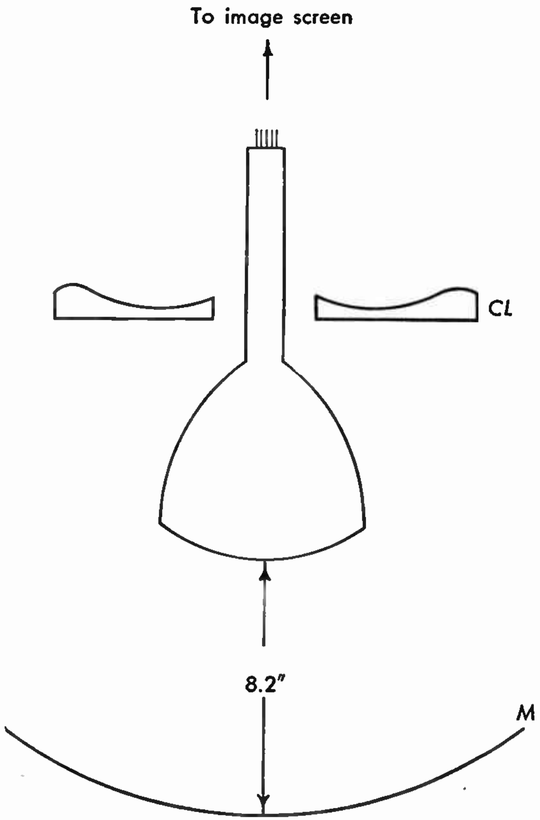

Fig. 11.45 Reflective projection system. CL = correcting lens, M = concave mirror.

11.45 A projection type television receiver uses the Schmidt optical arrangement of a spherical mirror M and a correcting lens CL Figure 11.45. The radius of curvature of the mirror is 13.7 inches, the distance of the kinescope screen from the mirror is 8.2 inches. A type 5TP4 projection kinescope is used. Disregarding the correcting lens find the distance of the image from the mirror, the size of the image and the magnification.

Solution:

Using the mirror formula

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f},$$

where d_i = the distance of the image from the mirror

d_o = the distance of the object from the mirror

f = the distance of the principal focus from the mirror (focal length),

and the fact that the principal focus of a spherical mirror is midway between the center of curvature and the vertex (center) of the mirror, i.e.

$$f = \frac{1}{2}r = \frac{13.7}{2} = 6.85 \text{ inches},$$

we have

$$\frac{1}{8.2} + \frac{1}{d_i} = \frac{1}{6.85}$$

$$\frac{1}{d_i} = \frac{1}{6.85} - \frac{1}{8.2} = \frac{8.2 - 6.85}{8.2 \times 6.85}$$

$$d_i = \frac{8.2 \times 6.85}{1.35} = 41.6 \text{ inches.} \quad \textit{Ans.}$$

Since the relative sizes of the object and the image depend upon their distances from the mirror we have the proportion

$$\frac{\text{Image size}}{\text{Object size}} = \frac{\text{image distance}}{\text{object distance}}$$

or

$$\frac{i}{o} = \frac{d_i}{d_o}.$$

Substituting $\frac{i}{5} = \frac{41.6}{8.2}$

and $i = \frac{41.6 \times 5}{8.2} = 25.4$ inches. *Ans.*

The linear magnification is

$$\frac{i}{o} = \frac{25.4}{5} = 5.08. \quad \textit{Ans.}$$

The area is magnified by a factor

$$5.08^2 = 25.8, \text{ approximately.} \quad \textit{Ans.}$$

Countdowns for Stated Number of Lines

11.46 A television system employs interlaced scanning with 60 fields per second. The pulse-timing unit master oscillator is tuned to the line frequency and followed by a doubler and odd-subharmonic multivibrators. The last multivibrator is synchronized with a 60-cycle power line sine wave. Find the frequencies of all stages for a 525- and a 625-line system.

Solution:

(a) For 525 lines.

The number of lines per second is found by multiplying the number of lines per field times 60.

$$f_L = 262.5 \times 60 = 15{,}750.$$

Since the master oscillator is tuned to the line frequency, the frequency of the master oscillator will be

$$f_{MO} = f_L = 15{,}750 \text{ cycles.} \quad \textit{Ans.}$$

The frequency of the doubler is

$$f_D = 2f_L = 2 \times 15{,}750 = 31{,}500 \text{ cycles.} \quad \textit{Ans.}$$

To find the countdown ratios of the following multivibrators it is necessary to find the prime factors of 31,500.

31,500 (7
4,500 (5
900 (5
180 (3
60

Hence the frequency of the divide-by-7 multivibrator is 4500 cycles. *Ans.*

The frequency of the first divide-by-5 multivibrator is 900 cycles. *Ans.*

The frequency of the second divide-by-5 multivibrator is 180 cycles. *Ans.*

The frequency of the divide-by-3 multivibrator is 60 cycles. *Ans.*

(b) For 625 lines.

The number of lines per field is 625/2 = 312.5 lines.

Hence $f_L = 312.5 \times 60 = 18{,}750$

$$f_{MD} = f. = 18{,}750 \text{ cycles.} \quad \textit{Ans.}$$

$$f_D = 2 \times 18{,}750 = 37{,}500 \text{ cycles.} \quad \textit{Ans.}$$

The prime factor analysis of 37,500 yields

37,500 (5
7500 (5
1500 (5
300 (5
60

There will be four subharmonic dividers, with frequencies of 7500, 1500, 300 and 60 cycles. *Ans.*

Scoping the Pulse-Timing Unit

11.47 The sweep frequency selector switch of an oscilloscope used to investigate the pulse-timing unit of a synchronizing-pulse generator is set for 180 cycles. How many cycles will appear on the screen for each stage of the pulse-timing unit of problem 11.46, designed for 525 lines?

Solution:

The duration of a time-base cycle (including the flyback time which will be assumed to be very short) is

$$t_{TB} = \frac{1}{180} \text{ second.}$$

The duration of a master-oscillator cycle is

$$t_{MO} = \frac{1}{15,750} \text{ second.}$$

The number of waves appearing on the screen is

$$n_{MO} = \frac{1}{180} \div \frac{1}{15,750} = \frac{1}{180} \times \frac{15,750}{1} = \frac{175}{2}$$

$$= 87.5 \text{ waves.} \quad \textit{Ans.}$$

The doubler will show

$$n_D = 87.5 \times 2 = 175 \text{ waves.} \quad \textit{Ans.}$$

The divide-by-7 will show

$$n_7 = \frac{1}{180} \div \frac{1}{4500} = \frac{4500}{180} = \frac{50}{2}$$

$$= 25 \text{ pulses.} \quad \textit{Ans.}$$

Also $175/7 = 25$ pulses.

The first divide-by-5 will show

$$n_5 = 25/5 = 5 \text{ pulses.} \quad \textit{Ans.}$$

The second divide-by-5 will show

$$n_5' = 5/5 = 1 \text{ pulse.} \quad \textit{Ans.}$$

The divide-by-3 will show

$$n_3 = 1/3 = 1/3 \text{ of a pulse,}$$

i.e. the selector switch must be set to a lower frequency to observe one or more pulses.

Measuring High-Frequency Television Pulses

11.48 To measure the duration of the horizontal synchronizing pulse on the bottom of the pulse a 15,750 cycle sine wave is used for the horizontal deflection of the oscilloscope and the measured pulse is applied to the vertical input and phased so that it occurs during the most linear portion of the applied sine wave as shown in Figure 11.48. If $p = 25$ millimeters and $d = 10$ centimeters what is the duration of p in terms of the whole cycle H and what is its actual duration in microseconds?

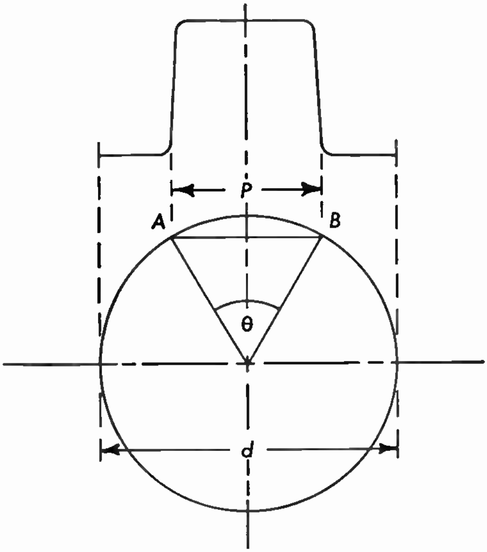

Fig. 11.48 Diagram illustrating the measuring of the duration of a pulse (see text).

Solution:

The duration of the pulse is the time necessary to sweep the arc AB or the angle θ.

Since $$\sin \theta/2 = \frac{25/2}{100/2} = 0.25$$

$$\theta/2 = \sin^{-1} 0.25 = 14.483°$$

$$\theta = 29.97 \text{ degrees} = \frac{29.97}{360} = 0.0832 \text{ cycle}$$

$$= 0.0832\,H. \quad \textit{Ans.}$$

The duration in microseconds is

$$t_{hp} = \frac{0.0832}{15{,}750} = 5.28 \text{ microseconds.} \quad \textit{Ans.}$$

Measuring Low-Frequency Television Pulses

11.49 To measure the pulse width of the vertical-blanking pulse of an R.M.A. synchronizing-signal generator a 60-cycle sine wave is used for the horizontal deflection of the oscilloscope, and the measured pulse is applied to the vertical input and phased so that it occurs during the most linear portion of the applied sine wave. Referring to Figure 11.48 with the horizontal amplifier adjusted to make $d = 10$ centimeters, what is the length of p if the blanking lasts for a period of 15 lines?

Solution:

The duration of the blanking pulse is 15 H, whereas the duration of the whole cycle is 1/60 of a second or 262.5 H, where H is the time from the start of one line to the start of the next line.

To find θ we use the proportion

$$\frac{\theta}{360} = \frac{15}{262.5}$$

and

$$\theta = \frac{15 \times 360}{262.5} = 20.4 \text{ degrees}$$

$$\sin \frac{20.4°}{2} = \frac{x}{5}, \text{ where } p = 2\,x.$$

$$0.176 = \frac{x}{5}$$

$$x = 0.88 \text{ and } p = 1.76 \text{ centimeters.} \quad \textit{Ans.}$$

Vertical "Burst"

11.50 One satisfactory system of separation of horizontal from vertical pulses employs amplitude separation for horizontal synchronization and frequency separation for vertical synchronization.* The vertical *r-f* "burst" is produced from a line-frequency blocking oscillator rich in harmonics, followed by a frequency multiplier tuned to the 17th harmonic, and by a doubler. At the receiver the discharge circuit of the vertical saw-tooth generator is triggered by the resonant voltage of an *i-f* type transformer. What is the resonant frequency of this transformer for a standard 525-line 60-field system?

* The Dumont Synchronizing Signal, Engineering Report for Panel No. 8NTSC, September 6, 1940. Allen B. Dumont Laboratories, Inc., Passaic, N. J., 1940.

Solution:

The line frequency is

$$f_L = 262.5 \times 60 = 15{,}750 \text{ cycles.}$$

The frequency of the first multiplier is

$$f_M = 15{,}750 \times 17 = 267{,}750 \text{ cycles.}$$

The frequency of the doubler is

$$f_D = 267{,}750 \times 2 = 535{,}500 \text{ cycles.}$$

The vertical burst to which the transformer is tuned is therefore

$$f_V = 535{,}500 \text{ cycles} = 535.5 \text{ kilocycles.} \quad \textit{Ans.}$$

Camera Viewing Angle

11.51 A television camera employs a lens of a focal length of 6.5 inches. Calculate the camera viewing angle for an average distance of 20 feet from the scene when the camera is equipped with a type 1850A iconoscope with a usable mosaic area of $3\frac{9}{16}$ x $4\frac{3}{4}$ inches.

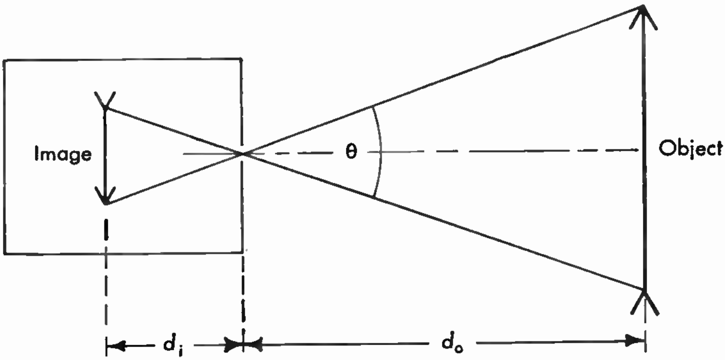

Fig. 11.51 Camera angle θ for a viewing distance of d_o feet.

Solution:

Using the lens formula

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f},$$

where d_o = distance of the object from the lens,

d_i = distance of the image from the lens,

f = focal length of the lens,

we have

$$\frac{1}{240} + \frac{1}{d_i} = \frac{1}{6.5},$$

where all distances are expressed in inches.

$$\frac{1}{d_i} = \frac{1}{6.5} - \frac{1}{240} = \frac{240 - 6.5}{240 \times 6.5}$$

$$d_i = \frac{240 \times 6.5}{233.5} = 6.68 \text{ inches},$$

which is the distance of the mosaic from the lens. Since the largest image size in a horizontal direction is $4\frac{3}{4}$ inches, the largest object size can be calculated by using the proportion

$$\frac{\text{Object size}}{\text{Image size}} = \frac{\text{object distance}}{\text{image distance}}$$

or

$$\frac{o}{i} = \frac{d_o}{d_i}.$$

Substituting

$$\frac{o}{4.75} = \frac{240}{6.68}$$

and

$$o = \frac{240 \times 4.75}{6.68} = 171 \text{ inches}.$$

We then have an isosceles triangle the base of which is 171 inches and the altitude of which is 240 inches. The camera viewing angle is the angle at the vertex of the triangle.

Since

$$\tan\frac{\theta}{2} = \frac{171/2}{240} = \frac{85.5}{240} = 0.356,$$

and

$$\frac{\theta}{2} = \tan^{-1} 0.356 = 19.6 \text{ degrees},$$

the camera viewing angle is

$$\theta = 2 \times 19.6 = 39.2 \text{ degrees.} \quad \textit{Ans.}$$

Gamma Response

11.52 An amplifier has an input of e_0 volts and an output of e_1 volts where $e_1 = 2\, e_0^{1.2}$. The output e_1 is fed into a second amplifier the output of which is e_2, where $e_2 = 1.5\, e_1^{0.8}$. Express the final output e_2 in terms of the original input e_0.

Solution:

Since $e_2 = 1.5\, e_1^{0.8}$ (1)

and $e_1 = 2\, e_0^{1.2}$ (2)

we obtain by substituting (2) in (1)

$$e_2 = 1.5\,(2\, e^{1.2})^{0.8}$$
$$= 1.5\,(2^{0.8} \times e_0^{1.2\times 0.8})$$
$$= 1.5 \times 2^{0.8} \times e_0^{0.96}.$$

The value N of $2^{0.8}$ can be found with the aid of logarithms, viz.,

$$N = 2^{0.8}$$
$$\log N = 0.8 \times \log 2 = 0.8 \times 0.3010$$
$$\log N = 0.2408$$

and $N = \text{antilog } 0.2408 = 1.741.$

Hence $e_2 = 1.5 \times 1.741 \times e_0^{0.96}$

$$e_2 = 2.62\, e_0^{0.96}. \quad \textit{Ans.}$$

Gamma-Unity Response

11.53 An amplifier has an input of e_0 volts and an output of e_1 volts where $e_1 = 2\, e_0^{1.2}$. The output is fed into a second amplifier the output of which is e_2 where $e_2 = 1.5\, e_1^{\gamma_1}$. Find the value of γ_1 and the equation of the final output in terms of the original input if the over-all response curve is a straight line.

Solution:

Since the over-all response curve is a straight line of the form $y = mx$ or $e_n = k_n\, e_0$ where x or e_0 has the exponent 1 we have

$$e_2 = 1.5\, e_1^{\gamma_1} = 1.5\,(2\, e_0^{1.2})^{\gamma_1}$$
$$= 1.5 \times 2^{\gamma_1}\, e_0^{1.2\gamma_1},$$

where $1.2\,\gamma_1 = 1,$

and $\gamma_1 = \dfrac{1}{1.2} = 0.833. \quad \textit{Ans.}$

Hence $e_2 = 1.5 \times 2^{0.833}\, e_0^{1.2\times 0.833}$

$$= 1.5 \times 1.78 \times e_0,$$
$$e_2 = 2.67\, e_0. \quad \textit{Ans.}$$

Figure of Merit of a Triode Voltage Amplifier

11.54 The following are the operating characteristics of an earlier (6J5) and a later type amplifier triode (6J6):

	6J5	6J6 single section	Units
Grid-to-plate capacitance	3.4	1.6	$\mu\mu$f
Output capacitance	3.4	0.4	$\mu\mu$f
Input capacitance	3.6	2.2	$\mu\mu$f
Mutual conductance	2600	5300	μmhos
Amplification factor	20	38	—

Assuming a gain of 5, what is the figure of merit of both tubes?

Solution:

(a) Type 6J5.

The figure of merit for voltage amplification is the ratio of the transconductance to the total tube capacitance. Due to the Miller effect the total tube capacitance is

$$C_T = C_{gk} + C_{pk} + C_{pg}\,(1 + G).$$

$$f.o.m. = \frac{g_m}{C_{gk} + C_{pk} + C_{pg}\,(1 + G)}$$

where g_m is in micromhos and the tube capacitance is in micromicrofarads.

$$f.o.m. = \frac{2600}{3.4 + 3.6 + 3.4\,(1 + 5)}$$

$$= \frac{2600}{7 + 20.4} = \frac{2600}{27.4} = 95. \quad \textit{Ans.}$$

(b) Type 6J6

$$f.o.m. = \frac{5300}{2.2 + 0.4 + 1.6\,(6)}$$

$$= \frac{5300}{2.6 + 9.6} = \frac{5300}{12.2} = 434. \quad \textit{Ans.}$$

Figure of Merit of Pentode Voltage Amplifiers

11.55 The earlier type amplifier pentode 6SJ7 has a grid-to-plate capacitance of 0.005 micromicrofarad, an input capacitance of 6.3 micromicrofarads, an output capacitance of 10 micromicrofarads, and a transconductance of 1650 micromhos; the later type 6AK5 has 0.01, 3.9 and 2.85 micromicrofarads, respectively, and a transconductance of 5100 micromhos. Calculate the figures of merit of both tubes.

Solution:

(a) Type 6SJ7.

Since the plate-to-grid capacitance of a pentode is negligible compared to the input and output capacitance the figure of merit becomes

$$f.o.m. = \frac{g_m}{C_i + C_o} = \frac{1650}{10 + 6.3} = \frac{1650}{16.3}$$

$$= 101. \quad \textit{Ans.}$$

(b) Type 6AK5.

$$f.o.m. = \frac{5100}{6.75} = 756. \quad \textit{Ans.}$$

Video Output Tubes

11.56 Amplifiers intended to develop as large an output voltage as possible to be used as the deflection voltage of cathode-ray tubes should have an output which is directly proportional to the plate current and inversely proportional to the plate-to-cathode or output capacitance. The earlier type 1852 has a plate current of 10 milliamperes and an output capacitance of 5 micromicrofarads. The later miniature type 6AK6 has a plate current of 15 milliamperes and an output capacitance of 4.2 micromicrofarads. Which of the two tubes is preferable as a video output tube?

Solution:

The ratio of the plate current to the output capacitance when the type 1852 is used is

$$\frac{I_p}{C_o} = \frac{10}{5} = 2.$$

The value of the ratio when using the type 6AK6 is

$$\frac{I_p'}{C_o'} = \frac{15}{4.2} = 3.57$$

The type 6AK6 is preferable. *Ans.*

Upper Frequency Limit of Amplifiers

11.57 Using the formula

$$f^* = \frac{g_m}{\pi\sqrt{C_i C_o}}$$

where f in cycles is the upper frequency limit of a tube if the gain is unity and a four-terminal filter coupling is used, g_m is in mhos and the capacitances in farads, calculate the upper frequency limit of the triode types 6J5, 6J6 and the pentodes types 6SJ7 and 6AK5. Use the data of problems 11.54 and 11.55.

Solution:

(a) Triode type 6J5

$$f_a = \frac{2600 \times 10^{-6}}{3.14\sqrt{3.4 \times 3.6 \times 10^{-24}}}$$

$$= \frac{2600 \times 10^{-6}}{3.14 \times 10^{-12} \times 3.5}$$

$$= 236 \times 10^6 \text{ cycles}$$

$$= 236 \text{ megacycles.} \quad \textit{Ans.}$$

(b) Triode type 6J6

$$f_b = \frac{5300 \times 10^{-6}}{3.14\sqrt{0.4 \times 2.2 \times 10^{-24}}}$$

$$= \frac{5300 \times 10^{-6}}{3.14 \times 10^{-12} \times 0.937}$$

$$= 1800 \text{ megacycles.} \quad \textit{Ans.}$$

(c) Pentode type 6SJ7

$$f_c = \frac{1650 \times 10^{-6}}{3.14\sqrt{10 \times 6.3 \times 10^{-24}}}$$

$$= \frac{1650 \times 10^{-6}}{3.14 \times 10^{-12} \times 7.94}$$

$$= 66 \text{ megacycles.} \quad \textit{Ans.}$$

* Emery, W. L., *Ultra-High-Frequency Radio Engineering*, New York, The Macmillan Company, 1944, p. 92.

(d) Pentode type 6AK5

$$f_d = \frac{5100 \times 10^{-6}}{3.14\sqrt{3.9 \times 2.85 \times 10^{-24}}}$$

$$= \frac{5100 \times 10^{-6}}{3.14 \times 10^{-12} \times 3.33}$$

$$= 488 \text{ megacycles.} \quad \textit{Ans.}$$

Direct and Reflected Signals

11.58 A reflected wave causes a blurred image by producing a picture element immediately following the picture element brought in by the direct wave. Using the scanning velocity calculated in problem 11.03, how much longer is the path over which the reflected wave travels as compared to the direct path? The screen is 9 × 12 inches.

Solution:

It is first necessary to calculate the diameter of a picture element on the given screen. Assuming that the beam diameter fits into the width of an active scanning line and that the number of active lines is 95 per cent of all lines, we obtain the diameter of a picture element by dividing the height of the picture by the number of active lines

$$d = \frac{9}{525 \times 0.95} = 0.018 \text{ inch} = 0.0458 \text{ centimeter.}$$

Since the horizontal velocity of the scanning beam is (problem 11.03)

$$v = 572{,}000 \text{ centimeters per second}$$

the time required to scan a distance of one picture element is

$$t = \frac{d}{v} = \frac{4.58 \times 10^{-2}}{5.72 \times 10^{5}} = 0.8 \times 10^{-7} \text{ second.}$$

During this time a radio wave will travel a distance of

$$s = 3 \times 10^{8} \times 0.8 \times 10^{-7} = 2.4 \times 10$$

$$= 24 \text{ meters} = 78.5 \text{ feet,}$$

which is the difference of the reflected path minus the direct path. *Ans.*

Direct-Current Restoration

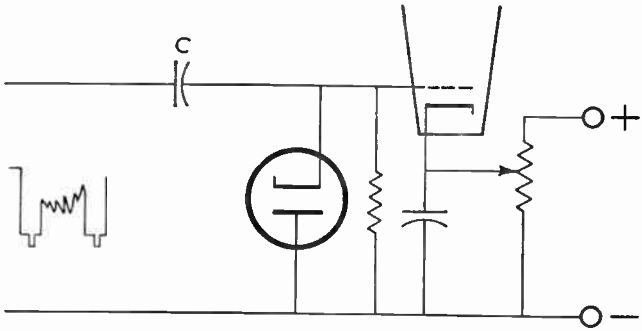

Fig. 11.59a D-c restorer using a diode tube.

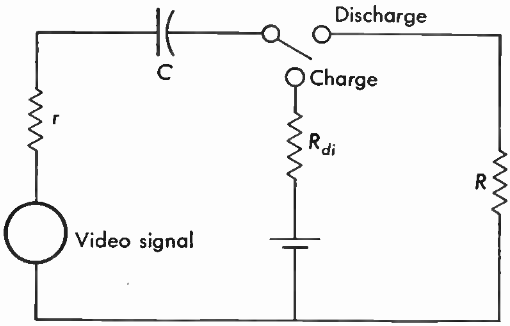

Fig. 11.59b Equivalent circuit of Fig. 11.59a.

11.59 The diode d-c restorer Figure 11.59a, the equivalent circuit of which is Figure 11.59b, has a coupling capacitor $C = 0.1$ microfarad, an internal resistance of the video output source $r = 3000$ ohms, a diode resistance during conduction $R_{di} = 4000$ ohms, and a discharge path when the diode is not conducting $R = 1$ megohm. Calculate the bias developed by the discharge current i_d across R for a white level of 30 volts below the top of the synchronizing signal and a grey level 10 volts below the synchronizing pulse.

Solution:

Diode current will only flow during the synchronizing pulse which lasts for a period of 0.08 H, yielding a charge time of

$$t_c = \frac{0.08}{15{,}750} = 5.14 \text{ microseconds.}$$

The charge-time constant is

$$C\,(r + R_{di}) = 0.1 \times 10^{-6} \times 7 \times 10^{3}$$
$$= 700 \text{ microseconds.}$$

Obviously the charge current flows well within the linear portion of the exponential charge curve, and since a linearly increasing voltage is due to a constant charge current Ohm's law may be used

$$i_{ch} = \frac{e_{ch}}{7000} \text{ ampere.}$$

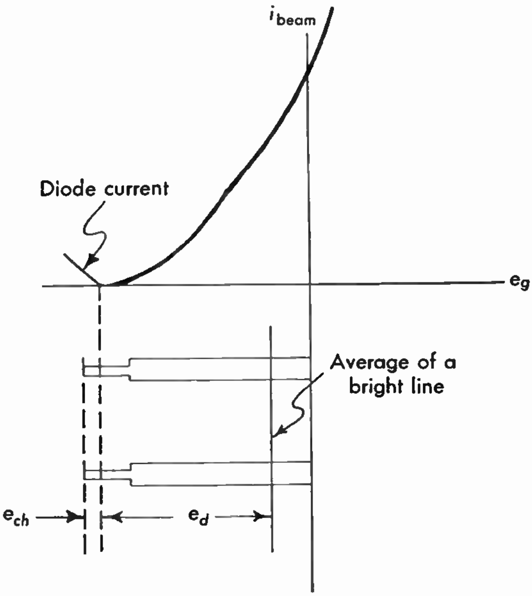

Fig. 11.59c Beam current-grid voltage characteristic of cathode-ray tube and bias conditions for a bright signal.

Similarly the duration of the discharge

$$t_d = \frac{0.92}{15{,}750} = 58.4 \text{ microseconds}$$

and the time constant is

$$C\,(R + r) \cong 100{,}000 \text{ microseconds}$$

yielding a discharge current of

$$i_d = \frac{e_d}{1003{,}000} \text{ ampere.}$$

For steady operation the quantity of charge flowing into the capacitor during the charge time must be equal to the quantity of charge leaving the capacitor during discharge, i.e.

$$i_d \times 58.4 = i_{ch} \times 5.14$$

or

$$\frac{58.5\ e_d}{1003{,}000} = \frac{5.14\ e_{ch}}{7000}$$

and

$$e_d \cong \frac{5.14 \times 10^6\ e_{ch}}{58.4 \times 7000} = 12.56\ e_{ch}$$

From Figure 11.59c it is seen that e_d is the bias developed due to rectification and e_{ch} is the voltage that charges the capacitor.

Since

$$\frac{e_d}{e_{ch}} = \frac{12.56}{1}$$

$$\frac{e_d}{e_d + e_{ch}} = \frac{12.56}{12.56 + 1} = 0.927$$

or the bias developed is 92.7 per cent of the peak amplitude.

For white $e_{dw} = 30 \times 0.927 = 27.8$ volts. *Ans.*

For grey $e_{dg} = 10 \times 0.927 = 9.27$ volts. *Ans.*

Line Pattern Testing of the Horizontal Oscillator

11.60 A sine wave of 630 kilocycles is fed into the detector of a television receiver. Calculate the number of vertical bars that will appear on the screen of the receiver which is set for a standard signal of 525 lines, 60 fields, employing interlaced scanning.

Solution:

The approximate number of vertical bars can be found by neglecting the flyback time. The number of lines per second is

$$n_l = 525 \times 30 = 15{,}750.$$

Since

$$\frac{630{,}000}{15{,}750} = 40$$

the signal generator will modulate a line 40 times and the number of vertical bars would be 40, if the flyback time were zero. However in an R.M.A. standard signal the horizontal blanking time is 0.18 H where H is the duration of trace plus flyback.

Hence there will be

$$40 \times 0.18 = 7.2 \text{ invisible bars}$$

and $$40 - 7.2 \cong 32.8 \text{ visible bars.} \quad \textit{Ans.}$$

Line-Pattern Testing of the Vertical Oscillator

11.61 A sine wave of 900 cycles is fed into the detector of a television receiver. Calculate the number of horizontal bars that will appear on the screen of the receiver which is set for a standard R.M.A. signal of 525 lines, 60 fields, employing interlaced scanning.

Solution:

The approximate number of horizontal bars can be found by neglecting the vertical retrace time. Since the number of vertical saw-tooth waves per second is 60, each saw-tooth wave will be modulated

$$\frac{900}{60} = 15 \text{ times,}$$

and there would be 15 horizontal bars if the flyback time were zero. However, in an R.M.A. standard signal the horizontal blanking time is 0.05 V where V is the duration of trace plus flyback.

Hence there will be

$$15 \times 0.95 = 14.25 \text{ visible bars.} \quad \textit{Ans.}$$

Television Receiver Intermediate and Oscillator Frequencies

11.62 The transmission standards of the Federal Communications Commission require a channel width of 6 megacycles; the visual carrier is located 4.5 megacycles lower in frequency than the aural center frequency; the aural center frequency is located 0.25 megacycles lower in frequency than the upper frequency limit. Give the visual carrier frequency, the aural center frequency of channel number 3 (60 to 66 megacycles) and find the frequency of the local oscillator and the visual intermediate frequency if the aural intermediate frequency transformer is tuned to 21.25 megacycles.

Solution:

The aural center frequency is

$$f_a = 66 - 0.25 = 65.75 \text{ megacycles.} \quad \textit{Ans.}$$

The visual carrier frequency is

$$f_v = 65.75 - 4.5 = 61.25 \text{ megacycles.} \quad \textit{Ans.}$$

The local oscillator frequency for this channel is

$$f_o = 65.75 + 21.25 = 87 \text{ megacycles.} \quad \textit{Ans.}$$

Hence the visual intermediate frequency is

$$i\text{-}f_v = 87 - 61.25 = 25.75 \text{ megacycles.} \quad \textit{Ans.}$$

Installing a Television Antenna

11.63 A television receiver has an input impedance of 300 ohms properly matched into a 300-ohm transmission line. The transmission line is to be connected to a 72-ohm dipole. If spaced-feeder lines of 150 ohms, 200 ohms, and 250 ohms are commercially available, find the correct impedance and length of the quarter-wave matching stub to be inserted between the antenna and the transmission line, when the receiver is tuned to 60 megacycles.

Solution:

The impedance of the quarter-wave matching stub

$$Z_o = \sqrt{Z_{ant} \times Z_{line}}$$
$$= \sqrt{300 \times 72} = 147 \text{ ohms.}$$

The 150-ohm twin-lead feeder would be satisfactory. *Ans.*

The length of the matching stub is

$$\lambda/4 = \frac{1}{4} \times \frac{3 \times 10^8}{60 \times 10^6} = \frac{1}{4} \times 5$$
$$= 1.25 \text{ meters} = 49 \text{ inches.} \quad \textit{Ans.}$$

Note: The electrical length will be somewhat shorter depending on the velocity of propagation of the type of line used.

12 *Measurements*

Wheatstone Bridge

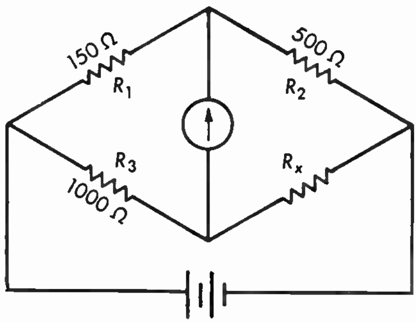

Fig. 12.01 Wheatstone bridge, as used to measure unknown resistances.

12.01 In Figure 12.01 find the value of the unknown resistor R_x.

Solution:

$$\frac{R_x}{500} = \frac{1000}{150}$$

$$R_x = \frac{500 \times 1000}{150} = 3333 \text{ ohms.} \quad \textit{Ans.}$$

Slide-Wire Bridge

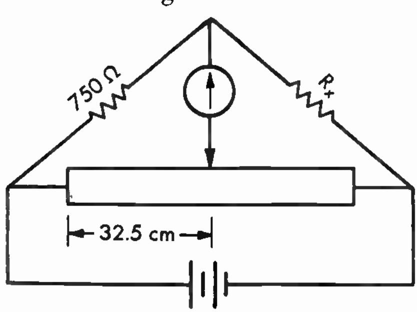

Fig. 12.02 Slide-wire bridge, consisting of a sliding arm making contact with a wire one meter long.

12.02 The slide-wire bridge of Figure 12.02 has a total length of 1 meter. Calculate R_x for the values indicated in Figure 12.02.

