From Atoms to Amperes

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ALSO OF INTEREST

PREFACE

In early days when lightning lit up the skies, mortals down below quaked in terror lest they should be destroyed by fire from heaven or by a thunderbolt, the weapon of the Gods. Electricity was then the enemy of man. Yet as the years have passed it has been discovered in other forms, it has been tamed and the enormous power harnessed. But to do this man has had to learn some of electricity's innermost secrets, not all, for Nature is elusive and still keeps much to herself. Electricity is now both friend and foe, a helpful servant yet at times a deadly killer.

Mostly we see electricity in terms of volts and amperes. Behind all this however is the content of matter itself, everything consisting of myriads of miniature universes with electrons whirling around within atoms just as the planets hurtle round the Sun. These atomic "solar systems" jostle with each other and join up in their own peculiar ways to produce all the different materials on earth. Then too, in an electric current, electrons are shaken loose from their normal habitats to speed through open spaces within matter like meteors flashing across the sky. Electricity is the very essence of our existence, what a pity it is never seen.

Herein we first remind ourselves of the difficulty of studying objects of infinitesimally small size and that so many of Nature's everyday tools are shrouded in mystery. A short foray into the past however shows how scientists have discovered what most of these do although not *how* they do it. This leads to the subject of electricity via the atom and electron. Then with a good basic understanding of this it becomes easier to get on with electrical generators, semiconductors and in fact, all other electrical devices.

For whom is the book written? Clearly for anybody with a tinge of curiosity. It makes easy reading even for those who have never got on well with the subject of physics or have forgotten all they ever knew. Especially has the level of mathematics required been deliberately kept low so a moderate knowledge of school algebra and a little basic trigonometry sees us through. As with most text books, this one is sprinkled

with calculations to add realism to the ideas put forward. These are especially for those among us who are in love with their calculators. However, skipping over the arithmetic and noting the answers only, detracts very little from the main message.

F. A. Wilson

CONTENTS

Chapter 1

AVOIDING THE DEEP END

We are not physicists whose mission in life is to dig deeply into the mysteries of the universe but people wishing to know something about what makes electricity tick. It is as well therefore to decide first just how deeply we wish to get involved. There is certainly no lack of information available but much of it requires an above average expertise in mathematics so things must be kept relatively simple. Also we need to look more carefully at some of the concepts previously accepted without much thought for the more we find out about electricity the more shaky some of our ideas become. An illustration of this is perhaps given by a look at the general idea of "space".

First thoughts are that space is what is seen on looking up into a clear sky. Beyond the earth's atmosphere there is nothing, just a vacuum. But apart from the satellites and the bits which drop off, can we truly expect to find a perfect vacuum up there? Indeed not, there are always a few odd particles around.

Astronauts in space communicate with the ground by radio. Stars emit waves and the Sun dispenses light and heat via space. Then there are the gravitational and magnetic fields of Jupiter, Neptune, Saturn and the rest which astrologers tell us shape our worthy characters. Unquestionably all these influences are present up there. Early scientists invented the *ether* for the transmission of radio and other waves but we are not so lucky. The old-fashioned ether has gone and it must be accepted that waves propagate through nothing. Perhaps space is not so empty after all.

What this little revelation indicates is that to think deeply about some aspects of Nature gives rise to many uncertainties. This is just the way of things so we leave the more complicated puzzles to the physicists and rely on the fact that our intimacy with the electron can still be improved by studying it at a modest level. There must be no struggling with those aspects which baffle understanding.

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Now this is hardly an encouraging start, exposing our ignorance of Nature's box of tricks. However, by observation and experiment over the centuries scientists have managed to set down many of the *rules* or *laws* by which Nature operates. She may keep us in the dark as to her motives but she cannot stop us prying into her working arrangements. This is what must be remembered when pictures are painted of happenings in the atomic world. These are no more than our inadequate way of visualizing things so infinitesimal in size as to be truly beyond human comprehension. Happily such pictures are becoming clearer and further experimentation tends to confirm that we are on the right track. This can be summed up by saying that phenomena such as force, charge, field and gravitation must be accepted as what Nature has provided and even though it is not possible to describe clearly what they *are*, much is known about what they *do*. In fact without this knowledge, modern electronic magic which continues apace, could never come to fruition. Keeping this in mind we make a start by studying the laws pertaining to the items in Nature's tool-kit for these laws tell us how and why electrons act as they do. Oddly enough we can best get to grips with the intricacies surrounding our tiny subject by applying these same laws to heavenly bodies and spacecraft first. But next some revision.

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Chapter 2

SHRINKING THE NUMBERS

To many of us one thousand pounds is a lot of money. But imagine a thousand packets, each containing one thousand pounds. There is one million, a lovely thought and human beings can handle such a number — just. Given larger numbers still, things tend to get difficult. Who can truly grasp the significance of the World population at around 5,000,000,000? As the number of noughts grows so does our confusion. Fortunately that does not prevent us from working with very large or small numbers. Scientists long ago decided that so many noughts everywhere made arithmetic unwieldy and so was born *Scientific Notation.*

Let us see the problem as it affects us here. This is a number which in physics crops up time and time again. Written "longhand" it is:

$$0.000,000,000,000,000,000,160,2$$

(the electron charge — more about this later). Believe it or not, there are other numbers commonly used twice this length. Scientific notation is a method of writing such numbers down in a shortened form with subsequent calculations greatly simplified although some accuracy is inevitably forfeited. A little revision first however:

$10 \times 10 = 100$ or 10^2
(ten squared, or to the power of 2)

$10 \times 10 \times 10 = 1000$ or 10^3
(ten cubed, or to the power of 3)

$1000 \times 1000 = 10^3 \times 10^3 = 1,000,000$ or 10^6
(ten to the sixth, or to the power of six)

so the multiples of 10 can be expressed as "powers of ten" using the little raised number or *exponent* (from Latin —

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placed outside) to indicate the power. Thus 10^9 stands for 1,000,000,000 which is 1 multiplied by 10, 9 times. Note also that $10^1 = 10$ and $10^0 = 1$.

Multiplying by 10 increases the exponent by 1, multiplying by 100 increases it by 2 etc. from which comes the general rule:

$$10^x \times 10^y = 10^{(x+y)}$$

Dividing by 10 decreases the exponent by 1, hence

$$10^x \div 10^y = 10^{(x-y)}$$

Let us test this out by multiplying 10,000 by 100.

$$10,000 = 10^4 \qquad 100 = 10^2$$

$$\therefore \quad 10^4 \times 10^2 = 10^{4+2} = 10^6 \text{ (one million)}$$

Conversely dividing 1,000,000 by 100 gives:

$$10^6 \div 10^2 = 10^{6-2} = 10^4$$

or equally,

$$10^6 \times 10^{-2} = 10^{6-2} = 10^4 \ .$$

So we move between division and multiplication by changing the sign of the exponent.

With numbers containing a decimal point, multiplying by 10 merely shifts the point one place to the right, dividing by 10 shifts it one place to the left, so

16.02 multiplied by 100 becomes 1602,

i.e. $\quad 16.02 \times 10^2 = 1602$

also $\quad$ 16.02 divided by $10^2 = 16.02 \times 10^{-2} = 0.1602$

and in the same way $16.02 \times 10^{-4} = 0.001602$,
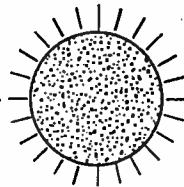
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Fig. 2.1 Large numbers and their expression in scientific notation.

a case of shifting decimal points and changing the exponent accordingly.

This is what scientific notation does so the World population of 5,000,000,000 becomes simply 5×10^9 and the almost unmanageable number for the electron becomes 16.02×10^{-20}.

To standardize the technique the number is usually *normalized* meaning that only one digit precedes the decimal point, 16.02×10^{-20} therefore changes to 1.602×10^{-19}.

Other examples of numbers quoted in scientific notation are:

$$854 = 8.54 \times 10^2$$

$$2847.3 = 2.8473 \times 10^3$$

$$0.00039 = 3.9 \times 10^{-4} .$$

This is where scientific calculators come in so readers owning one and perhaps finding little use for some of the extra facilities may now see that their money was well spent. Most scientific calculators and personal computers handle scientific notation directly. They cannot display exponents in the form of raised numbers but do so in other ways, for example 10^{19} might be displayed as 19 or E19, depending on the manufacturer. Also all entries may be automatically normalized.

If merely in possession of an ordinary pocket calculator, don't despair, a little mental arithmetic can take care of the exponents while the calculator handles any nasty looking fractions.

As we become involved with things atomic such fantastically small and large numbers will appear frequently. It is pleasantly surprising to find that although one cannot possibly have any conception as to their physical representation, they do begin to make sense with practice and in fact we ourselves then tend to think directly in scientific notation. Finally, so that we are not deceived about the magnitude of numbers represented by small exponents, Figure 2.1 is a reminder.

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Chapter 3

THE USE OF SI UNITS

For centuries the system of measurement used in the UK has been the *Imperial*. It has served us well, but with limitations especially in that multiples of the units are all of different values, for example, *12* inches to the foot, *3* feet to the yard and *5½* yards to the rod. It comes as no surprise therefore to find that the days of these units are well and truly over.

In France, changes were made when the country was getting back together again following the Revolution. There they introduced the *Metric* system which had the advantage of all multiples being the same, ten, an easy figure to work with. The basic unit of length is the *metre* (from Greek, *metron*, measure) which was supposedly one ten-millionth (10^{-7}) of the distance from the North Pole to the Equator, as shown in Figure 3.1. Having established this particular length, the unit of mass followed as that of one cubic centimetre of water at 0°C and known as the *gram*.

Scientists the world over soon discovered the benefits of the metric system and for them the basic units became the centimetre, gram and second, known as the CGS system. Subsequent improvements have brought about international agreement on the present system which is now based on the metre, kilogram and second [MKS (one kilogram = 1000 grams)]. Its title is in French and is "Système International d'Unités" (SI). SI is now accepted throughout a large part of the World.

It was not considered judicious to decimalize time because the hour, minute and second are so well established throughout the World. The circle also has been left with 360 degrees.

3.1 BASIC UNITS

Whichever system is used, its basic units can only be chosen arbitrarily as the French did when they decided on the length of the metre. The SI system has seven basic units but only the

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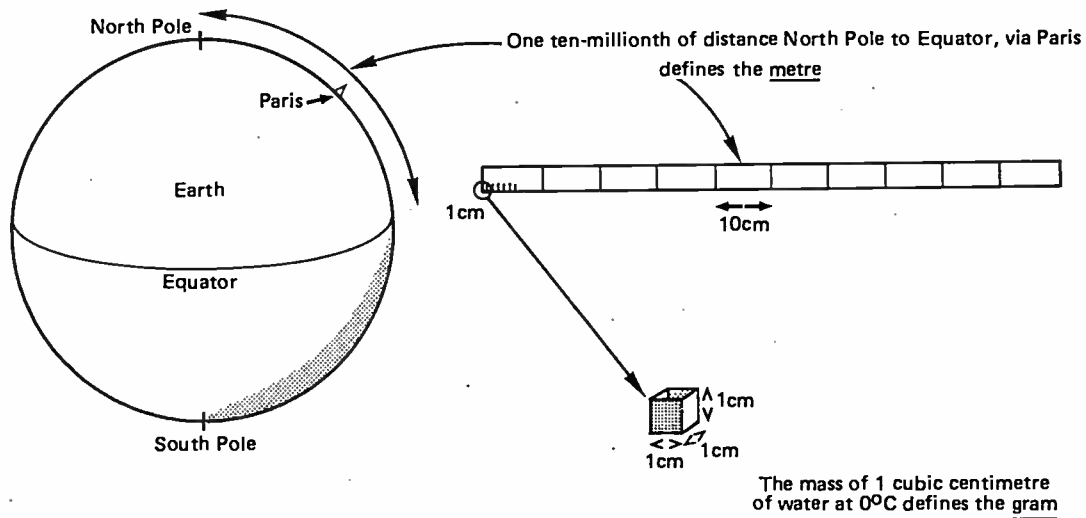
Fig. 3.1 Adoption of metre and gram by French Academy of Sciences, 1791.

first five are of interest here. They are shown in Table 3.1. From these many other units are derived and those which we need are defined and explained as we go.

Table 3.1 BASIC SI UNITS

QUANTITY	NAME OF UNIT	SYMBOL
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	kelvin	K

3.2 MULTIPLES AND SUB-MULTIPLES

Larger or smaller quantities of any basic SI unit are obtained by multiplying or dividing the unit by multiples of 10. An appropriate prefix is then added to the unit name. Those we are likely to need are given in Table 3.2.

Table 3.2 SI PREFIXES

MULTIPLICATION FACTOR	PREFIX	SYMBOL
$\times 10^9$ (a thousand million times)	giga	G
$\times 10^6$ (a million times)	mega	M
$\times 10^3$ (a thousand times)	kilo	k
$\times 10^{-2}$ (a hundredth)	centi	c
$\times 10^{-3}$ (a thousandth)	milli	m
$\times 10^{-6}$ (a millionth)	micro	μ
$\times 10^{-9}$ (a thousand millionth)	nano	n
$\times 10^{-12}$ (a million millionth)	pico	p

The original metric unit, the gram, has the symbol g so as shown in Table 3.2, the SI unit of one kilogram is represented by kg and, using symbols, 1 kg = 10^3g. One quantity not

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conforming exactly to the system, yet in common use, is the metric *tonne* which is equivalent to 1000 kg.

3.3 SOME EQUIVALENTS

For readers not yet fully metricated a few approximate equivalents may be helpful:

1 metre (m) = 39.37 inches (just over 1 yard)

2.54 centimetres (cm) = 1 inch

1 kilometre (km) is about five-eighths of 1 mile

there are about 2.2 pounds to 1 kilogram (kg)

there are nearly 30 grams (g) to 1 ounce

the metric tonne and Imperial ton are almost the same

1 litre (l) is about 1¾ pints.

Chapter 4

NATURE'S MOTIVATORS

Readers who achieved high grades in physics at school and have memories to match may write this chapter off as kid's stuff. On the other hand, for the rest of us, revision and understanding of the basic principles around which life revolves are essential. It is surprising how many of these maxims, unexciting when first studied, brighten up when uses are found for them. All that follows in this chapter is essential basic knowledge for truly getting to grips with electricity for such is the latter's complexity.

There is a scientist whose name has become a household word, Einstein. His fame arose mainly from his theories about the nature of space and time. These appeared to contradict all that was known at the time about how the universe functioned. Laws had been produced which apparently explained things perfectly well yet here was a physicist casting doubt on well-established principles. Time has since changed opinion in his favour. Fortunately Einstein's theories are effective only at speeds approaching that of light and therefore do not materially affect the laws we meet first because these deal mostly with things moving at relatively low speeds. We will see later that the electron is an incredibly fast mover whereupon Einstein's theories become of consequence.

Therefore remember that what follows next is not always strictly true but generally the modifications required by Einstein make little difference. We meet the gentleman again later.

4.1 FIRST SOME REVISION

This Section contains a few useful reminders. Some terms, speed and velocity, for example, may be used as synonymous in ordinary conversation. There it does not matter but in scientific study it does. It is important to get the terms right to be sure of saying exactly what we mean.

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4.1.1 Scalars and Vectors

A *scalar* quantity is one which has only magnitude and therefore is expressed by a single number. As an example, the temperature of bath water might be quoted as 38 degrees Celsius. This describes the temperature completely; no other information is required. Weight, length, area and volume are also examples.

A *vector* quantity on the other hand has both magnitude and direction. For example, quoting the speed of a ship in mid-Atlantic only tells us a limited amount. What is missing is the present position, and direction of travel. The magnitude and direction of any quantity are illustrated on paper by a single arrowed line drawn to scale and at the appropriate angle to some reference as shown in Figure 4.1(i).

When two or more conditions which are described by vectors act together, their resultant effect cannot be found by simple addition. This is best explained by example, bearing in mind that the mariner's speed in knots (nautical miles per hour) becomes metres per second in SI (m/s or ms^{-1} for short).

Suppose the ship is moving at 10 m/s (about 20 knots) in a northerly direction, represented by a vector as shown in Figure 4.1(ii). Further let a passenger run across the ship in a west—east direction at 2 m/s. To find the speed and direction of the passenger (who at the same time is moving north at 10 m/s and east at 2 m/s), the vectors are added as shown by completing the parallelogram (in this case a rectangle) and drawing the diagonal. All vectors emanate from a single point, O. The angle, θ and length of the resultant velocity R, can be measured if the vectors are drawn to scale, or alternatively calculated, in which case a scale drawing is unnecessary.

To calculate R we use Pythagoras' Theorem:

$$R^2 = 10^2 + 2^2 = 100 + 4$$

$$\therefore \quad R = \sqrt{104} = 10.2$$

$$\tan \theta = 2/10 = 0.2$$

$$\therefore \quad \theta = \tan^{-1} 0.2 = 11.3°$$
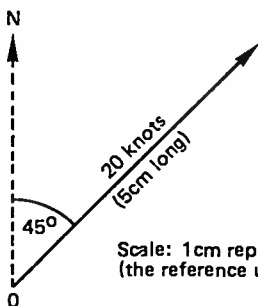
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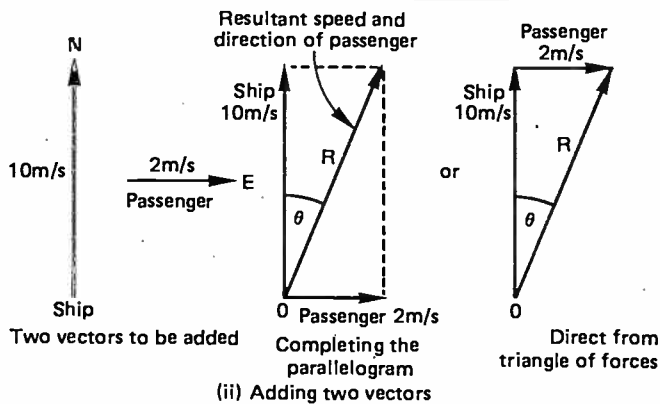
Fig. 4.1 Vectors.

The passenger is therefore effectively moving at 10.2 m/s in a direction 11.3° East of North.

The alternative construction employs a *triangle of forces*. Here the second vector (2 m/s) is added to the first at the end remote from the point 0. Note that the arrows on the two vectors run in the same direction round the triangle. The arrow on the third side (the resultant) is in the opposite direction.

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4.1.2 Speed and Velocity

Speed is expressed in magnitude only and is a scalar quantity. *Velocity* on the other hand involves both magnitude and direction and is accordingly a vector quantity. The ship in Figure 4.1(ii) has a velocity of 10 m/s due north but more simply a speed of 10 m/s.

4.1.3 Mass and Weight

These are often used interchangeably but they should not be. *Weight* refers to the gravitational pull exerted by the earth on a body. Even over the earth's surface the weight of an item varies and it certainly decreases with altitude, for example, in the extreme, the weight of an astronaut in space can be zero. The term *mass* is not so commonly used except in engineering where it describes the quantity of matter contained by a body but in an unvarying way. Mass is not easily defined, so we derive a definition through *inertia*. This is another rather nebulous quantity and is the reluctance of a body to alter its rate of motion. If it is moving it resists being accelerated, slowed or stopped, equally if at rest it resists being moved. A practical illustration is in the effort needed to push a football filled with air compared with that required for one filled with sand. The latter has greater mass and greater inertia. Our own inertia becomes evident on standing in a bus or tube train. With rapid acceleration or braking we fall backwards or forwards while trying to resist the change. Clearly there is a link between mass and inertia, so we are able to define mass by the acceleration imparted to a body when a certain push or pull is exerted on it. An exact formula for mass follows later. The unit of both mass and weight is the kilogram.

Later it will be found that, scientifically, weight is also measured as a force (of gravity). For the present, however there is a reminder provided by the astronaut whose weight decreases as the distance from earth increases but with the mass remaining constant.

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4.1.4 Time and Motion

There is a set of formulae, generally known as the *Equations of Motion*, some of which are needed later:

(i) velocity is the distance moved during a given time (the velocity must be constant):

$$v = s/t \quad \text{where } v = \text{velocity, } s = \text{distance, } t = \text{time}$$

$$\therefore \quad s = vt \ .$$

When the velocity is not constant but varies smoothly:

distance = average velocity × time

$$s = (u + v)/2 \times t \quad \text{where } u = \text{initial and } v = \text{final velocity.}$$

(ii) acceleration is the rate of change of velocity, i.e. change of velocity during a given time (the acceleration must be constant):

$$a = (v - u)/t \quad \text{where } a = \text{acceleration}$$

from which,

(iii) final velocity = initial velocity + (acceleration × time)

$$v = u + at \quad \text{also } at = v - u$$

(iv) formula for final velocity in terms of acceleration and distance:

from $s = (u + v)/2 \times t$

since $v = u + at \quad \therefore t = (v - u)/a$

$$\therefore \quad s = (v + u)(v - u)/2a$$

$$\therefore \quad s = (v^2 - u^2)/2a$$

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$$\therefore \quad 2as = v^2 - u^2$$

$$\therefore \quad v^2 = u^2 + 2as$$

4.1.5 Momentum

In a way momentum describes the vigour underlying the velocity of a body. A feather hitting a window at high velocity is unlikely to cause damage but a golf ball at the same velocity smashes the glass. Evidently something more than just velocity has done the damage and this is clearly the mass of the ball. Yet mass on its own is insufficient to break the glass if the ball velocity is very low. Hence the definition:

momentum = mass x velocity

or in shorthand,

$p = mv$

With velocity in the equation, momentum must be a vector quantity.

Sir Isaac Newton, the great English mathematician and physicist gave us his "Laws of Motion" and the *First Law of Motion* states that:

"a body will continue in a state of rest or of uniform motion in a straight line unless acted upon by some external force."

This is a profound doctrine and on earth we have no opportunity of seeing the law fully in action because all moving bodies are subject to the effects of friction and gravity. Looked at in another way the Law is also saying that:

"the momentum of a body is constant unless an external force acts."

The *Third Law of Motion* can be interpreted as:

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> "to every action there is an equal and opposite reaction".

Thus when holding an article still in the palm of the hand, its weight is exerting a force downwards on the hand, the hand is therefore obliged to exert an equal and upward force so that there is no motion.

These two Laws lead to the principle of *Conservation of Momentum:*

> "provided that no external force acts, the total momentum of a group of objects is constant".

This idea can be examined in a practical way by using the classic example of rifle and bullet. These two form a "group of objects" and while the rifle is being aimed the total momentum is zero because neither rifle nor bullet has velocity. When the trigger is pulled the explosion forces the bullet forward with a certain momentum (mass x velocity). Because the total momentum must remain at zero, the rifle recoils, i.e. it acquires a momentum in the opposite direction. The forward momentum of the bullet equals the backward momentum of the rifle.

This is when things fly apart. In the atomic world we are more interested in what happens when they collide. For collisions the Law still holds good, the total momentum is unchanged after the event. This can best be illustrated as follows:

A lorry of mass 5 tonnes travelling at 72 km per hour runs into the rear of a car of mass 1 tonne travelling in the same direction at 18 km/h. Assuming both engines cease to function after the collision and ignoring the effects of friction, what is the velocity of the combined vehicles immediately following the collision?

First restate the velocities in the units we require:

72 km/h = 72000/3600 = 20 m/s

18 km/h = 18000/3600 = 5 m/s

then:

Momentum of lorry before collision =

5000 kg x 20 m/s = 10^5 kg m/s

Momentum of car before collision =

1000 kg x 5 m/s = 5000 kg m/s

Total momentum = 105,000 kg m/s

Total mass of combined vehicles = 6000 kg

∴ Initial velocity after collision =

$$\frac{momentum}{mass} = \frac{105000}{6000} = 17.5 \text{ m/s}$$

In illustrating the principle of Conservation of Momentum the example also shows how some of the momentum from the lorry is suddenly added to that of the car:

Momentum of car after collision =

1000 x 17.5 = 17500 kg m/s

Momentum of lorry after collision =

5000 x 17.5 = 87500 kg m/s

showing that a momentum of 12,500 kg m/s has been transferred from the lorry to the car. As this takes place suddenly it is fortunate that the lorry driver has a seat belt and the car has headrests.

Momentum is a vector quantity yet in the above example this does not seem to have been taken seriously. This is because the two vehicles are travelling in the same direction. A little thought shows that the vector triangle [e.g. Fig.4.1 (ii)] flattens out into a straight line. In Figure 4.2(i) AB represents the momentum of the lorry and BC that of the car.

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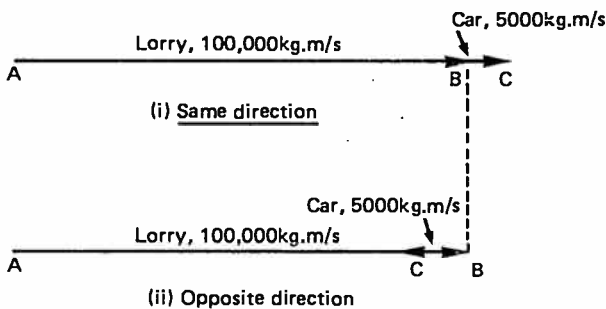
Fig. 4.2 Vector "triangles" when momenta are in-line.

The resultant momentum is the third side of the "flat" triangle, AC. Should the two vehicles have been travelling in opposite directions and meet head-on, (ii) of the Figure shows how the resultant is again AC. The principle has not been violated because, being in the opposite direction, one of the momenta is negative compared with the other.

4.2 FORCE

We must next expand on Newton's First Law (Sect.4.1.5) because the term "force" has been used without definition. *Force* is again one of Nature's tools, unseen, unheard but essential to motion. Nothing moves without force. Nothing stops without force. Movement with increasing or decreasing velocity signifies *acceleration* and the greater the force, the greater the acceleration. Conversely, the greater the mass of a body, the less the acceleration for mass has the property of inertia which opposes changes in motion (Sect.4.1.3), hence:

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$$\text{acceleration } \alpha \; \frac{\text{force}}{\text{mass}} \qquad (\alpha = \text{varies as})$$

This results in Newton's *Second Law of Motion:*

"Force is equal to mass multiplied by acceleration",

in shorthand: $\qquad F = m\,a$

so defining a force exactly. One unit of force is known appropriately as a *newton* (N) and it is that force which when applied to a mass of 1 kg accelerates it by 1 metre per second per second (m/s^2 or ms^{-2}).

A push or pull can come from any direction so force is a vector quantity.

Force is defined in terms of mass rather than weight because although on earth weight might seem to be more practical, physics is concerned with matters affecting the whole universe. Accordingly the definition must be in terms of a quantity which does not vary anywhere. A good illustration of this is through the force exerted by a rocket motor, force could hardly be defined in terms of weight in a situation where weight has no meaning.

As mentioned earlier, only Einstein has upset the Newtonian applecart and this is to such a tiny degree that we continue at present without him.

4.3 WORK

When a force moves a body then *work* is done and because force has been defined as above, work follows as:

work done $=$ force applied x distance moved

in shorthand: $\quad W = F\,s$

where $W =$ work, $F =$ force, $s =$ distance.

There is a special unit for work known as the *joule* (J — after James Prescott Joule, an English scientist) and one joule is the work done when a force of one newton acts through a distance of one metre.

Suppose a workman pushes a trolley by exerting a force of 80N. If the trolley is moved 50m, he has done 80N x 50m = 4000 newton-metres or joules of work.

We can get the newton into perspective by holding a 100g or 3½ oz weight steady in the hand. This needs a force of about 1N upwards on the weight. The workman exerting a force of 80N is therefore pushing with a force equivalent to carrying about an 8 kg or 18 lb weight.

4.4 ENERGY

Of Nature's artifices with which she holds the World together and keeps its parts moving, energy is the most fundamental, it is the *prime mover.* Not existing in any material sense, we have to explain it by what it does and so it is defined as the *capacity for doing work* and work can only be carried out if energy is expended. We ourselves may know energy best as that certain something in our muscles which enables us to do things, we wake up full of it and go to bed depleted. Because energy produces work, it is measured in the same units, joules.

There are many sources of energy for anything which is ultimately capable of doing work is one. They are categorized mainly as follows:

(i) *potential energy* — when a body is in such a position or condition that it can do work by changing either or both of these then it is said to possess potential (or stored) energy. An example of change of position is that of a brick carried to the top of a building. To suit our calculations it is said to possess energy simply because it can fall back to its original position. The work it is capable of doing is evident from the hole made in the ground. In this particular case the energy is *gravitational* and:

> "the gravitational potential energy of a body is the product of its weight (W) and height (h)".

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The symbol generally accepted for potential energy is E_p. Hence:

$$E_p = W \times h$$

and in such a calculation the reference level for h must be known, it is generally (but not necessarily) at ground level.

Change of condition is illustrated by a stretched spring on a chest expander, the energy reveals itself to the fitness enthusiast holding it.

(ii)   *kinetic energy* — this is the energy of motion, it is very important in studies of the atomic world because there practically everything is moving and therefore possessing energy. It is designated by E_k. By combining the various formulae obtained so far:

from Section 4.2:

$$F = m a$$

from Section 4.3:

$$W = F s$$

$$\therefore \quad W = m a s$$

from Section 4.1.4:

$$s = \frac{(u + v)}{2} \times t \quad \text{and} \quad a t = (v - u)$$

from which:

$$W = \frac{m(v + u)(v - u)}{2}$$

Simple algebra tells us that

$$(v + u)(v - u) = v^2 - u^2$$

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hence if an object starts from rest, $u = 0$ and so:

$$E_k = \frac{1}{2}m v^2 \text{ joules (J)}$$

an important relationship. Remember that the unit for mass is the kg and for velocity, m/s.

There are other forms of energy, all characterized by the fact that they can on their own or through an intermediate device, do work. A few are given below with one example only in each case of the work done:

electrical	— drives trains
chemical	— gunpowder shifts things
heat	— does our cooking
nuclear	— generates steam to drive turbines
radiant	— light moves a photometer needle
sound	— moves our ear drums.

4.4.1 Conservation of Energy

Energy is a phenomenon of Nature who has kindly provided us with a certain amount. This amount, whatever it is, does not change irrespective of the processes it goes through. Accordingly, energy is not used up and lost, it only changes its form. This is summed up by a Law entitled the *Conservation of Energy*, saying in brief that:

"energy can neither be created nor destroyed".

In practice therefore, provided that all our equations fit properly into the scheme of things, we can calculate how much energy exists somewhere, let things happen to it, then calculate the total energy again and get the same answer. The classic way of illustrating this is by considering the energy of a failing object, showing how the equations for potential and kinetic energies work together. Cunningly we ignore the effect of friction by air particles. The fact that an object is held somewhere above ground means that it has potential energy. It has no motion, therefore no kinetic energy. When

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the object is allowed to fall its p.e. steadily decreases as its height falls, changing into k.e. as the velocity increases. On hitting the ground the object has no p.e. because its height is zero but maximum k.e. because it has maximum velocity. The energy has changed from potential to kinetic and the equations show that the total energy is constant at any height.

4.5 FIELD

It is impossible to escape from hearing about the *field* of gravity but do we ever stop to think about what a field really is? Again, one of Nature's invisibles and intangibles yet a man who falls off a ladder soon realizes what power lurks in the gravitational field enveloping him. Although fields such as this affect us profoundly, we have yet to understand what they are so once more it is necessary to describe them in terms of what they do. Generally it can be said that when something produces a particular kind of field, then other somethings are affected by it. Gravity is not the only force which produces a field, we have yet to study electric and magnetic fields so:

> "a field is the region of influence exerted by a body in the space surrounding it".

Fields not only arise in different strengths but also have direction, for example, the field of gravity always gives rise to a downward force. We will see how to represent fields pictorially in Section 4.7.

4.6 GRAVITY

The attraction of gravity is one of Nature's artifices by which she keeps the universe in place. We are grateful for it otherwise our days here would be numbered.

The Italian scientist Galileo Galilei was one of the early experimenters with gravity. He dropped balls of different weights from the Leaning Tower of Pisa and found that they

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all reached the ground at the same time. However it is Newton who determined the basic law. He suggested that:

> "any two bodies attract each other with a force which is proportional to each of their masses and which varies inversely with the square of the distance between them".

In shorthand: $F \propto \dfrac{m_1 m_2}{d^2}$

where m_1 and m_2 are the masses and d the distance. This refers to any two bodies, the earth need not be one of them.

Nature did not do her design work with the SI system in mind so to make this formula useful for gravity calculations, a constant has to be introduced. This is known as the *gravitational constant*, G, the original value of which was first determined by Henry Cavendish, an English scientist. The value used now is:

$G = 6.67 \times 10^{-11}$ so that:

$$F = \frac{G m_1 m_2}{d^2} = \frac{6.67 \times 10^{-11} \, m_1 m_2}{d^2}$$

(r is sometimes used for distance in preference to d because in many calculations the distance happens to be the radius of a circle).

In the case of the earth, not only does it attract us but each of us also attracts it. The force is the same in both cases but with the inertia of the earth (Sect.4.1.3) somewhat greater than his, a parachutist for example goes down, we have yet to notice the earth rising to meet him.

4.6.1 Acceleration of Free Fall

It is now possible to expand on the observation made in Section 4.1.3 about weight being measured as a force. We have

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already decided that mass is a function of inertia for the harder it is to accelerate a body, the greater must be its mass. Then in Section 4.2 the formula from which force is calculated appears as $F = m\,a$. To measure the force of gravity therefore a known mass is taken and its acceleration measured as it falls from a height (but without friction due to the surrounding air). The accepted figure for the acceleration is 9.81 m/s^2, which naturally applies near the earth only. It is represented by g, hence:

force due to gravity $= m\,g$ and

gravitational force on an object of mass $m = m\,g = m \times 9.81\text{N}$ where m is in kg.

$\therefore$ weight of an object of mass $m = m\,g = m \times 9.81\text{N}$

from which the "weight" of an object in newtons is almost ten times its mass in kilograms.

4.7 CHARGE

This is one of the most important fundamentals of electricity, its driving force. Charge is a completely separate phenomenon from gravity with some similaries and one big difference, its strength. In the atomic world we will find infinitesimal specks of almost nothing, each carrying a charge capable of exerting a force millions and millions of times that of its gravity. Unbelievable at first perhaps but not when we consider what electricity can do. Sufficient here to say that:

"nearly all electrical phenomena arise from charges in motion or at rest".

Unlike gravity there are two different varieties of charge which historically have got themselves labelled *positive* and *negative* (+ve and —ve or just plain + and —). Positive is in no way superior to negative as might be imagined from the use of the words in everyday life. There is a golden rule:

"like charges repel, unlike charges attract".

With gravity there is a force of attraction only, never does repulsion occur. Also likes, in the sense that two objects both have mass, attract, with charges likes repel.

The above law infers that both types of charge must give rise to a field (Sect.4.5) and in the field of one charge a second feels a force acting on it. What exactly carries a charge in practice will be understood better when we enquire into the atom, here we can only imagine little pin points of something from which emanate the force associated with electricity. We can perhaps draw a naive but useful analogy from the rose for this has an aura of perfume around it. We cannot see nor touch this "field of smell", it is only evident when a very special object, a nose, is placed within it. So it is with charges, the field of one charge has no effect on anything except another charge. Thus without a nose a field of smell remains undetected, similarly without another charge, be it a like or unlike one, an electric field cannot be proved.

4.7.1 Coulomb's Law

Experience with gravity leads to the expectation that the mutual force between two charges [as in Fig.4.3(iii)] depends directly on their magnitudes and inversely on the distance between them. It was through a series of experiments that Charles Augustin de Coulomb (a French engineer and physicist) was able to expand on this reasoning and establish the true relationship. In so doing he laid the foundations of *electrostatics*, the theory of stationary electric charges. His law shows that the force F can be expressed in terms of two charges of magnitude Q_1 and Q_2 separated by a distance, d, as:

$$F = \frac{k\,Q_1\,Q_2}{d^2} \qquad \text{where k is a constant.}$$

By adopting the convention that a +ve charge is represented by $+Q$ and a —ve charge by $-Q$, then because in the formula
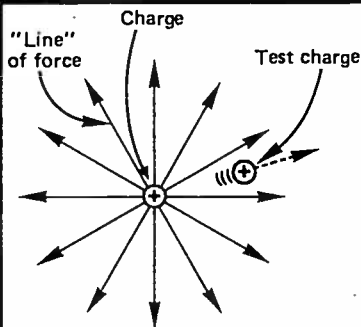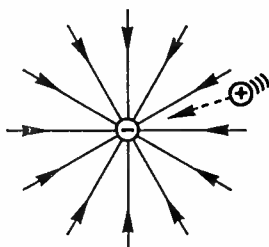
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Fig. 4.3 Electric fields.

both $\dot{Q}$'s are positive, F is a force of repulsion. It is also repulsion when both Q's are negative but $Q_1 \times -Q_2$ or $-Q_1 \times Q_2$ give $-F$, i.e. negative repulsion, in other words, attraction.

Named after him is the unit of charge, the *coulomb* (C) which we cannot define yet because it is in terms of the electrical engineer's unit, the ampere which has yet to be discussed.

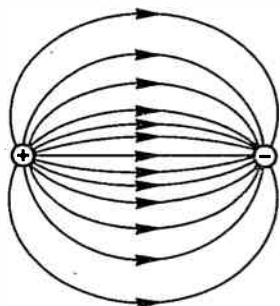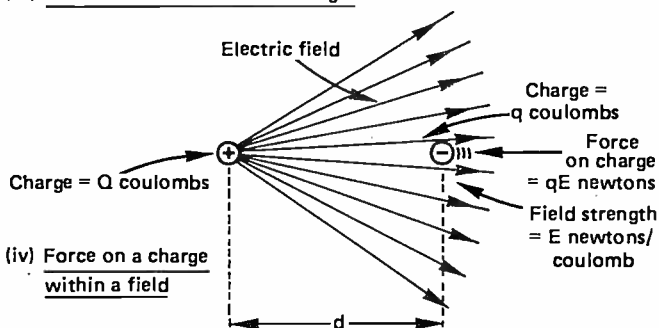To fit this equation into the modern system of units there has to be assigned an appropriate value for k. This works out to an odd-looking figure of 8.99×10^9. However it is more convenient in many electronics calculations for the constant to be in the denominator of the formula in a form including 4π so we get $1/4\pi\epsilon_0$ instead where ϵ_0 (ϵ is the Greek letter epsilon) is known as the *permittivity of free space.* This looks like rhetoric but in fact "permittivity" occurs frequently in electrostatics and in effect refers to the ability of a substance to store energy in an electric field. The substance in this case is "free space" or just plain "vacuum". It all adds up to the fact that ϵ_0 is an adjustment to make things work out satisfactorily. So for k we substitute $1/4\pi\epsilon_0$, hence:

$$\epsilon_0 = 1/4\pi k = 1/(4\pi \times 8.99 \times 10^9) = 8.85 \times 10^{-12}$$

so the practical formula becomes:

$$F = \frac{1}{4\pi\epsilon_0} \times \frac{Q_1 \, Q_2}{d^2}$$

and with Q_1 and Q_2 in coulombs, d in metres, F is expressed in newtons.

To get the unit, the coulomb into perspective, just imagine two charges, each of one coulomb separated by a distance of 10 cm, then:

$$F = \frac{1}{4\pi \times 8.85 \times 10^{-12} \times (0.1)^2} \simeq 9 \times 10^{11} \text{ newtons}$$

which, because 9.81N are equivalent to the force of gravity on

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1 kg (Sect.4.6), shows F to be equivalent to $9 \times 10^{11}/9.81$ $\simeq 9 \times 10^{10}$ kg or about 90 million tonnes weight — a force to be reckoned with! Clearly there are occasions when the microcoulomb (μC) is a more practical unit.

4.7.2 The Electric Field

Electric fields arising from single charges can be represented as shown in Figure 4.3(i) and (ii). The convention adopted is that the "lines of force" representing the field have arrows pointing in the direction in which a free *positive* charge would move. The closeness of the lines may be used as an indication of the strength of the field. As a single example, in (ii) of the Figure is sketched the field between two unlike charges.

In an electric field, consider a single charge of q coulombs. It is situated at a point where the strength of the field expressed in terms of the force exerted in newtons per coulomb is designated by E as in Figure 4.3(iv). The actual force exerted on the charge is therefore:

$F = q E$ newtons .

The electric field is the result of some other charge, say, of Q coulombs situated at a distance d from the charge q. Hence from Coulomb's Law (Sect.4.7.1):

$$F = \frac{qQ}{4\pi\epsilon_0 d^2} = qE$$

Therefore, *Electric Field Strength*,

$$E = \frac{Q}{4\pi\epsilon_0 d^2} \text{ newtons per coulomb (N/C)}$$

another important formula in electrostatics.

N and S = North and South poles

Compass needle

N   Bar magnet   S

(The arrows indicate the direction in which a hypothetical free North pole would move)
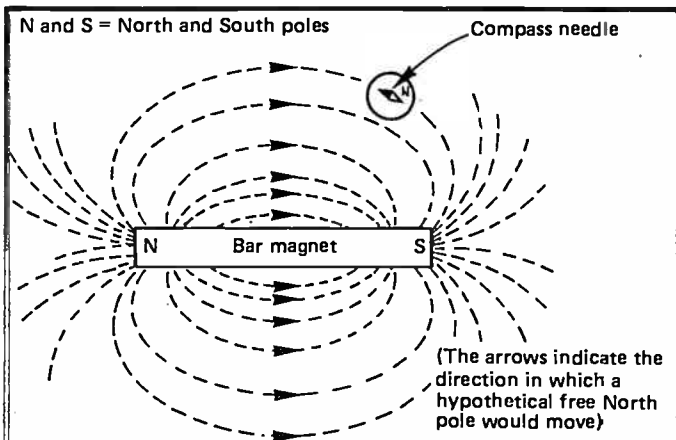
(i) Lines of magnetic force or flux of a bar magnet

Undisturbed compass needle

Showing attraction by unlike poles

N   S

Showing repulsion by like poles

S   N
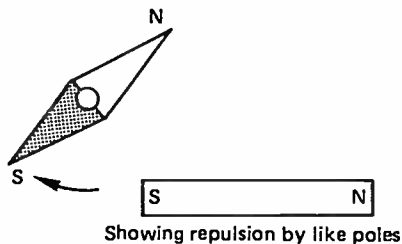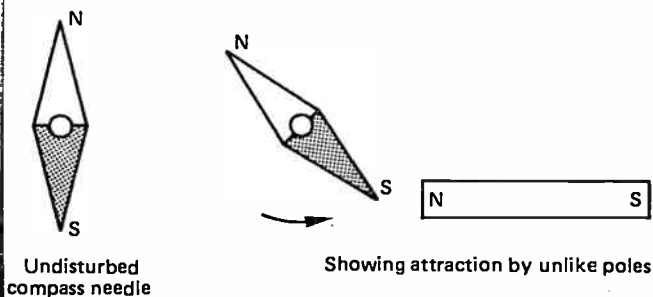
(ii) Like poles repel, unlike poles attract

Fig. 4.4 Magnetism.

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4.8 MAGNETISM

With some knowledge of charges and how they react we can next look at another unseen but indispensable phenomenon, *magnetism*. Looking beyond the simple tricks of magnets it soon becomes evident that magnetism is not a different phenomenon from charges but in fact is inextricably linked with them.

Magnetism is just one more of the forces of Nature and it only acts between charged particles *in motion*, not instead of the electric force but in addition to it. That atomic particles are charged and in constant motion even in solid materials will become evident later. What we must revise first however are the general rules under which magnetism operates.

4.8.1 Magnetic Poles and Fields

Few can have escaped the standard school experiment of covering an ordinary bar magnet with paper on which iron filings are sprinkled to make the magnet draw its own magnetic field. Figure 4.4(i) shows how it might look with well-behaved filings arranging themselves to demonstrate *lines of magnetic force* or *flux*, a convenient way of mapping out a field and similar to the technique used for electric fields. The action arises from near the ends of the magnets and these are labelled as North and South *poles*. It so happens that the same basic rule which applies to electric charges also applies to magnetic poles:

"like poles repel, unlike poles attract"

and this is easily proved with a bar magnet and pocket compass as shown in Figure 4.4(ii).

Whereas electric charges of either polarity can exist separately, magnetic poles cannot, a North must always have its South and vice versa.

The fact that the dotted lines in Figure 4.4(i) are called lines of magnetic *flux* (from Latin, *to flow*) gives the impression that something flows from N to S. It does not, the lines merely indicate the presence and direction of a field.

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By definition the arrows indicate the direction in which a hypothetical "free" North pole would move if placed within the field ("hypothetical" because a free North pole, i.e. one not linked with a South pole, cannot exist).

Certain materials, themselves unmagnetized, are attracted to either a N or S pole, these are chiefly iron, steel, nickel and cobalt. They are known as *ferromagnetic* materials (ferro from Latin = *iron*).

4.9 WAVES

Waves need no description for they are everywhere, always on the sea, sometimes in the bath. Both have motion through a medium which in this case is water. Two other examples are sound waves for which the medium is air and radio waves which propagate through many materials including air and also through space. How space can be a medium is difficult to visualize but we call it that for convenience. The underlying feature of all such waves is that they carry energy from one point to another in the medium. A floating cork is disturbed if a stone is thrown into the water nearby. The waves generated travel outwards and on reaching the cork the latter bobs up and down, a clear indication that energy has been supplied. Sound waves have the energy to move ear-drums and radio waves too have energy to excite the particles in an antenna.

We need to consider the *periodic* wave, that is, one in which the crests and troughs follow regularly. The simplest is the *sinusoidal* and fortunately many waveforms can be considered as such. The name arises because the graphical shape of the wave follows a sine curve as given from any book of trigonometrical tables. It is a useful exercise to plot such a curve, labelling the x-axis in degrees from 0 to 360 and plotting on the y-axis the appropriate value of the sine from the tables or a scientific calculator. Computer owners can easily program their machines to do the job for them. When the angle gets above 90°, the tables run out so if any angle is labelled θ, then for angles between:

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90° and 180° use $180 - \theta$ and the result is positive
180° and 270° use $\theta - 180$ and the result is negative
270° and 360° use $360 - \theta$ and the result is negative

There are other ways of illustrating a sine wave, the pendulum is perhaps the most imaginative. Figure 4.5(i) shows a pendulum equipped with an inked brush tracing out its own graph on a strip of paper moving at constant speed. The line drawn is almost a perfect sine wave.

