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Commentary

OUR sincere congratulations are due to our sister journal *The Engineer* which this month celebrates its centenary. It first appeared on January 4, 1856, and week by week ever since it has recorded progress and development in the vast field of engineering. Among the many features characterizing *The Engineer* is one which must surely be unique, namely, that for the last ninety years its editorship has been handed down from father to son and to grandson.

To mark the occasion, a special centenary issue is being published, the principal theme of which is a series of articles dealing with the influence of engineering discoveries over the period 1856-1956. The series deals with a number of topics and of principal interest to us are those of electronics, nuclear power and electricity.

In 1856 electronics was entirely non-existent and electricity was no more than a scientific novelty. Twenty-five years earlier—in 1831—Faraday had annunciated his laws of electromagnetism and had made his first magneto electric machine, but at the time *The Engineer* first saw the light of day the only use of electricity was for the railway telegraph systems. The world's first submarine cable had been laid across the English Channel a few years earlier but the transatlantic cable was yet to come.

There was no electrical engineering as such in 1856 for the sole reason that there was no useful load for the Faraday machine, except, perhaps, for electroplating purposes. The steam engine was supreme as the principal source of power.

The story of the progress of electricity, electronics and the latest development, nuclear power, is ably told in the centenary issue and it would be superfluous to add to it. But on such an occasion as a centenary it is permissible to look back and perhaps browse through the early issues of *The Engineer*.

One realizes, of course, the enormous progress that

has been made during the last century—the pace has never been equalled in human history—but nevertheless one is struck by the number of problems current in 1856 which still seem to be with us today.

The abatement of smoke was, for example, very much a problem in 1856 and the early issues of *The Engineer* deal extensively with this subject. There is the announcement of a special prize by the London Society for the Encouragement of Arts, Manufactures and Commerce "for the best essay on the means of preventing the nuisance of smoke arising from fires and furnaces; treating the subject practically; reviewing the various plans which have been put forth as remedies . . ." and the contribution by a Charles Wye Williams is adjudged the winner of the special gold medal. But the smoke problem is still with us and we must conclude that the said Mr. Williams had not provided a complete solution. His essay remains however—as an essay in good English.

Progress in many fields we have undoubtedly made during the last century, but dare it be said that some of the things we do in 1956 are not done as well or perhaps as speedily as they were in 1856? Coming nearer to home it would certainly appear so. For in *The Engineer* of February 22, 1856, there is a brief reply from the Editor to a correspondent in Durham assuring him that "*The Engineer* is published every Friday afternoon in time for the evening post and that he should receive his copy on Saturday".

We wonder, too, how the present day advertiser would regard the announcement by the Publisher of 1856 to the effect that "advertisements can be guaranteed insertion if delivered before eight o'clock on Thursday evening each week". There is no question that it takes considerably longer in 1956.

However, by the time *The Engineer* reaches its second centenary in 2056 we shall perhaps have regulated our affairs very much better.

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Strip Transmission Lines

By C. Bowness*, B.Sc., A.Inst.P.

This article describes basic laboratory work on the design, development and testing of strip transmission lines and makes comparisons with existing microwave components and techniques. A new method of making strip line components provides more consistent results and eases the measurement problem. Some possible uses of strip lines are suggested.

M licroWAVE systems involving the manufacture of high precision waveguide components are usually very expensive to produce. Although the cost in the production stage can sometimes be reduced by the use of some form of casting, the cost of experimental and prototype models is still high.

Coaxial line generally has better broad-band characteristics than waveguide, but is beset with constructional



difficulties. The problem of making a component such as a hybrid ring in coaxial line form cannot be solved without considerable expense. Moreover, a plug and socket which will join coaxial lines carrying X-band frequencies without producing considerable reflection of power is not yet available, so that a simple method of making microwave circuits at low cost is very desirable. The idea of using an "opened out" coaxial line has been under consideration for some years; basically, this consists of a flat strip or circular conductor held at a fixed distance above a flat extended ground plane. Various types have been suggested and the most important ones are described.

Work has been carried out in America on strip transmission lines and a list of some reports published is given¹⁻⁶. Strip lines have been made for both X-band and S-band; only brief reference is made in the literature to a strip line standing wave measuring gear, though random radiation troubles are reported to have caused erratic readings. The dielectric used was sometimes polystyrene, but in Arditi's¹ work fibreglass was normally used. The dielectric loss caused by the use of fibreglass is quite considerable, probably about 20 times that of polystyrene and, as most of the readings were taken through a strip-waveguide junction and a length of strip line, optimistic values of v.s.w.r. are to be expected.

In the S-band strip line, Grieg and Engelman² have described a hybrid ring with commercial coaxial sockets roughly soldered on to the input; this indicates that no attempt was made to obtain a high value of v.s.w.r. **Alternative Types of Strip Line**

Reference to Fig. 1 shows the following types of strip line:

- (a) Cross-section of the simplest form of strip line. This is a cylindrical conductor above a ground plane with air as the dielectric; the conductor has mechanical supports which may cause mismatches.
- (b) As (a) but the cylindrical conductor is replaced by a flat strip.
- (c) As (a) with a slab of dielectric as a spacer between the wire and the ground plane.
- (d) As (b) but with dielectric spacing.
- (e) As (d) but a further slab of dielectric has been added above the line to confine the fields closer to the strip and hence reduce radiation effects.
- (f) A further ground plane may be added to make the line a complete sandwich. This is intended to reduce still further any possible radiation.

Types (e) and (f) can clearly have cylindrical centre conductors instead of strips, if required.

Advantages and Disadvantages of the Various Types

Types (a) and (b) have theoretical low loss characteristics, but are mechanically impossible. Types (c) and (d) are the simplest and best from the manufacturing point of view.

Type (e), although somewhat more difficult to construct and manufacture than (d), may be of advantage in reducing the radiation. Type (f) has been suggested as the best for low radiation though there is reason to doubt this. It is difficult to construct and makes measurements of performance very difficult.

In general, it is clear that a strip conductor is the only one worthy of consideration, since it can be made as a metal foil stamping or by a photographic method; the line impedance can be altered simply by varying the width of the strip or its separation from the ground plane, and junctions present no difficulty. On the other hand, a cylindrical conductor cannot easily be formed into junctions or tapered when a slow change in impedance is required. The use of wire has one advantage over strip, viz. the absence of any sharp edges which can give rise to very strong local electric fields.

Compared with waveguide, strip lines should tend to have better performance over wide frequency bands as they approximate to coaxial cable and operate on the same fundamental TEM mode. Because of the small spacing required between the strip and the ground plane to produce the necessary low impedance, they are, however, suitable

^{*} E.M.I. Engineering Development Ltd.

only for low-power operation. High power tends to heat up the dielectric, because of the loss which occurs, and it can also rupture it.

Radiation from the line may prove troublesome in the open types, but is very difficult to measure accurately. A metal plane can be brought to within a centimetre or less of a well-made line without any visible effect on the power transmitted or on the v.s.w.r. Most of the radiation which occurred in our early experiments was attributed to discontinuities produced by the methods of manufacture as mentioned below.

Mechanical Difficulties

A method of making strip line in the laboratory had to be found and this was much more difficult than would appear. The major trouble arises from the electrical requirement that the metal should be bonded to the dielectric without air pockets between the two. The presence of air pockets, even though they may be only a few thousandths of an inch in thickness, introduces electrical discontinuities.

The initial choice of copper foil strip and brass sheet ground plane with polystyrene or polythene as the low-loss dielectric seemed fairly obvious. Copper foil 005in thick can be cut with reasonable accuracy to any desired shape and many adhesives, composite adhesives and solvents were tried to bond the foil to the dielectric. Owing to its waxy nature and flexibility, it was found impossible to obtain any real adhesion to polythene and attention was turned to polystyrene. This is another non-polar material and again no adequate adhesive could be found, so a chemical silvering process, involving the Rochelle salt method, was attempted on a laboratory scale. Thin silver films were obtained but these lacked durability because of the inadequacy of those cleaning agents which can be used with polystyrene.

The adhesives used in an attempt to bond the copper to the plastics were Durofix and polystyrene cement mixed in various proportions with polythene and polystyrene solvents. Bostik adhesives were not used as they may introduce unknown losses and tend to hold small pockets of air.

The problem of laboratory production was not solved until a conductive silver paint was found; this is manufactured by Melton Metallurgical Labs., Poole, in a grade specifically intended for application to plastics. With this paint, it was possible to paint strips 0 lin wide, with a resistance of $0.1\Omega/in$ or less.

A further trouble was encountered when uniform sheets of polystyrene were required. Clear sheets of consistent thickness were eventually obtained from Erinoid Limited.

In production, paint may have to be superseded by copper foil and the necessary bonding can, it is believed, be accomplished by first sulphonating the surface of the polystyrene to render it polar. Alternatively, if p.t.f.e. is used as the dielectric, the copper can be bonded by a sintering process.

Measuring Difficulties and Choice of Type of Line

The first lines constructed were of the type shown in Fig. 1(d). Considerable random radiation was found to occur and this type was therefore abandoned in favour of the sandwich line shown in Fig. 1(f). We have since confirmed, by the conducting paint method of manufacture, that nearly all this radiation was caused by poor metal-to-dielectric bonding and is not inherent in the open type of line.

The sandwich line was still found to radiate considerable power from along its edges. It was constructed by clamping together two lines of the type shown in Fig. 1(d), strip to strip. A small air-gap remains between the dielectric sheets and this may tend to induce radiation but the existence of spurious modes between the two ground planes is thought to be a more likely cause. Since the edges of the sandwich are high impedance points it is possible that the inverted TEno modes might be present. Two examples, TE_{10} and TE_{20} , are shown in Figs. 2(a) and (b) respectively. Full lines represent the electric field and dotted lines represent the magnetic field. Short-circuiting the edges would obviously stop the propagation of these modes but would transform the sandwich line into a rectangular section coaxial line in which the normal waveguide modes might exist. These spurious modes will radiate power and thus be attenuated owing to the finite impedance of free space across the edge of the sandwich. Any asymmetric structure placed in the sandwich line will tend to excite these modes.



Fig. 2. Possible field configurations of sandwich strip line



Fig. 3. Field configuration of an open strip line

When attempting to reduce radiation from our experimental lines, the ground plane widths were increased, up to thirty times the strip width, without appreciable improvement. It is now considered that reducing the width of the ground planes until the spurious modes were cut off might produce a radiation-free strip line. No experiments have yet been carried out on this type of line.