Solution:

Since 1 meter = 100 centimeters, the length of the wire under R_x is

$$L_x = 100 - 32.5 = 67.5 \text{ centimeters.}$$

We then have $\dfrac{R_x}{67.5} = \dfrac{750}{32.5}$,

and $R_x = \dfrac{750 \times 67.5}{32.5} = 1560$ ohms. *Ans.*

Kelvin Bridge

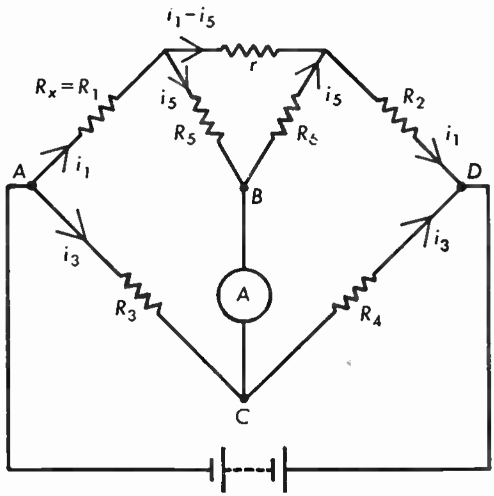

Fig. 12.03 Kelvin bridge, suitable for measuring the small resistance r.

12.03 Derive the balance condition for the Kelvin bridge of Figure 12.03.

Solution:

Since no current flows through the meter A when the bridge is balanced, the IR drop from A to B is equal to the IR drop from A to C; thus there is no potential difference between B and C. Also the currents through R_3 and R_4 are both i_3, the currents through R_1 and R_2 are both i_1, and the currents through R_5 and R_6 are both i_5.

We have

$$i_3 R_3 = i_1 R_1 + i_5 R_5 \tag{1}$$

and

$$i_3 R_4 = i_1 R_2 + i_5 R_6; \tag{2}$$

considering the parallel path r, R_5, R_6, we have by the shunt law

$$\frac{i_5}{i_1 - i_5} = \frac{r}{R_5 + R_6}, \tag{3}$$

which is used to express i_5 in terms of i_1, viz.,

cross-multiplying

$$i_5 R_5 + i_5 R_6 = i_1 r - i_5 r$$

$$i_5 r + i_5 R_5 + i_5 R_6 = i_1 r,$$

$$i_5 (r + R_5 + R_6) = i_1 r,$$

$$i_5 = i_1 \frac{r}{r + R_5 + R_6}.$$

Substituting the value of i_5 in (1) and (2) and factoring out i_1,

we obtain

$$i_3 R_3 = i_1 \left(R_1 + \frac{r R_5}{r + R_5 + R_6}\right)$$

and

$$i_3 R_4 = i_1 \left(R_2 + \frac{r R_6}{r + R_5 + R_6}\right).$$

Dividing the upper equation by the lower yields

$$\frac{R_3}{R_4} = \frac{R_1 + \dfrac{r R_5}{r + R_5 + R_6}}{R_2 + \dfrac{r R_6}{r + R_5 + R_6}}.$$

This is the equation for balance.

If we select the values so that

$$\frac{R_3}{R_4} = \frac{R_1}{R_2},$$

we obtain

$$\frac{R_1}{R_2} = \frac{R_1 + k R_5}{R_2 + k R_6},$$

where

$$k = \frac{r}{r + R_5 + R_6};$$

cross-multiplying

$$R_1 R_2 + k R_1 R_6 = R_1 R_2 + k R_2 R_5,$$

and $$k R_1 R_6 = k R_2 R_5.$$

Transposing $$\frac{R_1}{R_2} = \frac{k R_5}{k R_6} = \frac{R_5}{R_6}.$$

Or if $$\frac{R_1}{R_2} = \frac{R_3}{R_4},$$

then also $$\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{R_5}{R_6}. \quad \textit{Ans.}$$

This balance is independent of the junction resistance r.

Measuring Small Resistances

12.04 With the aid of the Kelvin bridge of Figure 12.03, the resistance R_x is measured with the following bridge resistances: $R_3 = 10$ ohms, $R_5 = 5$ ohms, $R_6 = 50$ ohms. Find the correct value for the arm R_4 and calculate R_x if R_2 has to be adjusted to 0.135 ohm for bridge balance.

Solution:

Using the balance equation derived in the preceding problem, viz.,

$$\frac{R_x}{R_2} = \frac{R_3}{R_4} = \frac{R_5}{R_6},$$

we have $$\frac{R_5}{R_6} = \frac{5}{50} = 0.1,$$

and $$\frac{R_3}{R_4} = 0.1,$$

or $$\frac{10}{R_4} = 0.1, \text{ and } R_4 = \frac{10}{0.1} = 100 \text{ ohms.} \quad \textit{Ans.}$$

Likewise $$\frac{R_x}{R_2} = 0.1,$$

and $$\frac{R_x}{0.135} = 0.1, \text{ and } R_x = 0.1 \times 0.135 = 0.0135 \text{ ohm.} \quad \textit{Ans.}$$

Ohmmeter Hookup

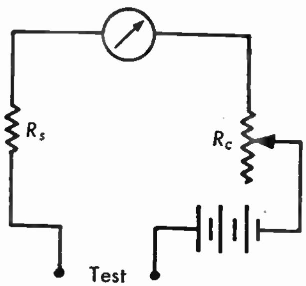

Fig. 12.05 Ohmmeter, consisting of an ammeter and series resistors.

12.05 A 0–1 milliammeter is used to make an ohmmeter as indicated in Figure 12.05. The series resistor R_s is 4000 ohms, the internal resistance of the meter is 100 ohms. What will be an adequate value for the variable calibrating resistor R_c if the voltage of the battery is 4.5 volts, and what will be the part not used in the circuit when the battery is new?

Solution:

Since the meter reads 1 milliampere full scale, the total resistance of the circuit is

$$R = \frac{4.5}{1 \times 10^{-3}} = 4500 \text{ ohms.}$$

A 1000-ohm variable resistor for R_c will be adequate. *Ans.*
The part not used in the circuit is equal to the total resistance available, viz.,

$$R_t = 4000 + 1000 + 100 = 5100 \text{ ohms,}$$

minus the resistor which produces a current of 1 milliampere, i.e., 4500 ohms. The unused resistance is

$$R_{un} = 5100 - 4500 = 600 \text{ ohms.} \quad \textit{Ans.}$$

Calibrating an Ohmmeter

12.06 What is the approximate resistance measured when the milliammeter of problem 12.05 reads 0, 0.25, 0.5, 0.75, and 1 milliampere? What is the direction of the reading?

Solution:

R_o will indicate too great a resistance to be measured by the ohmmeter (infinite). Since 4500 ohms are always present in the circuit, the resistance found by Ohm's law exceeds the measured resistance by 4500 ohms.

Therefore
$$R_{0.25} = \frac{4.5}{0.25 \times 10^{-3}} - 4500$$
$$= 18{,}000 - 4500 = 13{,}500 \text{ ohms.} \quad \textit{Ans.}$$
$$R_{0.5} = \frac{4.5}{0.5 \times 10^{-3}} - 4500$$
$$= 9000 - 4500 = 4500 \text{ ohms.} \quad \textit{Ans.}$$
$$R_{0.75} = \frac{4.5}{0.75 \times 10^{-3}} - 4500$$
$$= 6000 - 4500 = 1500 \text{ ohms.} \quad \textit{Ans.}$$
$$R_1 = \frac{4.5}{1 \times 10^{-3}} - 4500 = 0 \text{ ohms.} \quad \textit{Ans.}$$

The meter will read backward. *Ans.*

Measuring Resistance with a Voltmeter

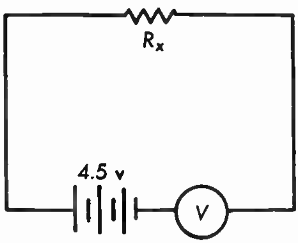

Fig. 12.07 Circuit for measuring resistance with a voltage-indicating meter.

12.07 In Figure 12.07, the instrument arrangement is shown which can be used to measure resistance with a voltmeter when an ohmmeter is not available. The reading of the 1000-ohms-per-volt meter is 2.7 volts on the 10-volt scale. What is the value of R_x?

Solution:

The resistance of the meter when the 10-volt scale is used is

$$r_m = 10 \times 1000 = 10{,}000 \text{ ohms.}$$

The current in the circuit is

$$I = \frac{4.5}{R_x + 10{,}000}.$$

Now the voltage drop across the meter resistance is 2.7 volts,

or $$E_r = 2.7.$$

Substituting, we obtain the equation

$$\frac{4.5}{R_x + 10{,}000} \times 10{,}000 = 2.7,$$

from which R can be calculated.

$$45{,}000 = 2.7\,R_x + 27{,}000$$

$$18{,}000 = 2.7\,R_x$$

$$R_x = \frac{18{,}000}{2.7} = 6660 \text{ ohms.} \quad \textit{Ans.}$$

Measuring Inductance with Voltmeter and Ammeter

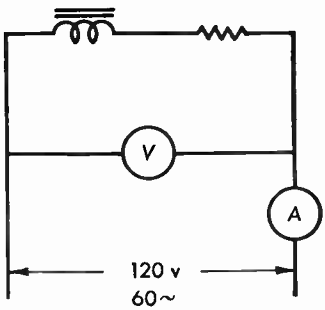

Fig. 12.08 Circuit for measuring inductance with a voltmeter and an ammeter.

12.08 The filter choke in Figure 12.08 has a resistance of 450 ohms. The voltmeter across the 60-cycle line reads 120 volts, the ammeter 21.5 milliamperes. What is the approximate inductance of the choke and its power factor?

Solution:

The impedance of the choke is

$$Z = \frac{E}{I} = \frac{120}{21.5 \times 10^{-3}} = 5580 \text{ ohms.}$$

The reactance of the choke is

$$X = \sqrt{5580^2 - 450^2} = 5550 \text{ ohms.}$$

From $X = 2\,\pi f L$

we obtain $L = \dfrac{X}{2\,\pi f} = \dfrac{5550}{377} = 14.71$ henries. *Ans.*

The power factor is

$$pf = \frac{R}{Z} = \frac{450}{5580} = 0.0807$$

$$= 8 \text{ per cent, approximately.} \quad \textit{Ans.}$$

Measuring Capacitance with the Slide-Wire Bridge

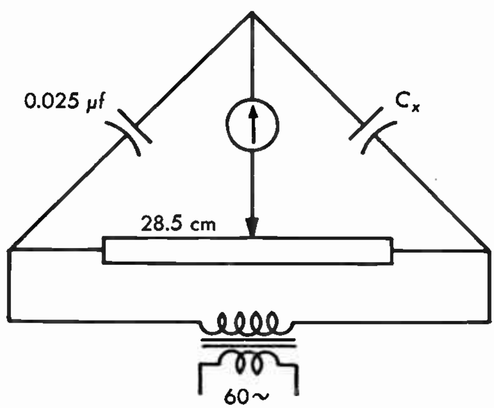

Fig. 12.09 Slide-wire bridge used to measure an unknown capacitance.

12.09 The slide-wire bridge of Figure 12.09 has a total length of 1 meter. Find the unknown capacitance C_x from the values indicated in Figure 12.09.

Solution:

Let the reactance of the 0.025-microfarad capacitor be X_1, then

$$\frac{X_1}{X_x} = \frac{28.5}{100 - 28.5} = \frac{28.5}{71.5}.$$

Now, $$X_1 \div X_x = \frac{1}{2\,\pi f\,C_1} \div \frac{1}{2\,\pi f\,C_x}$$

$$= \frac{1}{2\,\pi f\,C_1} \times \frac{2\,\pi f\,C_x}{1} = \frac{C_x}{C_1}.$$

Substituting $$\frac{C_x}{C_1} = \frac{28.5}{71.5},$$

and $$C_x = \frac{28.5 \times 0.025 \times 10^{-6}}{71.5}$$

$$= 0.01 \times 10^{-6}$$

$= 0.01$ microfarad, approximately. *Ans.*

Capacitance-Resistance Bridge

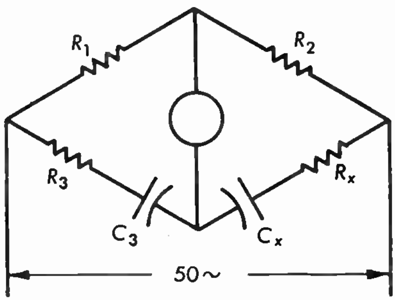

Fig. 12.10 Capacitance-resistance bridge, suitable for measuring unknown capacitance and power factor. When C_x and R_x are connected in *parallel* the circuit is referred to as a *Wien bridge* (see also problem 12.12).

12.10 In the capacitance-resistance bridge of Figure 12.10, $R_1 = 0.75$ megohm, $R_2 = 0.5$ megohm, $R_3 = 0.5$ megohm, $C_3 = 500$ micromicrofarads. The applied frequency is 500 cycles. Find the value of R_x and C_x when the indicator reads zero.

Solution:

(a) Impedance proportion

We have $$\frac{\mathring{Z}_x}{\mathring{Z}_2} = \frac{\mathring{Z}_3}{\mathring{Z}_1},$$

and $$\mathring{Z}_x = \frac{\mathring{Z}_2\,\mathring{Z}_3}{\mathring{Z}_1}.$$

For the evaluation of the above equation we need X_3.

$$X_3 = \frac{1}{2\,\pi\,f\,C_3}$$

$$= \frac{1}{6.28 \times 500 \times 500 \times 10^{-12}}$$

$$= 636{,}000 \text{ ohms.}$$

Substituting the known values, all in megohms,

we obtain $$\mathring{Z}_x = \frac{0.5 \times (0.5 - j\,0.636)}{0.75}$$

$$= \frac{0.25 - j\,0.318}{0.75}$$

$$= 0.333 - j\,0.424 \text{ vector megohms.}$$

The resistance is the real term of this quantity,

$$R_x = \frac{1}{3} \text{ megohm.} \quad \textit{Ans.}$$

To find C_x, we must calculate which capacitance will produce a capacitive reactance of 424,000 ohms.

Using $$C_x = \frac{1}{2\,\pi\,f\,X_x},$$

we obtain $$C_x = \frac{1}{6.28 \times 500 \times 424{,}000}$$

$$= 750 \text{ micromicrofarads.} \quad \textit{Ans.}$$

(b) Complex equation

The value sought may be found more quickly if the fact is used that in a complex equation of the form

$$a + j\,b = c + j\,d,$$

the real parts are equal, viz.,

$$a = c;$$

also the imaginary parts are equal,

$$b = d.$$

The bridge proportion is

$$\frac{0.5 - j\,X_3}{0.75} = \frac{R_x - j\,X_x}{0.5}$$

equating the real parts

$$\frac{0.5}{0.75} = \frac{R_x}{0.5},$$

from which

$$R_x = \frac{0.5 \times 0.5}{0.75} = \frac{0.25}{0.75}$$

$$= \frac{1}{3}\text{ megohm.} \quad \textit{Ans.}$$

Equating the imaginary parts, we have

$$\frac{X_3}{0.75} = \frac{X_x}{0.5}$$

and

$$X_x = \frac{0.5\, X_3}{0.75}$$

or

$$X_x = \frac{2}{3} X_3.$$

Since the capacitances are in inverse proportion to the reactances, we have

$$C_x = \frac{3}{2} \times C_3 = \frac{3}{2} \times 500$$

$$= 750\text{ micromicrofarads.} \quad \textit{Ans.}$$

Inductance of a Radio-Frequency Coil

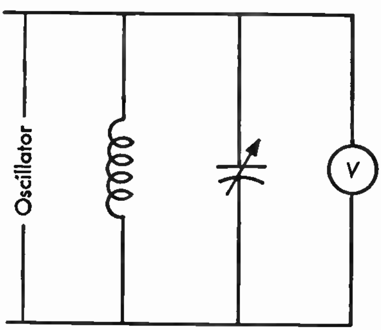

Fig. 12.11 Circuit for measuring the inductance of an r-f coil.

12.11 To measure the approximate inductance of a radio-frequency coil, a calibrated variable capacitor, a signal generator, and a vacuum-tube voltmeter are used in the arrangement of Figure 12.11. A frequency of 950 kilocycles is fed into the tank circuit, and the tank capacitor is adjusted to 260 micromicrofarads, at which point resonance is indicated by a maximum reading of the vacuum-tube voltmeter. Neglecting the distributed capacitance of the coil, calculate the inductance.

Solution:

Solving the formula

$$f = \frac{1}{2\,\pi\,\sqrt{L\,C}} \text{ for } L,$$

we obtain

$$f^2 = \frac{1}{4\,\pi^2\,L\,C}$$

from which

$$L = \frac{1}{4\,\pi^2\,f^2\,C}.$$

Substituting the known values, we have

$$L = \frac{1}{39.5 \times (950 \times 10^3)^2 \times 260 \times 10^{-12}}$$

$$= \frac{1}{39.5 \times 10 \times (9.5 \times 10^2 \times 10^3)^2 \times 2.6 \times 10^2 \times 10^{-12}}$$

$$= \frac{1}{3.95 \times 90 \times 2.6 \times 10} = 0.000108 \text{ henries}$$

$= 108$ microhenries, approximately. *Ans.*

Wien Bridge

12.12 An arrangement similar to Figure 12.10, *but with C_x and R_x connected in parallel* is used to determine the capacitance C_x when the resistors and the frequency of the source are known. With $R_1 = R_2 = R_x = 1000$ ohms, and $R_3 = 500$ ohms, the indicator reads zero when the frequency of the source is 15,750 cycles. Calculate C_x and C_3.

Solution:

Let the reactances of C_x and C_3 be X and X_3, respectively. The bridge is at balance when

$$\frac{\mathring{Z}_1}{\mathring{Z}_2} = \frac{\mathring{Z}_3}{\mathring{Z}_x}.$$

Substituting the known values

$$\frac{1000}{1000} = \frac{500 - j X_3}{\dfrac{(-j X)\,1000}{(-j X) + 1000}}$$

yields $$\frac{-j\,1000\,X}{1000 - j X} = 500 - j X_3$$

and $$-j\,1000\,X = (500 - j X_3)(1000 - j X).$$

Multiplying we obtain

$$-j\,1000\,X = 500{,}000 - j\,1000\,X_3 - j\,500\,X - X X_3$$

or, after transposing

$$X X_3 - j\,500\,X = 500{,}000 - j\,1000\,X_3.$$

Equating the real parts and the imaginary parts we obtain a pair of simultaneous equations:

$$X X_3 = 500{,}000 \qquad (1)$$

$$500\,X = 1000\,X_3, \qquad (2)$$

which are solved by the method of substitution.

From (2) $$X = 2\,X_3.$$

Substituting in (1)

$$(2\,X_3)\,X_3 = 500{,}000$$

$$X_3^2 = 250{,}000$$

$$X_3 = 500 \text{ ohms}$$

and $$X_3 = 1000 \text{ ohms}.$$

At 15,750 cycles

$$C_x = \frac{1}{2\,\pi f X} = 0.101 \text{ microfarad.} \quad \textit{Ans.}$$

and $$C_3 = 0.202 \text{ microfarad.} \quad \textit{Ans.}$$

Extending the Range of a Voltmeter

12.13 A 0-to-10-volt, 1000-ohms-per-volt meter is to be extended to read 100 volts full scale. What is the necessary multiplier resistor?

Solution:

The resistance of the meter is

$$R_i = 10 \times 1000 = 10{,}000 \text{ ohms.}$$

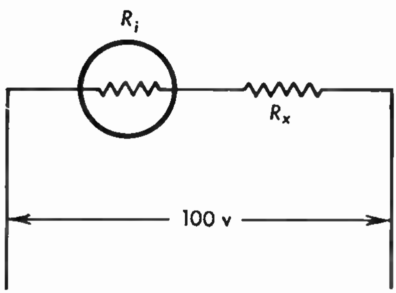

Fig. 12.13 Voltmeter with series resistor extending the range of the meter.

The multiplier for the 100-volt scale R_x must cause a drop of 90 volts. We have the proportion

$$\frac{R_x}{90} = \frac{10{,}000}{10},$$

and

$$R_x = \frac{90 \times 10{,}000}{10} = 90{,}000 \text{ ohms.} \quad \textit{Ans.}$$

Extending the Range of an Ammeter

12.14 A 0-to-1 milliammeter with an internal resistance of 30 ohms is to be extended to read 100 milliamperes full scale. What is the necessary shunt resistor?

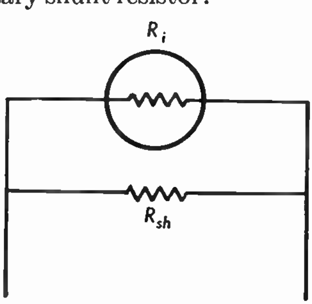

Fig. 12.14 Ammeter with shunt resistor extending the range of the meter.

Solution:

The current through the shunt resistor must be

$$I_{sh} = 100 - 1 = 99 \text{ milliamperes.}$$

Since the current is inversely proportional to the resistance, we have the proportion

$$\frac{R_{sh}}{30} = \frac{0.001}{0.099},$$

and $$R_{sh} = \frac{30 \times 0.001}{0.099} = 0.303 \text{ ohm.} \quad \textit{Ans.}$$

Volt-Milliammeter

12.15 It is desired to cause a 0–1 milliammeter with an internal resistance of 30 ohms to read volts also. Find the series resistor necessary to make it read 100 volts full scale.

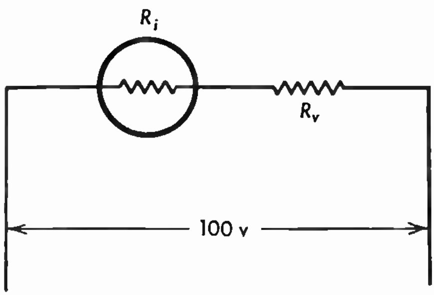

Fig. 12.15 Ammeter with series resistor to make the meter read voltage.

Solution:

The voltage drop across the meter resistance

$$E_m = 0.001 \times 30 = 0.03 \text{ volt.}$$

The series resistor R_v must produce a voltage drop of

$$E_v = 100 - 0.03 = 99.97 \text{ volts.}$$

Therefore, $$R_v = \frac{99.97}{0.001} = 99{,}970 \text{ ohms.} \quad \textit{Ans.}$$

Vacuum-Tube Voltmeter

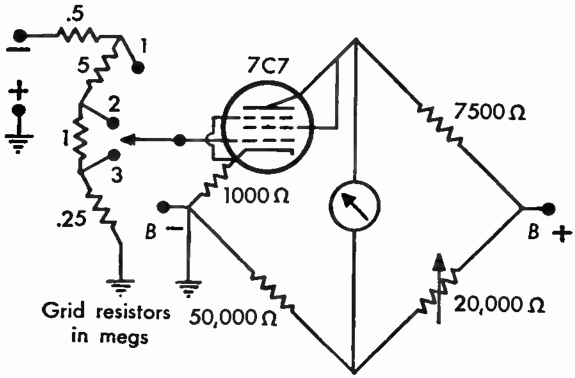

Fig. 12.16 Vacuum-tube voltmeter with range-selector switch.

12.16 In the vacuum-tube voltmeter circuit Figure 12.16, what is the equivalent resistance of the tube with no voltage applied to the grid, and with the balancing resistor reduced to 12,000 ohms for null indication? If the selector switch is at No. 1 position, the meter is calibrated for 5 volts. What range are positions No. 2 and No. 3?

Solution:

The meter works on the principle of a balanced-plate circuit, and the application of a negative voltage to the grid will upset the balance and cause a meter reading. At balance the equivalent resistance of the tube is found from the proportion

$$\frac{R_{tu} + 1000}{50{,}000} = \frac{7500}{12{,}000}$$

and $$R_{tu} + 1000 = \frac{75{,}000 \times 50{,}000}{12{,}000} = 312{,}000 \text{ ohms}$$

$$R_{tu} = 311{,}000 \text{ ohms, approximately.} \quad \textit{Ans.}$$

When the switch is at contact No. 1, the voltage applied to the grid is that portion of 5 volts which exists from No. 1 to ground.

$$E_g = 5 \times \frac{6.25}{6.75} = 4.62 \text{ volts.}$$

At contact No. 2, in order to maintain the grid voltage at 4.62 volts, the drop from No. 2 to ground must be 4.62 volts. Using the voltage-divider relation

$$4.62 = E_2 \frac{1.25}{6.75}$$

we find $$E_2 = \frac{4.62 \times 6.75}{1.25} = 25 \text{ volts.} \quad \textit{Ans.}$$

Likewise, at contact No. 3, we find from

$$4.62 = E_3 \frac{0.25}{6.75},$$

that $$E_3 = \frac{4.62 \times 6.75}{0.25} = 125 \text{ volts.} \quad \textit{Ans.}$$

Measuring Voltages with a Milliammeter

FCC Study Guide Question 3.02

12.17 In the diagram below, compute direct plate voltage, direct-current grid bias, and supply voltage.

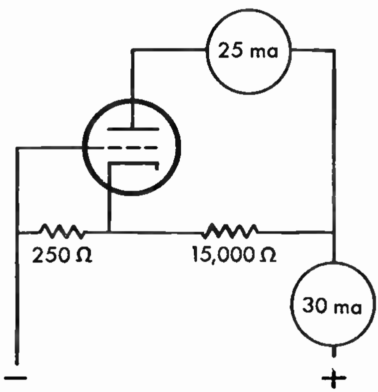

Fig. 12.17 Use of ammeter to find voltages when resistances are known.

Solution:

The plate voltage is determined by the voltage drop across the plate-cathode resistor of 15,000 ohms. The current through this resistor is

$$I_{15} = 30 - 25 = 5 \text{ milliamperes}$$

Thus $$E_p = 5 \times 10^{-3} \times 15 \times 10^3 = 75 \text{ volts.} \quad \textit{Ans.}$$

The grid voltage is determined by the voltage drop across the grid-cathode resistor of 250 ohms. The current through this resistor is 30 milliamperes.

Thus $E_g = 30 \times 10^{-3} \times 250 = 7.5$ volts. *Ans.*

The B-supply voltage is the voltage across both resistors connected across the B-supply,

$$E_b = 75 + 7.5 = 82.5 \text{ volts.} \quad \textit{Ans.}$$

Meter Shunt

FCC Study Guide Question 5.07

12.18 A milliammeter with a full-scale deflection of one milliampere and having an internal resistance of 25 ohms is used to measure an unknown current, by shunting the meter with a 4-ohm resistance. When the meter reads 0.4 milliampere, what is the actual value of current?

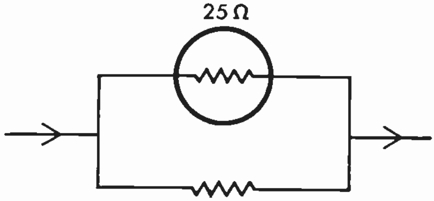

Fig. 12.18 Equivalent circuit of ammeter with internal resistance and shunt.

Solution:

Referring to Figure 12.18, it is obvious that the line current consists of the meter current plus the shunt current,

$$I_l = I_m + I_s.$$

Now the ratio of the shunt current to the meter current is

$$\frac{I_s}{I_m} = \frac{25}{4},$$

and

$$I_s = \frac{25\, I_m}{4}.$$

Substituting,

$$I_l = I_m + \frac{25\, I_m}{4}$$

$$= 0.4 + \frac{25 \times 0.4}{4}$$

$$= 0.4 + 2.5 = 2.9 \text{ milliamperes.} \quad \textit{Ans.}$$

Measuring Capacitance with a Voltmeter

12.19 To determine the capacitance of a capacitor, a 25,000-ohm resistor is connected in series with the capacitor and a 60-cycle line voltage is applied across this series circuit. A vacuum-tube voltmeter reads 82.5 volts across the resistor and 86.7 volts across the capacitor. Calculate the capacitance.

Solution:

The current of the series circuit is found by Ohm's law

$$I = \frac{82.5}{25,000} = 3.3 \text{ milliamperes.}$$

Thus $$X_c = \frac{E_L}{I} = \frac{86.7}{3.3 \times 10^{-3}} = 26,300 \text{ ohms.}$$

Using $$X_c = \frac{1}{2\,\pi\,f\,C},$$

we obtain $$C = \frac{1}{2\,\pi\,f\,X_c} = \frac{1}{377 \times 26,300}$$

$$= \frac{1}{3.77 \times 2.63 \times 10^6} \cong 0.1 \times 10^{-6}$$

$$= 0.1 \text{ microfarad.} \quad \textit{Ans.}$$

Measuring Inductance with a Voltmeter

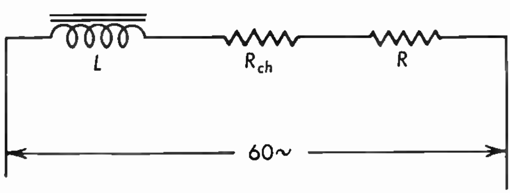

Fig. 12.20 Circuit illustrating the measuring of inductance with a voltmeter (see text).

12.20 To determine the inductance of a choke, a 2500-ohm resistor is connected in series with the choke, and the voltage of a 60-cycle line is impressed upon this series hookup. An a-c voltmeter reads 50 volts across the resistor and 113 volts across the choke. The resistance of the choke is 100 ohms. Calculate the inductance of the choke (a) neglecting its resistance, (b) including the resistance of the choke. What is the percentage error of (a)?

Solution:

(a) Neglecting the resistance of the choke

Since the current through the choke and the resistor are equal, we obtain:

$$I = \frac{50}{2500} \text{ and } I = \frac{113}{X_L}.$$

Thus
$$\frac{50}{2500} = \frac{113}{X_L},$$

and
$$X_L = \frac{113 \times 2500}{50} = 5650 \text{ ohms},$$

which corresponds to an inductance of

$$L = \frac{X_L}{2\,\pi\,f} = \frac{5650}{377} = 15 \text{ henries.} \quad \textit{Ans.}$$

(b) Including the resistance of the choke

The current is found by Ohm's law

$$I = \frac{50}{2500} = 0.02 \text{ ampere.}$$

The voltage drop across R_{ch} is

$$E_{ch} = 0.02 \times 100 = 2 \text{ volts}$$

$$E_L = \sqrt{113^2 - 2^2} = 112.9 \text{ volts},$$

and
$$X_L = \frac{E_L}{I_L} = \frac{112.9}{0.02} = 5645 \text{ ohms},$$

which corresponds to an inductance of

$$L = \frac{5645}{377} = 14.98 \text{ henries.} \quad \textit{Ans.}$$

$$\text{Error} = \frac{\text{difference}}{\text{true value}} = \frac{15 - 14.98}{14.86} = \frac{0.02}{14.98}$$

$$= 0.00133 = 0.133 \text{ per cent.} \quad \textit{Ans.}$$

Measuring Resistance with a Vacuum-Tube Voltmeter

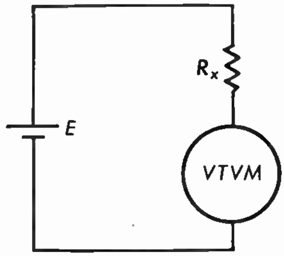

Fig. 12.21 Vacuum-tube voltmeter hookup for measuring resistance.

12.21 A vacuum-tube voltmeter with an input resistance of 10 megohms is used to measure an unknown resistor by the circuit indicated in Figure 12.21. $E = 6.3$ volts, the voltmeter reading is 1.2 volts. What is the resistance of R_x?

Solution:

We have the proportion

$$\frac{R_x}{E_x} = \frac{R_m}{E_m}.$$

But
$$E_x = E - E_m$$
$$= 6.3 - 1.2 = 5.1 \text{ volts}.$$

Substituting
$$\frac{R_x}{5.1} = \frac{10 \times 10^6}{1.2}$$

$$R_x = \frac{5.1 \times 10}{1.2} \times 10^6$$
$$= 42.5 \times 10^6 \text{ ohms}$$
$$= 42.5 \text{ megohms.} \quad \textit{Ans.}$$

Q of a Radio-Frequency Coil

12.22 The arrangement of Figure 12.11 is also used to determine the Q of the coil as follows: The oscillator is turned to an off-resonance position causing a lower reading of the vacuum-tube voltmeter. When the oscillator frequency is adjusted so as to make the vacuum-tube voltmeter read 70.7 per cent of the value which it indicated at resonance, the formula $Q = f/\Delta f$ can be used to determine Q, where

f = resonant frequency, Δf = the frequency change. If the oscillator reads 1035 kilocycles after the described adjustment, what is the Q of the coil?

Solution:

$$Q = \frac{f}{\Delta f} = \frac{950}{1035 - 950}$$

$$= \frac{950}{85} = 11.2. \quad \textit{Ans.}$$

13 *Industrial and Control Circuits*

Thyratron Frequency Limits

13.01 The typical deionization time of a thyratron is 1000 microseconds, the ionization time 10 microseconds. There are a few faster types with a deionization time of 100 microseconds and an ionization time of 20 microseconds. What is the maximum frequency of operation of a typical and a short deionization-type thyratron oscillator?

Solution:

If each cycle were to consist of ionization and deionization only, then the theoretical periods of both types would be

$$t_t = 1000 + 10 = 1010 \text{ microseconds,}$$

$$t_s = 100 + 20 = 120 \text{ microseconds.}$$

The corresponding frequencies would be

$$f_t = \frac{1}{t_t} = \frac{1}{1010 \times 10^{-6}}$$

$$= \frac{10^6}{1010} = 990 \text{ cycles.} \quad \textit{Ans.}$$

$$f_s = \frac{1}{t_s} = \frac{1}{120 \times 10^{-6}}$$

$$= \frac{10^6}{120} = 8333 \text{ cycles.} \quad \textit{Ans.}$$

Saw-tooth Wave Applied to Thyratron

13.02 The thyratron tube type 502-A is operated at a plate voltage of 350 volts; the control grid direct voltage at start of discharge is −3 volts. A saw-tooth wave of the shape shown in Figure 13.02 is applied between grid and cathode with the terminal voltage of the saw-tooth generator connected to the grid. What are the approximate active and inactive periods of the tube?