(i) Pendulum drawing a sine wave
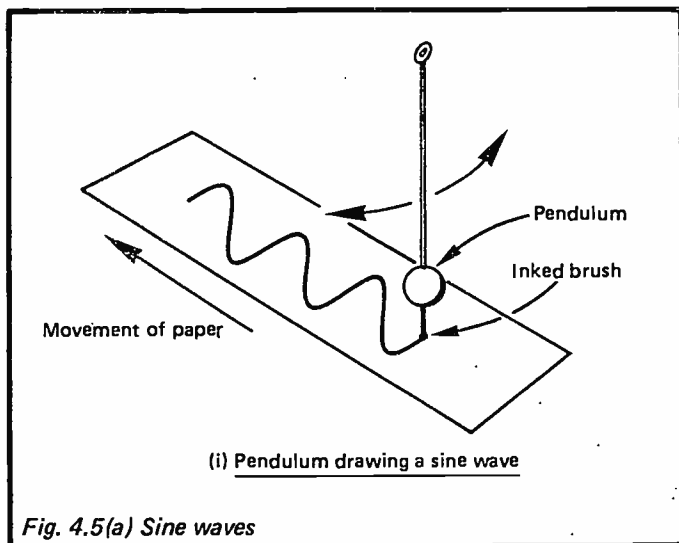
Fig. 4.5(a) Sine waves

Technically a wave is generated as shown in Figure 4.5(ii). In the circle on the left, OP is a vector, it has fixed length r but can rotate about O to point in any direction. From simple trigonometry the perpendicular PQ drawn from P onto the base line XX' is $r \sin \theta$ where θ is the angle of OP with XX'. As P rotates anticlockwise round the circle, θ changes from 0° to 360°. Plotting this movement on the adjacent graph by drawing P'Q' equal to PQ for each value of θ generates the sine wave. This also reminds us that the range of $\sin \theta$ is from 0 to 1 and between 180° and 360° it is negative. This is shown in

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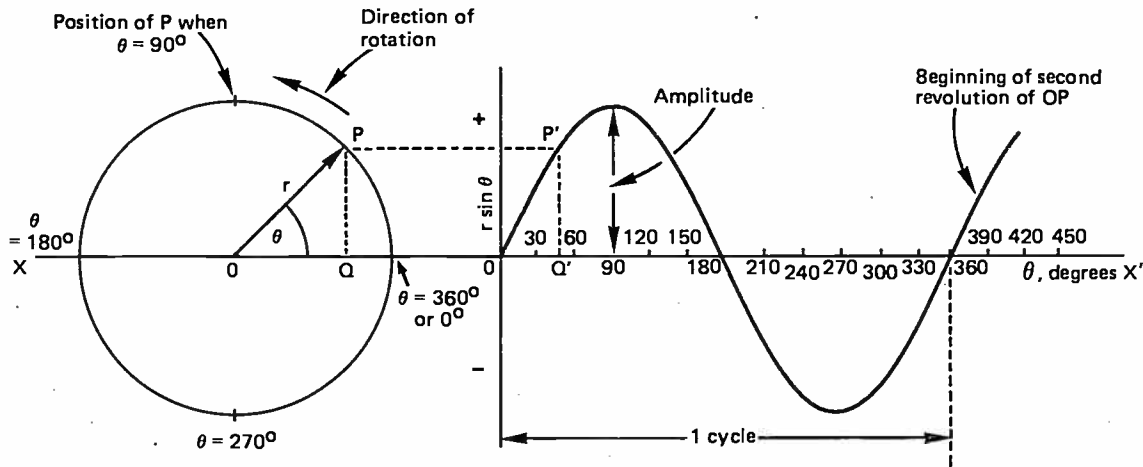
(ii) Drawing a sine wave from a rotating vector

Fig. 4.5(b)

the wave shape, a symmetrical and graceful curve. When it is moving as in the case of a wave on water or sound wave, it has a velocity and various terms are used to describe it.

(i) one *cycle* occurs between two successive points of the same value and at which the wave is varying in the same direction;

(ii) the period (T) is the time taken for one complete cycle to move past a given point;

(iii) the frequency (f) is the number of cycles completed per second. The unit is one cycle per second and it is known as the *hertz*, abbreviated to Hz (after the German physicist, Heinrich Hertz).

If the time taken for one complete cycle is T seconds then there must be $1/T$ cycles in one second, i.e. $f = 1/T$ Hz or $T = 1/f$ secs.

(iv) waves such as those of sound, radio and light travel at known velocities. It is therefore possible to calculate for any particular frequency the distance the wave travels during one cycle, i.e. during the periodic time. This distance is known as the *wavelength*, usually denoted by λ (Greek, lambda). Now from the equation of motion (Sect.4.1.4):

$$s = v\,t \qquad \text{(distance = average velocity} \times \text{time)}$$

in this particular case,

$$\lambda = v \times T \qquad \text{and since } T = 1/f$$

then $\lambda = v/f \qquad$ or $f = v/\lambda$,

a useful relationship applicable to all waves moving with constant velocity. Radio waves travel at the unbelievable speed of 3×10^8 m/s (over 7 times round the earth in one second) so as an example, a radio station transmitting on 200 kHz (kilohertz) has a wavelength of:

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$$v/f = \frac{3 \times 10^8 \text{ m/s}}{200 \times 10^3 \text{ Hz}} = 1500 \text{ metres.}$$

4.9.1 Standing Waves

A difficult concept to grasp in early studies is that of *standing waves*. They occur in the electronics world with electrical signals on wires but for our purpose we can study them via a bit of rope and string; but not just any string, that of a musical instrument.

It is a fact, although not always easy to demonstrate, that if a length of rope (say, 6 metres or so) is fixed at one end, held moderately taut at the other end and then given a jerk, a pulse or kink is formed. This travels along the rope to the fixed end from which it is "reflected" back to the starting point. Such an experiment demonstrates the underlying features of wave travel and reflection. The pulse is projected down the rope and we have to understand how it is reflected back from the fixed end.

Figure 4.6(i) shows a pulse arriving at a fixed support. During its journey down the rope the leading edge of the pulse continually applies a force to the rope immediately in front of it to lift it as shown in (ii) (ignore the effects of friction). To do this work it draws on the energy it possesses, originally supplied by the hand. When the pulse meets the support it exerts an upward force on it as shown in (iii). Now, by Newton's Third Law (Sect.4.1.5), the support exerts an equal and opposite force on the rope. Hence, as the energy of the arriving pulse is expended in applying an upward force on the support, the latter responds by applying an equal downward force. This duplicates the pulse and sends an inverted copy back along the rope as shown in (iv).

So far this is easy to visualize and with luck, to try out. We now replace the single pulse by a sine wave which in fact can be looked upon as a series of similar pulses alternating up and down. The effect of the support is the same, that is, to turn the wave upside down and send it back.

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(i) Pulse arriving at fixed support

Direction of pulse travel

Rope

Pulse

Support

When pulse reaches this position,

this section of rope has been lifted

Leading edge

(ii) Upward force exerted by leading edge of pulse

F

(iii) Leading edge of pulse applies upward force, F to support

F

Inverted pulse

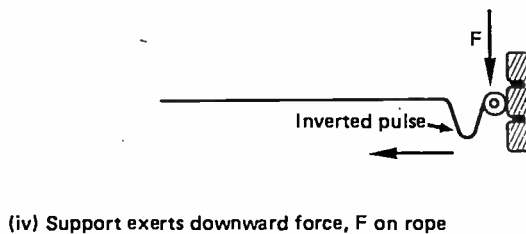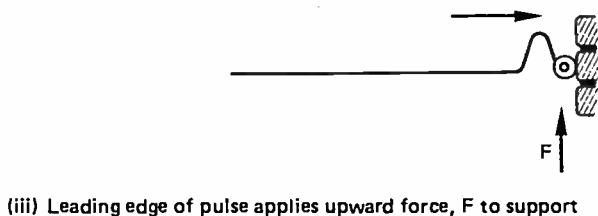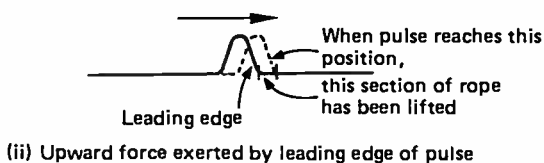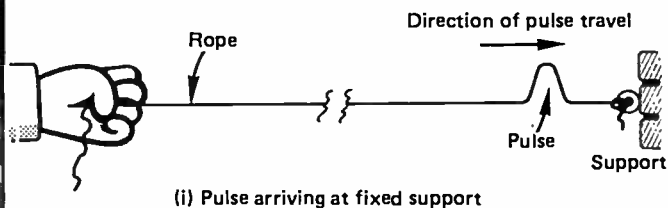**(iv) Support exerts downward force, F on rope**

Fig. 4.6 Reflection of pulse.

Consider next a string under tension as in a violin or cello. When plucked the string vibrates and from the above it would appear that a pulse or wave travels to the clamp. The string is clamped at both ends so the wave travels to and fro between the clamps with a reversal on each reflection. Incident and reflected waves exist together on the same length of string, seemingly a free-for-all but because the waves are all of the same length, an interesting pattern emerges.

To identify points on the waveform in Figure 4.5(ii), we label them according to the number of degrees through which OP has rotated. The wave reaches maximum +ve amplitude at 90° and maximum −ve amplitude at 270°. Reversing the wave is therefore equivalent to shifting it through 180°. This is also borne out by consideration of OP for any shift of this vector by 180° places it in an exactly opposite position.

Next consider a vibrating string at one of the end clamps as in Figure 4.7(i). Imagine the wave *a* to arrive, reaching the support when it is at 120°. Then on reversal by the clamp there arises waveform *b* starting at 120° ± 180° = 300° travelling *away* from the clamp. When two such waves act together, the net result at any point is their sum. This is plotted as the full-line curve (*s*) which therefore represents the actual displacement of the string. This wave is correctly zero at the clamp.

In (ii) of the Figure the summation of waves *a* and *b* is again carried out but a little later in time with wave *a* at 60° generating wave *b* at 240°. Wave *s* has now reversed but its zeros are in the same position with one at the clamp as there must be and again at the point marked *node* (Latin, *nodus*, a knot). This is the interesting feature, it does not matter where wave *a* is drawn, it and the reflected wave *b* always result in a *standing wave* for which points along the string of maximum and zero displacement are found. At the nodes the string does not vibrate at all.

We have only considered the clamp at one end. For the whole string a typical set of waveforms might be as in Figure 4.7(iii). Here two complete waves fit into the length (*l*) of the string, hence $\lambda = l/2$.

Pictorially, standing waves in a taut string are as in Figure 4.8. We have examined the last one (iv). Generally musical
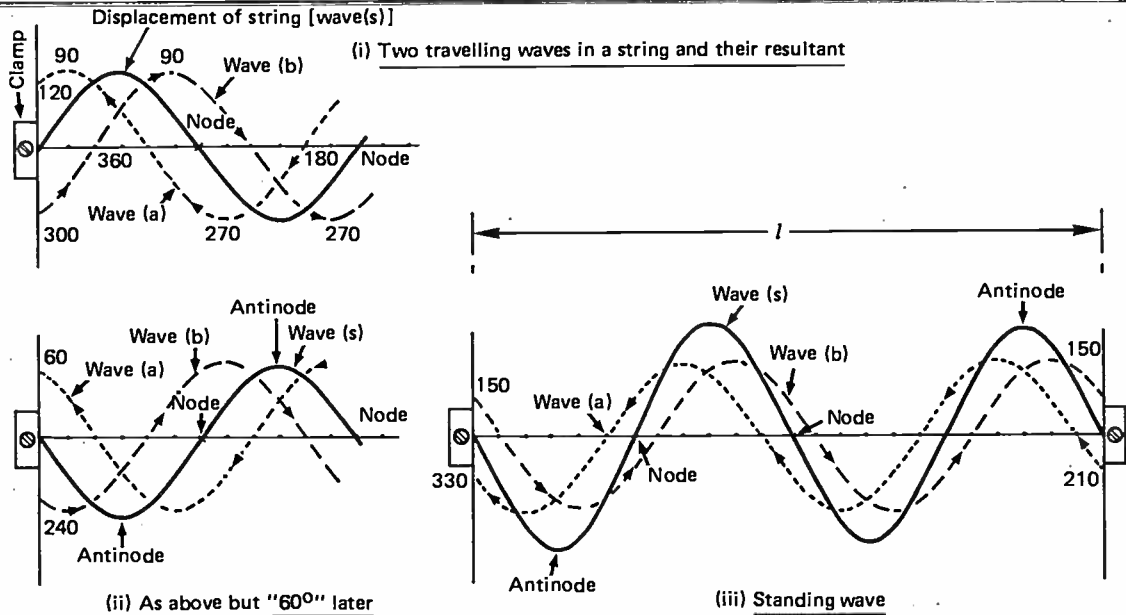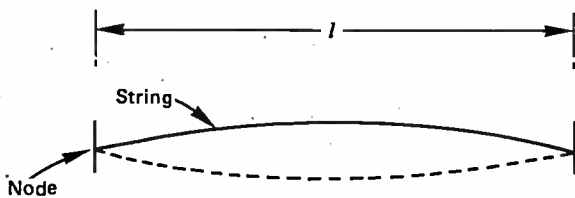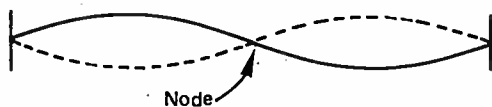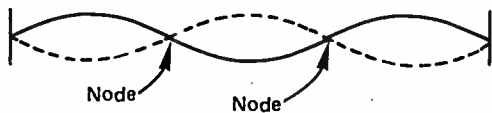
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Fig. 4.7 Standing waves in a taut string.

(i) The fundamental mode, $\lambda = 2l$

(ii) $\lambda = l$

(iii) $\lambda = \dfrac{2l}{3}$

(iv) $\lambda = \dfrac{l}{2}$

Fig. 4.8 Vibration of strings.

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instruments function in the *fundamental* mode of vibration, (i). This is easily demonstrated by clamping a $1 - 2$ metre length of fine string (ordinary, not necessarily musical) and plucking it after pulling it tight, the vibration as shown in the Figure can be seen. Other modes of vibration are not so easily set up in this way but this is of no consequence because from hereon we need only to concentrate on:

(i) there must be an exact number of half-wavelengths within the length of the string;
(ii) there are nodes at the two ends.

Understanding this helps in getting to grips with some of the more intriguing aspects of the atom.

4.10 LIGHT

Light is radiant energy generally exhibiting wave properties and *always* travelling at the same velocity (usually denoted by c) in a vacuum. The velocity was first calculated by the renowned Scottish physicist, James Clerk Maxwell. He, in the mid-eighteen hundreds developed the wave theory of light, thereby also indicating the existence of radio waves. His formula for the velocity of light is simple to use but not so easy to understand. The formula involves the *permittivity* or *electric constant* ϵ_0, and the *permeability* or *magnetic constant* μ_0, of free space. We have already met ϵ_0 with a value of 8.85×10^{-12}, μ_0 has not yet been considered so at this point it must be accepted at its face value, $4\pi \times 10^{-7}$. From Maxwell's formula, velocity of light:

$$c = \frac{1}{\sqrt{(\epsilon_0 \ \mu_0)}} = \frac{1}{\sqrt{(8.85 \times 10^{-12} \times 4\pi \times 10^{-7})}}$$

$$= 2.998 \times 10^8 \text{ m/s} .$$

The value is so close to 3×10^8 m/s that this is the figure generally used. Fortunately the velocity is almost the same in air.

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Around us things with velocity must have started from zero and then accelerated. Not so with light.

Compared with the vibrations of strings of up to a few thousand hertz, light frequencies are at the other end of the spectrum, reaching almost 10^{15} Hz. Later on there will be needed a further stretch of the imagination because light, although here being considered as a wave, at times can also be thought of as a stream of particles.

Generally it has become fashionable to specify light by its wavelength rather than frequency. A useful unit in the SI system is the *nanometre* (10^{-9} m). The rainbow proves that white light can be separated into a range of colours and this is shown approximately in Figure 4.9. Waves of these frequencies are capable of releasing electric charges within the eye which the brain translates as light, of colour according to the wavelength.
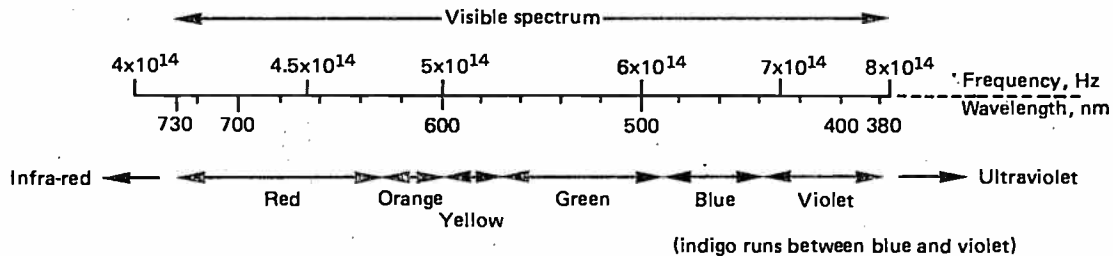
Fig. 4.9 The visible spectrum.

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Chapter 5

UP IN SPACE

To help in understanding the intricacies of the unseen small world there is something to be said for studying earth satellites first even though the magnitudes involved could not be more different. Both have bodies in *orbit* and are governed by the laws of Chapter 4. Many practical orbits although generally thought to be circular, are not so, they are in the form of an *ellipse* which might casually be described as a flattened circle. To avoid over complication, we consider circular orbits only.

Fig. 5.1 Object in orbit
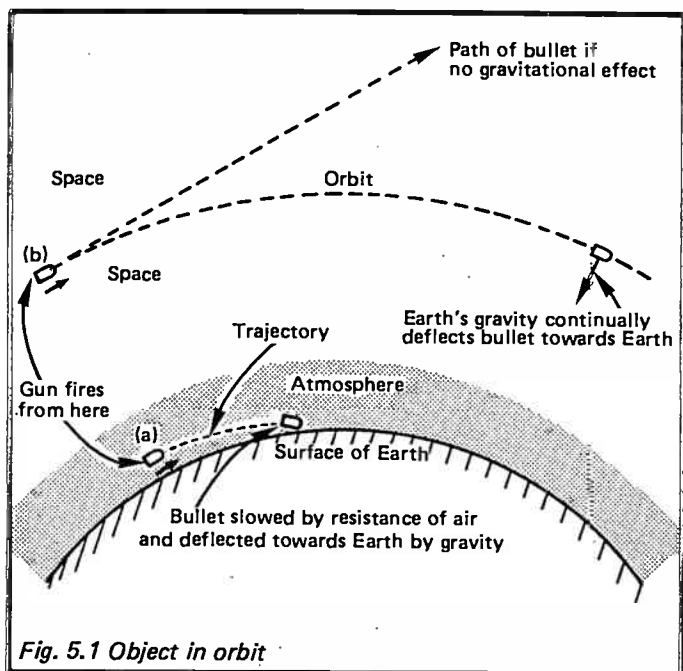
A brief and simplified explanation of the orbit follows from Figure 5.1. Consider a gun firing a bullet horizontally near the

surface of the earth as at (a). Air resistance and gravity slow the bullet and bring it down to ground with a trajectory as shown. At (b) somewhere up in space there is no air to create resistance and if the earth were not nearby a bullet would be able to continue in a straight line according to Newton's First Law (Sect.4.1.5). But the earth is there hence there is an external force of gravity acting, so as the bullet moves it is being continually deflected downwards. It is possible to choose a bullet speed commensurate with its distance from earth so that it can be forced into a circular orbit round the earth. It will then continue travelling in that orbit according to Newton's Law simply because there is nothing to stop it. This is how the Earth orbits the Sun and the Moon orbits the Earth. The next two Sections look at the principles in more detail.

5.1 MOTION IN A CIRCLE

The *radian* is a unit related directly to the radius of a circle and by using it mathematical analysis is frequently simplified. It is defined as the angle at the centre of a circle subtended by an arc of length equal to the radius. Figure 5.2(i) puts this into pictorial form.

The ratio of the circumference of a circle to its diameter works out to the odd-looking number 3.14159 . . . , universally represented by the Greek letter π (pi). Since the diameter of a circle is twice the radius, then:

$$\frac{\text{circumference}}{\text{diameter}} = \pi \quad \therefore \quad \frac{\text{circumference}}{\text{radius}} = 2\pi$$

showing that there are 2π radians in a circle. From this the relationship between degrees and radians follows as $360° = 2\pi$ radians, hence:

$$1 \text{ radian} = 360/2\pi \quad \simeq \quad 57.3° \ .$$

Next imagine that a body moves around the circumference

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Fig. 5.2 Motion in a circle.

Labels in the figure:

Length of circumference equal to radius, r

Angle subtended at centre by arc, $\theta = 1$ radian

A diameter, d

57.3°

θ

Radius, r

360°

(i) An angle of 1 radian

Time taken = t secs.

Body starts from here

$\theta = \dfrac{s}{r}$ radians

s metres
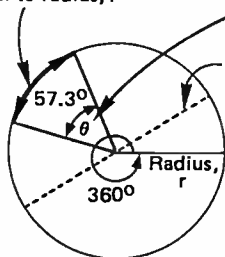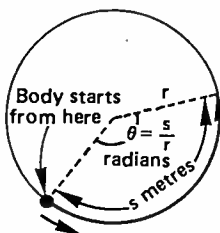
(ii) Velocity of a body travelling round the circumference

v_Q

O

r

Q

v_p

θ

r

P

C

δv

v_Q

D

B

v_p

$\dfrac{\theta}{2}$

δv

$\dfrac{\theta}{2}$

θ

A
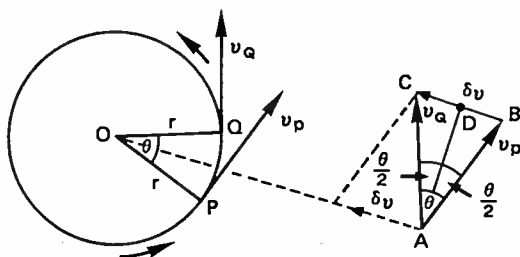
(iii) Vector triangle for changing velocity

with a velocity, v. In a certain time, t let it move a distance, s as shown in Figure 5.2(ii). The angle subtended at the centre, θ is s/r radians and the rate at which the angle changes, i.e. the *angular velocity* is the angle moved divided by the time taken. This is designated by ω (the Greek omega), hence

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$$\omega = \theta/t \quad \text{and since} \quad \theta = s/r, \quad \text{then} \quad \omega = s/rt.$$

Also $v = s/t$ (Sect. 4.1.4)

$\therefore \quad \omega = v/r \quad$ i.e. $v = \omega r$

showing that the velocity of a body round the circumference of a circle is equal to the angular velocity multiplied by the radius. This is not yet complete because the velocity contains direction which changes continually. It is therefore necessary to find how it is changing and we recall that changing velocity implies acceleration. This is the part which causes most consternation for, on first thoughts, how can a body with a constant speed have acceleration?

In (iii) of the Figure, let the body travel a short distance between two points on a circle, P and Q. It can be shown that at P its instantaneous direction of travel is along the tangent to the circle at P so can be represented by the vector v_P (Sect. 4.1.1). Again at Q there is the vector v_Q of length equal to v_P because the *speed* has not changed. The change in velocity is represented by δv (δ is the Greek delta, usually employed to indicate "a small change in"). It is determined from the vector diagram on the right of the Figure where v_P is *subtracted* from v_Q. To do this the vectors are drawn parallel to those on the circle, the triangle (or parallelogram) completed, with the vector labelled δv representing the difference between them. This is still in accordance with Figure 4.1(ii) for with the arrows of v_P and δv running in the same direction round the triangle, their resultant is the vector v_Q with its arrow in the opposite direction, i.e.:

$$v_P + \delta v = v_Q \quad \text{or} \quad v_Q - v_P = \delta v.$$

Simple geometry shows that the angle between v_P and v_Q on the vector diagram is θ. Next we add a line from A perpendicular to BC and cutting it at D — this bisects the angle θ. Then:

$$BD = AB \sin(\theta/2)$$

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$$\therefore \quad BC = 2\,AB\sin(\theta/2) \quad \text{or} \quad \delta v = 2\,v_P\sin(\theta/2)$$

i.e. $2\,v_P\sin(\theta/2)$ represents the change in velocity between P and Q.

The time taken by the body to move from P to Q is θ/ω seconds so the average change of velocity with time (the acceleration) is:

$$\frac{2\,v_P\sin(\theta/2)}{\theta/\omega} = \omega\,v_P \cdot \frac{\sin(\theta/2)}{\theta/2}.$$

Now it is a fact that the ratio $\dfrac{\sin(\theta/2)}{\theta/2}$

approaches unity as θ is reduced to zero. Hence when P and Q are *very* close together, δv becomes the continual change in the velocity, v. The rate of change of velocity with time becomes ωv and since $v = \omega r$:

$$\text{acceleration} = \omega^2 r \quad \text{or} \quad v^2/r.$$

Moreover from the vector diagram [Fig.5.2(iii)] it can be seen that the direction of δv is towards the centre of the circle, not away from it. When Q reaches P then AB and AC fall onto AD, becoming at right angles to the original direction of δv. The acceleration is therefore at right angles to the tangent of the circle at P, i.e. towards the centre.

From $F = m\,a$ (Sect.4.2),

$$F = m\,v^2/r$$

and this is known as the *centripetal* force (from Latin = centre-seeking). Summing up:

(i)   when a body moves in a circle or around a curve, even though at constant speed, its velocity and therefore momen-

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tum are not constant. The body in fact has acceleration towards the centre of the circle or curve:

acceleration, $a = v^2/r \ (\text{m/s}^2)$

(ii) accordingly a force, F must be acting on the body continually pushing it towards the centre:

$F = m v^2/\text{r}$ newtons (m in kg, r in metres).

One practical illustration will suffice. Suppose that a car of mass 1 tonne (1000 kg) negotiates a bend in the road of 200 m radius at 72 km/hr:

$m = 1000 \text{ kg} \qquad r = 200 \text{ m}$

$v = 72000/3600 = 20 \text{ m/s}$.

Centripetal acceleration is equal to:

$v^2/r = 400/200 = 2 \text{ m/s}^2$.

Centripetal force is equal to:

$m a = 1000 \times 2 = 2000 \text{ N} \quad (2 \text{ kN})$.

As the car moves, although it would prefer to travel in a straight line, a force of 2 kN is acting (at that particular velocity) to keep it moving round the bend instead. This force is provided by the friction between the tyres and the road. If the centripetal force required exceeds the maximum which tyre friction can apply, the car skids. The likelihood of skidding is therefore greater as the car velocity increases (F varies as v^2) and also with wet and slippery roads when tyre friction is less — as every motorist knows. When a car negotiates a tight bend at high speed its passengers tend to go straight on and therefore feel a force on themselves pushing outwards from the bend. This is referred to as the *centrifugal* force (Latin, centre-fleeing).

These formulae are of paramount important in the study of objects in orbit for even the minute electron is one.

5.2 MOTION IN ORBIT

The preceding Section proves the requirement of a centripetal force to pull a body out of straight-line into circular motion. Next it is possible to determine the important conditions required to maintain a body in orbit, that is to circle continually around some other larger body. Only movement within space is considered. Accordingly, with no braking effect from air or other gases, motion, once started continues indefinitely according to Newton's First Law (Sect.4.1.5). But firstly let us refresh our memories with regard to Newton's Second Law (Sect.4.2). In general:

Force = mass × acceleration

so changing the formula for gravitational calculations and using g for the acceleration due to gravity (Sect.4.6.1):

Force due to gravity $w = m\,g$.

Now if a body such as an artificial satellite as in Figure 5.3, is to move in a circular orbit around the earth, then a centripetal force is required to pull it away from its straight-ahead path onto the orbital path. This is naturally supplied by gravitational attraction and in satellite placing the problem is to calculate the height and velocity which together give rise to the orbit required. With excessive centripetal force the satellite will move in ever-decreasing circles and eventually fall to earth, with too little the opposite occurs and the satellite is lost.

Labelling the acceleration due to gravity at the satellite height, g_s and r_s the radius of the satellite orbit (= radius of earth, r_e + height of satellite above earth, h — see Fig.5.3):

then $w = m\,g_s$

and the centripetal force,

$F = m\,v^2/r_s$ (Sect.5.1).

These must balance, so

$$m\,g_s = m\,v^2/r_s$$

i.e. $g_s = v^2/r_s$ or $v = \sqrt{(g_s\,r_s)}$.

Take as an example an orbit 1720 km up. It is already known that:

$$r_e = 6.37 \times 10^6 \text{ metres}$$

(this is the average value, the earth is not a perfect sphere)

$$g = 9.81 \text{ m/s}^2 \text{ (at the earth's surface)}.$$

Because g_s varies inversely as the square of the distance (Sect. 4.6), then:

$$g_s = (r_e/r_s)^2 \times 9.81$$

and for

$$h = 1.72 \times 10^6 \text{ m},$$

$$r_s = r_e + h = (6.37 + 1.72) \times 10^6 = 8.09 \times 10^6 \text{ m}.$$

Now, since

$$v = \sqrt{(g_s\,r_s)}$$

$$\therefore \quad v = \sqrt{\frac{r_e^2}{r_s^2} \times 9.81 \times r_s}$$

$$= \sqrt{\frac{r_e^2 \times 9.81}{r_s}}$$

and in this case:

$$v = \sqrt{\frac{(6.37 \times 10^6)^2 \times 9.81}{8.09 \times 10^6}} = 7.015 \times 10^3 \text{ m/s}$$

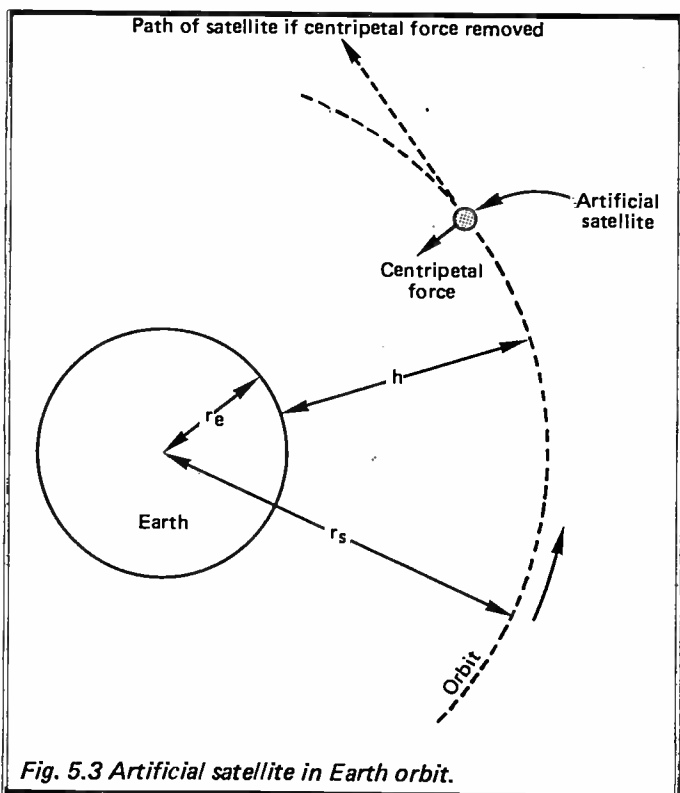*Fig. 5.3 Artificial satellite in Earth orbit.*

i.e. $v = 7.015 \times 3600 = 25250$ km/h .

The *period*, which is the time taken for one complete orbit:

$$= \frac{\text{length of orbit}}{\text{speed of satellite}} = \frac{2\pi \, r_s}{v} = \frac{2\pi \times 8.09 \times 10^6 \, h}{25250 \times 10^3}$$

$$= 2 \text{ hours.}$$

And we have done very well because such an orbit has actually

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been used with both velocity and period almost exactly as calculated.

More experience with these formulae shows that as satellite height increases the velocity required for a stable orbit decreases and the orbit period increases.

This is as far as we need to go in the study of motion, in fact the reader may well wonder what this has to do with electricity anyway. But electricity is based on motion and the same rules which govern the planets apply equally deep down in matter. We are about to study particles so small and moving at speeds so great that the imagination gives up. That need not deter us however for armed with scientific notation (Chapter 2) these quantities can be handled with ease.

THE ATOMIC WORLD

The Greeks had a word for it, *atomos*, meaning indivisible. Well over 2000 years ago the Greek philosopher Democritus proclaimed his theory of the formation of the universe as the coming together of "atomos". He suggested that this was the smallest particle, hence the naming of "indivisible". Democritus did not have his own way for long for Aristotle introduced new ideas that everything was a mixture of Earth, Air, Fire and Water. Full marks to Democritus, he was certainly nearer the truth. Subsequently little philosophical progress was made until around the 17th Century from which time the understanding of fundamental particles has advanced steadily.

When we first study the world of the atom our greatest difficulty may be in believing it all. But twinges of scepticism can be forgiven for who can gaze at a piece of stone with the conviction that it is in fact mostly space. Who can really think in terms of millions of orbits in a millionth of a second? But we are not alone, some scientists themselves were still unhappy about atomic theory right up to the turn of the century. Since then however their diligence has unearthed so much more evidence of the structure of matter that nowadays the existence of atoms and their constituent particles is in little doubt. In fact with a modern electron microscope having a magnification of one hundred million or more, clusters of atoms can be seen but not what goes on inside them. We therefore have to learn from simplified models or sketches based on guesswork from the general fund of knowledge which exists at present. Crude though these pictures may be, they can lead to a well-based understanding of matter in general and electricity in particular.

6.1 THE ATOM

An atom has many parallels with a well-organized universe except for size. There is a central *nucleus* (Latin, *kernel*) as

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(i) The solar system

(ii) The atomic system (atomic number 8, Oxygen)
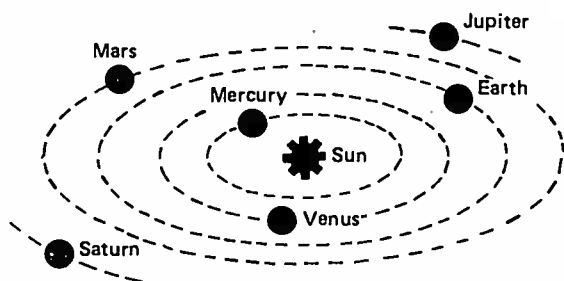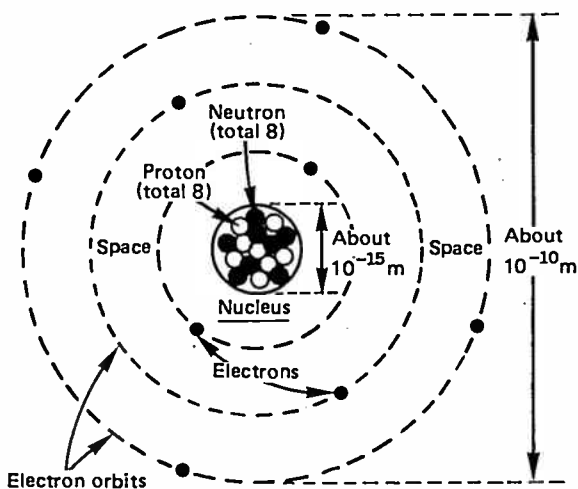
(Not to scale — atom diameter to nucleus diameter = 100,000:1)

Fig. 6.1 Orbiting a central nucleus.

we have in the Sun, around which the planets move in orbit as shown diagrammatically in Figure 6.1(i). The orbits need not be in the same plane as suggested by the Figure and although in reality elliptical, remember that for simplicity in calculations we treat them as circular. In (ii) is shown the basic idea of an atom, a central nucleus surrounded by a number of electrons in orbit. As with the solar system the orbits are in space and the mechanical principles involved are similar to those developed in Section 5.2 for an earth satellite.

In each and every atom the electrons are identical and all atoms, however different, have the same basic structure. Again a caution about the realism of our diagrams. For a start an electron is not a small ball as usually depicted, it is a minute "cloud" of electrical charge (Sect.4.7). Although so much is known about the effects of charges, there is uncertainty as to how to picture them. As a little ball is the simplest on paper but it might be preferable to visualize the electron as an infinitesimal ball of fluff or cotton wool. The same applies to the nucleus. The typical dimensions given on Figure 6.1(ii) also illustrate the impossibility of drawing to scale for the nucleus has a diameter which is a minute fraction of that of the complete atom. In effect, to show the nucleus as a 1 mm dot would require a sheet of paper 100 metres wide to include the whole atom. This incidently is useful to recall when getting to grips with the statement that most of the atom is space, just like the universe.

Other complications are that atoms *vibrate* with an exceedingly fast to and fro motion which only ceases at extremely low temperatures. Moreover the electron *spins* as it rotates in orbit, rather like a top. There is a parallel with the earth which spins round once per day. These features are worth bearing in mind as we progress although more about them follows.

6.1.1 The Electron

This Section contains a brief resumé only of the electron's characteristics for such an important particle warrants a

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chapter on its own and one follows (Chapter 8). The electron charge is labelled negative (Sect.4.7) and has a value of 1.602×10^{-19} coulombs (Sect.4.7.1), generally denoted by e. This may seem an insignificant amount but Nature can move an incredibly large number of electrons together with ease and their charges are additive. Even this tiny speck of almost nothing must have some mass, this works out to 9.109×10^{-31} kg so roughly 6×10^{25} electrons weigh the same as a grain of wheat. Figure 2.1 helps to get such numbers as these into perspective.

6.1.2 The Nucleus

This is a complex arrangement of different particles bound together, see Figure 6.1(ii). The particles of greatest interest to us are known as *protons*, each having *exactly* the same magnitude of charge as an electron, but positive. An electrically neutral atom has the same number of protons as electrons so that the charges balance. The attractive force between +ve protons and −ve electrons provides the centripetal force necessary to constrain the electrons to their orbits (Sect.5.2). Atoms range in the number of protons from 1 up to just over 100, each atom is different and the number of protons gives the atom its label, called the *atomic* or *proton number*, Z. Some of these numbers are lined up with the more familiar names of the *chemical elements* in Section 6.2.

Although a proton has the same charge (but of opposite sign) as an electron, it has a much greater mass of 1.6725×10^{-27} kg, it is therefore $(1.6725 \times 10^{-27})/(9.109 \times 10^{-31})$ = 1836 times heavier. On this account alone it is evident that most of the mass of an atom resides in the nucleus. In addition the nucleus contains other particles known as *neutrons*. These have almost the same mass as the proton but have no charge. The number of neutrons in a particular atom is equal to or slightly more than the number of protons. It is thought that in some way neutrons hold the protons in position for otherwise they would fly apart because of their like charges. The binding energies between protons and neutrons can be over one hundred thousand times greater than that between an electron and its nucleus. For this reason electrons

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can be removed from atoms relatively easily whereas separating protons from neutrons for nuclear fission or explosion is extremely difficult. In fact one might be excused a feeling of insecurity on reading about the enormity of the forces between atomic particles. Have no fear for they normally exist close together and their fields cancel with no net external effect. Nature has arranged for a perfect balance between such forces which is not easily upset.

Figure 6.1(ii) shows an atom with 8 protons and therefore 8 electrons, it is the atom of oxygen, one of the constituents of air. Then from Section 6.1.1:

total charge of protons

$$= 8 \times 1.602 \times 10^{-19} \text{C} = 1.2816 \times 10^{-18} \text{C}$$

total charge of electrons

$$= 8 \times -1.602 \times 10^{-19} \text{C} = -1.2816 \times 10^{-18} \text{C}$$

total charge of neutrons

$$= 0$$

hence the atom is electrically neutral as are all atoms with their full complement of electrons.

6.1.3 Weighing the Atom

It is instructive to try out some of the methods available for estimating the weight or mass (in this case the two terms are used synonymously) of any atom. *Atomic weight* or *mass* is well known to chemists and is likely to be quoted in most Periodic Tables found in text books (e.g. see Table 6.1 in Sect.6.2). It is not expressed directly in grams or kilograms but as a number of *atomic mass units* (u). This basic unit is accepted internationally and has a value:

$$1u = 1.6606 \times 10^{-27} \text{ kg}$$

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and the reason for the choice of this particular value need not concern us. So if, for example, the Table quotes the atomic weight or atomic mass of oxygen as 16, this is taken to mean 16 atomic mass units ($16u$) and the oxygen atom therefore has an actual mass of:

$$16 \times 1.6606 \times 10^{-27} \text{ kg} = 2.657 \times 10^{-26} \text{ kg} .$$

Alternatively we might turn to an Italian physicist named Amadeo Avogadro for help. His name is well known to chemists and he produced what is now called *Avogadro's Number*. The gist of the technique is quite simple. Avogadro's Number (N) is 6.022×10^{23} and it is the number of atoms in a *gram-atom* which is the atomic weight expressed in grams. From this:

mass or weight of a single atom $=$ (atomic weight)$/N$.

Taking oxygen again as an example, atomic weight $= 16$, then mass of oxygen atom is equal to:

$$16/N = 16/(6.022 \times 10^{23}) = 2.657 \times 10^{-23} \text{ g} .$$

Note that this is in grams which when converted to the standard unit of kilograms becomes 2.657×10^{-26} kg, precisely in agreement with the first method. This is to be expected for the two methods are not so very different for:

atomic weight $\times u$ $=$ atomic weight$/N$,

i.e. $u = 1/N$ or $N = 1/u$

and so, working in grams,

$$N = 1/(1.6606 \times 10^{-24}) = 6.022 \times 10^{23} .$$

exactly as stated by Avogadro.

We could however do the work for ourselves by adding up the masses of the separate parts. As shown in Figure 6.1(ii) the oxygen atom consists of 8 protons, 8 neutrons and 8

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electrons. Given that:

$$\text{mass of proton} = 1.6725 \times 10^{-27} \text{ kg}$$
$$\text{mass of neutron} = 1.6748 \times 10^{-27} \text{ kg}$$
$$\text{mass of electron} = 9.109 \times 10^{-31} \text{ kg,}$$

then, total mass of nucleus is equal to:

$$(8 \times 1.6725 \times 10^{-27}) + (8 \times 1.6748 \times 10^{-27})$$

$$= 2.6778 \times 10^{-26} \text{ kg}$$

and total mass of electrons is equal to:

$$8 \times 9.109 \times 10^{-31} = 7.2872 \times 10^{-30} \text{ kg}$$

$\therefore$ combined mass of atom $= 2.679 \times 10^{-26}$ kg

and evidently the mass of the electrons adds a mere 0.03% to that of the nucleus. This result is slightly higher than those above which is why the term "combined mass of atom" is used rather than simply "mass of atom". The reason is that the true mass is less than the sum of the masses of the constituents. Some of the mass is in the form of energy binding the particles of the nucleus together. That of all things mass and energy are interchangeable was one of the staggering revelations of Einstein. We will learn more about this gentleman later (Sect.8.5.4) and need go no further here for fear of becoming entangled in the complexities of nuclear physics.

6.1.4 Measuring the Atom

Calculation of the dimensions of atoms is a tricky business unless several assumptions are made. Accordingly let us assume that in solids the atoms are tightly packed in the simple lattice of adjacent rows and columns as suggested in Figure 6.2(i). This time we choose copper as an example.

The density of a metal element (these are considered in detail in Sect.6.2) is an indication of the weight of its atoms

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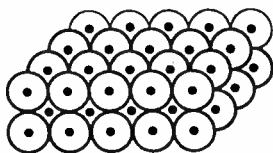
(i) Typical arrangement of atoms in a solid

Volume taken up by one atom = d^3

d

d

d
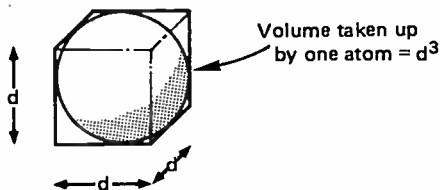
(ii) Space required by one atom

Fig. 6.2 Atoms in a lattice.

and how tightly they are packed. Densities are quoted in tables and that for copper is 8.96 grams per cubic centimetre (g/cm³). For comparison, lead, a heavier metal has a density of 11.34 g/cm³ and gold, even heavier, 19.3 g/cm³. The atomic weight of copper is 63.546. Therefore:

mass of single atom of copper = $63.546 \times u$ (Sect.6.1.3)

= $63.546 \times 1.6606 \times 10^{-27}$ kg = 1.055×10^{-25} kg

or 1.055×10^{-22} g

hence, 1 cm³ of copper contains:

$8.96/(1.055 \times 10^{-22}) = 8.49 \times 10^{22}$

or nearly 10^{23} atoms.

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The diameter of an atom can be taken as that of the outer-most orbit. Call it the "effective diameter", d. Then as Figure 6.2(ii) shows, the volume taken up by a single atom is d^3 and the number of such volumes in 1 cm^3 is $1/d^3$ where d is in cm. Then:

$$1/d^3 = 8.49 \times 10^{22}$$

$$\therefore \quad d = (1.1779 \times 10^{-23})^{1/3} = 2.275 \times 10^{-8} \text{ cm}$$

$$= 2.275 \times 10^{-10} \text{ m}.$$

This is more than a little approximate because of the assumption of a particular lattice structure. By taking into account the actual lattice arrangement employed, physicists are able to produce more accurate figures.

6.1.5 The Forces Within

Just as the solar system does not disintegrate thanks to gravitational attraction, neither does the atom of its own accord. However, although gravitational forces are present within the atom, they are negligible compared with the electric forces. This we now prove so that in subsequent considerations, gravity can be excluded with no twinge of conscience.

The simplest atom has the atomic number 1 (the gas hydrogen). It has one proton only and therefore one electron, in this particular case there are no neutrons. It has been determined that:

proton:- mass = 1.673×10^{-27} kg,

charge = $+1.602 \times 10^{-19}$ C

electron:- mass = 9.109×10^{-31} kg,

charge = -1.602×10^{-19} C

and given that the radius of the orbit is 5.3×10^{-11} m, then

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from Section 4.6, the gravitational force between proton and electron:

$$F_G = (G m_1 m_2)/d^2$$

and here d = radius of orbit

$$\therefore \quad F_G = \frac{6.67 \times 10^{-11} \times 1.673 \times 10^{-27} \times 9.109 \times 10^{-31}}{(5.3 \times 10^{-11})^2}$$

$$= 3.62 \times 10^{-47} \text{ newton}$$

a force of attaction as gravitational forces always are. On the other hand, the electric force between proton and electron (Sect.4.7.1):

$$F_E = \frac{1}{4\pi \epsilon_0} \times \frac{Q_1 Q_2}{d^2}$$

$$\therefore \quad F_E = \frac{+1.602 \times 10^{-19} \times -1.602 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times (5.3 \times 10^{-11})^2}$$

$$= -8.2 \times 10^{-8} \text{ newton},$$

again a force of attraction as indicated by the minus sign.