When conducting silver paint became available, attention was again focused on the simpler type of line shown in Fig. 1(d) and it was found that radiation no longer caused any serious trouble. The approximate field configuration is shown in Fig. 3 and a probe coupled into the field, parallel to the plane, as shown in Fig. 10, was used to measure v.s.w.r. A standing wave measuring gear having a tunable crystal probe which is moved along the length of the strip, was used to measure the admittance of various components. The standing wave patterns obtained on the laboratory-made lines were not very consistent, but this was to be expected since the strip may have varied quite considerably in thickness and width, and any local protuberances tend to cause voltage peaks and, hence, radiation. As a rough guide, v.s.w.r.'s of the order 0.9 can be measured with an accuracy of approximately ± 0.05 and the phase can be measured with an accuracy of 5 per cent, or better. Although this degree of precision is not all that could be desired, it is sufficient for preliminary design work.

From a consideration of the impedance required and the practical results obtained from the experimental lines, the following dimensions were chosen:

Ground plane	lin wide (minimum)
Strip	0 1in wide
Dielectric	1/16in thick (polystyrene)

Electrical Properties of the Strip Line

No accurate theory for strip lines is available; the fundamental mode may be termed TEM but, in the open types of line, the wave travels half in dielectric and half in air and consequently the wavelength falls between these two corresponding wavelengths. In one case, air, strip line and dielectric wavelengths were in the ratio 1:0.69:0.63.

On two separate occasions, a sudden change in the apparent strip line wavelength was noted. The wavelength, as measured by the movement of a probe above the line, increased by approximately 20 per cent as a perfect match was approached. This change is thought to be due to a surface wave which can propagate along an open strip line. The standing wave pattern of such a wave is normally masked by that of the dominant mode and is only apparent when the line is matched for the latter.

The impedance of the line is dependent on its physical dimensions, the dielectric constant of the insulator and the corresponding fringing fields at the edge of the strip. A simple analysis, given below, involving the use of a measured constant for the fringing capacitance of the line, gives:

$$Z_{\rm s} = \frac{10^{\rm s}}{3\sqrt{\epsilon} \left(8\cdot 83w/d + 7\right)} \Omega$$

where ϵ = dielectric constant of the insulator

w =width of strip

d = dielectric thickness, i.e. distance from strip to ground plane.

This formula is for a line of the type shown in Fig. 1(d) and implies the following restrictions:

(a) that the strip is thin compared with its width, and (b) that $w \ge d$

This formula gives the impedance of the strip used as approximately 100Ω .

An estimated cross-section of the field distribution is shown in Fig. 3; full lines represent the electric field and dotted lines, the magnetic field.

The impedance formula was obtained as follows:

Capacitance of strip to ground plane = $\epsilon w/4\pi d \, esu/cm$ = 8.83 $\epsilon w/d \, pF/m$

Barrett³ has made some measurements on the fringing capacitance of such lines and a very approximate value is $7\varepsilon pF/m$.

Hence, capacitance = $\epsilon(8.83w/d+7)$ pF/m

Now, for an h.f. line, the velocity of propagation

 $= \sqrt[7]{(LC)}$ $= \frac{1}{V_{\epsilon}} (3 \times 10^8) \text{m/sec.}$ Also $Z_{\circ} = \sqrt{(L/C)}$ These give $Z_{\circ} = \frac{V_{\epsilon}}{C} \frac{10^{-8}}{3} \Omega$ Thus, $Z_{\circ} = \frac{10^4}{3 \sqrt{\epsilon} (8 \cdot 83 w/d + 7)} \Omega$

Components Designed in Strip Line Form

Several components have been designed in open strip line form with a view to building a complete r.f. receiver. If the technique were to be adopted for an actual microwave system, a fuller study of discontinuities such as gaps in the line and steps in its width would be essential. It was decided to concentrate on making actual components



Fig. 4. Strip line termination

rather than start a more fundamental research programme, since the investigation was only a preliminary one.

The following components were designed:

WAVEGUIDE TO STRIP LINE JUNCTION

Since all the readily available measuring gear and all the sources of microwave power are designed for use with waveguide, this was the first essential component. The use of intermediate coaxial cable was originally ruled out on the grounds of the mismatch produced by any coaxial plug at these frequencies. An attempt to use the end of a strip line as a pick-up probe in the waveguide was abandoned because of the radiation which occurred from the point where the strip leaves the waveguide. Later, a satisfactory junction, formed of a bar and post transition to a coaxial line and a coaxial to strip line junction, was made. It was found that the length of the coaxial line could be reduced until the strip line lay on the top of the waveguide; this formed a satisfactory junction and occupied a minimum of space.

MATCHED TERMINATION

This presented less difficulty than was at first expected. Any high-loss material placed over the strip produces a small mismatch with considerable attenuation. A few inches of Morganite resistive board, placed in contact with the line in such a manner that its leading edge crosses the line obliquely (see Fig. 4), provides a v.s.w.r. of approximately 0.9 or better, provided the angle is optimum and contact with the strip is good.

For a production component a film of graphite deposited over the line with the end of the strip connected to the ground plane would no doubt prove suitable. Such films

are common in printed circuit work and should present no difficulty.

CRYSTAL HOLDER

This was designed to take the CVX3053 coaxial silicon crystal valve. A transition to coaxial line was made, as in the case of the waveguide junction, by taking the strip out at right-angles through the ground plane and joining the outer of the coaxial line to the ground plane. Initially, a holder with the crystal shunted across the line was tried and a coaxial stub, shorted by a Gecalloy piston was built above the line and used to take the if output from the crystal. Unfortunately, the r.f. coupling to this stub was too small for it to be used as a matching device, so the crystal was used in series with the line, and necessitated the provision of a d.c. return path for the crystal current elsewhere in the circuit. This requirement can easily be



Fig. 5. A strip line hybrid ring

met when bar and post junctions or matched terminations are included in the system.

The termination of a strip abruptly above a ground plane, gives rise to an electrical open-circuit. For this reason, the line was continued for approximately half a wavelength beyond the junction of the strip and coaxial inner conductor and a circular enlargement of the strip was made about this junction. (See Fig. 10.) Variation of the stub length varies the susceptance, whereas variation in the diameter of the circular enlargement mainly varies the conductance at the junction. By adjusting these, a v.s.w.r. of better than 0.95 was obtained with a mean crystal and a crystal current of 0.6mA. A batch of crystals placed in this holder then gave v.s.w.r.'s of between 0.8 and 1.0. As an aid to compactness, the i.f. lead is taken from the back of the crystal in this type of holder.

HYBRID RING

This may be formed by joining four strip lines at quarter wavelength intervals on to a ring having an impedance $\sqrt{2}$ times that of the lines and a mean circumference of one and a half wavelengths. In practice it is difficult to make the junctions as close or rings as narrow as this while maintaining uniformity, so that the spacing was increased and the ring impedance decreased. The lines taper to a lower impedance before entering the ring (see Fig. 5), to avoid the need for narrower width of ring to obtain impedance matching. In production, such tapers could be dispensed with and a small strip width used for the ring. The electrical path lengths round the ring are also shown. A signal arriving by arm A will split, and the power from these two paths will travel round the ring, arriving in phase at B and at D and out of phase at c; similarly a signal arriving by arm c divides at the junction with the ring and the signals are in phase at B and at D but out of phase at A. In addition, a signal from C arrives at B and D in phase whereas one from A arrives out of phase, hence, this may be used as a balanced mixer by feeding in signal and local oscillator via arms A and C and placing crystal holders at B and D.





At the design frequency, power injected at arm \land divides equally between arms B and D but in arm c the power was found to be 33dB down. Over a 12 per cent band, the difference in level at the two arms (B and D) was always less than 10 per cent of the mean level. The v.s.w.r. looking into arm \land , with the other three arms terminated in a matched load, was between 0.7 and 0.9 over a range of 1 000Mc/s in the X-band.

POWER DIVIDER

This takes the form of a Y junction with the three arms of equal impedance. It was found that any smooth transition from one line into two has a reasonably good v.s.w.r. and the precise final shape arrived at is, no doubt, only one of many. As is to be expected, the power division is equal when geometrical symmetry is maintained, provided the arms are loaded identically.

DIRECTIONAL COUPLER

A directional coupler of the type shown in Fig. 6 was also investigated and found to be quite satisfactory. The parameters which affect the coupling are the separation x

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of the two lines, the length L over which this separation is maintained and the angle θ between the arms. In general, when all arms are matched, power incident in arm (1) leaves arm (2), arm (3) and, to a much lesser extent, arm (4). The directivity, as measured by the difference in power level in arms (3) and (4), was found to depend mainly on the angle θ ; the smaller the angle, the greater the directivity and vice versa (up to 90°).

Over twenty different configurations were made and the coupling factor of each was measured, and the results are best seen by reference to Fig. 7. The curves must be asymptotic to the L = O axis and to the line giving a coupling of -3 dB, since, if the length L is great enough, the lines will share the power equally. The graph indicates that the greater the angle, the more critical is the length for small couplings. The effect of varying the



Fig. 8. A possible 3dB strip line coupler

separation x for a length L of 1.5 in was as follows:

xin	Coupling – dB
0.05	6
0.10	7
0.15	9

If a coupling closer than -10dB is required, it should also be possible to obtain it by using the type of directional coupler shown in Fig. 8. By varying the width d, a coupling of -3dB should be obtainable with this system.

COMPLETE R.F. SYSTEM

This is shown in Fig. 9 and is made up of the following parts:

Waveguide-to-strip line junction,

Directive feeds with 16dB and 19dB coupling,

Power dividers,

Short resistive loads,

Hybrid ring, and

Crystal holders.

Figs. 9 and 11 show sheets of Morganite resistive board used as loads. These resistive loads have the advantage that one can slide them along the line in order to see whether or not they are matched to the line in a manner similar to that adopted when checking resistive terminations in a waveguide.

The 3dB difference between the values of the forward coupling of the two directional couplers is illustrated in Fig. 11 where all the meters are seen to be giving approximately equal readings, though one coupler feeds only one crystal holder, while the other feeds two; the equality of power division brought about by the hybrid ring is also indicated by the readings on the two left-hand meters. It will

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Fig. 9. A complete r.f. system

be noted that one arm of this hybrid ring is not terminated in any way because so little power goes into this arm and that no termination is necessary. In a complete r.f. head this arm would carry the received signal.

Comparison With Waveguide

The small size of strip transmission line should give it many advantages over its waveguide counterpart. It is basically small and can be bent in comparatively small radii without causing undue reflection at the bend. Its total thickness need be only 0.125in, but for use on 3cm a working clearance above the strip of 0.5in is advisable.

For a strip 0-lin wide and having a dielectric 0-0625in thick, the ground plane should be about 1in wide. This means that for a single line a cross-section of $1 \text{in} \times 0.5 \text{in}$ is desired; this is approximately the same as a waveguide handling the same frequency. On the other hand, when a complete system is considered, the comparative space required can be made much less. The complete system shown in Fig. 9 could easily fit on to a sheet 6in square by reducing the length of the lines, and the components themselves are by no means as small as they could be. The resulting sheet could also be bent to suit the space available, providing the radius of bending were not too sharp.