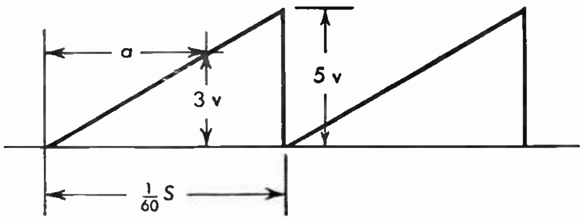

Fig. 13.02 Saw-tooth wave as applied between grid and cathode of the thyratron tube in problem 13.02.

Solution:

The period a in Figure 13.02 is the time during which the grid is less negative than -3 volts. Therefore a is the active period of the tube. We have the proportion of similar triangles

$$5 : \frac{1}{60} = 3 : a$$

and
$$a = \frac{3}{60 \times 5} = 0.01 \text{ second.} \quad \textit{Ans.}$$

The inactive period is

$$i = \frac{1}{60} - \frac{1}{100} = \frac{5}{300} - \frac{3}{300} = \frac{2}{300}$$
$$= 0.00667 \text{ second.} \quad \textit{Ans.}$$

Note: The active period will be somewhat longer because of the de-ionization time.

Delayed Firing

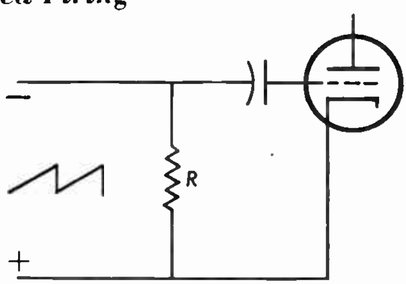

Fig. 13.03 Saw-tooth wave applied to thyratron tube through an R-C circuit.

13.03 If the saw-tooth voltage of problem 13.02 is applied to the grid of the thyratron through a resistance-capacitance circuit of the type shown in Figure 13.03, with R = 500,000 ohms and C = 0.01 microfarad, how long will the firing of the tube be delayed after the instant at which C is charged to the peak value of the saw-tooth wave?

Solution:

The delay time is the time the capacitor will need to discharge to 3 volts.

Using $$e = E\,\epsilon^{-\frac{t}{RC}}$$

with $$RC = 5 \times 10^5 \times 0.01 \times 10^{-6} = 0.005,$$

we have $$3 = 5\,\epsilon^{-\frac{t}{0.005}},$$

$$0.6 = \epsilon^{-\frac{t}{0.005}},$$

$$\log 0.6 = -\frac{t}{0.005}\log \epsilon,$$

$$0.778 - 1 = -\frac{t}{0.005}\,0.4343,$$

$$-0.222 = -t \times 86.5,$$

and $$t = \frac{0.222}{86.5} = 0.00256 \text{ second.}$$

The firing will be delayed by 2.5 milliseconds, approximately. *Ans.*

Neon Relaxation Oscillator

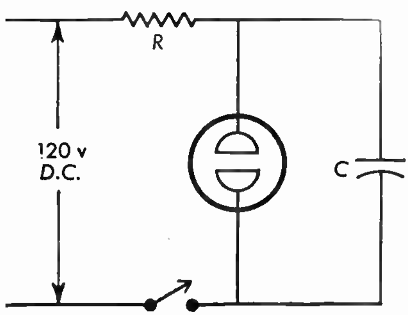

Fig. 13.04 Saw-tooth oscillator using a neon bulb discharge path.

13.04 A neon relaxation oscillator of the type shown in Figure 13.04 operates at an ignition voltage of 85 volts and an extinction voltage of 55 volts. If $R = 500{,}000$ ohms and $C = 0.02$ microfarad, what time will elapse between the instant of closing the switch and the ignition of the tube?

Solution:

The instantaneous voltage of a charging capacitor is

$$e = E\,(1 - \epsilon^{-\frac{t}{RC}}).$$

But $$RC = 500{,}000 \times 0.02 \times 10^{-6} = 0.01,$$

and $$e = 85 \text{ volts.}$$

Substituting $$85 = 120 - 120\,\epsilon^{-\frac{t}{0.01}},$$

$$120\,\epsilon^{-100\,t} = 35,$$

$$\epsilon^{-100\,t} = 0.292,$$

$$-100\,t \times 0.4343 = \log 0.292$$

$$-43.43\,t = 0.4654 - 1$$

$$-43.43\,t = -0.5346$$

$$t = \frac{0.5346}{43.43} = 0.0123 \text{ second.} \quad \textit{Ans.}$$

Frequency of Neon Oscillator

13.05 Calculate the frequency of the relaxation oscillator of problem 13.04.

Solution:

A simple way to find the approximate frequency is to calculate the time which elapses between the charge to the extinction voltage and the charge to the ignition voltage, assuming that the discharge through the neon bulb is instantaneous.

From problem 13.04, $$t_{85} = 0.0123 \text{ second}$$

To find t_{55} we substitute:

$$55 = 120 - 120\,\epsilon^{-100\,t},$$

$$120\,\epsilon^{-100} = 65,$$

$$\epsilon^{-100\,t} = 0.542;$$

now $$\log 0.542 = 0.734 - 1 = -0.266,$$

and $$\log \epsilon = 0.4343.$$

Therefore $$-100\,t \times 0.4343 = -0.266$$

and $$t_{55} = \frac{0.266}{43.43} = 0.00613 \text{ second.}$$

The period of oscillation is

$$t_{85} - t_{55} = 0.0123 - 0.00613$$
$$= 0.00617 \text{ second,}$$

i.e., the duration of 1 cycle. To find the number of cycles per second this fraction is divided into 1,

$$f = \frac{1}{0.00617} = \frac{1}{6.17 \times 10^{-3}}$$
$$= 0.162 \times 10^{3} = 162 \text{ cycles.} \quad \textit{Ans.}$$

Practical Discharge Time in Terms of RC-Seconds

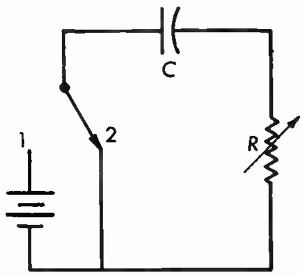

Fig. 13.06 Equivalent circuit of a charge and discharge path of a capacitor.

13.06 The capacitor C in Figure 13.06, having been charged to E volts, is to be discharged by throwing the switch to position No. 2. If a residual charge of 0.75 per cent of the original charge is considered to be a practical discharge, after how many $R\,C$-time constants will the capacitor be practically discharged?

Solution:

Using $$e = E\,\epsilon^{-\frac{t}{RC}}$$

and $$e = 0.0075\,E,\ t = x\,R\,C$$

we have $$0.0075\,E = E\,\epsilon^{-\frac{xRC}{RC}}.$$

Simplifying

$$0.0075 = \epsilon^{-x}$$

$$\log 0.0075 = \log \epsilon^{-x} = -x \log \epsilon$$

$$\bar{2}.1249 = 0.4343\, x$$

$$x = \frac{2.1249}{0.4343} = 4.9$$

At $t = 5\,R\,C$ seconds the capacitor will be discharged to approximately 0.75 per cent of the full charge. *Ans.*

Discharge Time in Seconds

13.07 Using the rule found in problem 13.06, if $R = 500{,}000$ ohms and $C = 10$ microfarads, what is the practical discharge time in seconds?

Solution:

$$t = 5\,R\,C = 5 \times 5 \times 10^5 \times 10 \times 10^{-6}$$

$$= 25 \text{ seconds.}$$

Timing Resistor

13.08 What should be the minimum setting of the variable resistor R in problem 13.07, if a discharge time varying from 0.1 second to 25 seconds is to be realized?

Solution:

Using $t = 5\,R\,C$,

we obtain $0.1 = 5\,R \times 10 \times 10^{-6}$

and $\dfrac{10^{-1}}{5 \times 10^{-5}} = R.$

$$R = 0.2 \times 10^4$$

$$= 2000 \text{ ohms, approximately.}$$ *Ans.*

Ignitron Demand Current

13.09 A supply voltage of 350 volts rms is used in a welder-ignitron circuit to satisfy a power demand of 850 kilovolt-amperes. Calculate the average current demand during any conducting cycle.

Solution:

The required line current is

$$I_L = \frac{850{,}000}{350}$$

$$= 2430 \text{ amperes rms}$$

$$= 2430 \times \sqrt{2} = 3430 \text{ amperes peak.}$$

The average current demand over a conducting cycle is determined by the fact that each tube is essentially a half-wave rectifier with an average of approximately 0.3185 of the peak (half of 0.637, which is the full-wave average). Therefore

$$I_{av} = 3430 \times 0.3185 = 1080 \text{ amperes.} \quad \textit{Ans.}$$

Ignition Loss to Cooling System

13.10 The ignitron type FG-238-B has an average current rating of 200 amperes and an arc drop of approximately 17 volts. What is the equivalent resistance of the arc, and how many kilowatts should the water-cooling system be designed to dissipate?

Solution:

The arc, causing a voltage drop of 17 volts at 200 amperes, has an equivalent resistance of

$$R = \frac{E}{I} = \frac{17}{200} = 8.5 \times 10^{-2} = 0.085 \text{ ohm.} \quad \textit{Ans.}$$

The power dissipation of the arc is

$$P = I^2 R = 200^2 \times 0.085$$

$$= 4 \times 10^4 \times 8.5 \times 10^{-2} = 34 \times 10^2 \text{ watts}$$

$$= 3.4 \text{ kilowatts.} \quad \textit{Ans.}$$

Welding Time of an Ignitron

13.11 The demand in kilovolt-amperes of a welder ignitron circuit is within the rating of the ignitron type WL-651/656 or its equivalent type FG-235-A. The following additional data are taken from the information of the manufacturer for the above ignitron and for the conditions in problem 13.09:

Average anode current	92 amperes
Maximum time of averaging anode current	9.8 seconds

Find the permissible welding time of the tube in seconds and in cycles of à 60-cycle line.

Solution:

During each averaging time the tube has a time-current product of

$$TI = 92 \times 9.8 = 902 \text{ ampere-seconds},$$

which is to say that during each averaging time the tube could conduct 902 amperes for the duration of 1 second, or 451 amperes for the duration of 2 seconds, etc. The welding time, i.e., the time the tube can conduct the demand current of 1080 amperes, is of course less than 1 second. The ratio of the weld to 1 second is

$$W_t/1 = 902/1080$$

The welding time therefore is

$$W_t = \frac{902}{1080} = 0.828 \text{ second.} \quad \textit{Ans.}$$

The welding time expressed in cycles is the number of cycles during which the tube can be active in each averaging time. Since each second has 60 cycles

$$W_f/60 = 0.828/1$$

$$W_f = 0.828 \times 60 = 49 \text{ cycles, approximately.} \quad \textit{Ans.}$$

Cycle-Duration of Required Welds

13.12 In problem 13.11, if it is desired to make 12 welds during each averaging time, how many cycles of a 60-cycle line can be used for each weld and what is the duration of each weld in seconds?

Solution:

The number of cycles of tube conduction permissible for each weld is, conservatively

$$W_{12} = \frac{49}{12} \cong 4 \text{ cycles.} \quad \textit{Ans.}$$

The duration of a weld is 4/60 second, or in decimals,

$$W_t' = \frac{4}{60} = 0.067 \text{ second.} \quad \textit{Ans.}$$

Duty of a Welder Ignitron

13.13 What is the duty in per cent of the welder ignitron in problem 13.12?

Solution:

The duty is the ratio of the active to the total time. In the above case the total time in cycles is

$$T_t = 9.8 \times 60 = 588 \text{ cycles.}$$

The active time of 12 welds, each taking 4 cycles, is

$$T_a = 12 \times 4 = 48 \text{ cycles.}$$

The duty therefore is

$$D = \frac{T_a}{T_t} = \frac{48}{588} = 0.0817 = 8.17 \text{ per cent.} \quad \textit{Ans.}$$

Phototube Current Amplification

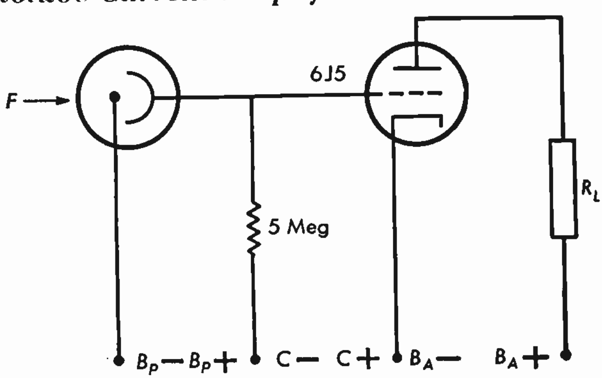

Fig. 13.14a Phototube with amplifier.

13.14 In the phototube circuit of Figure 13.14a, $F = 0.2$ lumen, $R_g = 5$ megohms, the sensitivity of the type 929 phototube is 45 microamperes per lumen. The amplification factor of the type 6J5 is 20, and its plate resistance is 7000 ohms. The load has a resistance of

10,000 ohms. Calculate the current increase in the load caused by an incident light flux of 0.02 lumen, and the current amplification of the circuit.

Solution:

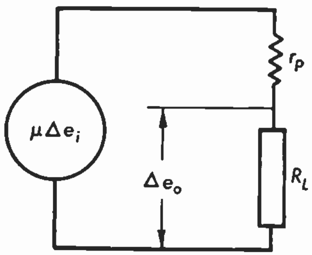

Fig. 13.14b Equivalent circuit of 13.14a.

Figure 13.14b is the equivalent circuit used to calculate the output voltage upon which the output current depends.

$$\Delta e_o = \mu \Delta e_i \frac{R_l}{R_l + r_p}$$

The only unknown is Δe_i. This is the voltage change across R_g caused by the luminous flux.
The current change caused by 0.2 lumen at a sensitivity of 45 micro-amperes per lumen is

$$\begin{aligned}\Delta i_i &= 45 \times 10^{-6} \times 0.02\\ &= 0.9 \times 10^{-6} \text{ ampere}\end{aligned}$$

The input voltage change across R_g therefore is

$$\begin{aligned}\Delta e_i &= \Delta i_i \times R_g\\ &= 0.9 \times 10^{-6} \times 5 \times 10^{6}\\ &= 4.5 \text{ volts.}\end{aligned}$$

Substituting, we obtain an output voltage change of

$$\begin{aligned}\Delta e_o &= 20 \times 4.5 \times \frac{10{,}000}{17{,}000}\\ &= 52.9 \text{ volts.}\end{aligned}$$

This will cause a current-output change of

$$\Delta i_o = \frac{\Delta e_o}{R_L} = \frac{52.9}{10^4}$$

$$= 5290 \text{ microamperes.} \quad \textit{Ans.}$$

The current amplification is

$$\text{Current amplification} = \frac{\Delta i_o}{\Delta i_i} = \frac{5290}{0.9} = 5880. \quad \textit{Ans.}$$

SECTION II *Problems for Further Practice*

1 *Circuit Components*

2-1.01 What is the resistance of a 1-mile line of copper wire, using No. 12 AWG?

2-1.02 What is the length of No. 25 copper wire necessary to make a shunt of 0.1 ohm?

2-1.03 What would be the length of the shunt in problem 2-1.02 if constantan wire were used?

2-1.04 What would be the length of the shunt in problem 2-1.02 if nichrome wire were used?

2-1.05 If the shunt in problem 2-1.02 is to be 10 inches long, what is the diameter of the wire to be used? What is the approximate AWG number?

2-1.06 What will be the resistance increase in per cent of the shunt in problem 2-1.02 at a temperature of 125F?

2-1.07 What is the resistance of 2 feet of No. 26 copper wire at 50 megacycles?

2-1.08 What is the d-c resistance of the conductor in problem 2-1.07? What is the ratio of the two resistances?

2-1.09 Can a capacitor designed for 450 volts maximum working voltage be used across a transformer winding of 300 volts rms?

2-1.10 What is the total capacitance of a tuning capacitance of 450 micromicrofarads shunted by a trimmer capacitor of 50 micromicrofarads?

2-1.11 A 10- to 40-micromicrofarad antenna capacitor and a 30- to 140-micromicrofarad tuning capacitor are available. What ranges of capacitance can be produced?

2-1.12 A 0.0004-microfarad mica capacitor is found to have an equivalent series resistance of 0.5 ohm at 6600 kilocycles; what is the power factor of the capacitor?

2-1.13 What is the equivalent shunt resistance of the capacitor in problem 2-1.12?

2-1.14 A paper capacitor consists of a foil-covered paper strip 6 centimeters wide and 2.5 meters long. The paper has a thickness of 0.0025 centimeter and a dielectric constant of 2.8. What is the capacitance?

2-1.15 If capacitors of 2, 3, and 4 microfarads are connected in parallel, what is the total capacitance?

2-1.16 If capacitors of 5, 6, and 7 microfarads are connected in series, what is the total capacitance?

2-1.17 Having available a number of capacitors rated at 400 volts and 2 microfarads each, how many of these capacitors would be necessary to obtain a combination rated at 800 volts, 5 microfarads?

2-1.18 What is the charge stored in a 3-microfarad capacitor with a potential difference of 85 volts across the plates?

2-1.19 What is the capacitance of a twelve-plate capacitor with a plate area of $2\frac{1}{2}$ square inches and a distance between the plates of 0.1 inch?

2-1.20 How many plates of the capacitor in problem 2-1.19 must be removed to reduce the capacitance to 30 micromicrofarads?

2-1.21 Two inductors with an inductance of 15 and 20 henries are connected

a) series aiding,
b) series opposing,
c) parallel aiding,
d) parallel opposing.

The coupling coefficient $k = 0.6$. What is the total inductance in each case?

2-1.22 If the mutual inductance between two coils is 0.08 henry, and the coils have inductances of 0.2 and 0.8 henry respectively, what is the coefficient of coupling?

2-1.23 What is the inductance of a single layer air-core inductor with an inside diameter of 1.5 inches using 70 turns of No. 25 enamel-covered wire, close-wound? (No. 25 enamel AWG has 51.7 turns per inch.)

2-1.24 What is the inductance of a coil having the same specification as the one in problem 2-1.23, but having three layers of windings?

2-1.25 Using the values in problem 2-1.23, calculate the inside diameter of the coil for an inductance of 144 microhenries.

2-1.26 What turns ratio should a transformer have which is to be used to match a source impedance of 500 ohms to a load of 12 ohms?

2-1.27 In a transformer having a turns ratio of 9 to 1, working into a load impedance of 1800 ohms and out of a circuit having an impedance of 15 ohms, what value of resistance may be connected across the load to effect an impedance match?

2-1.28 Two single-layer coils have equal inductances. One coil has a 70-turn winding 3 inches long and is 2 inches in diameter; the second is 2.5 inches long and 1.5 inches in diameter. Calculate the number of turns of the second coil.

2-1.29 What would be the inductance of the single-layer coil in problem 2-1.23 if the number of turns were doubled?

2-1.30 If a transformer has a primary voltage of 4200 volts and a secondary voltage of 220 volts, and has an efficiency of 95 per cent when delivering 22 amperes of secondary current, what is the value of primary current?

2-1.31 Three single-phase transformers, each with a ratio of 1 to 12 are connected across a 220-volt three-phase line, primaries in delta. If the secondaries are connected in Y, what is the secondary line voltage?

2-1.32 What is the secondary voltage of a transformer which has a primary voltage of 120, primary turns 210, and secondary turns 50?

2-1.33 The heater winding of a transformer is rated 7.5 volts and 3 amperes; it consists of 15 turns. How many turns should be added to change the voltage rating to 10 volts? What should be the value of the new current rating?

2-1.34 The type 80 tube draws 2 amperes at 5 volts from the secondary of a transformer. What is the primary current (a) neglecting the losses, (b) at an efficiency of 91 per cent? The primary voltage is 120 volts.

2-1.35 The secondary load of a step-down transformer with a turns ratio of 1 to 9 is 550 ohms. What is the impedance looking into the primary?

2-1.36 What turns ratio will be required to couple a 4-ohm voice coil to a plate load of 12,000 ohms?

2-1.37 The secondary voltage of a transformer dropped from 650 volts no-load voltage to 610 volts under full load. What is the voltage regulation of the transformer?

2 *Direct-Current Circuits*

2-2.01 A light bulb is marked: 110 volts, 60 watts. What current will it draw from the line?

2-2.02 A resistance cord of 350 ohms is used as a dropping resistor in a 150-milliampere-filament string. What is the voltage drop of the cord and what is the combined drop of the filaments?

2-2.03 What is the resistance of the bulb in problem 2-2.01?

2-2.04 A toaster element draws 2.5 amperes from a 117-volt line. What are the resistance and its power consumption?

2-2.05 A 0–1-milliampere d'Arsonval meter with an internal resistance of 32 ohms is used to make voltmeter reading 150 volts full scale. What is the value of the series dropping resistor?

2-2.06 A bleeder resistor draws 7.5 milliamperes from a 275-volt power supply. Find the resistance and the power dissipation.

2-2.07 In a radio-frequency amplifier stage having a plate voltage of 1250 volts, a plate current of 150 milliamperes, a grid current of 15 milliamperes, and a grid-leak resistance of 7500 ohms, what is the value of the operating grid bias?

2-2.08 Find the total resistance of 25, 45, and 60 ohms, all connected in parallel.

2-2.09 Four resistances of 5, 7, 9, and 11 ohms are connected in series across a 4-cell Edison battery. Find the voltage across each resistor (Voltage of 1 cell is 1.37 volts).

2-2.10 Find the value of the resistors of the voltage divider of Figure 2.15 for negligible current drain by the load and a current drain of 12 milliamperes by the bleeder.

2-2.11 In Figure 2.19a, $R_1 = 50$ ohms, $R_2 = 4$ ohms, $R_3 = 8$ ohms, $R_4 = 5$ ohms, $R_5 = 10$ ohms, $R_6 = 3$ ohms. Find the voltage across R_5.

2-2.12 In the unbalanced-bridge circuit of Figure 2.22a, $R_1 = 1$ ohm, $R_2 = 3$ ohms, $R_3 = 3$ ohms, $R_4 = 5$ ohms, $R_5 = 6$ ohms. Assuming an electromotive force of 100 volts, find the total resistance, the total current, and the current through R_4.

2-2.13 In problem 2-2.12 find the total current with the aid of a delta-star transformation. Also find the current through the shunt resistor R_4.

2-2.14 In the circuit of Figure 2.24a, $E = 100$ volts, $R_1 = 4$ ohms, $R_2 = 5$ ohms, $R_3 = 6$ ohms, $R_4 = 7$ ohms. Find I'.

2-2.15 Find the current through R_4 in problem 2-2.14 with the aid of Thévenin's theorem.

2-2.16 Solve problem 2-2.12 with the aid of Thévenin's theorem.

3 *Alternating-Current Circuits*

2-3.01 What is the wavelength of a 750-kilocycle signal; of a 3.5-megacycle signal?

2-3.02 What is the period of a 3.5-meter wave?

2-3.03 What is the instantaneous value of an alternating current at 260° of its cycle? The peak value is 7 amperes.

2-3.04 What is the instantaneous value of a 60-cycle alternating voltage of 90 volts peak value 3000 microseconds after the beginning of the cycle?

2-3.05 A full-wave rectified pulsating unidirectional voltage has a peak value of 295 volts. Calculate its d-c component.

2-3.06 What is the effective voltage resulting from a 50-volt d-c source and a 120-volt 60-cycle alternating voltage in series?

2-3.07 In a capacitive-resistive circuit the current leads the voltage by 45 degrees. The frequency is 60 cycles and the peak values of voltage and current are 100 volts and 2 amperes, respectively. Write the equations of the current and the voltage, indicating the phase difference. At the time when the voltage is 75 volts, what is the current?

2-3.08 What is the line current of a single-phase, 7-horsepower alternating-current motor, when operating from a 120-volt line at full rated load and at a power factor of 0.8, and 95 per cent efficiency?

2-3.09 A voltge of 60 volts at 120 cycles exists across a series circuit consisting of a capacitor of 5 microfarads and a resistor of 200 ohms. What is the power dissipated by the resistor?

2-3.10 A current of 600 milliamperes is flowing in a series circuit of 375 ohms resistance and unknown inductance. The voltage across the circuit is 250 volts. What is the value of the inductance if the frequency is 120 cycles?

2-3.11 A series circuit consists of a 0.75-megohm resistor, a 5-micromicrofarad capacitor, and a 150-millihenry inductor. A radio-frequency signal of 750 kilocycles is applied across the circuit, producing a voltage of 5 volts across the resistor. What is the voltage of the applied signal?

2-3.12 In Figure 3.13, R_1 = 400 ohms, R_2 = 310 ohms, the reactances of the other circuit components are:

$$X_{c_1} = 25 \text{ ohms}, \quad X_{l_1} = 900 \text{ ohms},$$

$$X_{c_2} = 700 \text{ ohms and } X_{l_2} = 60 \text{ ohms}.$$

Find the voltage across the 900-ohm inductive reactance.

2-3.13 In problem 2-3.12 find the phase difference between the voltage and the current, and the power factor of the circuit.

2-3.14 In Figure 3.15, C = 6 microfarads, R = 600 ohms, f = 50 cycles. Find the equivalent series circuit and the power factor.

2-3.15 In Figure 3.16, X_c = 6000 ohms, X_l = 5500 ohms, I = 12 microamperes. Find the total impedance and the voltage E.

2-3.16 In Figure 3.17, L = 20 millihenries, R = 60,000 ohms; the frequency f = 650 kilocycles. Find the equivalent series circuit and the power factor.

2-3.17 In Figure 3.18, L = 6 henries, R = 6000 ohms, C = 1 microfarad, and f = 240 cycles. Find the equivalent series circuit, the absolute value of the impedance, and the phase angle.

2-3.18 In Figure 3.20, C = 400 micromicrofarads, R_1 = 60 ohms, L = 85 microhenries, R_2 = 65 ohms, and f = 1.7 megacycles. Find the equivalent series circuit, the absolute value of the impedance, and the phase angle.

2-3.19 In Figure 3.21, $L_1 = 0.003$ henry, $L_2 = 0.004$ henry, $L_3 = 0.002$ henry, $R_1 = 500$ ohms, $R_2 = 300$ ohms, $C_4 = 0.02$ microfarad, $C_5 = 0.04$ microfarad, $E = 120$ volts at 7500 cycles. Find the total current I and the power factor.

2-3.20 A series resonant circuit consists of 45 millihenries inductance and 0.006 microfarad capacitance. What is the resonant frequency?

2-3.21 A series resonant circuit tuned to 16,000 kilocycles has a resistance of 50 ohms and an inductance of 75 microhenries. A voltage of 8 volts is applied across the circuit. Calculate the voltage across the resistor and the capacitor.

2-3.22 What capacitance will tune to resonance with a 25-henry choke at 60 cycles?

2-3.23 An inductor of 60 microhenries has a figure of merit of $Q = 22.5$ at 4 megacycles. Find the effective resistance of the coil at this frequency?

2-3.24 A tank circuit consists of an inductance of 25 microhenries, a capacitance of 60 micromicrofarads, and a coil resistance of 30 ohms. What is the resonant frequency, the impedance at resonance, and the Q of the circuit?

2-3.25 A radio-frequency voltage of 60 volts exists across the capacitor plates of a tank circuit, and a current of 0.001 ampere is flowing from and to the tank; the 75-micromicrofarad capacitor is. tuned to 3.5 megacycles. What is the Q of the circuit?

2-3.26 A tank circuit tuning to 7.5 megacycles has an impedance of 85,000 ohms. The capacitance is 15 micromicrofarads. What are the effective resistance of the inductor and the Q of the circuit?

2-3.27 An inductance of 75 microhenries with an effective resistance of 30 ohms is to be tuned to a frequency of 3500 kilocycles. What capacitor must be used? Calculate the circuit Q and the total impedance of the circuit.

2-3.28 An inductance of 100 microhenries is used with a 40-micromicrofarad capacitance in a parallel-resonant circuit. The resistance of the coil is 30 ohms. The tank circuit is loaded with 7500 ohms resistance. Find the total impedance and the Q of the circuit with and without load.

2-3.29 A loaded parallel-resonant circuit, tuned to 8.5 megacycles, has a total impedance of 10,000 ohms and an over-all $Q = 10$. Calculate the coil resistance and the value of the inductance and the capacitance.

2-3.30 Design a filter to reject all frequencies lower than 300 cycles, to work into a 750-ohm terminal impedance.

2-3.31 Design a filter with a terminal impedance of 3500 ohms to reject all frequencies lower than 5 megacycles.

2-3.32 Design a tone control to attenuate all frequencies higher than 9000 cycles, working into a 7500-ohm impedance.

2-3.33 Design a wave filter to pass only 550 to 590 kilocycles, to work into an impedance of 25,000 ohms.

2-3.34 A local station, 850 to 860 kilocycles, is to be rejected by a filter working into a tuned circuit with an impedance of 20,000 ohms. Design the band-elimination filter.

4 *Vacuum-Tube Fundamentals*

2-4.01 Calculate the thermionic emission of an oxide-coated cathode for a temperature of 1100K, with material constants $A = 0.006$, and $b = 11{,}500$.

2-4.02 The following readings were observed in a space-charge limited 2-electrode tube:

$$I = 60 \text{ milliamperes}, \ E_p = 75 \text{ volts}.$$

What current can be expected when a plate voltage of 100 volts is applied?

2-4.03 The triode type 6P5 has the following typical operation data, obtained from the information of the manufacturer:

plate voltage	100 volts
plate current	2.5 milliamperes
grid voltage	−5 volts
amplification factor	13.8

If the plate voltage were increased to 125 volts, what plate current could be expected?

2-4.04 The circuit of Figure 4.04 is used to examine the tube T. It is found that with a grid voltage of -7 volts and a plate voltage of 225 volts, the milliammeter reads 8 milliamperes; an increase of the plate voltage to 260 volts causes a rise of the plate current to 11 milliamperes. The plate voltage is then decreased to its original value and the grid bias is changed to -5 volts. The milliammeter again reads 11 milliamperes. What is the amplification factor of the tube?

2-4.05 In problem 2-4.04, what mutual conductance would be indicated with the readings taken?

2-4.06 An input voltage of 2 volts peak is impressed on the grid of the tube in problem 2-4.04. If a plate resistor of 6000 ohms is used in the plate circuit, what is the output voltage?

2-4.07 What is the voltage amplification in problem 2-4.06?

2-4.08 What would be the voltage amplification in problem 2-4.06 if the plate resistor were 60,000 ohms?

2-4.09 Using the $E_p I_p$ characteristics of the type 6C5 of Figure 4.11, find the x and y intercepts of the load line for a plate load of $R = 18{,}000$ ohms, and plot the load line by joining these points. The supply voltage is 250 volts.

2-4.10 In problem 2-4.09, we decide to choose -6 volts bias as the operating point. What is the plate voltage at zero signal? What is the plate current at zero signal?

2-4.11 Assuming a signal of 2 volts peak at the grid in problem 2-4.09, what would be the maximum and minimum values of the plate voltage?

2-4.12 What is the direct plate voltage of a resistance-coupled amplifier stage which has a plate-supply voltage of 210 volts, a plate current of 1 milliampere, and a plate-load resistance of 80,000 ohms?

2-4.13 In a radio-frequency amplifier stage having a plate voltage of 1250 volts, a plate current of 150 milliamperes, a grid current of 12 milliamperes, and a grid-leak resistance of 5000 ohms, what is the value of the operating grid bias?

2-4.14 The following operating data for type 6L6 tube are taken from the tube manual: $E_p = 350$ volts, $E_{sc} = 250$ volts, $E_g = -18$ volts, $I_p = 54$ milliamperes, $I_{sc} = 2.5$ milliamperes, maximum power output 10.8 watts, r_p = 33,000 ohms, g_m = 5200 micromhos, R_p = 4200 ohms. Only 280 volts of plate voltage are available. Calculate the other operating data.

2-4.15 From the tube manual the following data are taken for the triode type 6C5: E_p = 250 volts, E_g = −8 volts, I_p = 8 milliamperes, r_p = 10,000 ohms, amplification factor = 20, g_m = 2000 micromhos. Calculate the operating data for E_p' = 180 volts.

2-4.16 A power-amplifier tube has a transconductance of 1500 micromhos, a plate resistance of 5000 ohms, a load resistance of 12,000 ohms. What will be the power output at an input of 32 volts peak?

2-4.17 In problem 2-4.16, what should be the value of the plate load resistor for maximum output and what would be the maximum power output in watts?

2-4.18 In problem 2-4.16, what should be the value of the plate load resistor for maximum undistorted power and what will be the power output in this case?

2-4.19 A power tube has a power output of 5.2 watts while the plate current and plate voltage readings are 275 volts and 55 milliamperes, respectively. What is the plate efficiency?

2-4.20 The direct-current input power to the final amplifier stage is exactly 1600 volts and 750 milliamperes. The antenna resistance is 8.5 ohms and the antenna current is 9.5 amperes. What is the plate efficiency of the final amplifier?

2-4.21 What is the stage amplification obtained with a single triode operating with the following constants:

plate voltage	250	volts
plate current	20	milliamperes
plate impedance	6000	ohms
load impedance	13,000	ohms
grid bias	5.4	volts
amplification factor	19	

5 *Amplifiers*

2-5.01 In the resistance-coupled amplifier stage of Figure 5.01a, R_p = 300,000 ohms, C = 0.01 microfarad, R_g = 0.75 megohm, the effective shunt capacitance C_s = 75 micromicrofarads, the plate resistance r_p = 250,000 ohms, and the amplification factor μ = 70. Calculate the voltage amplification at a frequency of 1000 cycles per second. Do not use formulas. Neglect the effect of cathode resistor and capacitor.

2-5.02 Calculate the voltage amplification of the resistance-coupled amplifier stage in problem 2-5.01 at a frequency of 10,000 cycles.