Hence the electric force is $(8.2 \times 10^{-8})/(3.62 \times 10^{-47})$ = 2.27×10^{39} times greater than the gravitational force. So, although gravity is a powerful force as we know it, within the atom it is truly insignificant compared with the mighty electric force.

6.2 THE ELEMENTS

Of the myriads of different substances abounding on Earth just over 100 are described as *elements* because none can be resolved into anything simpler. As an example, a constituent

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of air, oxygen, is an element for it consists of oxygen and nothing else. The gas comprises only atoms of the form shown in Figure 6.1(ii). In the same way the gas hydrogen is an element. But when these two gases are combined under suitable conditions, the gaseous nature disappears, resulting in a drop or so of water. Hence water is *not* an element because it comprises other substances. Thus:

(i)   each element differs from all other elements
(ii) within an element all atoms are the same
(iii) an atom is the smallest particle of an element which can exist on its own.

A few of the elements commonly encountered are shown in Table 6.1. The last column shows that the number of neutrons is equal to or slightly more than the number of protons. The complete list of elements is given by the *Periodic Table* to be found in any chemistry book. From these elements all other matter is derived. We look at this again when the construction of the atom has been taken a step further.
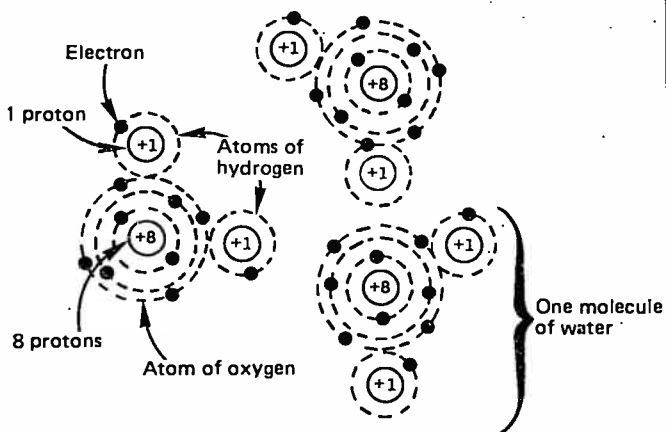
6.3 MOLECULES AND MATERIALS

The general description of a *material* as anything which has mass and occupies space makes the number of different materials on Earth almost limitless. We have discovered the secrets of the 100 or so elements, all other materials comprise groups or combinations of these. They are in units called *molecules*. A molecule is therefore a group of atoms bonded together with the characteristics of a particular substance or material. It is the smallest quantity of a material which can exist as such.

As an example, a molecule of water (the chemist's H_2O) is built up from two atoms of the element hydrogen (atomic number 1) with one atom of oxygen (atomic number 8) — see Figure 6.3(i). Water has its own unique characteristics and shows none of those of its constituent atoms (they are both gases). To take a second example, common salt, known
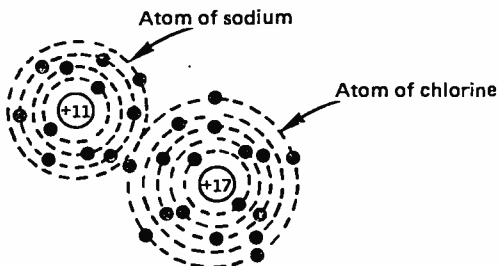
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TABLE 6.1 ATOMIC NUMBERS AND WEIGHTS OF SOME ELEMENTS

ELEMENT	DESCRIPTION	ATOMIC NUMBER	ATOMIC WEIGHT	NO. OF NEUTRONS
Aluminium	light, silvery metal	13	27	14
Carbon	non-metallic (graphite, coal, diamond)	6	12	6
Chromium	bright, hard metal	24	52	28
Copper	reddish malleable metal	29	63.5	34
Gold	yellow malleable metal	79	197	118
Hydrogen	light inflammable gas	1	1	0
Iron	silver-white metal of high strength	26	55.8	30
Neon	inert gas used in electric signs	10	20.2	10
Oxygen	colourless, odourless gas	8	16	16

(Note: Atomic number = number of protons = number of electrons)

Fig. 6.3 Diagrammatic representation of molecules.

Within the figure:

Electron

1 proton

Atoms of hydrogen

8 protons

Atom of oxygen

One molecule of water

(i) Molecules of water

Atom of sodium

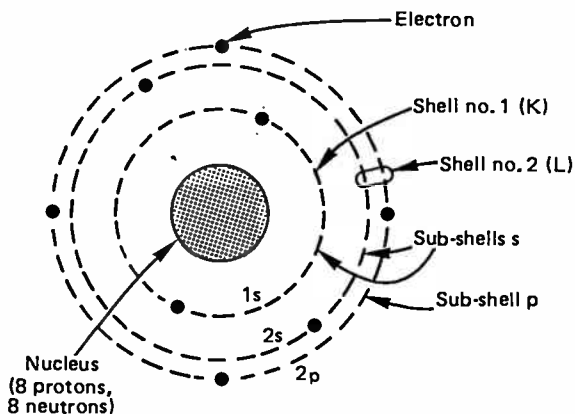Atom of chlorine

(ii) A single molecule of salt

to the chemist as sodium chloride, comprises molecules each made up of one atom of the element sodium (atomic number 11) with one atom of the element chlorine (atomic number 17) — Figure 6.3(ii). Sodium is an unfriendly metal because it reacts violently with water, chlorine is a poisonous greenish-yellow gas, yet from the atoms of both comes a safe, edible compound. Hence it is not possible to tell from the physical characteristics of the elements concerned what will be those of the compound they make. Any numbers of elementary atoms can combine to form molecules, some are therefore extremely complex. It is evident that modern thinking considers *everything* to be composed of nothing other than atoms.

We now advance from the statement in Section 6.1 that most of the atom is space to the notion that most of matter also is space. When things look solid to us their molecules are closely packed and they hold together by powerful forces, yet the actual particles concerned occupy only the tiniest fraction of the whole. In liquids the molecules are not so closely packed so the forces binding them together are weaker. Accordingly they move more easily relative to one another which is why a liquid flows. With gases the mobility is greater still so in fact space accounts for even more of the volume.
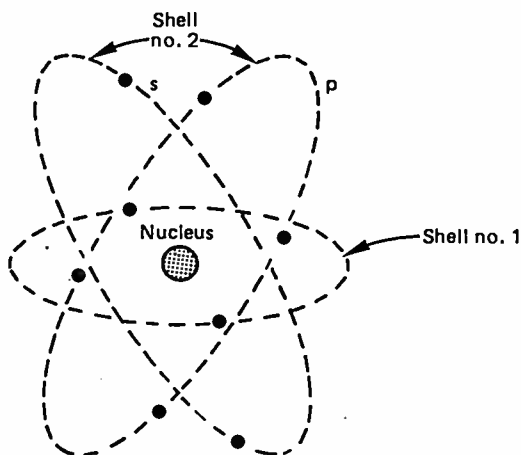
So far the atom has been considered as a complete unit with the number of its electrons exactly equal to the number of protons. However many things can happen to upset the atomic applecart so that atoms can join up to make molecules. Moreover stripping electrons away from atoms is the first step in electricity itself.

6.3.1 Valency

We need now to expand on the idea illustrated typically in Figure 6.1 that several electrons can follow each other in a single orbit. In fact each orbit is likely to be a complex arrangement containing other orbits. We still consider the basic orbit as circular and it is more correctly known as a *shell*. The orbits within a shell are known as *sub-shells*. The
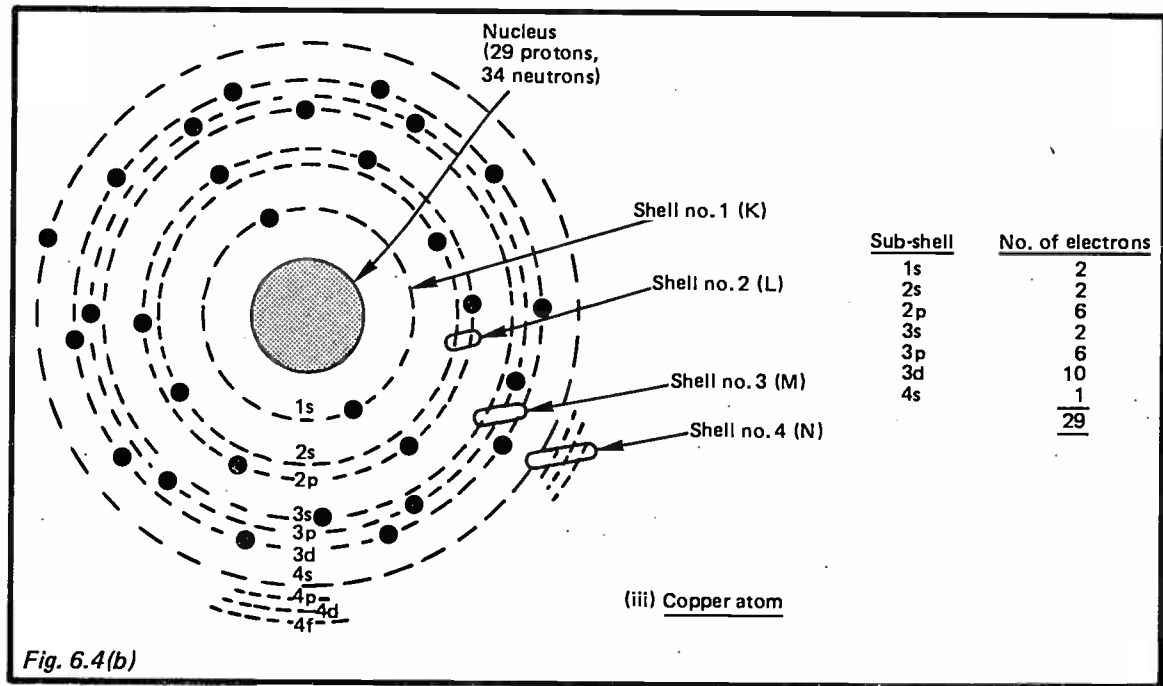
Fig. 6.4(a) Atomic arrangement of shells and sub-shells.

Labels in figure (i) Oxygen atom:
- Electron
- Shell no. 1 (K)
- Shell no. 2 (L)
- Sub-shells s
- Sub-shell p
- 1s
- 2s
- 2p
- Nucleus (8 protons, 8 neutrons)

Labels in figure (ii) An imaginative attempt to show (i) in 3-D:
- Shell no. 2
- s
- p
- Nucleus
- Shell no. 1

Nucleus
(29 protons,
34 neutrons)

Shell no. 1 (K)

Shell no. 2 (L)

Shell no. 3 (M)

Shell no. 4 (N)

Sub-shell	No. of electrons
1s	2
2s	2
2p	6
3s	2
3p	6
3d	10
4s	1
	29

1s
2s
2p
3s
3p
3d
4s
4p
4d
4f

(iii) Copper atom

Fig. 6.4(b)

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shells are numbered outwards from the nucleus and are usually designated by the letters K — Q with the sub-shells labelled s, p, d or f as shown for the oxygen atom of Figure 6.1 now redrawn as Figure 6.4(i) and (ii). This is repeated in (iii) of the same Figure for an atom with a higher atomic number and of considerable interest electrically, copper (atomic number 29). On these diagrams everything looks precise but considering the rate at which electrons move it may be better to visualize the shells more as circles of cloud or mist.

Imagine the confusion in a similar diagram for an atom with many more electrons, for example lead with 82. Happily this can be avoided because from now on we can ignore what there is inside the atom except for the nucleus and the outer shell. The reason is that it is only the electrons in the outer shell which take part in any extraneous activities. They are the ones which join up with similarly located electrons of other elements to form compounds. They are also the ones which escape from atoms to make an electric current. Very briefly, and taking the copper atom as an example [Fig.6.4(iii)], the single electron in the outer shell is screened from the pull of the nucleus by the 28 electrons orbiting in between. It is also at a greater distance from the nucleus, therefore the force holding it is less (Sect.4.7.1). The outermost electrons are known as the *valence* electrons (a term handed down by the chemists and derived from the Latin *valere*, to be well or strong). Henceforth therefore our diagrams can be simplified by including only those features which matter, as for example as shown in Figure 6.5. This pictures the electrons in the outermost shell and the relevant charges only. These can be considered separately from the rest because the electrons and protons *not* shown balance to give no net charge.

Section 6.2 mentions the Periodic Table, "periodic" because elements have a habit of turning up in groups having similar chemical and physical features. Of interest is the fact that within these groups adjacent elements are separated by atomic numbers 8, 18 or 32 as for example:

helium (2), neon (10), argon (18), krypton (36), xenon (54) and radon (86) are all *inert* gases, meaning that they are not anxious to associate with other elements. Those which are

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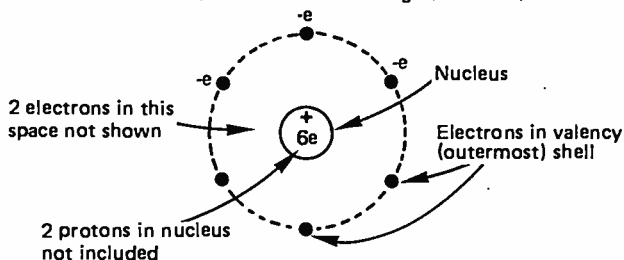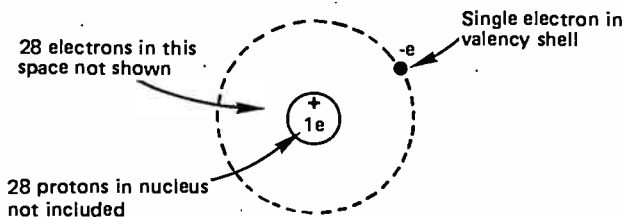
Fig. 6.5 Representation of atoms with valence electrons only.

especially active in the sense that they form molecules by some liaison between the valence electrons are in two other groups:

hydrogen (1), fluorine (9), chlorine (17), bromine (35), iron (53) — members of this group have a distinct tendency to accept electrons. They are called *halogens* to indicate that they form molecules with atoms of metals:

lithium (3), sodium (11), potassium (19), rubidium (37), caesium (55) — these are the ones which freely part with an electron. They are called the *alkali-metals* because certain of the molecules formed are alkaline.

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In all these the 8, 18, 32 rule is inviolate. Also it is evident that elements within the same group have no wish to form molecules but two elements from different groups can do so because their requirements are complementary. Sodium chloride (Fig.6.3) is an example.

While on this subject but not with regard to the formation of molecules, it is worth noting one more group following the rule:

copper (29), silver (47), gold (79) — these metals are especially suitable for conducting electricity. They are sometimes referred to as the copper group.

The ability to form molecules is not exclusive to the halogen and alkali-metal groups, the copper group must also be added. Clearly this is a complex subject so we must be satisfied with at least seeing some order in the atomic numbering scheme. *How* atoms actually get together to produce molecules follows in equally simplified form in the next two sections.

6.3.2 Ionization

An atom with its full complement of electrons is electrically neutral, that is, even though there are powerful forces within, the positive and negative charges balance out exactly. Take one electron away however and the atom becomes positive because it has one more proton than electrons. Add an electron instead and it goes negative.
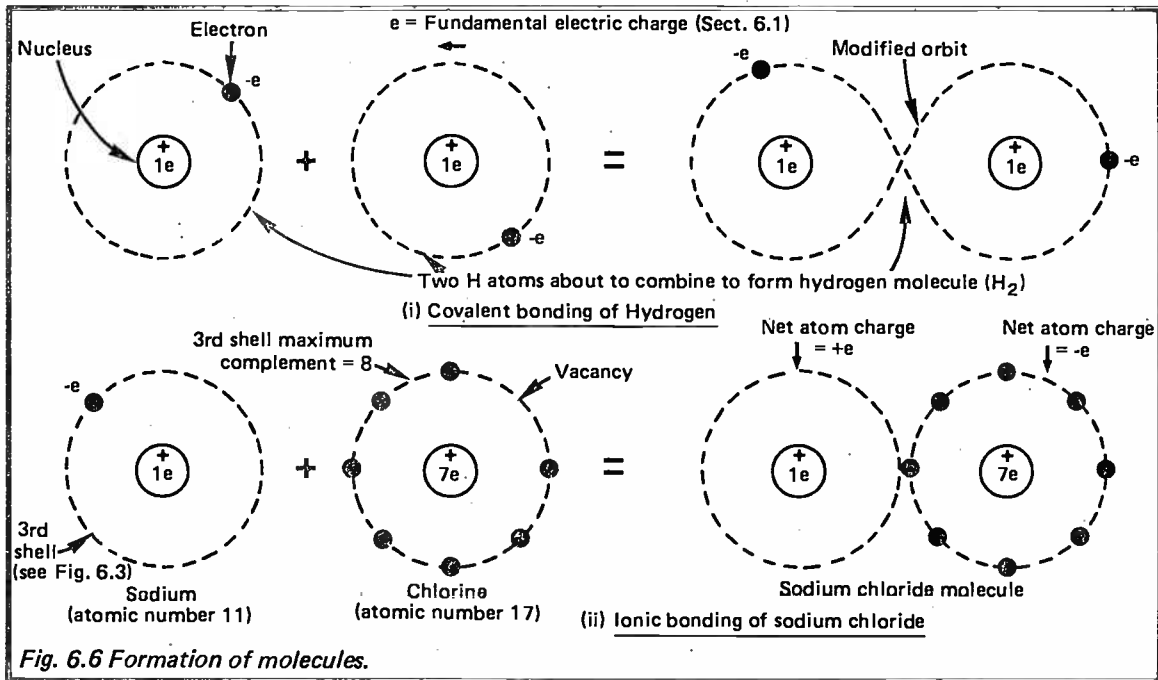
In both cases the atom becomes a charged particle and is called an *ion* (Greek = to go or wander) and the process is known as *ionization*. Removing one electron does not change an element into one of lower atomic number because it is the number of protons which determines the element, not the electrons (Sect.6.1.2). The protons are notoriously stable so it is possible to tamper with the valency electrons without changing the basic nature of the element. Thus for example, oxygen normally has 8 electrons, removing one produces an oxygen ion (atomic number 8 with 7 electrons), not a nitrogen atom (atomic number 7 with 7 electrons).

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Section 6.3.1 indicates that only those electrons in the outermost shell (the valence electrons) can be released. We will see later how much energy is required to free an electron from its orbit. This energy can be supplied in different ways by anything which is capable of doing sufficient work e.g. heat, light, electric fields, collision with other particles. Energy is one of the fundamentals of electricity so is discussed more fully later. Sufficient here to note that given the energy, electrons can not only be thrown completely free from an orbit, they can also change orbit although not to a lower orbit number (nearer to the nucleus). This is because atoms have no fancy for interference with any of their electrons other than the valence ones.

6.3.3 Chemical Bonding

This is the way in which atoms of elements get together to form molecules (Sect.6.3). One method is by *electron sharing* and this is most easily demonstrated by the hydrogen atom. This atom (atomic number 1) does not normally exist on its own but as hydrogen molecules, each comprising two atoms. It is an example of a molecule of an element, not of a compound because the latter comprises two or more different elements. When two elementary atoms are sufficiently close to each other their valence shells become modified in such a way that both electrons share the modified orbit and are in motion around both nuclei. Figure 6.6(i) shows a simplified idea of the arrangement and it is obvious why hydrogen has been chosen for the demonstration. The more complex arrangements for higher atomic number atoms, especially when several different ones combine to form a single molecule, would defy illustration. The two positive nuclei do not fly apart because the presence of the shared electrons binds them together. The bond is therefore effected by the charges of valency electrons, hence the mechanism is known as *covalent bonding*.

Ionic bonding is somewhat different and occurs when one or more electrons escape from one atom and enter the outermost (valence) shell of another nearby atom. In so doing, the

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Fig. 6.6 Formation of molecules.

Labels within figure:

Nucleus
Electron
-e
e = Fundamental electric charge (Sect. 6.1)
Modified orbit

Two H atoms about to combine to form hydrogen molecule (H₂)

(i) Covalent bonding of Hydrogen

3rd shell maximum complement = 8
Vacancy
-e
Net atom charge = +e
Net atom charge = -e

3rd shell (see Fig. 6.3)
Sodium (atomic number 11)
Chlorine (atomic number 17)
Sodium chloride molecule
(ii) Ionic bonding of sodium chloride

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losing atom becomes a positive ion while the gaining one goes negative. The two therefore attract each other and come together to form a molecule. An example is given by the compound mentioned in Section 6.3, sodium chloride. Figure 6.6(ii) shows how a sodium atom which normally has one electron only in the valence shell reacts with a chlorine atom which has seven. Because 10/11 (0.909) of the nucleus charge (+11) is cancelled out by the intervening electrons, the single electron of the sodium atom is easily disengaged. On the other hand the chlorine valence electrons are shielded by only 10/17 (0.588) of the nucleus charge, they are therefore less likely to be released. Also because of this lower screening, a free electron is easily absorbed into the single vacancy. The positive and negative ions so formed therefore attract each other and link up.

There are other atomic techniques of chemical bonding but we have looked at the two most easily understood. The approach is simplified because a full appreciation of bonding must include consideration of energy levels for it is only through changes in energy that an electron accomplishes its many tasks.

6.3.4 The Energy of Heat

Although energy is discussed in fair detail later, heat plays such a major part in energy equations that it is worthwhile considering a few of the basic characteristics first. Although heat is one of the fundamentals crucial to life it does in fact defy clear explanation. Even the statement that it is a *form* of energy leave some doubts. But certainly heat and energy are closely related for heat is often the outcome of other energy conversions. As an example the kinetic energy of a vehicle is reduced when the brakes are applied and heat is generated in them.

Atoms and molecules in all matter are continually in a state of vibration and in liquids and gases they are also in motion. They therefore possess kinetic energy. If heat is added the motion is increased so the kinetic energy increases also. Equally if heat is removed the motion is reduced until

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at absolute zero temperature it ceases altogether, this is at $-273°$ Celsius or $0°$ Kelvin (K). Thus one way of describing heat is that in matter it can be perceived as the kinetic energy of atomic and molecular motion. Temperature is a measure of the *average* of this energy. The word "average" is used because at any instant individual particles do not have the same energy.

With this theory the practical effects of heat can be explained. For example, metals expand when heated owing to the increased space required when atoms have greater vibration. Increase in pressure of a gas when its temperature rises also follows, take a motor car tyre as an example. At some given temperature the air molecules are far apart and in constant random motion. They continually hit the walls of the tyre and bounce off, the many millions of tiny but powerful thumps create the pressure. When the tyre gets hot the additional kinetic energy given to the molecules increases their hammering and the pressure rises.

The behaviour of gases was studied by Robert Boyle, an English chemist and later by Jacques Charles, a French scientist. From their work it can be stated that:

> "the average kinetic energy of the molecules of a gas is proportional to its absolute temperature"

i.e. $ke_{av} \propto T$

where T is the *thermodynamic* (absolute) temperature in degrees Kelvin (degrees Celsius + 273).

A useful formula follows from the work of Ludwig Boltzmann (an Austrian physicist) whose well known constant linking energy and temperature, k, has a value of 1.38×10^{-23} joules per degree Kelvin (J/K). It gives us a precise relationship between the kinetic energy of a molecule and the temperature:

$ke_{av} = 3/2 \, kT$ joules

and of interest is the fact that no m appears in the equation, hence the ke is independent of the mass of a molecule. This implies that at a given temperature all molecules have the same

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ke irrespective of their masses.

Calculating the average ke of a single molecule at, say, 20°C is straightforward:

$$20°C = (20 + 273)° \text{ Kelvin}$$

hence

$$ke_{av} = 3/2 \ kT = 3/2 \times 1.38 \times 10^{-23} \times 293$$

$$= 6.065 \times 10^{-21} \text{ J}.$$

a very small quantity indeed but then this is for one molecule only.

At atomic level heat is transferred by *conduction* and *radiation*. A short length of metal rod when heated at one end becomes hot at the opposite end, the "flow" of heat is by conduction. When heat is applied to one end the molecules or atoms there receive kinetic energy according to the formula above. Since ke = $\frac{1}{2}m v^2$ an increase in ke must result in a greater velocity of vibration. Although in a metal the *mean* position of each atom is fixed, greater vibration of any one particle creates greater havoc in those with which it comes into contact. Accordingly through successive collisions energy is conducted along the rod with a consequent temperature rise. Thus heat only "flows" from a body at a higher temperature to one at a lower temperature. There is no transference at the same temperature because all particles have the same ke hence energy is not passed on.

Radiation of heat is by electromagnetic wave. Intimate details of such a wave are reserved for later studies but briefly it is a waveform which in the open travels at the speed of light and has both electrical and magnetic properties. Light, X-rays, radio and television waves are of this form.

Standing in the sunshine and feeling warmed is all the evidence needed that heat arrives by radiation. It is not necessary that waves result in visibility, radiation of heat also occurs in the infra-red region of the spectrum (see Fig.4.9). The radiation supplies energy to the molecules so according to the above formula, there is a rise in temperature.

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In the kitchen pots are heated on gas or electric cookers mainly by conduction, food is heated in a microwave cooker by radiation.

6.3.5 Molecules in Motion

Electricity travels through all sorts of materials with varying degrees of success. It can also escape from them so it is propitious that we understand the posturing of molecules and atoms in the three well-known states of matter, solid, liquid and gaseous. Given high enough temperatures solid materials melt and become liquid, even gaseous. With temperatures sufficiently low gases may become liquid, even solid. Atoms therefore have the least mobility in a solid, more in a liquid, and most in gases. To examine this in a little more depth we choose water, not because it is within the electrical scene to any great extent but because it is well known in all three states — ice, water and steam.

Firstly comes the question as to how atoms behave towards each other when free to move. We can only look at this in a general way for atoms and molecules differ over such a wide range, furthermore the full scientific reasoning is heavy going. Looking back on Figure 6.6(ii) for example and remembering that all shells except the outermost have been omitted, it is clear that at any instant the various +ve and −ve charges are distributed over a molecule in a manner which depends on the construction of the molecule itself. Now for a group of similar charges a centre point can be found at which the net charge can be assumed to act. Generally the centre points for the two types of charge coincide and the molecule is then classed as *non-polar* as opposed to *polar* (having poles) when the centres do not coincide. In non-polar molecules therefore the two types of charge balance out and the molecule as a whole is electrically neutral. Hence it would at first appear that such molecules have no effect on each other and therefore can exist in a higgledy-piggledy fashion. This is not so because although a molecule may be electrically neutral overall, it has a variation of charge within it. It is also perhaps obvious that two or more atoms within a molecule or in

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adjacent molecules cannot squeeze into each other so that their individual inner electron shells mesh because the balance of charges would be upset and the electron "clouds" repel. In fact if atoms could fall into one another our whole world would collapse! Clearly atoms and molecules must repel if too close. The theory underlying this is complicated so we accept the results from scientists and the general conclusion is summed up in Figure 6.7. When the separation d is very small
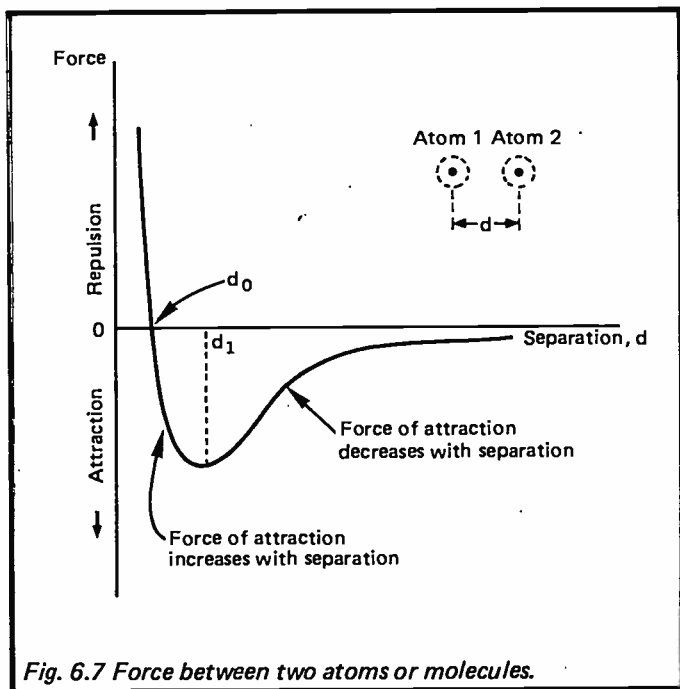

Fig. 6.7 Force between two atoms or molecules.

there is a force of repulsion, increasing rapidly with small decreases in d. This shows that atoms repel each other forcefully if they are squeezed together. At some small value of d, say d_0 (which varies from atom to atom) the force is neither repulsive nor attractive so molecules can live easily with each other this way. If on the other hand they are

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pulled apart the force changes to one of attraction and it is evident from the steepness of the graph around d_0 that any tiny change in d is met with a great force trying to restore d to d_0. However, beyond d the force of attraction diminishes with separation, tending towards zero at large values of d as might be expected. Thus within a solid, molecules and their component atoms are so arranged that each molecule is surrounded by others yet all keeping their prescribed distances. The material has a mechanical strength according to the distance d_0 and the shape of the curve for the particular molecules (as typically in Fig.6.7).

Take a thin piece of copper wire as an example. Pull on it gently and we are attempting to increase d from d_0 for most molecules. The wire stretches slightly, let go and it restores because the slight increase in d brings on a powerful force of molecular attraction. It is "elastic" because no molecular bonds have been broken. Pull harder and the wire breaks simply because bonds have been severed.

Next back to water. In its solid state the molecules are locked into position as outlined above. Adding heat to the particles in ice gives them the energy they need to move around and even break some of the bonds, hence water can flow. Adding still more heat creates sufficient agitation among the molecules that the force of attraction is insufficient to hold them together so they fly apart. Those escaping from the surface of the water are steam having the high mobility of a gas (note how quickly the smell of a gas pervades a room). From the total number of molecules making up a quantity of water only a tiny fraction escapes, hence the much lower density of the steam.

We should now have just sufficient understanding of the atomic affairs of matter for getting to grips with the bare bones of electricity. This follows in the next Chapter but further development of some of the more complex features is delayed until Chapter 8. Rome was not built in a day.

Chapter 7

ELECTRICITY IN OUTLINE

Patience is at last to be rewarded for we are now able to move on to the more practical aspects. The early part of the Chapter concentrates on those slightly confusing concepts which underly the general everyday practicalities. This is necessary because these are facets which are sometimes skipped over and therefore not properly understood, thereby doing little to create a firm base on which more advanced studies can be founded. To those already versed in electronic engineering this infers studying conductivity and resistivity first before going on to Ohm's Law which sometimes partially eclipses them. Then in Chapter 8, already knowing the practical outcome, the reasons for studying energy and the electron will be more clear. Cart before the horse, yes, but with a purpose.

By now, knowing that electrons can be freed from the orbits of certain atoms, we might relate the separation of the atom into positive and negative charged particles with the + and − of the everyday battery. This would be correct for basically the work of a battery is to separate electrons from their atoms and direct the differently charged particles to the appropriate terminals. In all batteries there is some metal even if only in the connecting wires. Accordingly we examine how and why electrons move around and along in metals first. The idea is to get a simplified view of electricity, simplified because many questions are left unanswered and things may not always be quite what they seem. Later chapters help in clarification but not completely because Nature does not share her innermost secrets with us with any great enthusiasm.

7.1 CONDUCTION

Electric (or electronic) *current* is a flow of charge from one point to another. Charge is that of millions of electrons, hence is negative and although it flows through other materials, gases

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or even a vacuum, generally it is within a metal, then known as a *conductor*. Certain metals are eminently suited to the conduction of electrons, of the better known metallic elements, silver is the best, followed closely by copper then gold, chromium and aluminium in that order. For most electrical work copper reigns supreme. Aluminium has its uses but because copper is in more general use, this is the metal we choose for an example.

When a metal allows electrons to flow through it easily, it is said to have high *conductivity*. This is due to its having a copious supply of free electrons available, free in the sense that they are not permanently attached to atoms but are able to move around. Copper we recall from Figure 6.4(iii) has only one electron in its valence shell so any atom can give up one electron and itself become a positive ion (Sect.6.3.2). In a solid, as is copper, the atoms and ions are locked into position in a *lattice*. A lattice is the geometric form in which the atoms are arranged and varies with the material. Within the lattice structure the electrons are free to move. Pictorially a piece of copper might be represented as in Figure 7.1. The electrons move randomly at very high speeds in all directions. In their travels they are attracted towards +ve ions and repelled by other electrons but overall there is no net movement of charge.

Some amplification of electron random movement will not come amiss. Such movement implies that any of the speeds or directions is as likely to occur as any other. The term "likely to occur" may be exemplified by the roulette wheel which relies entirely on the fact that any number is as likely to come up as any other. Hence for many millions of electrons with randomly distributed velocities the average velocity is zero because the average in any direction is exactly counterbalanced by that in the opposite direction. Accordingly there is no general drift in any direction. The velocities are all different because each electron has its own separate history.

Consider a single electron thrown free from its orbit by perhaps collision with another one. We recall that the electron is not a neat little ball but better thought of as a tiny bead of fluff or "cloud" of charge. Collision is therefore not like the hard surfaces of two marbles crashing together. Marbles have

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Fig. 7.1 Electrons in a metal.

Labels in figure: Copper atom (neutral), Copper ion (+), Free electron (−), A, B
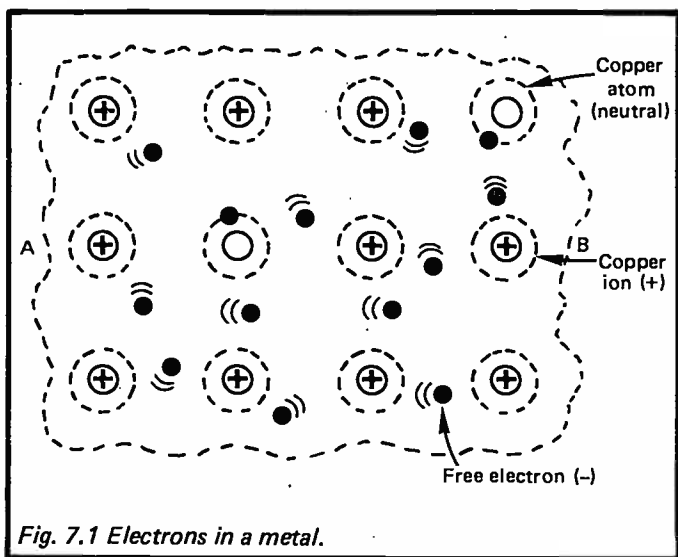
no built-in charges which repel so electron collisions must be considered as more sophisticated affairs, not just hard knocks. Our electron then, full of kinetic energy (Sect.4.4) flies into the spaces between the atoms and ions with a speed in some direction. It will however quickly encounter another particle and either be deflected or collide. Such collisions are *elastic* because the colliding particles do not stick together and the total kinetic energy is unchanged. As we are reminded by Section 4.1.5, the total momentum of two particles is unchanged by collision, it is redistributed, usually with one particle gaining at the expense of the other. Since momentum is equal to mass x velocity and the masses are constant, there is a redistribution of velocity.

An illustration of the use of elastic collision comes from the snooker or billiards enthusiast. His or her aim is to drive a ball in such a direction and with the right amount of energy for it to engage in elastic collision with a second ball. Without perhaps realizing it, allowance is also made for the effect of friction of the cloth fibres on the ball. If the player gets things

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right the two colliding balls share the momentum of the first, bounce off each other and hopefully proceed as planned. On impact some of the kinetic energy of the first ball is changed into elastic potential energy as the balls are deformed. This energy is returned as they regain their shapes and force themselves apart. With electrons both are moving as they collide but the principles are the same. Incidently if the balls were to be changed for some made of putty or soap the hapless player would be demonstrating *inelastic* collision.

Accordingly, following a collision our electron will either gain or lose velocity and almost certainly change direction. This process continues until eventually the electron is caught in the outermost shell of a positive ion and held there but with the likelihood of being released again later.

In Figure 7.1, if a −ve charge is applied to, say, end A with a +ve charge at B, an electric field (Sect.4.5) is created through the metal and this affects all particles. The ions are fixed but their electrons are free, hence they *drift* towards B. If electrons are continually supplied into the end A and attracted out at B, then an electric current flows and it is reasonable to suppose that this current depends on the average velocity of the electrons.

The difference between random movement and that in an electric field is demonstrated by Figure 7.2. Each arrow attached to an electron represents its speed and direction, (simulated) random in (i) and with added vectors representing the effect of a field shown in (ii) and (iii). In this Figure the "direction" of the electric field is not shown because the arrows would point in the opposite direction to that in which the electrons are encouraged to move (Sect.4.7.2). The anomaly arises from the fact that when people first tried to sort out polarities and directions they decided that current flow was from +ve to −ve but nowadays, as Figure 7.2 shows, we are confident that they got it wrong. Modern thinking as expressed here clearly considers that in metals the electron is the charge carrier and therefore electric current flows from −ve to +ve. Notwithstanding this, there is still talk of the "conventional" direction of current flow, i.e. from +ve to −ve.

Within an electric field a force is exerted equally on all electrons (Sect.4.7.2) of:

No electric field

(i) Random movement

Added field vector →

−   +

(ii) Velocities in weak electric field

Resultant velocity

Vector representing original velocity

Vector representing added velocity due to electric field

Added field vector →

≡   +
                   +
                   +

(iii) Velocities in strong electric field
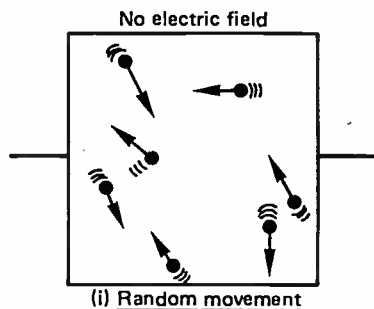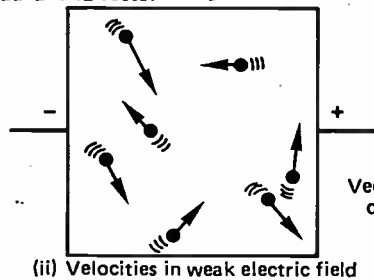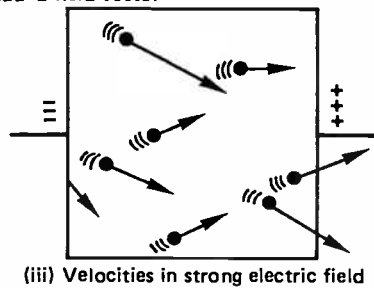
● = electron (positive ions not shown)

Fig. 7.2 Electron drift.

$$F = eE$$

where e is the electron charge and E the strength of the field and because

$$a = F/m$$

where a = acceleration and m = mass (Sect.4.2), then electrons will have an acceleration proportional to the strength of the field and in a direction towards the +ve end.

The magnitude of an electric current is the *rate* at which charge passes a given point, a current is therefore something which is flowing continuously just as water flows in a stream. When charge in the shape of one or more electrons moves then obviously in the calculation of current, time is a factor. Hence:

$$\text{current} = \frac{\text{quantity of charge}}{\text{time interval}} = \frac{Q}{t} = \frac{ne}{t}$$

where n is the number of electrons passing the given point during the interval of time t. e is the electron charge $(1.602 \times 10^{-19} C$ — Sect.6.1.1).

Electrical engineers measure electric current in *amperes* (after André Ampère, a French mathematician and physicist) such that:

one ampere (A) is the current when one coulomb of charge flows past a given point in one second.

Since charge = $n \times e$

then $n = \dfrac{\text{charge}}{e} = \dfrac{1C}{1.602 \times 10^{-19} C}$

i.e. $n = 6.24 \times 10^{18}$,

so for one ampere of current, 6.24×10^{18} electrons on *average* are jostling past every second. This same number enters the

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wire at one end and leaves at the other end — the same number, not the same electrons.

Taking a practical example, a 60 watt electric lamp on a 240 V mains passes 0.25 A (or 250 milliamperes) so in this case $6.24 \times 10^{18} \times 0.25 = 1.56 \times 10^{18}$ electrons flow through each second.

These are enormous numbers from which one might assume that for currents of this order to flow through a piece of wire, all the electrons would need to be recruited for the job. Let us dream up some figures to see if this is so. What follows next is full of ifs and buts but nevertheless is instructive.

We use as an example a one metre length of ordinary copper wire such as might be used in a house lighting circuit and which has a cross-sectional area of 1 square millimetre (10^{-2} cm^2). For convenience we work in cubic centimetres so the wire has a volume of 1 cm^3. In Section 6.1.4 this volume is shown to contain nearly 10^{23} atoms, an astonishing number for such a small length of wire.

Guessing that for copper every atom gives up its single valence electron indicates that the metre length of wire could contain up to nearly 10^{23} free electrons, hence only one in $10^{23}/(1.56 \times 10^{18})$, i.e. only one in every 64,000 electrons is actually moving along the wire in each second. If the current is increased by application of a greater electric field then more electrons become involved. If it is increased greatly then it is possible for the agitation of the electrons through their added accelerations to cause heat and we have built ourselves an electric fire.

7.2 POTENTIAL DIFFERENCE

From what has been discovered so far it is apparent that for an electric current to flow between two points:

(i)   there must be a conducting path between them;
(ii) there must be an electric field between them.

So far all the electron movements considered have been caused by some unspecified + and − conveniently put on a

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diagram. These represent a difference in charge across two points for creating an electric field between them (e.g. Fig. 7.2). This casual approach cannot continue for surely we must next specify the difference in charge and then find the relationship with the current which flows (Sect.7.1). One thing is already certain which is that greater charge differences cause greater currents to flow but is this a linear relationship?

Figure 7.2 reminds us that a charge difference between two points has the capacity to do work on the electrons by providing a force capable of moving them along ($W = F.s$ — Sect. 4.3). Moving a charge relative to another charge changes the potential energy of the first charge as does raising or lowering a weight above earth. In the case of the weight it is the vertical distance which counts, horizontal moves do not because no extra work is done against gravity. Similarly with charges, work is only done in the direction of the field. This is shown in more detail in Figure 7.3 where the vector triangle (Sect. 4.1.1) shows that because the work done in relation to the field from C to B is zero, the actual work is measured over the distance d (A to C).

Work done in moving charge from A to B
= work done A to C + work done C to B

Fig. 7.3 Work done in moving a charge in an electric field.

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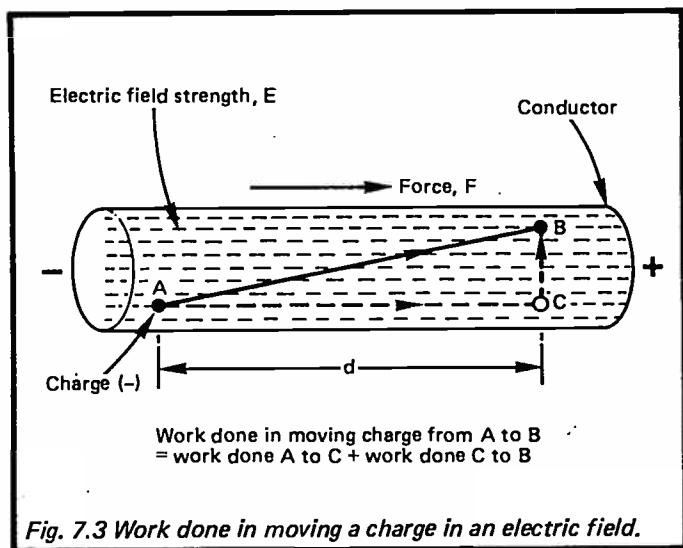
Now the force F on a charge q is $q\,E$ (Sect.4.7.2), hence: work done in moving the charge from A to B,

$$W = F\,d = q\,E\,d$$

and this work results in a difference in potential energy of the same amount since work done and energy involved are the same.

We need a unit which expresses this potential difference irrespective of the value of the charge q and this is given conveniently by a unit so well known to all, the *volt* (V) [after Alessandro Volta, an Italian physicist]. It can be defined as the ratio between the work done (or energy involved) in moving a charge between two points in a uniform electric field to the value of that charge. In short this is *work per unit charge*. Hence:

Electric potential difference (V) is equal to:

$$\frac{\text{Work done}}{\text{charge}} = \frac{W}{q} = \frac{q\,E\,d}{q} = E\,d$$

i.e. the strength of the field multiplied by the distance moved. In practical units,

$$1 \text{ volt (1V)} = \frac{1 \text{ joule}}{1 \text{ coulomb}} \quad (\text{J/C})$$

and we now have a second way of specifying an electric field:

$$E = V/d \quad (\text{V/m})$$

Also $W = q\,V$ hence, change in energy $= q\,V$

so an electric charge of 1 coulomb moving through a potential difference of 1 volt gains 1 joule of energy.

The volt is a convenient unit to use in electric circuit calculations because it avoids the involvement of charge or

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energy magnitudes. Nevertheless before moving on we might pause to use the basic equations to produce a rather surprising result.

In a t.v. tube the electrons are ejected from a source at the rear end of the tube which is evacuated so that they do not collide with air molecules. They are accelerated towards a plate (see Fig.7.4) which attracts them with its positive potential of 750 V. The odd-looking mixture of thick and
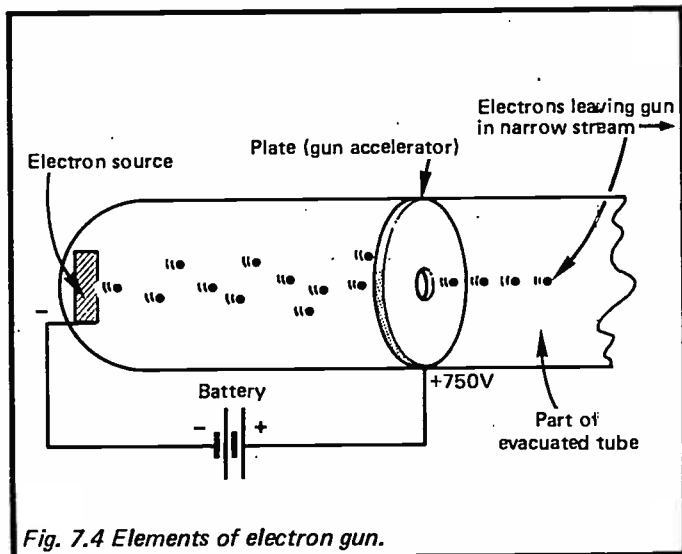
Fig. 7.4 Elements of electron gun.

thin vertical lines in the Figure is the symbol for a *battery*. Batteries are not discussed until Chapter 10 so all we need to know about them at this stage is that they continually force electrons away from the +ve terminal towards the −ve terminal. Showing a battery on a diagram is therefore a convenient way of indicating a charge or potential difference.