Strip lines show a great saving in cost since a complete r.f. unit could be produced at a tooling cost of about £10 and each unit would then cost only a few shillings; even a very complicated system would not exceed this cost by very much. Appendages such as crystal holders would

Fig. 10. Method of measuring v.s.w.r.

Fig. 11. Strip line in operation



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be an additional expense but, even so, a considerable saving over waveguide costs is effected. Development costs also should be low, once the technique has been established and some of the theoretical investigations carried out. The saving in development cost is the result of the ease with which this line can be handled in the laboratory. When a waveguide component is modified, it often involves a considerable amount of accurate machining and reassembly, but a strip component can be modified in a few minutes by judicious scraping or by painting with conducting paint.

Losses in Strip Transmission Line

Using polystyrene as the dielectric and conducting paint for both ground plane and strip, the measured loss along the line was 4.5dB/m. This method of manufacture involves a d.c. resistance of about $10\Omega/m$ in the strip and $1.5\Omega/m$ in the ground plane and the r.f. loss would be less if both strip and plane were made of copper sheet or some other good conductor.

The effect of placing a resistive carbon film of $400\Omega/$ square resistivity on top of the strip is to increase the loss along the line to the order of 200dB/m, thus a film of lower resistivity can be used to make short matched terminations.

From normal transmission line theory, the loss due to the dielectric is

$$\alpha_{d} = 4.34 \ GZ_{\circ} \ dB/m$$

$$= 4.34 \ \omega CZ_{\circ} \ tan\delta \ dB/m$$

$$= \frac{4.34 \ \omega V \epsilon}{3 \ \times \ 10^{-8}} \ tan\delta \ dB/m$$

$$= 9.09 \ \times \ 10^{-8} \ f(V \epsilon) \ tan\delta \ dB$$

Hence, for any line with polystyrene dielectric in which $\epsilon = 2.55$ and tan $\delta = 0.0005$, the dielectric loss at, say, 9kMc/s will be 0.65dB/m. This value will be modified because the wave travels partly in air and partly in dielectric.

/m

In addition to this loss, there will be a resistive loss α_r . The theory gives $\alpha_r = 4.34 \ R/Z_0 \ dB/m$, but the r.f. resistance R is not easily measurable. In most coaxial lines, α_r is of the same order as α_d which implies a total loss of about 1.3dB/m.

This discrepancy between this value and the measured value of 4.5dB/m is probably caused by greater resistive losses. Approximate measurements of the power radiated from the strip were made with the aid of a matched horn pick-up. The radiation per inch of line, from the line alone, was found to be approximately 60dB down on the power carried. At the waveguide-to-strip junction, the measured radiation was 45dB down on the power carried by the junction. Designing these junctions for low radiation would doubtless improve this value. No excessive radiation was found to come from a crystal holder junction.

Production and Tolerances

The production method best suited to strip line is that used for other printed circuits. A large scale black and white drawing of the complete layout could be reduced photographically to the size required. This would then be used in a photo-etching process for prototype production, any corrections being made on the large-scale original.

Tolerances, which do not appear to be very tight, should cause no difficulty since there is no reason why every article made should not be identical, so that, once the prototype has been corrected, no further trouble should occur.

Production of crystal-holder and coaxial-cable or waveguide junctions would still use standard methods, but these should generally be very simple. When used in a complete system having printed circuits elsewhere, the extra cost of the strip line becomes comparatively negligible.

Applications

The main applications of strip line are in cases where low cost is of more importance than perfect performance. In conjunction with a specially designed klystron or klystron cavity and an aerial of dipole, helical or possibly strip line type, the cost of the entire r.f. unit would be enormously reduced compared with that of the equivalent waveguide system. Moreover, as has already been explained, the trouble with the high tolerances normally required on such equipment would largely be obviated.

In most applications, the problem of the choice of a dielectric arises. For general low-temperature specifications, copper or silver on polystyrene is an obvious choice and, when tropical or more severe specifications have to be met. the choice of dielectric material becomes the limiting factor. In order to minimize the dielectric losses of the line, the choice is necessarily limited and, at high temperatures, this leaves only such dielectrics as p.t.f.e or the American plastic Kel F; these are somewhat waxy in nature and may involve some bonding troubles during manufacture. By the use of p.t.f.e. the melting point of the dielectric is no longer a limiting factor in high-temperature operation.

Conclusions

From the experimental work carried out, it would appear that strip transmission lines may be used to advantage in certain microwave systems. In their present form, they are limited to those systems in which loss is not of prime importance and in which a certain amount of stray radiation can be tolerated. Another factor which may need to be considered is that of noise, particularly if the system is to be subject to vibration, but no measurements have been carried out by us and no reference to any such measurements has been found.

An advantage of strip line is the comparative ease of manufacture and design and, hence, its low cost. Moreover, the space occupied by a strip system could be considerably less than that occupied by a similar waveguide system. At present the main disadvantages are the absence of a klystron specifically designed for connexion to a strip line and the lack of any suitable method of making aerials. At the moment, both the input and output are necessarily of waveguide or coaxial cable form and this implies the use of junctions, which tend to narrow the bandwidth available with this technique. However, these disadvantages are expected to be overcome in the very near future.

Acknowledgment

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REFERENCES

- 1. ARDITI, M. Experimental Determination of the Properties of Microstrip Components. Conv. Inst. Radio Engrs., Pt. 10, 27 (1953).
- GRIEG, D. D., ENGELMAN, H. F. Microstrip, A New Transmission Technique. Proc. Inst. Radio Engrs. 40, 1644 (1952).
- 3. BARRETT, R. M. Etched Sheets Save as Microwave Components. *Electronics* 25, 114 (1952).
- BARNES, M. H., BARRETT, R. M. Microwave Printed Circuits. Radio Televis. News 46, 1631 (1952).
- ASSADOURIAN, F., RIMAI, E. Simplified Theory of Micro-Strip Transmission Systems. Proc. Inst. Radio Engrs. 40, 1651 (1952). KOSTRIZA, J. A. Microstrip Components. Proc. Inst. Radio Engrs. 40, 1658 (1952).

The Recording of High-Speed Single Stroke Electrical Transients

(Part 1)

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The article summarizes developments which have taken place in the recording of high speed single stroke electrical transients since the last review paper on this subject was published by Miller and Robinson in 1936.

Three aspects have been considered, namely, the progress in design of cathode-ray tubes, development of the design of the auxiliary circuits and improvements in photographic techniques.

No substantial changes in oscillograph technique with conventional continuously-pumped tubes of the cold-cathode type have occurred during the period of the review. The subject matter is, therefore, centred on the development of the techniques involved with high speed sealed-off cathoderay tubes and with the continuously evacuated micro-oscillographs.

A DETAILED account of the growth of cathode-ray oscillography in general was given in 1925 by MacGregor-Morris and Mines¹ and the further development of high-speed oscillography in particular was described in 1936 by Miller and Robinson². It is the object of this article to review the progress made during the past 20 years or so in the field of high speed transient oscillography.

Three aspects are considered, namely, progress in the design of high speed oscillograph tubes, development in the design of the auxiliary circuits and improvements in the photographic technique.

The continuously pumped form of cathode-ray tube, although used extensively in the past, has now largely been replaced by the more convenient sealed-off tube. No substantial change in oscillograph technique with continuously pumped tubes of the cold-cathode type has taken place since the last review paper³ was written. Oscillographs of this type are described elsewhere^{3,4,5} and are not, therefore, considered in detail here.

THE OSCILLOGRAPH WITH CONTINUOUSLY-PUMPED CATHODE-RAY TUBE WITH COLD CATHODE

For historical interest it may be recalled that the first oscillograph cathode-ray tube was constructed in 1897 by Braun⁶, but it was suitable only for low-speed work. The specialized technique of high-speed oscillography began in 1914 when Dufour^{7,8} published the design of his modified form of cathode-ray tube which initiated a considerable amount of work by a number of investigators. The consequently rapid development culminated in practical constructions of continuously-pumped cold-cathode tubes operating with high accelerator voltage and internal recording of the trace on photographic emulsion, examples of which have been described by Burch and Whelpton³, Miller and Robinson⁴ and again in an E.R.A. Report⁵.

THE OSCILLOGRAPH WITH SEALED-OFF CATHODE-RAY TUBE WITH HOT CATHODE

The advent of thermionic cathodes stimulated a trend to simplify the design and operation of high-speed oscillographs by using sealed-off cathode-ray tubes. The first sealed-off cathode-ray tube with hot-cathode was made in 1918 by Samson⁹ but was, however, unsuitable for any oscillograph work because of an unsatisfactory design of the electron gun. In 1929 George¹⁰ described a continuously-pumped tube with hot-cathode which, on account of its electron-optical design features (namely an electric immersion lens at the cathode and an electric final focusing lens), may be regarded as the forerunner of the practical sealed-off construction which began to emerge in subsequent years.

The first report on the use of a commercially available type of hot-cathode sealed-off tube with electric final focusing in a high-speed single-stroke transient oscillograph was given in 1938 in a detailed description by McGillewie¹¹. Electric final focusing appeared also in the sealed-off tube specially constructed in 1939 for high-speed single-stroke transient oscillograph work by Katz and Westendorf¹² Considerable improvements in the technique of recording high-speed single-stroke transients with hot-cathode sealed-off tubes were made more recently by Bauer and Nethercot¹³ in 1949 and subsequently in 1950 by White¹⁴, in both cases the tubes used are of a commercially available type with electric final focusing.

Although magnetic final focusing has been used for many years in continuously-pumped high-speed transient oscillograph tubes, it does not appear to have been applied to sealed-off tubes for high-speed transient oscillographs until fairly recently¹⁵.

HIGH-SPEED OSCILLOGRAPH TUBES WITH POST-DEFLEXION ACCELERATION

High velocity of the beam electrons when impinging upon the screen is one of the principal factors underlying the high writing-speed performance of transient oscillograph tubes. Post-deflexion acceleration, or intensification as it is also sometimes called, is the principle applied to obtain a tube performance which combines the otherwise conflicting conditions of high electron velocity, and thus high writing speed, and high sensitivity of beam deflexion. The principle of post-deflexion acceleration was first applied in 1928 by Sommerfeld¹⁶ in a continuously-pumped tube. In 1938 Bigalke¹⁷ reported the construction of a sealed-off hot-cathode tube with post-deflexion acceleration, a development rapidly followed by a number of further constructions^{18,19,20,21,22}.