2-5.03 Calculate the voltage amplification of the circuit of problem 2-5.01 at 50 cycles.

2-5.04 Using the formula derived in problem 5.04, calculate the medium-frequency response of the resistance-coupled amplifier stage, problem 2-5.01.

2-5.05 Using the formula derived in problem 5.06, calculate the high-frequency response of the resistance-coupled amplifier stage, problem 2-5.01.

2-5.06 Using the formula derived in problem 5.08, calculate the low-frequency response of the resistance-coupled amplifier stage, problem 2-5.01.

2-5.07 A radio receiver is inoperative because of an open coupling capacitor, the rating of which cannot be read nor measured. If the ohmmeter registers 500,000 ohms grid-leak resistance, what would be an adequate value of coupling capacitance?

2-5.08 While looking for the reason for distortion in a radio receiver, it is found that the audio stage has a leaky coupling capacitor with a leakage resistance of 15 megohms. The plate resistor is 0.25 megohm, and the grid-leak resistor is 1 megohm. What is the value of the positive voltage caused by this leakage at the grid of the tube V_2 in Figure 5.11, if the plate supply is 250 volts?

2-5.09 A transmitting triode requires a grid bias of −75 volts. If this voltage is obtained by the grid-leak resistor, and the direct grid current is 4.2 milliamperes, what will be the value of the resistance and what power rating will be adequate?

2-5.10 A power-amplifier triode requires a grid voltage of −45 volts, obtained by means of the voltage drop through a resistor in the plate return lead. The plate current is 42 milliamperes. Find the value of the bias resistor, the by-pass capacitor and determine practical power and voltage ratings, respectively.

2-5.11 A preamplifier having a 600-ohm output is connected to a microphone so that the power output is −35 decibels. Assuming the mixer system to have a loss of 12 decibels, what is the voltage amplification necessary in the line amplifier in order to feed +10 decibels into the transmitter line?

2-5.12 If the power output of a modulator is decreased from 1200 watts to 20 watts, how is the power loss expressed in decibels?

2-5.13 If 100 per cent modulation is obtained with an input level of 75 decibels, what percentage of modulation will be obtained with an input level of 55 decibels?

2-5.14 If an audio-frequency amplifier has an over-all gain of 45 decibels and the output is 7.5 watts, what is the input?

2-5.15 Which of the two operating conditions of a triode works with a greater power sensitivity?

1) Plate voltage 220 volts, grid signal rms 25 volts, output 1.2 watts.
2) Plate voltage 300 volts, grid signal rms 45 volts, output 2.2 watts.

2-5.16 Which of the two operating conditions of a pentode works with a greater sensitivity?

1) $E_p = 260$ volts, $E_g = 18$ volts, $P = 3.5$ watts,
2) $E'_p = 310$ volts, $E'_g = 22$ volts, $P' = 4.5$ watts.

2-5.17 From the load line drawn in problem 4.10 calculate the power output of the amplifier working with a zero-signal bias of −5 volts.

2-5.18 What is the second-harmonic distortion of the class A amplifier working under the conditions of problem 2-5.17?

2-5.19 The circuit of Figure 5.23 represents the equivalent circuit of a transformer-coupled audio-frequency amplifier, where R_1 is the plate resistance plus primary resistance, L_1 the primary inductance, L_2 the secondary leakage inductance, R_2 the reflected secondary resistance, C the total shunt capacitance. If $R_1 = 12{,}500$ ohms, $L_1 = 42$ henries, $L_2 = 0.45$ henry, $R_2 = 1200$ ohms, $C = 1300$ micromicrofarads, calculate the voltage amplification at 1000 cycles, assuming a 1 to 1 transformer ratio and an amplification factor of 16.

2-5.20 In Figure 5.23, problem 5.23, calculate the voltage amplification at 50 cycles.

2-5.21 What will be the voltage amplification of the transformer-coupled amplifier, the equivalent circuit of which is shown in Figure 5.23, problem 5.23, at a frequency of 5000 cycles?

2-5.22 The circuit of Figure 5.26 represents the equivalent circuit of an impedance-coupled audio-frequency amplifier where R_p is the plate resistance, L is the coupling inductance, C is the coupling capacitance, R_g is the grid resistance, C_s is the shunt capacitance (combined output capacitance of the first tube, input capacitance of the second tube, and distributed capacitance of the winding). If $r_p = 100{,}000$ ohms, $L = 350$ henries, $C = 0.05$ microfarad, $R_g = 0.75$ megohm, and $C_s = 300$ micromicrofarads, calculate the voltage amplification at 1000 cycles and an amplification factor $\mu = 60$.

2-5.23 Express the voltage amplification of problem 2-5.22 in decibels.

2-5.24 An amplifier has an over-all voltage amplification of 4000. The input voltage is obtained across a 0.75-megohm resistor, the output voltage is fed into a 610-ohm transmission line. What is the power gain in decibels?

2-5.25 Find the value of the cathode by-pass capacitor to shunt a 950-ohm resistor

1) for broadcast frequencies (550 kilocycles),
2) for audio frequencies (400 cycles).

2-5.26 A plate tank circuit tuned to 2500 kilocycles is shunt-fed from a power supply through a radio-frequency choke coil. The impedance of the tank circuit is 6000 ohms. Disregarding the distributed capacitance of the choke coil, what should be its inductance?

2-5.27 A power-amplifier pentode works with a grid bias of -16 volts while a screen current of 6.2 milliamperes and a plate current of 40 milliamperes are flowing. What is the value of the cathode resistor and what is its rating?

2-5.28 What should be the value of the cathode resistor in problem 2-5.27 if 2 tubes are used in push-pull?

2-5.29 A pentode amplifier is operated at a plate voltage of 260 volts, a screen voltage of 110 volts, a plate current of 9.6 milliamperes and a screen current of 2.1 milliamperes. What are the resistance and the power rating of the screen dropping resistor if the screen and the plate obtain their voltage from the same supply?

2-5.30 A transmitting triode when used as a class A audio-frequency amplifier has an undistorted power output of 4 watts when the direct current is 32 milliamperes and the peak audio-frequency grid voltage is 52 volts. When used as a class B push-pull amplifier the power output is 46 watts, while the direct plate current is 135 milliamperes and the peak radio-frequency grid voltage is 325 volts; when operated as a class C radio-frequency amplifier the power output is 26 watts, the direct plate current is 68 milliamperes and the peak radio-frequency grid voltage is 250 volts. The applied plate voltage is 585 volts throughout. Calculate the plate efficiencies.

2-5.31 What is the maximum permissible rms value of audio-frequency voltage which can be applied to the grid of a class A audio-frequency amplifier which has a grid bias of 12.5 volts.

2-5.32 Two beam power tubes are operated as a push-pull class AB_2 amplifier. The manufacturer recommends a plate voltage of 350 volts, a screen voltage of 260 volts, a peak signal input of 68 volts. A power pentode tube is used for the driver stage, triode-connected. It operates at 240 volts plate voltage with a recommended plate resistance of 4000 ohms and a power output of 0.82 watts. What is the turns ratio of the input transformer?

2-5.33 The amplification factor of the duo-triode tube used in the circuit of Figure 5.37a is 35, the plate resistance 11,000 ohms. The tube is used as a phase inverter. Where should the 0.5-megohm resistor be tapped to provide a balanced signal input to the push-pull circuit if the plate load resistor is 200,000 ohms?

2-5.34 The plate dissipation of a power-amplifier tube is 12 watts. Can two tubes be used for a class B audio amplifier with a power output of 40 watts? What is the maximum power obtainable if an efficiency of 45 per cent is realized?

2-5.35 What is the correct value of negative grid bias for operation as a class B amplifier for a vacuum tube of the following characteristics: plate voltage 1200, plate current 127 milliamperes, filament voltage 4 volts, filament current 5.4 amperes, mutual conductance 8000 micromhos, and amplification factor 35?

2-5.36 A triode transmitting tube operating with a plate voltage of 1175 volts has a filament voltage of 10, a filament current of 3.25 amperes, and a plate current of 150 milliamperes. The amplification factor is 28. What value of control-grid bias must be used for operation as a class C amplifier?

2-5.37 One set of operating conditions of a radio-frequency amplifier pentode is as follows:

direct plate voltage	1500	volts
direct grid voltage	−90	volts
radio-frequency grid voltage	190	volts peak
direct plate current	160	milliamperes
direct screen current	15	milliamperes
direct grid current	27	milliamperes
power output	160	watts
cutoff bias	−18	volts

What is the total space current flowing away from the filament?

2-5.38 In problem 5-5.37, what is the minimum permissible value of the direct plate voltage during the active part of the cycle, and what is the output rms signal?

2-5.39 In problem 5-5.37, calculate the angle in degrees during which the tube is conducting under the given operating conditions.

2-5.40 In problem 5-5.37, what is the average grid impedance exhibited by the tube under the given operating conditions?

2-5.41 The following typical operation is given in the tube manual for the radio-frequency amplifier type 808:

direct plate voltage	1250	volts
direct grid voltage	−225	volts
radio-frequency grid voltage	360	volts peak
direct plate current	100	milliamperes
direct grid current	32	milliamperes
grid resistor	7000	ohms
driving power	10.5	watts
power output	105	watts

Assuming that the circuit $Q = 10$, calculate the inductance and capacitance necessary to tune the circuit to 4.5 megacycles.

2-5.42 Given a class C amplifier with a plate voltage of 1200 volts and a plate current of 160 milliamperes which is to be modulated by a class A amplifier with a plate voltage of 2000 volts, plate current of 200 milliamperes, and a plate resistance of 17,000 ohms; what is the proper turns ratio for the coupling transformer?

6 *Oscillators*

2-6.01 An inductance of 32 microhenries is used for a Hartley-type vacuum-tube oscillator, tuned to a frequency of 6500 kilocycles. Find the value of the capacitor.

2-6.02 An inductor of 32 microhenries is used for a Colpitts oscillator. A fixed capacitor of 25 micromicrofarads is available. What is the value of the other capacitor required to tune the circuit to 7500 kilocycles?

2-6.03 The circuit of Figure 6.03 is the equivalent circuit of a tuned-plate oscillator. The inductance is 80 microhenries with an effective resistance of 60 ohms. The capacitance is 120 micromicrofarads and the plate resistance is 8000 ohms. Find the value of the resonant frequency

(a) neglecting the plate resistance and the coil resistance,
(b) including the plate resistance and the coil resistance.

2-6.04 In problem 2-6.03, by what per cent does the frequency calculated by including the plate resistance and the coil resistance exceed the frequency found by the simpler formulas?

2-6.05 Figure 6.06 represents the equivalent circuit of a crystal-controlled oscillator. If $L = 4$ henries, $C = 6$ micromicrofarads, $R = 7000$ ohms and $C' = 0.06$ micromicrofarad, what change in frequency could be accomplished by connecting a 10-micromicrofarad capacitor from the grid terminal to the cathode terminal of the crystal?

2-6.06 Calculate the approximate Q of the crystal described in problem 2-6.05.

2-6.07 An X-cut crystal having a temperature coefficient of -16 parts per million per degree centigrade is rated 212.65 kilocycles at

18C. What will be the change in cycles if the temperature falls to 16C?

2-6.08 A transmitter is operating on 6000 kilocycles, using a 1200-kilocycle crystal with a temperature coefficient of −4 cycles per megacycle per degree centigrade. If the crystal temperature increases 7.5 degrees centigrade, what is the change in the output frequency of the transmitter?

2-6.09 The radio-frequency amplifier in Figure 6.11 has a grid input impedance of 5500 ohms to V_2 and a load of 700 ohms for V_1. What is a suitable value for the coupling capacitor if the circuit is tuned to 7 megacycles?

2-6.10 Disregarding the method of coupling, what is the value of the inductance and capacitance in both the grid and the plate circuits, if a Q of 10 is provided for each circuit of problem 2-6.09?

2-6.11 From the tube manual we obtain the following typical operating conditions for the radio-frequency power amplifier and oscillator triode type 801:

direct plate voltage	500	volts
direct grid voltage	−125	volts
peak radio-frequency grid voltage	235	volts
direct plate current	65	milliamperes
direct grid current	15	milliamperes
grid resistor	8300	ohms
driving power	3.5	watts
power output	20	watts

Calculate the values of the capacitance and inductance of a Hartley oscillator tuned to a frequency of 1600 kilocycles with a circuit $Q = 35$, working as close as possible under the typical operating conditions.

7 *Transmitters*

2-7.01 If a transmitter is modulated 90 per cent by a sinusoidal tone, what percentage increase in antenna current will occur?

2-7.02 A ship's transmitter has an antenna current of 8 amperes using A1 emission. What would be the antenna current when this transmitter is 90 per cent modulated by sinusoidal modulation?

2-7.03 The direct-current plate input to a modulated class C amplifier, with an efficiency of 65 per cent, is 225 watts. What value of sinusoidal audio-frequency power is required in order to insure 100 per cent modulation, 50 per cent modulation?

2-7.04 Which of the two signals is more effective:

(a) a carrier amplitude of 70 volts, 80 per cent modulated, or
(b) a carrier amplitude of 56 volts, 100 per cent modulated?

2-7.05 What is the total bandwidth of a transmitter using A2 emission with a modulating frequency of 750 cycles and a carrier frequency of 1750 kilocycles? What are the upper and the lower frequencies?

2-7.06 In 90 per cent amplitude modulation, what is the ratio of peak antenna current to unmodulated antenna current?

2-7.07 In 90 per cent modulation, what is the ratio of instantaneous peak antenna power to unmodulated antenna power?

2-7.08 At 90 per cent modulation, what percentage of the total output power is in the sidebands?

2-7.09 If a vertical antenna has a resistance of 400 ohms and a reactance of zero at its base, and an antenna power input of 12 kilowatts, what is the peak voltage to ground under conditions of 100 per cent modulation?

2-7.10 A transmitter has an output of 150 watts. The efficiency of the final modulated-amplifier stage is 55 per cent. Assuming that the modulator has an efficiency of 66 per cent, what plate input to the modulator is necessary for 100 per cent modulation of this transmitter?

2-7.11 The output current of a transmitter is 10 amperes when the unmodulated carrier is radiated. With modulation applied the current increases to 17.5 amperes. What is the per cent modulation?

2-7.12 The 60-kilowatt output stage of a broadcast transmitter having a final amplifier efficiency of 33 per cent, has a plate current of 12 amperes. If the water-cooling system leakage-current meter reads 13 milliamperes, what is the resistance of the water system from plate to ground?

2-7.13 A 55-kilowatt transmitter employs 6 tubes in push-pull parallel in the final class B linear stage, operating with a 55-kilowatt output and an efficiency of 35 per cent. Assuming that all of the heat radiation is to the water-cooling system, what amount of power must be dissipated from each tube?

2-7.14 A 1700-kilocycle carrier is modulated by a 1000-cycle note. If the note contains harmonics up to the 6th, what are the frequency ranges of interference?

2-7.15 A transmitting tube is operated at a plate voltage of 1800 volts and a plate current of 75 milliamperes. Calculate the power which must be supplied by the modulator for a 100 per cent sinusoidal plate modulation.

2-7.16 A power amplifier operates class C with a plate voltage of 1350 volts. a direct plate current of 175 milliamperes and a d-c grid bias of -150 volts. What is the modulation impedance for these conditions?

2-7.17 A class C amplifier operating at a plate voltage of 2200 volts is to be modulated by a class B modulator with a plate supply of 1350 volts, a rated power output of 310 watts, and a rated plate-to-plate load of 6600 ohms. If the coupling is accomplished by an output transformer, calculate its proper turns ratio for 100 per cent modulation.

2-7.18 A station has an assigned frequency of 2400 kilocycles and a frequency tolerance of plus or minus 0.05 per cent. The oscillator operates at $\frac{1}{8}$ the output frequency. What is the maximum permitted deviation of the oscillator frequency, in cycles, which will not exceed the tolerance?

2-7.19 A radio-frequency amplifier tube has a rated plate dissipation of 45 watts. If used for grid modulation what will be the approximate unmodulated radio-frequency output for the tube?

2-7.20 Using the data in problem 2-7.19, what would be the approximate carrier power that can be expected for plate modulation?

2-7.21 A frequency-modulated carrier operates between 92,750 and 92,790 kilocycles. What is the carrier frequency and what is the frequency deviation?

2-7.22 A 45-megacycle carrier is frequency-modulated so as to have a channel width of 100 kilocycles. What will be the modulation index

(a) for a 500-cycle note, (b) for 3200-cycle telephone service (c) for 10,000-cycle music?

2-7.23 The phase-splitting circuit *RC* of the reactance-tube modulator in Figure 7.27 is to be designed so that the circuit is practically purely resistive and the carrier through *C* leads the voltage by no more than 0.75 degree. If $C = 25$ micromicrofarads and the oscillator operates at a frequency of 6 megacycles, what should be the value of *R*?

2-7.24 A frequency-modulation transmitter is to operate at a frequency of 56.8 megacycles with a bandwidth of 100 kilocycles. If the output frequency is the result of a frequency multiplication by 8, what frequency deviation should the reactance-tube oscillator be able to produce, and to which frequency will its tank circuit be tuned?

2-7.25 In problem 2-7.24, find the highest audio frequency for which the reactance-tube modulator exhibits a modulation index of $m = 5$?

2-7.26 What should be the frequency limits of a 56.8-megacycle frequency-modulation broadcast station if it is desired to obtain a minimum modulation index of $m = 5$?

2-7.27 Show why it is impractical to operate a frequency-modulation station in the regular broadcast band, illustrating the problem for the case of a station with an assigned frequency of 1050 kilocycles.

8 *Receivers*

2-8.01 In the tuning circuit of Figure 8.01, $C_1 = 60$ micromicrofarads, $C_2 = 30$ micromicrofarads, $C = 15$ to 100 micromicrofarads, $L = 60$ microhenries. What is the frequency range of the tuning circuit?

(a) with the capacitance *C* only
(b) with *C* and C_2 in the circuit, and C_1 short-circuited
(c) with all three in the circuit

2-8.02 A superheterodyne receiver works with intermediate-frequency stages tuned to 175 kilocycles. If a signal of 1550 kilocycles is to be received, what could be the frequency of the oscillator, and what would be the image frequency of the lower of the two possible oscillator frequencies?

2-8.03 If a superheterodyne receiver is tuned to a desired signal at 1200 kilocycles, and its conversion oscillator is operating at 1656 kilocycles, what would be the frequency of an incoming signal which would possibly cause image reception?

2-8.04 A superheterodyne receiver, having an intermediate frequency of 456 kilocycles and tuned to a broadcast station on 1100 kilocycles, is receiving severe interference from an image signal. What is the frequency of the interfering station?

2-8.05 A superheterodyne receiver is tuned to 1410 kilocycles and the intermediate frequency is 176 kilocycles. What is the frequency of the mixer-oscillator?

2-8.06 If a superheterodyne receiver is receiving a signal of 610 kilocycles and the mixing oscillator is tuned to 1066 kilocycles, what is the intermediate frequency?

2-8.07 The oscillator circuit of a broadcast receiver, operating at an intermediate frequency of 176 kilocycles, is capacitance-tuned and the maximum capacitance is 365 micromicrofarads. What is the inductance of the oscillator coil, and what is the lowest capacitance of the variable capacitor?

2-8.08 The diode detector of Figure 8.10 has a plate resistance of 3000 ohms; the input peak signal E_i = 15 volts. If a load resistor of 0.75 megohm is used, the ammeter reads 18 microamperes. What is the efficiency of the detector?

2-8.09 In problem 2-8.08, what is the power which the diode draws from the input circuit?

2-8.10 What is the equivalent input resistance of the diode in problem 4-8.08?

2-8.11 The antenna-bias-type volume control of Figure 8.15 has a value of 15,000 ohms and is variable only from A to B. The 6K7 tube is operated under the following conditions: E_p = 180 volts, E_s = 75 volts, I_p = 4 milliamperes, I_s = 1 milliampere, and E_g = −3 volts. Calculate the value of the variable range of the tapped volume control. If this type of volume control were not available for replacement, what would be a fair value for R_1 and R_{ab} employing two separate parts.

2-8.12 In the tuning-eye circuit of Figure 8.17, R_g = 2 megohms, I_p = 240 microamperes, R_p = 1 megohm, $B+$ = 250 volts and E_g = 0.1 volt. What is the potential difference between the ray-control electrode and the target?

2-8.13 An amplifier has an input signal of 3.5 volts and an output of 40 volts; 1/45 the output is fed back to the input in opposite phase to the input. What is the new gain; what is the gain-reduction factor?

2-8.14 In problem 2-8.13, if 8 per cent harmonic distortion was present without negative feedback, how much harmonic distortion is present with feedback?

2-8.15 An amplifier has a voltage input of 8 volts peak and a voltage output of 146 volts for the audio-frequency band, except in the region of 3500 cycles where the output is 250 volts for the same signal input. If 5 per cent of the output voltage is applied to the input in opposite phase, what is the frequency distortion with and without feedback?

2-8.16 In the circuit of Figure 8.21, R_1 = 75,000 ohms, R_2 = 3000 ohms. Calculate the feedback factor β.

2-8.17 In the circuit of Figure 8.22a, C_1 and C_2 are large enough so as to offer negligible reactance at audio frequencies; r_p = 120,000 ohms, R_L = 0.25 megohms, R_g = 0.75 megohms. What value of the feedback resistor R_f will provide 10 per cent negative feedback?

2-8.18 In the circuit of Figure 8.23, R_1 = 75,000 ohms, R_f = 5000 ohms; the coupling capacitor C is large enough to offer only negligible reactance at audio frequencies. Calculate the feedback factor β.

2-8.19 In the circuit of Figure 8.27a, R_p = 350 ohms, R_s = 0.9ohms, R_{vc} = 6 ohms, the step-down ratio is 20 to 1. What part of the total power output of 6 watts will be transferred to the loudspeaker?

2-8.20 In problem 2-8.19, what is the efficiency of the output transformer?

9 *Power Supplies*

2-9.01 The rectifier type 25Z5 is used in the rectifier circuit of Figure 9.01a. The line voltage is 117 volts, 60 cycles; L = 20 henries, C_1 = C_2 = 20 microfarads. The resistance of the choke is 50 ohms.

The d-c load draws 50 milliamperes from the rectifier. What is the approximate ripple voltage at $A\ B$; at $C\ D$?

2-9.02 The rectifier type 80 is used in the rectifier circuit of Figure 9.03a. The line voltage is 117 volts, 60 cycles; the secondary high voltage is 800 volts from plate to plate; $L = 15$ henries, 2000 ohms, $C_1 = C_2 = 12$ microfarads. The d-c load draws 80 milliamperes from the rectifier. What is the approximate ripple voltage at $A\ B$; at $C\ D$?

2-9.03 The rectifier circuit of Figure 9.06 is connected to a 60-cycle a-c line. The secondary voltage is 550 volts peak per plate. $L_1 = 15$ henries, 500 ohms, $L_2 = 15$ henries, 300 ohms, $C_1 = C_2 = 10$ microfarads. The equivalent load resistance across the output capacitor is 6000 ohms. Calculate the approximate per cent ripple across the input and the output capacitors. Do not use formulas.

2-9.04 Calculate the per cent ripple across the input and the output capacitors in problem 2-9.03 with the aid of the ripple-filter formulas.

2-9.05 A full-wave choke-input rectifier, working from a 60-cycle line is to deliver a direct output voltage of 280 volts and a current of 55 milliamperes. What is the critical inductance?

2-9.06 An audio-frequency amplifier has a plate supply of 350 volts and a current drain from the power supply of 90 milliamperes at full signal, and 20 milliamperes at no signal. Calculate the value of a bleeder resistor which will prevent the current from falling below 25 per cent of the maximum drain.

2-9.07 The rectifier circuit of Figure 9.10 has a choke resistance $R_{ch} = 250$ ohms and a load demand of $I_L = 120$ milliamperes. A type 5U4 rectifier tube is used; the following data are obtained from the manufacturer:

D-c load in milliamperes	Direct volts at A
0	630
30	590
120	510
150	490

Calculate the voltage regulation without and with a bleeder current of 30 milliamperes. Also find the resistance and the power dissipation of the bleeder resistor.

2-9.08 In problem 2-9.06, between what critical values should the swinging choke vary between the no-signal current and the full-signal current?

2-9.09 A choke-input, 2-section, full-wave rectifier works into an effective load of 5000 ohms. Neglecting the d-c drops, determine the first filter so as to provide a percentage ripple of 2 per cent. What should be the inductance of the second choke if both capacitors are equal and a final ripple of 0.02 per cent is required?

2-9.10 In Figure 9.13, the output voltage of the power supply E_{dc} = 300 volts. The two sections of the bleeder are: R_1 = 4000 ohms and R_2 = 3000 ohms. The reading of the ammeter is 60 milliamperes. What is the equivalent resistance of the load R_L?

2-9.11 The plate voltage of the input tube of a high gain public-address amplifier is obtained through a decoupling filter. The ripple frequency is 100 cycles. A resistor of 7500 ohms is used in the decoupling filter. Determine the value of the capacitor which will reduce the ripple voltage to 1 per cent of the value at the output of the power supply.

2-9.12 A power supply using mercury-vapor rectifier tubes works with a peak voltage of 2000 volts from plate to plate, and a load current of 180 milliamperes. A choke-input, 2-section filter is used, with choke resistances of 400 and 350 ohms. Calculate the direct output voltage.

2-9.13 A power supply using mercury-vapor rectifier tubes works into a load of 250 milliamperes. What transformer voltage is necessary to produce a direct output voltage of 450 volts, if a choke-input, 2-section filter is used with choke resistances of 350 and 300 ohms?

2-9.14 A universal-type superheterodyne receiver employs the following tubes: 6SK7, 6SA7, 6SK7, 6SQ7, 25L6 and 25Z5. The line voltage is 120 volts. Calculate the necessary value of the lamp-cord resistor R in series with the filament string, and the power lost in the cord resistor.

2-9.15 The following tubes are used in a tuned-radio-frequency receiver of the transformerless type: 6K7, 6K7, 6J7, 25L6 and 25Z5. Provisions are made for a type 46 pilot light (6.3 volts, 0.25 amperes).

Calculate the resistances and the power dissipations of the series dropping resistor R_s and the pilot shunt resistor R_p.

2-9.16 A dynamotor-type power supply of a sound-truck amplifier consumes 450 watts from two storage batteries connected in series, 6.3 volts each. The internal resistance of each battery is 0.01 ohm. What is the voltage at the input to the power supply while the amplifier is "on"?

2-9.17 In the filament circuit of problem 9.15, show how a burnt-out pilot lamp will affect the A-supply of the receiver.

2-9.18 A three-way portable receiver (a-c, d-c, battery) uses a series filament hookup of the following tubes: 1N5, 1A7, 1N5, 1H5, 1A5. The tubes are d-c heated from a 117Z6 tube. The current drain of the tubes is 0.05 amperes and the rectifier output voltage 120 volts. What is the value of the series dropping resistor and what is the wasted power?

2-9.19 A rectifier beam power amplifier type 32L7 is to be substituted by using a selenium rectifier type NC-5 and a simple beam power amplifier. Find the resistance and the wattage dissipation of the series filament-dropping resistor.

10 *Antennas and Transmission Lines*

2-10.01 If the "end effect" reduces the length of the antenna by 5 per cent, calculate the length of a Hertz antenna for 7 and for 30 megacycles.

2-10.02 What is the resonant frequency of a Hertz antenna 32.5 feet long?

2-10.03 What must be the height of a vertical radiator 1/2 wavelength high if the operating frequency is 2700 kilocycles?

2-10.04 If the antenna current is 8.2 amperes for 4.5 kilowatts, what is the current necessary for a power of 2 kilowatts?

2-10.05 What is the antenna current when a transmitter is delivering 1200 watts into an antenna having a resistance of 15 ohms?

2-10.06 If the reading of the ammeter connected at the base of a Marconi antenna is increased to 1.67 times its original value, what is the increase in output power?

2-10.07 If the day input power to a broadcast station antenna having a resistance of 16 ohms is 1800 watts, what would be the night input power if the antenna current were cut in half?

2-10.08 A 2-wire transmission line consists of No. 10 wire AWG; the distance between the centers is 8 inches. What is the characteristic impedance of the line?

2-10.09 The outer conductor of a coaxial transmission line consists of copper tubing 0.1 inch thick, with an outside diameter of 1.75 inches. The diameter of the inner conductor is 0.5 inch. What is the characteristic impedance of the line?

2-10.10 An antenna is being fed by a properly terminated 2-wire transmission line. The current in the line at the output end is 3.2 amperes. The surge impedance of the line is 610 ohms. How much power is being supplied to the line?

2-10.11 If the daytime transmission-line current of a 12-kilowatt transmitter is 13 amperes, and the transmitter is required to reduce to 7.5 kilowatts at sunset, what is the new value of transmission-line current?

2-10.12 The power input to a 67-ohm concentric transmission line is 4000 watts. What is the peak voltage between the inner conductor and the sheath?

2-10.13 A long transmission line delivers 12 kilowatts into an antenna; at the transmitter end the line current is 6 amperes and at the coupling house it is 5.7 amperes. Assuming the line current to be properly terminated and the losses in the coupling system negligible, what is the power lost in the line?

2-10.14 What is the attenuation in decibels of the transmission line in problem 2-10.08 for a frequency of 25 megacycles?

2-10.15 If the line in problem 2-10.14 were 950 feet long, what would be the efficiency of the transmission?

2-10.16 What is the attenuation of the line in problem 2-10.09 at 25 megacycles?

2-10.17 What is the line loss in decibels, if the line in problem 2-10.16 is 500 feet long?

2-10.18 How long at most can be the line in problem 2-10.16, if an efficiency of not less than 80 per cent is to be realized?

2-10.19 Find the series equivalent of the characteristic impedance of a line, the circuit constants of which at 1000 cycles were found to be: $R = 12$ ohms, $L = 3.6$ millihenries, $C = 0.01$ microfarad, $G = 0.3$ micromho.

11 *Television*

2-11.01 Disregarding the flyback time what is the speed of the scanning beam at the end of the beam if the picture frame is 10 inches wide and a 441-line system is used with 60 frames per second and interlaced scanning?

2-11.02 What is the number of picture elements scanned per second in problem 2-11.01, if each line contains 400 picture elements and 40 lines are inactive?

2-11.03 Prove the formula $r = 3.57\sqrt{h}$, where r is the radius of the horizon in kilometers, h the height in meters.

2-11.04 What area will be covered by a television antenna, located on a hill with an effective height of 1250 feet?

2-11.05 What is the maximum distance between two television-relay stations if one antenna has an effective height of 575 feet and the other 350 feet?

2-11.06 A cathode-ray tube has a beam length $L = 20$ centimeters, measured from the center of the deflecting plates to the screen, and a voltage $e_d = 600$ volts between the deflecting plates which are separated by a distance $h = 0.4$ centimeter and have a length of $l = 3$ centimeters. Find the deflection D at the fluorescent screen if the applied second anode voltage is $E = 10{,}000$ volts. Also find the deflection sensitivity for the stated operating conditions.

2-11.07 A cathode-ray tube has a beam length of $L = 20$ centimeters, measured from the center of the deflecting field to the screen, an axial length $l = 2.5$ centimeters of the region of the uniform magnetic field. Calculate the flux density that exists in the magnetic field when the beam is deflected a distance of $D = 8$ centimeters when a second anode voltage of $E = 6000$ volts is applied.

2-11.08 The total gain obtainable from a seven-stage multiplier such as is used in an image orthicon is 200 to 500. Calculate the secondary emission ratio of each stage corresponding to the two stated values of gain.

2-11.09 A person was tested and found to have a minimum viewing angle of 1.4 minutes. What is the smallest distance the person will be able to distinguish on a screen 40 feet away from his eye?

2-11.10 The test pattern of a television station indicates a vertical resolution of 350 picture elements. What is the critical viewing distance for a person whose minimum viewing angles is one minute if the screen is 13.5 x 18 inches?

2-11.11 A television raster contains 525 lines with a vertical blanking time of 0.05 V, with a tolerance of $+0.03$ and -0.0, where V is the time from the start of one field to the start of the next field. What is the critical distance for seeing the screen as a homogeneous rectangle of light if the minimum viewing angle is 1 minute and the screen is 13.5 x 18 inches?

2-11.12 The amplified synchronizing pulses of a television signal are fed in composite form to the differentiating circuit of Figure 11.19. If a system of 421 lines and 60 fields employing interlaced scanning is used, what will be the percentage output of the line-scanning signal, if the fundamental frequency only is considered?

2-11.13 The amplified synchronizing pulses of a television signal are fed in composite form to an integrating circuit (Figure 11.20). In a system employing 421 lines per frame and 60 fields per second, what will be the percentage output of the line-scanning signal, if the fundamental frequency only is considered?

2-11.14 Using the formulas 11.1 and 11.12 on charge and discharge of a capacitor in a resistance-capacitance circuit calculate the charge and discharge voltage across the capacitor e_c in terms of the applied voltage E after times of 1.5 RC, and 2.5 RC seconds, respectively.

2-11.15 Using the universal time-constant chart find the voltage of the integrated output Figure 11.20b at the peaks A, B, C, D, E, and F and at the troughs between these points. The input wave has the form of Figure 11.23a, but the duration of the narrow pulses is 0.5 RC and the duration of the wide pulses is 3 RC second.

2-11.16 Find the voltage of the differentiated output Figure 11.23c at the points A, A', etc. to FG'. The input wave has the form of

Figure 11.23a, but the duration of the narrow pulses is 0.5 RC and the duration of the wide pulses is 3 RC second.

2-11.17 In Figure 11.25 E_s = 10 volts, C_1 = 4 microfarads, C_2 = 9 microfarads. Find the voltage E_2 across the plates of the capacitor C_2.