The plate in the Figure has a small hole in the centre through which electrons pass and the arrangement forms an *electron gun*, a device for producing a narrow stream of high-velocity electrons. Let v be the velocity at which electrons leave the gun. The basic kinetic energy equation is now used

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because the electrons are being given motion:

energy gained by single electron $=$ work done $(W) = q\,V$

$\therefore \quad W = 1.602 \times 10^{-19}C \times 750V = 1.2 \times 10^{-16}$ joules

hence:

k.e. $= W = \frac{1}{2}m\,v^2$.

(Sect.4.4 – for q and m see Sect.6.1.1).

$$\therefore \quad v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(1.2 \times 10^{-16})J}{(9.109 \times 10^{-31})kg}}$$

$$= 1.62 \times 10^7 \text{ m/s}$$

i.e. over sixteen thousand kilometres per second. Incredible but as far as we know, true. The kinetic energy of each electron when it strikes the screen falls to zero ($v = 0$, $\therefore$ $\frac{1}{2}m\,v^2 = 0$) and reappears as heat which causes the screen fluorescent material to glow.

What has been done so far in this Chapter is summed up in Figure 7.5. Even the most vivid imagination will have difficulty in transforming the few particles shown into the millions of millions which actually take part. Also it must not be forgotten that the electrons do not move parallel to the edges of the conductor as shown but drift as suggested in Figure 7.2.

7.3 THE ELECTRIC CIRCUIT

In Figure 7.5 as it is drawn there would clearly be a rush of electrons from the surplus on the left to the deficiency on the right. It would be all over in a trice. Suppose however that a battery constantly supplies electrons to the left-hand end and extracts them from the right. Through the conductor would be a continuous flow of charge, i.e. an electric current. At any

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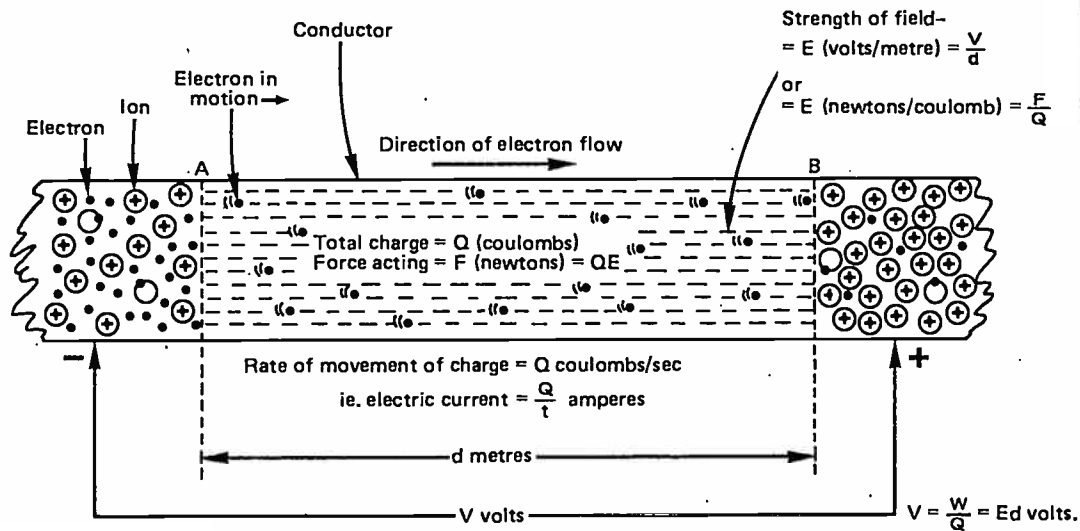
Fig. 7.5 Current flow in a conductor.

point along the conductor the current is the same for if it were not so, any build-up of charge would create its own electric field and oppose further arrivals, thus automatically cancelling itself out. On the other hand, deficiency of charge would swing that particular section of the wire positive so attracting more electrons, again becoming self-cancelling. From this it follows that once electrons are set in motion there must be somewhere for them to go and this only happens when there is a complete *electrical circuit*. A working circuit consists of a source of potential difference (such as a battery) connected to a loop of wires and electrical components so that there is a continuous unbroken electron path. One simple circuit is that of a hand torch as sketched in Figure 7.6(i) with the equivalent electrical diagram in (ii).

The battery is one of several different types of electrical potential difference generators. Basically all of these convert some kind of energy into electrical energy. Nothing happens unless there is a complete circuit connected to the two battery terminals as in the Figure. This particular circuit is not effective until the switch is closed whereupon the electric field created by the p.d. of the battery is able to permeate the whole circuit and cause electron drift as in Figure 7.2. Electrons are ejected from the −ve terminal of the battery, pass through the switch and lamp back to the +ve terminal.

Figure 7.6(iii) shows a water pump working in similar fashion. When the tap is opened the pump is able to create a flow of water round the pipe circuit and the *rate* of flow (in say, litres per minute) is the same throughout. As with the rate of flow of charge (current) this must be so for there cannot be a build-up of water in an already full pipe, neither can there be a depletion.

In electrical engineering the circuit current is measured by insertion of a measuring device known as an *ammeter* [ampere + meter − Fig.7.6(iv)]. Because it is part of the circuit the same current flows through it as for the rest of the circuit. A pointer on the scale indicates the current in amperes (A), milliamperes (mA) or microamperes (µA) [Sect.3.2]. The current in a hand-torch circuit employing a 4.5 V battery might be around 500 mA, for a small pocket-torch with a

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Arrows indicate direction of electron flow

Switch - press for "on"

When switch is pressed, metal spring makes contact with reflector

Metal strip

Metal spring connected to metal strip

Metal reflector

Battery

Plastic case
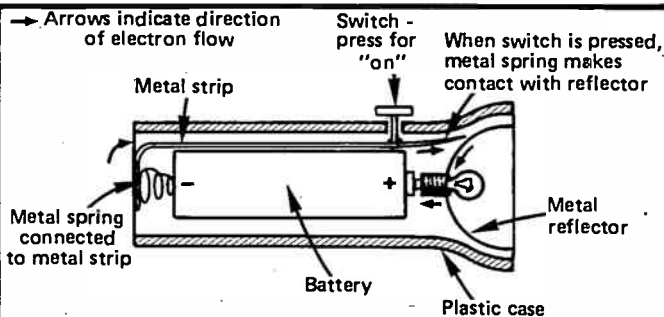
(i) Elements of a hand-torch

Electron flow when switch closed

Switch

Lamp

Connecting wire

(ii) The electrical circuit

− + 4.5V

Battery

Tap

Pump

(iii) Water analogy

mA or A

Ammeter
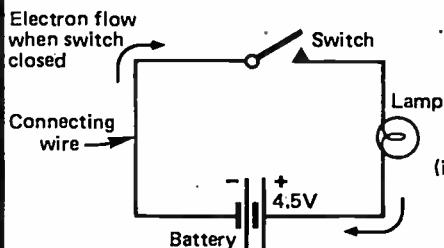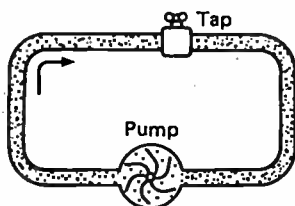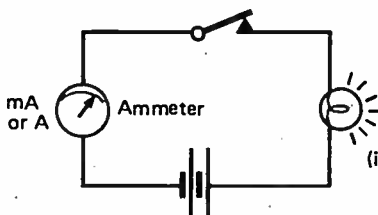
(iv) Measuring the current

Fig. 7.6 Simple electrical circuit.

3 V battery, about half this.

7.3.1 Conductivity

The *conductivity* of a conductor is an expression of its ability to pass an electric current when a potential difference is applied. It is dependent on electron *mobility*. For any material the higher the p.d. applied across it, the greater the current flow because the enhanced electric field applies a greater force to the electrons. The famous German physicist, Georg Simon Ohm, was the first to establish by measurement that, provided that the temperature and other physical conditions are kept constant:

> "the steady current flowing through a conductor is *directly proportional* to the potential difference across its ends".

This expresses the relationship between current and potential difference but how much current actually flows depends not only on the p.d. but also on the conductivity and the dimensions of the conductor (or other material). Conductivity is given the symbol σ (the Greek lower case *sigma*). In Figure 7.7(i) it might be considered as being given by I/V amperes per volt but this is for a particular shape and volume of material. Hence for an overall unit applicable to any uniformly shaped material irrespective of its dimensions, allowance is made for the cross-sectional area A and length l. This is because greater cross-sectional areas carry proportionally greater currents and the electric field varies inversely with length. This gives a formula for the conductivity as:

$$\sigma = \frac{I}{A} \times \frac{1}{V/l} = \frac{I}{A} \times \frac{l}{V}$$

This formula and the units of its components lead to the unit for conductivity which is *amperes per square metre per volt per metre*!
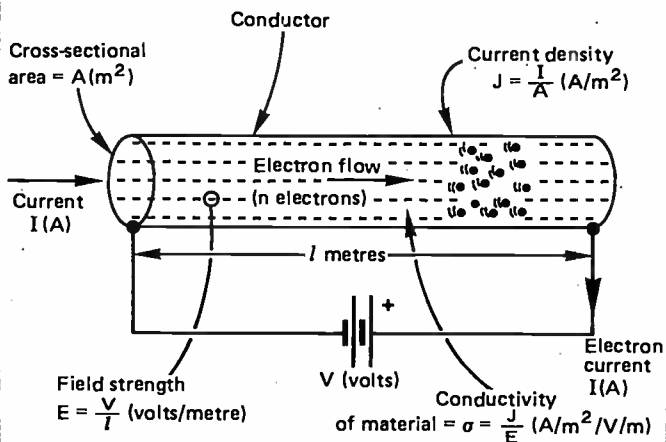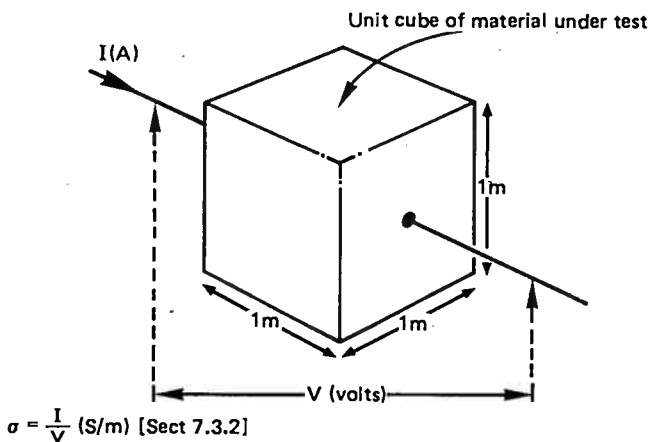
Fig. 7.7 Conductivity.

Happily there is a condensed version as will be seen later in Section 7.3.2.

Now I/A is the *current density* which is normally represented by J (A/m^2) and V/l is the electric field strength, E (V/m), so $l/V = 1/E$. Hence

$$\sigma = J/E \quad \text{or} \quad J = \sigma E.$$

Knowing the conductivity and the field strength therefore enables the current density to be calculated. Let us try this out for 100 m length of copper wire of uniform circular cross-section, 1.6 mm diameter connected across a 1.5 V battery. The conductivity σ, of copper is quoted as 5.8×10^7 $A/m^2/V/m$.

Area of cross-section of wire,

$$A = \pi r^2 = \pi \times 0.8^2 \simeq 2 \text{ mm}^2 = 2 \times 10^{-6} \text{ m}^2 .$$

The electric field strength is 1.5 V running through 100 m of wire, i.e.

$$E = 1.5/100 = 1.5 \times 10^{-2} \text{ V/m} .$$

Then, current density, $J = \sigma E = 5.8 \times 10^7 \times 1.5 \times 10^{-2}$ amperes per square metre of cross section.

$\therefore$ Wire current (through 2×10^{-6} m^2 cross-section)

$$= JA = 5.8 \times 10^7 \times 1.5 \times 10^{-2} \times 2 \times 10^{-6}$$

$$= 5.8 \times 3 \times 10^{-1} = 1.74 \text{ A} .$$

Looking a little more closely into the meaning of conductivity we see that copper normally passes 5.8×10^7 amperes of current through a block of one square metre cross-sectional area per metre length and per volt applied. This gives a clue as to a practical way of defining the unit. It is conveniently expressed by the ratio I/V for a unit cube of a material as shown in Figure 7.7(ii). Obviously this is not the

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practical way of measuring σ for taking silver as an example, its metre cube would weigh over 10 tonnes. Measurements are therefore made on a small sample of known dimensions from which σ is calculated.

Table 7.1 quotes some typical conductivities. The figures are approximate.

TABLE 7.1 CONDUCTIVITIES AT 20°C

MATERIAL	σ (A/m²/V/m)
Silver	6.25×10^7
Copper	5.8×10^7
Gold	4.1×10^7
Chromium	3.85×10^7
Aluminium	3.82×10^7
Iron	8.3×10^6
Lead	4.5×10^6

We have assumed that conductivity depends on the mobility of the electrons but so far have not yet determined what the relationship is. The mobility, μ (the Greek lower case *mu*) expresses how much movement occurs when an electric field is applied. Technically it is defined as the ratio of the drift velocity (v) to the intensity of the electric field (E), i.e.

$$\mu = v/E$$

from which,

drift velocity, $v = \mu E$

showing that the drift velocity increases with the electron mobility as might be expected. Electron mobility may be a slightly difficult concept to grasp so reference back to Section 7.1 which develops a simplified idea of electron movement may be profitable.

In Section 8.4 drift velocity is considered in detail and we bring forward a formula from this Section:

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$$\text{drift velocity}, v = \frac{I}{neA} \text{ m/s}$$

where e is the electron charge, n is the number of free electrons per m^3, I is the current and A the cross-sectional area of the conductor. Since

$$J = I/A \quad \text{then} \quad v = J/n e$$

and since

$$J = \sigma.E \quad \text{then} \quad v = \sigma E/n e$$

$$\therefore \quad v/E = \sigma/n e .$$

But v/E is the mobility μ, hence

$$\mu = \sigma/n e \quad \text{or} \quad \sigma = \mu n e$$

showing that the conductivity, σ depends not only on the electron mobility, μ but also on the total available charge. It is possible to measure n for a given material, hence by also measuring σ, the mobility can be determined.

In the practical world conductivity is less popular for analysing electric circuits than its opposite number *resistivity*, this is discussed next.

7.3.2 Resistivity

Such a condition as all atoms in a material giving up electrons to act as charge carriers is the ideal, it seldom happens that way. Atoms can be lavish, sparing or downright mean. The first we have met, these are the atom of conductors. The last, having no desire to part with their electrons allow practically no current flow and are the atoms of *insulators*, typical of which are most plastics and porcelain. Those which are not enthusiastic about letting go yet do so to a certain extent are in a class known as *semiconductors* (semi = half), important

materials in the manufacture of transistors (Chapter 9). In order, the materials are said to have high, medium or low conductivity. Generally however we work the other way round and talk in terms of *resistivity*, that is the capability of *resisting* current flow. Not such an odd idea really for it has certain advantages and modern circuit analysis uses it almost exclusively. Accordingly a good conducting material has low resistivity and an insulating material high. In a sense therefore resistivity is a measure of how determined the atoms of a material are to retain their electrons. We will understand the mechanism of this better from Chapter 8.

The relationship between conductivity and resistivity is straightforward, each is simply the reciprocal of the other. Resistivity is given the symbol ρ (the Greek lower case *rho*), hence:

$$\rho = 1/\sigma \quad \text{and} \quad \sigma = 1/\rho .$$

In fact we just turn upside down everything we know about conductivity and hey presto there is resistivity.

As with conductivity, the resistivity of any material can be expressed for a unit cube of that material so that resistivities of different materials can be directly compared. The important unit concerned with resistance to current flow is known as the *ohm* (after Georg Simon Ohm who introduced it). It is the standard, internationally agreed unit having the symbol Ω (the Greek capital *omega*). Resistivity is measured in ohms per metre cube or briefly, in ohm-metres (Ωm) as shown in Figure 7.8. It is an expression of how great a charge difference is required across opposite faces of the cube to cause a certain current to flow, i.e.

$$\rho = V/I \text{ ohms per metre cube}$$

where V is in volts, expressing the charge or potential difference (Sect.7.2) and I is the electron flow or current in amperes (Sect.7.1).

To get things into perspective, Table 7.2 shows approximate resistivities for a few conductors and insulators. Silver is therefore the best conductor in the list because it has the

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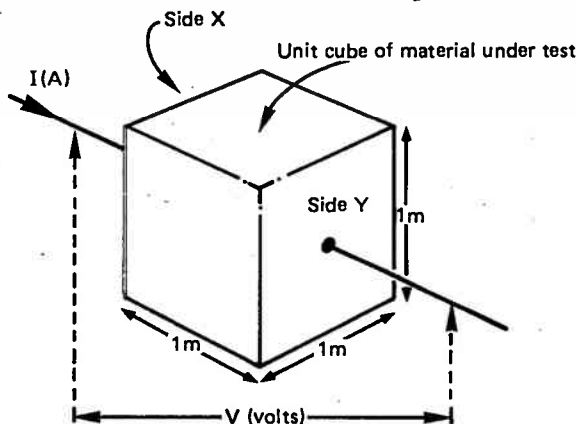
Fig. 7.8 Resistivity.

TABLE 7.2 RESISTIVITIES AT 20°C

MATERIAL	$\rho\ (\Omega m)$
CONDUCTORS	
Silver	$1.6\ \times 10^{-8}$
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Chromium	$2.6\ \times 10^{-8}$
Aluminium	2.62×10^{-8}
Iron	$10-14 \times 10^{-8}$
Lead	$22\ \ \times 10^{-8}$
INSULATORS	
Glass	$10^{10} - 10^{14}$
Ceramics	around 10^{14}
P.V.C.	around 10^{14}
Polyethylene	around 10^{17}
Rubbers	around 10^{15}

lowest resistivity which agrees with its having the highest conductivity as shown in Table 7.1. The Table refers to pure metals only. Generally impurities in metals increase the resistivity for although the number of electrons available for conduction is almost the same, impurity atoms within the lattice reduce their mobility.

The electrical difference between conductors and insulators is illustrated by resistivity ratios. As an example, glass at say, 10^{12} Ωm compared with copper at 1.7×10^{-8} Ωm has a resistivity $10^{12}/(1.7 \times 10^{-8}) \simeq 6 \times 10^{19}$ times as great. We saw in Section 7.1 that for one ampere of current 6.24×10^{18} electrons flow per second so if a p.d. is maintained across a metre cube of copper of such value that one ampere flows then the same p.d. will give rise to $(6.24 \times 10^{18})/(6 \times 10^{19})$ $\simeq 0.1$ electrons flowing per second through a metre cube of glass. There cannot be 0.1 of an electron so it must indicate one electron entering and one leaving the cube on average every 10 seconds. But never must we be persuaded that such calculations have any pretence to accuracy.

Having introduced the ohm, it is possible to see how the unit for conductivity has developed. Earlier the easily remembered unit, the *mho* was in use as the "reciprocal" of the ohm. Nowadays however the SI has introduced the *siemens* (S — after Ernst Werner von Siemens, a German electrical engineer) so whereas resistivity is expressed in Ωm, conductivity is in S/m.

7.3.3 Resistance and the Inevitable Ohm's Law

Conductivity and resistivity are reciprocal features of a metal or other current-carrying material. The value is intrinsic to the *material* and so is irrespective of shape or volume. Accordingly two samples of say, copper have similar conductivities and resistivities because they are of the same metal but unless they are of exactly the same shape and dimensions they will affect current flow differently. In this case the electrical effect of each sample is assessed from its *conductance* or *resistance*. (Note the change from "ivity" to "ance". The former signifies a property of the material itself, the latter a property of a piece of any shape or size).

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Let us concentrate on resistance for reasons given earlier. Looking again at the metre cube of Figure 7.8, it has a certain value of resistance per metre (i.e. between sides X and Y). The electric field extends over 1 metre so has a strength $E = V/l = V$ volts/metre. Now if a second similar metre cube is added and the current flows through both in succession then l becomes 2 metres and E is reduced to $V/2$ volts/metre. Since the current is proportional to the electric field strength (Sect.7.2), then the current is halved and accordingly the resistance to current flow must have doubled. Similarly for any other number of cubes hence the resistance varies directly as the length of the path (i.e. $R \propto l$) — provided that the area of cross-section remains constant. Also if the cube is halved so that sides X and Y are reduced to 0.5 m^2 then the current is halved because there is only half the number of free electrons available. With the current halved the resistance must have doubled so leading to $R \propto 1/A$, i.e. the resistance varies inversely as the area of cross-section.

In terms of l and A together,

$$R \propto l/A$$

Now for a unit cube,

$$R = \rho = V/I \text{ (Fig.7.8)}$$

hence for any other uniform shape (e.g. a wire or bar):

$$R = \frac{\rho \, l}{A} \text{ ohms}.$$

The resistance of a material can also be determined from the current which flows when a p.d. is applied across it, thus:

$$\text{Resistance} = \frac{\text{potential difference}}{\text{current}} \text{ i.e. } R = \frac{V}{I}.$$

Resistance therefore has the dimension of volts per metre but

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the more convenient unit, the ohm is used such that:

$$1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}} \, .$$

We have now studied *Ohm's Law* which surely must be engraved on the heart of every electrical and electronics engineer.

Ohm's Law states that $I = V/R$ where I is in amperes, V in volts and R in ohms. Hence

$$V = IR \quad \text{and} \quad R = V/I.$$

Each of the quantities involved (amperes, volts and ohms) can be measured in practice with an appropriate meter.

In Section 7.3.1 a 100m length of copper wire of diameter 1.6mm $(A \simeq 2 \times 10^{-6} \text{ m}^2)$ is connected to a 1.5 V battery. The current flowing is calculated via the conductivity and found to be 1.74 amperes. We should get the same answer via resistivity and Ohm's Law.

$$\rho = 1.72 \times 10^{-8} \ \Omega\text{m} \, .$$

Knowing the resistivity of copper it is possible to calculate the resistance of this particular length of copper wire from $R = \rho l/A$ ohms, remembering that l and A must be in m and m^2 respectively. Then

$$R = \frac{1.72 \times 10^{-8} \times 100}{2 \times 10^{-6}} = 0.86 \ \Omega$$

and from Ohm's Law,

$$I = V/R = 1.5/0.86 = 1.74 \text{ A}$$

showing that:

(i) knowing the resistivity of the metal used, the actual

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resistance of any wire of known dimensions can be calculated; (ii) through Ohm's Law, knowing the p.d. applied (V) and the resistance (R), the current (I) can be calculated.

A typical bench test of this is shown in Figure 7.9. For reasonable accuracy the ammeter should add negligible resistance to the circuit otherwise a lower current flows.

7.3.4 Electromotive Force

To most people the term "electrical generator" conjures up a picture of a huge machine in a power station or the device which generates electricity for the car. In electronics however a generator is any device which is capable of converting energy supplied to it into an electrical form, that is, to maintain a potential difference. A few examples of practical generators are: (i) the battery (the originating energy is chemical), (ii) dynamo (mechanical), (iii) photocell (light), (iv) microphone (sound) and (v) thermocouple (heat). They are considered in detail in Chapter 10.

Each is rated according to its capability of imparting electrical energy to a circuit via the electric field set up. The term used is *electromotive force* (e.m.f.) and it is defined as the energy converted to electrical form per unit charge flowing, hence:

$$\text{e.m.f.} = \frac{\text{energy}}{Q}$$

where Q is the charge flowing, or

electrical energy supplied $= Q \times$ e.m.f.

Section 7.2 shows that the practical unit, the volt, is defined in terms of energy per charge, e.m.f. is similar therefore it too is measured in volts. Although so far p.d. and e.m.f. seem to be the same, there is in fact a subtle difference. E.m.f. is an expression of energy *conversion* whereas p.d. is that of energy *provision*. Obviously p.d. is derived from a source of

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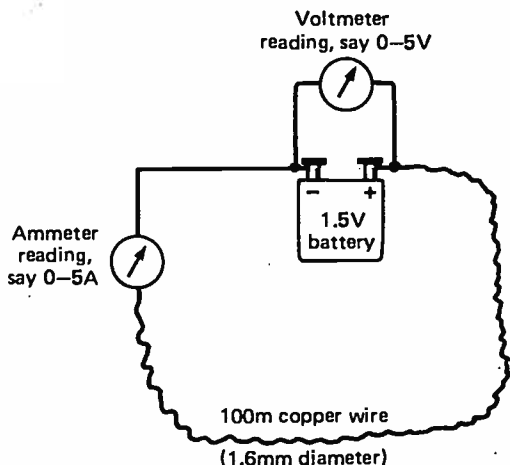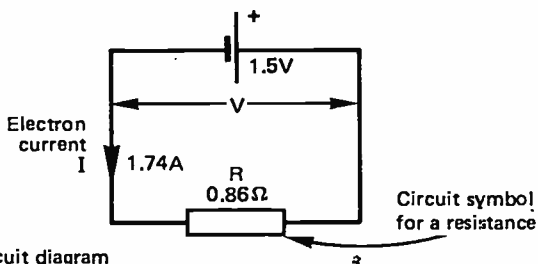
Fig. 7.9 Testing Ohm's Law.

e.m.f. for without a generator taking in some kind of energy and redistributing it round the circuit in the form of electrical energy, no p.d. can exist. The difference between the two terms comes to light on the realization that there are energy losses as heat within the generator itself. These losses are accounted for in the electrical circuit by assigning a certain value of resistance to the generator. A dynamo for example, has windings of copper wire which possess resistance. Similarly

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all other generators have *internal resistance* of some form or other. Hence the circuit diagram to replace Figure 7.9(ii) is as in Figure 7.10 (i) where the battery now comprises two components, the generator and separately its internal resistance which for demonstration is shown as 0.115 ohms.

Fig. 7.10 Electromotive force and potential difference.
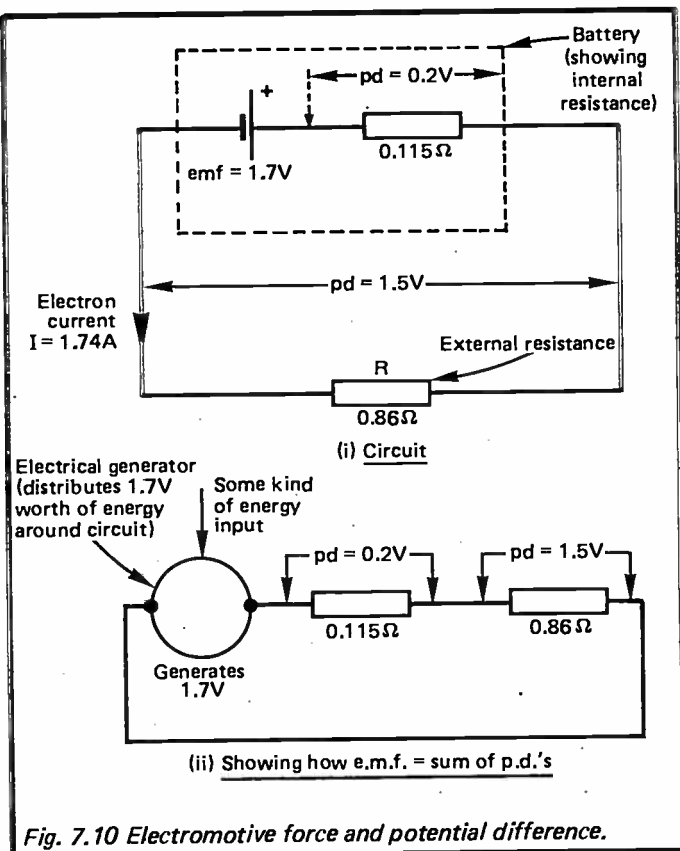
The e.m.f. now works out to be 1.7 V and because the internal resistance of the battery is 0.115 Ω, then a p.d. is developed across it of:

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$$V = I \times R = 1.74 \times 0.115 = 0.2 \text{ V}.$$

In other words 0.2 V of the available 1.7 V is used up in driving the current of 1.74 A through the internal resistance. When this particular current flows therefore the battery terminal p.d. (or just plain "voltage") is $1.7 - 0.2 = 1.5$ V which is the value left to drive a current of 1.74 A through a resistance of 0.86 Ω. Figure 7.10(ii) shows the same circuit redrawn to emphasize the important features:

(i) the e.m.f. is the only driving force in a circuit and its magnitude in volts is a measure of its ability to set up a current;

(ii) in a circuit the e.m.f. is equal to the sum of the p.d.'s;

(iii) the p.d. across the the terminals of an electrical generator is always less than the e.m.f. because of the internal resistance.

We leave circuit analysis at this point because there are many standard text books ready to take over.

7.4 ELECTROLYTES

Water on its own is a poor conductor of electricity but when small quantities of certain substances are dissolved in it, conduction increases. Such solutions and the substances which are used are called *electrolytes*. The last syllable in this term, "lyte" is taken from the Greek meaning to release or set free and the significance of this will be seen when the process is understood. We have become used to the fact that when an atom or molecule loses one or more electrons it becomes a positive ion. Conversely the gain of one or more electrons creates a negative ion and these are met mainly in solutions. Section 6.3 gives ideas as to how atoms combine to form molecules, next we find that the molecules of certain compounds split up into ions as soon as the compound is dissolved in water. The process is best illustrated by the simple chemical compound, sodium chloride, known to all as common salt, the chemist's NaCl. The constituent atoms are

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sodium (Na, atomic number 11) and chlorine (Cl, atomic number 17).

Fig. 7.11 Ionization of sodium chloride.
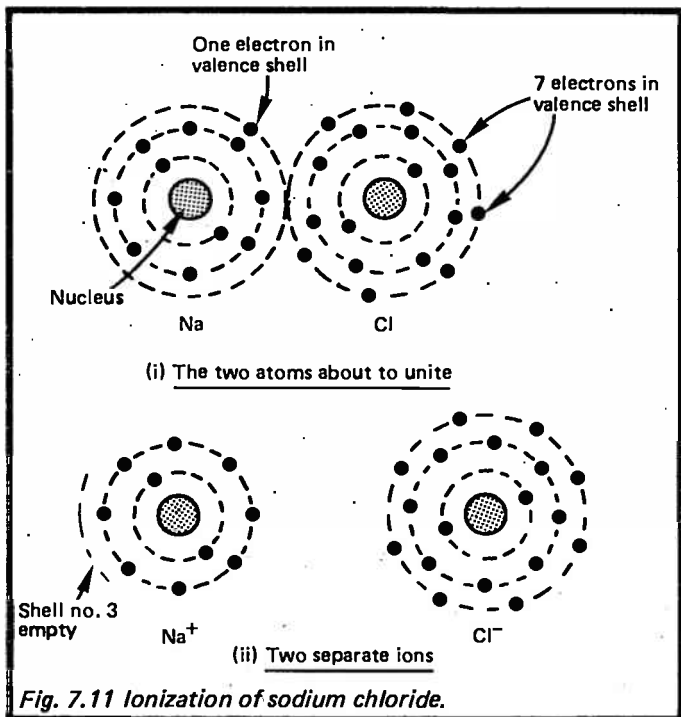
The molecule about to be formed is pictured in Figure 7.11(i). Compared with chlorine the sodium atom easily loses its valence electron, chlorine on the other hand prefers to have its valence shell (3rd) completed so readily takes on one electron. The result is that within the molecule an electron has shifted from the sodium atom to the chlorine. On dissolving the salt in water the molecule breaks up and two separate ions are freed, as shown in Figure 7.11(ii). We label the sodium ion Na$^+$ to indicate the loss of one electron and the chlorine ion Cl$^-$ to show its gain. Molecules are constantly separating into ions in

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this way but as the ions move freely within the liquid there is always the possibility of oppositely charged ones meeting to form an uncharged molecule. There is therefore a steady production of ions together with recombination. The process is dependent on the total number of molecules present. This is extremely high hence ample free ions are always available as current carriers.

Figure 7.11(ii) may give the impression that the molecule has separated into sodium and chlorine. This is not so, a positive sodium ion is just that, it has no similarity with the metal sodium. The same applies to the chlorine ion, this also only exists in solution and it bears no relationship to the gas, chlorine. The added electron must be removed for this to happen.

With a good supply of both +ve and −ve current carriers in the electrolyte the conductivity increases, for example, that of tap water when a little sodium chloride is added easily increases a hundredfold.

In Figure 7.12(i) the electric field within the solution created by the potentials on the *anode* (+ve electrode) and *cathode* (−ve electrode) causes the ions to drift accordingly i.e. the Cl^- ions to the anode and the Na^+ ions to the cathode. On reaching the anode a Cl^- ion is relieved of its extra electron by the relatively high +ve charge there and so is no longer an ion but a chlorine atom. This is an element which can exist on its own, hence bubbles of chlorine gas rise from the anode. The whole process in a nutshell is therefore:

(i)   sodium and chlorine are both elements but their atoms can marry to produce sodium chloride molecules;

(ii)   dissolved in water, sodium chloride molecules split up into two separate ions: $NaCl \rightarrow Na^+$ and Cl^-;

(iii)   the chlorine ion Cl^- is attracted by the anode, its spare electron is seized and taken away towards the +ve terminal of the battery;

(iv)   a Cl^- ion losing one electron becomes Cl, the element chlorine.

The value of this? Well at least we have discovered how to produce chlorine gas from common salt in an *electrolytic cell.*

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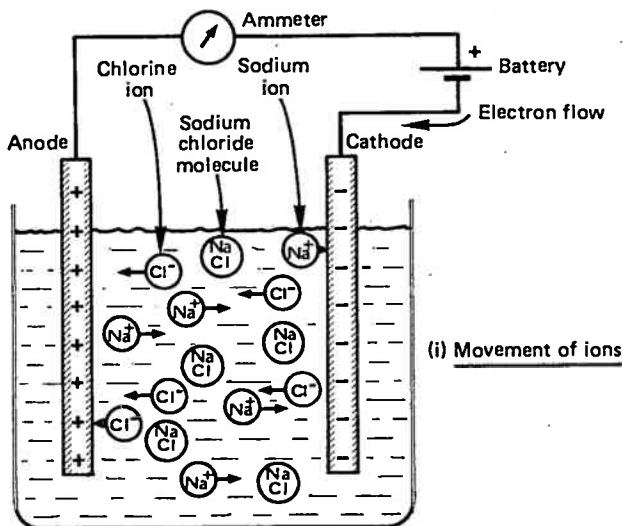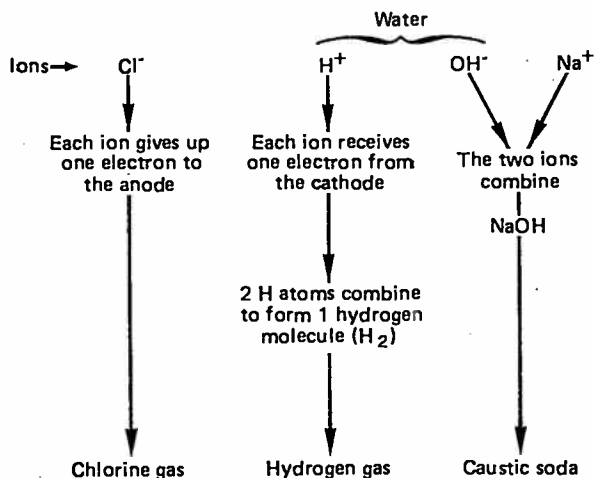
Fig. 7.12 Conduction in a sodium chloride electrolyte.

In similar fashion the Na^+ ions reach the $-$ve cathode and are forced to fill their electron vacancies because of the overwhelming number available. Ordinarily one would expect atoms of sodium to arise but in this particular case there is the complication that the water itself also ionizes. The molecule of water consists of two atoms of hydrogen linked with one of oxygen (H_2O) and this ionizes into H^+ and OH^- (hydroxide). The H^+ ions are present with the Na^+ ions at the cathode but the H^+ have greater attraction for electrons so becoming hydrogen gas and leaving many Na^+ ions unchanged. However because the solution now contains Na^+ and OH^- ions, subsequent evaporation of the water reverses the ionization process and shifts an electron from the OH^- to the Na^+, producing a new compound, sodium hydroxide, NaOH (caustic soda).

This digression into chemistry has been made to show a practical outcome of electrolysis. From common salt which abounds in plenty, the gases chlorine and hydrogen and the solid, caustic soda are obtained, all chemicals much used in industry. The process is illustrated by Figure 7.12(ii).

7.5 NOISE

Within a conductor "free" electrons move at high speeds in all directions, linking up with positive ions and breaking away from them in a purely random manner. At each end of the conductor, the net charge is a function of the number of electrons and +ve ions near the end and their distances from it. Because this is a random process (i.e. the numbers of particles and their locations change in an unpredictable way — Sect.7.1), then at any instant it is unlikely that there will be exactly the same charge at both ends. Hence there is a minute charge or potential difference across the two ends of the conductor and this is when nothing else is connected to it. Figure 7.13(i) and (ii) show two typical conditions, the second occurring after a time interval $t_2 - t_1$. At (iii) in the Figure this action is extended over a longer period of time. The graph has been drawn by the simple expedient of using random numbers (obtainable from tables, computers and

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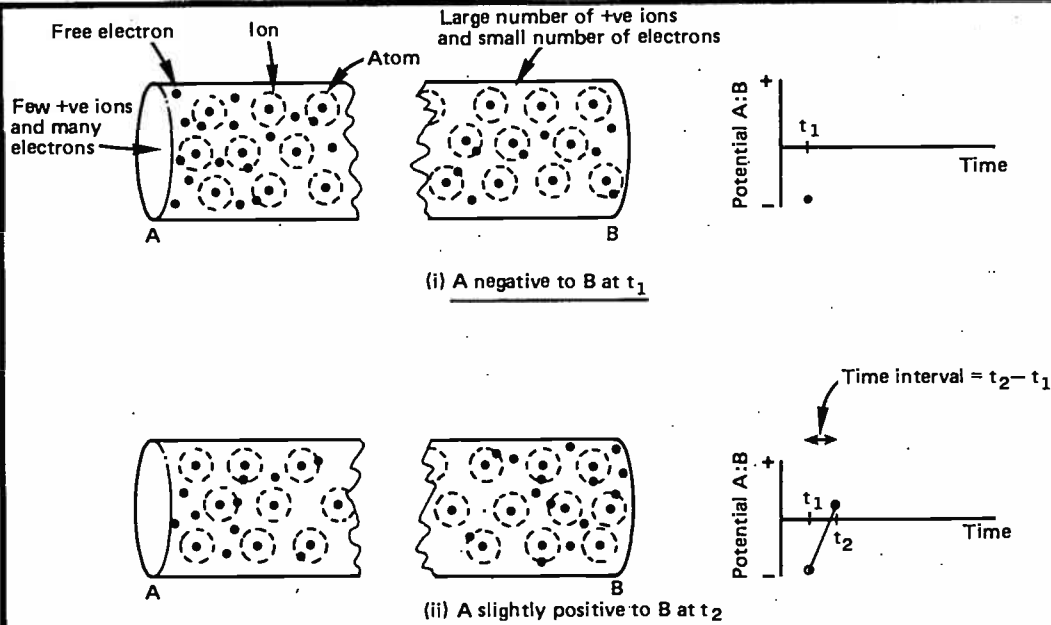
Fig. 7.13(a) Noise voltage across a conductor.

some calculators) to give a realistic impression. Averaged over a period of time many times that of $(t_2 - t_1)$ an answer of around zero can be expected.

In practice a noise voltage reveals itself as a hiss from a loudspeaker when it is fed by an amplifier of high gain. It is also responsible for the multitude of white flashes on a tv screen when the antenna (aerial) is disconnected.

Generally in a conductor the number of free electrons and their velocities increase with temperature because the added heat provides them with more energy. Accordingly with more free and more energetic electrons available the end charges and their differences are greater, hence the noise voltage increases with temperature. There is a well-known formula for calculating this voltage. Development of the formula is beyond us here although at least we know part of it having studied the relationship between particle kinetic energy and absolute temperature (Boltzmann's Constant — Sect.6.3.4).
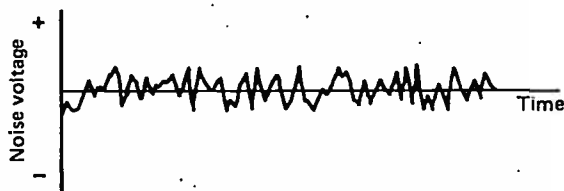
(iii) Typical noise voltage waveform

Fig. 7.13(b)

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Chapter 8

MAINLY CONCERNING THE ELECTRON

In the foregoing chapters it is assumed that all electrons have the same characteristics and behave similarly, there being only one variety. Rather than taking this for granted it is instructive first to look back on the main experiments which gave scientists confidence in the notion.

Although much had been discovered about electricity earlier, it was not until 1897 that the English physicist J. J. Thomson was able to prove that such particles as electrons (as they were later called) actually existed. Once the scene had been set for the electron to make itself known, work by others went on unceasingly, not only to back up Thomson's findings but also to progress further in unravelling more atomic mysteries. Studying the discoveries of the past enables us to understand better many facets of electron behaviour. This is the main purpose of the Chapter.

8.1 EARLY IDENTIFICATION

The various electrical breakthroughs which had already been made enabled Thomson to set up an apparatus based on principles still much used today in the cathode-ray or television tube. Essentially the apparatus consisted of an evacuated glass vessel as shown in Figure 8.1(i). Earlier experiments had shown that a stream of rays or particles could be projected through an evacuated tube by being generated at a negative cathode and collected at a positive anode using a high direct voltage. Thomson's apparatus passed the rays from the cathode to a positively biassed electrode, S_1 so accelerating the ray or particles with some passing on through the slit. At S_2 there was further acceleration with all that passed through the S_2 slit eventually reaching the zinc sulphide coating on the end of the tube. Zinc sulphide had already been found to fluoresce with these rays so a spot of light appeared on the screen. Then, with a potential applied across the plates P_1 and

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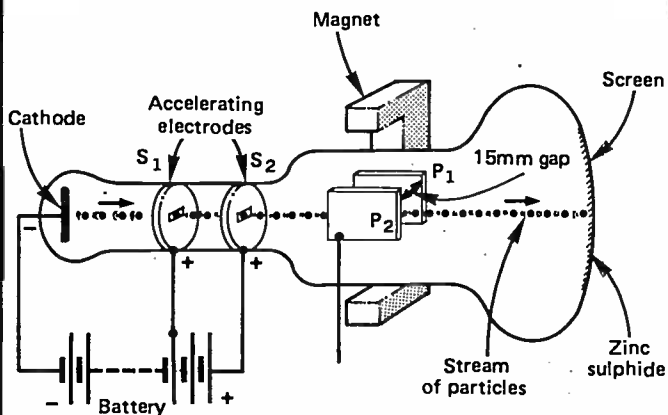
(i) The basic apparatus
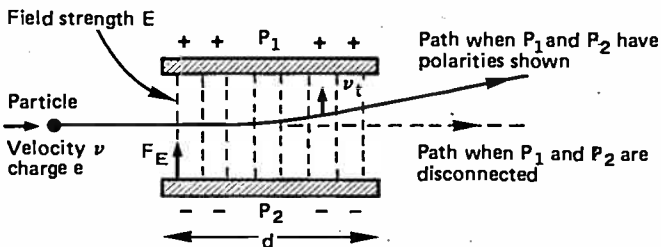
(ii) Deflection of particle in electrostatic field
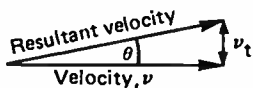
(iii) Vector diagram showing relative velocities

Fig. 8.1
J. J. Thomson's experiment to prove existence of electron.

P_2, Thomson found that there was deflexion away from the negative plate and towards the positive, such deflexion being evident on the screen as a movement of the spot. This added further proof that the charge carried by the ray or particles was negative. Because under the same conditions on P_1 and P_2 the deflexion was always the same it appeared that the stream consisted of particles all similar in mass and charge.

By placing a magnet as shown in the Figure so that its field occupied the same position and length as that of the plates, it was found possible for the magnetic field to cancel the electrostatic and so reduce the deflexion of the spot to zero. From this the fundamental nature of the particles can be proved to show that they are all similar and have the same charge to mass ratio, as follows.

Consider a charged particle travelling between the plates P_1 and P_2 towards the screen. When P_1 and P_2 have a potential across them then a force acts on the particle pushing it away from one plate towards the other. It therefore not only has a longitudinal velocity, v (along the tube) but also a transverse one v_t (across the tube). This is shown in Figure 8.1(ii).

Now the force F_E on a particle carrying a charge e in an electric field of strength E is eE (Sect.4.7.2). This is the transverse force deflecting the particle. Note that the longitudinal velocity does not enter into the equation. However for a magnetic field it does, so:

force F_M on a particle carrying a charge e in a magnetic field of flux density $B = e\,v\,B$ (this is discussed in more detail in Sect.8.6.4).

By arranging the magnet [see Fig.8.1(i)] so that the electric and magnetic field path lengths in the direction of travel are the same, (d) and adjusting either field so that the net deflexion is zero, then

$$F_E = F_M ,$$

hence

$$eE = e\,v\,B \qquad \therefore v = E/B$$

and since

$$F_E = e E \quad \text{and} \quad F_E = m a$$

where m = mass of particle and a = its acceleration (Sect.4.2), then

$$a = F_E/m = e E/m$$

and if the length of the deflecting path = d, then time particle is under the influence of F_E or F_M, $t = d/v$ (Sect.4.1.4). Hence transverse velocity,

$$v_t = a \, t = \frac{e E}{m} \times \frac{d}{v}$$

A triangle of forces helps us to see how the particle is deflected; see Figure 8.1(iii) and refer back to Section 4.1.1 if necessary.