HIGH-SPEED OSCILLOGRAPH TUBES WITH HIGH-VOLTAGE SIGNAL DEFLECTOR

In certain high-voltage investigations the customary potential divider with its attendant errors may be dispensed with if special designs of deflector systems of very low

^{*} Formerly University of Manchester, now with Brush Electrical Engineering Company Limited. † Formerly University of Manchester, now with Electrical and Musical Industries Limited.

sensitivity are employed. A continuously-pumped tube with a signal deflector of this type was described in 1933 by Messner²³. In 1936 Nuttall²⁴ reported a modification of the deflector to a dual deflector system in a continuously pumped tube, one signal deflector of low deflexion sensitivity being provided for high-voltage signals and the second signal deflector of higher deflexion sensitivity for signals of lower voltage. Sealed-off tubes with very low deflexion sensitivities for the direct recording of high voltage signals have been constructed recently by Hergenrother and Rudenberg²⁵ and with a dual deflector system by Feinberg et al¹⁵.

TRACE DISTORTION

The faithfulness of the waveform of the trace is limited by the cut-off frequency of the signal deflector of the oscillograph tube which, in turn, is determined by the transittime of the beam electrons in the deflector field and by the signal distortion caused by the impedance of the deflector leads and the impedance mismatch between deflector and signal line. Investigations into trace distortion due to electron transit time were made in 1925 by MacGregor-Morris and Mines¹ and subsequently by Hintenberger²⁶, Hollmann²⁷, and Klemperer and Wollf²⁸.

The effort to increase the cut-off frequency of signal deflexion has moved along two distinct and independent lines of approach, one of which is characterized merely by a modification of the deflector design. In 1945 Rudenberg²⁹ analysed theoretically the improvement in cut-off frequency to be achieved with a twin-wire deflector. To overcome the low deflexion sensitivity of the straight twin-wire deflector, Bartlett and Davies³⁰ suggested in 1946 a folded twin-wire deflector, thus introducing the principle of travelling-wave deflexion. The construction of a tube of this type was reported in 1950 by Owaki et al³¹. In 1952 Smith et al³² described a tube with a distributed helix as signal deflector and with post-deflexion acceleration to give increased writing speed.

The other line of approach to the problem of increasing cut-off frequency is based on a radical change of design of the oscillograph tube, where the physical size of the deflectors and hence electron transit time and signal deflector capacitance are considerably reduced. The so-called microoscillograph was first constructed in 1939 by von Ardenne³³, and was further developed in 1946 by Lee³⁴ and again in 1954 by Fert et al³⁵.

Design Requirements for High Speed Single Stroke Transient Oscillograph Cathode-Ray Tubes With Hot-Cathode

There are four fundamental features of performance to be achieved in the design of a cathode-ray tube with hotcathode for high-speed single-stroke transient oscillography. These apply both to tubes of the sealed-off type with a luminescent screen and external photographic recording which are used in the customary types of oscillographs, and to those of the continuously pumped type with internal trace recording on photographic emulsion used in the recently established micro-oscillograph technique.

The four common features of tube performance are:

(1) High kinetic energy of the beam electrons when impinging upon the luminescent screen or on the photographic emulsion, respectively, in order to achieve deep electron penetration.

(2) Small spot size of the electron beam combined with a high current density to give high temporal resolution of the trace, and thus high maximum writing speed of the tube. (3) A signal deflector designed to give the desired uniform frequency response.

(4) Means to prevent interference in the deflector systems between the signal line and time-base generator circuits.

The problems of obtaining high kinetic electron energy and, at the same time, a small spot size combined with high current density, may be resolved by electron-optical considerations in the design of the electron gun. The high electron velocity necessary to produce the required kinetic energy is obtained in a straight electron gun by the use of a high accelerator voltage, and in a gun with postdeflexion acceleration by a high post-deflexion acceleration voltage. Guns with post-deflexion acceleration are usually designed with electric final focusing, whereas straight electron guns may be obtained with either electric or magnetic field focusing. Despite the fact that electron guns with a magnetic final focusing lens will usually produce a finer spot than those with electric focusing, the latter system is frequently employed in the construction of this type of cathode-ray tube.

Two factors must be considered if the frequency response of the oscillograph is to be uniform up to a high value of frequency. In the first instance, care has to be taken that the deflector leads are designed with uniformly distributed capacitance and inductance values so that the characteristic impedance of the line is sensibly constant along its length and reflections are, thereby, avoided. Secondly, consideration has to be given to the transit time of the beam electrons in the deflector field.

Interference between the signal line and time-base generator circuits is largely caused by a capacitive coupling between the signal deflector and time-base deflector plates. This capacitive coupling, and hence the magnitude of interference between the two circuits, can be minimized by providing a screen between the two deflector plate systems.

The design of the luminescent screen is a feature applying only to tubes of the sealed-off type. The screen forms the link between the electronic and photographic sections of the oscillograph and has, therefore, to be an efficient convertor of electron beam energy such that the light energy produced is of suitable wavelength to give an optimum actinic effect on a high speed photographic emulsion. The cathodoluminescent efficiency of the screen is a function of a number of factors such as the chemical composition of the screen phosphor, the method of its preparation, its crystal structure, particle size and temperature, the method of screen laying, absence or presence of metal backing of the screen (usually in the form of a thin aluminium film), and the kinetic energy of the beam electrons as well as the magnitude of beam current density and the velocity of the motion of the spot across the screen^{36,37,38,39}. Silver-activated zinc-sulphide giving a blue light of high actinic power has been found to be the most suitable screen material for the photographic recording of high-speed transients⁴⁰.

Cathode-Ray Tubes

CONVENTIONAL HOT-CATHODE SEALED-OFF MACRO-OSCILLO-GRAPH TUBES

A typical contemporary example of the more successful macro-oscillograph tubes of the straight-acceleration type available in this country is the G.E.C. type 908BCC, shown in Fig. 1(a). This tube has a spot size of the order of 0.2 to 0.25mm diameter at an accelerating voltage of 10kV and a typical maximum writing speed under these conditions of about $3 \times 10^{\circ}$ cm/sec. The deflector sensitivities are of the order of 7.5×10^{-2} and 4×10^{-2} mm/V so that signals of not less than about 100V are required in order to produce

a reasonable deflexion, together with a sweep voltage of about 2.5 kV.

Writing speeds in excess of 10° cm/sec with a spot size of about 0.4mm have been obtained⁴¹ with a Ferranti type 06/3P sealed-off tube operated at 10kV. The signal deflexion sensitivity at this voltage was about 4×10^{-2} mm/V. This tube has also been used with accelerating voltages of up to 30kV with a corresponding increase in writing speed up to an estimated maximum of something in excess of 3×10^{10} cm/sec⁴¹.

The G.E.C. high-speed oscillograph tube type 1608BCCA is an example of the post-deflector accelerator technique and is shown in Fig. 1(b). A typical spot size, claimed by the makers, is 0.45 to 0.5mm diameter and a maximum writing speed of 10° cm/sec, in consequence is somewhat less than that obtained with the type 908BCC tube. The deflexion sensitivity, however, is some three times greater and



(a) (b) Fig. 1(a). G.E.C. cathode-ray tube type 908BCC. (b) G.E.C. type 1608BCCA with post deflexion acceleration

reasonable oscillograms can be obtained with signal voltages of about 40V and a sweep voltage of about 700V.

The physical limitations imposed on the design of the post-deflector electrodes results in some non-uniformity of sensitivity over the tube face. Robinson and Allen⁴² have described how these errors may be allowed for in the interpretation of an oscillogram by photographing a series of orthogonal equipotentials, but this is only accurate at low frequencies. Hollmann²⁷ has suggested a method of calibration at high frequencies which involves tiny hairpin cathodes functioning as peak valve voltmeters mounted close to the deflector plates.

Many oscillograph tubes both with and without postdeflector accelerator electrodes have been described in the literature^{43,44} and have performances much the same as the tubes described above.

TUBES WITH HIGH-VOLTAGE DEFLECTOR SYSTEMS

An alternative to the use of potential dividers with oscillograph tubes of conventional design for the display of high voltage transients is to employ oscillograph tubes which have very low deflexion sensitivities^{23,24}.

The high voltage deflector system of such a tube, described recently by Hergenrother and Rudenberg²⁵ has a

cylindrical high voltage electrode of 5 to 6mm diameter enclosed on three sides by an earthed screen of about 30mm diameter. An additional earthed electrode is arranged opposite the open slot and so shaped that the leakage field between this and the high voltage electrode produces full scale deflexion of the electron beam with a signal voltage of about 100kV. The oscillograph was designed to operate with unidirectional signals and the deflector system was accordingly offset from the tube axis in order to fully utilize the screen area. The tube was magnetically focused and a deflecting coil was used to hold the stationary beam at the side of the tube. Operating at an accelerating voltage of 18kV the time sweeps range from 0.2 to $20\mu \text{sec/cm}$. No details of writing speed, spot size, or the bandwidth of the deflector system are given.

A low sensitivity tube of somewhat similar design¹⁵ has a dual deflector system so that both high and low voltage pulses can be displayed.

TUBES WITH TRANSMISSION LINE DEFLECTOR SYSTEMS

The macro-oscillograph tube with the conventional deflector system is unsuitable for the recording of very high speed transients of, say, less than 10^{-s} sec duration because of signal distortion due to transit time effects and the lumped impedance presented by the deflector plates.

When the time taken by an electron to pass through the deflector plates becomes of the same order of magnitude as the time duration of the applied transient (or of the periodic time of a continuous wave) an appreciable reduction in sensitivity occurs. MacGregor-Morris and Mines¹ and later Hollmann⁴⁵ give as the ratio of deflexion A_{ω} at a frequency ω to the d.c. deflexion A_{0}

$$A_{\omega}/A_{\circ} = \frac{\sin \omega T/2}{\omega T/2}$$

where T is the transit time of the electrons through the deflecting region. This expression is valid only when the plates are closely spaced in relation to their length and are small compared to the wavelength of the deflecting voltage, end effects ignored and the deflector plates considered to be equipotentials^{26,28}. Thus single frequency waves are attenuated, but undistorted in waveshape whereas transients are both attenuated and distorted.

A later investigation²⁷ of the effect of entrance and exit stray fields proved that the reduction in sensitivity was more pronounced for a more non-uniform field.

The transit time for a G.E.C. tube type 908BCC is about 7×10^{-10} sec so that theoretically the deflexion becomes zero at a frequency of about 1 500Mc/s and is attenuated by 15 per cent at 500Mc/s. The G.E.C. type 1608BCC with a transit time of about 1.4×10^{-9} sec has a zero deflexion at a frequency of about 750Mc/s and an attenuation of about 55 per cent at 500Mc/s.

TWIN-WIRE DEFLECTOR SYSTEM

A twin-wire deflector system has been proposed by Rudenberg²⁹ which results in an appreciable reduction in electron transit time and, at the same time, presents to the electrical transient a distributed impedance in the form of a transmission line. It is thus possible to match the deflector system with the connecting leads and test apparatus and to terminate the line in its characteristic impedance at the far side of the tube.