2-11.18 In the electronic counter circuit Figure 11.26 the signal voltage E_s = 100 volts, C_1 = 0.06 microfarads and C_2 = 3 microfarads. Find the voltage that will be built up across C_2 after the source voltage E_s is applied twice by closing the key K.

2-11.19 Calculate the voltage built up across C_2 in problem 11.26 after 3 pulses and after 6 pulses.

2-11.20 Find the voltage increase across C_2 in problem 11.26 caused by the 8th pulse applied to the counting circuit.

2-11.21 A pulse counter of the type of Figure 11.26, problem 11.26 is to be designed to operate at a range where the incremental voltage is more than 1 per cent of applied pulse voltage. What is the permissible number of pulses?

2-11.22 Calculate the number of pulses necessary to build up 20 volts or slightly more across the capacitor C_2 in problem 11.26?

2-11.23 A television receiver employs a high-voltage bleeder circuit of the type of Figure 11.34, where R_1 = R_2 = 475,000 ohms, R_3 = 2.7 megohms, R_4 = 1 megohm, R_5 = 2.2 megohms and R_6 = R_7 = 1 megohm. The voltage at the cathode of the rectifier is +2600 volts. Find the variable ranges of voltage of the focus control and of the horizontal and vertical centering controls. Also find the potential difference of the focus control voltage and of the centering voltage with respect to ground when the controls are at midposition.

2-11.24 In the video amplifier of Figure 11.38a, R_p = 4800 ohms, C_c = 0.05 microfarad, R_g = 820 kilohms, C_i = 10 micromicrofarads, C_o = 5 micromicrofarads, L_p = 41 microhenries, L_c = 178 microhenries, the tube V_1 has a g_m = 5200 micromhos. Calculate the voltage gain at 4 megacycles.

2-11.25 In the cathode follower circuit of Figure 11.40a, R_k = 1750 ohms, r_p = 15,000 ohms, μ = 35. Find the stage gain of the circuit.

2-11.26 With the aid of the formulas derived in problem 11.41, find the gain and the output impedance of the cathode-follower circuit of problem 2-11.25.

2-11.27 The Fourier analysis of the saw-tooth wave in Figure 11.43 yields the following harmonic components:

$$y = \frac{2E}{\pi}\left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x \cdots\right)$$

Find the amplitudes of the 6th and 7th component assuming $E = 1$ volt.

2-11.28 A symmetrical triangular pulse is given by the equation

$$y = kE + \frac{2E}{k\pi^2}\sum_{n=1}^{n=\infty}\frac{1}{n^2}(\sin nk\pi)^2 \cos nx$$

where k is a proper fraction expressing the duration of the pulse as a fraction of a cycle.

n the order of the harmonic

E the amplitude of the pulse.

Find the amplitudes of the fundamental and second harmonic and write their equations for a pulse voltage of $E = 10$ volts and a pulse duration of 36 degrees.

2-11.29 A projection-type television receiver uses the Schmidt optical arrangement of a spherical mirror M and a correcting lens CL Figure 11.45. The radius of curvature of the mirror is 10.58 inches, the distance of the kinescope screen from the mirror is 6.1 inches. A type 5TP4 projection kinescope is used. Disregarding the correcting lens find the distance of the image from the mirror, the size of the image and magnification.

2-11.30 A television system employs interlaced scanning with 50 fields per second. The pulse-timing unit master oscillator is tuned to the line frequency and followed by a doubler and odd-subharmonic multivibrators. The last multivibrator is synchronized with a 50-cycle power line sine wave. Find the frequencies of all stages for a 625-line system.

2-11.31 The sweep frequency selector switch of an oscilloscope used to investigate the pulse-timing unit of a synchronizing-pulse generator is set for 240 cycles. How many cycles will appear on the screen for each stage of the pulse timing unit designed for 525 lines?

2-11.32 To measure the duration of the horizontal syn. pulse on the bottom of the pulse a 15,750 cycle sine wave for the horizontal deflection of the oscilloscope and the mea. pulse is applied to the vertical input and phased so that it occ during the most linear portion of the applied sine wave as shown in Figure 11.48. If $p = 3$ centimeters and $d = 10$ centimeters, what is the duration of p in terms of the whole cycle H and what is its actual duration in microseconds?

2-11.33 To measure the pulse width of the vertical-blanking pulse of an R.M.A. synchronizing-signal generator a 60-cycle sine wave is used for the horizontal deflection of the oscilloscope, and the measured pulse is applied to the vertical input and phased so that it occurs during the most linear portion of the applied sine wave. Referring to Figure 11.48, with the horizontal amplifier adjusted to make $d = 10$ centimeters, what is the length of p if the blanking lasts for a period of 14 lines?

2-11.34 A television camera employs a lens of a focal length of 5 inches. Calculate the camera viewing angle for an average distance of 20 feet from the scene when the camera is equipped with a type 1850A iconoscope with a usable mosaic area of $3\frac{9}{16} \times 4\frac{3}{4}$ inches.

2-11.35 The following are the operating characteristics of an earlier and a later type amplifier triode:

	6C5	6C4	Units
Grid-to-plate capacitance	2.0	1.6	$\mu\mu$f
Output capacitance	11.0	1.3	$\mu\mu$f
Input capacitance	3.0	1.8	$\mu\mu$f
Mutual conductance	2000	3100	μmhos
Amplification factor	20	19.5	—

Assuming a gain of 5 what is the figure of merit of both tubes?

2-11.36 The earlier type amplifier pentode type 57 has a grid-to-plate capacitance of 0.007 micromicrofarad, an input capacitance of 5 micromicrofarads, an output capacitance of 6.5 micromicrofarads and a transconductance of 1225 micromhos; the later type 6AG5 has 0.025, 6.5 and 1.8 micromicrofarads respectively and a transconductance of 5000 micromhos. Calculate the figures of merit of both tubes.

2-11.37 Amplifiers intended to develop as large an output voltage as possible to be used as the deflection voltage of cathode-ray tubes should have an output which is directly proportional to the plate current and inversely proportional to the plate-to-cathode or output capacitance. The earlier type 6V6 has a plate current of 47 milliamperes and an output capacitance of 9 micromicrofarads. The later miniature type 6AQ5 has a plate current of 45 milliamperes and an output capacitance of 6.0 micromicrofarads. Which of the two tubes is preferable for a video output tube?

2-11.38 Using the formula

$$f = \frac{g_m}{\pi \sqrt{C_i C_o}}$$

where f in cycles is the upper frequency limit of a tube if the gain is unity and a four-terminal filter coupling is used, g_m is in mhos and the capacitances in farads, calculate the upper frequency limit of the triode types 6C5, 6C4 and the pentodes types 57 and 6AG5. Use the data of problems 2-11.35 and 2-11.36.

2-11.39 Using the scanning velocities calculated in problem 11.03 find the difference of the reflected path minus the direct path causing a ghost image following the direct image at a distance of 1 centimeter on a 9″ x 12″ screen.

2-11.40 A sine wave of 500 kilocycles is fed into the detector of a television receiver. Calculate the number of vertical bars that will appear on the screen of the receiver which is set for a standard signal of 525 lines, 60 fields, employing interlaced scanning.

2-11.41 A sine wave of 1000 cycles is fed into the detector of a television receiver. Calculate the number of horizontal bars that will appear on the screen of the receiver which is set for a standard R.M.A. signal of 525 lines, 60 fields, employing interlaced scanning.

2-11.42 The transmission standards of the Federal Communications Commission require a channel width of 6 megacycles; the visual carrier is located 4.5 megacycles lower in frequency than the aural center frequency; the aural center frequency is located 0.25 megacycles lower in frequency than the upper frequency limit. Give the visual carrier frequency, the aural center frequency of channel number 5 (76 to 82 megacycles) and find the frequency of the local oscillator and the visual intermediate frequency if the aural intermediate frequency transformer is tuned to 21.25 megacycles.

12 *Measurements*

2-12.01 In the Wheatstone bridge in Figure 12.01, $R_1 = 175$ ohms, $R_2 = 225$ ohms, $R_3 = 75$ ohms; find the unknown resistor R_x.

2-12.02 The slide-wire bridge of Figure 12.02 has a total length of 1 meter. Calculate R_x for the values indicated in Figure 12.02, if 75 ohms are added to R_1.

2-12.03 With the aid of the Kelvin bridge in Figure 12.03, the resistance R_x is measured with the following bridge resistances: $R_3 = 12$ ohms, $R_5 = 48$ ohms, $R_6 = 8$ ohms. Find the correct value for the arm R_4 and calculate R_x if R_2 must be adjusted to 0.35 ohm for bridge balance.

2-12.04 What is the approximate resistance measured when the milliammeter of problem 12.06 reads 0.2, 0.4, 0.6, 0.8 milliampere?

2-12.05 In Figure 12.07 is shown the instrument arrangement which can be used to measure resistance with a voltmeter when an ohmmeter is not available. The reading of the 1000 ohms-per-volt meter is 2.7 volts on the 25-volt scale. What is the value of R_x?

2-12.06 The filter choke in Figure 12.08 has a resistance of 300 ohms. The voltmeter across the 50-cycle line reads 120 volts, the ammeter 19 milliamperes. What is the approximate inductance of the choke and what is its power factor?

2-12.07 The slide-wire bridge in Figure 12.09 has a total length of 1 meter. Find the unknown capacitance C_x from the values indicated in Figure 12.09, for a frequency of 50 cycles.

2-12.08 In the capacitance-resistance bridge of Figure 12.10, $R_1 = 1$ megohm, $R_2 = 0.5$ megohm, $R_3 = 0.75$ megohm, $C_3 = 750$ micromicrofarads. The applied frequency is 500 cycles. Find the value of R_x and C_x when the indicator reads zero.

2-12.09 To measure the approximate inductance of a radio-frequency coil, a calibrated variable capacitor, a signal generator, and a vacuum-tube voltmeter are used in the arrangement of Figure 12.11.

A frequency of 1000 kilocycles is fed into the tank circuit and the tank capacitor is adjusted to 220 micromicrofarads, at which point resonance is indicated by a maximum reading of the vacuum-tube voltmeter. Neglecting the distributed capacitance of the coil, calculate the inductance.

2-12.10 A 0-to-10 volt, 1000 ohms-per-volt meter is to be extended to read 250 volts full scale. What is the value of the necessary multiplier resistor?

2-12.11 A 0-to-1 milliammeter with an internal resistance of 30 ohms is to be extended to read 250 milliamperes at full scale. What is the value of the necessary shunt resistor?

2-12.12 It is desired to make a 0-to-1 milliammeter with an internal resistance of 30 ohms read volts also. Find the value of the series resistor necessary to make it read 250 volts full scale.

2-12.13 A milliammeter with a full-scale deflection of one milliampere and an internal resistance of 25 ohms is used to measure an unknown current by shunting the milliammeter with a 0.25-ohm resistance. When the meter reads 0.4 milliamperes, what is the actual value of current?

2-12.14 To determine the capacitance of a capacitor, a 20,000 ohm resistor is connected in series with the capacitor, and a 120-volt, 60-cycle line voltage is applied across this series circuit. A vacuum-tube voltmeter reads 75 volts across the resistor and 86.7 volts across the capacitor. Calculate the capacitance.

2-12.15 To determine the inductance of a choke, a 3000 ohm resistor is connected in series with the choke, and the voltage of a 120-volt, 60-cycle line is impressed upon this series hookup. An a-c voltmeter reads 55 volts across the resistor. The resistance of the choke is 100 ohms. Calculate the inductance of the choke (a) neglecting its resistance, and (b) including its resistance. What is the percentage error of (a)?

2-12.16 A vacuum-tube voltmeter with an input resistance of 10 megohms is used to measure an unknown resistor by using the circuit indicated in Figure 12.21, in which E = 4.5 volts and the voltmeter reading is 3.2 volts. What is the resistance of R_x?

13 *Industrial and Control Circuits*

2-13.01 If the saw-tooth voltage of problem 13.02 is applied to the grid of the thyratron through a resistance-capacitance circuit of the type shown in Figure 13.03, with R = 75,000 ohms and C = 0.0125 microfarad, how long will the firing of the tube be delayed after the instant at which C is charged to the peak value of the saw-tooth wave?

2-13.02 A neon relaxation oscillator of the type shown in Figure 13.04 operates at an ignition voltage of 80 volts and an extinction voltage of 60 volts. If R = 750,000 ohms and C = 0.015 microfarad, what time will elapse between the instant of closing the switch and the ignition of the tube?

2-13.03 Calculate the frequency of the relaxation oscillator of problem 2-13.02.

2-13.04 The capacitor C in Figure 13.06, having been charged to E volts, is to be discharged by throwing the switch to position No. 2. If a residual charge of 1 per cent of the original charge is considered a practical discharge, after how many RC-time constants will the capacitor be practically discharged?

2-13.05 Using the rule found in problem 2-13.04, if R = 750,000 ohms and C = 8 microfarads, what is the practical discharge time in seconds?

2-13.06 What should be the minimum setting of the variable resistor R in problem 2-13.05 if a discharge time varying from 0.1 second to the time found in the previous problem is to be realized?

2-13.07 A supply voltage of 450 volts rms is used in a welder-ignitron circuit to satisfy a power demand of 950 kilovolt-amperes. Calculate the average current demand during any conducting cycle.

2-13.08 An ignitron has an average current rating of 250 amperes and an arc drop of approximately 17 volts. What is the equivalent resistance of the arc, and how many kilowatts should the water-cooling system be designed to dissipate?

SECTION III *Some Important Tools of Radio Mathematics*

1 *Powers of 10*

Many persons find difficulty in visualizing the amounts indicated by such large numbers as millions, billions, or greater numbers, or by such small numbers as millionths, billionths, or smaller numbers. If *one billion* is written 1,000,000,000 or if *one-billionth* is written 0.000,000,001 for use in solution of problems, the task of counting ciphers for the purpose of determining the position of the decimal point becomes arduous and time-consuming.

These difficulties are avoided by use of exponents to denote the powers of 10. In this notation, 1,000,000,000 becomes 10^9—the exponent is the same as the number of ciphers which follow 1 when the number is written out in full. Correspondingly, 0.000,000,001 becomes 10^{-9}—the negative exponent is one unit more than the number of ciphers which follow the decimal point and precede 1—or $(n + 1)$ times the number of ciphers. Obviously $10^{-9} = 1/10^9$ because $10^{-9} \times 10^9 = 10^0 = 1$.

These simple rules will suffice to express decadic numbers such as 0.01, 10, 1000 as a power of 10. But any other number can be expressed as a multiple of a power of 10. Thus $2350 = 2.35 \times 1000 = 2.35 \times 10^3$ or 23.5×10^2 or 235×10. In this notation the given number is factored in two factors:

a) the basic number, e.g. 2.35
b) the decimal multiplier, e.g. 10^2.

It is convenient to make the basic number *smaller than 10* because small basic numbers make it easier to obtain a mental approximation of the result expected. Thus $2{,}510{,}000 = 2.51 \times 10^6$ rather than 25.1×10^5; also $0.00325 = 3.25 \times 10^{-3}$ rather than 325×10^{-5}.

A. Positive exponent forms and prefixes

million $= 10^6 =$ mega- or meg- (megacycle, megohm) hundred thousand = 10^5

ten thousand = 10^4

thousand = 10^3 = kilo- or kil- (kilocycle, kilohm)

hundred = 10^2

ten = 10^1 = deca- (decagram)

B. Zero exponent

unit = $10^0 = 1$

C. Negative exponent forms and prefixes

tenth = 10^{-1} = deci- (decimeter)

hundredth = 10^{-2} = centi- (centimeter)

thousandth = 10^{-3} = milli- (milliampere)

ten-thousandth = 10^{-4}

hundred-thousandths = 10^{-5}

millionth = 10^{-6} = micro- (microfarad)

The convenience in computation secured by use of this notation is shown in the following example:

Multiply 2,350,000,000 by 0.000,000,042

In powers of 10, the problem becomes

$$(2.35 \times 10^9) \times (4.2 \times 10^{-8}) = 2.35 \times 4.2 \times 10$$
$$= 98.70$$

2 *The Slide Rule*

There are other aids and devices which are admittedly more accurate for computing radio problems, such as logarithm tables or calculating machines. But none can be compared to the slide rule for performing speedy operations.

The slide rule contains two languages. One is an arithmetical, the other a geometrical language. The arithmetical expressions appear in the form of *numbers*. These are printed on the slide rule; for example, the number 2. The geometrical expressions appear in the form of *distances*; for instance, the distance of the number 2 from the number 1. This distance is about one third of the whole slide rule scale, or more accurately 0.301 of the whole scale. The distance of the number 3 from 1 is about one half of the whole scale or more accurately, 0.477. A look into a logarithm table will show that

$$\log 2 = 0.301$$
$$\log 3 = 0.477.$$

It is clear that the slide rule is a graphical logarithm table. Whenever a distance of the slider is added to a distance of the fixed part two logarithms are added. Since

$$\log (a \times b) = \log a + \log b,$$

a multiplication is most conveniently performed on the slide rule by adding the distances, i.e. the logarithms of the two factors.

The proverbial accuracy of the slide rule is due more to the admiration of the layman who watches the expert at work than to the recognition of one who knows the slide rule. The accuracy of the slide rule is approximately one part per thousand, which is illustrated by the fact that numbers a little less than one thousand are readable as 3-place numbers, such as 998 or 887; numbers a little more than one thousand are readable as 4-place numbers, such as 1,001 or 1,115. In both cases the accuracy is about one part per thousand or 0.001, or

0.1 per cent. This accuracy, though not great, is entirely sufficient in most phases of electronic practice, because electronic parts seldom have a closer tolerance than 1 per cent. Only a few problems in this book need be solved more accurately than a slide rule will permit; these are problems involving frequency calculations. If it were not for the invention of the crystal oscillator, the laws regulating frequency tolerance could not be as strict as they are. A common tank circuit cannot be designed to work with a high degree of stability.

There are several inexpensive books on the slide rule (see reference list 000), and the leading manufacturers include a free book on the use of the particular slide rule which the buyer selects.

An interesting attempt to illustrate the two languages of the slide rule is the Macmillan Table Slide Rule, invented by J. P. Ballantine.

The rules for multiplication and division, and almost all other rules for slide rule computations are so simple that they can be explained in a few minutes. However it is not the remembering of these rules but rather the reading of the slide rule scales which the author has observed to cause most difficulties for the beginner. Patience will easily overcome this initial obstacle and the slide rule will soon become the great timesaver which makes it indispensable for radio problems.

3 *The j-Operator*

Definition

The letter j is an abbreviation for the quantity $\sqrt{-1}$. This quantity or any multiple of it is called an *imaginary* number, whereas any multiple of 1 is called a *real* number. Thus, 2, 15, -4.6, and $\sqrt{2}$ are real numbers; but $\sqrt{-25}$, $\sqrt{-2}$ and $-\sqrt{-4}$ are imaginary numbers, which can also be written $j\,5$, $j\sqrt{2}$ and $-j\,2$.

A real number and an imaginary number added make a *complex* number, e.g., $3 + 2\sqrt{-1}$ or $3 + j\,2$.

Although these numbers are called *imaginary* and *complex*, their usefulness is so real that they help to design radar and to construct bridges, and the only thing complex about them is the word *complex*.

The Concept of Direction

The first man who wrote a negative number thereby made an excursion into the realm of unreality;—no one ever saw negative things. And yet, given the interpretation of direction, negative numbers become real. Thus, 5 amperes of current flow will become -5 amperes if the leads connecting the load to the battery are reversed. The mercury column of a thermometer can move above zero in a positive direction or below zero in a negative direction.

But the idea that positive and negative numbers are the only real numbers is as fallacious as an implication that there are only two directions in our world. C. F. Gauss gave a clear graphical interpretation of the imaginary numbers. He demonstrated that each time a number is multiplied by j its direction changes by $+90°$, just as multiplication by -1 produces a change of direction by 180°.

Referring to Figure 3-6 we observe:

1 Multiplying the number $+1 \times j$ will rotate the distance 1 by 90° in a positive direction, arriving at $+j\,1$ at B.

2 Multiplying the quantity $+j\,1 \times j$ will again rotate the distance 1 by 90° in a positive direction, arriving at -1 at C. This is in accordance with arithmetical fact, since

$$j \times j = \sqrt{-1} \times \sqrt{-1} = -1.$$

3 By multiplying the quantity $-1 \times j$, we produce a further shift by $+90°$ and arrive at $-j\,1$ at D.

4 A multiplication of $-j\,1 \times j$ yields $-j^2$. But in No. 2 it was found that $j^2 = -1$; therefore $-j^2 = -(-1) = +1$. We have then arrived back at the starting point A at $+1$.

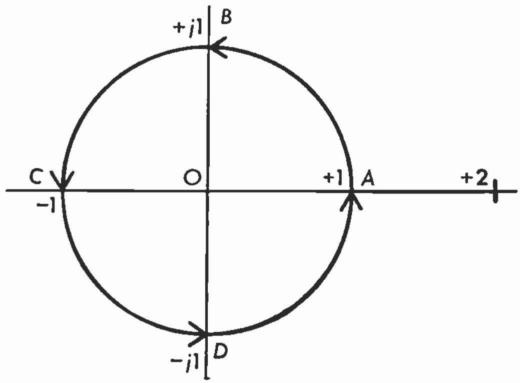

Fig. 3-6 Rotating vectors in the complex plane.

We now have four directions, the *real* right-left direction and the *imaginary* up-down direction.

Gauss demonstrated that all vectors in between the four rectangular directions can be interpreted as obtained from component vectors, situated on the real and the imaginary axes. These components add up like forces according to the parallelogram law of mechanics. Figure 3-7 represents the complex quantity $(2 + j\,3)$, its magnitude being determined by the diagonal of the parallelogram and its direction by the angle θ. A quantity defined in both magnitude and direction is called a *vector* quantity, whereas a quantity defined in magnitude only is called a *scalar* quantity. The numbers 2, 15, 0.003, 1/4 are scalar quantities, whereas the quantities -1, $+17$, $j\,6$, $-j\,0.18$, $(9 + j\,3)$ are vector quantities, because both their magnitudes and directions are given.

The magnitude of the vector $(2 + j\,3)$ in Figure 3-2 is easily found as the hypotenuse of the right triangle, the sides of which are the component rectangular vectors $+2$ and $+j\,3$.

$$\sqrt{2^2 + 3^2} = 3.6 \text{ units.}$$

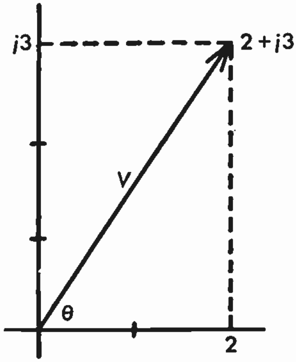

Fig. 3-7 Vector representation of a complex member.

The direction is determined by the *phase angle* θ, which is found from the relation

$$\tan\theta = \frac{3}{2} = 1.5, \text{ and } \theta = \tan^{-1} 1.5 = 56.3^\circ$$

A vector can thus be uniquely defined either by its rectangular components, which is written in *j*-notation

$$\mathring{V} = 2 + j\,3$$

or by its magnitude and phase angle, which is written

$$\mathring{V} = 3.6\underline{/56.3^\circ}.$$

The first form is the *rectangular* form, and the second is the *polar* form of a vector. The relation between both forms is given by

$$a + j\,b = \sqrt{a^2 + b^2}\ \underline{/\tan^{-1}\frac{a}{b}}$$

or in terms of electrical circuit constants

$$R + j\,X = \sqrt{R^2 + X^2}\ \underline{/\tan^{-1}\frac{X}{R}}$$

Example:

Write the vectors V_1, V_2, V_3 and V_4 in Figure 3-8 in rectangular form. The distance of the real component is 4 units and the length of the imaginary component is 3 units in all four cases; the vectors will differ in the signs of the rectangular components.

$$V_1 = 4 + j\,3$$
$$V_2 = -4 + j\,3$$
$$V_3 = -4 - j\,3$$
$$V_4 = 4 - j\,3.$$

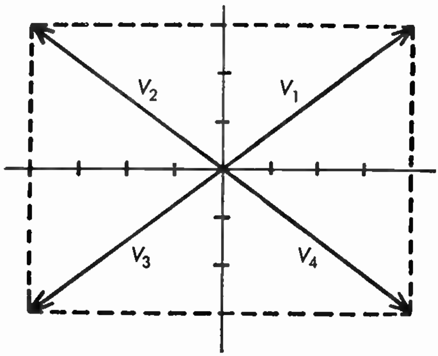

Fig. 3-8 Four vectors having the same absolute rectangular components.

Powers of j

$$j^1 = \sqrt{-1} = j,$$
$$j^2 = (\sqrt{-1})^2 = -1,$$
$$j^3 = j^2 \times j = -1 \times j = -j,$$
$$j^4 = j^2 \times j^2 = (-1)^2 = +1,$$
$$j^5 = j^4 \times j = 1 \times j = j \text{ etc.}$$

Multiplication Table of Real Operators

	+	−
+	+	−
−	−	+

Rule:

The product of like signs is "plus," the product of unlike signs is "minus."

Multiplication Table of j-Operators

From the powers of j it follows that $+j \times +j = j^2 = -1$. Thus $j^2\,74$ will be identical with -74. Similarly, $-j \times +j = -j^2 = +1$.

	$+j$	$-j$
$+j$	$-$	$+$
$-j$	$+$	$-$

Rule:

The product of two like j's is -1, the product of two unlike j's is $+1$.

The Fundamental Operations of Rectangular Vectors

Addition

$$(R_1 + j\,X_1) + (R_2 + j\,X_2)$$
$$= (R_1 + R_2) + j\,(X_1 + X_2).$$

EXAMPLE:

$$(5 + j\,3) + (6 - j\,8)$$
$$= 5 + 6 + j\,(3 - 8)$$
$$= 11 - j\,5. \quad \textit{Ans.}$$

Subtraction

$$(R_1 + j\,X_1) - (R_2 + j\,X_2)$$
$$= (R_1 - R_2) + j\,(X_1 - X_2)$$

EXAMPLE:

$$(5 + j\,3) - (6 - j\,8)$$
$$= (5 - 6) + j\,[3 - (-8)]$$
$$= -1 + j\,11. \quad \textit{Ans.}$$

Multiplication

$$(R_1 + j\,X_1)\,(R_2 + j\,X_2)$$
$$= R_1 R_2 + j^2 X_1 X_2 + j\,(R_1 X_2 + R_2 X_1)$$
$$= (R_1 R_2 - X_1 X_2) + j\,(R_1 X_2 + R_2 X_1)$$

EXAMPLE:

$$(5 + j\,3)\,(6 - j\,8)$$
$$= 30 - j^2\,24 + j\,(-40 + 18)$$
$$= 30 + 24 + j\,(-22)$$
$$= 54 - j\,22. \quad \textit{Ans.}$$

Division

$$\frac{R_1 + j\,X_1}{R_2 + j\,X_2} = \frac{R_1 + j\,X_1}{R_2 + j\,X_2} \times \frac{R_2 - j\,X_2}{R_2 - j\,X_2}$$

The French mathematician Cauchy called the expression $R_2 - j\,X_2$ a *conjugate* number of $R_2 + j\,X_2$ and vice versa. The pair of conjugate numbers consists of two binomials differing only by the sign of the second term of the binomial. The first step in performing the division

$$(R_1 + j\,X_1) \div (R_2 + j\,X_2)$$

is to write it in fractional form and to multiply numerator and denominator by the conjugate denominator, in this case $R_2 - j\,X_2$.

We obtain
$$\frac{R_1 + j\,X_1}{R_2 + j\,X_2} \times \frac{R_2 - j\,X_2}{R_2 - j\,X_2}$$

$$= \frac{R_1\,R_2 + X_1\,X_2 + j\,(R_2\,X_1 - R_1\,X_2)}{R^2 - j^2\,X^2}$$

$$= \frac{R_1\,R_2 + X_1\,X_2}{R^2 + X^2} + \frac{j\,(R_2\,X_1 - R_1\,X_2)}{R^2 + X^2}$$

Since $-j^2 = +1$, the denominator is real and consists of the *sum* of the squares of the original denominator.

EXAMPLE:

$$\frac{54 - j\,22}{5 + j\,3} = \frac{54 - j\,22}{5 + j\,3} \times \frac{5 - j\,3}{5 - j\,3}$$

$$= \frac{270 - j\,110 - j\,162 + j^2\,66}{5^2 + 3^2}.$$

Since $+j^2 = -1$, we obtain

$$\frac{270 - 66 - j\,272}{34} = \frac{204}{34} - j\,\frac{272}{74} = 6 - j\,8. \quad \textit{Ans.}$$

Practical Application of the j-Operator

For reasons which are explained by the theory of alternating currents, the impedance of a series circuit consisting of resistance and reactance is denoted

$$\mathring{Z} = R + j\,X \quad \text{or} \quad \mathring{Z} = R - j\,X,$$

where X is the absolute difference between the inductive and capacitive reactance. $+j\,X$ indicates that the inductive reactance is greater, $-j\,X$ that the capacitive reactance predominates.

EXAMPLES:

1 Find the absolute value of the total impedance and the phase angle of a series circuit consisting of 500 ohms capacitive reactance, 1000 ohms inductive reactance and 250 ohms resistance.

$$\mathring{Z} = 250 + j\,1000 - j\,500 = 250 + j\,500$$

$$|Z| = \sqrt{250^2 + 500^2} = 560 \text{ ohms}$$

$$\theta = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{500}{250} = \tan^{-1} 2 = 63.5°$$

2 A circuit consists of two parallel branches. One branch contains 300 ohms resistance and 400 ohms inductive reactance, the other branch contains 400 ohms resistance and 300 ohms capacitive reactance. Find the equivalent series circuit, and from the latter the absolute impedance and the phase angle.

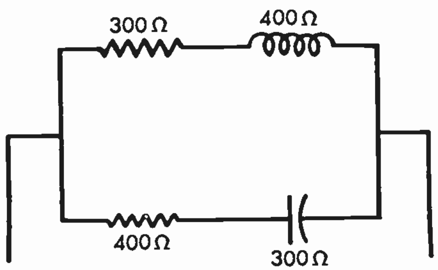

Fig. 3-9 Parallel impedances.

$$\frac{1}{\mathring{Z}_T} = \frac{1}{\mathring{Z}_1} + \frac{1}{\mathring{Z}_2} = \frac{\mathring{Z}_1 + \mathring{Z}_2}{\mathring{Z}_1\,\mathring{Z}_2}$$

and
$$\mathring{Z}_T = \frac{\mathring{Z}_1\,\mathring{Z}_2}{\mathring{Z}_1 + \mathring{Z}_2}.$$

The numerator

$$\mathring{Z}_1\,\mathring{Z}_2 = (300 + j\,400)\,(400 - j\,300)$$
$$= 240{,}000 + j\,70{,}000.$$

The denominator

$$\mathring{Z}_1 + \mathring{Z}_2 = 300 + j\,400 + 400 - j\,300$$
$$= 700 + j\,100.$$

Therefore $$\mathring{Z}_T = \frac{240{,}000 + j\,70{,}000}{700 + j\,100}$$
$$= 100 \times \frac{24 + j\,7}{7 + j\,1}.$$

Multiplying numerator and denominator by the conjugate number $(7 - j\,1)$

$$\mathring{Z}_T = 100 \times \frac{168 + j\,49 - j\,24 + 7}{7^2 + 1^2}$$
$$= \frac{100}{50}(175 + j\,25) = 350 + j\,50. \quad \textit{Ans.}$$

The equivalent circuit contains 350 ohms pure resistance and 50 ohms inductive reactance.

The absolute value of the impedance is

$$Z_T = \sqrt{350^2 + 50^2} = 354 \text{ ohms.} \quad \textit{Ans.}$$

The phase angle is $\theta = \tan^{-1}\dfrac{50}{350} = \tan^{-1} 0.143 = 8.1°$. *Ans.*

4 *Polar Vectors*

As was demonstrated in the chapter on the j-operator, a vector quantity is uniquely defined either by its rectangular components (j-notation) or by its magnitude and phase (polar notation). In the first case a horizontal axis (the real axis) and a vertical axis (the imaginary axis) are used as a reference system. Thus the vector quantity $2 + j\,3$ represents a vector resulting from a horizontal component of $+2$ units and a vertical component of $+3$ units.

The other system of vector representation employs no axes but (1) a reference vector v_1 and (2) a pole P. The vector quantity $3.6\underline{/56.3^\circ}$ is identical with the above mentioned vector $2 + j\,3$.

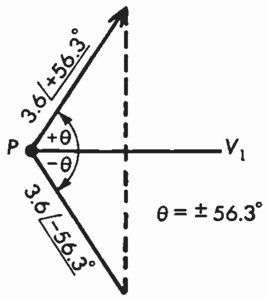

Fig. 3-10 Two polar vectors of the same magnitude having a positive and a negative phase angle.

It is defined by: (1) its magnitude 3.6 units, measured from the pole, (2) its phase angle $\underline{/56.3^\circ}$, measured from the reference vector in a counterclockwise direction. The dotted vector is equal in magnitude, but its phase is $\underline{/-56.3^\circ}$, also sometimes written $\overline{/56.3^\circ}$ or $\overline{/-56.3^\circ}$. It is measured from the reference vector in a clockwise direction.

In electrical circuits, a positive phase angle indicates that the circuit is resistive and predominatly inductive, i.e., the inductive reactance is greater than the capacitive reactance. A negative angle indicates that the circuit is resistive and predominantly capacitive. $\underline{/+90^\circ}$ indicates pure inductance, $\underline{/-90^\circ}$ indicates pure capacitance.

The j-operator, though a powerful tool in the solution of steady-state alternating-current circuits, is in many cases not the most rapid key to the solution of a problem and in some cases it is entirely inadequate. The well-known formula for example, for the characteristic impedance of the transmission line—

$$Z_o = \sqrt{\frac{R + j\,X_L}{G + j/X_c}}$$

cannot be evaluated conveniently without the aid of polar vectors, and, as will be shown below, vectors are most rapidly multiplied and divided when expressed in polar form.