From this it is evident that the deflexion takes place through an angle θ where

$$\tan \theta = v_t/v \quad \therefore \tan \theta = \frac{e E d}{m v^2} \, .$$

Now θ is measured in radians (Sect.5.1) and tables show that up to about $10°$ or 0.1745 radians, $\tan \theta$ is almost the same as θ (the difference is not more than 1%). Accordingly we can conveniently say that:

$$\theta = \frac{e E d}{m v^2} \, .$$

Furthermore, as already shown, since $v = E/B$, then

$$\theta = \frac{e E d}{m(E/B)^2} = \frac{e d B^2}{m E} \quad \therefore \frac{e}{m} = \frac{\theta E}{d B^2}$$

E, B and d are known (d in the original experiment was about

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5cm), the deflexion can be measured for θ to be calculated, so a value for the specific charge e/m follows.

With different gases at low pressure in the tube or with various metals for the electrodes, e/m was found to be reasonably constant, hence presumably the particles were not constituents of the gases or metals so must be negative particles fundamental to all matter. This was the breakthrough.

Work went on apace after this with e/m being measured by other methods and the results had a striking similarity. From all the work there remained little doubt that the particles were constituents of all atoms. Earlier in 1881 G. J. Stoney had suggested the name "electron" for the elementary electrical particle and soon the name was adopted. The name arises from the Greek *elektron* for "amber" which had already been found to develop electrostatic forces when suitably rubbed.

The currently accepted value for the charge/mass ratio of the electron is:

$$e/m_e = 1.7588 \times 10^{11} \text{ coulombs per kilogram.}$$

Note the subscript e which has crept in and attached itself to the m. m_e is known as the *electron rest mass* which is a reminder that we have yet to take into account Einstein's work. Briefly he has shown that mass is not constant, it can change at high velocities; a rather disturbing notion which we worry about later.

8.2 CHARGE

Thomson and others had made attempts to measure the charge of an electron with fair success in that the results were in the right order. However it was left to R. A. Millikan and his colleagues in the USA to refine the earlier measurements and so produce a more acceptable figure. By no means could the work be classed as simple and straightforward, it started in 1909 and continued as a series of tests for several years. Certainly this work has had a profound influence on the development of atomic physics.

Very briefly Millikan and his co-workers sprayed droplets of oil from an atomizer as shown in outline in Figure 8.2. In this process each droplet acquired a charge due to friction (Sect.10.5). A single droplet between the two plates could be selected and observed through a telescopic device and its velocity measured. With no potential on the plates the droplet falls due to gravity but against the resistance of the air, ultimately reaching a final velocity v_0. On the other hand, with a potential on the plates as shown, i.e. the upper plate positive, there is an upward force on the droplet because it has a negative charge and is within an electric field (Sect.4.7.2). By adjusting the potential the droplet could be held in suspension or even caused to rise against gravity, with a velocity, say, v_1. Knowing the values of V and d (see Fig.8.2), the viscosity of the air and also the two velocities v_0 and v_1, through various formulae the charge on any droplet was calculated.
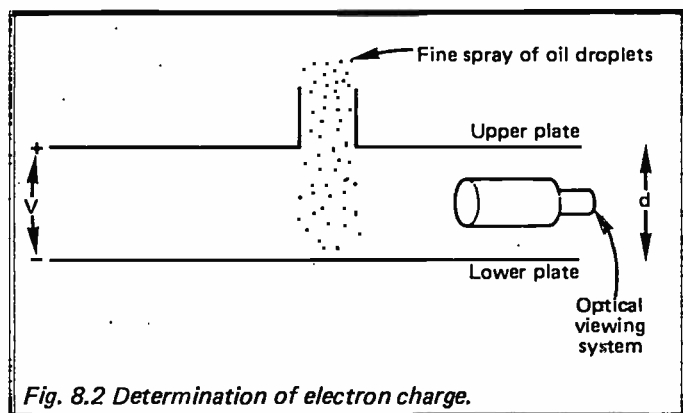
Fig. 8.2 Determination of electron charge.

Throughout the experiment which in fact involved thousands of measurements, never was the charge found to be less than about 1.6×10^{-19} coulomb, what is more it was always an integer (whole number) multiple of this value. The indication was that:

(i) a single electron has a charge of the value quoted;
(ii) greater charges are the result of two or more electrons.

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Thus Thomson's work was confirmed and a value obtained for the charge. In electronics the electron charge (sometimes called the *elementary charge*) is invariably designated by *e* (nothing to do with e, the base of natural logarithms).

Subsequent experiments with refinements and modifications have produced the reasonably consistent value:

$$e = 1.6021 \times 10^{-19} \text{ coulomb.}$$

We must ever be mindful of the fact that scientific endeavour continues unceasingly. More recently, in experiments aimed at discovering more about the nature of fundamental particles, especially within the nucleus, has come the idea of *quarks*, a name used to describe a purely hypothetical component of an elementary particle. The supposition that quarks exist is made simply as a starting point for further investigation. This is in the realm of atomic physics so does not concern us here except that the idea of quarks has now widened to include the electron and already claims have arisen that particles with a "fractional charge" exist. However other experimenters have been unable to confirm this and many still remain in agreement with Millikan. Certainly such controversy is not easily resolved, hence for the time being, with such an aura of uncertainty, we can only continue our discussions with the fund of knowledge we have. Accordingly for the present we remain convinced that the electron charge is indivisible.

8.3 MASS

In Section 8.1 is given the charge/mass ratio for the electron as $e/m_e = 1.7588 \times 10^{11}$ with both *e* and m_e then being unknown quantities. We now have a value for *e*, so from the ratio, m_e follows simply from $e/(1.7588 \times 10^{11})$ i.e.

$$m_e = \frac{1.6021 \times 10^{-19}}{1.7588 \times 10^{11}} = 9.109 \times 10^{-31} \text{ kilogram}$$

(or 9.109×10^{-28} g).

Our high speed speck of charge therefore does have some weight for 10^{27} of them add up to about one gram (some thirtieth of an ounce). Going back to Figure 2.1 might help us to get this number into perspective.

8.4 ELECTRON VELOCITY IN A CONDUCTOR

Many have the impression that electricity moves fast. After all it takes no time for the room to light up after the switch is operated nor for a bell to ring when the button is pushed. We can look at this by first obtaining an estimate of the velocity of the electrons along a wire when a current flows. Section 7.1 draws a mental picture of electron drift so already the idea of electrons streaking along a wire like pellets from a shotgun must be receding. Figure 8.3(i) is a sketch of a length of wire of cross-sectional area A m^2. If the average velocity of the electrons along the wire is v m/s, then the length of wire travelled in 1 second is v metres and this is shown shaded in the diagram as the length XY. Then:

volume of length $XY = A \, v$ m^3

and if the wire contains n free electrons per m^3, then the number in length $XY = n \, A \, v$ (for n see Sect. 6.1.4).

Since this number of electrons is moving v metres per second, this is the number passing any point such as X or Y in 1 second. In fact this is the total charge passing a point per second, i.e. the current, I. For the electron with charge e (= 1.602×10^{-19} C):

$I = n \, e \, A \, v$ amperes   and $v = I/(n \, e \, A)$ m/s

so the electron velocity along the wire increases with the current and decreases with the wire cross-sectional area. It also decreases as the number of available electrons increases. Here is a practical example:

In a computer is a very fine copper wire of diameter

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Fig. 8.3 Electron velocity in a conductor.
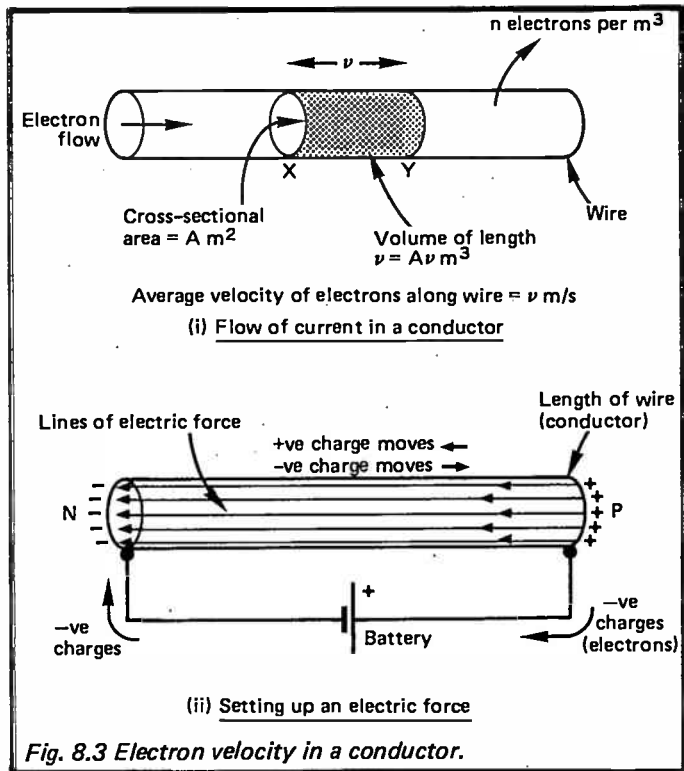
0.04mm (about SWG 48 to the old hands). A current of 1mA flows through the wire.

$$A = 1.26 \times 10^{-9} \ m^2 \ .$$

Then

$$v = \frac{0.001}{10^{29} \times 1.602 \times 10^{-19} \times 1.26 \times 10^{-9}}$$

$$\simeq 5 \times 10^{-5} \ m/s$$

or about 1mm in 20 seconds. From this, electricity is certainly not one of the fastest things on earth. This shows just how comparatively slow the electron *flow* is even though individual electron velocities are extremely high. This may be difficult to appreciate so it is worthwhile illustrating what happens through a couple of situations having some semblance. It is the *electric force* which is propagated almost immediately (approaching the speed of light) when a potential difference is applied. Its effect is felt evenly along the wire as shown pictorially in Figure 8.3. There is a charge or potential difference existing across the battery terminals and this is applied to the N and P ends of the conductor. Lines of force are therefore set up as shown, hence electron drift takes place from N to P. At the two ends the displacement of charges appears to be instantaneous although no electron will have made the full journey for some time. This might be illustrated by comparison with the normal mains water supply. The pumping station maintains pressure all along the pipes so that when a tap is turned on, water flows out yet the water which at the same time is entering the pipes at the pumping station could take days to reach the tap.

Just in case it is not perfectly clear from this analogy, let's try again:

1 metre of the 0.04mm diameter wire has a volume of 1.26×10^{-9} m^3, therefore the number of free electrons contained is equal to

$$(1.26 \times 10^{-9}) \times 10^{29} = 1.26 \times 10^{20} .$$

In one second 1mA is equivalent to the movement of

$$(6.24 \times 10^{18})/1000 = 6.24 \times 10^{15} \text{ electrons,}$$

as shown in Section 7.1.

Hence in one second only 1 in $(1.26 \times 10^{20})/(6.24 \times 10^{15})$, i.e. 1 in 20,000 of the total electrons in the wire is effective. This means that in one second for a current of 1mA, 6.24×10^{15} electrons have entered at N with the same number removed from P but this is only 1 in 20,000 of the free electrons in the

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wire. The process might be illustrated by likening the wire to a barrel full of peas, each pea representing an electron. Current flow is demonstrated by continual removal of handfuls of peas from the bottom bung-hole and replacing them at the top. Movement of peas down the barrel is very slow and the process goes on for a long time before a pea from the top reaches the bottom, yet there is a continual flow of peas.

8.4.1 Conductivity Again

Having now studied electron drift it is possible to develop a simplified formula for conductivity. It is necessary to make several assumptions, nevertheless the formula is useful because it illustrates mathematically some factors often taken for granted. What is more, it helps in getting better acquainted with the realities of electron movement.

Again consider the conductor of Figure 8.3(i) with its n free electrons per unit volume. These are streaking about in all directions and in such a free-for-all, collisions are inevitable. Because of the random nature of such turmoil the distance an electron travels between collisions varies but we postulate an average length of path in a straight line for all the electrons and call it d. This is known as the *mean free path*. The average time between collisions is therefore d/v where v is the average electron velocity. Summing this up, there are:

n free electrons per unit volume with mean free path, d, mean free time, t and mean velocity, v. The word "mean" (or "average") occurs frequently because we are considering a group of electrons together.

From Section 6.3.4 the average k.e. = $3/2$ kT where k = Boltzmann's constant and T = absolute temperature, and since k.e. is given by $\frac{1}{2} m v^2$, then

$$\frac{1}{2} m v^2 = 3/2 \, k \, T \quad \text{i.e.} \quad v = \sqrt{\frac{3 \, k \, T}{m}}$$

and since $t = d/v$, then

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$$t = d \sqrt{\frac{m}{3k\,T}}\ .$$

Consider a length, l of the conductor with a potential difference, V across its ends.

Electric field strength,

$$E = V/l$$

and the force acting on an electron in the direction of the field,

$$F = e\,E = e\,V/l$$

Therefore acceleration of an electron in the direction of the field,

$$a = F/m = e\,v/l\,m\ ,$$

which is for an average time, t.

Now assuming that on average, after a collision the velocity in the direction of the field is zero, then from the equations of motion (Sect.4.1.4), the final velocity = $a\,t$ and the average drift velocity,

$$v_d = a\,t/2$$

$$\therefore \quad v_d = \frac{e\,V}{l\,m} \times d \sqrt{\frac{m}{3\,k\,T}} \times \frac{1}{2}\ .$$

$$= \frac{e\,V\,d}{2\,l\,m} \times \frac{\sqrt{m}}{\sqrt{3k\,T}} = \frac{e\,V\,d}{2l\,\sqrt{3k\,m\,t}}\ .$$

From the previous section, the current, $I = n\,e\,A\,v_d$

$$\therefore \quad I = n\,e\,A\,\frac{e\,V\,d}{2l\,\sqrt{(3k\,m\,T)}} = \frac{n\,e^2\,A\,d}{2l\,\sqrt{(3k\,m\,T)}} \times V$$

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Hence from Ohm's Law (Sect.7.3.3):

Resistance of wire,

$$R = V/I = \frac{2l\sqrt{(3k\,m\,T)}}{n\,e^2\,A\,d}$$

and from $R = \rho\,l/A$ (Sect.7.3.3), resistivity, $\rho = A\,R/l$

$$\therefore \quad \rho = \frac{2\sqrt{(3k\,m\,T)}}{n\,e^2\,d}$$

and conductivity,

$$\sigma = 1/\rho = \frac{n\,e^2\,d}{2\sqrt{(3k\,m\,T)}}$$

or

$$\frac{e^2}{2\sqrt{(3k\,m)}} \times \frac{n\,d}{\sqrt{T}} \quad .$$

Now

$$\frac{e^2}{2\sqrt{(3k\,m)}}$$

has a constant value of

$$\frac{(1.602 \times 10^{-19})^2}{2\sqrt{(3 \times 1.38 \times 10^{-23} \times 9.109 \times 10^{-31})}}$$

$$= 2.09 \times 10^{-12}$$

$$\therefore \quad \sigma = 2.09 \times 10^{-12} \times \frac{n\,d}{\sqrt{T}} \quad ,$$

proving that the conductivity, σ varies:

  (i) directly with the number of free electrons
 (ii) directly with the length of the mean free electron path
(iii) inversely as the square root of the absolute temperature.

From our knowledge of electron activity, (i) and (ii) make sense and there is ample practical evidence in support of (iii) for generally the conductance of a conductor falls as temperature rises.

From this little burst of mathematical enterprise we see a little more of the wood for the trees but we must not ignore the many assumptions made. Nevertheless, seemingly being on the right track, it may also be instructive to estimate the mean free electron path, d for copper from a knowledge of its resistivity, ρ:

for copper, $\rho = 1.72 \times 10^{-8}$ at 20°C (Sect.7.3.2) and
        $n = 8.49 \times 10^{28}$ per m^3 (Sect.6.1.4) on the assumption that every atom gives up its valence electron.

$$T = 293 \text{ K } (20^{\circ}\text{C})$$

$\therefore$ since $\rho = \dfrac{\sqrt{T}}{2.09 \times 10^{-12} \, n \, d}$

then

$$d = \dfrac{\sqrt{T}}{2.09 \times 10^{-12} \, n \, \rho}$$

$\therefore \quad d = \dfrac{\sqrt{293}}{2.09 \times 10^{-12} \times 8.49 \times 10^{28} \times 1.72 \times 10^{-8}}$

$\quad\quad\quad = 5.6 \times 10^{-9}$ metres.

Section 6.1.4 estimates the diameter of the copper atom as 2.275×10^{-10} m, hence according to our formula the mean

free path (i.e. the distance an electron travels on average between collisions) is about 25 times the diameter of a single copper atom.

As mentioned, the assumption is that every atom gives up its electron. If this is not so, then n is smaller and the formula shows that d increases. This is to be expected for with fewer electrons milling around, collisions are less likely hence t increases also.

8.4.2 Superconductivity

A phenomenon which is giving scientists a hard time but has promise of great reward is that of *superconductivity*, a condition in a material of virtually zero resistance and is in fact not just *super* but almost infinite conductivity. It is seemingly a cure for many electronic ills but it is not without drawbacks for the material has to be at a very low temperature.

In 1911 Kamerlingh Onnes, a Dutch physicist, first observed superconductivity in mercury but only when cooled to within a few degrees of absolute zero $(-273°C)$. He found that once a current was set up in a circuit of mercury without disturbance, it continued to flow for many hours. As we now know, at normal temperatures, in common with all other materials, mercury has resistance. In any resistance a current dissipates power and hence would die away if no energy in the form of a p.d. were available to maintain it. Under superconductive conditions however the resistance decreases to around one thousand million millionth (10^{-15}) of that at room temperature. Under such conditions power losses are truly negligible and a current once started experiences no opposition. The discovery was (and still is) of such great importance that two years later Onnes was awarded a Nobel prize for his low temperature work.

Many other metal elements, alloys and compounds exhibit the effect, each having its own *critical* or *transition temperature*, i.e. that temperature above which the material is resistive and below which it is superconductive. Materials which have extremely low transition temperatures are useful in nuclear

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research but the fact that considerable power is required for cooling them means that their use in the more practical world is hardly economic for example, in electrical power engineering. In power distribution all resistance losses are wasteful and result in additional fuel being consumed at the power station. However continual research is discovering materials which exhibit superconductivity at much higher temperatures. In fact present scientific development is moving so fast that temperatures higher than 100K ($-173°C$ — the home freezer works at about $-18°C$) are now feasible with the cooling accomplished by liquid nitrogen. Cooling is therefore becoming less of a restriction. Claims are even made of superconductivity at above room temperature. Even if this is in fact achieved there still remains the difficulty of forming wires from the materials, most of which unfortunately fall into the "rare earths" group (atomic numbers 57 — 71), none of them being "drawn" easily as with a metal such as copper.

The mechanism through which superconductivity arises is not yet well understood. It is generally suggested that the vibration of atoms and ions (Sect.6.1) ceases below a certain temperature and under such becalmed conditions the electrons are less likely to suffer collisions and capture by motionless ions. They therefore experience a less eventful journey and progress freely through the lattice structure (Sect.7.1).

8.5 ENERGY, WAVES AND PARTICLES

Chapter 7 discusses electricity in terms of the escapades of electrons as though they are merely particles carrying (or of) negative charges. By so doing we are able to formulate a useful picture of what electricity is and how it moves. In scientific language we are studying *electrodynamics.* "Dynamics" implies motion and central to this is energy, the stuff which provides the driving force behind all that happens. When dealing with electrons for example, because they all have the same mass, energy can be considered as the vitality of their velocities (k.e $= \frac{1}{2}mv^2$). As shown in Section 4.1.5, when vehicles collide the new distribution of momentum means that their velocities change, hence although

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the total energy remains the same, it too is redistributed. Electron energies are therefore at random and considering that they can collide at any angle means that their directions of movement are at random also. Summing this up therefore, electricity is apparently tiny "packets" of charge in motion and when control is exercised over the normal random nature of the motion then an electric current flows. Fine, but in reaching this conclusion we may have accepted without a second thought the way in which electrons gain their freedom for it all looks so simple, give them sufficient energy and they're off. That is how it is but for a better understanding of the whole process we must next expand our thinking to embrace two almost unbelievable facts:

(i)   that energy does not move smoothly but in "packets"
(ii) that electrons, although undoubtedly particles are also associated with wave motion (Sect.4.9).

To appreciate that particles behave like waves and vice versa needs quite a stretch of the imagination. Disbelief may be assuaged however on learning that the theory, known as *quantum electrodynamics* or just *quantum theory* is able to underpin all the laws so far written about physics and chemistry.

A *quantum* is a discrete unit or packet of energy and the aim now is to find out how such packets are delivered and what happens to the electrons when they arrive. Many famous scientific gentlemen have contributed to the theory as it stands today. In the technical papers they have produced the mathematics and reasoning are such that we ordinary mortals give up in despair, nevertheless it is possible to glean sufficient from their conclusions to at least develop a sense of what is going on.

8.5.1 The Electron-Volt

Because in atomic considerations the joule is a comparatively large unit of energy, a unit referring to a single electron is frequently used. One electron-volt (eV) is the energy acquired

by an electron when accelerated through a potential difference of one volt. Hence:

work done = $q\,V$

and since

$q = 1.602 \times 10^{-19}\,C$ and $V = 1$ volt,

then $1\,eV = 1.602 \times 10^{-19}$ coulomb-volts, i.e. joules.

(Note that although volt appears in the name of the unit, eV is a unit of *energy*, not voltage.)

8.5.2 Energy in Orbit

Section 5.2 shows how satellites and heavenly bodies stay in orbit with a centripetal force provided by gravitational attraction. For an electron orbiting an atom nucleus the centripetal force consists of the attractive force created by charges plus gravitation. Section 6.1.3 however gives ample justification for disregarding the latter, hence the centripetal force can be taken simply as the electric force, F_e where, from Coulomb's Law (Sect.4.7.1):

$$F_e = \frac{1}{4\pi\,\epsilon_0} \times \frac{Q_1\,Q_2}{d^2} \quad .$$

Now Q_1 and Q_2 are both equal to e, the charge of proton or electron (Sect.6.1.1) and substituting k for $1/4\pi\,\epsilon_0$ (= 8.99 $\times 10^9$ N m^2/C^2 — Sect.4.7.1 and note this is not the Boltzmann's constant, k) leaves:

$$F_e = k\,\frac{e^2}{r^2} \quad ,$$

where d now becomes r, the radius of orbit.

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Also, now working in terms of motion, the centripetal force:

$$F_c = \frac{m v^2}{r} \quad \text{(Sect.5.1) and since } F_c = F_e,$$

then

$$\frac{m v^2}{r} = \frac{k e^2}{r^2}.$$

from which:

$$v = \sqrt{\frac{k e^2}{m r}}$$

k, e and m are known so given the radius of the orbit, r, the electron velocity, v can be calculated. The formula shows that as the radius of orbit increases, the velocity falls. We have yet to determine r.

Next consider two like charges Q_1 and Q_2 separated by a distance, d and let one of them be fixed, say Q_1 with Q_2 mobile. When a body is moved to a higher position above Earth and thereby gains in potential energy, the surface of the Earth is usually the reference location (at which the p.e. is zero). It might at first seem sensible when considering the p.e. of an electron to make the reference location the nucleus and what is more there seems to be nothing else available. However to complete the analogy with a body above Earth we need p.e. = 0 at the reference location as shown in Figure 8.4(i). This condition arises for the electron only when d = infinity, a seemingly odd concept to use but one which does serve the purpose well. Accordingly imagine the charge Q_2 [Fig.8.4(ii)] to be at infinity or at least some distance away (note that 1mm to an atom is equivalent to several hundred kilometres to a cricket ball). As it moves nearer to Q_1, the work done in moving it *against* the electric field can be added up bit by bit until at a distance d, it can be shown that because the total work done is in fact the potential energy gained:

$$\text{p.e.} = \frac{k Q_1 Q_2}{d}.$$

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Fig. 8.4 Reference locations for potential energy calculations.
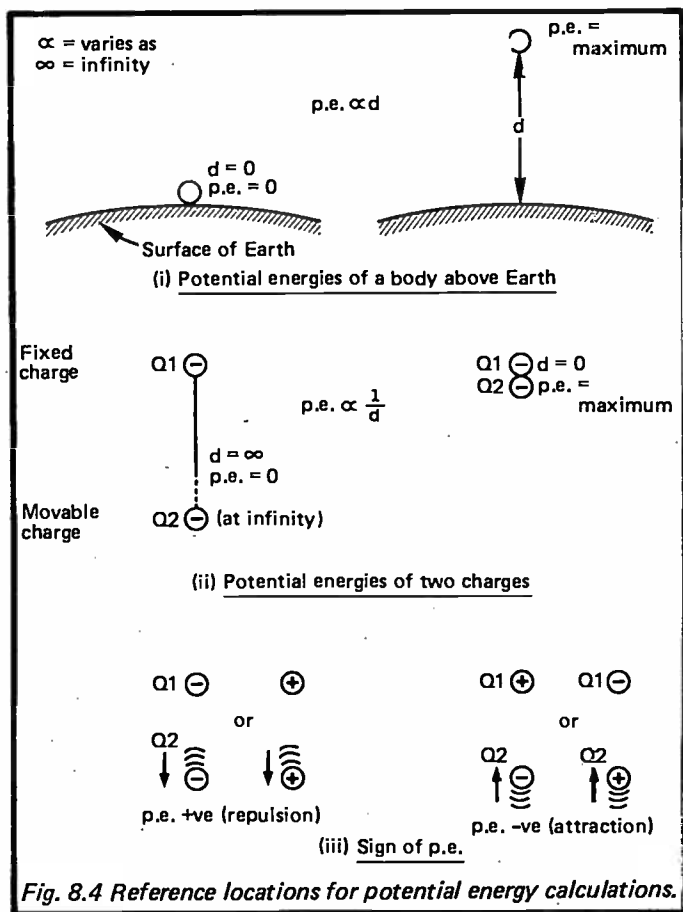
That Q_2 now has potential energy is demonstrated by the fact that if it is allowed its freedom, it will move away from Q_1 (like charges) with its potential energy becoming kinetic. Figure 8.4(iii) recalls from Section 4.7.1 that if both Q_1 and Q_2 have the same sign the p.e. is +ve so corresponding to a force of repulsion between them. For opposite charge signs the p.e. is −ve, arising from attraction. When Q_1 and Q_2 are

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proton and electron:

p.e. $= -(k\,e^2)/r$ (the minus sign for opposite charges).

Next consider the simplest of all atoms, hydrogen (one proton, one electron). Its total energy due to the orbiting electron is the sum of k.e. and p.e. (Sect.4.4.1). The k.e. is given by $\frac{1}{2}\,m\,v^2$, hence the total energy, $E = \frac{1}{2}\,m\,v^2 + [-(k\,e^2)/r]$ and since $v = \sqrt{[(k\,e^2)/(m\,r)]}$, then:

$$E = \frac{1}{2}\,m \times \frac{k\,e^2}{m\,r} - \frac{k\,e^2}{r} = \frac{k\,e^2}{r} \times (\tfrac{1}{2}-1)$$

$$= -\frac{k\,e^2}{2\,r}$$

The negative total energy indicates attraction between proton and electron.

The question now arises as to what we have achieved by only considering the least complicated atom. Quite a lot, for remembering that −ve energy represents attraction, then if E were to become zero, the attraction must also be zero. Under this condition the electron would no longer be held by the nucleus and would fall out of orbit, more so if E were +ve so indicating a force of repulsion. With these formulae therefore it is now possible to calculate the +ve energy which must be supplied to an electron to free it from its orbit. So far we have developed two equations:

$$v = \sqrt{\frac{k\,e^2}{m\,r}} \quad \text{and} \quad E = -\frac{k\,e^2}{2r}$$

but there are three unknown quantities, v, r and E, hence for a complete solution another equation is required. Here experimental work comes to our aid which has found the energy actually required to split the hydrogen electron from its nucleus i.e. take it out of orbit. The magic figure is 13.6 eV

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so indicating that the normal energy of the electron is -13.6 eV (adding 13.6 to this value makes E zero), then since:

$$E = -\frac{k\,e^2}{2r} \quad \therefore r = -\frac{k\,e^2}{2E}$$

i.e.
$$r = \frac{-(8.99 \times 10^9 \ \mathrm{N\,m^2/C^2}) \times (1.602 \times 10^{-19} \ \mathrm{C})^2}{2 \times [-13.6 \times 1.602 \times 10^{-19} \ \mathrm{J\,(N\,m)}]}$$

(see Sects.4.4 and 4.7.1 if the units do not make sense)

$$\therefore r = \frac{8.99 \times 10^9 \times 1.602 \times 10^{-19}}{2 \times 13.6} \ \mathrm{m} = 5.295 \times 10^{-11} \ \mathrm{m}$$

i.e. about *one twentieth of one millionth of a millimetre.*

With a value for the radius of orbit, the electron velocity follows for:

$$v = \sqrt{\frac{k\,e^2}{m\,r}} = e\sqrt{\frac{k}{m\,r}}$$

$$= 1.602 \times 10^{-19} \mathrm{C} \sqrt{\frac{(8.99 \times 10^9 \ \mathrm{N\,m^2/C^2})}{(9.109 \times 10^{-31}\,\mathrm{kg} \times 5.295 \times 10^{-11})\,\mathrm{m}}}$$

(see also Sect.6.1.1 for m).

$$\therefore v = 2.187 \times 10^6 \ \mathrm{m/s},$$

i.e. *over two thousand kilometres in one second.*

The figures obtained for r and v are almost unbelievable, more so that we can actually calculate them. Happily they do tie in with the results from so much other experimental work that we can now move on, convinced of being on the right track.

8.5.3 Letting Some Light In

It all started with the difficulty scientists had in explaining the emission of light from a very hot metal such as iron which changes from "red-hot" to "white-hot" as its temperature is raised. The ideas then current about the nature of light were inadequate. Then a German scientist, Max Planck found that he could predict what was observed if the radiation of light were considered as though emitted in tiny bursts or "packets" of energy. This was a revolutionary concept but it seemed to work and it gave rise to the first relationship involving both energy and waves.

Planck's formula is simply;

$$E = hf$$

where E is the energy of a single burst or *quantum* (amount or portion — plural quanta), f is the light frequency (Fig.4.9) and h is a constant which is known today as *Planck's Constant*, of value 6.626×10^{-34} joule-second. In this unit for h time is involved because it arises in f.

From Figure 4.9 we take the range of light frequencies as from 4.1×10^{14} Hz (lowest red) to 7.9×10^{14} Hz (highest violet) and so calculate the light quantum or *photon* (from Greek, *phos* = light) energy:

for red $E_R = 6.626 \times 10^{-34} \times 4.1 \times 10^{14} = 2.72 \times 10^{-19}$ J

or in electron-volts

$$\frac{2.72 \times 10^{-19}}{1.602 \times 10^{-19}} \simeq 1.7 \text{ eV} .$$

Similarly for violet:

$$E_V = \frac{6.626 \times 10^{-34} \times 7.9 \times 10^{14}}{1.602 \times 10^{-19}} \simeq 3.3 \text{ eV} ,$$

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i.e. visible light photon energies range from 1.7 to 3.3 eV.

The formula could hardly be shorter, but it has a profound message. The energy of a quantum is proportional to the *frequency* of the light, not its intensity. We will see experimental evidence to support this in the next section.

8.5.4 Enter Einstein

Einstein went one step further. Although Planck's ideas were with regard to the emission of light, the *photoelectric effect* still defied satisfactory explanation. This is the effect light can have on the emission of electrons from the surface of a metal. In 1905 Einstein extended Planck's theories by suggesting that not only was light emitted and absorbed in discrete amounts but that all the energy of a photon can be absorbed by a single electron. Having received the full energy of a photon, an electron is then known as a *photoelectron.*

Energy must be supplied to an electron at the surface of a metal for it to escape from the metal itself. The amount of energy required for this to happen is known as the *work function* (ϕ — the Greek lower case *phi*) and it is conveniently measured in electron-volts (Sect.8.5.1). If therefore a photon of energy E acts upon an electron, then an amount ϕ is used in freeing the electron from the surface of the metal. The energy left over is $E - \phi$ and this is available to increase the k.e. Since $E = hf$, Einstein suggested that:

> maximum k.e. of *emitted* photoelectron = $hf - \phi$
> (generally k.e.'s are less than this because of losses through collisions before emission).

An interesting result of this which is confirmed by experiment is that when the whole of a photon energy is expended in providing the work function, there is none left over to accelerate the freed electron. It therefore falls back into the metal which has become positively charged. In this case:

$$hf - \phi = 0 \quad \text{hence } f = \phi/h$$

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showing that there is a light frequency below which emission is not possible.

Such an experiment which checks the validity of Einstein's hypothesis is illustrated in Figure 8.5. Photoelectrons are

Electron flow

Vacuum

Collecting electrode

Photoelectrons

Incident light (variable in frequency and intensity)

Ammeter

Metal plate

Variable voltage

An arrow through a component symbol ↗ indicates that the value or output is variable

Fig. 8.5 Energy of photoelectrons.
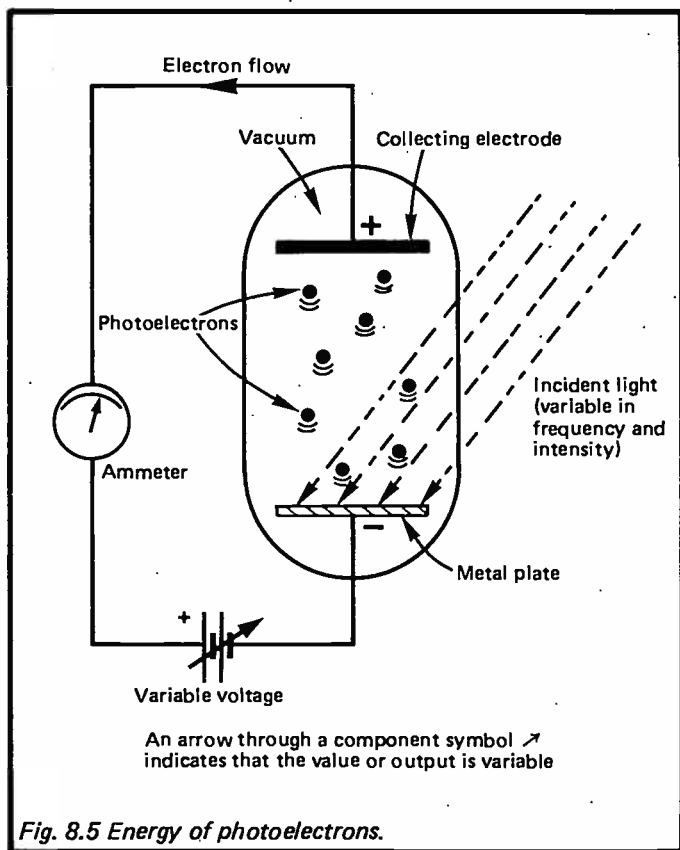
emitted from the metal plate and are encouraged across the space to the collecting electrode. Suppose that the electric field is reversed by making the collecting electrode slightly negative with respect to the metal plate (i.e. the battery is

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reversed). The electric field now acts to repel the photo-electrons. When the field is weak, those released by the incident light with high kinetic energies overcome the retarding force and still reach the collecting electrode even though it is negative. The number which does so is shown as current on the ammeter. As the field is increased in strength however, photoelectrons with the lower velocities are repelled before they reach the collecting electrode and are driven back. Thus the collecting electrode voltage which just reduces the current to zero corresponds to the maximum photoelectron energy. The substitution of different light frequencies and intensities (colour and brightness in television terms) proves that photo-electron energy is not dependent on light intensity. On the other hand a graph relating their maximum k.e.'s with light frequency is a straight line so proving that E_{max} varies directly with frequency. In more colourful terms a dim violet light (say, $f = 7.5 \times 10^{14}$ Hz — Fig.4.9) gives rise to more energetic photoelectrons than does a powerful red light (say, $f = 4.5 \times 10^{14}$ Hz).

As an example, suppose that the metal plate used in Figure 8.5 is zinc which has a work function (ϕ) of 3.73 (eV). The minimum (threshold) frequency (f_0) of light which will cause photoemission is therefore:

$$f_0 = \frac{\phi}{h} = \frac{3.73 \times 1.602 \times 10^{-19}}{6.626 \times 10^{-34}}$$

(ϕ must be brought to joules), therefore

$$f_0 = 9 \times 10^{14} \text{ Hz}$$

and by reference to the diagram of the visible spectrum in Figure 4.9 it is evident that ordinary visible light will not set free electrons from zinc but ultraviolet light will. To calculate the maximum kinetic energy at any other frequency, say 10^{16} Hz, it is first necessary to find the total energy from $E = h f$, i.e. $6.626 \times 10^{-34} \times 10^{16} = 6.626 \times 10^{-18}$ J. From this is subtracted the energy required to release the electron from the surface of the metal (for zinc,

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$3.73 \times 1.602 \times 10^{-19}$ J). Hence there remains:

$$(6.626 \times 10^{-18}) - (3.73 \times 1.602 \times 10^{-19})$$

$$\simeq 6 \times 10^{-18} \text{ joules or } 37.6 \text{ eV}$$

as the kinetic energy which speeds the electron across the tube.

The electron velocity follows from:

$$\text{k.e.} = \tfrac{1}{2} m v^2 \quad \text{i.e. } v = \sqrt{\frac{2 \times \text{k.e.}}{m}}$$

$$\therefore \quad v = \sqrt{\frac{2 \times 6 \times 10^{-18}}{9.109 \times 10^{-31}}} = 3.63 \times 10^6 \text{ m/s}.$$

This excursion into Einstein's work together with the experimental evidence helps to build up confidence in accepting the apparent paradox of the association of the electron with both particles and waves. There is a wave theory for light and also a quantum theory. Each can satisfactorily explain particular phenomena and so the two theories although seemingly very different, are in fact complementary. We will see how the two theories coalesce as this Chapter unfolds.

Einstein did not stop there. He went on to show that mass and energy are related and that each can be converted into the other. Although undoubtedly a startling pronouncement at the time, its validity has been proved over and over again. This is seen in action when nuclear reactions act upon mass and the resulting energy release is enormous. The simple equation Einstein gave us belies the profound effect it had on the world of his day. It is:

$$E = m c^2$$

where E is the energy, m is the mass and c the speed of light.

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We have so far avoided this equation because its outcome would have had practically no effect. As the velocity of a particle rises, its kinetic energy increases so producing an increase in mass of E/c^2. Now c^2 is equal to 9×10^{16} m/s, such a large number that the velocity must be almost at that of light for any effect to show. Nevertheless Einstein has since been proved to be correct. His ideas are needed here because light is being brought into the discussion and it always travels at c.

From this formula together with Planck's arises a relationship between the mass of a photon and the frequency of the light it represents for:

$$E = h f \quad \text{and} \quad E = m c^2, \quad \text{hence} \quad h f = m c^2$$

$$\therefore \quad m = h f/c^2 \quad \text{or} \quad m = k f$$

since h/c^2 has constant value, showing that the *mass* of a photon is directly proportional to the frequency.

8.5.5 Matter Waves

Fanciful though the title of this Section may seem, it is not so to physicists. In fact in 1924 Louis Victor de Broglie (a French physicist) published a hypothesis suggesting that *any* moving object has an associated wavelength. He based this on the argument that if light can act both as particles and also as waves, then why should not electrons which are particles also act as waves? He called them *matter* or *pilot waves* in the sense that the wave guides or steers the particle. From this the idea could be extended to all moving objects.

We can trace this through, beginning with the formula for the mass of a photon, $m = h f/c^2$.

Recalling that wavelength and frequency of light are related by $\lambda = c/f$, then, momentum (p) of a photon travelling at c is equal to

$$m \times v = \frac{h f}{c^2} \times c = \frac{h f}{c}$$

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and since $f/c = 1/\lambda$, then

$p = h/\lambda$ i.e. $\lambda = h/p$

also since

$p = m \times v$ then $\lambda = h/mv$

This is the expression for the "de Broglie wavelength".

With tongue in cheek we might be tempted to suggest that if any moving object has its own wavelength, what about, say, a cricket ball? How can a wave *guide* such an object? Consider one of mass 0.156 kg travelling at 50 km/h.

$50 \text{ km/h} = (50 \times 1000)/3600 = 13.89 \text{ m/s}$.

Then λ for cricket ball is equal to

$$\frac{h}{mv} = \frac{6.626 \times 10^{-34} \text{ J s}}{0.156 \text{ kg} \times 13.89 \text{ m/s}} \simeq 3 \times 10^{-34} \text{ m}$$

and obviously a ball with a diameter over 10^{32} times that of the wavelength is not going to change its path in any detectable way. A peculiar notion but worth looking at. It will certainly make more sense at atomic level where masses are significantly smaller.

8.5.6 Orbital Waves

Section 8.5.2 develops some interesting data for the hydrogen electron:

(i)   the energy required to free an electron from its orbit = 13.6 eV
(ii)   radius of orbit, $r = 5.295 \times 10^{-11}$ m
(iii)   electron velocity, $v = 2.187 \times 10^6$ m/s.

We can use these figures in an introduction to quantum theory.

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The length of the hydrogen orbit $= 2\pi r = 2\pi \times 5.295 \times 10^{-11} = 3.3 \times 10^{-10}$ m. The "de Broglie wavelength",

$$\lambda = \frac{h}{m\,v} = \frac{6.626 \times 10^{-34} \text{ J s}}{9.109 \times 10^{-31} \text{ kg} \times 2.187 \times 10^6 \text{ m/s}}$$

$$= 3.3 \times 10^{-10} \text{ m},$$

exactly the same and calculated from two entirely different approaches. A link within the apparent inconsistency of wave versus particle motion is beginning to show. The orbit of the hydrogen electron has a length exactly that of the de Broglie wave hence the end of the wave must be joined to its beginning. Referring to Section 4.9.1 we note that the amplitude of a vibrating string must be zero at a node. In the case of an electron orbit both the beginning and end of the wave are at the same node. The electron orbit can be likened to a circular wire loop which is set in vibration in the same way as a musical string. Whatever the mode of vibration of the loop, there must be a whole number of wavelengths round it so that, going round the loop, the end of the last wave joins the beginning of the first. If this does not happen then subsequent vibrations do not reinforce the earlier ones and the vibration is damped. For the electron orbit this is illustrated in Figure 8.6 in the best way we can. The Figure shows as an example normal circular orbits modified by a single cycle of a waveform and again by three cycles. The wave amplitudes are obviously exaggerated for the purpose of illustration. Call the number of complete waves, n. Then for a wavelength, λ:

circumference of orbit $= n\,\lambda$

but in terms of the radius of the orbit, circumference $= 2\pi r$, hence

$n.\lambda = 2\pi r$

and as seen above, for the wave to join back to itself, n must be a whole number. It is called the orbit *quantum number*

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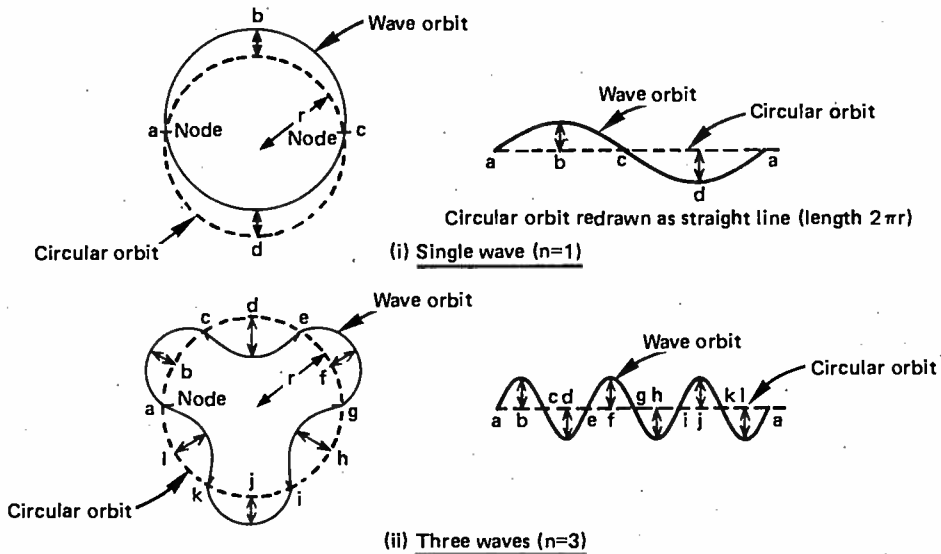
Fig. 8.6 Orbital waves.

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and labelling the orbit radius according to the quantum number by r_n, we get:

$$n\lambda = 2\pi r_n \quad \text{where} \quad n = 1, 2, 3, \text{etc.}$$

We will also find that the quantum theory predicts that a shell cannot have more than $2n^2$ electrons in it.

8.5.7 Energy Levels

So far, based on the hydrogen atom, it is possible to see how waves might fit into the orbit of an electron. It is also evident from Section 8.5.4 that electrons accept energy only in discrete packets or quanta. We have yet to discover what happens to an electron when such energy is given to it. Accordingly a little juggling with the formulae so far produced will be rewarding.

From Section 8.5.2 it is apparent that the electron velocity, v is related to the orbit radius, r by

$$v = \sqrt{\frac{k\,e^2}{m\,r}},$$

so for any particular radius, r_n and now taking into consideration the principle of de Broglie waves ($\lambda = h/m\,v$), then:

$$\lambda = \frac{h}{m\,\sqrt{[(k.e^2)/(m.r_n)]}} = \frac{h}{\sqrt{[(m.k\,e^2)/r_n]}}$$

$$= \frac{h\,\sqrt{r_n}}{e\,\sqrt{m\,k}}$$

and from the orbit quantum number formula (Sect.8.5.6),

$$n\lambda = 2\pi r_n$$

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we get:

$$\frac{n \, h \, \sqrt{r_n}}{e \, \sqrt{m \, k}} = 2\pi r_n$$

and squaring both sides to get rid of the square root signs:

$$\frac{n^2 \, h^2 \, r_n}{e^2 \, m \, k} = 4\pi^2 \, r_n{}^2$$

hence

$$r_n = \frac{n^2 \, h^2}{4\pi^2 \, e^2 \, m \, k} \ ,$$

never forgetting that n can only be 1, 2, 3, etc. (Sect.8.5.6). Also, when $n = 1$:

$$\frac{h^2}{e^2 \, m \, k} = 4\pi^2 \, r_1 \quad \therefore \quad r_1 = \frac{h^2}{4\pi^2 \, e^2 \, m \, k}$$

hence $r_n = n^2 \, r_1$ (where $n = 1, 2, 3$, etc.), and we realize that there can be several orbits:

$n = 1 \ldots \ldots$ orbit radius r_1
$n = 2 \ldots \ldots$ orbit radius $r_2 = 4r_1$
$n = 3 \ldots \ldots$ orbit radius $r_3 = 9r_1$
$n = 4 \ldots \ldots$ orbit radius $r_4 = 16r_1$ and so on.