Such a tube has been constructed¹⁵ with which a writing speed of 6.6×10^{10} cm/sec has been recorded. Over a central screen area of about 4cm diameter a writing speed of 4×10^{10} cm/sec was obtained with a spot size 0.2mm, which corresponds to a writing speed of 2×10^{12} spot widths per second. The deflector system was estimated to

have a d.c. sensitivity of 7×10^{-3} mm/V at 25kV and about 30 per cent of this value at 10 000Mc/s.

A similar system has been constructed independently by Mackay⁴⁶, who used a twin tape so shaped that an expanded time scale was obtained. This resulted in an increased temporal resolution without the corresponding reduction in trace density.

TRAVELLING WAVE DEFLECTOR SYSTEMS

The low deflexion sensitivity of the twin wire deflector system described in the previous section limits the application of this tube to problems in which large deflector voltages (several kV) are available.

By folding such a transmission line so that the velocity of the signal in the direction of the beam is equal to the velocity of the beam electrons, the principle described by Haeff⁴⁷ of arranging a deflecting field to travel in phase with an electron beam may be utilized to give an increased deflexion sensitivity.

A cathode-ray tube deflector system designed on this principle, as proposed by Bartlett and Davies³⁰, was constructed by Owaki et al³¹ who estimate that the sensitivity would be constant to frequencies of the order of 20 000Mc/s falling to 30 per cent at 100 000Mc/s. However, radiation and reflections at the glass seals were ignored, as were the effects of finite line length and the transit time through one fold, and it is certain that these will have an appreciable effect on the deflexion sensitivity at such high frequencies. The tube was used at an accelerating potential of 2.5kV for the observation of Lissajous figures produced by transit distortion of the second kind⁴⁵ and for frequency analysis of continuous waves by Hollmann's method of inversion spectrograms⁴⁵. Its application to the study of transient phenomena was indicated, but no practical investigations were made.

The deflector system described by Smith et al³² consists of a helix flattened on one side and fabricated from 1cm metal strip, mounted inside a cylinder similarly flattened, with the electron beam passing through the gap between the flattened portions. The line so formed has a characteristic impedance of 50Ω and the transit time per turn of the helix is 0.17×10^{-9} sec at an electron velocity corresponding to an acceleration voltage of 10kV. The reduction in deflexion sensitivity so obtained was about 5 per cent at 1000 Mc/s. In order to give increased brightness, a post-deflexion acceleration voltage of 25kV was used and the resulting deflexion sensitivity was about 4V/mm with a spot size of 0.15mm. The writing speed was claimed to 10^{11} spot widths per second or $1.5 \times 10^{\circ}$ cm/sec.

Hollmann⁴⁸ proposed a travelling wave deflector system in which three pairs of deflector plates, of such a length that the transit time through one pair is equal to half the period of the sine wave to be displayed, were connected so that the deflexions produced by each are additive. The range of frequencies over which such a scheme can be used is rather small and it is therefore of limited use for transient analysis.

MICRO-OSCILLOGRAPH TUBES

A reduction in the size of the deflector system will, necessarily, result in a low deflexion sensitivity, but by reducing the diameter of the electron beam the overall resolution can compare favourably with the more conventional tubes if microscope techniques are used to view the resulting oscillograms.

Von Ardenne³³ constructed a two-beam micro-oscillograph in which the spot size was reduced to about 10^{-3} mm by careful design of the magnetic focusing assembly. The deflector plates were reduced in size by about a factor of 10, and the resulting oscillograms by a factor of 100. A six-beam tube of more recent design⁴⁹ incorporates electric and magnetic focusing, and has a spot diameter of 10^{-2} mm, but the oscillograph has a spring driven camera as timebase and is therefore unsuitable, in this form, for the recording of high-speed transients.

Lee³⁴ used these principles in the design of a three-beam continuously evacuated oscillograph. Each deflecting system has plate dimensions 5×4 mm with a spacing of 3.6mm and is separately screened. To assist in the production of a very small spot size the focusing lens was arranged between the deflector plates and the photographic emulsion.

The calculated reduction in sensitivity due to the transit time effect given by Lee has since been shown by Hollmann²⁷ to be in error, and should be reduced by about 50 per cent at 10 000Mc/s. This revised estimate may still require modification, since at such high frequencies several effects such as resonances of local cavities, radiation fields

	TABLE 1	
Characteristics	of some High Speed	Oscillograph Tubes.

							the second se	
	1 G.E.C. 908всс	-2 FERRANTI 06/3p (at 15kV)	3 FERRANTI 06/3p (at 25kV)	4 g.e.c. 1608bcca	5 G.E.C. VCRX357A	6 LEE'S OSCILLO- GRAPH ³⁴	7 smith's oscillo- graph ³²	8 Ferranti Type 06/20p
Pre-deflexion accelerating voltage (kV)	10	15	25	3	3	50	10	25*
Post-deflexion accelerat- ing voltage (kV)			_	7	5		25	.—
Signal deflector imped- ance (ohms)		-	_		-	-	50	250
Spot size (mm)	0.2-0.25	0.3	0.5	0.45-0.5	0.9	0.01	0.15	0.5
Useful screen diameter (spot widths)	400	420	630	350	100	about 180	about [*] 150	630†
Maximum writing speed (spot widths/sec $\times 10^{\circ}$)	130	1,20	700	20	. 2	200	100	2,000
Signal deflector sensitivity (volts/spot width)	3	10	11	2	2.8	10	0.6	30

* The manufacturer's recommended maximum working voltage is 20kV for this

† 200 at maximum writing speed.

and the fact that the deflector plates are not equipotentials, have been neglected.

Lee obtained recordings of a single sweep, 10 000Mc/s oscillation but the quality of the oscillogram was not good and showed some evidence of non-sinusoidality, due probably to overlapping of the focusing and deflecting fields. The sensitivity of the tube was about 10⁻³mm/V and spot size 10⁻²mm at an accelerating voltage of 50kV. The maximum writing speed under these conditions was $2 \times 10^{\circ}$ cm/sec, or 0.2 spot widths per pico-second.

A more recent design has been described by Fert et al³⁵ in which magnetic focusing is again used to give a small spot size. The tube is continuously evacuated and has an accelerating voltage of 50 to 60kV. The four beams are deflected independently in the vertical direction by an electric system and have a common magnetic deflecting system in the horizontal direction. A writing speed at the film of $6mm/\mu sec$ is claimed.

Pierce⁵⁰ has described an electrode construction in which the several deflector plates form the lumped capacitances of a low-pass filter. The plates are connected by inductance and the network terminated within the tube by its low frequency characteristic impedance of 75Ω . The system was designed to have a cut-off frequency of 1330Mc/s, but due to a change in characteristic impedance with frequency and insufficiently close matching of components, the response is considered uncertain much above 500Mc/s. A deflexion sensitivity of 1.3V/mm with a spot size of 0.02mm was claimed by Pierce, but because of the low beam current and accelerating voltage (2kV) the tube was found suitable only for the recording of recurrent waveforms.

The characteristics of a number of high speed oscillograph tubes are summarized in Table 1.

REFERENCES

- 1. MACGREGOR-MORRIS, J. T., MINES, R. Measurements in Electrical Engineering by Means of Cathode-Rays. J. Instn. Elect. Engrs. 63, 1056 (1925).
- (1925).
 MILLER, J. L., ROBINSON, J. E. L. The High-Speed Cathode-Ray Oscillograph. Rep. Phys. Soc. Progr. Phys. 2, 259 (1936).
 BURCH, F. P., WHELPTON, R. V. The Technique of the High-Speed Cathode-Ray Oscillograph. J. Instn. Elect. Engrs. 71, 380 (1932).
 MILLER, J. L., ROBINSON, J. E. L. The Design and Operation of a High-Speed Cathode-Ray Oscillograph. J. Instn. Elect. Engrs. 74, 511 (1934).
 Surge Phenomena. Rep. Brit. Elect. Res. Ass. S/T35, 146 (1941).
 BRAUN, F. Über ein Verfahren zur Demonstration und zum Studium des Zeitlichen Verlaufes variabler Strome. Ann. Phys. Chem., Lpz. 60, 552 (1897).
 DUFOUR, Z. Sur un Oscillographe Cathodique. C.R. Acad. Sci., Paris, 158, 1339 (1914).

- 7. DUFOUR, Z. 1339 (1914). DUFOUR, A. Oscillographe Cathodique. J. Phys. Radium 1, 147 (1920).
 SAMSON, C. Uber ein Braunsches Rohr mit Clubbertein.

- 10.
- DUFOUR, A. Oscillographe Cathodique. J. Phys. Radium I, 147 (1920). SAMSON. C. Uber ein Braunsches Rohr mit Gluhkathode und einige Anwendungen desselben. Ann. Phys., Lpz. 55, 608 (1918). GEORGE, R. H. A New Type of Hot Cathode Oscillograph and its Application to the Automatic Recording of Lightning and Switching Surges. Trans. Amer. Inst. Elect Engrs. 48, 884 (1929). MCGILLEWIE, D. I. The Use of the High Vacuum Cathode-Ray Tube for Recording High-Speed Transient Phenomena. J. Inst. Elect. Engrs. 83, 657 (1938).

- KATZ, H., WESTENDORF, E. Erreichung hoher Schreibgeschwindigkeiten mit einer abgeschmolzenen rein elektrostatisch arbeitenden Braunschen Rohre. Z. Tech. Phys. 20, 209 (1939).; 22. 121 (1941).
 BAUER, R. E., NETHERCOT, W. A New Oscillograph for the Recording of Very Fast Electrical Transients. Rep. Brit. Elect. Res. Ass. U/T.115 (1949).
 WHITE, E. L. The Design, Construction and Performance of a New High-Speed Cathode-Ray Oscillograph for Recording High Voltage Surges. Rep. Brit. Elect. Res. Ass. SIT70 (1950).
 FEINBERG, R., JACKSON, B., HARDY, D. R. Design Features in Cathode-Ray Tubes for the Recording of High Speed Transients. (To be published.)
 SOMMERFELD, E. Uber einen Kathodenoszillographen hoher Spannungs-empfindlichkeit. Arch. Elektrotech. 20, 607 (1928).
 BIGALKE, A. Nachbeschleunigungs-Elektronenstrahl-Oszillograph. Z. Tech. Phys. 19, 163 (1938).
 DE GIER, J. A Cathode-Ray Tube with Post-Acceleration. Philips Tech. Rev. 5.

- DE GIER, J. A Cathode-Ray Tube with Post-Acceleration. Philips Tech. Rev. 5, 245 (1940).
- CHRISTALDI, P. S. Cathode-Ray Tubes and Their Applications. Proc. Inst. Radio Engrs. 33, 373 (1945).
 LEMPERT, I. E., FELDT, R. The 5 RP Multiband Tube: An Intensifier-Type Cathode-Ray Tube for High Voltage Operation. Proc. Inst. Radio Engrs. 34, Cathode-Ray Tube for High Voltage Operation.
- Cathode-Ray Tube for High Voltage Operation. Proc. Inst. Radio Engrs. 34, 432 (1946).
 WHITE, W. G. Cathode-Ray Tubes with Post-Deflexion Acceleration. Electronic Engng. 21, 75 (1949).
 ALLARD, L. S. An "Ideal" Post-Deflexion Accelerator C.R.T. Electronic Engng. 22, 461 (1950).
- MESSNER, M. Ein Kathodenstrahloszillograph zur unmittelbaren Aufnahme hoher Gleich-und Wechselspannungen. Arch. Elektrotech. 22, 335 (1933).