Translating j-Vectors into Polar Vectors

1 *General solution*

The problem calls for finding

a) the magnitude
b) the phase

of the polar vector from the given j-notation $R + j\,X$.
It was shown in the previous article of this section that the magnitude

is $$|Z| = \sqrt{R^2 + X^2}$$

and the phase angle is $$\theta = \tan^{-1} \frac{X}{R}.$$

Therefore $$R + j\,X = \sqrt{R^2 + X^2} \,\underline{/\tan^{-1} \frac{X}{R}}$$

2 *With the aid of the conversion table*

The author has worked out a vector conversion table which is included in Section IV of this book. The reader is referred to the introductory remarks of the table.

EXAMPLE:

Express $50 - j\,75$ in polar notation.

(a) By the formula

The magnitude is

$$|Z| = \sqrt{50^2 + 75^2}$$
$$= \sqrt{2500 + 5625} = \sqrt{8125} = 90.2 \text{ ohms.}$$

The phase angle is

$$\theta = \tan^{-1}\frac{-75}{50} = -\tan^{-1} 1.5 = -56.3^\circ.$$

Therefore

$$50 - j\,75 = 902\underline{/-56.3^\circ}. \quad \textit{Ans.}$$

(b) Using the vector conversion table

Under $X/R = 1.5$ find $\theta = 56.3^\circ$, and $|Z| = 1.2\,X = 1.2 \times 75 = 90.2$ ohms.

Translating Polar Vectors into j-Vectors

1 *General solution*

If $Z\underline{/\theta}$ is the vector to be expressed in its rectangular components, then the problem calls for finding

a) The adjacent side
b) the opposite side

of a right triangle of which the hypothenuse Z and the angle θ are known. We have

$$\frac{R}{Z} = \cos\theta, \quad \text{and} \quad R = Z\cos\theta,$$

$$\frac{X}{Z} = \sin\theta, \quad \text{and} \quad X = Z\sin\theta.$$

Therefore $$Z\underline{/\theta} = Z\cos\theta + j\,Z\sin\theta.$$

2 *With the aid of the conversion table*

The reader is referred to the introductory remarks of the vector conversion table.

EXAMPLE:

Express $5000\underline{/28.3^\circ}$ in j-notation.

(a) By the formula

$$R = 5000\cos 28.3^\circ$$
$$= 5000 \times 0.88 = 4400 \text{ ohms},$$
$$X = 5000\sin 28.3^\circ$$
$$= 5000 \times 0.474 = 370 \text{ ohms}.$$

Therefore $$5000\underline{/28.3^\circ} = 4400 + j\,2370. \quad \textit{Ans.}$$

(b) Using the vector conversion table

Under 28.3° find $Z = 2.1\ X$, and $X = 5000/2.0 = 2370$, and in the same line $X/R = 0.538$, and $R = 2370/0.538 = 4400$ ohms.

Multiplication and Division of Polar Vectors

The simple rules for multiplication and division of polar vectors are derived as follows:

$$Z_1\underline{/\theta_1} \times Z_2\underline{/\theta_2}$$

$$= (Z_1 \cos\theta_1 + j \sin\theta_1)(Z_2 \cos\theta_2 + j \sin\theta_2)$$

$$= Z_1 Z_2 \cos\theta_1 \cos\theta_2 + j Z_1 Z_2 \sin\theta_1 \cos\theta_2$$

$$+ j Z_1 Z_2 \cos\theta_1 \sin\theta_2 + j^2 Z_1 Z_2 \sin\theta_1 \sin\theta_2;$$

but $\quad +j^2 = -1,$

therefore

$$= Z_1 Z_2 (\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) + j(\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2)$$

$$= Z_1 Z_2 \cos(\theta_1 + \theta_2) + j \sin(\theta_1 + \theta_2)$$

$$= Z_1 Z_2\underline{/\theta_1 + \theta_2}.$$

It can likewise be shown that

$$\frac{Z_1\underline{/\theta_1}}{Z_2\underline{/\theta_2}} = \frac{Z_1}{Z_2}\underline{/\theta_1 - \theta_2}.$$

Rule:

Polar vectors are *multiplied* by multiplying the magnitudes and adding the phase angles. Polar vectors are *divided* by dividing the magnitudes and subtracting the phase angles; the phase angle of the divisor is the subtrahend.

EXAMPLES:

1 Multiply $(5\underline{/20°}) \times (10\underline{/30°})$.

$$(5\underline{/20°}) \times (10\underline{/30°})$$
$$= 5 \times 10\underline{/20° + 30°} = 50\underline{/50°}. \quad \textit{Ans.}$$

2 Divide $(5\underline{/20°}) \div (10\underline{/30°})$.

$$\frac{5\underline{/20°}}{10\underline{/30°}} = \frac{5}{10}\underline{/20° - 30°} = 0.5\underline{/-10°} \quad \textit{Ans.}$$

Powers and Roots of Polar Vectors

From $$(Z\underline{/\theta}) \times (Z\underline{/\theta}) = Z^2\underline{/2\theta}$$

we can conclude that—

$$(Z\underline{/\theta})^n = Z^n\underline{/n\theta},$$

an equation which is known as Demoivre's theorem.

If $n = \frac{1}{2}$, we have—

$$(Z\underline{/\theta})^{\frac{1}{2}} = Z^{\frac{1}{2}}\underline{/\tfrac{1}{2}\theta}$$

or

$$\sqrt{Z\underline{/\theta}} = \sqrt{Z}\underline{/\tfrac{\theta}{2}}.$$

Rule:

A polar vector is squared, cubed etc. by squaring, cubing etc. the magnitude and multiplying the phase angle, by 2, 3, etc.

The square root, cube root, etc., of a polar vector is extracted by extracting the root of the magnitude and dividing the phase angle by 2, 3 etc.

EXAMPLES:

1 Square $5\underline{/25°}$.

$$(5\underline{/25°})^2 = 5^2\underline{/2 \times 25°} = 25\underline{/50°}. \quad \textit{Ans.}$$

2 Extract the square root of $5\underline{/25°}$.

$$\sqrt{5\underline{/25°}} = \sqrt{5}\underline{/\tfrac{25°}{2}} = 2.24\underline{/12.5°} \quad \textit{Ans.}$$

Solution of Problems Involving Parallel Impedances

Whenever the equivalent impedance of impedances in parallel is calculated, one or more multiplications and division of vector quantities must be performed. Polar vectors are therefore adequate for the solution of such problems.

EXAMPLE:

Given the two impedances $Z_1 = 300 + j\,400$ and $Z_2 = 400 - j\,300$. Calculate the total impedance and the phase angle of the two impedances connected in parallel. (Note that this problem was solved by j-vectors in the previous article of this section.)

$$\mathring{Z}_t = \frac{\mathring{Z}_1 \times \mathring{Z}_2}{\mathring{Z}_1 + \mathring{Z}_2}$$

The denominator vector $\mathring{Z}_1 + \mathring{Z}_2 = 700 + j\,100$.

The absolute value of the total impedance is

$$|Z_t| = \frac{|Z_1|\ |Z_2|}{|Z_d|} = \frac{500 \times 500}{707} = 354 \text{ ohms.} \quad \textit{Ans.}$$

The phase angle is

$$\theta = \tan^{-1}\frac{400}{300} + \tan^{-1}\frac{-300}{400} - \tan^{-1}\frac{100}{700}$$
$$= \tan^{-1} 1.33 - \tan^{-1} 0.75 - \tan^{-1} 0.143$$
$$= 53.1° - 36.9° - 8.1° = 8.0°. \quad \textit{Ans.}$$

SECTION IV *Formulas and '*

1 *Electronic Formulas*

1 *Circuit Components*

1.1 *Resistors*

1.11 Resistance of a substance of uniform cross section

$$R = \frac{\rho l}{A},$$

where R = resistance in ohms,
ρ = resistivity,
l = length,
A = area.

If l is in centimeters, A is in square centimeters and ρ is in ohms per centimeter. If l is in feet, A is in circular mils, and ρ in ohms per circular-mil-foot.

1.12 Resistance of wires

$$R = \frac{\rho l}{d^2},$$

Where R = resistance in ohms,
ρ = ohms per circular-mil-foot,
d = diameter in mils.

1.13 Circular units

1.131 1 mil = 0.001 inch
1.132 1 square mil = 1.273 circular mils
1.133 1 circular mil = 0.7854 square mil

1.14 Resistance at any temperature

$$R_T = R_t\,[1 + \alpha_t\,(T - t)],$$

where R_T, R_t = resistance in ohms at $T°$ or $t°$,
α_t = temperature coefficient at $t°$.

1.141 Conversion of temperatures

1.1411 $C = \frac{5}{9}(F - 32)$

1.1412 $F = \frac{9}{5}(C + 32)$

where C = degrees centigrade,

F = degrees Fahrenheit.

1.15 High-frequency resistance

$$R_f = \frac{83.2\sqrt{f}}{d},$$

where R_f = ohms per centimeter,

f = cycles,

d = diameter in centimeters.

1.16 Resistors in series

$$R_t = R_1 + R_2 + R_3 + \cdots.$$

1.17 Resistors in parallel

1.171 Conductance equation

$$G_t = G_1 + G_2 + G_3 + \cdots,$$

where $G = \frac{1}{R}$ in mhos;

or $$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \cdots.$$

1.172 Product-sum formula for 2 resistors

$$R_t = \frac{R_1 \times R_2}{R_1 + R_2}$$

1.173 Simplified formula for more than 2 resistors

$$R_t = \frac{R_1}{1 + R_1/R_2 + R_1/R_3 + R_1/R_4 \cdots}$$

R_1 could be any of the branch resistances, but it is most convenient to let the largest resistance be R_1.

1.2 *Capacitors*

1.21 Working voltage

$$V_{max} = E_{rms} \times 1.414,$$

where V_{max} = maximum working voltage of the capacitor,
E_{rms} = effective voltage applied across the capacitor.

1.22 Power factor

$$pf = \frac{R_s}{X},$$

where R_s = equivalent series resistance,
X = reactance at the given frequency.

1.23 Shunt resistance

$$R_{sh} = \frac{X}{pf},$$

where R_{sh} = equivalent shunt resistance,
X = reactance,
pf = power factor.

1.24 Capacitance of parallel plate capacitor

$$C = \frac{0.0884 \times K \times A\,(n-1)}{d},$$

where C = capacitance in micromicrofarads,
K = dielectric constant,
A = area in square centimeters,
d = distance between plates in centimeters,
(If A and d are in inches, change 0.0884 to 0.224)
n = number of plates.

1.25 Capacitors in series

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots.$$

1.251 Two capacitors in series

$$C_t = \frac{C_1 \times C_2}{C_1 + C_2}$$

1.26 Capacitors in parallel

$$C_t = C_1 + C_2 + C_3 + \cdots.$$

1.27 Charge of a capacitor

$$Q = E \times C,$$

where Q = charge in coulombs,
E = volts across capacitor,
C = capacitance in farads.

1.3 *Inductors and Transformers*

1.31 Combined inductors

1.311 Inductors in series

$$L_t = L_1 + L_2 + L_3 + \cdots$$

1.312 Inductors in parallel

$$\frac{1}{L_t} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

1.313 Coupled inductance

1.3131 Series aiding

$$L_t = L_1 + L_2 + 2M$$

where M = mutual inductance.

1.3132 Series opposing

$$L_t = L_1 + L_2 - 2M$$

1.3133 Parallel aiding

$$L_t = \frac{1}{\dfrac{1}{L_1 + M} + \dfrac{1}{L_2 + M}}$$

1.3134 Parallel opposing

$$L_t = \frac{1}{\dfrac{1}{L_1 - M} + \dfrac{1}{L_2 - M}}$$

1.3135 Mutual inductance

$$M = k\sqrt{L_1 L_2},$$

where k = coupling coefficient.

1.32 Inductance of an air-core coil

$$L = \frac{0.2 \times D^2 \times N^2}{3\,D + 9\,l + 10\,d},$$

where L = inductance in microhenries,
D = diameter of the coil in inches,
l = length of winding in inches,
N = total number of turns,
d = coil depth in inches,
(can be omitted for single-layer coils).

1.33 Step-up and step-down transformers

1.331 Voltage transformers

$$\frac{E_p}{E_s} = \frac{N_p}{N_s},$$

where E_p = voltage across the primary,
E_s = voltage across the secondary,
N_p = primary turns,
N_s = secondary turns.

1.332 Current transformers

$$\frac{I_p}{I_s} = \frac{N_s}{N_p},$$

where I_p = current through primary,
I_s = current through secondary,
N_p = primary turns,
N_s = secondary turns.

1.333 Impedance transformers

$$\frac{Z_p}{Z_s} = \frac{N_p^2}{N_s^2},$$

where Z_p = impedance of the primary,
Z_s = impedance of the secondary,
N_p = primary turns,
N_s = secondary turns.

1.334 Transformer efficiency

$$\eta = \frac{P_s}{P_p},$$

where P_p = the power input to the primary,
P_s = the power output of the secondary.

1.335 Three-phase transformer

1.3351 Y-connection

$$E_L = E_\Phi \times \sqrt{3},$$

where E_L = voltage across the line,
E_Φ = voltage in one phase.

1.3352 Δ-connection

$$I_L = I_\Phi \sqrt{3},$$

where I_L = current in each line,
I_Φ = current in each phase.

1.34 Transformer design

$$E = \frac{4.44\, f\, B\, A\, t}{10^8},$$

where E = volts of circuit,
f = cycles of line,
B = lines per inch,
(75,000 for 60 cycles
50,000 for 20 cycles, approximately.)
A = area of circuit in square inches,
t = number of turns.

1.35 Voltage regulation

$$G = \frac{E_n - E_l}{E_l}$$

Voltage regulation is a proper fraction, usually expressed in per cent,

E_l = voltage under load,
E_n = voltage under no-load.

2 *Direct-Current Circuits*

2.1 *Ohm's Law*

I = current in amperes,
E = volts across R,
R = resistance in ohms,
P = power in watts.

Current formulas: $I = \frac{E}{R} = \sqrt{\frac{P}{R}} = \frac{P}{E}$

Voltage formulas: $E = I\,R = \frac{P}{I} = \sqrt{P\,R}$

Power formulas: $P = E\,I = I^2 R = \frac{E^2}{R}$

2.11 Efficiency

$$\eta = \frac{P_o}{P_i},$$

where P_i = input power in watts,
P_o = output power in watts.

2.2 *Circuit Theorems*

The following theorems are also valid for alternating-current circuits if the resistance values are replaced by *impedance* values, and the currents and the voltages by *vector currents* and *vector voltages*, respectively.

2.21 Kirchhoff's laws

2.211 Kirchhoff's first law:

The algebraic sum of the currents at any junction of conductors is always zero.

2.212 Kirchhoff's second law:

The algebraic sum of voltages around a closed circuit is always zero.

2.22 Shunt law:

When a voltage is applied across two resistances connected in parallel, the ratio of the currents is inversely proportional to the ratio of the resistances.

or $\frac{I_1}{I_2} = \frac{R_2}{R_1}$.

2.23 Superposition theorem:

The current in a conductor is the sum of the currents which would be produced by each voltage of the network, with the other voltages short-circuited.

2.24 Thévenin's theorem:

Any linear network containing one or more sources of voltage with a load connected across two terminals can be replaced by a generator with an equivalent voltage E and an internal resistance r.

E is the voltage that would exist under no-load conditions.

r is the internal resistance that would exist in the generator if all sources of voltage were short-circuited.

2.25 Delta-star transformation

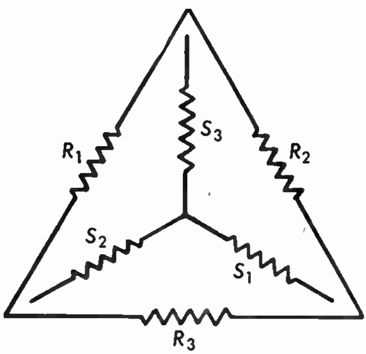

Fig. 4-2.25 Equivalent delta and star impedance elements.

2.251 Star formulas

$$S_1 = \frac{R_2 R_3}{R_1 + R_2 + R_3},$$

$$S_2 = \frac{R_1 R_3}{R_1 + R_2 + R_3},$$

$$S_3 = \frac{R_1 R_2}{R_1 + R_2 + R_3}.$$

2.252 Delta formulas

$$R_1 = \frac{S_1 + S_2 + S_3}{S_1},$$

$$R_2 = \frac{S_1 + S_2 + S_3}{S_2},$$

$$R_3 = \frac{S_1 + S_2 + S_3}{S_3}.$$

2.26 Potentiometer rule:

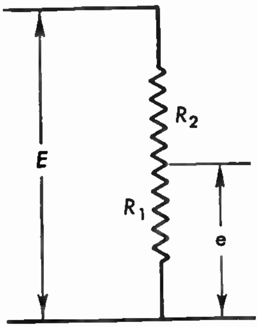

Fig. 4-2.26 Voltage division by tap in load resistor.

The voltage e across a part of a potentiometer is equal to the voltage E across the whole potentiometer, times a proper fraction whose numerator is the resistance across e, and whose denominator is the whole resistance of the potentiometer:

$$e = E \times \frac{R_1}{R_1 + R_2}$$

2.3 *Batteries*

e = electromotive force of 1 cell,
R = load resistance,
r = internal resistance of 1 cell,
I = current through R,
s = number of cells in series,
p = number of cells (banks) in parallel.

2.31 One cell

$$I = \frac{e}{R + r}$$

2.32 Cells in series

$$I = \frac{se}{R + sr}$$

2.33 Cells in parallel

$$I = \frac{e}{R + (r/p)}$$

2.34 Series-parallel connection

$$I = \frac{se}{R + (sr/p)}$$

3 *Alternating-Current Circuits*

3.1 *Ohm's Law*

The formulas given under 2.1 are valid for alternating current if the circuit is purely resistive. If the circuit contains reactive and resistive components, we obtain:

3.11 $\mathring{I} = \dfrac{E}{\mathring{Z}}$ or $|I| = \dfrac{E}{|Z|}$

where $\mathring{Z}$ is impedance in vector ohms
$|Z|$ the absolute value of $\mathring{Z}$ in ohms.

3.12 $P = E \times I \times \cos\theta,$

where the power factor is

3.121 $\cos\theta = \dfrac{R}{|Z|}.$

3.122 Phase angle

$$\theta = \tan^{-1}\frac{X}{R}.$$

3.2 *Instantaneous Values*

3.21 Current

$i = I \sin 2\pi f t,$

where i = current, t seconds after the beginning of a cycle,
I = peak current,
f = frequency in cycles,
t = seconds,
$2\pi f t$ = angle in radians.

3.211 Angle conversions

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees} = 57.3°$$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian} = 0.01745 \text{ radian}$$

3.22 Voltage (referring to 3.21)

$$e = E \sin (2 \pi f t \pm \theta)$$

where e = voltage, t seconds after the beginning of a current cycle.
E = peak voltage,
θ = angle of lead or lag between current and voltage.

3.3 *Average and Effective Values*

3.31 Effective or rms values

$$E_{rms} = E/\sqrt{2} \times = 0.707\ E$$

$$I_{rms} = I/\sqrt{2} \times = 0.707\ I$$

3.32 Average values

$$E_{av} = \frac{2}{\pi} E = 0.637\ E$$

$$I_{av} = \frac{2}{\pi} I = 0.637\ I$$

3.33 Combined alternating and direct current

$$E_{rms} = \sqrt{E^2_{d\text{-}c} + E^2_{a\text{-}c}}$$

3.4 *Reactance and Impedance*

3.41 Capacitive reactance

$$X_c = \frac{1}{2 \pi f C}$$

where X_c = capacitive reactance in ohms,
f = frequency in cycles,
C = capacitance in farads.

3.42 Inductive reactance

$$X_l = 2 \pi f L$$

3.43 Impedance of series circuit including resistance, capacitance, and inductance

3.431 $\mathring{Z} = R + j\,X$ vector ohms

3.432 $|Z| = \sqrt{R^2 + X^2}$ ohms

where $X = X_l - X_c$.

3.44 Mho values.

3.441 A-c conductance (resistive mhos)

$$G = \frac{R}{X^2 + R^2} \text{ mhos}$$

3.442 Susceptance (reactive mhos)

$$B = \frac{X}{X^2 + R^2} \text{ mhos}$$ (positive when capacitive, negative when inductive)

3.443 Admittance

$$Y = \frac{1}{Z} \text{ mhos.}$$

3.45 Impedance of parallel circuit including resistance, capacitance, and inductance

3.451 $\frac{1}{\mathring{Z}_t} = \frac{1}{\mathring{Z}_1} + \frac{1}{\mathring{Z}_2} + \frac{1}{\mathring{Z}_3} \cdots$

3.452 $\mathring{Y}_t = G_t - j\,B_t$

3.453 $|Y_t| = \sqrt{G_t^2 + B_t^2}$

3.45 Polar form of $\mathring{Z}$ (see also Vector-Conversion Table)

$|Z|\underline{/\theta} = |Z|\,(\cos\theta + j\sin\theta)$,

where $\theta = \tan^{-1}\frac{X}{R}$.

3.5 *Resonant Circuits*

3.51 Series resonant circuit

3.511 Resonant frequency

$$f = \frac{1}{2\,\pi\,\sqrt{L\,C}}$$

3.512 Merit

$$Q = \frac{X_l}{R}$$

where X_l = inductive reactance in ohms,
f = frequency in cycles,
L = inductance in henries,
C = capacitance in farads.

3.513 Q of resonant circuit

$$Q = \frac{f_{res}}{f_{off} - f_{res}}$$

where f_{res} = resonant frequency causing a voltage E across the tuned circuit,
f_{off} = frequency off resonance causing a voltage 0.707 E across the detuned circuit.

3.514 Voltage across inductance or capacitance at resonance

$$E_R = E\,Q,$$

where E = voltage applied to circuit.

3.515 Impedance at resonance

$$Z_R = R.$$

3.52 Parallel resonant circuit

3.521 Resonant frequency (with fair Q)

$$f = \frac{1}{2\,\pi\,\sqrt{L\,C}}$$

3.522 Merit

$$Q = \frac{X_l}{R_f},$$

where R_f is the series equivalent resistance of the tank circuit at the frequency f.

3.523 Impedance at resonance

$$Z_r = \frac{X^2}{R} = Q\,X = \frac{L}{R\,C}$$

3.534 Loaded tank impedance

$$Z_t = \frac{Z_r R_l}{Z_r + R_l}$$

3.53 Wavelength and frequency

3.531 $f = \frac{3 \times 10^8}{\lambda}$,

where f = frequency in cycles
λ = wavelength in meters

3.532 $\lambda = \frac{3 \times 10^8}{f}$

3.6 *Filters*

3.61 High-pass filter

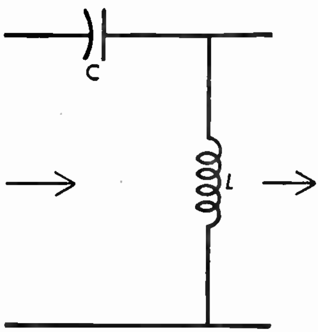

Fig. 4-3.61 High-pass filter.

3.611 $C = \frac{1}{4 \pi f R}$,

3.612 $L = \frac{R}{4 \pi f}$,

3.613 $R = \sqrt{\frac{L}{C}}$,

where C = series capacitance,
L = shunt inductance,
R = terminating resistance,
f = cutoff frequency.

3.62 Low-pass filter

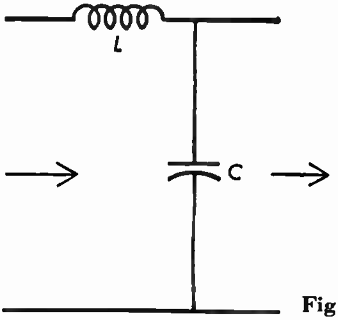

Fig. 4-3.62 Low-pass filter.

3.621 $C = \frac{1}{\pi f R}$,

3.622 $L = \frac{R}{\pi f}$,

3.623 $R = \sqrt{\frac{L}{C}}$,

where C = shunt capacitance,
L = series inductance,
R = terminating resistance,
f = cutoff frequency.

3.63 Band-pass filter

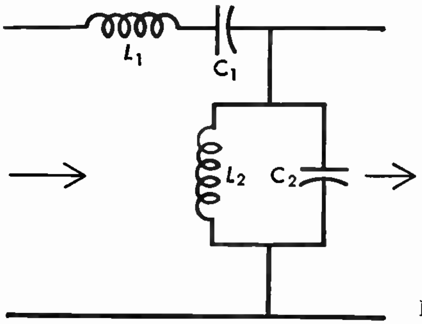

Fig. 4-3.63 Band-pass filter.

3.631 $C_1 = \frac{f_2 - f_1}{4 \pi f_1 f_2 R}$

3.632 $C_2 = \frac{1}{\pi (f_2 - f_1) R}$

3.633 $L_1 = \dfrac{R}{\pi (f_2 - f_1)}$

3.634 $L_2 = \dfrac{(f_2 - f_1) R}{4 \pi f_1 f_2}$

where f_1 = lower cutoff frequency
f_2 = upper cutoff frequency.

3.64 Band-elimination filter

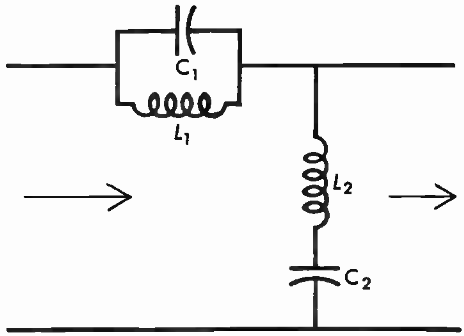

Fig. 4-3.64 Band-elimination filter (wave trap).

3.641 $C = \dfrac{1}{4 \pi (f_2 - f_1) R}$

3.642 $C_2 = \dfrac{f_2 - f_1}{\pi f_1 f_2 R}$

3.643 $L_1 = \dfrac{(f_2 - f_1) R}{\pi f_1 f_2}$

3.644 $L_2 = \dfrac{R}{4 \pi (f_2 - f_1)}$

where f_1 = lower cutoff frequency,
f_2 = upper cutoff frequency.

3.7 *Circuit Theorems*

(See note under 2.2.)

4 *Electronic Fundamentals*

4.1 *Electron Emission*

4.11 Dushman's equation

$$I = A T^2 \epsilon^{\frac{-b_0}{T}},$$

where I = current in amperes per square centimeter,
A = a material constant,
T = absolute temperature in degrees Kelvin,
b_0 = quantity characterizing the electron-emitting material,
ϵ = 2.7182.

4.12 Plate current in 2-electrode tube, limited by the space-charge effect.

$$I = K E_p^{3/2},$$

where K = the perveance, a constant determined by the geometry of the tube,
E_p = plate voltage with respect to cathode,
I = current in amperes.

4.13 Space current in triodes

$$I_p + I_g = K\left(E_g + \frac{E_p}{\mu}\right)^{\frac{3}{2}}$$

where I_p = plate current,
I_g = grid current,
K = the perveance, a constant determined by the geometry of the tube,
E_g = grid voltage,
E_p = plate voltage.

4.14 Space current in tetrode, beam, and pentode tubes

$$I_p + I_s + I_g = K\left(E_g + \frac{E_s}{\mu_s}\right)^{\frac{3}{2}}$$

where I_p = plate current,
I_s = screen current,
I_g = grid current,
K = the perveance, a constant determined by the geometry of the tube,
E_g = grid voltage,
E_s = screen voltage,
μ_s = cutoff amplification factor.

4.2 *Tube Parameters*

4.21 Amplification factor

$$\mu = \frac{\Delta E_p}{\Delta E_g} \text{ (with } I_p \text{ constant)}$$

where ΔE_p = change in plate voltage,
ΔE_g = change in grid voltage.

4.22 Plate resistance

$$r_p = \frac{\Delta E_p}{\Delta I_p} \text{ (with } E_g \text{ constant)}$$

where ΔI_p = change in plate current,
ΔE_p = change in plate voltage.

4.23 Mutual conductance

$$g_m = \frac{\Delta I_p}{\Delta E_g} \text{ (with } E_p \text{ constant)} = \frac{\mu}{r_p}$$

5 *Amplifiers*

5.1 *Voltage Amplifiers*

5.11 Output voltage across resistive plate load

$$e_o = -\mu\, e_g \frac{R_p}{R_p + r_p}$$

where e_o = output voltage,
μ = amplification factor,
$-e_g$ = signal voltage on grid,
R_p = plate-load resistor,
r_p = plate resistance.

5.12 Output voltage across plate-load impedance Z.

$$e_o = -\mu\, e_g \frac{Z}{Z + r_p}.$$

5.13 Voltage amplification

$$\text{voltage amplification} = \frac{e_o}{e_g}.$$

5.14 Biasing and by-passing

5.141 Cathode-bias resistor

$$R_c = \frac{E_g}{I_c}$$

where R_c = cathode bias resistor in ohms,
E_g = required grid bias in volts,
I_c = total cathode current in amperes.

5.142 By-pass capacitor

$$C \cong \frac{1}{2\,\pi f\,(R/10)}$$

where C = capacitance,
f = lowest frequency to be by-passed,
R = resistance across C.

5.143 Radio-frequency choke

$$L \cong \frac{10\,Z}{2\,\pi f},$$

where L = inductance of radio-frequency choke,
Z = impedance across L,
f = lowest frequency to be rejected.

5.2 *Power Amplifiers*

5.21 Power in R_p

$$P = \left(\frac{\mu\, e_g}{r_p + R_p}\right)^2$$

5.22 Maximum power output

$$P_{max} = \frac{(\mu\, e_g)^2}{4\, r_p}$$

5.23 Maximum undistorted power output when $R_p = 2\, r_p$

$$P_{undist.} = \frac{2\,(\mu\, E_g)^2}{9\, r_p}$$

5.24 Approximate audio-frequency power output of a single pentode

$$P = 0.33 \times I_p \times E_p,$$

where I_p = direct plate current in amperes,
E_p = direct plate voltage in volts.

5.25 Plate efficiency

$$\eta = \frac{P}{E_p \times I_p}$$

5.26 Power sensitivity

$$\text{Power sensitivity} = \frac{P}{E_g^2},$$

where E_g = input signal in rms volts.

5.3 *Graphical Analysis*

5.31 Load-line equations

5.311 Explicit equation for plate voltage

$$e_p = E_b - i_p R_p,$$

where e_p = plate voltage in volts,
E_b = plate-supply voltage in volts,
i_p = plate current in amperes,
R_p = load resistor in ohms.

5.312 Explicit equation for plate current

$$i_p = -\frac{1}{R_p} e_p + \frac{E_b}{R_p}$$

5.313 Power output

$$P = \frac{(I_{max} - I_{min})(E_{max} - E_{min})}{8}$$

5.314 Second harmonic distortion

$$D_2 = \frac{(I_{max} + I_{min}) 2 - I_o}{I_{max} - I_{min}},$$

where I_o = zero-signal plate current in amperes.

5.4 *Frequency Response of Resistance-Coupled Amplifier*

5.40 Medium-frequency response

$$G_m = g_m \times R',$$

where g_m = transconductance in mhos,

R' = R_p, R_g, and r_p in parallel.

5.42 High-frequency response

$$G_h = \frac{G_m}{\sqrt{1 + (R'/X_s)^2}},$$

where G_m = medium-frequency response

R' = as in 5.41

X_s = capacitive reactance of the total shunt capacitance.

5.43 Low-frequency response

$$G_l = \frac{G_m}{\sqrt{1 + (X/R'')^2}},$$

where G_m = medium-frequency response

X = reactance of coupling capacitor

R'' = equivalent resistance obtained by the parallel combination of R_p and r_p in series with R_g, viz.,

$$R'' = \frac{R_p \times r_p}{R_p + r_p} + R_g.$$

5.44 Audio-frequency coupling capacitor time constant

$$C\,R_g \cong \frac{1}{250}$$

where C = coupling capacitor in microfarads,

R_g = grid resistor in ohms.

5.5 *Gain in Decibels*

5.51 Power gain

$$N_{db} = 10 \log \frac{P_1}{P_2},$$

where N_{db} = gain in decibels,

P_1 = larger power,

P_2 = smaller power.

If P_1 is the output, N_{db} is positive; if P_1 is the input, N_{db} is negative, indicating a loss.

5.52 Voltage gain

$$N_{db} = 20 \log \frac{E_1}{E_2}$$

5.53 Current gain

$$N_{db} = 20 \log \frac{I_1}{I_2}$$

6 *Oscillators*

6.1 *Tuned-Plate Oscillator*

$$f = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{r + r_p}{r_p}},$$

where L = inductance of the plate tank circuit,
C = capacitance of the plate tank circuit,
r = resistance of the inductor,
r_p = plate resistance.

6.2 *Armstrong Oscillator*

$$f = \frac{1}{2\pi\sqrt{C\dfrac{L r_p + L'R}{r_p}}},$$

where L = inductance of the tank coil,
C = tuning capacitance,
L' = inductance of the tickler coil,
R = resistance of the tank coil,
r_p = plate resistance.

6.3 *Hartley Oscillator*

$$f = \frac{1}{2\pi\sqrt{LC}},$$

where L = total inductance across C,
C = tuning capacitance.

6.4 *Colpitts Oscillator*

$$f = \frac{1}{2\pi\sqrt{L\dfrac{C_1 C_2}{C_1 + C_2}}}$$

7 Transmitters

7.1 *Amplitude Modulation*

7.11 Current during modulation

$$I_{mod} = I_c \sqrt{1 + \frac{m^2}{2}},$$

where I_{mod} = current during modulation,
I_c = carrier current,
m = degree of modulation.