This clearly shows that electrons can move into certain alternative orbits of greater radius, implying that the electron moves outward from the nucleus. The next question is obviously, how is this accomplished?

Going back to Section 8.5.2 again:

$$E = -(k \, e^2)/2r$$

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so writing E_n for the energy in the appropriate orbits

$$E_n = -(k\,e^2)/2r_n$$

but

$$r_n = \frac{n^2\,h^2}{4\pi^2\,e^2\,m\,k}$$

$$\therefore \quad E_n = -\frac{k\,e^2}{2} \times \frac{4\pi^2\,e^2\,m\,k}{n^2\,h^2} = -\frac{2\pi^2\,e^4\,k^2\,m}{n^2\,h^2}$$

(where $n = 1, 2, 3$, etc.) and when $n = 1$,

$$E_1 = \frac{-2\pi^2\,e^4\,k^2\,m}{h^2}$$

hence $E_n = E_1/n^2$ (where $n = 1, 2, 3$, etc.), so now we can not only calculate the radii of the alternative orbits but also the energies the electrons must have for stability within them. These are known as the *energy levels* of a particular atom.

Is there something unexpected here though, for if E_n is inversely proportional to n^2 it seems that energy must be subtracted from an electron to make it move to a larger orbit. However Section 8.5.2 shows that the way we put signs to energy is that for attraction the energy works out to be negative, hence the greater the negative number, the greater the attraction. The opposite applies for repulsion. Between the nucleus and the electrons of an atom there is normally attraction and in the case of hydrogen the energy level in the innermost orbit is -13.6 eV. Any energy added reduces this number so indicating less attraction to the nucleus.

When the valence electrons are in their innermost orbits ($n = 1$), the atom is said to be in the *ground state* and when $n = 2, 3, 4$ etc. these are the *excited states*. We put this all together in Figure 8.7 for which the calculations are shown in Table 8.1. Just as a reminder, these are obtained as follows, ($k = 8.99 \times 10^9$, $m = 9.109 \times 10^{-31}$ kg, $e = 1.602 \times 10^{-19}$ C):

TABLE 8.1 QUANTUM NUMBERS AND ENERGY LEVELS FOR HYDROGEN ATOM

quantum number n (1)	atom state (2)	orbit radius r_n (m) (3)	energy level E_n (eV) (4)	electron velocity v_n (m/s) (5)	electron k.e. (eV) (6)	electron p.e. (eV) (7)
1	ground	5.30×10^{-11}	-13.6	2.19×10^6	13.6	-27.2
2	excited	2.12×10^{-10}	-3.40	1.09×10^6	3.40	-6.8
3	excited	4.77×10^{-10}	-1.51	7.29×10^5	1.51	-3.02
4	excited	8.47×10^{-10}	-0.85	5.47×10^5	0.85	-1.70
5	excited	1.32×10^{-9}	-0.54	4.37×10^5	0.54	-1.08
∞	ionized	no orbit — electron is freed	0 or +	—	—	—
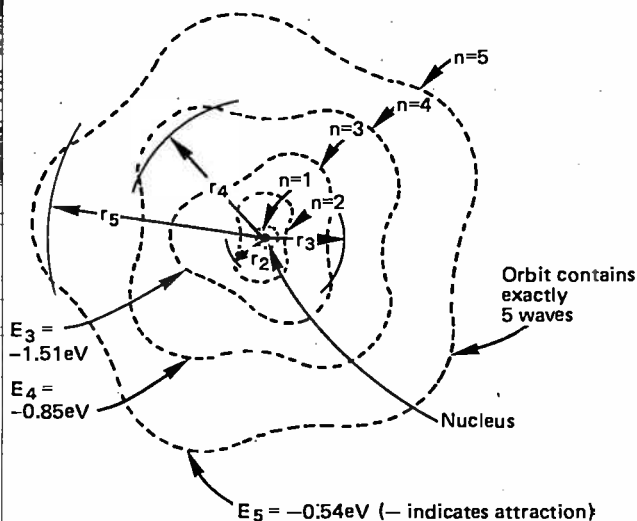
Fig. 8.7 Electron orbits in the hydrogen atom.

orbit radius, $r_n = n^2 . r_1$

energy level, $E_n = E_1/n^2$ (eV)   (= k.e. + p.e.)

electron velocity, $v_n = e \sqrt{[k/(m\,r_n)]} = 15.915/\sqrt{r_n}$ (m/s)

k.e. of electron $= \frac{1}{2}m\,v_n^2 = 2.843 \times 10^{-12} \times v_n^2$ (eV)

p.e. of electron $= -(k\,e^2)/r_n = -(1.44 \times 10^{-9})/r_n$ (eV)

The Table on page 151 only quotes quantum numbers up to 5 because any electron in such an orbit, far away from its nucleus, requires only 0.54 eV of energy to set it free. Hence any gain in energy is likely to be sufficient for escape. $n = \infty$ simply expresses the fact that there is no numbered orbit, the

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electron is released from the pull of the nucleus and is free. The energy we recall, is at all times the sum of kinetic and potential. Columns (6) and (7) in the Table show how these two energies change with orbit but always add up to E_n.

It is more than likely that some readers will have had difficulty with the earlier Section (8.5.2) dealing with the two types of energy. The idea of a minus sign on energy may well be confusing for how can anything be full of negative energy? It might even seem like a mathematical fiddle but we do need to account in some way for the effects of both attraction and repulsion. Let us clarify what we have done by examining the figures in Table 8.1 more closely. The Table is developed from quantum theory and is intended to show that valence electrons can only accept or dispose of the exact amounts of energy required for a move to a different orbit, unless ionization takes place. We start by considering the various orbital positions of the electron in terms of the mean radii of the orbits and then examine the changes in p.e.

It has already been decided that "ground level" for the p.e. is at an infinite distance from the nucleus, i.e. p.e. = 0 at infinity. We can therefore look at the figures in a different way by suggesting that in the ground state, whatever energy the electron has, it is short of 27.2 eV of p.e. to be able to leave home. This is because it needs to do work to shift its position sufficiently far out so as to overcome the attractive force of the nucleus and as the Table shows, when the electron does get farther away its need of p.e. for freedom gets less.

The story however is not simply one of potential energy, the electron orbits at high velocity and has kinetic energy, the figures are given in column (6). K.e. is energy of motion and having nothing to do with forces of attraction or repulsion, is always positive. From the formula for maintaining an electron in orbit (Sect.5.2), the k.e. is inversely proportional to r so the farther away from the nucleus, the less the k.e. required. However this is more than offset by the increase in p.e. (shown by the figures as a decrease in −ve p.e.) and in fact the increase in p.e. is double the decrease in k.e. This follows from the formula in Section 8.5.2 which shows that the total energy E (E_n in this particular case) = $-k\,e^2/2r$.

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Also the potential energy,

$$E_P = -k\,e^2/r$$

hence:

$$E = E_P/2 \quad \text{and since} \quad E = E_P + E_K$$

(where E_K = kinetic energy), then

$$E_P/2 = E_P + E_K \quad \text{from which} \quad E_P = -2E_K$$

as Table 8.1 shows in practical figures.

Overall therefore for larger orbits the electron has greater energy. This is summed up in column (4) where the total energy required for electron freedom from the ground state is 13.6 eV, falling for example to 0.85 eV at orbit 4, i.e. when the electron has gained 12.75 eV. Thus column 4 could have been labelled "energy required for electron escape" and in this case the minus signs are redundant.

The system used for rating energy (i.e. p.e. = 0 at infinity) has the advantage that all valence electrons are considered as having zero energy at the ionization level. Unfortunately this makes the energy at the ground state ($n = 1$) and intervening levels ($n = 2, 3, 4$, etc.) negative and we are already conscious of the fact that this for the newcomer (and many of the older hands too) can be confusing. Just to help those who are still in trouble, Figure 8.8 shows the results of Table 8.1 pictorially and in the rectangle adds figures for the total energy which is now expressed relative to zero for the atom in the ground state. This may give a more acceptable picture of the energy levels. Readers who wish can hereafter forget all about p.e., k.e. and the + and − signs and simply remember that, starting from the ground state, the provision of energy from electric fields, heat etc. enables the electron to jump to orbits at greater distances from the nucleus. The energy required for ionization varies from atom to atom, for hydrogen as we have seen, it is 13.6 eV but for example for copper it is 7.72 eV, for gold, 9.22 eV.

When energy is added it is kinetic. This is interchangeable with potential energy which the electron requires for the work it has to do against the attraction of the nucleus as it moves

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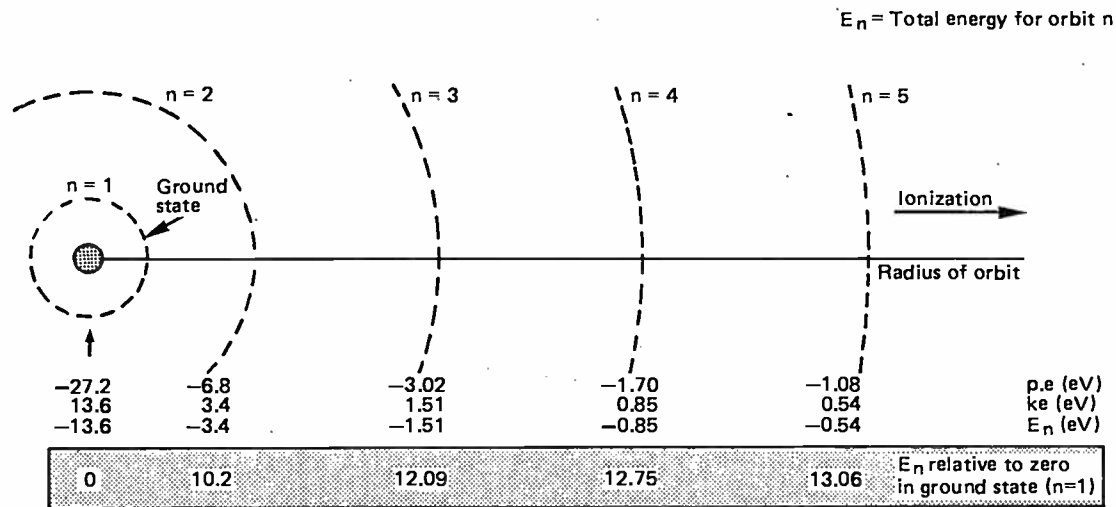
Fig. 8.8 Energy levels for hydrogen atom.

farther away. The final balance between p.e. and k.e. satisfies the requirements of the particular orbit into which the electron is moving.

The frequency of vibration of the electron about its mean circular orbit is easily calculated. Again taking quantum number 4 as an example, the length of orbit is equal to:

$$2\pi r_n = 2\pi \times 8.47 \times 10^{-10} \text{ m} = 5.32 \times 10^{-9} \text{ m.}$$

There are 4 complete waves in one orbit, therefore:

$$\lambda = (5.32 \times 10^{-9})/4 = 1.33 \times 10^{-9} \text{ m.}$$

The electron velocity, v_n is given in the Table as 5.47×10^5 m/s, hence

$$f = v/\lambda = (5.47 \times 10^5)/(1.33 \times 10^{-9})$$

$$= 4.11 \times 10^{14} \text{ Hz.}$$

The word "orbit" and the drawings we use tend to create an impression of neat, circular motion. Right from the start we have known that in fact the orbits are elliptical and now that we have discovered that somehow there is also a wave motion involved, it is perhaps better if we move on to the term "orbital". This, when used as a noun means "that part of the region surrounding a nucleus in which a particular electron is likely to be found".

8.5.8 The Excited Atom

The excursion into quantum theory, brief though it may be, enables us to see how and why valence electrons break free to become charge carriers. Hydrogen of course cannot be used as an example of a conductor but its atomic simplicity is ideal for use in explanations. Normally hydrogen's single electron is at its lowest energy level with the atom in the ground state.

Boltzmann again comes to our rescue to put figures to this statement. His Distribution Law illustrates the statistical

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distribution of particles at different energy levels. In general terms it shows the energy conditions within a material which is undisturbed by external influences except heat as:

$$n = n_0 \exp - [(E - E_0)/k\,T]$$

where E and E_0 are the energies in joules and n and n_0 are the numbers of particles at the two levels. [exp (x) is a convenient way of printing e (the base of natural logarithms) to the power of x i.e. e^x.]

$k\,T$ we know from Section 6.3.4 and its presence underlines the fact that heat provides energy so resulting in more atoms being in excited states. But at say, room temperature, 293 K, modifying the formula to use quantum number subscripts for E and n:

from Table 8.1,

$$E_2 - E_1 = 10.2\ \text{eV} = 10.2 \times 1.602 \times 10^{-19}$$

$$= 1.63 \times 10^{-18}\ \text{J}$$

$$k\,T = 1.38 \times 10^{-23}\ \text{J/K} \times 293\ \text{K} = 4.04 \times 10^{-21}\ \text{J}$$

therefore:

$$n_2/n_1 = \exp - [(1.63 \times 10^{-18})/(4.04 \times 10^{-21})]$$

$$= \exp(-403) \simeq 10^{-175}$$

showing that at room temperature the number of atoms we can expect to find on average at level 2 is $1/10^{175}$ of the number in the ground state (level 1). This is such an incredibly small number that it might as well be called zero so proving the point that hydrogen normally has all its atoms in the ground state.

10.2 eV is rather a large energy change and it occurs with only a few elements. Hence to see what else the formula has to tell, when for example, at room temperature, $E_2 - E_1 = 0.5$ eV, it would be found that one in every 400 million atoms

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would be excited to the higher energy level. Raising the temperature to a mere 200° C increases the number to about one in 200,000. This does not seem many but never forget that there are always a lot of atoms around.

The formula proves another interesting fact. If the energy difference between the two levels is extremely small or the temperature is very high, the exponent of e tends to zero. Anything raised to the power of 0 is equal to 1 at which n_2 would be equal to n_1. However because E_2 must be greater than E_1, e^0 cannot occur so on average n_2 will always be less than n_1.

Energy may be gained (or lost) by the atom from various sources, for example, vigorous encounters with other atoms, heat which increases kinetic energy, from light and electric fields. Also when two atoms move close together their valence electrons may be affected by the changes in electric field and energy may be transferred from one to the other.

The time intervals in which these things happen are extremely small. Experimental work shows that atoms generally lose their newly acquired energy within about one-hundredth of a microsecond (10^{-8} s).

Because an electron can only absorb the exact amount of energy required to move it out to a larger orbit, then should the hydrogen electron receive 10.2 eV of energy, it would be found in the next orbit ($n = 2$ on Fig.8.7). Unless energy is continuously available, the atom may then revert to the ground state with the consequent emission of energy, for example in the form of a photon of light (Sect.8.5.3). If on the other hand more energy is being supplied the electron may shift to even higher orbits, each with less attraction to the nucleus because of the increase in orbit radius. Give the electron of hydrogen 13.6 eV or more, the total energy becomes algebraically zero or a positive value, indicating that there is no longer attraction to the nucleus and the electron is therefore completely free. What we find with the hydrogen atom applies to other atoms but unfortunately analysis then gets too complicated for us to handle.

In materials all this may take place within an overriding electric field produced by an externally applied p.d. A free electron then experiences drift and it can at any time collide

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with another electron or merge with an ion with an appropriate change in energy.

The availability or radiation of energy in discrete quanta is something beyond human experience or understanding. All that can be said is that the general theory of waves and particles here satisfactorily explains many phenomena. But we are only scratching the surface of a most complex subject.

8.5.9 Thermionic Emission

We touched upon the emission of electrons from the surface of a metal in Section 8.5.4 in discussing Einstein's work on photoelectricity. This is one particular way in which electrons escape but what we have yet to decide is why they do not jump out of the metal under normal conditions. What is there to stop them? After all, a football flying around in all directions and continually being fed with lumps of energy by the players, must go over the line sometime. This clearly does not happen to any great extent with electrons otherwise metals would develop a positive charge and no longer be conductors. The answer is comparatively simple, the relatively few electrons which do escape are held back by the +ve charge they leave behind and themselves congregate at the surface and repel the emission of more. So, effectively a few do escape but the *space charge* they set up limits the release of others. It is a dynamic process, electrons with sufficient kinetic energy continually join the escapers while others go back home. Overall at room temperature an equilibrium occurs and because the space charge is so close to the material it can really be considered as part of it. Figure 8.9 does its best to illustrate this, there is no neat line indicating the surface of the material because at atomic level it is far from smooth but rather more like a rocky coastline.

The question next arises as to how emission from the surface of a conductor can be increased. Provided that an electrical circuit exists to replenish electrons lost, apart from the use of radiant light energy they can be extracted from the surface by either:

(i) providing them with sufficient energy by heat or
(ii) using a strong +ve electric field

or both together. What concerns us first then is the degree to which heat can provide the necessary kinetic energy.
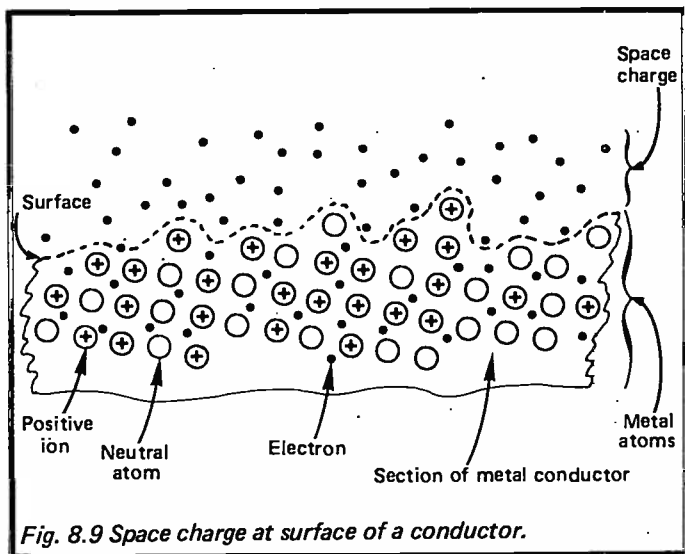
Fig. 8.9 Space charge at surface of a conductor.

Apart from the energy needed to escape from an atom, a definite additional amount must be imparted to the electron to enable it to overcome the retarding force of the space charge electric field. Much research has been carried out on *thermionic emission*, especially before the advent of semi-conductors for then all radios, televisions and amplifiers depended on this for operation of diodes, valves and cathode-ray tubes. The latter still use thermionic emission for a copious supply of electrons to illuminate the tube face. A useful formula, known as Richardson's Law (after Sir Owen Williams Richardson, an English physicist — the Law is also known as the Richardson-Dushman Equation) shows that the emission current density in amperes per square centimetre (for A/m^2 multiply by 10^4),

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$$J = A T^2 \exp{-(\phi/kT)}$$

where A is a constant for the particular material, ϕ is its work function, T is the temperature in °K and k is Boltzmann's constant.

The formula assumes that the emitted electrons have somewhere to go and also are replaced.

We might have guessed from Section 6.3.4 that k T would appear somewhere in the formula because it is the part which relates temperature and kinetic energy. The formula is complicated so we will not dig too deeply, however, it has an important message. Firstly, although discussed earlier, we may need a reminder about ϕ, the electron *work function*. It is the energy which must be supplied to an electron to enable it to cross over the surface barrier (the space charge) of a metal. It is normally expressed in electron-volts.

For the formula therefore k must also be expressed in electron-volts so for the value of 1.38×10^{-23} joules per degree Kelvin is substituted

$$\frac{1.38 \times 10^{-23}}{1.602 \times 10^{-19}} = 8.61 \times 10^{-5} \text{ eV per degree Kelvin.}$$

We can use the formula to compare the emission from a heated wire with that from one at room temperature. Tantalum is a metal which is favoured for its emissive properties and for it A is approximately 40 and ϕ, 4.1. Hence at, say 2400 K:

$$J = 40 \times 2400^2 \exp{-\left[\frac{4.1}{8.61 \times 10^{-5} \times 2400}\right]} \text{ A/cm}^2$$

(Users of scientific calculators may prefer to evaluate the exponent first, i.e.

$$-\frac{4.1}{8.61 \times 10^{-5} \times 2400}$$

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Next use the EXP or e^x key and finally multiply by 2400^2 and 40.)

The exponent of e works out to -19.84

$$\therefore \quad J = 40 \times 2400^2 \times e^{-19.84} = 0.56 \text{ A/cm}^2$$

meaning that over each square centimetre of surface area of tantalum heated to 2400 K, the electrons have been given sufficient energy that 0.56 A can be emitted, i.e. nearly 3×10^{18} electrons escaping per cm^2 each second (1 ampere is equivalent to approximately 6.24×10^{18} electrons/sec — Sect. 7.1).

Next, at room temperature, say 20° C or 293 K:

$$J = 40 \times 293^2 \exp - \left[\frac{4.1}{8.61 \times 10^{-5} \times 293} \right]$$

$$= 9 \times 10^{-65} \text{ A/cm}^2$$

showing that not even one electron has much chance of being emitted. In fact for this metal a temperature of some 790 K is needed before *on average* a single electron will escape per sq. cm per second. The work function for any metal does not exceed 6 and using practical values for A the formula shows that similar conditions apply generally, i.e. emission at room temperature is virtually zero yet at some elevated temperature it can be appreciable.

In practice, as for example, in a cathode-ray tube, special oxide materials are used which have low values of ϕ (<2) and therefore are capable of useful emission at lower temperatures. Such materials may either be coated onto a heating wire or onto a separate cylinder surrounding a heater as shown in Figure 8.10. The emissive material forms a cathode (—ve) and a cylindrical anode is placed round the assembly and held at a high positive potential. The electric field between cathode and anode reduces the negative charge at the emitter surface thereby lowering the escape energy required. It also removes electrons which have sufficient energy to overcome the space charge and accelerates them towards the

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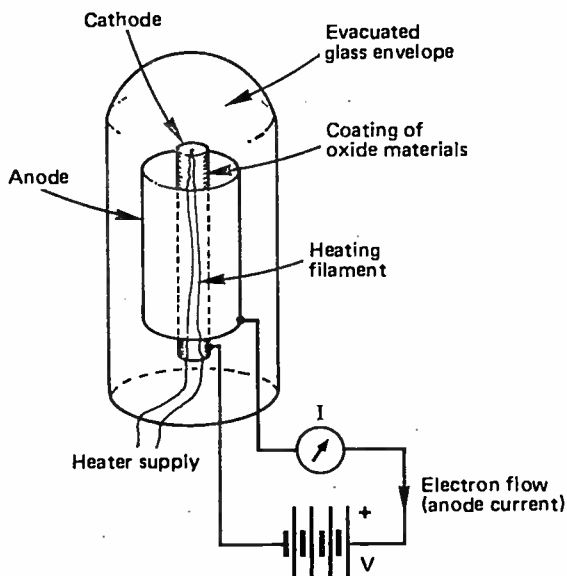
Fig. 8.10 Thermionic emission in diode.

anode to form the *anode current.* Because the envelope is evacuated the electrons have an unimpeded passage across the space between cathode and anode. The *diode* shown not only demonstrates the practical side of thermionic emission but also reminds us that:

(i)   electric current can flow through a vacuum
(ii) in metals only negative electricity is mobile.

8.5.10 Energy and Einstein Again

We can round off Section 8.5 with a look at an everyday event in which Einstein's energy equation has something to say.

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We use the formula for the ultimate speed of electrons in a cathode-ray tube as developed in Section 7.2 (see Fig.7.4). In a device such as this, high electron speeds are required so that the energy given up on impact on the fluorescent coating of the screen is sufficient to cause it to glow. Section 7.2 shows that the work done (W) in moving an electron through a p.d. of $V = q V$, q in this case being the electron charge of 1.602×10^{-19} C. The work done results in kinetic energy hence:

$$q v = \tfrac{1}{2} m v^2$$

where m is the electron mass and v its final velocity.

$$\therefore \quad v = \sqrt{[(2q\,V)/m]} = \sqrt{[2q/m]} \times \sqrt{V}$$

$$\therefore \quad v = \sqrt{\frac{2 \times 1.602 \times 10^{-19} \text{ C}}{9.109 \times 10^{-31} \text{ kg}}} \times \sqrt{V}$$

$$= 5.93 \times 10^5 \sqrt{V} \text{ m/s} .$$

Accordingly a tube using an anode voltage of, say 25,000 (and this is not unusual) gets its electrons up to a velocity of $5.93 \times 10^5 . \sqrt{25000}$ m/s = 9.38×10^7 m/s (nearly 100,000 km/s), i.e. approaching one-third of the speed of light.

The accelerating p.d. (V) operates over the distance between the cathode and the anode (e.g. S_2 in Fig.8.1) so this is the speed at which the electrons leave the anode. It is also the speed at which they strike the screen because the tube is evacuated hence, ignoring gravity, Newton's First Law of Motion (Sect.4.1.5) is effective. The answer is of course approximate because we assumed that each electron leaves the cathode with zero velocity which is hardly true.

It is now evident that classical theory can let us down when we are dealing with high particle velocities. Recalling the remarks in Section 8.5.4 about Einstein's famous equation which shows that the energy of velocity results in an increase in mass, here we have $v \simeq 10^8$ m/s, so:

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increase in mass is equal to:

$$E/c^2 = q \, V/c^2 = \frac{1.602 \times 10^{-19} \times 25000}{(3 \times 10^8)^2}$$

$$= 4.45 \times 10^{-32} \text{ kg},$$

so the electron mass has increased from 9.109×10^{-31} kg to 9.55×10^{-31} kg, an appreciable amount so putting our k.e. calculation in error. Scientists working with particle accelerators would consider such a speed as leisurely, accordingly in all their work Einstein's corrections are essential. We touch on his corrections again in the next Section when trying to discover what magnetism is all about.

8.6 CHARGES IN MOTION AND MAGNETISM

Many of us grow up with the idea that there is electric charge and there is magnetism, two separate but somehow linked entities. There are rules regarding them and when it comes to generators and motors we get our fingers in a twist in deciding which way current, field and motion go. When later we study the electromagnetic wave we discover that electric and magnetic fields are not only linked in some way but are in fact in a close partnership, moreover one does not exist without the other. It is helpful indeed for us to explore this unity to understand more about the fundamental force, the electric charge and how such charges have a different effect on each other when in relative motion compared with that when they are stationary. This implies that we need to go further than Coulomb's Law (Sect.4.7.1) which holds for charges at rest but not for those in motion. Obviously velocity has now to be taken into consideration and to do so is not as easy a task as it at first appears. However let us commence by establishing that what we call *magnetism* is merely a convenient way of handling the special force which occurs between *charges in motion.*

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Fig. 8.11 Forces between wires carrying currents.
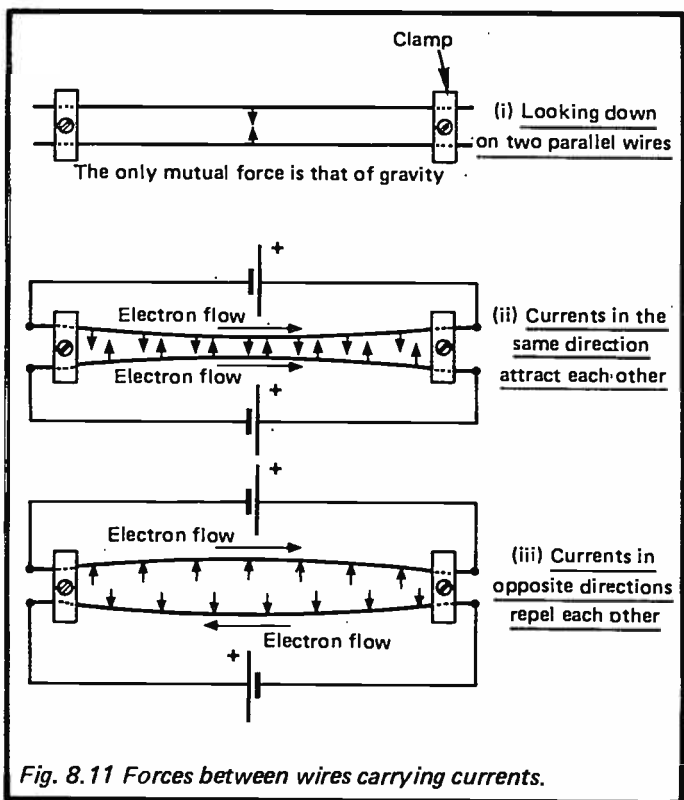
Consider the two parallel wires in Figure 8.11(i). There is an attractive force between the wires due to gravity (Sect. 4.6) but this is so weak as to be relatively ineffectual. In (ii) currents are caused to flow along the two wires in the same direction and we find an attractive force between them. This cannot be the normal electrical force because neither wire exhibits a charge when current is flowing in it. We simply say that it is the magnetic effect of the currents. But the only difference between (i) and (ii) or (iii) in the Figure is that in the latter electrons are moving along the wires. There is only one conclusion therefore which is that what is called magnetic

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effect is in reality a force somehow arising from charges in motion. Like gravity, charge is one of Nature's *fundamental* devices in that it is not the result of something else. Magnetism on the other hand is not fundamental because it arises from charge.

8.6.1 Frames of Reference

Technically a *frame of reference* is a system of geometrical axes for defining position, it can be put more broadly as the place from which observations are made. There are many ways of illustrating a frame of reference but for a simple, everyday one, consider two trains on parallel tracks. Suppose both are heading in the same direction, at the same speed, 40 km/h, neck and neck so to speak. Without realizing it we have just quoted direction and speed from a frame of reference on the ground for this is the one which is generally used. However for a passenger in train A whose frame of reference is his or her seat on the train, the velocity of train B is zero, it must be because a passenger opposite in train B moves neither backward nor forward. This condition is summed up in Figure 8.12(i).

Next let the trains be travelling in opposite directions at the same speed of 40 km/h and passing each other as in (ii) of the Figure. The assumption is that a passenger on either train would see the other passing at the sum of the two speeds, 80 km/h. The condition is summed up in Figure 8.12(ii). This shows how choice of frame of reference can materially affect conclusions as to velocity. Moreover these are not the only frames of reference (ground and passenger seat), an observer could be in an aircraft, in a passing car or even on the Moon, all have motion relative to all others. This is one revelation of Einstein's which has had a resounding impact on physics. He said that it is not possible to determine *absolute* motion (i.e. without relation to other things) because there is no single fixed frame of reference in the Universe, everything is moving relative to everything else.

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* = relative to ground

Train A v = 40km/h *

Train B v = 40km/h *
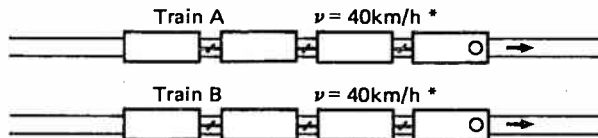
From frame of reference on ground, train A is moving to right at 40km/h
From frame of reference in train B, train A is stationary

(i) Trains running in same direction

Train A v = 40km/h *

Train B v = 40km/h *
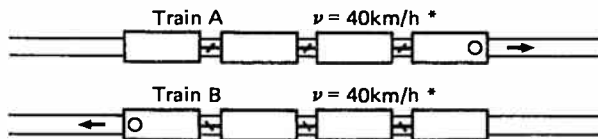
From frame of reference on ground, trains are moving right and left at 40km/h
From frame of reference in train A, train B is moving in opposite direction at 80km/h
From frame of reference in train B, train A is moving in opposite direction at 80km/h

(ii) Trains running in opposite directions

Fig. 8.12 Frames of reference.

8.6.2 The Lorentz Contraction

The train problem looks simple enough, just a matter of common sense. However Einstein complicated even this by showing that two observers in relative motion cannot agree on their measurements of the distance between any particular pair of points. Let us approach this by considering a spacecraft travelling away from the Sun at, say 100,000 km/s and suppose that it has apparatus on board by which the Sun's light rays which are travelling past the spacecraft can be measured. We would assume that with the spacecraft moving at 100,000 km/s and the light rays travelling in the same direction at 300,000 km/s, the measurement would infer that the rays were travelling past the spacecraft at 200,000 km/s. But no, the measurement would in fact show 300,000 km/s, so common sense has let us down.

Strange though it may seem we can only conclude that distance and time but not the velocity of light, are not absolute but are dependent on the relative motion of the observer and what is being measured. In other words there must be a correction to the answer and there is, for both time and distance. It is the latter which we need here and it is given by the *Lorentz Contraction*, as stated by the Dutch physicist, Hendrike Anton Lorentz as:

$$L = L_0 \sqrt{[1 - (v^2/c^2)]}$$

where L is the length measured with the object in motion, L_0 is the length measured with the object at rest, v is the velocity of *relative* motion and c is the velocity of light.

Evidently c is constant, showing how basic it is to this phenomenon for it does not vary whatever the frame of reference.

Let us go back to our earthly example of a train in motion observed from the ground. In this case $v = 40$ km/h, then:

$$L = L_0 \sqrt{[1 - (v^2/c^2)]}$$

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$$= L_0 \sqrt{1 - \frac{40^2}{(300{,}000 \times 3600)^2}}$$

which is almost exactly L_0, showing that at earthly speeds the contraction in length is unmeasurable.

At the speed of launch of a rocket at 40,000 km/h,

$$L = L_0 \times 0.999\ 999\ 999\ 3 \ .$$

Here a contraction in length can be detected but only just. Clearly the Lorentz Contraction is ineffective in the daily round — until we begin to study the electron!

The formula developed in Section 8.5.10 will show that an electron, accelerated through 10,000 volts can reach 5.93×10^7 m/s. At this speed:

$$L = L_0 \sqrt{1 - \frac{(5.93 \times 10^7)^2}{(3 \times 10^8)^2}} = L_0 \times 0.98$$

so the contraction is beginning to show. At higher velocities than this it becomes more significant and if we could ultimately reach the speed of light, $L = 0$. Things cannot disappear so the formula indicates that nothing travels as fast as light.

8.6.3 The Magnetic Force

In Figure 8.13(i) we look inside the two parallel wires of Figure 8.11(ii) except that in conductor 1 one electron only is picked out for examination of the forces acting on it. To this electron:

(i) the −ve charges in conductor 2 are at rest, there is therefore no Lorentz contraction

(ii) the +ve charges in conductor 2 have a relative velocity of v, there is therefore a contraction.
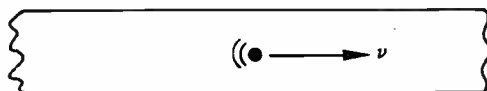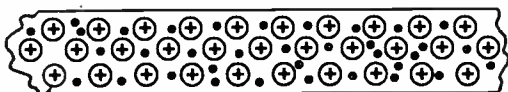
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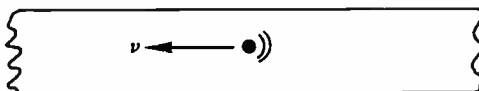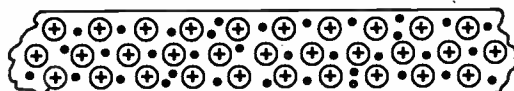
Fig. 8.13 Relative motion of particles.

Accordingly from the point of view of the electrons in conductor 1, the distances between the +ve charges in conductor 2 are less because of the contraction, hence a greater concentration of charge arises. This leads to an *attractive electric force* arising entirely from the relative motions. The net effect is doubled because similar conditions apply to all electrons in conductor 2. Furthermore, considering the flow of charge in conductor 1 to be reversed as in Figure 8.13(ii), the electron in this conductor finds that:

(i) the —ve charges in conductor 2 have a relative velocity of $2v$

(ii) the +ve charges in conductor 2 have a relative velocity of v.

(i) leads to repulsion double that of the electric force above whereas (ii) leads to attraction equal to that of the electric force above, the net result being a *repulsive electric force* of similar magnitude to the attractive force for the currents in the same direction. These forces are over and above the normal interaction of charges and are in fact the *magnetic force* between the two wires.

Recalling Section 8.4 which gives an example of the mean velocity of the electrons in a conductor as 1mm in 20s it is clear that the Lorentz contraction will be something infinitesimally small. However the electric forces are extremely powerful and the total number of electrons involved is very great, e.g. for 1 ampere, 6.24×10^{18} flow per second. From this it can be deduced that the magnetic force is directly proportional to the current for the latter is simply an expression for the number of electrons involved. Unless therefore the current is very high, the magnetic force between two parallel wires is relatively weak but fortunately many artifices for producing strong ones have been developed but always founded on these principles.

8.6.4 The Magnetic Field

We have already developed a formula for the magnitude E of an electric field in terms of the force, F_e exerted on a charge

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of magnitude, q (Sect.4.7.2) as:

$$E = F_e/q$$

and the directions of both E and F_e are the same.

B is the adopted symbol for the strength of a magnetic field (i.e. the *magnetic flux density*) and because we are dealing with charges in motion, v is used for the charge velocity. It is an experimental fact and one which can be proved theoretically (but not by us) that any magnetic force is proportional to the charge velocity, hence:

force, F_m on a charge, $q = q\,v$

$\therefore \quad B = F_m/q\,v$

when the direction of B is at right angles to the velocity (as in the parallel wires above) as shown in Figure 8.14(i) for a *negative* charge. This illustrates the basic principle of the electric motor for the forces on the moving electrons in a wire add up to a much greater force on the wire itself. Hence a wire in a magnetic field can do work when a current flows through it. More generally as shown in (ii):

$$F_m = B\,q\,v\sin\theta$$

and (iii) shows that when the direction of B is parallel to that of v, $F_m = 0$.

For similar forces therefore:

$$E = \frac{F}{q} \text{ (electric field)}, \quad B = \frac{F}{q\,v\sin\theta} \text{ (magnetic field)}.$$

The unit of magnetic flux density is the tesla (T — after Nikola Tesla, a Croation-born engineer) and:

> "one tesla is the magnetic flux density which creates a force of one newton on a charge of one coulomb moving at one metre per second perpendicular to the field"

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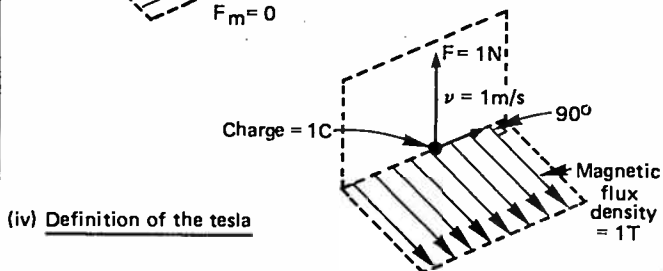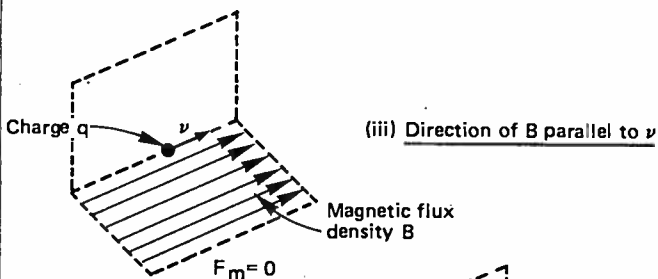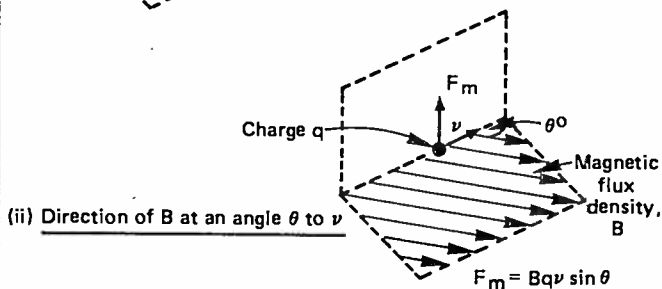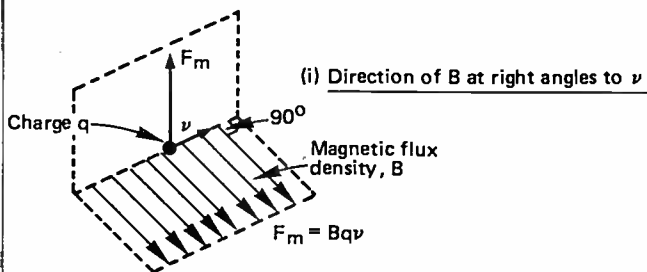
Fig. 8.14 Force on charge moving in a magnetic field.

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as shown in Figure 8.14(iv).

The Earth's magnetic field is some $3-4 \times 10^{-5}$ T whereas a good permanent magnet has a flux density around 1.0 T.

8.7 ELECTRON SPIN AND MAGNETISM

The previous Section examines the relationship between magnetism and electric current. We have yet to account for the fact that magnetic effects arise where no current is flowing, for example in a permanent magnet.

In the early 1920's when several physicists began to study the effects of magnetism at atomic level it was suggested that electrons might possibly behave as though spinning on axes through their centres while at the same time orbiting the nucleus. So much experimental evidence has since arisen endorsing this concept that the existence of electron spin is now generally accepted. There is a similarity in this with the Earth spinning on its axis once per day while orbiting the Sun once per year. A spinning electron acts as a tiny magnet and we turn to the Earth again for an analogy because this spins and also exhibits magnetic poles. Electron spin has two directions as suggested in Figure 8.15 which again conveniently uses the familiar ball to represent an electron. The spin is the same for all electrons and just as positive and negative charges can cancel out so can the two directions of spin result in mutual magnetic cancellation.

Our studies of energy levels has created the impression that atoms naturally move to the ground state if the supply of energy fails them. Recalling the atomic arrangement of shells and sub-shells (e.g. Fig.6.4), the question naturally arises as to why in atoms with more than one shell do the electrons not "fall" into shells nearer the nucleus, ultimately perhaps into the K ($n = 1$) shell. This clearly cannot happen otherwise we might as well tear up what we have done so far and start again. Wolfgang Pauli (an Austrian physicist) sorted this out with his *exclusion principle* which states:

"no orbital in any shell may be occupied by more than two electrons and these must have opposite spins".

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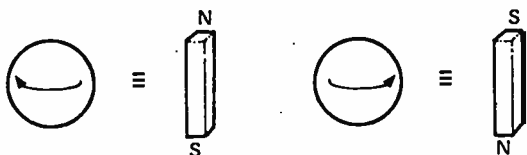
(i) A spinning electron behaves like a tiny magnet
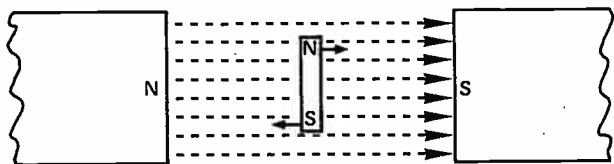
(ii) When a magnet is placed in a magnetic field, forces act on it

Fig. 8.15 Electron spin.

This takes us another step forward in visualizing the atom for according to Pauli, shells and sub-shells may be divided into orbitals, each containing not more than two electrons. Continuing to show orbitals as circular, Figure 8.16 gives a new impression of the sodium atom (compare with Fig.6.3) and it is evident why sodium has been chosen as an example for say, lead with 82 electrons would have filled the page! The Figure shows that to accommodate the Pauli principle, sub-shell p of shell 2 (L) which contains 6 electrons, itself has to be sub-divided into 3 orbitals, each of 2, oppositely spinning electrons.

All electron spins of sodium as shown in Figure 8.16 cancel out except for the outermost single one. An atom on its own therefore exhibits a weak magnetic field but the whole metal does not become a powerful magnet because the fields are in random directions. On the other hand in certain materials the fields may be aligned with that of an external magnetic field as is happening to a tiny magnet in Figure 8.15(ii). This is especially likely to occur when there are many

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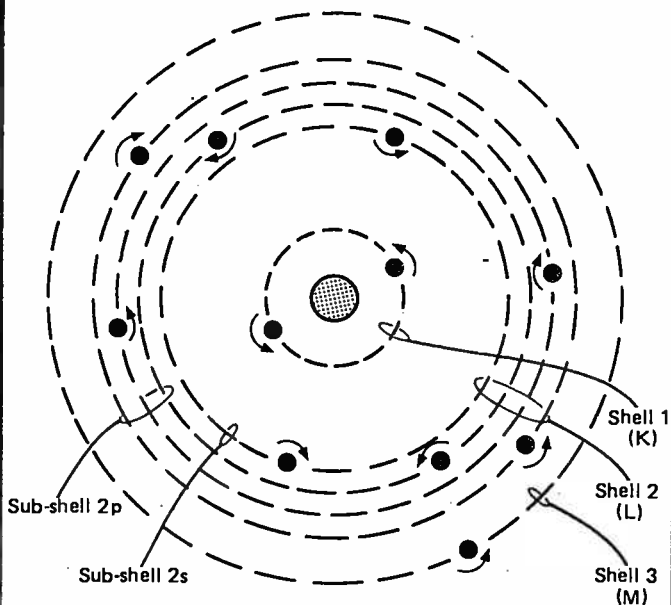
Sodium:

n	1	2	2	3
Shell	K	L	L	M
Sub-shell	s	s	p	s
Electrons	2	2	6	1

Fig. 8.16 Electron spin in sodium atom.

unpaired electron spins. Iron (atomic number 26) is an example, its atomic structure is as follows:

Shell	1(K)	2 (L)		3 (M)			4 (N)		
Sub-shell	s	s	p	s	p	d	s	p	d	f
Number of electrons	2	2	6	2	6	6	2	—	—	—

It so happens that of the electrons in the 3 d sub-shell, 5 have one-way spins with only one in the opposite direction so the net magnetic effect is greater than normal. It has also been found that groups of these atoms form themselves into *domains* (spheres of influence) each having total enhanced magnetism. When all domains are randomly distributed the overall magnetic effect is zero but when an external magnetic field is applied the domain magnetic poles are aligned and a magnet is produced.   This suggests a mechanism through which certain materials are magnetizable and the idea is invaluable in studies of electromagnetism.

We do not probe further into the theory of electron spin for this requires too great a depth of knowledge of quantum theory.   What matters to us most about electron spin is:

(i) it gives rise to a magnetic field as is found in a permanent magnet [see Fig.4.4(i)]

(ii) when an external magnetic field is applied the electron experiences a force on it.

Pausing to consider that from its own frame of reference a bound electron sees the nucleus moving with a high relative velocity and that electron spin is added to this in a complex manner, then no wonder that we tend to put all these separate phenomena together, work out the more practical rules governing their *net* effects and simply label it all "magnetism".