- NUTTALL, A. K. A Cathode-Ray Oscillograph for the Direct Measurement of High Voltage Transients. J. Instn. Elect. Engrs. 78, 229 (1936).
- HERGENROTHER, R. C., RUDENBERG, H. G. A Direct-Reading Oscilloscope for 100kV Pulses. Proc. Instn. Radio Engrs. 41, 896 (1953).
 HINTENBERGER, H. Über das Verhalten der Braunschen Rohre bei sehr hohen Frequenzen. Z. Tech. Phys. 18, 256 (1937).
- 27.
- HOLLMANN, H. E. The Dynamic Sensitivity and Calibration of Cathode-Ray Oscilloscopes at Very High Frequencies. Proc. Inst. Radio Engrs. 38, 32 Oscillo: (1950).
- KLEMPERER, H., WOLFF, O. Die Verzerrungen im Kathodenoszillographen bei hohen Messgeschwindigkeiten. Arch. Elektrotech. 26, 495 (1932). 28. 29.
- RUDENBERG, H. G. Deflexion Sensitivity of Parallel-Wire Lines in Cathode-Ray Oscillographs. J. Appl. Phys. 16, 279 (1945). BARTLETT, J. G., DAVIES, G. T. The Design of High-Speed Oscillographs. J. Instn. Elect. Engrs. 93, Pt. 3A, 1304 (1946). 30.
- OWARI, K., TERCHATA, S., HADA, T., NAKAMURA, T. The Travelling Wave Cathode-Ray Tube. Proc. Inst. Radio Engrs. 38, 1172 (1950).
 SMITH, S. T., TALBOT, R. V., SMITH, Jr., C. H. Cathode-Ray Tube for Recording High-Speed Transients. Proc. Inst. Radio Engrs. 40, 297 (1952).
- VON ARDENNE, M. Der Elektronen-Mikrooszillograph. Hochfrequenztechnik u Elektroakustic. 54, 181 (1939).
- LEEKTOAKUSIC. 24, 161 (1939).
 LEE, G. M. A. Three-Beam Oscillograph for Recording at Frequencies up to 10 000 Megacycles. Proc. Inst. Radio Engrs. 34, 121W (1946).
 FERT, J., LAGASSE, J., OLLE, J. Sur la Réalisation d'un oscillographe cathodique a plusieurs faisceaux. C.R. Acad. Sci., Paris, 238, 59 (1954).
 HOPKINSON, R. G. An Examination of Cathode-Ray Tube Screen Characteristics. J. Instn. Elect. Engrs 93, Pt. 3A, 779 (1946).
 Roy A. G. Schutter, and Science and With Cothode Ray Electronic Science and Science an
- BRIL, A. On the Saturation of Fluorescence with Cathode-Ray Excitation. *Physica*. 15, 361 (1949). 37.
- BRIL, A., KLASENS, H. A. Intrinsic Efficiencies of Phosphors under Cathode-Ray Excitation. *Philips Res. Rep.* 7, 401 (1952). 38.
- 39.
- Hull, C. G. A. Applied Cathodoluminescence. Brit. J. Appl. Phys. Supplement No. 4. S6. (1955).
 HOPKINSON, R. G. The Photography of Cathode-Ray Tube Traces. J. Instn. Elect. Engrs. 93, Pt. 3A, 808 (1946).
 JACKSON, B. The Recording of High-Speed Electrical Transient Phenomena M.Sc. Thesis, Manchester University (1953). 40
- 41.

- M.Sc. Thesis, Manchester University (1953).
 42. ROBINSON, G., VAN ALLEN, R. Precision Measurements with a Cathode-Ray Oscilloscope. Rev. Sci. Instrum. 23, 701 (1952).
 43. CRAGGS, J. D., MEEK, J. M. High Voltage Laboratory Technique. (Butterworth, 1954).
 44. LEWIS, I. A. D., WEELS, F. H. Millimicrosecond Pulse Techniques. (Pergamon Press, 1954).
 45. HOLLMANN, H. E., Braun Tube at Very High Frequencies. Hochfrequenz-technic u Elektroakustic 40, 97 (1932).
 46. MACKAY, D. M. Some New Aspects of High-Speed Oscillography. Proc. Phys. Soc., Lond. 61, 235 (1948).
 47. HAFFE, A. V. Device for Detection and Measurement of High Frequency.
- HAEFF, A. V. Device for Detection and Measurement of High Frequency Currents. U.S. Patent 2 064 469 (1936).
- 48. HOLLMANN, H. E. The Travelling Wave Cathode-Ray Tube. Proc. Inst. Radio Engrs. 39, 194 (1951).
- VON ARDENNE, M. A Six-Trace Cathode-Ray Micro-Oscillograph. Wireless Engr. 19, 231 (1942). 50. PIERCE, J. R. Travelling Wave Oscilloscope. Electronics 22, 97 (Nov. 1949).

(To be continued)

Royal Society Antarctic Expedition

The MV "Tottan" sailed from Southampton on 22 Novem-ber with the ten men who form the advance party of the Royal Society International Geophysical Year Antarctic Expedition¹. The main object is to prepare for the installation of a scientific-research station on the Antarctic mainland, on the shores of the Weddell Sea, south of the Fakkland Islands. The expedition commander, Surgeon Lieutenant-Commander David Dalgliesh, R.N., will carry instructions to reach, if possible, the Vahsel Bay region (78°S, 35°W) which is one of great scientific interest for geophysical studies. If this advance party succeeds in for geophysical studies. If this advance party succeeds in for geophysical studies. If this advance party succeeds in erecting the research station this year a further party, 18 strong, consisting mainly of scientific research workers, will sail a year hence, to relieve the advance party, and complete the Royal Society's Antarctic Station for the International Geophysical Year (July 1957—December 1958). It is hoped that this station will be established in close proximity to the Weddell Sea base of the Trans-Antarctic December 1951 in the month or how MY

Expedition which left earlier in the month on board MV "Theron". The "Tottan" will sail via the Cape Verde Islands. Rio de Janeiro and her final call will be in South Georgia. She is expected to arrive in the Weddell Sea a few days after the "Theron". The station will be one of 22 International Geophysical Stations being set up on the Antarctic mainland itself by the Commonwealth, the United States of America, Russia and other countries.

America, Russia and other countries. The three scientists in the advance party are Major G. E. Watson, R.E.M.E., an electronic engineer loaned by the War Office. Dr. Stanley Evans, a radio-astronomer from the Jodrell Bank Experimental Station, Manchester University, and Mr. D. W. S. Limbert, of the Meteorological Office, Harrow. The party will send back reports on the operation of their equipment in the extreme cold to guide the preparations for the main expedition in 1957.

the main expedition in 1957.

United Kingdom Programme in the International Geophysical Year. Elec-tronic Engng. 27, 457 (1955).

A Harmonic Generator with Tuning Fork Drive

By W. A. Penton*, A.M.Brit.I.R.E., and G. W. G. Court[†], B.Sc., A.M.I.E.E., A.Inst.P.

A circuit is described which generates harmonic frequencies from a fundamental provided by a simple valve-maintained tuning fork. A suitable waveform is generated from the fundamental sinusoid and its harmonics are selected by a frequency-selective amplifier. Harmonics up to and including the fourth are readily obtained from a fundamental frequency of 560c/s, with a fre-quency stability of the order of 1 part in 10⁴ per °C. The range can be extended to harmonics up to the 10th.

REQUIREMENT arose at this laboratory for a highstability audio source suitable for the calibration of a frequency-measuring device at four spot frequencies. The frequencies were the fundamental 560c/s and its harmonic multiples up to and including the fourth. The complete unit was required to be readily transportable and simple and reliable in use.

It was known that an electronically-maintained tuning fork, without control of temperature or amplitude of oscillation, would have a stability adequate for the requirement. One solution to the problem was therefore to use a set of four such tuning-fork oscillators. However, it is difficult to design a simple tuning-fork oscillator for a frequency as high as 2kc/s. The harmonic relationship of the required frequencies suggested the use of one fork only, at the fundamental frequency, and the selection of the harmonics from a suitable waveform derived from the fundamental sinusoid.

The circuit to be described performs this task adequately. and has formed the basis of a unit to be developed in a commercial laboratory. The filter used to isolate the desired harmonics is a four-section RC oscillator with sufficient negative feedback applied to prevent oscillation.

General Design

TUNING FORK

The problem was to produce sinusoidal harmonic frequencies up to the fourth from a valve-maintained tuning fork. Earlier work in this laboratory had shown the good frequency stability of such a source. A simple fork, machined from mild steel, driven by headphone magnet and coil assemblies, and without control over temperature or over amplitude of oscillation can give a considerable improvement in frequency stability over uncontrolled LC or RC oscillators. The temperature coefficient of the fork has by far the major effect on its stability, and measurement has shown this coefficient to approximate one part in 10⁴ per °C temperature change.

The output of such a fork is rarely a pure sinusoid, especially if the output is taken at high level from the anode of the maintaining valve. Although this distorted waveform has a certain harmonic content it is difficult to reproduce and control precisely, and in order to produce a repeatable practical circuit it is necessary to generate a reproducible waveform at the fundamental frequency. This waveform is chosen to have an appropriate harmonic content. It is convenient to start with a good sinusoidal waveform which is obtained from the grid circuit of the fork maintaining valve and amplified to an appropriate level in a separate stage. (Fig. 1, V_{1a.b}).

MULTIPLYING AND FILTER CIRCUITS

The next operation is the conversion of this sinusoid into a waveform which by prior theoretical analysis is known to have a suitable harmonic content, i.e. high level harmonics up to the fourth. For the present application higher order harmonics were not important.

Finally this waveform is passed through a filter circuit tunable to each appropriate harmonic frequency and the harmonic sinusoid delivered to the output terminals through a cathode-follower. In the circuit used, the filter stage V, is a phase-shift oscillator circuit with sufficient negative feedback to just prevent oscillation, thus providing a frequency selective amplifier, controlled by the RC constants of the feedback network.

It should be noted that the minimum requirement for the filter is that it should separate one harmonic frequency from the next, so that a comparatively low Q suffices.

