$$
\begin{aligned}
& 3 \times 1{,}000 \times 17 \times 10{,}000 \\
= {} & 3 \times 10^3 \times 17 \times 10^4 \\
= {} & 3 \times 17 \times 10^3 \times 10^4 \\
= {} & 3 \times 17 \times 10^7 \\
= {} & 51 \times 10^7 \\
= {} & 510{,}000{,}000
\end{aligned}
$$

Indices

Here is a series of numbers offered as a typical example:

$0.0001 = \frac{1}{10000} = 10^{-4} \times 1$

$0.001 = \frac{1}{1000} = 10^{-3} \times 1$

$0.01 = \frac{1}{100} = 10^{-2} \times 1$

$0.1 = \frac{1}{10} = 10^{-1} \times 1$

and

1.0	= 1.0	$= 10^0 \times 1$
10	= 10	$= 10^1 \times 1$
100	$= 10 \times 10$	$= 10^2 \times 1$
1000	$= 10 \times 10 \times 10$	$= 10^3 \times 1$
10,000	$= 10 \times 10 \times 10 \times 10$	$= 10^4 \times 1$
100,000	$= 10 \times 10 \times 10 \times 10 \times 10$	$= 10^5 \times 1$
1,000,000	$= 10 \times 10 \times 10 \times 10 \times 10 \times 10$	$= 10^6 \times 1$

You may notice that the number of 'O's following the 1. in the left-hand column series of numbers is the same as those in the centre column, that is it indicates the number of tens in the answer. Typically, $10 \times 10 \times 10 \times 10 = 10{,}000$, ie four O's after the 1. By showing this as 10^4, all we are meaning is that this number (10^4) is 10 multiplied by itself four times, and is a simple shorthand way of writing large numbers. As can be seen above it is much easier to write 10^6 than 1,000,000 and also takes up less space. It also helps accuracy since you can easily miss a 'O' when counting such large numbers but the little number at the top of the 10 (this little number is called in index and two or more are called indices) always tells you how many O's there should be.

In order to understand fully the system, here is another look at the sequence in more detail. We have shown $10 \times 10 \times 10$ as 10^3 and 10×10 as 10^2 so we can say there are three Os in the first sum which is 1,000 and two in the second which is 100. Logically therefore 10 on its own has only one 'O' so we can show it as 10^1, and since 1 has no 'O's we can show it as 10^0.

An interesting feature now arises. If $100 = 10^2$ and $1{,}000 = 10^3$ $100 \times 1000 = 100{,}000 = 10^5$ but $10^2 \times 10^3$ does not equal 10^6 but 10^5. Do *not* multiply the indices, add them. A further example would be:

$1{,}000 \times 10{,}000 = 10{,}000{,}000$
$= 10^7 \times 1$

$10^3 \times 10^4 = 10^{3+4} = 10^7 \times 1$

This technique enables the handling of very large numbers with a fairly simple operation and since it can deal with numbers as small as one millionth of a millionth of a millionth and as large as 1,000,000,000,000, the importance of this technique cannot be over-emphasised.

Take for example a multiplication of $3{,}000 \times 170{,}000$, we would rewrite this as:

$3 \times 1{,}000 \times 17 \times 10{,}000$
$= 3 \times 10^3 \times 17 \times 10^4$
$= 3 \times 17 \times 10^3 \times 10^4$
$= 3 \times 17 \times 10^7$
$= 51 \times 10^7$
$= 510{,}000{,}000$
(Note 7 O's).

Here is an example combining knowledge of decimals with knowledge of indices:

$3{,}140 \times 175{,}600$
$= 3.14 \times 1000 \times 1.756 \times 100{,}000$
$= 3.14 \times 10^3 \times 1.756 \times 10^5$
$= 3.14 \times 1.756 \times 10^8$
$= 5.51384 \times 10^8 = 551{,}384{,}000$
or 5 with 8 'O's
or $5 \times 100{,}000{,}000$ plus a bit
$= 500{,}000{,}000$ plus a bit.

Obviously such calculations are normally done on a calculator but the above shows how to get the actual value of the answer. The calculator only gives a string of numbers with no indication of whether it is in thousands or millions, because it does not often show the position of the decimal point.

The whole technique is about manipulating numbers, and using the numbers just like a mechanic uses his tools. We are arranging the numbers to do things for us.

Since $1 = 10^0$ and $10 = 10^1$ then any number between 1 and 10 must have a value of index between 0 and 1. For example if $10^0 = 1$ and $10^1 = 10$ then 5 must have a value between 10^0 and $10^{1.0}$ (between 0 and 1). The actual value is .698; and $100^{.698} = 5$. (0.698 is a value between 0 and 1.) The actual index for any number between 1 and 10 is not a direct relationship but is a relationship which alters in a special form. This relationship is called a logarithmic relationship. The actual index value is called the logarithm of the number and actual logarithms for numbers of 1-10 are shown below:

1 = 0.000
2 = 0.301
3 = 0.477
4 = 0.602
5 = 0.698
6 = 0.778
7 = 0.845
8 = 0.903
9 = 0.954
10 = 1.000

Referring back to these original notes we use the indices in multiplication exactly as described previously. $2 \times 2 = 4$ or $100^{.301} \times 100^{.301} = 100^{.602}$ and $2 \times 2 \times 2 = 8$ or $10^{.301} \times 10^{.301} \times 10^{.301} = 100^{.903}$

Next month: square roots, logarithms and decibels.