8.8 A NEW VIEW

We are now beginning to form a rather different picture of the atom from what is standard in electronics text books. Another look at Figure 8.7 shows how complicated things have become. For example no longer does the hydrogen atom have the simple arrangement of nucleus and one orbit but now it appears that its electron may be sliding in and out of several alternative orbits, each with its own different wave motion. Speeds are incredible and moreover electrons are spinning like tops. All other atoms are even more complicated. How on earth can one really see all this in the mind's eye, let alone on a diagram?

Our difficulties are shared by the experts. As is seen in this Chapter, electrons only behave as though with a wave motion, they are not waves themselves but then again they are not particles either although we have little choice but to draw them as such. Nevertheless the fallibility in our drawings need not shame us for we could never see for ourselves an undisturbed electron even if we were that clever with magnification. This is because humans need light to see and those little photons would be absorbed and upset everything.

Werner Karl Heisenberg (a German physicist) took a philosophical attitude to this science and developed his *Uncertainty Principle.* This in essence states that it is impossible to measure simultaneously both the momentum and the position of an atomic particle with any pretence to accuracy. But there is no uncertainty with us, definitely we will not dig any deeper!

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Chapter 9

ELECTRONIC CONDUCTION IN SOLIDS

The difference between solids, liquids and gases has mostly to do with atom or molecule mobility. Taking solids and gases as the extremes, molecules of a gas are in the atomic sense few and far between compared with solids in which they (or atoms in the case of elements) are tightly packed and immobile. It is with solids that we next expand our knowledge of conduction, i.e. the flow of electrons when confronted by an electric field. Firstly however some revision, especially with regard to metals:

(i) a conductor contains "free" electrons, i.e. they have accumulated sufficient energy to break away from atoms;

(ii) with no external interference the electrons move randomly in all directions, changing their energies as collisions occur and there is no net development of charge;

(iii) an electric field applied across the ends of the conductor is propagated from end to end at almost the speed of light;

(iv) the field accelerates all free electrons so that they tend to move towards the positive pole. This is an electron drift at a relatively low speed along the conductor and is the current. It is the same at all points in a circuit because charge cannot build up;

(v) the current depends on the strength of the electric field (potential difference), the mobility of the electrons in that particular material and the number of them available. This is summed up in a practical manner by Ohm's Law.

Scanty though it may be, the knowledge we now have of the quantum theory allows us to understand better the basic principles of semiconductors, one of the outstanding developments in electronics.

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9.1 ENERGY BANDS

In most solids the formation is *crystalline*, meaning that the atoms are fixed in a regular 3-dimensional arrangement as shown in its simplest form in Figure 9.1. These basic units are built up into larger crystals, tightly bound together in a lattice form, so giving the material its strength. Where such a crystalline nature is not evident the material is said to be amorphous (shapeless). Resin and soft plastics are of this form. Our interest lies in the crystalline structure for the non-crystalline materials are generally not conductors.
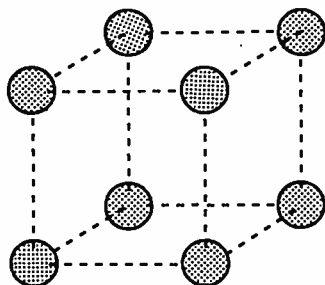
Fig. 9.1 Atoms in a single cubic formation.

Discussion of energy levels brought us to Table 8.1 for the hydrogen atom. As an *energy level diagram* the results are displayed in Figure 9.2(i). This shows that the electron, although normally at ground state, on being given precise quanta of energy, can move out to larger orbits labelled by the quantum numbers 2–5 but never to an orbit in between. Above 5 it has been decided that because only a relatively small amount of energy is required (0.54 eV), the electron is likely to leave home. This is for a single atom on its own, it is necessary next to consider the millions of atoms locked together in a crystalline lattice. When atoms are close together

13.06 —— n = 5
12.75 —— n = 4
12.09 —— n = 3

10.20 —— n = 2

Energy, eV

0 ———— Ground
                n = 1
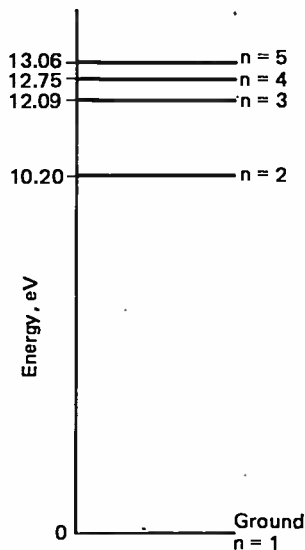
(i) Energy level diagram for hydrogen electron

Energy

{ Energy band for valence electrons
← Ground state

n = 3(M)

n = 2(L)   } Energy levels of inner electrons

n = 1(K)
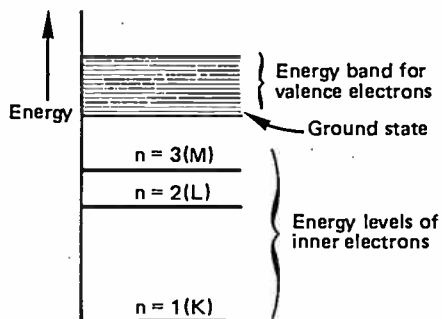
(ii) Energy diagram for copper atoms in crystalline lattice

Energy

← Electrons free

} Conduction band

} Valence band

(Inner electrons omitted)   ← Electrons bound to atoms
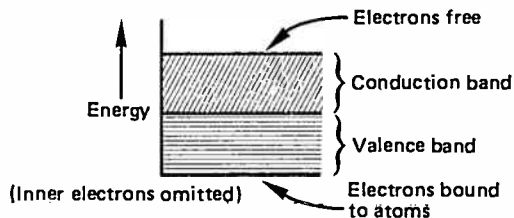
(iii) Valence and conduction bands

Fig. 9.2 Energy level and energy band diagrams.

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their charges interact and modify the electric fields in which the valence electrons move. The possible orbitals are therefore different from the single atom case and in fact each orbital may embrace several atoms. With molecules the electric field configuration is even more complex. Pauli's exclusion principle must still hold good (i.e. that no more than 2 electrons can occupy an orbital — and these must have opposite spins), hence many orbitals and accompanying energy levels arise. The result is that whereas for one atom we have been concerned with energy *levels*, for many atoms together levels broaden out into *bands*. Figure 9.2(i) therefore gives way to (ii) which is for the more complex atoms with inner electron shells. Copper is again used as an example.

Because the inner electrons take no part in the conduction process, they need not be included in conductivity discussions. By so doing it is now possible to compare the few precise energy levels of the single hydrogen electron in (i) with the energy band for the valence electrons of the atoms of copper in (ii). This is known as the valence band and it contains a multitude of separate energy levels. The Figure shows the unexcited condition in which there are few so-called "free" electrons.

Heat however adds energy and another band is needed to accommodate those electrons with sufficient energy to escape from their atoms and be available for conduction. This is known as the *conduction band*. In it the electrons are no longer the valence ones of atoms but are free and are accordingly called *conduction electrons*. An example is shown in (iii) of the Figure and in summary:

(i) electrons with energies in the valence band are held by the nuclei in valence orbitals;

(ii) those with energies in the conduction band have sufficient for freedom.

In copper, as has been noted before, at room temperature there are many electrons in the conduction band. Nevertheless, generally, with no external supply of energy and especially at low temperatures, electron energy is such that most remain in the valence band.

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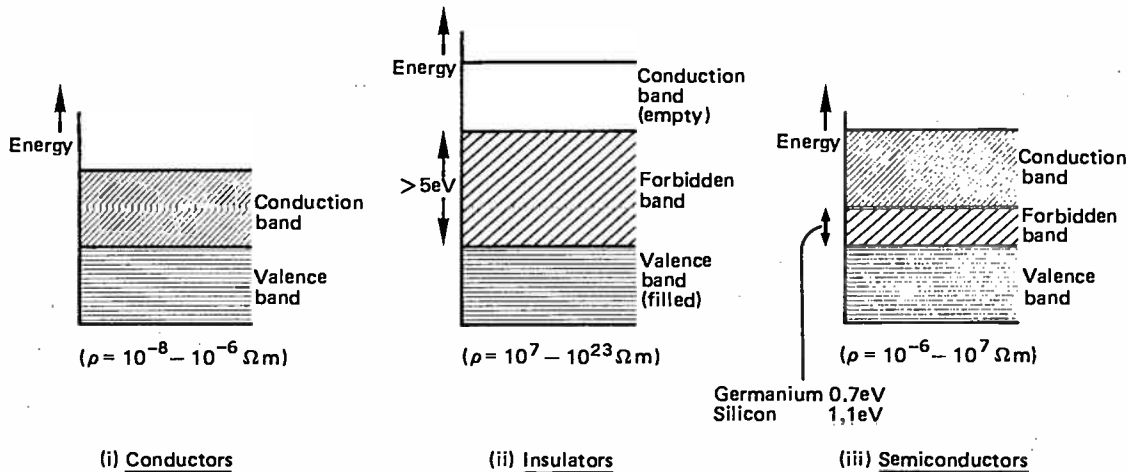
Fig. 9.3 Energy bands.

9.2 CONDUCTORS

For conduction a good supply of electrons is required with energy levels such that in Figure 9.2(iii) they would be within the conduction band. In conductors therefore as Figure 9.3(i) shows, the valence and conduction bands join and they may even overlap. Electrons already in the conduction band are available as current carriers and very little energy is required to lift more valence electrons into the band while there are vacancies at any level (remember Pauli). In practice in most metals there are ample vacant levels because there is only one valence electron per atom, Pauli allows two, hence half the possible levels can be considered as unoccupied and able to take on an electron. Accordingly only a small external influence is required to move valence electrons into vacant levels within the conduction band. This is akin to a double-decker bus where only a small amount of extra energy is required to reach vacant seats on the upper deck. Table 9.1 quotes the atomic arrangements for some good metallic conductors showing that all have a single electron in the outermost orbital. Remember that when more than 2 electrons are shown in a sub-shell, it must be sub-divided into a sufficient number of orbitals to satisfy the Pauli principle. So now we have a new impression of what goes on in an electrical conductor. Electrons gain and lose energy and therefore continually move between the two energy bands and when in the conduction band they are free to act as current carriers. An electric field gives them energy, hence more move up energywise into the conduction band and current flow is proportional to the strength of the field (i.e. to the p.d. applied).

9.3 INSULATORS

The energy band diagram is shown in Figure 9.3(ii). The valence band is full but for electrons to reach the conduction band they must from the energy point of view jump the gap labelled *forbidden band* or *zone*. This band contains the energy levels which quantum theory proves the electron

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TABLE 9.1 ELECTRON POSITIONS IN METALLIC CONDUCTORS

Metal	Atomic Number	Shell 1 (K)	Shell 2 (L) sub-shell		Shell 3 (M) sub-shell			Shell 4 (N) sub-shell				Shell 5 (O) sub-shell			Shell 6 (P) sub-shell		
			s	p	s	p	d	s	p	d	f	s	p	d	s	p	d
Aluminium	13	2	2	6	2	1											
Chromium	24	2	2	6	2	6	5	1									
Copper	29	2	2	6	2	6	10	1									
Silver	47	2	2	6	2	6	10	2	6	10		1					
Gold	79	2	2	6	2	6	10	2	6	10	14	2	6	10	1		

cannot have. Conductors therefore have no or very small forbidden bands, on the other hand insulators are distinguished by their having forbidden bands extending over a wide range of energy levels (the double-decker bus again but somebody has taken the stairs away). The gap between valence and conduction bands is generally more than 5 eV which greatly exceeds the energy which can normally be made available not only thermally but also by an electric field for it is equivalent to accelerating each electron through a p.d. of 5 V. This may seem a small amount until atomic distances are considered for which 5 V leads to very high electric field strengths indeed. Hence in practice, voltages upwards of 10,000 per mm may be required before electrons can be lifted across the forbidden band into the conduction band, whereupon the insulating material has broken down.

Also, given sufficient heat, electrons can be encouraged to jump the gap. An example is given by glass which if heated nearly to melting point falls greatly in its resistivity. This shows that electrons have gained sufficient heat energy to be able to move directly into the conduction band.

The practical outcome of the forbidden band is shown by an insulated copper wire in which the electrical conductivity of the wire itself is as much as 10^{20} times that of the surrounding insulation.

9.4 SEMICONDUCTORS

The energy diagram for semiconductors which are neither good conductors nor good insulators but somewhere in between is now perhaps obvious. A typical arrangement is shown in Figure 9.3(iii). In the Figure the extremes of the ranges of resistivities for each type are also quoted. Resistivity is not as easy a quantity to get on with as is resistance so it is helpful if we recall (from Sects.7.3.2/3 and 8.4.1) that whereas resistivity applies to a particular material generally, resistance refers to a certain quantity of that material of some particular shape. The link between them is that resistivity is the resistance between opposite faces of a one metre cube of the material. It may seem odd perhaps that in a chapter discussing

conductivity, some explanations are in terms of resistivity, this is because the latter finds greater favour in electronics work. Remember too that each is the reciprocal of the other. Semiconductors therefore have resistivities higher than for any conductor but lower than for any insulator. Figure 9.3 also shows that conductivity or resistivity of a material can be estimated from the width of the forbidden energy band.

The semiconductor forbidden band is about 1 eV wide. Although at low temperatures practically no electrons will have sufficient energy to jump the gap and move up into the conduction band, at room temperature the extra energy introduced by heat allows a small proportion of them to do so. Silicon and germanium are the two semiconductors in which we are most interested and the former has a wider forbidden band hence at the same temperature its resistivity is greater at around 10^{-3} Ωm compared with germanium at about 5 x 10^{-7} Ωm.

9.4.1 Doping

The above figures refer to silicon and germanium in their *pure* states, the conductivity is known as *intrinsic* because it belongs naturally to them. In the pure state there is nothing exciting about these metals but when they are *doped* with *impurities*, things begin to happen. These two words usually have somewhat unpleasant connotations, the first with horse and dog, the second, as the dictionary says, with dirt and adulteration. They are both words however which in technology have clear and expressive meanings.

Silicon and germanium are both *tetravalent* elements, meaning that their valency shells contain 4 electrons. The atomic number of silicon is 14 with shells, 1, 2 and 3 (K, L and M) containing 2, 8 and 4 electrons respectively. For germanium with an atomic number of 32, its shells 1–4 (K–N) contain 2, 8, 18 and 4 electrons. The crystalline structure is cubical and each atom is linked or bound with 4 others through the intermingling of the valence electrons as pictured in Figure 9.4. The Figure may take some sorting out but taking the bottom left-hand cubical arrangement, there is an

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Fig. 9.4 Arrangement of atoms in a cubical lattice.
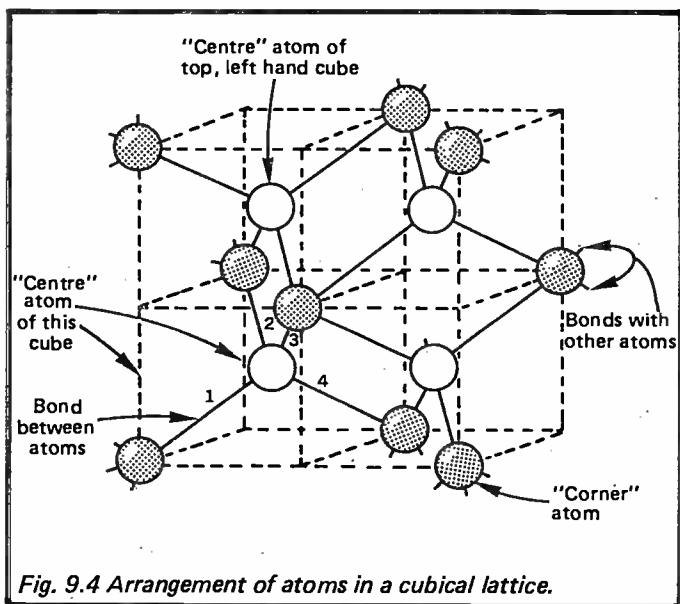
atom at the centre linked with other atoms situated at the cube corners. Each of the 6 faces of the cube has 2 atoms diagonally opposed, 4 atoms only are required. The centre atom has therefore 4 bonds with 4 other atoms (marked 1—4 in the Figure). Only 4 cubical arrangements are shown, but on realizing that each "corner" atom is bonded to 4 "centre" atoms of ongoing cubes (not shown otherwise we get lost in a sea of circles and dotted lines), the way in which every atom is bonded to 4 others becomes evident. For greater simplicity the two-dimensional view of Figure 9.5(i) can be used and this shows the bonds in more detail. Each bond contains two electrons, one contributed by each of the two atoms concerned. It is not possible to portray how a bond is constructed (even if we knew) for the electron is such a very fast mover that it is impossible to catch up with it. Also it has its built-in wave motion and its path is modified by adjacent charges. Hence Figure 9.5 must be accepted as no more than guesswork.

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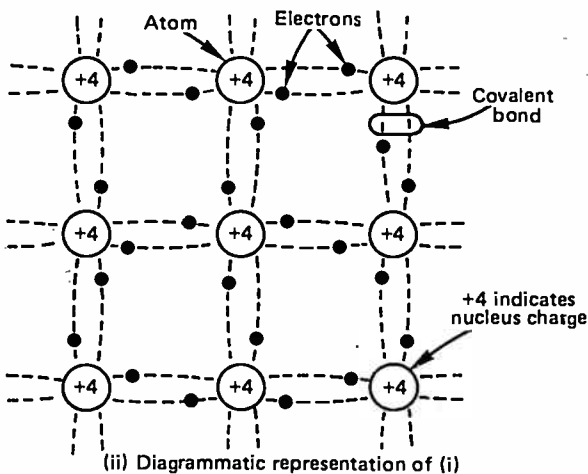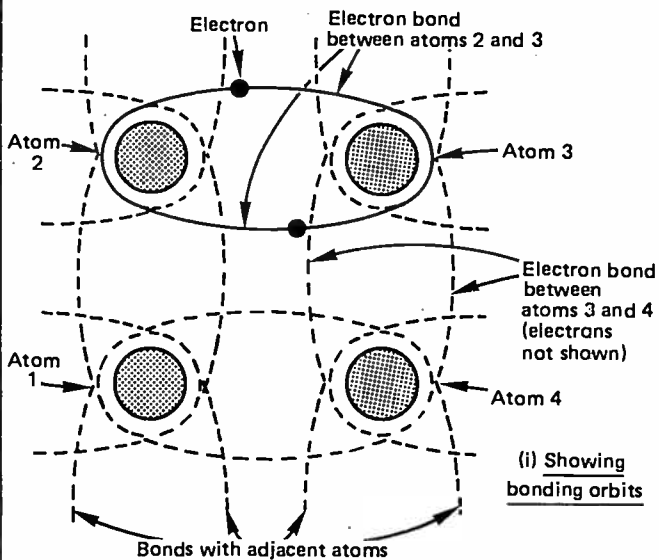
Fig. 9.5 Covalent bonding in tetravalent elements.

(i) Showing bonding orbits

- Electron
- Electron bond between atoms 2 and 3
- Atom 2
- Atom 3
- Electron bond between atoms 3 and 4 (electrons not shown)
- Atom 1
- Atom 4
- Bonds with adjacent atoms

(ii) Diagrammatic representation of (i)

- Atom
- Electrons
- Covalent bond
- +4 indicates nucleus charge

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Having said that, we go on to simplify matters even further as in Figure 9.5(ii).

The bonds through which the atoms are held together by the binding forces of shared electrons are known as *covalent* (co = jointly or mutually). Although each atom has a share in one electron from each of 4 neighbours, it also shares its own 4 valence electrons. Nothing is added nor taken away, hence the whole crystal is electrically neutral. The fact that the Figure shows no free electrons might give the impression that the crystal is an insulator, not a semiconductor. However, considering that only one in about 10^{10} atoms needs to free an electron as a charge carrier in a semiconductor, we can truthfully say that most of the crystal structure can be portrayed as shown.

So much for intrinsic semiconductors, pure and unadulterated. What makes them a blessing in disguise is the fact that their conductivities can be greatly increased and from this all the magic of diodes and transistors is derived. For the original work we have to thank William Bradford Shockley and his colleagues of the Bell Telephone Laboratories (U.S.A.) who gave the World the benefits of their inventiveness way back in 1948.

Semiconductors acquire their increased conductivities through the process of doping (with impurities), in plain language, the addition of tiny quantities of other elements, hence the pure semiconductor is no longer so. The artful move is to use tri- or pentavalent (3 or 5) elements alloyed with the semiconductor. When in the manufacturing process the semiconductor cools from the molten state in which the atoms are reasonably mobile, the impurity locates its own atoms within the crystal lattice structure. Figure 9.6(i) illustrates how conductivity increases when a pentavalent element is used such as:

phosphorus — atomic number 15,
   5 valence electrons in the 3rd (M) shell
arsenic — atomic number 33,
   5 valence electrons in the 4th (N) shell
antimony — atomic number 51,
   5 valence electrons in the 5th (O) shell.

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Fig. 9.6 n-type and p-type semiconductors.

Labels in figure:

Atoms of semiconductor (eg. silicon)

Pair of electrons forming one bond

Atom of donor (eg. Arsenic)

Surplus electron (one per donor atom)

(i) n-type

Atom of acceptor (eg. Indium)

Incomplete bond (one electron only)

(ii) p-type
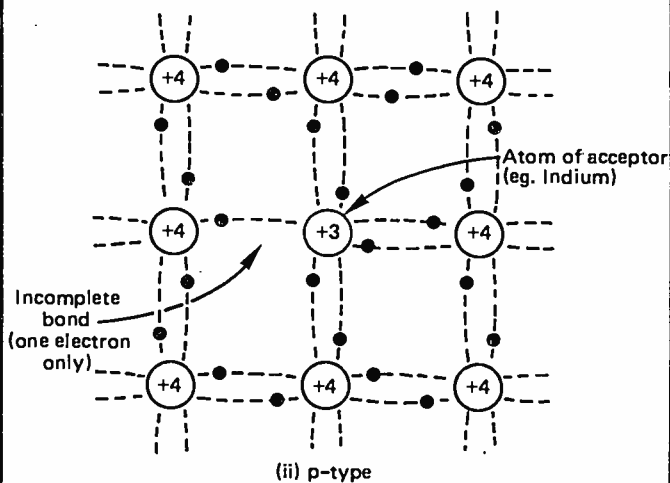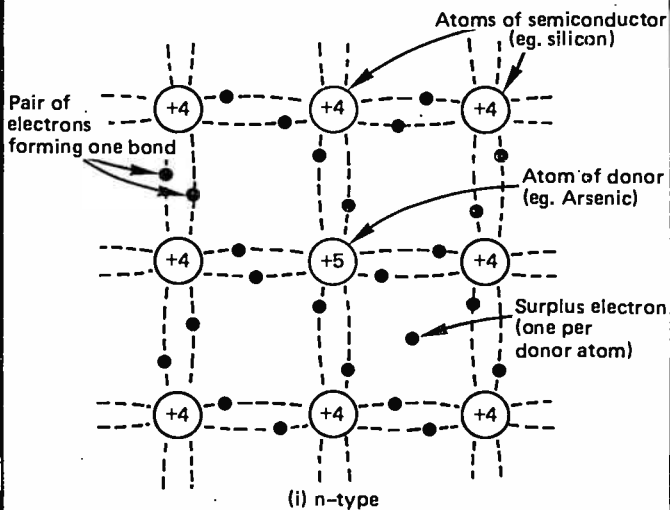
(Figures shown on atoms indicate positive charge of nucleus)

The Figure shows one single atom of, for example, arsenic locked within a lattice of silicon. Four of the five valence electrons of the arsenic form covalent bonds with the silicon atoms but the fifth electron has no home and is thrown spare. All places are taken so it has nowhere to go unless heat energy releases others and creates vacancies. This free electron is therefore available as a charge carrier and accordingly the conductivity of the silicon is increased. The amount of the impurity element added is very small indeed for if it is considered that in the pure semiconductor about one in every 10^{10} atoms contributes a conduction electron at room temperature, then an impurity addition of only one atom per 10^7 silicon atoms increases the conductivity about 1000 times. Because the impurity adds electrons to the semiconductor it is known as a *do(n)or* and the resulting doped semiconductor is classed as *n-type* because it has a preponderance of (n)egative charges.

Alternatively as (ii) in the Figure shows, a trivalent impurity can be added as:

boron — atomic number 5,
    3 valence electrons in the 2nd (L) shell
gallium — atomic number 31,
    3 valence electrons in the 4th (N) shell
indium — atomic number 49,
    3 valence electrons in the 5th (O) shell.

To take its place among the semiconductor atoms the impurity needs one more electron for full linkage. If one is not available, a "vacancy" is created as shown in the Figure.

Alternatively an electron could be absorbed from another atom elsewhere but in either case the result is that a positive ion is created, the number of ions being the same as the number of impurity atoms added. Because the semiconductor now has a surplus of (p)ositive charges it is classed as *p-type* and the impurity element as an *acce(p)tor* because it accepts an electron from the crystal.
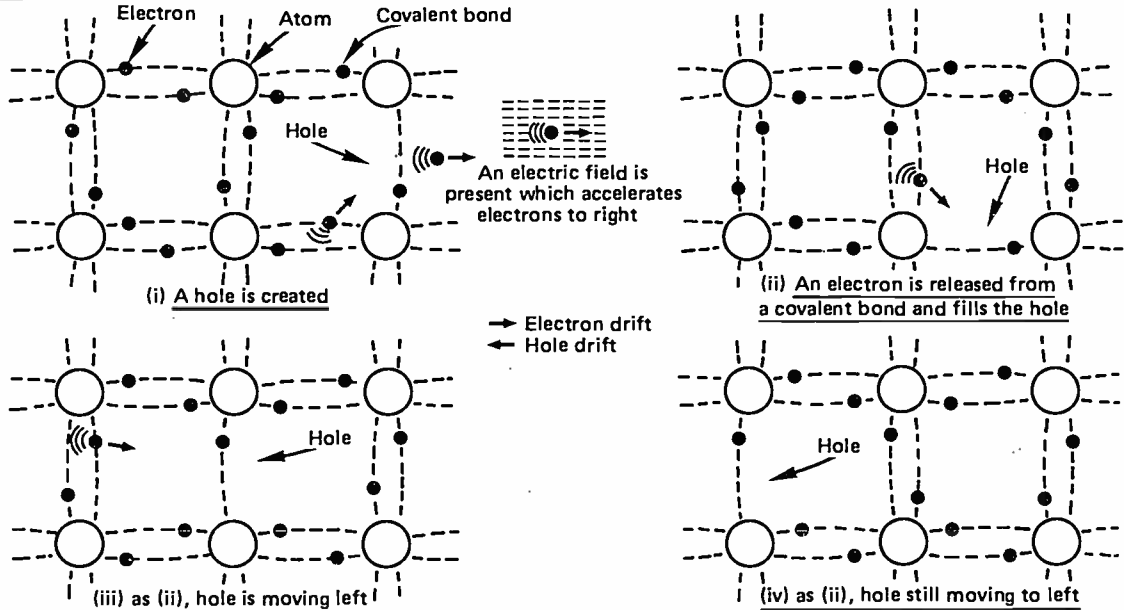
Fig. 9.7 Hole drift.

9.4.2 The Atomic Golf Course

Messrs. Shockley and Co. not only invented the transistor, they invented *holes* along with it. They needed some way of handling theoretically a *deficiency* of electrons as in the case of p-type material. Strictly speaking, a hole is merely nothing with something round it (as on the golf course). The semiconductor hole is not of this form, it relates instead to an atom within a lattice which is without one of its electrons and therefore is positively charged. This atom cannot move so what it is important next to appreciate is the idea (i) of a semiconductor "hole" being able to move (ii) that electrons popping in and out of holes and going one way constitute a positive current going the other way. A bit of a fiddle perhaps but useful. Firstly, how do holes move?

Consider a queue of customers at a counter. Customer 1 is served, moves away and leaves a "hole". Customer 2 moves up and fills the "hole" and in doing so leaves another. All move up one by one and as they do so the "hole" moves backwards to the end of the queue. Figure 9.7 illustrates how the process might be imagined to take place at atomic level.

In the Figure a hole implies that one of the bonded atoms is ionized, hence hole drift is synonymous with positive charge drift. When an electron escapes from an atom in this way an *electron-hole pair* is created. With no external field applied therefore electron-hole pairs are being continually created and also lost through recombination.

9.4.3 The Energies

The usefulness of energy level diagrams can be extended by drawing electrons and holes at the levels where they are most likely to occur. This is done in Figure 9.8 which is effectively the semiconductor diagram of Figure 9.3 modified by the two doping processes. The electrons of the impurity atoms happen to have energies somewhat higher than those in the normal valence band and for n-type their energy level is a mere 0.01 to 0.1 eV below the conduction band as shown in (i). Here we can talk in terms of an energy level rather than a band because

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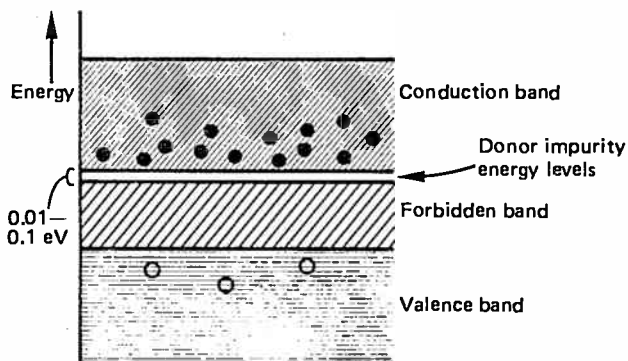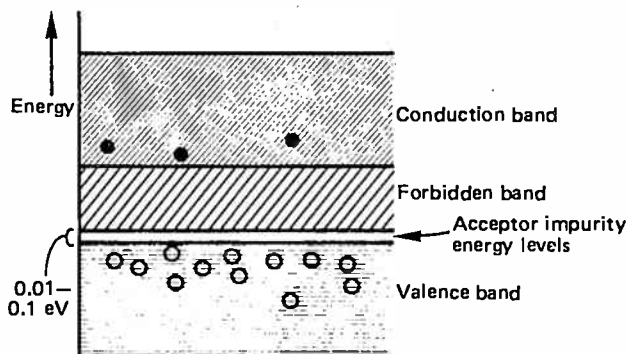
Fig. 9.8 Energy bands for impurity semiconductors.

the impurity atoms are relatively few and far between, hence their valence electrons do not interact and spread the energies over a band (Sect.9.1). With such an insignificant forbidden band for the donor surplus electrons, these easily gain the energy needed to cross into the conduction band, with the

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result that there are many electrons there. A few holes are shown in the valence band because heat also gives some electrons sufficient energy for freedom.

With the p-type impurity there is the opposite effect. The level for the acceptor electrons is only just above the valence band, hence few electrons are able to cross the forbidden band. Holes created by the doping process are left in the valence band, holes, of course, being simply the result of a deficiency of electrons.

The electrons in n-type and holes in p-type as shown in Figure 9.8 are known as *majority carriers*, they are the additional current carriers generated by the doping process. There will also be a smaller number of opposite polarity carriers around (i.e. holes in n-type, electrons in p-type) caused by the continual formation (with recombination) of electron-hole pairs, these are termed *minority carriers*. Their numbers increase with temperature.

Neither n- nor p-type exhibits any charge overall because it is electrically neutral, the atoms have their correct complement of electrons, wherever they are.

9.4.4 The P-N Junction

Things really begin to happen when n-type and p-type join forces. The movement of charges can be traced with the help of Figure 9.9 which at (ii) and (iii) shows diagrammatically n-type and p-type semiconductors in intimate contact. By this is meant that they are not separate parts pressed together but "grown" as a single crystal. Hence the line marked "junction" is not clean cut as shown but a region through which n-type gradually gives way to p-type and vice versa.

Let us avoid for the moment the complication of the minority carriers and only consider the majority ones, i.e. the added electrons in the n region and the extra holes created in the p region. Also only the impurity atoms are shown and these randomly because this is how they are in practice among the more disciplined tetravalent (pure semiconductor) atoms. In the Figure, (i) is simply a reminder of what we have done so far. In the n-type the donor atoms are positive ions, each

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n

p

Hole

Acceptor atom
(has acquired
one
extra electron
to complete
covalent bond)

Donor atom (has
given up one electron)
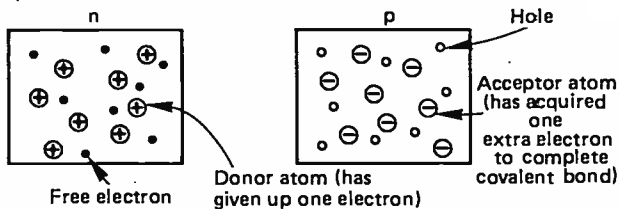
Free electron

(Impurity atoms and majority carriers only)

(i) n and p materials not in contact

Junction

n

p
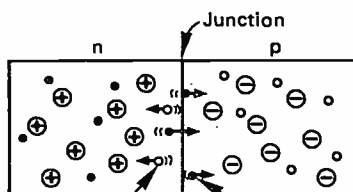
(ii) Materials in contact
(p-n junction)

Some holes drift
into n region

Some electrons drift into p region

This side of
junction goes +ve

This side of junction
goes —ve

Potential across depletion
layer equivalent to battery
as shown

A few tenths of 1 V

n

p

Acceptor atoms
repel electrons to left

Donor atoms repel
holes to right
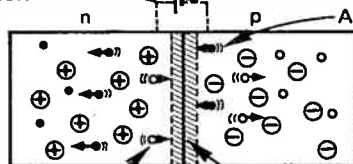
Very few charge carriers
within this region

Depletion layer
($\approx 10^{-6}$ m)

(iii) p-n junction at equilibrium

Fig. 9.9 p-n junction.

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atom having given up one of its 5 electrons. The free electrons move at random within the crystal. In the p-type the acceptor atoms are negative ions because each has gained an electron thus leaving a hole somewhere and the holes are considered to be mobile as Figure 9.7 shows.

When n and p materials are "grown" into each other the random movement of the majority carriers inevitably means that some electrons from the n pass through the junction into the p and equally holes from the p find their way across into the n. This is illustrated in Figure 9.9(ii). There will be some recombination, but apart from this, on the n side of the junction a positive charge builds up because electrons have been lost and holes gained. On the p side of the junction the gain of electrons and loss of holes creates a negative charge. These charges produce a *potential barrier* which tends to oppose further diffusion of majority carriers across the junction. Only those with above average energy levels are able to overcome the effect of the barrier and when they do they in fact increase the barrier potential so discouraging crossings even more. Figure 9.9 can only give a sketchy picture of what actually goes on in an extremely lively situation with electrons escaping from and joining covalent bonds, colliding, gaining and losing energy and skipping across the junction, all at the unbelievably high velocities we have already calculated.

As mentioned earlier, not shown are the minority carriers (the normal electron-hole pairs created from the energy of heat) so electrons from these in the p region will be *assisted* across the junction, similarly with holes from the n region. Hence the whole system settles down to a value for the potential barrier such that its electric field discourages majority carriers but aids the smaller number of minority carriers in crossing the junction to the extent that majority and minority diffusion currents are equal. Being in opposition, these currents cancel. This condition must apply so that there is no overall unbalance of charge between the n and p regions. Both n and p as in (i) of the Figure were originally electrically neutral, hence together they must remain so (otherwise somehow we have built ourselves a battery!).

Figure 9.9(iii) expands on this to show the zone at the junction in which the electric field of the potential barrier acts.

It is known as the *depletion layer*, appropriately because it is depleted of charge carriers. The resistivity of the depletion layer is therefore high, the overall outcome being two low resistivity regions separated by a very narrow one of high resistivity. It might appear that we are almost back to Figure 9.9(i) but this is only part of the story.

9.4.5 Biassing

Biassing is used as a technical term in its general sense as being synonymous with "influencing". It involves connection of an external p.d. across the two regions either to assist or oppose the potential barrier voltage.

Figure 9.10 illustrates the story of *semiconductor diode* (two electrodes) biassing. In (i) the n-p diode is shown with no bias as in Figure 9.9(iii). Added to this is a kind of energy diagram to illustrate the additional energy (E_h) required by the majority electrons in the n-type to cross the depletion layer by overcoming the force exerted against them by the potential barrier. The action is similar in the opposite direction for holes crossing from p to n, but electrons only are considered in the Figure for the sake of clarity. We imagine the electrons need to climb an "energy hill" to be able to drift from n into p. With no bias, as in (i) very few have sufficient energy to do so.

In (ii) an external p.d. is connected with +ve to n and −ve to p. The electric field created across the diode assists the potential barrier, for example, increasing its value from a small fraction of one volt to several volts. The potential or energy hill is accordingly increased and with it the width of the depletion layer. The Figure shows that E_h is increased to the level that practically no electrons can find sufficient energy to climb the hill into the p region.

Not shown, but inevitably there, are the minority carriers, the electrons of these moving from p to n. Their numbers are comparatively small and at a given temperature, constant. The total minority current is known as the *reverse saturation current*. Except for this, with the bias as shown, we are even closer to Figure 9.9(i). The diode is said to be *reverse biassed.*
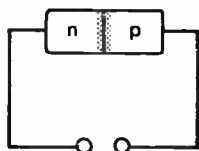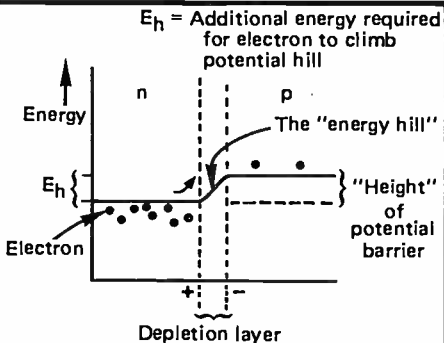
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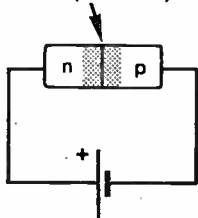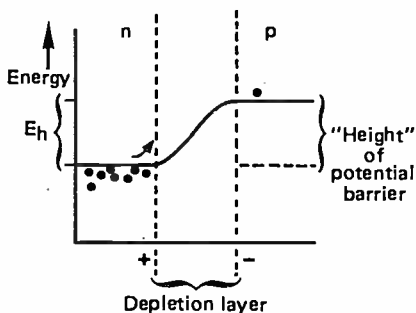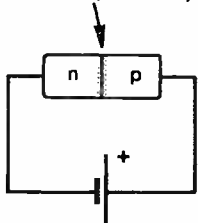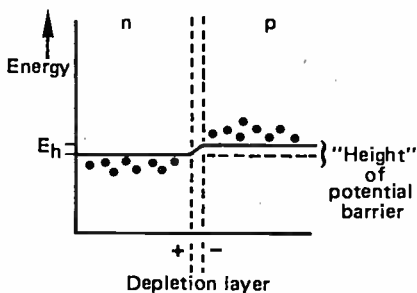
Fig. 9.10 Biassing semiconductor diodes.

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Again there is a similar story to tell with regard to the holes.

Forward biassing changes the electrical conditions considerably. With this, as (iii) of the Figure shows, the externally applied potential opposes the potential barrier. Electrons in the n-type are driven by the field of the external potential towards the junction, holes similarly, with the result that the depletion layer is reduced. The smaller energy hill thus allows majority carriers of lower energy to climb it, i.e. cross the depletion layer. Because electrons and holes are driven together considerable recombination takes place, made up by a supply of electrons into the n-type from the battery and equally by its removal of electrons from the p-type, thereby creating a supply of holes. The diode is now of low resistivity throughout.

If the charge of an electron is designated by $-e$, then the charge of a hole is $+e$. If n_e and n_h are the numbers of electrons and holes respectively passing a particular point per second, then because electron and hole currents are in opposite directions:

$$\text{total current} = n_h \, e - (-n_e \, e) = (n_e + n_h) \, e$$

(again ignoring the minority carriers).

This is the semiconductor diode so well known to all circuit builders and it is especially useful for turning alternating waveforms into unidirectional ones.

9.4.6 Transistors

What we have learned about the semiconductor diode stands us in good stead in understanding the action of another device with a tremendous impact on modern living, the *transistor*. This is used mainly as an electrically driven on/off *switch* or as an *amplifier*, a device which can transform a tiny electrical current into a larger one.

Transistors are basically two semiconductor diodes connected back-to-back n-p with p-n or p-n with n-p, the two similar zones in the centre of the sandwich merging into one, i.e. n-p-n or p-n-p. Both types do the same jobs but n-p-n is more
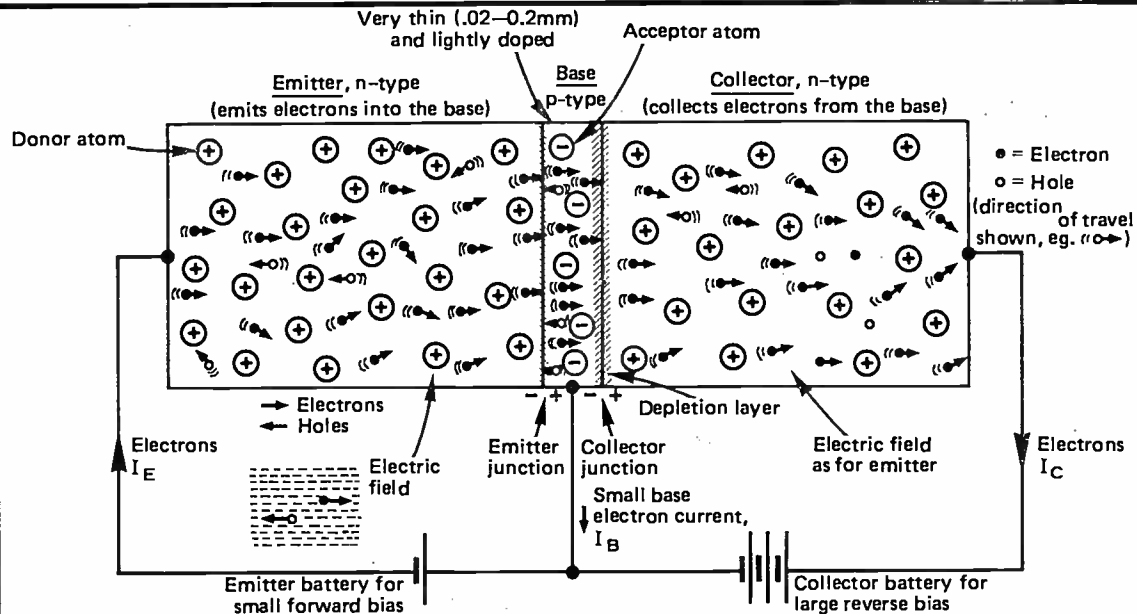
Fig. 9.11 n-p-n transistor.

common, this therefore is the one taken as an example in Figure 9.11. The Figure shows an *emitter* consisting of heavily doped n material, a *collector* also of n material and sandwiched between them a very thin and lightly doped *base* of p material. This creates two junctions, the *emitter junction* and the *collector junction*. Biassing of these junctions in this elementary circuit is shown as being provided by two separate batteries.

Starting with the emitter, the action follows that of Figure 9.10(iii). The negative potential of the battery drives electrons into the left-hand side of the n region and so repels majority electrons to the right. There are plenty of them because of the heavy doping and they have no problem in crossing the emitter junction because the electric field of the emitter battery not only reduces the potential barrier but also gives them extra energy. We cannot revise too often how energy is provided by an electric field, so:

Force (F) acting on an electron in an electric field of strength

$$E = e\,E$$

hence acceleration (a) of electron is equal to

$$F/m = e\,E/m$$

where m = electron mass.

Over a time interval, t, final velocity, $v = u + a\,t$ where u = initial velocity

$$\therefore \quad v = u + \frac{e\,E\,t}{m}$$

from which the gain in kinetic energy can be calculated from:

$$\tfrac{1}{2}m\,v^2 - \tfrac{1}{2}m\,u^2 \quad \text{i.e. } \tfrac{1}{2}m(v^2 - u^2)$$

and this gain in k.e. needs to be sufficient for the electron to climb the "energy hill."

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The excess of electrons fed into the emitter increases the number of recombinations but this is kept small by the material being heavily doped so that there are already many more electrons available than holes.

Once within the base region one would expect from a comparison with the diode that the +ve pole of the emitter battery would gather in the electrons which have crossed the junction. To a small degree this happens so some electrons flow out of the base. However the base is made sufficiently thin that its width is somewhat less than the mean free electron path (i.e. the average distance an electron travels before recombining with a hole). In addition the light doping of the base ensures that there are not many holes around anyway. In simple terms therefore, most of the electrons shoot across the base into the collector region. Thereafter collisions with the majority electrons of the collector cause some of the imported electrons to lose energy, leaving them with insufficient to climb back over the reverse-bias "energy hill" [see Fig.9.10(ii)]. This infers that once through the collector junction the electron has little chance of returning, so helping to keep the base current low. The electrons are now within the electric field of the collector battery, the field is in such a direction that they continue to be accelerated to the right and are eventually removed by the attraction of the +ve pole.

The net result is that as Figure 9.11 stands, there is a constant stream of electrons diffusing from emitter through to the collector with a small current flowing out of the base (some 1% or less). Hence:

$$I_E = I_C + I_B .$$

Current control is achieved by altering the emitter battery potential to vary the emitter junction potential barrier which also automatically affects the base current. Hence a lower emitter battery voltage reduces both base and collector currents and a higher voltage raises them. There is also a relationship therefore between base and both emitter and collector currents in that a small change in base current creates a much greater change in the main currents.

For p-n-p transistors everything above applies provided that we read hole for electron and electron for hole and also change over +ve and —ve.

Finally a note on the symbols used for semiconductor devices is appropriate. Those for the devices discussed are shown in Figure 9.12 and it is obvious that in each case the arrow points in the wrong direction for electron current. Rectifying diodes were around long before the semiconductor variety was invented and in accordance with the mistaken idea that current flowed from + to —, the arrow on the symbol

Fig. 9.12 Symbols for semiconductor devices.
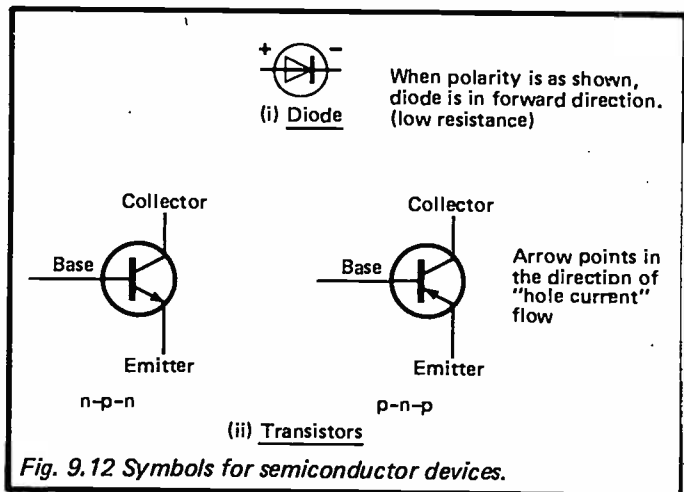
pointed that way for the forward direction. However with the notion that "positive holes" move, no change in the diode symbol was necessary provided that transistor symbols indicated current flow similarly, i.e. with the arrow pointing in the direction of "hole current" flow. We must never forget however that "hole flow" is simply an agreeable way of illustrating the movement of positive charge but in a solid material the only mobile particle i.e. with both mass and velocity and therefore energy, is the electron. The conception of holes does not conflict with the idea that current flows from — to +, it is merely an extension of it.