Analysis giving the harmonic content of various waveforms is known, e.g. Terman¹. Calculation shows that for the particular requirement, suitable waveforms are a short square pulse, with mark-space ratio of the order 1:10, a pulse of similar duration forming part of a sine wave, or a sawtooth, Relative harmonic amplitudes up to the fourth for these waveforms are shown in Table 1.

l	R	T.	F	1.
h	D	ж.		

TA

		RELATIVE AMPLITUDES								
WAVEFORM	FUNDA-	2nd	3rd	4th						
	MENTAL	harmonic	Harmonic	harmonic						
Short square pulse	0.50	0.19	0.12	0.15						
Sawtooth	0·11	0·11	0·10	0·10						
	0·64	0·32	0·21	0·16						

The above table demonstrates the theoretical advantage of the first two waveforms, and a practical trial was made of each. It was found that the relative amplitudes derived from theory could be realized, but other effects also occurred in practice. A critical adjustment of the filter stage near to the threshold of oscillation is undesirable, and at less critical settings the effective Q of the filter becomes insufficient to prevent some breakthrough of the pulse waveform. This superimposes on the harmonic sinusoids to produce a distorted waveform. The feedback adjustment was judged too critical for an instrument to be used by relatively unskilled operators, even though good waveforms could actually be produced from the short-pulse waveforms. The sawtooth waveform was next examined. With

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the sawtooth, due to the absence of a steep leading and trailing edge in this waveform, there was no trouble experienced with breakthrough. The feedback setting was not particularly critical, and despite its lower content of high-order harmonics it appeared the best choice for the final instrument.

It was found in practice that due to variation in gain with frequency of the filter circuit, there was a tendency to self-compensation for the loss in level with the higher harmonics. It is quite possible to arrange such a filter stage for a predetermined gain/frequency characteristic. In this case an increase of gain with frequency proves an asset. The instrument can be adjusted quite easily to deliver approximately constant amplitude harmonics up to and including the third, with a reduction in amplitude of about 30 per cent for the fourth harmonic. The sawtooth also has a slight advantage in the ease with which it may be injected into the filter stage, as discussed in the section which follows. anode waveform of V_{2b} is a positive-going square wave, the duration of which is governed by the time that the grid of V_{2a} is held above the critical level. By adjustment of the potential of V_{2a} grid the circuit will generate short square waves of adjustable length, at the anode of V_{2b} .

A capacitor is connected from V_{2b} anode to earth. As V_{2b} is cut off, the capacitor will charge through the anode resistor, and the anode potential will rise; as V_{2b} is switched on again the capacitor will discharge and the anode potential will fall. Thus at V_{2b} anode a sawtooth waveform is generated and its times of rise and fall may be adjusted by the on-off times of V_{2b} , and thus by the setting of the grid potential of V_{2a} . The actual rates of rise and fall are determined by the circuit constants. The constants shown in Fig. 1 are satisfactory for the particular design.

The output waveform from V_{2b} is fed to the screen grid of V_{3} , which value acts with its tunable *RC* feedback loop as a variable frequency filter. Adjustable negative feedback



Fig. 1. The complete circuit

Circuit Description

The design basis of the instrument has been dealt with in the preceding paragraphs. The complete circuit finally used is shown in Fig. 1.

The tuning fork itself follows a design by Lamson². Its fundamental frequency is $560c/s \pm 0.25c/s$ at 20°C. The primary uncertainty of frequency and the temperature coefficient of the fork being known, it may be shown that the design requirement of an accuracy of 0.2 per cent in generated frequencies may be met over a range of temperature of about 13°C without temperature control of the fork. In the instrument itself the fork is mounted in a thermally-lagged compartment and a panel thermometer registering temperature of the fork compartment is provided so that the instrument is not used without correction should the permissible temperature range be exceeded.

The fork drive and pick-up coils are standard headphone coils and magnets. V_{1a} drives the fork, and from its grid is taken the near-sinusoidal waveform of the fork, at a level of a few volts r.m.s. This is amplified in V_{1b} . The sinusoid appearing in the anode circuit of V_{1b} is applied to the grid of V_{2a} . This valve is normally cut off by current in V_{2b} raising the common cathode potential above that set by the potentiometer to which the grid V_{2a} is attached. The action of this circuit, when V_{2a} grid is raised above the critical level, is to switch rapidly over so that V_{2a} is drawing current and V_{2b} is cut off. This condition persists until V_{2a} grid is brought below a new critical level. Thus, the in V_s cathode permits the filter to be set to the nonoscillating condition, the whole then acting as a high-gain amplifier at frequencies determined by the *RC* values of the network. The values shown cover a range from about 500c/s to 2 600c/s by variation of *C* alone.

When the filter is tuned over its range, the various harmonics in the waveform injected into the screen grid of V_3 appear at the output of the cathode-follower V_4 . Adjustment of the filter is not very critical. Depending upon the signal level into the filter, there is some small output signal throughout the tuning range, but the level peaks sharply at the appropriate frequencies. As the filter is adjusted to move, say, from the second harmonic to the third, the output signal falls greatly in amplitude but remains constant in form until it changes to the third harmonic waveform, then increases rapidly to maximum level.

The tuning shaft is indexed by a simple detent mechanism to correspond to the required harmonic. The front panel dial is appropriately marked for each frequency.

The normally-closed push-button switch in the grid circuit of V_{1b} , is provided so that before using a harmonic for calibration purposes the drive from the fork can be cut off. This checks that the filter is stable, and that the frequency being produced is actually derived from the fork and not because the filter has shifted to a self-oscillating condition. In practice, this has never been found to occur by accident, but is not impossible if the filter feedback control should be set too near the oscillation threshold.



Fig. 2. Typical waveforms (a) Sawtooth plus 4th harmonic. (b) Sawtooth plus 2nd harmonic. (c) Fork output plus fundamental. (d) Fork output plus 3rd harmonic.

Universal Flight Trainer

A new "universal" instrument flight trainer has been developed by the Flight Simulator Division of Redifon Limited and is now in production. Completely selfcontained it is electro-mechanically operated and incorporates analogue computers.

Though considerably more advanced than a primary ground trainer it is less comprehensive than a flight simulator, which for economical, operational or other reasons, is not available to all aircraft operators. Before designing the equipment Redifon engineers held extensive consultations with leading Commonwealth and European airlines. From an analysis of their findings, the final design specification was prepared.

The cockpit, representing that of a typical multi-engined airliner, accommodates a captain and first pilot and provides a full coverage of every aspect of instrument flying.

The first two trainers have been ordered by Sabena for delivery early in 1956 and will be used for training crews on the Douglas DC-7C and the Convair 440 Metropolitan. The former will be the first general purpose trainer to embody the Sperry integrated flight system.

A subsequent model will be delivered to KLM, who will make use of the equipment in converting pilots to the new Lockheed 1049G Super Constellation.

The KLM trainer will be the first of its type to provide cloud and collision warning radar training facilities—one of the many services which can be supplied with the equipment as an optional extra.

Seated at the trainer's controls, pilots will be able to practice take-offs, approaches. and normal flight and holding manoeuvres at a fraction of the cost of training in the air.

To add to the realism of "flying" the trainer, certain handling characteristics, the instrument layout and the disposition of most of the controls can be made to duplicate the features of a specific aircraft.

Training will be in the hands of a qualified instructor who shares the "cabin" with the trainee crew. He will be able to observe closely their reactions as he feeds their

Results

Some typical waveforms are shown in Fig. 2. Two illustrations show the relationship between the fork frequency (at top) and various harmonics, and the other two show the sawtooth generated from the fork frequency and the associated harmonics. It will be seen that the output waveforms are good sinusoids and that harmonic amplitudes are comparable. The output level of each harmonic may be from about 30 to 50V peak.

It was found not difficult to produce harmonics up to the tenth, free from distortion, though at this order of harmonic only a few volts appear at the output.

Acknowledgment

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REFERENCES

- 1. TERMAN, F. E. Radio Engineers' Handbook (McGraw-Hill).
- 2. LAMSON, H. W. A Simple and Precise Standard of Musical Pitch. J. Acoust-Soc. Amer. 7 (July 1935).

instruments with fault conditions and failures from his control console.

Full track recording facilities are provided, giving small scale presentation of a large area for route flying and navigational exercises and large-scale presentation of local areas for observing holding patterns, radio range and other procedures. An approach recorder for monitoring the most critical of all aircraft manoeuvres, the final approach, is also supplied.

Crystal Controlled Radar for Jan Smuts Airport

The Jan Smuts Airport, the largest civil airport in the Union of South Africa, is to be equipped with the latest Marconi multi-purpose radar.

Marconi's are to supply their Type S.232 radar, together with four display consoles, ancillary equipment and test gear. The equipment is generally similar to that manufactured for use at London Airport.

The Type S.232, which was announced early this year, is claimed to be the only crystal controlled radar of its type in the world. Operating in the 500 to 610Mc/s band, it is virtually unaffected by rain and cloud, conditions which can on occasions hamper operations on the centrimetric bands.

A further important feature of this equipment is the m.t.i. (moving target indicator) system, which is more effective in removing unwanted clutter and permanent echoes, and more stable in operation than any other previous type. This efficient m.t.i. system enables the equipment to be used both for long-range surveillance and for close control of aircraft within the airport terminal area. Ranges of from $\frac{3}{4}$ mile to more than 100 miles are obtained, and thus by the use of two or more p.p.i. displays (up to eight can be used with one aerial head) the equipment can perform several functions simultaneously. In practice, all airport radar requirements except precision approach radar (p.a.r.) can be met.

Due to the use of crystal control, the oscillators in the type S.232 are devoid of frequency drift. The m.t.i. is likewise completely stable. At Jan Smuts airport the radar head will be remotely operated from the control tower about $1\frac{1}{2}$ miles away.

Noise Characteristics of Pulse-Slope-Modulation

By Jajneswar Das*

Signal-to-noise ratio in a p.s.m. system has been calculated by the method indicated by Daloraine, Labin and Goldman. Signal-to-noise ratio in the output of the audio amplifier has been found to be 27.4db for the input peak signal-to-noise ratio (random noise only) of 6db only. Experimental determination of the signal-to-noise ratio confirms the theoretical results. The signal-to-noise ratio is found to improve considerably with the increase of the high frequency bandwidth of the system. The results of the impulse-noise tests show a further improvement in the output ratio. Pentode slicer circuits with proper high frequency compensation and "Boxcar" aemodulators have been used to eliminate the noise and other distortions.

THE noise characteristics of different p.t.m. and p.c.m. systems have been investigated by different authors. The noise characteristics of pulse-slope-modulation, reported earlier by the author¹ can also be evaluated by following the general method indicated by Daloraine, Labin² and Goldman³ (1948). As has been shown by these authors, the process of slicing removes all noise at the base and the top of the pulses so long as the slicing level is above the peak noise voltage. The only noise, that remains in the signal appreciably, is in the form of phase or slope modulation of the pulses by the noise voltages occuring during the rise-time of the pulses.