7.12 Sideband power

$$P_{sb} = 0.5\, m^2 P_c,$$

where P_{sb} = sideband power,
m = degree of modulation,
P_c = carrier power.

7.13 Bandwidth during modulation

$$f_w = 2 f_m,$$

where f_w = bandwidth,
f_m = highest modulation frequency.

7.2 *Frequency Modulation*

7.21 Frequency deviation

$$F_d = \frac{F - f}{2},$$

where F = upper frequency limit,
f = lower frequency limit.

7.22 Modulation index

$$M_i = \frac{F - f}{2 f_m} = \frac{F_d}{f_m},$$

where F = upper-frequency limit,
f = lower-frequency limit,
f_m = modulation frequency.

7.23 Audio frequency in frequency modulation

$$f_m = \frac{F_d}{M_i} = \frac{F - f}{2 M_i}$$

8 *Receivers*

8.1 *Detectors*

8.11 Efficiency

$$\eta = \frac{E_{dc}}{E_{cm}},$$

where E_{dc} = voltage developed across load resistance,
E_{cm} = peak amplitude of applied carrier.

8.111 Input resistance

$$R_{eff} = \frac{R}{\eta},$$

where R_{eff} = equivalent resistance offered by the diode plus load to applied signal,
R = load resistance,
η = efficiency.

8.12 Sound-frequency voltage

8.121 Diode detector

$$E_d = \eta \times m \times E_i,$$

where E_d = audio-frequency voltage, across the load,
η = efficiency,
m = degree of modulation,
E_i = peak input signal.

8.122 $E_p = \eta \times \mu \times m \times E_i,$

where E_p = audio-frequency voltage across load,
η = efficiency,
m = degree of modulation,
E_i = carrier amplitude.

9 *Power Supplies*

9.1 *Capacitor Input*

9.11 Ripple voltage across input capacitor

$$E_{a\text{-}c} = \frac{I\sqrt{2}}{2\pi f C}, \text{ (approximately)},$$

where I = load direct current,
f = ripply frequency,
C = capacitance of input capacitor.

9.2 *Choke Input*

9.21 Fourier analysis of a full-wave rectified pulsating direct voltage

$$y = \frac{2}{\pi} E \left(1 + \frac{2}{2^2 - 1} \cos 2x - \frac{2}{4^2 - 1} \cos 4x + \frac{2}{6^2 - 1} \cos 6x \cdots \right)$$

where E = peak of applied alternating current.

9.22 Ripple voltage at output of one filter section

$$E'_{a\text{-}c} = E_{a\text{-}c} \times \frac{1}{(2\pi f)^2 L_1 C_1},$$

where $E'_{a\text{-}c}$ = ripple voltage across C,
$E_{a\text{-}c}$ = ripple voltage at input,
L_1 = inductance,
C_1 = capacitance.

9.23 Ripple voltage at output of second filter section

$$E''_{a\text{-}c} = E_{a\text{-}c} \times \frac{1}{(2\pi f)^4 \times L_1 C_1 L_2 C_2}$$

9.24 Critical inductance

$$L = \frac{R}{1130},$$

where L = input inductance,
R = effective load resistance.

10 *Antennas and Transmission Lines*

10.11 Hertz antenna in meters

$$L = \frac{1}{2} \times \frac{3 \times 10^8}{f},$$

where L = length in meters,
f = frequency in cycles.

10.12 Hertz antenna in inches

$$L = \frac{5906}{f_m},$$

where L = length in inches,
f_m = frequency in megacycles.

10.21 Two-wire open-air transmission line

$$Z = 276 \log \frac{s}{r},$$

where Z = characteristic impedance in ohms,
s = spacing between the wire centers in inches,
r = radius of the conductor in inches.

10.22 Concentric transmission line

$$Z = 138 \log \frac{D}{d}$$

where Z = characteristic impedance in ohms,
D = inside diameter of the outside conductor in inches,
d = outside diameter of the inside conductor in inches.

10.23 Resistance of coaxial transmission line

$$R = 0.1 \left(\frac{1}{d} + \frac{1}{D}\right) \sqrt{F},$$

where R = resistance in ohms per 100 feet,
d and D as in 10.22,
F = frequency in megacycles.

10.24 Resistance of open 2-wire copper line

$$R = \frac{\sqrt{F}}{5\,d},$$

where R = resistance in ohms per 100 feet,
F = frequency in megacycles,
d = diameter of conductor in inches.

10.25 Attenuation in decibels

$$A = 4.35 \frac{R}{Z},$$

A = attenuation in decibels per 100 feet,
R = resistance in ohms per 100 feet,
Z = characteristic impedance in ohms.

10.26 Characteristic impedance

10.261 $Z_o = \sqrt{\dfrac{R + j\,2\,\pi\,f\,L}{G + j\,2\,\pi\,f\,C}}$

where $R + j\,2\,\pi\,f\,L$ = series impedance per unit length,

$G + j\,2\,\pi\,f\,C$ = shunt admittance per unit length.

10.262 $Z_o = \sqrt{Z_{Ls}\,Z_{Lo}}$,

where Z_{Ls} = input impedance when the load end is shorted out,

Z_{Lo} = input impedance when the load end is open.

10.263 $Z_o = \sqrt{Z_1\,Z_2}$,

where Z_o = impedance of quarter-wave line, matching Z_1 to Z_2.

11 *Transient Circuits*

11.1 *Resistor and Capacitor*

11.11 Charge of a capacitor

$e_c = E\,(1 - \epsilon^{-\frac{t}{RC}})$,

where e_c = voltage across C at time t seconds after closing the switch,

E = applied voltage,

R = resistance (in ohms) in series with C,

C = capacitance in farads.

11.12 Discharge of a capacitor

$e_c = E\,\epsilon^{-\frac{t}{RC}}$

11.21 Time constant

$t = R\,C$,

where t = time required for voltage to fall to $1/\epsilon$ of its initial value or to rise to $(1 - 1/\epsilon)$ of its final value.

12 *Constants worth remembering*

12.01 $X_{L_{60}} = 377$ ohms
where $X_{L_{60}}$ = reactance of a 1-henry inductor at a frequency of 60 cycles.

12.02 $X_{L_{1kc}} = 6280$ ohms = reactance of a 1-henry inductor at a frequency of 1 kilocycle.

12.03 $X_{C_{60}} = 2650$ ohms
where $X_{C_{60}}$ = reactance of a 1-microfarad capacitor at a frequency of 60 cycles.

12.04 $X_{C_{1kc}} = 159$ ohms = reactance of a 1-microfarad capacitor at a frequency of 1 kilocycle.

12.05 $\epsilon = 2.1782$

12.06 $\dfrac{1}{\epsilon} = 0.368$

12.07 $1 - \dfrac{1}{\epsilon} = 0.632$

12.08 $\log \epsilon = 0.4343$

2 Tables

TABLE I *Copper-Wire Table, American Wire Gauge (B. and S.)*

B & S Gauge No.	Diam. in Mils at 20C	Area Circular Mils	Ohms per 1,000 Ft. 25°C, 77°F.	Approx. Pounds per 1,000 Ft.
1	289.3	83,690.0	0.1264	253
2	257.6	66,370.0	0.1593	201
3	229.4	52,640.0	0.2009	159
4	204.3	41,740.0	0.2533	126
5	181.9	33,100.0	0.3195	100
6	162.0	26,250.0	0.4028	79
7	144.3	20,820.0	0.5080	63
8	128.5	16,510.0	0.6405	50
9	114.4	13,090.0	0.8077	40
10	101.9	10,380.0	1.018	31
11	90.74	8,234.0	1.284	25
12	80.81	6,530.0	1.619	20
13	71.96	5,178.0	2.042	15.7
14	64.08	4,107.0	2.575	12.4
15	57.07	3,257.0	3.247	9.8
16	50.82	2,583.0	4.094	7.8
17	45.26	2,048.0	5.163	6.2
18	40.30	1,624.0	6.510	4.9
19	35.89	1,288.0	8.210	3.9
20	31.96	1,022.0	10.35	3.1
21	28.46	810.1	13.05	2.5
22	25.35	642.4	16.46	1.9
23	22.57	509.5	20.76	1.5
24	20.10	404.0	26.17	1.2
25	17.90	320.4	33.00	0.97
26	15.94	254.1	41.62	0.77
27	14.20	201.5	52.48	0.61
28	12.64	159.8	66.17	0.48
29	11.26	126.7	83.44	0.38
30	10.03	100.5	105.2	0.30
31	8.93	79.70	132.7	0.24
32	7.95	63.21	167.3	0.19
33	7.08	50.13	211.0	0.15
34	6.31	39.75	266.0	0.12
35	5.62	31.52	335.5	0.095
36	5.00	25.00	423.0	0.076
37	4.45	19.83	533.4	0.060
38	3.96	15.72	672.6	0.048
39	3.53	12.47	848.1	0.038
40	3.14	9.89	1,069.0	0.030
41	2.80	7.84	1,323.0	0.0229
42	2.50	6.22	1,667.0	0.0189
43	2.22	4.93	2,105.0	0.0153
44	1.98	3.91	2,655.0	0.0121

TABLE II *Specific Resistances and Temperature Coefficients*

Material	Ohms per circular-mil-foot at 20C	Temperature coefficient per degree C at 20C
Aluminum	17.0	0.0049
Brass	45	.002
Constantan	295	.0000
Copper	10.4	.00393
German silver	200 to 290	.00027
Gold	14.7	.0034
Iron	59	.006
Lead	132	.004
Mercury	575	.00089
Nichrome	600	.0002
Nickel	47	.005
Silver	9.8	.004
Tin	69	.0042
Tungsten	33	.0045
Zinc	36	0.0035

Combined use of tables I and II

EXAMPLE:

Find the resistance of 1000 feet of No. 10 tin wire at a temperature of 75C.

Solution:

Using $R = \frac{\rho l}{d^2}$

we find d^2 from the third column of table I and ρ from the second column of table II for 20C.

At 20C

$$R_{20} = \frac{69 \times 1000}{10{,}380} = 6.65 \text{ ohms.}$$

Using formula 1.14, sect. IV and the value of α at 20C from the third column of table II we have

$$\begin{aligned} R_{75} &= R_{20}[1 + \alpha_{20}(T - t)] \\ &= 6.65[1 + 0.0042(75 - 20)] \\ &= 6.65[1 + 0.0042 \times 55] \\ &= 6.65[1 + 0.231] \\ &= 6.65 \times 1.231 = 8.18 \text{ ohms.} \quad \textit{Ans.} \end{aligned}$$

TABLE III *Common Logarithm Table*

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
62	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

TABLE III *Common Logarithm Table* (CONTINUED)

N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8917
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

TABLE IV *Natural Sines and Cosines**

NOTE. For cosines use right-hand column of degrees and lower line of tenths.

Deg	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
0°	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	89
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	88
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506	87
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680	86
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854	85
5	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028	84
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201	83
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374	82
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547	81
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719	80°
10°	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891	79
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062	78
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232	77
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402	76
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571	75
15	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740	74
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907	73
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074	72
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239	71
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404	70°
20°	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567	69
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730	68
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891	67
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051	66
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210	65
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368	64
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524	63
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679	62
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833	61
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985	60°
30°	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5900	0.5105	0.5120	0.5135	59
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284	58
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432	57
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577	56
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721	55
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864	54
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004	53
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143	52
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280	51
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414	50°
40°	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547	49
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678	48
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807	47
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934	46
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059	45
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg

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TABLE IV *Natural Sines and Cosines* (CONCLUDED)*

Deg	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181	44
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302	43
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420	42
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536	41
49	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649	40°
50°	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760	39
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869	38
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976	37
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080	36
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181	35
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281	34
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377	33
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471	32
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563	31
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652	30°
60°	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738	29
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821	28
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902	27
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980	26
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056	25
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128	24
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198	23
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265	22
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330	21
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391	20°
70°	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449	19
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505	18
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558	17
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608	16
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655	15
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699	14
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740	13
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778	12
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813	11
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845	10°
80°	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874	9
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900	8
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923	7
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943	6
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960	5
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	4
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985	3
87	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993	2
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998	1
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000	0°
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg

TABLE IV *Natural Tangents and Cotangents**

NOTE. For cotangents, use right-hand column of degrees and lower line of tenths.

Deg	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
0°	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	89
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	88
2	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507	87
3	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682	86
4	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857	85
5	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033	84
6	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210	83
7	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388	82
8	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566	81
9	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745	80°
10°	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926	79
11	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107	78
12	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290	77
13	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475	76
14	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2643	0.2661	75
15	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849	74
16	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038	73
17	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230	72
18	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424	71
19	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620	70°
20°	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819	69
21	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020	68
22	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224	67
23	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431	66
24	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642	65
25	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856	64
26	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073	63
27	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295	62
28	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520	61
29	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750	60°
30°	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985	59
31	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224	58
32	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469	57
33	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720	56
34	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976	55
35	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239	54
36	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508	53
37	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785	52
38	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069	51
39	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361	50°
40°	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662	49
41	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972	48
42	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293	47
43	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623	46
44	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965	45
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg

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TABLE IV *Natural Tangents and Cotangents* (CONCLUDED)*

Deg	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
45	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	44
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	43
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	42
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	41
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	40°
50°	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	39
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	38
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222	37
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	36
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	35
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	34
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	33
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	32
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	31
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	30°
60°	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	29
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	28
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	27
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	26
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	25
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355	24
66	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445	23
67	2.3599	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627	22
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916	21
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326	20°
70°	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878	19
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595	18
72	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	2.2506	17
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646	16
74	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062	15
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812	14
76	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972	13
77	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646	12
78	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970	11
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140	10°
80°	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2433	9
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264	8
82	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285	7
83	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572	6
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20	5
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95	4
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46	3
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27	2
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08	1
89	57.29	63.66	71.62	81.85	94.49	114.6	143.2	191.0	286.5	573.0	0°
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg

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TABLE V *Vector Conversion Table*

The following table can be used to change the form of vector quantities from j-notation to polar notation, and from polar notation to j-notation.

The table contains three columns:

The first column is the ratio $\frac{X}{R}$, in many cases the Q of the circuit.

The second column is the argument or the phase of the polar vector; it is based upon the equation

$$\theta = \tan^{-1}\frac{X}{R}.$$

The third column contains the modulus or the absolute magnitude of the vector in terms of X; it is based upon the equation

$$Z = X \csc\left(\tan^{-1}\frac{X}{R}\right).$$

Example 1

Express $\mathring{Z} = 3000 - j\,4000$ in polar notation.

STEP 1. $\frac{X}{R} = \frac{4000}{3000} = 1.3333.$

STEP 2. The phase angle is found in the line where $\frac{X}{R}$ equals or nearly equals 1.333. It is 53.1°, but since the j-term is negative, $\theta = -53.1°$.

STEP 3. In the third column we find

$$|Z| = 1.25\,X = 1.25 \times 4000 = 5000 \text{ ohms.}$$

Therefore $3000 - j\,4000 = 5000\underline{/-53.1°}.$

Example 2

Express $\mathring{Z} = 5000\underline{/-53.1°}$ in j-notation.

STEP 1. For the given phase angle of 53.1°, we find in the third column $1.25\,X = 5000$. From this $X = 5000/1.25 = 4000$ ohms.

STEP 2. In the same line we find

$$\frac{4000}{R} = 1.332, \text{ from which } R = \frac{4000}{1.332} = 3000 \text{ ohms.}$$

Since θ is negative we have a negative j-term.

Therefore $5000\underline{/-53.1°} = 3000 - j\,4000.$

No interpolation was performed in the above examples and the multiplications and divisions were performed on a 10-inch slide rule. However, if greater accuracy is desired, interpolations may be made and a larger slide rule, logarithm tables, or a calculating machine may be employed.

The tables also can be used to find the hypotenuse of a right triangle without extracting the square root of the sum of the squares of the sides. Thus if $I_L = 1$ ampere and $I_R = 775$ milliamperes, it is not necessary to calculate

$$|I_T| = \sqrt{1000^2 + 775^2},$$

but to find the ratio $775/1000 = 0.775$, and where the X/R ratio is 0.775 we find $Z = 1.6316\ X$, which indicates that

$$|I_T| = 1.6316 \times 775 = 1266 \text{ milliamperes.}$$

$\frac{X}{R}$	θ degrees	Z	$\frac{X}{R}$	θ degrees	Z
0.00000	0.0	$Z = R$	0.06116	3.5	16.380 X
.00175	.1	572.96 X	.06291	3.6	15.926 X
.00349	.2	286.48 X	.06467	3.7	15.496 X
.00524	.3	190.98 X	.06642	3.8	15.089 X
.00698	.4	143.24 X	.06817	3.9	14.702 X
0.00873	0.5	114.59 X	0.06993	4.0	14.335 X
0.01047	0.6	95.495 X	0.07168	4.1	13.986 X
.01222	0.7	81.853 X	.07344	4.2	13.654 X
.01396	0.8	71.622 X	.07519	4.3	13.337 X
.01571	0.9	63.664 X	.07694	4.4	13.034 X
.01746	1.0	57.299 X	0.07870	4.5	12.745 X
0.01920	1.1	52.090 X	0.08046	4.6	12.469 X
0.02095	1.2	47.750 X	.08221	4.7	12.204 X
.02269	1.3	44.077 X	.08397	4.8	11.950 X
.02444	1.4	40.930 X	.08573	4.9	11.707 X
.02618	1.5	38.201 X	.08749	5.0	11.474 X
.02793	1.6	35.814 X	0.08925	5.1	11.249 X
0.02968	1.7	33.708 X	0.09101	5.2	11.033 X
0.03143	1.8	31.836 X	.09277	5.3	10.826 X
.03317	1.9	30.161 X	.09453	5.4	10.626 X
.03492	2.0	28.654 X	.09629	5.5	10.433 X
.03667	2.1	27.290 X	.09805	5.6	10.248 X
0.03842	2.2	26.050 X	0.09981	5.7	10.068 X
0.04016	2.3	24.918 X	0.10158	5.8	9.8955 X
.04191	2.4	23.880 X	.10344	5.9	9.7283 X
.04366	2.5	22.925 X	.10510	6.0	9.5668 X
.04541	2.6	22.044 X	.10687	6.1	9.4105 X
.04716	2.7	21.228 X	0.10863	6.2	9.2593 X
0.04891	2.8	20.471 X	0.11040	6.3	9.1129 X
0.05066	2.9	19.766 X	.11217	6.4	8.9711 X
.05241	3.0	19.107 X	.11394	6.5	8.8337 X
.05416	3.1	18.491 X	.11570	6.6	8.7004 X
.05591	3.2	17.914 X	.11747	6.7	8.5711 X
.05766	3.3	17.372 X	0.11924	6.8	8.4457 X
0.05941	3.4	16.861 X			

$\frac{X}{R}$	θ degrees	Z
0.12107	6.9	8.3238 X
.12278	7.0	8.2055 X
.12456	7.1	8.0905 X
.12663	7.2	7.9787 X
.12810	7.3	7.8700 X
0.12988	7.4	7.7642 X
0.13165	7.5	7.6613 X
.13343	7.6	7.5611 X
.13521	7.7	7.4634 X
.13698	7.8	7.3683 X
0.13876	7.9	7.2756 X
0.14054	8.0	7.1853 X
.14232	8.1	7.0972 X
.14410	8.2	7.0112 X
.14588	8.3	6.9273 X
.14767	8.4	6.8454 X
0.14945	8.5	6.7755 X
0.15124	8.6	6.6874 X
.15302	8.7	6.6111 X
.15481	8.8	6.5365 X
.15660	8.9	6.4637 X
0.15838	9.0	6.3924 X
0.16017	9.1	6.3228 X
.16196	9.2	6.2546 X
.16376	9.3	6.1880 X
.16555	9.4	6.1227 X
.16734	9.5	6.0588 X
0.16914	9.6	5.9963 X
0.17093	9.7	5.9351 X
.17273	9.8	5.8751 X
.17453	9.9	5.8163 X
.17633	10.0	5.7588 X
.17813	10.1	5.7023 X
0.17993	10.2	5.6470 X
0.18173	10.3	5.5928 X
.18353	10.4	5.5396 X
.18534	10.5	5.4874 X
.18714	10.6	5.4362 X
0.18895	10.7	5.3860 X
0.19076	10.8	5.3367 X
.19257	10.9	5.2883 X
.19438	11.0	5.2408 X
.19619	11.1	5.1942 X
.19800	11.2	5.1484 X
0.19982	11.3	5.1034 X
0.20163	11.4	5.0593 X
.20345	11.5	5.0158 X
.20527	11.6	5.9732 X
.20709	11.7	5.9313 X
0.20891	11.8	5.8901 X
0.21073	11.9	5.8496 X
.21256	12.0	4.8097 X
.21438	12.1	4.7706 X
.21621	12.2	4.7320 X
.21803	12.3	4.6942 X
0.21986	12.4	4.6569 X
0.22169	12.5	4.6201 X
.22353	12.6	4.5841 X
.22536	12.7	4.5486 X
.22719	12.8	4.5137 X
0.22903	12.9	4.4793 X
0.23087	13.0	4.4454 X
.23270	13.1	4.4121 X
.23455	13.2	4.3792 X
.23693	13.3	4.3469 X
0.23823	13.4	4.3150 X
0.24008	13.5	4.2836 X
.24192	13.6	4.2527 X
.24377	13.7	4.2223 X
.24562	13.8	4.1923 X
.24747	13.9	4.1627 X
0.24933	14.0	4.1336 X
0.25118	14.1	4.1048 X
.25304	14.2	4.0765 X
.25490	14.3	4.0486 X
.25676	14.4	4.0211 X
0.25862	14.5	3.9939 X
0.26048	14.6	3.9672 X
.26234	14.7	3.9408 X
.26421	14.8	3.9147 X
.26608	14.9	3.8890 X
.26795	15.0	3.8637 X
0.26982	15.1	3.8387 X
0.27169	15.2	3.8140 X
.27357	15.3	3.7897 X
.27554	15.4	3.7657 X
.27732	15.5	3.7420 X
0.27920	15.6	3.7186 X
0.28109	15.7	3.6955 X
.28297	15.8	3.6727 X
.28486	15.9	3.6502 X
.28674	16.0	3.6279 X
0.28863	16.1	3.6060 X
0.29053	16.2	3.5843 X
.29242	16.3	3.5629 X
.29432	16.4	3.5418 X
.29621	16.5	3.5209 X
0.29811	16.6	3.5003 X

$\frac{X}{R}$	θ degrees	Z
0.30001	16.7	3.4799 X
.30192	16.8	3.4598 X
.30382	16.9	3.4399 X
.30573	17.0	3.4203 X
.30764	17.1	3.4009 X
0.30995	17.2	3.3817 X
0.31146	17.3	3.3627 X
.31338	17.4	3.3440 X
.31530	17.5	3.3255 X
.31722	17.6	3.3072 X
0.31914	17.7	3.2891 X
0.32106	17.8	3.2712 X
.32299	17.9	3.2535 X
.32492	18.0	3.2361 X
.32685	18.1	3.2188 X
0.32878	18.2	3.2017 X
0.33072	18.3	3.1848 X
.33265	18.4	3.1681 X
.33459	18.5	3.1515 X
.33654	18.6	3.1352 X
0.33848	18.7	3.1190 X
0.34043	18.8	3.1030 X
.34238	18.9	3.0872 X
.34433	19.0	3.0715 X
.34628	19.1	3.0561 X
0.34824	19.2	3.0407 X
0.35019	19.3	3.0256 X
.35215	19.4	3.0106 X
.35412	19.5	2.9957 X
.35608	19.6	2.9810 X
0.35805	19.7	2.9665 X
0.36002	19.8	2.9521 X
.36199	19.9	2.9379 X
.36397	20.0	2.9238 X
.36595	20.1	2.9098 X
.36793	20.2	2.8960 X
0.36991	20.3	2.8824 X
0.37190	20.4	2.8688 X
.37388	20.5	2.8554 X
.37587	20.6	2.8422 X
.37787	20.7	2.8290 X
0.37986	20.8	2.8160 X
0.38186	20.9	2.8032 X
.38386	21.0	2.7904 X
.38587	21.1	2.7778 X
.38787	21.2	2.7653 X
0.38988	21.3	2.7529 X

$\frac{X}{R}$	θ degrees	Z
0.39189	21.4	2.7406 X
.39391	21.5	2.7285 X
.39593	21.6	2.7165 X
.39795	21.7	2.7045 X
0.39997	21.8	2.6927 X
0.40200	21.9	2.6810 X
.40403	22.0	2.6695 X
.40606	22.1	2.6580 X
0.40809	22.2	2.6466 X
0.41013	22.3	2.6353 X
.41217	22.4	2.6242 X
.41421	22.5	2.6131 X
.41626	22.6	2.6022 X
0.41831	22.7	2.5913 X
0.42036	22.8	2.5805 X
.42242	22.9	2.5699 X
.42447	23.0	2.5593 X
.42654	23.1	2.5488 X
0.42860	23.2	2.5384 X
0.43067	23.3	2.5281 X
.43274	23.4	2.5179 X
.43481	23.5	2.5078 X
.43689	23.6	2.4978 X
0.43897	23.7	2.4879 X
0.44105	23.8	2.4780 X
.44314	23.9	2.4683 X
.44523	24.0	2.4586 X
.44732	24.1	2.4490 X
0.44942	24.2	2.4395 X
0.45152	24.3	2.4300 X
.45362	24.4	2.4207 X
.45573	24.5	2.4114 X
.45783	24.6	2.4022 X
0.45995	24.7	2.3931 X
0.46206	24.8	2.3841 X
.46418	24.9	2.3751 X
.46631	25.0	2.3662 X
0.46843	25.1	2.3574 X
0.47056	25.2	2.3486 X
.47270	25.3	2.3399 X
.47483	25.4	2.3313 X
.47697	25.5	2.3228 X
0.47912	25.6	2.3143 X
0.48127	25.7	2.3059 X
.48342	25.8	2.2976 X
.48557	25.9	2.2894 X
.48773	26.0	2.2812 X
0.48989	26.1	2.2730 X

$\frac{X}{R}$	θ degrees	Z	$\frac{X}{R}$	θ degrees	Z
0.49206	26.2	2.2650 X	**0.60086**	31.0	1.9416 X
.49423	26.3	2.2570 X	.60324	31.1	1.9360 X
.49640	26.4	2.2490 X	.60562	31.2	1.9304 X
0.49858	26.5	2.2411 X	0.60801	31.3	1.9248 X
0.50076	26.6	2.2333 X	**0.61040**	31.4	1.9193 X
.50295	26.7	2.2256 X	.61280	31.5	1.9139 X
.50514	26.8	2.2179 X	.61520	31.6	1.9084 X
.50733	26.9	2.2103 X	0.61761	31.7	1.9030 X
0.50952	27.0	2.2027 X	**0.62003**	31.8	1.8977 X
0.51172	27.1	2.1952 X	.62244	31.9	1.8924 X
.51393	27.2	2.1877 X	.62487	32.0	1.8871 X
.51614	27.3	2.1803 X	.62730	32.1	1.8818 X
0.51835	27.4	2.1730 X	0.62973	32.2	1.8766 X
0.52057	27.5	2.1657 X	**0.63217**	32.3	1.8714 X
.52279	27.6	2.1584 X	.63462	32.4	1.8663 X
.52501	27.7	2.1513 X	.63707	32.5	1.8611 X
.52724	27.8	2.1441 X	0.63953	32.6	1.8561 X
0.52947	27.9	2.1371 X	**0.64199**	32.7	1.8510 X
0.53171	28.0	2.1300 X	.64466	32.8	1.8460 X
.53395	28.1	2.1231 X	.64693	32.9	1.8410 X
.53619	28.2	2.1162 X	0.64941	33.0	1.8361 X
0.53844	28.3	2.1093 X	**0.65189**	33.1	1.8311 X
0.54070	28.4	2.1025 X	.65438	33.2	1.8263 X
.54295	28.5	2.0957 X	.65688	33.3	1.8214 X
.54522	28.6	2.0890 X	0.65938	33.4	1.8166 X
.54748	28.7	2.0824 X	**0.66188**	33.5	1.8118 X
0.54975	28.8	2.0757 X	.66440	33.6	1.8070 X
0.55203	28.9	2.0692 X	.66692	33.7	1.8023 X
.55431	29.0	2.0627 X	0.66944	33.8	1.7976 X
.55659	29.1	2.0562 X	**0.67197**	33.9	1.7929 X
0.55888	29.2	2.0498 X	.67451	34.0	1.7883 X
0.56117	29.3	2.0434 X	.67705	34.1	1.7837 X
.56347	29.4	2.0370 X	0.67160	34.2	1.7791 X
.56577	29.5	2.0308 X	**0.68215**	34.3	1.7745 X
0.56808	29.6	2.0245 X	.68471	34.4	1.7700 X
0.57039	29.7	2.0183 X	.68728	34.5	1.7655 X
.57270	29.8	2.0122 X	0.68985	34.6	1.7610 X
.57502	29.9	2.0061 X	**0.69243**	34.7	1.7566 X
.57735	30.0	2.0000 X	.69502	34.8	1.7522 X
0.57968	30.1	1.9940 X	0.69761	34.9	1.7478 X
0.58201	30.2	1.9880 X	**0.70021**	35.0	1.7434 X
.58435	30.3	1.9820 X	.70281	35.1	1.7391 X
.58670	30.4	1.9761 X	.70542	35.2	1.7348 X
0.58904	30.5	1.9703 X	0.70804	35.3	1.7305 X
0.59140	30.6	1.9645 X	**0.71066**	35.4	1.7263 X
.59376	30.7	1.9587 X	.71329	35.5	1.7220 X
.59612	30.8	1.9530 X	.71593	35.6	1.7178 X
0.59849	30.9	1.9473 X	0.71857	35.7	1.7137 X

$\frac{X}{R}$	θ degrees	Z
0.72122	35.8	1.7095 X
.72388	35.9	1.7054 X
.72654	36.0	1.7013 X
0.72921	36.1	1.6972 X
0.73189	36.2	1.6932 X
.73457	36.3	1.6891 X
.73726	36.4	1.6851 X
0.73996	36.5	1.6812 X
0.74266	36.6	1.6772 X
.74538	36.7	1.6733 X
0.74809	36.8	1.6694 X
0.75082	36.9	1.6655 X
.75355	37.0	1.6616 X
.75629	37.1	1.6578 X
0.75904	37.2	1.6540 X
0.76179	37.3	1.6502 X
.76546	37.4	1.6464 X
0.76733	37.5	1.6427 X
0.77010	37.6	1.6389 X
.77289	37.7	1.6352 X
.77568	37.8	1.6316 X
0.77848	37.9	1.6279 X
0.78128	38.0	1.6243 X
.78410	38.1	1.6206 X
.78692	38.2	1.6170 X
0.78975	38.3	1.6135 X
0.79259	38.4	1.6099 X
.79543	38.5	1.6064 X
0.79829	38.6	1.6029 X
0.80115	38.7	1.5994 X
.80402	38.8	1.5959 X
.80690	38.9	1.5924 X
0.80978	39.0	1.5890 X
0.81268	39.1	1.5856 X
.81558	39.2	1.5822 X
0.81849	39.3	1.5788 X
0.82141	39.4	1.5755 X
.82434	39.5	1.5721 X
0.82727	39.6	1.5688 X
0.83022	39.7	1.5655 X
.83317	39.8	1.5622 X
.83613	39.9	1.5590 X
0.83910	40.0	1.5557 X
0.84208	40.1	1.5525 X
.84506	40.2	1.5493 X
0.84806	40.3	1.5461 X

$\frac{X}{R}$	θ degrees	Z
0.85107	40.4	1.5429 X
.85408	40.5	1.5398 X
0.85710	40.6	1.5366 X
0.86013	40.7	1.5335 X
.86318	40.8	1.5304 X
.86623	40.9	1.5273 X
0.86929	41.0	1.5242 X
0.87235	41.1	1.5212 X
.87543	41.2	1.5182 X
0.87852	41.3	1.5151 X
0.88162	41.4	1.5121 X
.88472	41.5	1.5092 X
0.88784	41.6	1.5062 X
0.89097	41.7	1.5032 X
.89410	41.8	1.5003 X
0.89725	41.9	1.4974 X
0.90040	42.0	1.4945 X
.90357	42.1	1.4916 X
.90674	42.2	1.4887 X
0.90993	42.3	1.4858 X
0.91312	42.4	1.4830 X
.91633	42.5	1.4802 X
0.91955	42.6	1.4774 X
0.92277	42.7	1.4746 X
.92601	42.8	1.4718 X
0.92926	42.9	1.4690 X
0.93251	43.0	1.4663 X
.93578	43.1	1.4635 X
0.93906	43.2	1.4608 X
0.94235	43.3	1.4581 X
.94565	43.4	1.4554 X
0.94896	43.5	1.4527 X
0.95229	43.6	1.4501 X
.95562	43.7	1.4474 X
0.95896	43.8	1.4448 X
0.96232	43.9	1.4422 X
.96569	44.0	1.4395 X
0.96907	44.1	1.4370 X
0.97246	44.2	1.4344 X
.97586	44.3	1.4318 X
0.97927	44.4	1.4292 X
0.98270	44.5	1.4267 X
.98613	44.6	1.4242 X
0.98958	44.7	1.4217 X
0.99304	44.8	1.4192 X
0.99651	44.9	1.4167 X