O what a tangled web we weave!

Chapter 10

ELECTRICAL GENERATORS

Generators are devices which change one form of energy into electrical energy. This takes its form as a potential difference across the generator terminals capable of driving a current round a circuit. Generators have a range of achievement which is very great, from the mighty power station machines to the photocell with a tiny output, capable perhaps of doing no more than moving a needle on a meter.

At last a case arises where current flow is from +ve terminal to −ve terminal for this is what happens *within* a generator. Figure 10.1 shows the reasoning.

Electron flow round the circuit, −ve terminal to +ve terminal
Electron flow within the generator, +ve terminal to −ve terminal

Electrical
circuit

Electron
flow

Electrical
generator

−

Electrons

+

Generator
terminal

Fig. 10.1 Current through a generator.
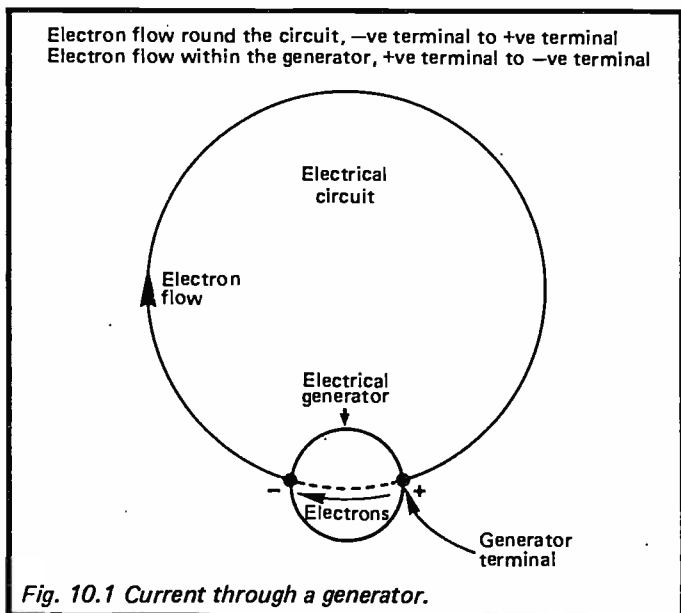
Pressure and friction are two less obvious generators which are offshoots of the general class of mechanical ones. Electron

flow from pressure is only considered briefly in this Chapter but on the other hand electricity from friction is looked at in greater depth because most of us are subject to its vagaries and it can be explained in terms with which we are already familiar.

10.1 CHEMICAL (BATTERIES)

The electric battery began with the Italian physicist, Alessandro Volta who in 1799 produced his *voltaic cell* consisting of copper and zinc plates immersed in dilute sulphuric acid. Subsequently a French chemist, Georges Leclanché developed an improved cell and this is named after him. The Leclanché cell worked better but it also employed a liquid solution which was messy. Nowadays this type of cell turns up everywhere in hand torches, transistor radios and other portable equipment as the "dry battery". This is still based on Leclanché's ideas but the liquid solution with its nasty habit of leaking and corroding everything around it has given way to a damp but unspillable paste. The solution mentioned with regard to both types of cell (voltaic and Leclanché) is called an *electrolyte* as already mentioned in Section 7.4.

The elements of the dry cell are shown in Figure 10.2(i). The outer zinc case contains the paste electrolyte of ammonium chloride (NH_4Cl). This case is the negative pole and is consumed during operation of the cell. The positive pole is a carbon rod, surrounded by a *depolarizer* which is a chemical mixture for removing unwanted products. It is the depolarizer which makes the chemistry of the cell more than a little complicated. Accordingly we use a simplified explanation of the ionic arrangements, but it is one which tells the essential part of the story.

Figure 10.2(ii) shows that the electrolyte molecules ionize by shifting one electron from each NH_4 group of atoms to the chlorine atom so producing NH_4^+ and Cl^- ions. Where the zinc is in contact with the electrolyte the chemical action causes zinc atoms to be ionized, each giving up 2 electrons to the electrode and passing into the solution as Zn^{2+} ions (2+ = minus 2 electrons). Continual production of these +ve ions

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Terminals

Carbon rod

Depolarizer (carbon black or graphite mixed with manganese dioxide powder)

Zinc case

Paste electrolyte

(i) Dry cell

External circuit

Electron flow

Ionization components of electrolyte

Carbon electrode

Zinc electrode

Zinc ion combining with 2 chlorine ions

$(ZnCl_2)$

Depolarizer

Chemical action at surface of zinc
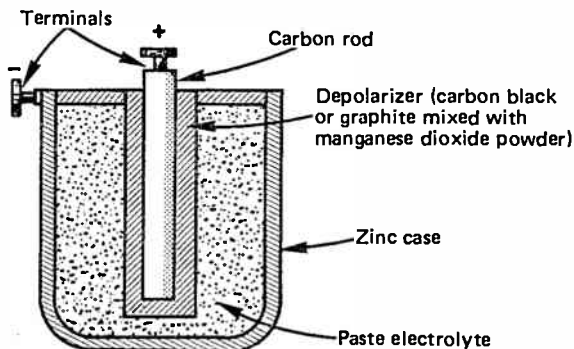
Electrolyte (ammonium chloride, NH_4Cl)
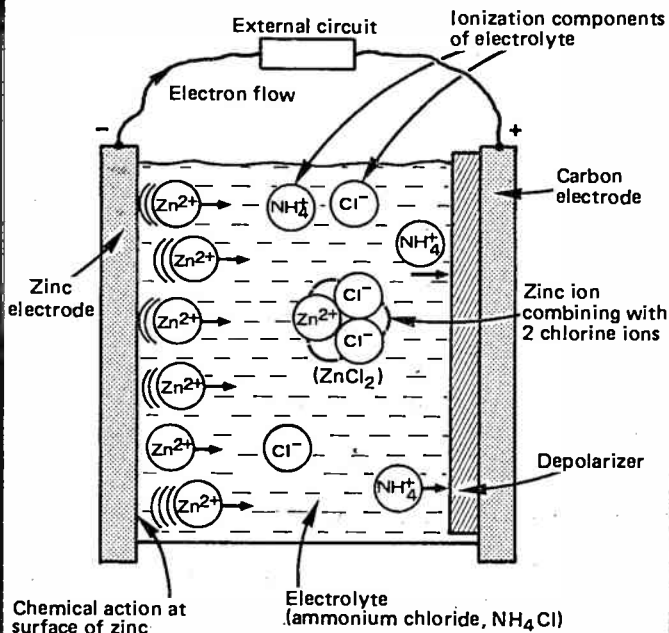
(ii) Ionic action

Fig. 10.2 Leclanché dry cell.

creates a flow of them into the solution where they combine with Cl^- ions so that:

$$Zn^{2+} + 2Cl^- \rightarrow ZnCl_2 ,$$
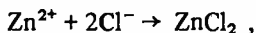
i.e. neutral zinc chloride. This leaves NH_4^+ ions which are repelled by the Zn^{2+} ions towards the carbon electrode at which they accept electrons:

$$2NH_4^+ + 2H_2O \rightarrow 2NH_4OH + H_2$$

(NH_4 ions) + (water) $\rightarrow$ (ammonium hydroxide) + (hydrogen)
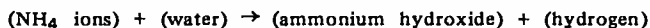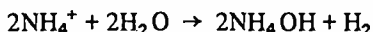
The products of this last reaction need not concern us except that this is where the depolarizer comes in. Sufficient here to show that the nub of the action is at the zinc case where it is the chemical action between the zinc and the electrolyte which provides the main driving force by supplying electrons to the —ve terminal. The cell eventually fails when the zinc case is corroded away or the electrolyte dries up.

On open circuit the cell has a voltage when new of about 1.6, falling to $1.2 - 1.4$ when discharged. These voltages are irrespective of battery size although naturally the larger cells are capable of delivering a greater total electrical energy.

Many other types of cell exist, all basically transforming chemical energy into electrical energy by ionization within an electrolyte. The secondary cell (e.g. car battery) does not function on this principle, it merely *stores* electrical energy presented to it chemically, so it is not a *generator*.

10.2 MECHANICAL (GENERATORS)

As early as 1819 Hans Oersted, a Danish physicist, showed that current in a wire gives rise to a magnetic field around the wire and we ourselves have explored this phenomenon in the context of two parallel wires (Sect.8.6). Later in 1831 Michael Faraday, the celebrated English scientist, examined the idea that a magnetic field could produce a current. What

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he discovered was that changing or moving magnetic fields around a conductor produce a current but not when there is no motion. This basic experiment of his has led to the electric generator, the type of machine which produces nearly all the electric power in the world today. Faraday's discovery may be summed up in terms of "lines of force" as:

"an electromotive force is generated within a conductor whenever it cuts lines of magnetic flux".

In Section 8.6.4 we considered three mutually perpendicular vectors representing force, velocity and magnetic field acting on an electron. The result is that the total effect on all the electrons within a conductor appears as a force on the conductor at right angles to its length. Figure 8.14(i) still applies but now the conductor is turned round so that the force acting on each electron when the *conductor* moves causes the electrons to drift *along* it, so becoming a current. This assumes that there is a complete circuit so that current can flow. Accordingly a similar Figure except that the vectors are labelled differently, can be used to illustrate the generator principle and this is shown in Figure 10.3(i). The process is aptly named *electromagnetic induction* and Figure 10.3(ii) shows this in a more practical form.

(Readers who are conversant with Fleming's Rules and even remember that it is the right hand rule for generators may have successfully wriggled their fingers into the appropriate directions only to find that this is not in agreement with Figure 10.3. It must be remembered however that in considering the electron we are committed to the true direction of current flow, not the *conventional*. Agreement will be found by switching to the left hand.)

From the formula developed in Section 8.6.4:

$$F = B q v ,$$

Suppose an electron moves under the influence of F a distance l along the conductor, then:

$$\text{Work done} = F \times l = B q.l v .$$

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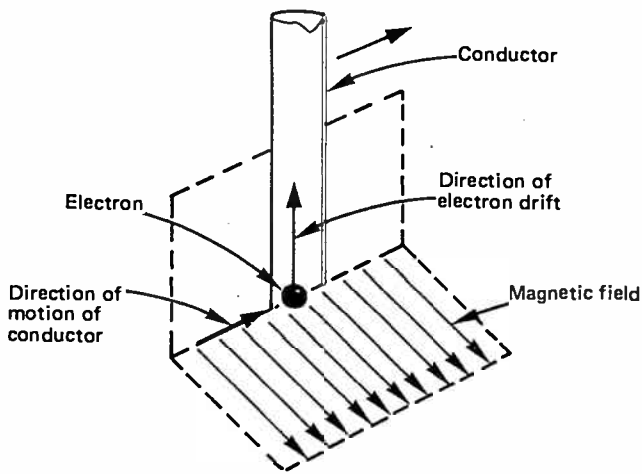
(i) Forces on an electron
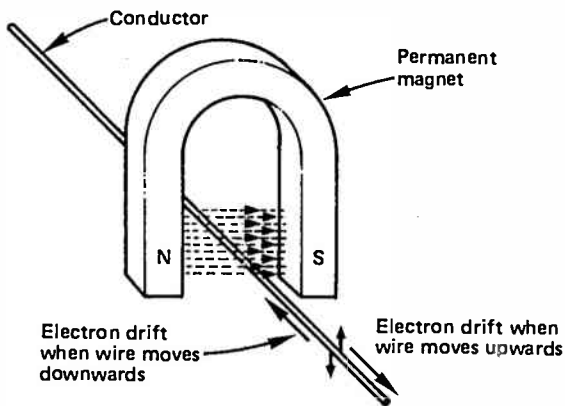
(ii) An elementary generator

Fig. 10.3 Electromagnetic induction.

In Section 7.2 it is shown that:

potential difference = work done/charge = W/q

hence voltage developed across length l is equal to:

$(B\,q\,l\,v)/q = B\,l\,v$

and this is the induced e.m.f. (e) which causes a current to flow in an external circuit.

Should the conductor not move at right angles to the field then the equation is modified to:

$e = B\,l\,v\sin\theta$ V

as might be gathered from Figure 8.14(ii).

In a practical generator the wires rotate through the magnetic flux which is made as high as is conveniently possible. The length of wire is increased by using many turns on a coil, by so doing the e.m.f.'s induced in each turn are additive.

10.2.1 Microphones

Not instantly recognizable as mechanical generators are certain types of microphone for these are capable of transforming the energy of a sound wave, small though it may be, into electrical energy. A sound wave consists of pressure variations in the air at audio frequencies (about 20 − 20,000 Hz − Sect.4.9) which travel at around 344 m/s.

A microphone *diaphragm* in the path of a sound wave is caused to vibrate at the frequency of the wave. By suitable coupling the centre or apex of the diaphragm may:

(i) move a coil of wire within a magnetic field on the principle shown in Figure 10.3 thereby creating electron flow in the turns of the coil in sympathy with the sound wave (*moving-coil* microphone);

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(ii) apply a varying pressure to a special type of *crystal* which responds with a shift of electrons within it thereby creating a potential difference across two of the crystal faces (*piezoelectric* microphone – piezo from Greek, "to press"). The electron movement only occurs in a few materials and the mechanism is not easily explained.

A third type of microphone, again based on Figure 10.3 is the *ribbon*. In this the diaphragm is a light corrugated metal ribbon which vibrates directly from the action of a sound wave. By letting the ribbon take the place of the conductor in Figure 10.3(i) it is evident that when held in a strong magnetic field, electron flow in the ribbon is according to its velocity and direction of motion.

10.3 HEAT (THERMOCOUPLES)

Heat gives energy to atomic particles but it is not generally responsible directly for current flow except for example in the case of the *thermocouple*. With this device we are no longer talking in terms of powerful generators but, as with the microphone, something very much smaller. Way back in 1821 Thomas Johann Seebeck (a German physicist) found that if two dissimilar wires were joined in a loop as in Figure 10.4(i), with one junction cold and the other heated, a tiny current could be detected. Copper and iron are suggested in the sketch but many other combinations show a similar effect. To account for the current, electrical energy must be derived from some outside source and that can only be the heat.

It requires more quantum theory than we have at our disposal for a full explanation of this. "Seebeck effect" so we must be satisfied with a simplified one. We saw in Section 8.5.9 when studying thermionic emission that electrons need extra energy to escape from the surface of a metal, this being related to the work function (ϕ). So what is likely to happen with metal to metal rather than metal to space? Firstly the two metals must have different work functions, say, metal A with work function ϕ_A and metal B with ϕ_B. Let metal A have the higher work function. This means that metal A
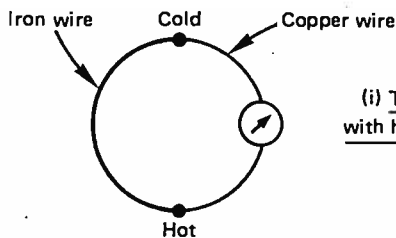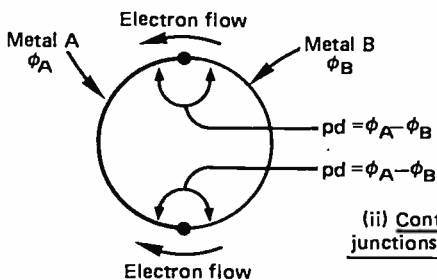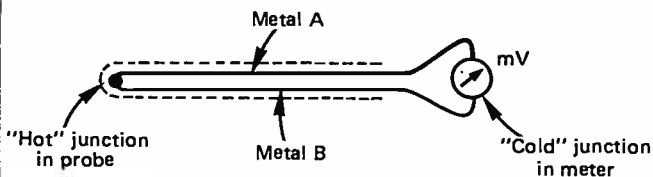
Fig. 10.4 Thermocouples.

electrons need to be given more energy than those of B for escape. Accordingly on an energy-level diagram metal B has more higher energy levels filled, consequently when in contact with A more electrons will pass from B to A than in the opposite direction.

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A passage of electrons across the junction infers that a potential difference exists and this is known as the *contact potential*. Because an electron-volt is the energy required to accelerate an electron through a p.d. of 1 volt, this potential has a value of $\phi_A - \phi_B$ as shown in Figure 10.4(ii). A practical example is given by copper (ϕ = 4.26 eV) and silver (ϕ = 3.56 eV). Their contact potential is therefore $4.26 - 3.56 = 0.7$ volts.

Now let heat be applied to one of the junctions. The cancellation of contact potentials which occurs when both junctions are at the same temperature no longer applies [see Fig. 10.4(ii)]. The fact that the two contact potentials are now different means that an e.m.f. exists. Unfortunately this is the complicated part, that is, to find the answer as to why heat changes a contact potential. There are actually several effects which add up to the complete story, one major one being that the electrons at the hot junction have more energy than those at the cold. We leave it at that until our quantum theory is more advanced and conclude by looking at the thermocouple arrangement for measuring temperature in Figure 10.4(iii). With a copper-iron thermocouple, a temperature difference of 100°C is likely to produce an e.m.f. of around 0.7 V. Special metal alloys have been developed to give e.m.f.'s many times this value.

10.4 LIGHT (PHOTOVOLTAIC CELLS)

The theory of light as a generator of electric current by invigorating electrons with sufficient energy to escape from the surface of a metal is covered in Section 8.5.4. Using Planck's formula and knowing the work function of a particular metal enables us to examine its capabilities of photoelectron emission from:

$$\text{maximum k.e. of emission} = hf - \phi$$

where h is Planck's constant, f is the light frequency and ϕ is the work function, the right hand side of the equation being simply energy of photon minus work function.

A *photovoltaic cell* is one which produces an electric current at the junction of two substances when acted upon by light, a commonly used form of which is the *photo-diode*. This relies on the fact that light falling on a semiconducting material generates electron-hole pairs through the energy of photons in the normal way. From Figure 9.9(iii) it can be appreciated that at the junction the electrons and holes are separated by the potential barrier acting across the depletion layer. This Figure shows the majority carriers only but now we are considering minority carriers which move in the opposite direction, they comprise the normal, comparatively small, reverse saturation current (Sect.9.4.5). In Figure 9.10(i) is shown the energy hill (E_h) which must be climbed by an electron for it to be capable of crossing the depletion layer. It is around 0.7 eV for germanium and about 1.1 eV for silicon. With the photon energy range for visible light as approximately 1.7 to 3.3 eV (Sect.8.5.3), absorption of one photon by an electron can clearly give it ample energy for the journey across. Accordingly when light is present some of the additional electrons set free in the p region gain sufficient energy to sweep across the depletion layer, equally with holes created in the n region. The total charge flow is the *photo-current* which is in addition to the reverse saturation current. The photocurrent can therefore be attributed only to the energy obtained from the incident light, hence the photo-diode is a generator.

A typical example for a silicon photodiode is an increase from some 0.1 μA (*dark current*) to up to 300 μA (*light current*) for reasonably strong illumination. Hence one of the uses of a photodiode is as a solar cell.

10.5 FRICTION

The simple experiment of rubbing a plastic ruler vigorously on a piece of flannel or on the sleeve and finding that it then picks up tiny pieces of paper demonstrates how friction can create a charge. The charge on the ruler is *static* for unless there is a path for it to "leak" away, it remains there. We are now involved in *electrostatics* and the first thing to realise is

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that static electricity is the same kind as current electricity except that it stays put.

It might be suggested that gravity accounts for the attraction exerted by the ruler but clearly it is far too weak. It must therefore be due to charges and it is easy to imagine how the ruler picks up a charge through friction if use is made of Figure 8.9. This is a reminder of how rough a surface is at atomic level. Hence rubbing two such surfaces together transfers surface electrons from one to the other, they get caught up so to speak. But which way do they go, flannel to ruler or ruler to flannel?

In this particular case electrons are collected by the ruler which therefore exhibits a negative charge while clearly the flannel must be left with an equal positive charge. Which pairs of materials when rubbed together do what is answered in the theory of electrostatics but this is not something we need to understand. However, the question as to why the ruler attracts pieces of paper which presumably are electrically neutral ought to be answered for it appears to be inconsistent with the basic principles of electronic charge. Figure 10.5

Fig. 10.5 Electrostatic induction.

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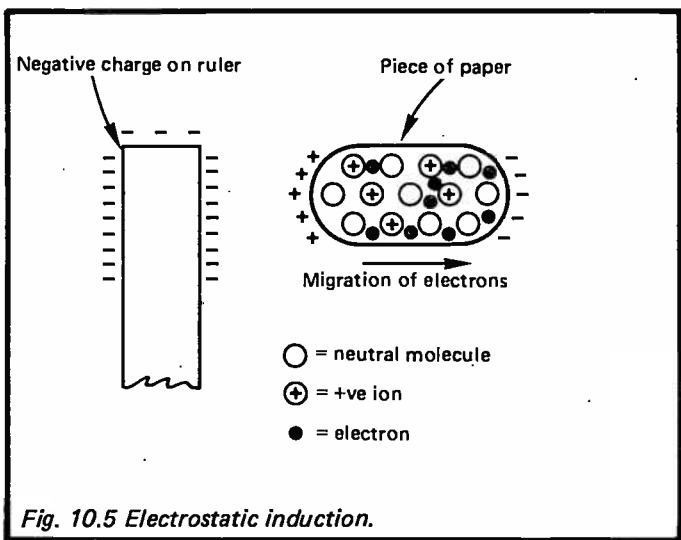
shows the ruler close to a piece of paper. The −ve potential residing on the ruler is high, it would be measured in hundreds or even thousands of volts. Its field therefore is capable of ionizing some of the molecules of the paper and the electrons so freed are repelled away from the ruler, leaving a surplus of +ve ions nearer to it. The result is an attractive force on the +ve ions greater than the repulsive force on the electrons because the latter are farther away. We could reverse all polarities and find the effect to be the same. Creating a charge on a normally neutral material is called *electrostatic induction*.

What interests the electronics engineer more perhaps are the electrical magnitudes which can exist. The fact that pulling off a nylon or similar man-made fibre garment gives rise to a crackling noise indicates that such high voltages have developed that the insulation of the intervening air has broken down (i.e. even though an insulator, it has become ionized — Sect.9.3). Many thousands of volts may be present but only minute currents can flow so there is no danger to life.

Clearly the number of electrons rubbed off one material onto another is comparatively small, hence the total charge is small. That on a ruler might be only 10^{-12} C and on the greater area of a nylon garment, perhaps 10^{-10} C. The charge on the garment could therefore be due to:

$$\frac{10^{-10}}{1.602 \times 10^{-19}} \simeq 6 \times 10^{8} \text{ electrons,}$$

not a very large number in the atomic sense.

The electric field strength developed when there is, say, a 0.1 mm gap (d) between the garments (hardly a practical proposition but we have to assume something!) is, from Sections 4.7.2 and 7.2:

$$E = \frac{Q}{4\pi \epsilon_0 d^2} = \frac{10^{-10}}{4\pi \times 8.85 \times 10^{-12} \times (10^{-4})^2}$$

$$\simeq 9 \times 10^{7} \text{ V/m,}$$

i.e. 9,000 V across the gap.

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To people who are accustomed to a few volts and not many more milliamperes such figures may come as a surprise. What is equally interesting however is just how quickly such charges can "leak" away. Suppose the p.d. of 9,000 V due to a charge of 10^{-10} C meets what we would normally consider to be an *insulating* path of 100 megohms (10^8 Ω) then, from Ohm's Law:

$$I = \frac{9000}{10^8} = 9 \times 10^{-5} \text{ A}$$

and since (from Sect.7.1)

$$I = \frac{Q}{t}$$

then

$$t = \frac{Q}{I} = \frac{10^{-10}}{(9 \times 10^{-5})} = 1.1 \times 10^{-6} \text{ s} = 1.1 \ \mu\text{s} ,$$

meaning that our so-called insulating path can actually conduct away this charge in about one microsecond.

Let us always be mindful however of the many approximations made!

Chapter 11

ELECTRICITY IN ACTION

Many of the foregoing discussions and calculations tell of happenings which are hardly believable yet intuitively we feel that it all makes sense. Accordingly our misgivings will be lessened if finally we enquire into a few examples of the translation of theoretical principles into practical devices.

One of the most useful demonstrations of the basic principles of electron activity is provided by the cathode-ray tube. This has already been discussed in various earlier Sections, namely:

Section 8.1 (Fig.8.1) — shows the deflexion of an electron by electric and magnetic fields

Section 8.5.9 (Fig.8.10) is concerned with thermionic emission from a heated cathode and how it can be utilized

Section 8.5.10 links electron energy with velocity.

Some other developments which have arisen from the fertile imaginations and inventiveness of the world's scientists make up the remainder of this Chapter. They are based on what we have learned so far.

11.1 ELECTRIC LIGHTING (FILAMENT)

Research continues apace in designing electric lamps of greater efficiencies. It all started with the knowledge that passing a current through a conductor can generate sufficient heat to make it glow white hot, so giving off light. Air must be excluded from the process otherwise the conductor burns away. The heating follows from the fact that a potential difference applied across a conductor raises the energy levels of its electrons, their random motions increase and as is seen in Section 6.3.4, heat which is the kinetic energy of particle

motion, is therefore generated. With a sufficiently high p.d. applied, most electrons are maintained at the higher levels of energy with heat as the result. However when electrons do drop back from the higher levels, the energy is emitted as photons (Sect.8.5.3) and provided that energy transitions are of the order of electron-volts, e.g. $1.7 - 3.3$, visible light is emitted. This is arranged according to Ohm's Law whereby for a given applied voltage the filament length and resistance when hot are such that the desired conditions are obtained. These are that the filament should not be at such a high temperature that it melts, conversely not at such a low temperature that light tending to the red end of the spectrum is emitted (see Fig.4.9).

11.2 ELECTRIC LIGHTING (GASES)

The glow of a gas, for example the sign-maker's neon, arises from the fact that the gas atoms are capable of absorbing or emitting photons of only a certain definite size. Hence for each gas there are fixed energy levels and light is emitted on change from one level to a lower one. Every gas therefore has its own characteristic "glow" when this process takes place. Neon has a reddish glow, sodium is yellow as used for street lighting. The requirement therefore for this to happen is to maintain a supply of energy in a suitable form.

A gas is normally an insulator so it is first necessary to create a supply of electrons in it so that a current may be made to flow. Consider two metal electrodes spaced apart within a glass tube, evacuated except for a small amount of gas which has been added. Let a potential difference V be connected as in Figure 11.1(i). Inert gases are used, for being such they do not corrode the metal electrodes. From Section 8.5.9 on thermionic emission little is expected to happen at low values of V because the cathode is not heated. A small flow only of electrons occurs, made up of the few which gain sufficient energy from heat or light to overcome the space charge. These are then collected by the anode. As V is increased, this current increases but to no great extent. At a certain value, V_i however, depending on the gas, the

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Cathode

Inert gas at low pressure

−

Anode

+

Electron flow (I)

V
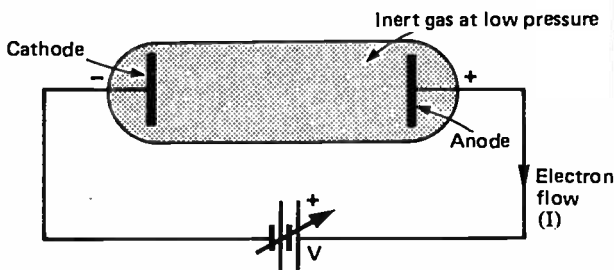
(i) General arrangement for ionization of a gas

I

With gas

With vacuum

V_i

V
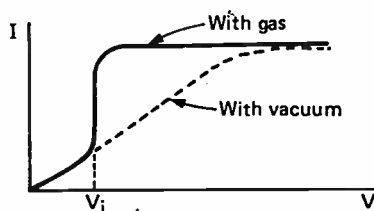
(ii) Typical current-voltage characteristic

Cathode (emits electrons)

Neutral gas atom or molecule

+ve, ion

−

+

Ion striking cathode (produces secondary emission of electrons)

Space charge

Electron about to collide with gas atom or molecule

Anode (collects electrons)

Electron (attracted by anode)
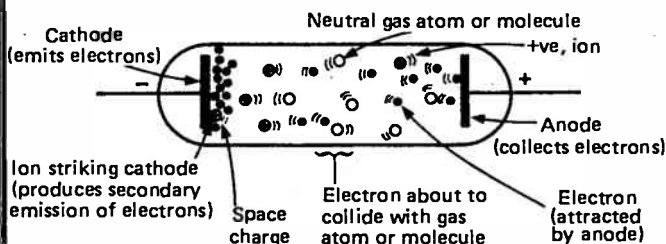
(iii) Ionization process

Fig. 11.1 Conduction in gas at low pressure.

current suddenly rises as shown in (ii) of the Figure and the gas glows with its characteristic colour.

Of interest therefore is what happens at V_i and one important factor must be that the space charge surrounding the cathode is reduced. This can only happen through neutralization by +ve charges and these can only arise by the creation of

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+ve ions. As V increases towards V_i, the accelerations imparted to the comparatively few electrons within the interelectrode space increase. Eventually their velocities are such that on encountering a neutral gas atom (or molecule — Sect.6.3), one or more electrons are freed by collision. These also are accelerated by the field and cause more collisions. The effect is cumulative but clearly does not occur until V reaches a value such that the electron energies are capable of causing ionization by collision. Not only therefore does the supply of free electrons increase, some of which form an anode current but also the atom or molecule ions drift towards the cathode. Here they reduce the space charge by their presence and by removing electrons from it so that they become neutral particles again. But we have yet to account for the fact that a relatively few gas ions can have such a great effect on the space charge. Just to get things into some sort of perspective, take the gas neon (an element, atomic number 10) as an example. Its atomic weight is given as 20.179.

Then mass of single neon atom is equal to:

$$20.179 \times 1.6606 \times 10^{-27} = 3.35 \times 10^{-26} \text{ kg (Sect.6.1.3)}$$

After collision and losing say, one electron (of mass 9.109×10^{-31} kg), the neon ion has a mass of $(3.35 \times 10^{-26}) - (9.109 \times 10^{-31})$ kg.

Hence, compared with the electron its mass is:

$$\frac{(3.35 \times 10^{-26}) - (9.109 \times 10^{-31})}{9.109 \times 10^{-31}} = 3.68 \times 10^4$$

i.e. nearly 37,000 times heavier.

Suppose that after collision the neon ion and separated electron both start from rest (an assumption we must make or things get out of hand) and that they move through the same p.d. Both particles therefore gain the same k.e. Hence, using subscripts i for ion and e for electron:

$$\tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2} m_e v_e^2 \quad \therefore v_e^2 / v_i^2 = m_i / m_e$$

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$$\therefore \quad \nu_e/\nu_i = \sqrt{[m_i/m_e]} = \sqrt{[3.68 \times 10^4]} = 192,$$

showing that the electron reaches a velocity nearly 200 times that of the ion. The leisurely pace of the latter means that the time they are around and +ve before recombination and neutral is comparatively long hence their effect in neutralizing the space charge is that much greater. The few collisions which occur at first therefore have a disproportionately greater effect, the space charge is quickly reduced and a much greater emission of electrons follows.

With little space charge, the +ve ions are collected by the cathode, so adding to the current. These heavy ions "bombard" the cathode with a further release of electrons there (see also Sect.11.4). The whole process is summed up pictorially in Figure 11.1(iii).

Once therefore ionization commences, it is self-sustaining and the gas continues to glow as atoms revert to the ground state and photons are emitted.

11.2.1 Fluorescent Lighting

A fluorescent tube operates on the same basic principle but with refinements. The two electrodes are tungsten wires which are heated on start-up by passing a current through them. This increases the tube current (Sect.8.5.9) and ensures starting. Subsequently when the glow discharge is present the heating current is switched off, the wire electrodes then being maintained at a suitable temperature by the tube current itself. Because the discharge is self-feeding, a controlling device is used to prevent excessive current, for a system of ionization by collision fuelled by a constant electric field must be self-destructive.

The gas used in a fluorescent lamp is usually a vapour of mercury, an element which ionizes at 7.1 eV compared with neon at 21.6 eV. This enables lower supply voltages to be used for the same length of tube. The characteristic glow of the vapour comprises mainly violet and ultra-violet light which is not generally satisfactory for lighting. Accordingly the inside of the tube is coated with a fluorescent powder.

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Energy from light generated in the tube excites atoms in the coating and these emit photons as energy levels slip lower. Now however, the light is in a more desirable part of the visible spectrum.

11.3 LASERS

The laser is particularly useful as an example of atomic behaviour because it brings together both particles and waves. It works by exciting atoms to a high energy level and then compelling them to drop back to the ground level en masse, emitting light as they do so. The result is an intense narrow beam of light. Laser stands for "Light Amplification by the Stimulated Emission of Radiation", an acronym which describes the basic action well. The idea behind the development arose long ago with Einstein who was then involved with Planck's radiation equation ($E = hf$) but it was not until 1960 that Theodore Harold Maiman (an American physicist) developed a working model. There are gas, liquid and solid lasers, we choose just one of the latter for its more easily understood chemistry.

In Section 8.5.8 Boltzmann's Distribution Law shows that generally, undisturbed atoms rest in the ground state, having least energy. They can be excited as shown in that Section but normally revert to the ground state within about 10^{-8} seconds with a photon emitted at each energy transition. However with some there is also a *metastable* state, i.e. stable unless unduly disturbed. With little interference, atoms remain in this state for some 10^{-3} s before reverting, hence remain excited for about 100,000 times as long. This now gives us three energy levels to contend with, E_1, the ground state, E_2 the metastable and E_3 the fully excited state as shown in Figure 11.2(i).

An atom changing its energy level emits or absorbs radiation at a frequency according to Planck of E/h. The idea of stimulated emission begins with placing the atoms in a radiation field of the same frequency. Let us consider a material with energy levels E_1 and E_3 only. Because there are many more atoms at E_1 than at E_3, absorption of energy by atoms

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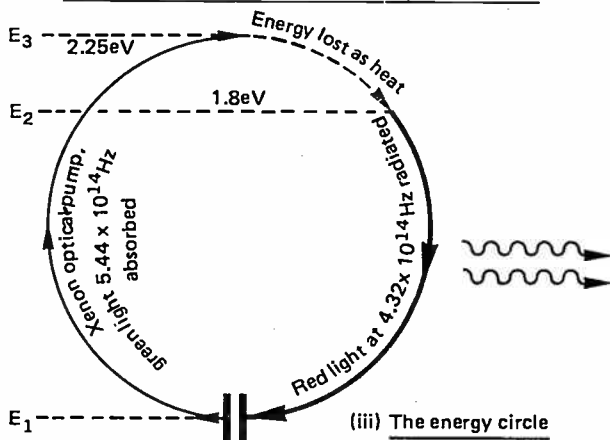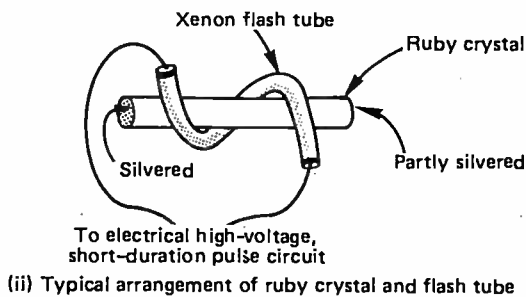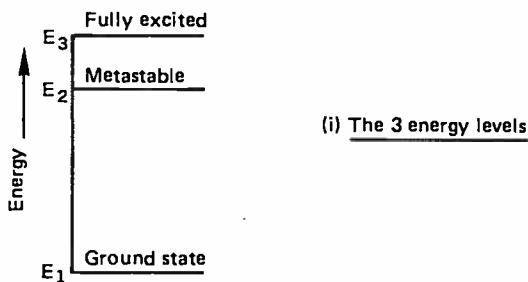
Fig. 11.2 Lasers.

at E_1 will predominate over the emission of energy by those at E_3. The latter enhance a radiation field because of the similarity in frequency but being relatively small in number and with so much absorption taking place simultaneously, so far stimulation has got us nowhere. What is needed is an arrangement by which there are more excited atoms than ground state ones. This is known as *population inversion* ("population" is the statistician's term for the total number of things in a given place). The ruby laser is a device which achieves this.

One type of ruby laser uses a rod-shaped ruby crystal some 4cm long and 0.5cm diameter. One end of the rod is fully silvered, the other partially as in Figure 11.2(ii). Ruby crystals can be made artificially and need to contain a tiny percentage (about 0.05%) of chromium ions to give the characteristic reddish pink colour. The chromium ions have a convenient metastable state and it is through this that light amplification is obtained. Adjacent to or surrounding the ruby rod is a xenon flash tube working on the principles developed in Section 11.2. An electrical circuit (not shown) pulses the tube at high voltage and short duration to produce intense flashes of light. The method is known as *optical pumping* and its purpose is to raise (or pump up) the chromium ions from level E_1 to E_3.

The characteristic colour of xenon gas is green at a frequency of 5.44×10^{14} Hz (see Fig.4.9). The energy level at E_3 is therefore:

$$E_3 = hf = \frac{(6.626 \times 10^{-34}) \times (5.44 \times 10^{14})}{1.602 \times 10^{-19}}$$

$$= 2.25 \text{ eV}$$

hence each chromium ion absorbing one photon of energy from the xenon light receives 2.25 eV. From level E_3 the ions quickly fall to E_2 by giving up energy in the form of heat to the crystal atoms (in about 10^{-8} secs.). At this point the ion population is inverted (for up to 10^{-3} secs.), i.e. there are many more ions at the higher level E_2 than at E_1. The energy situation is shown in Figure 11.2(iii).

A few ions from E_2 spontaneously fall to E_1, emitting photons of frequency 4.32×10^{14} Hz (from $f = (E_2 - E_1)/h$), their characteristic red light. This light within the ruby rod is at exactly the right frequency to stimulate all other ions at E_2 to release their 1.8 eV of energy as radiation. The result is an avalanche process through which an intense pulse of light is emitted from the partly silvered end of the rod. The length of the pulse is generally of the order of microseconds but lasers have been constructed having a pulse length of a mere 10^{-14} s. These are useful for examining molecular motion where for example, a molecule rotates once in about 10^{-12} s.

In Section 4.9.1 is the conclusion that standing waves are set up in a string an exact number of half-wavelengths long. Similar conditions apply to light within the ruby rod so the rod is made an integral number of half-wavelengths long at the emitted photon frequency. The repeated photon reflexions between the ends of the rod therefore take the form of an optical standing wave which therefore controls further emissions. Accordingly that which is emitted is at a single frequency (or wavelength) only. The process of pumping to E_3, drop to E_2 and emission on falling to E_1 is repeated for each flash of the xenon tube. The ruby laser is only one of many types with the principle of population inversion common to all.

11.4 SECONDARY EMISSION

Figure 8.5 shows in essence a photoelectric cell, a device which allows a current to flow according to the amount of light illuminating the cathode. When the light is poor there are not many photons around so the current flow is hardly detectable and moreover it could be partially swamped by noise currents (Sect.7.5). Direct amplification is of no avail because the noise is amplified too. Use of the phenomenon of *secondary emission* avoids these difficulties by increasing the electron flow up to many thousands of times.

From the basic equations, electrons within an electric field gain energy because of the acceleration imparted to them.
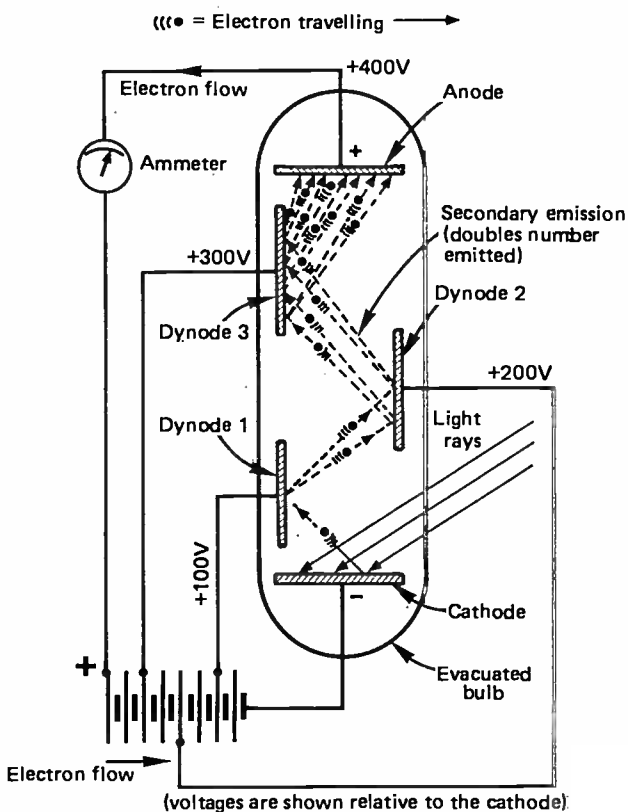
Fig. 11.3 Electron multiplication.

Given sufficient velocity by this means therefore electrons "bombarding" the surface of a solid, share or give up their energies so that more than one other electron (usually around 4 but up to 2—3 times this number) finds sufficient energy to overcome the potential barrier of the space charge and be emitted (Sect.8.5.4). The elements of a practical photocell with electron multiplication are shown in Figure 11.3 which is simply Figure 8.5 with the added electrodes and voltage supplies.

In operation, light photons arriving at the cathode release electrons (Sect.8.5.4). These have relatively low velocities but are given acceleration and are attracted to the first *dynode* by the +ve potential on it of 100 V or so. Bombardment of the dynode by these "photoelectrons" gives rise to secondary emission so resulting in a greater flow of electrons which are now accelerated and attracted by a further 100 V +ve difference on dynode 2. This again emits more secondary electrons. The process continues until finally the electron stream is collected by the anode after electron multiplication at each dynode. The Figure shows 3 dynodes only but in practice up to 10 is quite normal.

Multipliers are assessed by their *secondary emission ratio* (or *secondary emission coefficient*) which is the average number of secondary electrons emitted per impinging primary electron. Thus if each of 10 dynodes has a coefficient of say, 4, there is an overall multiplication of 4^{10}, i.e. 1.05×10^6 showing that for each electron emitted from the cathode over one million are expected to reach the anode and this is in no way the best which can be achieved.

11.5 ELECTROPLATING

A step further in exploiting the use of ionization in liquids is in *electroplating*, the deposition of a metal onto another metal or conducting material. In copper plating for example, a copper anode and the object to be plated are immersed in a solution of copper sulphate, $CuSO_4$ (see Fig.7.12). The copper sulphate molecule ionizes into Cu^{2+} and SO_4^{2-}, indicating that two electrons are transferred. Water ions

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complicate the process but the main requirement is met in that each Cu^{2+} ion on reaching the negatively charged item to be plated accepts two electrons from it and becomes a copper (Cu) atom, consequently the metal slowly builds up over the surface. The Cu^{2+} ions in the electrolyte are not exhausted by the process because further ions are produced at the anode.

Copper plating is seldom used in practice for beautifying items as with silver, chromium or gold, but for *refining* copper. Impure copper is used at the anode but only pure copper is deposited at the cathode.

Silver plating is accomplished similarly. The anode is a block of silver, the cathode is the item to be plated. A typical electrolyte is silver nitrate, $AgNO_3$ which ionizes into Ag^+ and NO_3^-.

Generally for plating the electrolyte must dissociate into two ions, the positive one of which is related to the metal to be deposited.

11.6 AND OTHERS

There are two other items which are especially worthy of mention but have not been described in detail. The electric motor and generator need no introduction to electrical engineers, how these machines are constructed is widely known. Although mention of them is made in Sections 8.6.4 (motor) and 10.2 (generator), a little amplification may be useful to show the link between them. The basic principle is in fact the same for both types of machine. This is that when an electron moves through a magnetic field ("cuts lines of magnetic flux"), a force is developed on it. In the motor an external p.d. creates electron movement in the form of a current in a conductor, through a magnetic field. The force exerted on the electrons is at right angles to their direction of motion and therefore across the conductor as can be deduced from Figure 8.14(i). Since the electrons are trapped within the conductor, the forces acting on them are transferred to the conductor. Electrical energy from the p.d. is therefore transformed into the mechanical energy of conductor movement.

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With a generator, by moving a conductor through a magnetic field the electrons contained within the conductor are obliged to move similarly. The direction of the force on them in this case is along the conductor hence electron drift occurs, so constituting a current. Figure 10.3(i) illustrates this. Mechanical energy required to move the conductor within the field is transformed into electrical energy.

The *electromagnetic* or radio wave is a little difficult for most to get to grips with but what we have learned about electric and magnetic fields and how magnetism arises from charges in motion should be of inestimable value. But the wave, its idiosyncrasies and how the Earth's atmosphere plays tricks with it, well, that's another story.

APPENDIX

Physical Constants Used in the Main Text

Quantity	Symbol	Value	Unit Symbol
Acceleration of free fall	g	9.81	m/s^2
Atomic mass unit	u	1.6606×10^{-27}	kg
Avogadro's No.	N	6.022×10^{23}	
Boltzmann Constant	K	1.38×10^{-23}	J/K
Electron charge	$-e$	1.602×10^{-19}	C
Electron rest mass	m_e	9.109×10^{-31}	kg
Electron charge/ mass ratio	e/m_e	1.7588×10^{11}	C/kg
Free Space — permeability (magnetic constant)	μ_0	$4\pi \times 10^{-7}$	H/m
Free Space — permittivity (electric constant)	ϵ_0	8.85×10^{-12}	F/m
Gravitational Constant	G	6.67×10^{-11}	$N\,m^2/kg^2$
Neutron rest mass	m_n	1.6748×10^{-27}	kg
Proton rest mass	m_p	1.6725×10^{-27}	kg
Planck Constant	h	6.626×10^{-34}	J s
Velocity of light	c	2.998×10^8	m/s

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