$\frac{X}{R}$	θ degrees	Z	$\frac{X}{R}$	θ degrees	Z
1.0000	45.0	1.4142 X	1.2045	50.3	1.2997 X
1.0035	45.1	1.4117 X	1.2088	50.4	1.2978 X
1.0070	45.2	1.4093 X	1.2131	50.5	1.2960 X
1.0105	45.3	1.4069 X	1.2174	50.6	1.2941 X
1.0141	45.4	1.4040 X	1.2218	50.7	1.2922 X
1.0176	45.5	1.4020 X	1.2261	50.8	1.2904 X
1.0212	45.6	1.3996 X	1.2305	50.9	1.2886 X
1.0247	45.7	1.3972 X	1.2349	51.0	1.2867 X
1.0283	45.8	1.3949 X	1.2393	51.1	1.2849 X
1.0319	45.9	1.3925 X	1.2437	51.2	1.2831 X
1.0355	46.0	1.3902 X	1.2482	51.3	1.2813 X
1.0391	46.1	1.3878 X	1.2527	51.4	1.2795 X
1.0428	46.2	1.3855 X	1.2572	51.5	1.2778 X
1.0464	46.3	1.3832 X	1.2617	51.6	1.2760 X
1.0501	46.4	1.3809 X	1.2662	51.7	1.2742 X
1.0538	46.5	1.3786 X	1.2708	51.8	1.2725 X
1.0575	46.6	1.3763 X	1.2753	51.9	1.2707 X
1.0612	46.7	1.3740 X	1.2799	52.0	1.2690 X
1.0649	46.8	1.3718 X	1.2845	52.1	1.2673 X
1.0686	46.9	1.3695 X	1.2892	52.2	1.2656 X
1.0724	47.0	1.3673 X	1.2938	52.3	1.2639 X
1.0761	47.1	1.3651 X	1.2985	52.4	1.2622 X
1.0799	47.2	1.3629 X	1.3032	52.5	1.2605 X
1.0837	47.3	1.3607 X	1.3079	52.6	1.2588 X
1.0875	47.4	1.3585 X	1.3127	52.7	1.2571 X
1.0913	47.5	1.3563 X	1.3174	52.8	1.2554 X
1.0951	47.6	1.3542 X	1.3222	52.9	1.2538 X
1.0990	47.7	1.3520 X	1.3270	53.0	1.2521 X
1.1028	47.8	1.3499 X	1.3319	53.1	1.2505 X
1.1067	47.9	1.3477 X	1.3367	53.2	1.2488 X
1.1106	48.0	1.3456 X	1.3416	53.3	1.2472 X
1.1145	48.1	1.3435 X	1.3465	53.4	1.2456 X
1.1184	48.2	1.3414 X	1.3514	53.5	1.2440 X
1.1224	48.3	1.3393 X	1.3564	53.6	1.2424 X
1.1263	48.4	1.3372 X	1.3613	53.7	1.2408 X
1.1303	48.5	1.3352 X	1.3663	53.8	1.2392 X
1.1343	48.6	1.3331 X	1.3713	53.9	1.2376 X
1.1383	48.7	1.3311 X	1.3764	54.0	1.2361 X
1.1423	48.8	1.3290 X	1.3814	54.1	1.2345 X
1.1463	48.9	1.3270 X	1.3865	54.2	1.2329 X
1.1504	49.0	1.3250 X	1.3916	54.3	1.2314 X
1.1544	49.1	1.3230 X	1.3968	54.4	1.2298 X
1.1585	49.2	1.3210 X	1.4019	54.5	1.2283 X
1.1626	49.3	1.3190 X	1.4071	54.6	1.2268 X
1.1667	49.4	1.3170 X	1.4123	54.7	1.2253 X
1.1708	49.5	1.3151 X	1.4176	54.8	1.2238 X
1.1750	49.6	1.3131 X	1.4228	54.9	1.2223 X
1.1791	49.7	1.3112 X	1.4281	55.0	1.2208 X
1.1833	49.8	1.3092 X	1.4335	55.1	1.2193 X
1.1875	49.9	1.3073 X	1.4388	55.2	1.2178 X
1.1917	50.0	1.3054 X	1.4442	55.3	1.2163 X
1.1960	50.1	1.3035 X	1.4496	55.4	1.2149 X
1.2002	50.2	1.3016 X	1.4550	55.5	1.2134 X

$\frac{X}{R}$	θ degrees	Z
1.4605	55.6	1.2119 X
1.4659	55.7	1.2105 X
1.4714	55.8	1.2091 X
1.4770	55.9	1.2076 X
1.4826	56.0	1.2062 X
1.4881	56.1	1.2048 X
1.4938	56.2	1.2034 X
1.4994	56.3	1.2020 X
1.5051	56.4	1.2006 X
1.5108	56.5	1.1992 X
1.5166	56.6	1.1978 X
1.5223	56.7	1.1964 X
1.5282	56.8	1.1951 X
1.5340	56.9	1.1937 X
1.5399	57.0	1.1924 X
1.5458	57.1	1.1910 X
1.5517	57.2	1.1897 X
1.5577	57.3	1.1883 X
1.5636	57.4	1.1870 X
1.5697	57.5	1.1857 X
1.5757	57.6	1.1844 X
1.5818	57.7	1.1831 X
1.5880	57.8	1.1818 X
1.5941	57.9	1.1805 X
1.6003	58.0	1.1792 X
1.6006	58.1	1.1779 X
1.6128	58.2	1.1766 X
1.6191	58.3	1.1753 X
1.6255	58.4	1.1741 X
1.6318	58.5	1.1728 X
1.6383	58.6	1.1716 X
1.6447	58.7	1.1703 X
1.6512	58.8	1.1691 X
1.6577	58.9	1.1678 X
1.6643	59.0	1.1666 X
1.6709	59.1	1.1654 X
1.6775	59.2	1.1642 X
1.6842	59.3	1.1630 X
1.6909	59.4	1.1618 X
1.6977	59.5	1.1606 X
1.7044	59.6	1.1594 X
1.7113	59.7	1.1582 X
1.7182	59.8	1.1570 X
1.7251	59.9	1.1559 X
1.7320	60.0	1.1547 X
1.7390	60.1	1.1535 X
1.7461	60.2	1.1524 X
1.7532	60.3	1.1512 X
1.7603	60.4	1.1501 X
1.7675	60.5	1.1489 X
1.7747	60.6	1.1478 X
1.7820	60.7	1.1467 X

$\frac{X}{R}$	θ degrees	Z
1.7893	60.8	1.1456 X
1.7966	60.9	1.1445 X
1.8040	61.0	1.1433 X
1.8115	61.1	1.1422 X
1.8190	61.2	1.1411 X
1.8265	61.3	1.1401 X
1.8341	61.4	1.1390 X
1.8418	61.5	1.1379 X
1.8495	61.6	1.1368 X
1.8572	61.7	1.1357 X
1.8650	61.8	1.1347 X
1.8728	61.9	1.1336 X
1.8807	62.0	1.1326 X
1.8887	62.1	1.1315 X
1.8967	62.2	1.1305 X
1.9047	62.3	1.1294 X
1.9128	62.4	1.1284 X
1.9210	62.5	1.1274 X
1.9292	62.6	1.1264 X
1.9375	62.7	1.1253 X
1.9458	62.8	1.1243 X
1.9542	62.9	1.1233 X
1.9626	63.0	1.1223 X
1.9711	63.1	1.1213 X
1.9797	63.2	1.1203 X
1.9883	63.3	1.1193 X
1.9969	63.4	1.1184 X
2.0057	63.5	1.1174 X
2.0145	63.6	1.1164 X
2.0233	63.7	1.1155 X
2.0323	63.8	1.1145 X
2.0412	63.9	1.1135 X
2.0503	64.0	1.1126 X
2.0594	64.1	1.1116 X
2.0686	64.2	1.1107 X
2.0778	64.3	1.1098 X
2.0872	64.4	1.1088 X
2.0965	64.5	1.1079 X
2.1060	64.6	1.1070 X
2.1115	64.7	1.1061 X
2.1251	64.8	1.1052 X
2.1348	64.9	1.1043 X
2.1445	65.0	1.1034 X
2.1543	65.1	1.1025 X
2.1642	65.2	1.1016 X
2.1741	65.3	1.1007 X
2.1842	65.4	1.0998 X
2.1943	65.5	1.0989 X
2.2045	65.6	1.0981 X
2.2147	65.7	1.0972 X
2.2251	65.8	1.0963 X

$\frac{X}{R}$	θ degrees	Z
2.2355	65.9	1.0955 X
2.2460	66.0	1.0946 X
2.2566	66.1	1.0938 X
2.2673	66.2	1.0929 X
2.2781	66.3	1.0921 X
2.2889	66.4	1.0913 X
2.2998	66.5	1.0904 X
2.3109	66.6	1.0896 X
2.3220	66.7	1.0888 X
2.3332	66.8	1.0880 X
2.3445	66.9	1.0872 X
2.3558	67.0	1.0864 X
2.3673	67.1	1.0855 X
2.3789	67.2	1.0847 X
2.3906	67.3	1.0840 X
2.4023	67.4	1.0832 X
2.4142	67.5	1.0824 X
2.4262	67.6	1.0816 X
2.4382	67.7	1.0808 X
2.4504	67.8	1.0801 X
2.4627	67.9	1.0793 X
2.4751	68.0	1.0785 X
2.4876	68.1	1.0778 X
2.5002	68.2	1.0770 X
2.5129	68.3	1.0763 X
2.5257	68.4	1.0755 X
2.5386	68.5	1.0748 X
2.5517	68.6	1.0740 X
2.5649	68.7	1.0733 X
2.5781	68.8	1.0726 X
2.5916	68.9	1.0719 X
2.6051	69.0	1.0711 X
2.6187	69.1	1.0704 X
2.6325	69.2	1.0697 X
2.6464	69.3	1.0690 X
2.6604	69.4	1.0683 X
2.6746	69.5	1.0676 X
2.6889	69.6	1.0669 X
2.7033	69.7	1.0662 X
2.7179	69.8	1.0655 X
2.7326	69.9	1.0648 X
2.7475	70.0	1.0642 X
2.7625	70.1	1.0635 X
2.7776	70.2	1.0628 X
2.7929	70.3	1.0622 X
2.8083	70.4	1.0615 X
2.8239	70.5	1.0608 X
2.8396	70.6	1.0602 X
2.8555	70.7	1.0595 X
2.8716	70.8	1.0589 X
2.8878	70.9	1.0582 X
2.9042	71.0	1.0576 X
2.9208	71.1	1.0570 X
2.9375	71.2	1.0563 X
2.9544	71.3	1.0557 X
2.9714	71.4	1.0551 X
2.9887	71.5	1.0545 X
3.0061	71.6	1.0539 X
3.0237	71.7	1.0533 X
3.0415	71.8	1.0527 X
3.0595	71.9	1.0521 X
3.0777	72.0	1.0515 X
3.0960	72.1	1.0509 X
3.1146	72.2	1.0503 X
3.1334	72.3	1.0497 X
3.1524	72.4	1.0491 X
3.1716	72.5	1.0485 X
3.1910	72.6	1.0479 X
3.2106	72.7	1.0474 X
3.2305	72.8	1.0468 X
3.2505	72.9	1.0462 X
3.2708	73.0	1.0457 X
3.2914	73.1	1.0451 X
3.3121	73.2	1.0466 X
3.3332	73.3	1.0440 X
3.3544	73.4	1.0435 X
3.3759	73.5	1.0429 X
3.3977	73.6	1.0424 X
3.4197	73.7	1.0419 X
3.4420	73.8	1.0413 X
3.4646	73.9	1.0408 X
3.4874	74.0	1.0403 X
3.5105	74.1	1.0398 X
3.5339	74.2	1.0393 X
3.5576	74.3	1.0387 X
3.5816	74.4	1.0382 X
3.6059	74.5	1.0377 X
3.6305	74.6	1.0372 X
3.6554	74.7	1.0367 X
3.6806	74.8	1.0362 X
3.7062	74.9	1.0358 X
3.7320	75.0	1.0353 X
3.7583	75.1	1.0348 X
3.7848	75.2	1.0343 X
3.8118	75.3	1.0338 X
3.8390	75.4	1.0334 X
3.8667	75.5	1.0329 X
3.8947	75.6	1.0324 X
3.9231	75.7	1.0320 X
3.9520	75.8	1.0315 X
3.9812	75.9	1.0311 X

$\frac{X}{R}$	θ degrees	Z
4.0108	76.0	1.0306 X
4.0408	76.1	1.0302 X
4.0713	76.2	1.0297 X
4.1022	76.3	1.0293 X
4.1335	76.4	1.0288 X
4.1653	76.5	1.0284 X
4.1976	76.6	1.0280 X
4.2303	76.7	1.0276 X
4.2635	76.8	1.0271 X
4.2972	76.9	1.0267 X
4.3315	77.0	1.0263 X
4.3662	77.1	1.0259 X
4.4015	77.2	1.0225 X
4.4373	77.3	1.0251 X
4.4737	77.4	1.0247 X
4.5107	77.5	1.0243 X
4.5483	77.6	1.0239 X
4.5864	77.7	1.0235 X
4.6252	77.8	1.0231 X
4.6646	77.9	1.0227 X
4.7046	78.0	1.0223 X
4.7453	78.1	1.0220 X
4.7867	78.2	1.0216 X
4.8288	78.3	1.0212 X
4.8716	78.4	1.0208 X
4.9151	78.5	1.0205 X
4.9594	78.6	1.0201 X
5.0045	78.7	1.0198 X
5.0504	78.8	1.0194 X
5.0970	78.9	1.0191 X
5.1445	79.0	1.0187 X
5.1929	79.1	1.0184 X
5.2422	79.2	1.0180 X
5.2923	79.3	1.0177 X
5.3434	79.4	1.0174 X
5.3955	79.5	1.0170 X
5.4486	79.6	1.0167 X
5.5026	79.7	1.0164 X
5.5578	79.8	1.0160 X
5.6140	79.9	1.0157 X
5.6713	80.0	1.0154 X
5.7297	80.1	1.0151 X
5.7894	80.2	1.0148 X
5.8502	80.3	1.0145 X
5.9123	80.4	1.0142 X
5.9758	80.5	1.0139 X
6.0405	80.6	1.0136 X
6.1066	80.7	1.0133 X
6.1742	80.8	1.0130 X
6.2432	80.9	1.0127 X
6.3137	81.0	1.0125 X
6.3859	81.1	1.0122 X
6.4596	81.2	1.0119 X
6.5350	81.3	1.0116 X
6.6122	81.4	1.0114 X
6.6911	81.5	1.0111 X
6.7720	81.6	1.0108 X
6.8547	81.7	1.0106 X
6.9395	81.8	1.0103 X
7.0264	81.9	1.0101 X
7.1154	82.0	1.0098 X
7.2066	82.1	1.0096 X
7.3002	82.2	1.0093 X
7.3961	82.3	1.0091 X
7.4946	82.4	1.0089 X
7.5957	82.5	1.0086 X
7.6996	82.6	1.0084 X
7.8062	82.7	1.0082 X
7.9158	82.8	1.0079 X
8.0285	82.9	1.0077 X
8.1443	83.0	1.0075 X
8.2635	83.1	1.0073 X
8.3862	83.2	1.0071 X
8.5126	83.3	1.0069 X
8.6427	83.4	1.0067 X
8.7769	83.5	1.0065 X
8.9152	83.6	1.0063 X
9.0579	83.7	1.0061 X
9.2051	83.8	1.0059 X
9.3572	83.9	1.0057 X
9.5144	84.0	1.0055 X
9.6768	84.1	1.0053 X
9.8448	84.2	1.0051 X
10.019	84.3	1.0050 X
10.199	84.4	1.0048 X
10.385	84.5	1.0046 X
10.579	84.6	1.0044 X
10.780	84.7	1.0043 X
10.988	84.8	1.0041 X
11.205	84.9	1.0040 X
11.430	85.0	1.0038 X
11.664	85.1	1.0037 X
11.909	85.2	1.0035 X
12.163	85.3	1.0034 X
12.429	85.4	1.0032 X
12.706	85.5	1.0031 X
12.996	85.6	1.0029 X

$\frac{X}{R}$	θ degrees	Z	$\frac{X}{R}$	θ degrees	Z
13.229	85.7	1.0028 X	27.271	87.9	1.0007 X
13.617	85.8	1.0027 X	28.636	88.0	1 0006 X
13.951	85.9	1.0026 X	30.145	88.1	1.0005 X
14.301	86.0	1.0024 X	31.820	88.2	1.0005 X
14.668	86.1	1.0023 X	33.693	88.3	1.0004 X
15.056	86.2	1.0022 X	35.800	88.4	1.0004 X
15.464	86.3	1.0021 X	38.188	88.5	1.0003 X
15.894	86.4	1.0020 X	40.917	88.6	1.0003 X
16.350	86.5	1.0019 X	44.066	88.7	1.0002 X
16.832	86.6	1.0018 X	47.739	88.8	1.0002 X
17.343	86.7	1.0017 X	52.081	88.9	1.0002 X
17.886	86.8	1.0016 X	57.290	89.0	1.0001 X
18.464	86.9	1.0015 X	63.657	89.1	1.0001 X
19.081	87.0	1.0014 X	71.615	89.2	1.0001 X
19.740	87.1	1.0013 X	81.847	89.3	1.0001 X
20.446	87.2	1.0012 X	95.489	89.4	1.0000 X
21.205	87.3	1.0011 X	114.59	89.5	1.0000 X
22.022	87.4	1.0010 X	143.24	89.6	1.0000 X
23.904	87.5	1.0009 X	190.98	89.7	1.0000 X
23.859	87.6	1.0009 X	286.48	89.8	1.0000 X
24.898	87.7	1.0008 X	572.96	89.9	1.0000 X
26.031	87.8	1.0007 X	R = 0	90.0	1.0000 X

APPENDIX *References*
Answers to Section II
Index

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Answers to Section II

1 *Circuit Components*

2-1.01 8.41 ohms.
2-1.02 3.08 feet.
2-1.03 1.3 inches.
2-1.04 0.644 inch.
2-1.05 9.31 mils, No. 31 AWG.
2-1.06 12.4 per cent.
2-1.07 0.888 ohm.
2-1.08 0.0818 ohm, 10.9 to 1.
2-1.09 yes.
2-1.10 500 micromicrofarads.
2-1.11 10 to 40 micromicrofarads, 30 to 140 micromicrofarads, 7.5 to 31.1 micromicrofarads, 40 to 180 micromicrofarads.
2-1.12 0.83 per cent.
2-1.13 7300 ohms.
2-1.14 0.1485 microfarads.
2-1.15 9 microfarads.
2-1.16 1.96 microfarads.
2-1.17 10 capacitors.
2-1.18 255 microcoulombs.
2-1.19 61.7 micromicrofarads.
2-1.20 6 plates.
2-1.21 a) 55.8 henries, b) 14.2 henries, c) 13.85 henries, d) 3.1 henries.
2-1.22 20 per cent.
2-1.23 132 microhenries.
2-1.24 1.15 millihenries.
2-1.25 1.57 inches.
2-1.26 6.45 to 1.
2-1.27 3740 ohms.
2-1.28 84.4 turns.
2-1.29 528 microhenries.
2-1.30 1.21 amperes.
2-1.31 4570 volts.
2-1.32 28.6 volts.
2-1.33 5 turns, 2.25 amperes.
2-1.34 0.0833 ampere, 0.0915 ampere.
2-1.35 44,550 ohms.
2-1.36 54.7 to 1.
2-1.37 6.55 per cent.

2 *Direct-Current Circuits*

2-2.01 0.545 ampere.
2-2.02 52.5 vols, 67.5 volts.
2-2.03 202 ohms.
2-2.04 46.8 ohms, 292 watts.
2-2.05 149,968 ohms.
2-2.06 37,600 ohms, 2.06 watts.
2-2.07 —112.5 volts.
2-2.08 12.7 ohms.
2-2.09 0.856 volt, 1.2 volts, 1.54 volts, 1.9 volts.
2-2.10 20,800 ohms, 8330 ohms, 12,500 ohms.
2-2.11 19.5 volts.
2-2.12 3.062 ohms, 32.7 amperes, 4.86 amperes.
2-2.13 32.7 amperes, 4.86 amperes.
2-2.14 4.7 amperes.
2-2.15 7.38 amperes.
2-2.16 3.062 ohms, 32.7 amperes, 4.86 amperes.

3 *Alternating-Current Circuits*

2-3.01 400 meters, 85.7 meters.
2-3.02 0.01168 microsecond.
2-3.03 −6.89 amperes.
2-3.04 81.5 volts.

2-3.05 188 volts.

2-3.06 130 volts.

2-3.07 $e = 100 \sin 377\, t$,
$i = 2 \sin (377\, t + 45°)$,
0.129 or 1.99 amperes.

2-3.08 57.2 amperes.

2-3.09 6.5 watts.

2-3.10 0.252 henries.

2-3.11 6.75 volts.

2-3.12 144.3 volts.

2-3.13 18.3 degrees, voltage leading, 95 per cent.

2-3.14 $262 - j\,296$ vector ohms, 66.3 per cent.

2-3.15 66,000 ohms, 0.792 volts.

2-3.16 $38{,}900 + j\,27{,}950$ vector ohms, 80.5% per cent.

2-3.17 $84 - j\,704$ vector ohms, 710 ohms, 83.2 degrees.

2-3.18 $114 - j\,299$ vector ohms, 320 ohms, −69.2 degrees.

2-3.19 84 milliamperes, 74 per cent.

2-3.20 9.7 kilocycles.

2-3.21 8 volts, 1206 volts.

2-3.22 0.28 microfarad.

2-3.23 67 ohms.

2-3.24 41.2 megacycles, 13,900 ohms, 21.6.

2-3.25 99.

2-3.26 23.5 ohms, 60.

2-3.27 27.5 micromicrofarads, 55, 90,750 ohms.

2-3.28 6880 ohms, 83,500 ohms,
4.35 with load,
52.7 without load.

2-3.29 100 ohms, 18.75 microhenries
18.75 micromicrofarads.

2-3.30 0.1985 henries,
0.354 microfarad.

2-3.31 55.8 microhenries,
4.55 micromicrofarads.

2-3.32 0.266 henry,
0.00471 microfarad.

2-3.33 L_1 0.199 henry,
$L_2 = 245$ microhenries,
$C_1 = 0.392$ micromicrofarad,
$C_2 = 319$ micromicrofarads.

2-3.34 $L_1 = 87$ microhenries,
$L_2 = 0.159$ henry,
$C_1 = 398$ micromicrofarads,
$C_2 = 0.218$ micromicrofarad.

4 *Vacuum-Tube Fundamentals*

2-4.01 209 milliamperes per square centimeter.

2-4.02 92.5 milliamperes.

2-4.03 6.1 milliamperes.

2-4.04 17.5

2-4.05 1500 micromhos.

2-4.06 11.88 volts.

2-4.07 5.94.

2-4.08 14.7.

2-4.09 250 volts, 13.9 milliamperes.

2-4.10 170 volts, 4.4 milliamperes.

2-4.11 135 volts, 185 volts.

2-4.12 130 volts.

2-4.13 60 volts.

2-4.14 $E'_{sc} = 200$ volts,
$E'_g = -14.4$ volts,
$I'_p = 39$ milliamperes.
$I'_{sc} = 1.75$ milliamperes,
$r'_p = 37{,}200$ ohms,
$R'_p = 4730$ ohms,
$g'_m = 4710$ micromhos,
$p' = 6.15$ watts.

2-4.15 $E'_g = 5.75$ volts,
$I'_p = 4.88$ milliamperes,
$r'_p = 12{,}000$ ohms,
$g'_m = 1700$ micromhos.

2-4.16 1.2 watts.

2-4.17 5000 ohms, 1.44 watts.

2-4.18 10,000 ohms, 1.28 watts.

2-4.19 34.4 per cent.

2-4.20 63.7 per cent.

2-4.21 13.

5 *Amplifiers*

2-5.01 32.3
2-5.02 28.3.
2-5.03 30.4.
2-5.04 30.2.
2-5.05 28.3.
2-5.06 30.4.
2-5.07 0.008 microfarad.
2-5.08 15.4 volts.
2-5.09 17,850 ohms, 0.315 watt dissipation, 1 watt rating.
2-5.10 1070 ohms, 4 watts, 29.7 microfarads, 100 volts.
2-5.11 707.
2-5.12 −17.8 decibels.
2-5.13 10 per cent.
2-5.14 0.2375 milliwatt.
2-5.15 The first.
2-5.16 The first.
2-5.17 0.104 watt.
2-5.18 7.8 per cent.
2-5.19 15.75.
2-5.20 11.4.
2-5.21 22.8.
2-5.22 52.1.
2-5.23 33.6 decibels.
2-5.24 102.9 decibels.
2-5.25 0.00304 microfarad, 4.2 microfarads.
2-5.26 3.82 millihenries.
2-5.27 346 ohms, 2 watts.
2-5.28 173 ohms, 5 watts.
2-5.29 71,300 ohms, 1 watt.
2-5.30 21.7 per cent, 57.5 per cent, 65.3 per cent.
2-5.31 8.83 volts.
2-5.32 1.19 to 1.
2-5.33 15,400 ohms.
2-5.34 No. 19.6 watts.
2-5.35 34.3 volts.
2-5.36 −105 volts.
2-5.37 202 milliamperes.
2-5.38 190 volts, 927 volts.
2-5.39 135.5 degrees.
2-5.40 3910 ohms.
2-5.41 21.4 microhenries, 58 micromicrofarads.
2-5.42 2.13 to 1.

6 *Oscillators*

2-6.01 18.7 micromicrofarads.
2-6.02 32.3 micromicrofarads.
2-6.03 1624.4 kilocycles, 1636.6 kilocycles.
2-6.04 0.375 per cent.
2-6.05 1.5 kilocycles approximately.
2-6.06 11.75.
2-6.07 6.8 cycles.
2-6.08 180 cycles.
2-6.09 41.4 micromicrofarads.
2-6.10 Grid: 12.5 microhenries, 41.4 micromicrofarads; Plate: 15.9 microhenries, 32.4 micromicrofarads.
2-6.11 33.5 microhenries, 295 micromicrofarads.

7 *Transmitters*

2-7.01 18.6 per cent.
2-7.02 9.5 amperes.
2-7.03 112.5 watts, 28.1 watts.
2-7.04 Equal signals after detection.
2-7.05 1.5 kilocycles, 1750.75 kilocycles, 1749.25 kilocycles.
2-7.06 1.9 to 1.
2-7.07 3.6 to 1.
2-7.08 28.8 per cent.
2-7.09 4380 volts.
2-7.10 206 watts.
2-7.11 75 per cent.
2-7.12 1.16 megohms.
2-7.13 17 kilowatts.
2-7.14 1694 to 1699 kilocycles, 1701 to 1706 kilocycles.
2-7.15 67.4 watts.

2-7.16 7700 ohms.
2-7.17 1.15 to 1.
2-7.18 150 cycles.
2-7.19 24.2 watts.
2-7.20 64 watts.
2-7.21 92.77 megacycles, 20 kilocycles.
2-7.22 100, 15.6, 5.
2-7.23 81,100 ohms.
2-7.24 6.25 kilocycles, 7.1 megacycles.
2-7.25 10 kilocycles.
2-7.26 56.725 to 56.876 megacycles.
2-7.27 —

8 *Receivers*

2-8.01 (a) 5320 to 2050 kilocycles,
(b) 3060 to 1800 kilocycles,
(c) 4060 to 3210 kilocycles.
2-8.02 Oscillator frequency: 1725 or 1375 kilocycles.
Image frequency: 1200 kilocycles.
2-8.03 2112 kilocycles or 4512 kilocycles.
2-8.04 2012 kilocycles.
2-8.05 1586 kilocycles.
2-8.06 456 kilocycles.
2-8.07 132 microhenries, 68.4 micromicrofarads.
2-8.08 90 per cent.
2-8.09 270 microwatts.
2-8.10 416,000 ohms.
2-8.11 14,430 ohms, 15,000 ohms, 600 ohms.
2-8.12 240 volts.
2-8.13 9.12, 0.797.
2-8.14 6.38 per cent.
2-8.15 70.8 per cent, 20 per cent.
2-8.16 3.85 per cent.
2-8.17 657,000 ohms.
2-8.18 6.25 per cent.
2-8.19 4.65 watts.
2-8.20 77.5 per cent.

9 *Power Supplies*

2-9.01 9.4 volts, 0.167 volt.
2-9.02 12.5 volts, 0.121 volt.
2-9.03 0.604 per cent, 0.00754 per cent.
2-9.04 0.598 per cent, 0.0075 per cent.
2-9.05 4.51 henries.
2-9.06 105,000 ohms, 1.16 watts.
2-9.07 31.3 per cent, 28.7 per cent, 19,400 ohms, 17.5 watts.
2-9.08 13.25 henries, 3.32 henries.
2-9.09 4.42 henries, 9.45 microfarads, 4.42 henries.
2-9.10 428 ohms.
2-9.11 21.25 microfarads.
2-9.12 487 volts.
2-9.13 1392 volts r.m.s.
2-9.14 150 ohms, 13.5 watts.
2-9.15 150 ohms, 126 ohms, 13.5 watts, 0.315 watt.
2-9.16 11.84 volts.
2-9.17 $I = 0.237$ ampere, etc.
2-9.18 2250 ohms, 5.62 watts.
2-9.19 255 ohms, 5.7 watts.

10 *Antennas*

2-10.01 66.8 feet, 15.6 feet.
2-10.02 14.42 megacycles.
2-10.03 182 feet.
2-10.04 5.46 amperes.
2-10.05 8.94 amperes.
2-10.06 2.79 times.
2-10.07 450 watts.
2-10.08 610 ohms.
2-10.09 68 ohms.
2-10.10 6.22 kilowatts.
2-10.11 10.3 amperes.
2-10.12 733 volts.

2-10.13 1300 watts.
2-10.14 0.07 decibel per 100 feet.
2-10.15 86.5 per cent.
2-10.16 0.0121 decibel per 100 feet.
2-10.17 0.0605 decibel.
2-10.18 8000 feet.
2-10.19 $620 - j\,154$ vector ohms.

11 *Television*

2-11.01 7500 mi/hour.
2-11.02 4.8×10^6 picture elements.
2-11.03 use radius of earth = 6370 kilometers.
2-11.04 5950 square miles.
2-11.05 52.5 miles.
2-11.06 4.5 centimeters, 133 volts per centimeter.
2-11.07 41.7 lines square centimeter.
2-11.08 2.14 to 1; 2.42 to 1.
2-11.09 0.0559 inch.
2-11.10 22.2 feet.
2-11.11 7.8 feet.
2-11.12 97 per cent.
2-11.13 28 per cent, 7.8 per cent.
2-11.14 77.7 per cent, 22.3 per cent; 91.79 per cent, 8.21 per cent.
2-11.15 0, 3.9, 0.195, 4.02, 0.201, 9.5, 5.8, 9.8, 5.8, 9.8, 5.97, 7.54, 0.714, 4.33, 412, all in volts.
2-11.16 0, 10, 6.1, −3.9, −0.195, 9.805, 5.98, −4.02, −0.201, 9.79, 0.5, −9.5, −5.8, 4.2, 0.2, −9.8, −5.97, 4.03, 2.46, −7.54, −0.714, 9.286, 5.67, −4.33, −0.412 all in volts.
2-11.17 3.08 volts.
2-11.18 3.8 volts.
2-11.19 7.1 volts, 13.78 volts.
2-11.20 2.06 volts.
2-11.21 35.
2-11.22 9.
2-11.23 Voltage across focus control: 417 volts (from 918 volts to 1335 volts), midposition 1226.5 volts; voltage across position controls: 136.5 volts, midposition 2534 volts.
2-11.24 22.
2-11.25 0.785.
2-11.26 0.784, 337 ohms.
2-11.27 0.106 volt, 0.091 volt.
2-11.28 1.92 volts, 1.75 volts.
2-11.29 40 inches, 32.8 inches, 6.56 linear magnification, 43.1 area magnification.
2-11.30 Master oscillator 15,625 cycles, doubler 31,250 cycles, countdowns: 6250, 1250, 250, 50 cycles.
2-11.31 Master oscillator
65.75 waves
doubler
131.5 waves
first divide-by-7
18.75 waves
first divide-by-5
3.75 waves;
The second divide-by-5 and the divide-by-3 cannot be observed, at the sweep frequency of 240 cycles.
2-11.32 0.0969 H, 6.15 microseconds.
2-11.33 1.69 centimeters.
2-11.34 50 degrees.
2-11.35 77,235.
2-11.36 106.6, 602.
2-11.37 5.22, 7.5; type 6AQ5 is preferable.
2-11.38 111 megacycles, 548 megacycles, 68.5 megacycles, 467 megacycles.
2-11.39 1713 feet.
2-11.40 26 visible bars.
2-11.41 15.8 visible bars.
2-11.42 77.25 megacycles, 81.75 megacycles, 103 megacycles, 25.75 megacycles.

12 *Measurements*

2-12.01 96.5 ohms.
2-12.02 1710 ohms.
2-12.03 2 ohms, 2.1 ohms.
2-12.04 18,000 ohms, 6700 ohms, 3000 ohms, 1125 ohms.
2-12.05 16,700 ohms.
2-12.06 20 henries, 4.75 per cent.
2-12.07 0.01 microfarad.
2-12.08 0.375 megohm, 1500 micromicrofarads.
2-12.09 115 microhenries.
2-12.10 240,000 ohms.
2-12.11 0.1205 ohm.
2-12.12 249,970 ohms.
2-12.13 40.4 milliamperes.
2-12.14 0.114 microfarad.
2-12.15 15.42 henries, 15.31, 0.8 per cent approximately.
2-12.16 4.06 megohms.

13 *Industrial and Control Circuits*

2-13.01 479 microseconds.
2-13.02 0.01352 second.
2-13.03 175 cycles.
2-13.04 4.6 RC second.
2-13.05 27.6 seconds.
2-13.06 2720 ohms.
2-13.07 950 amperes.
2-13.08 0.068 ohm, 4.25 kilowatts.

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