The slope-modulation of the pulses had been made possible by varying the rise-time of pulses of constant width according to the frequency and the amplitude of the modulating frequency. The modulating signal was recovered by differentiation and "Boxcar" demodulation of the slope-modulated pulses. Slicer circuits were used in the receiver to improve the signal-to-noise ratio in the audio output.

Theoretical conclusions have been corroborated with experimental results and the overall noise characteristics of p.s.m. is shown to be better than those of f.m. and slightly worse than those of p.t.m.

Analysis

The following analysis is for the random noise and for a slicing level equal to half the signal pulse height. If f_c is the bandwidth of the receiver, the rise-time of the noise pulses will be equal to $1/(2f_c)$ (= t_r) approximately. Taking the ideal case of triangular noise pulses, the duration of pulses will be equal to $2t_r$ or more.

On inspecting certain geometric constructions (Fig. 1) of the resultant slopes of the signal pulses due to the presence of noise, it is evident that the maximum slope variation due to noise will occur when the disturbing noise pulse starts at the time where the signal pulse edge meets the slicing level, i.e. at $t = (a_0/2)$ and where N = (A/2) and duration of the noise pulse is equal to $2t_{\rm r}$.

If $\tan \theta_N$ is the new pulse slope, then: $\tan \theta_N = \frac{(\tan \theta \cdot t_r) + N}{(\tan \theta_N - t_r) + N}$

$$=\frac{[A/(2a_{0}f_{c})]+\dot{N}}{1/(2f_{c})}=\frac{A+2a_{0}f_{c}N}{a_{0}}$$

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ELECTRONIC ENGINEERING

If $\tan \theta_{\text{mean}}$ denotes the unmodulated pulse slope and $\tan \theta_{\text{max}}$ denotes the condition on maximum modulation, then:

Noise modulation =
$$K(\tan \theta_N - \tan \theta_{mean})$$

$$= K(A/a_0 + 2f_cN - (A/a_0)) - 2f_cNK$$

Maximum signal modulation = $K(\tan \theta_{\max} - \tan \theta_{\max})$ = $K(A/t_{\pi} - (A/a_{\pi}))$

$$(2Af_0 - (A/g_0))K$$

$$\frac{\text{Max noise output}}{\text{Max signal output}} = \frac{2f_c N}{2Af_c - (A/a_c)}$$

$$= (N/A) \times \left(\frac{f_{\rm c}}{f_{\rm c} - (1/2\alpha_{\rm o})}\right) \dots (1)$$

where:

$$a_0 = \frac{2t_r T}{T + t_r}$$

$$K = \text{constant}$$

$$an \theta_{\text{max}} = (A/t_r) = 2f_c A$$

$$an \theta_{\text{mean}} = \frac{\tan \theta_{\text{max}} + \tan \theta_{\text{min}}}{2}$$

$$= A/2(1/t_r + (1/T))$$

 $= A/a_0$

The noise output given by equation (1) is reduced due to the following reasons.

- (a) Probability of cccurrence of peak noise pulse is less than unity.
- (b) Probability of coincidence of phases between peak

Fig. 1. Resultant slope due to noise plus signal $A = amplitude \text{ of signal pulse; } \alpha_n = mean \text{ rise time: } t_p = rise \text{ time of } noise \text{ pulse; } N = peak amplitude of noise.}$



noise pulse and the signal pulse is less than unity.

(c) Noise output reduction due to the limited bandwidth of the a.f. amplifier.

The consideration of the above factors will result in noise output reduction as follows:

If the noise pulses were ideal rectangular pulses, it is seen from the geometrical construction of noise plus signal wave that the pulse edges will be shifted due to noise, but there will not be any slope variation (Fig. 2).



Fig. 2. Effect of rectaugular noise pulses on p.s.m.



Fig. 3. Probability distribution of noise voltage

As the process of demodulation is such that random time shifts of pulse edges do not produce any audio output, it is evident that these rectangular noise pulses will not contribute any audio noise output.

If the bandwidth is sufficiently large, the random noise voltage appears as a superposition of an average voltage and a fine grained fluctuation voltage. The average voltage level can be considered as the r.m.s. noise voltage as this has maximum probability of occurrence and gives rise to effective noise output due to random phase shift in pulse-time-modulation². In our case, the noise voltage to be considered is the fine grained fluctuation voltage which is in excess of the r.m.s. voltage and produces both time shift and slope variation of the pulse edges.

The probability distribution of noise voltage has been shown⁴ to be of the form $p(V)dv = V \cdot \exp(-V^2/2) \cdot dv$ and is plotted in Fig. 3.

Where V = ratio of the envelope voltage to r.m.s. noise voltage (prior to detection)

and p(v) dv = probability of an instantaneous noise voltage lying between V and (V + dv).

The probability of this envelope voltage exceeding a particular voltage V_e is given by:

$$p_{\mathbb{N}} = \int_{\mathbf{v}_{e}}^{\infty} V \cdot \exp(-V^{2}/2)$$
$$= \exp(-Vc^{2}/2)$$

The shaded portion of the graph is equal to p_N , where the lower limit V_o is equal to 1.

. Probability of noise voltage exceeding r.m.s. value = $e^{-1/2}$

$$= 0.6066$$

Further, the average value of this ratio $V = \left(\frac{\text{envelope voltage}}{\text{r.m.s. noise voltage}}\right)$, so long as it is greater than

one, is given by:

 \therefore The average value of the peak noise voltage under above condition = 1.656 N (r.m.s.)

and the average excess value of the peak noise voltage = 0.656 N (r.m.s.)

Theory and experiments show that the "crest factor" (= ratio of the amplitude of highest peaks to the r.m.s. value of the amplitude) of random noise lies betwen 3.4 and 4.5. A mean value of 4 can be safely taken for the average crest factor. This shows that if N is the peak noise voltage, then r.m.s. noise voltage is N/4, whereas the slicing level is always above the peak value of N. Therefore, the disturbing peak noise voltage in this case is given by:

$$N_{\rm eff} = 0.656 \ N_{\rm r.m.s.}$$
$$= 0.164 \ N_{\rm peak}$$

In the analysis above, signal and noise amplitudes during the rise-time of the pulse edges have been added together. This assumes that the voltages are in phase. But with the pulse generators used in practice, the phases of successive pulses are essentially random and hence all possible values of phase difference between the signal pulse voltage and noise voltage are possible in the range of 0 to π . Kretzmer⁵ has shown by means of an auto-correlation method that the mean effective value of the time-shift in p.t.m. is equal

to 0.7 of the maximum time-shift. Similarly N_{eff} in our case becomes equal to (0.164 × 0.7) N_{peak} . We have so far dealt with positive slopes of the noise pulses only. But there is an equal probability of positive and negative slopes of the noise pulses occurring at the slicing instants and a complete noise wave in the output will comprise of both positive and negative crests. Thus the effective noise modulation $\Delta \tan \theta_N$ is halved and its value is equal to $(2f_e \cdot N_{eff})/2$ (neglecting the constant K).

$$\therefore (\Delta \tan \theta_{\rm N})_{\rm eff} = f_{\rm c} \times 0.164 \times 0.7 \times N_{\rm peak}$$
$$= 0.1148 f_{\rm c} N_{\rm peak} \qquad (2$$

NOISE REDUCTION DUE TO LIMITED A.F. BANDWITH

Fourier expansion of the trapezoidal pulses may be written as:

$$f(t) = \frac{A(T - (a/2))}{T_o} + 2A/T_o \sum_{n=1}^{\infty} \left\{ \frac{\sin \omega_o n(T-t)}{\omega_o n} + \frac{\cos \omega_o n(a-t)}{a\omega_o^2 n^2} - \frac{\cos \omega_o nt}{a\omega_o^2 n^2} \right\}$$

where a = rise-time of the pulses

A = amplitude of the pulses

T =pulse width

 $T_o = (2\pi/\omega_o)$ = repetition time-period of the pulses. The process of detection of the slope-modulated signal consists of differentiation and then audio demodulation by a "Boxcar" pulse-lengthener circuit.

It has been shown elsewhere⁶ that the differentiated signal may be written as:

$$f'(t) = (A/T_{o}) + (A/\pi na) \sum_{n=1}^{\infty} \left[\sin \omega_{0} nt - \sin \omega_{0} n(t-a) \right]$$

= 1/a $\left\{ (Aa/T_{o}) + (A/\pi n) \sum_{n=1}^{\infty} [\sin \omega_{0} nt - \sin \omega_{0} n(t-a)] \right\}$
...(3)

If the original signals were slope-modulated by noise, then the differentiated signal becomes : F(t) =

$$\frac{(1+m_{\rm N}\sin\omega_{\rm N}t)}{a_{\rm o}} \left\{ \frac{Aa_{\rm o}}{T_{\rm o}(1+m_{\rm N}\sin\omega_{\rm N}t)} + A/\pi n \sum_{\rm n=1}^{\infty} \left[\sin\omega_{\rm o}nt - \sin\omega_{\rm o}n \left(t - \frac{a_{\rm o}}{1+m_{\rm N}\sin\omega_{\rm N}t} \right) \right] \right\}$$

since :

 $a = \frac{a_0}{1 + m_{\rm N} \sin \omega_{\rm N} t}$

where $m_N =$ modulation index for noise

$$s = noise$$
 angular frequency.

The characteristic of a "Boxcar" pulse-lengthener circuit⁷ is such that it removes all time modulation (p.l.m. and p.p.m.) and the demodulated output is only amplitude modulated. Therefore the final demodulated output is given by:

$$F'(t) = \frac{(1 + m_{\rm N} \sin \omega_{\rm N} t)}{\left\{\frac{Aa_{\rm o}}{T_{\rm o}} + \frac{A}{\pi n} \sum_{n=1}^{\infty} \left[\sin \omega_{\rm o} nt - \sin \omega_{\rm o} n(t - a_{\rm o})\right]\right\}}$$

$$=\frac{(\tan\theta_{\rm o}\pm\Delta\tan\theta_{\rm N})}{(1+1)^{1+1}}$$

 $(\Delta$

$$\left\{\frac{A\alpha_{o}}{T_{o}} + \frac{2A}{\pi n}\sum_{n=1}^{\infty}\left[\sin \frac{\omega_{o}n\alpha_{o}}{2}\cos \omega_{o}n(t-\alpha_{o}/2)\right]\right\}.$$
 (5)

It is thus seen that the noise power is in the terms:

$$\frac{\tan \theta_{\rm N} \times \alpha_{\rm o}}{T_{\rm o}} \text{ and} \\ \left[\frac{(\Delta \tan \theta_{\rm N}) \times 2 \sin (\omega_{\rm o} n \alpha_{\rm o}/2)}{\pi n} \right] \cos \omega_{\rm o} n (t - (\alpha_{\rm o}/2))$$

and the total noise power is contributed by the amplitude modulation of the zero frequency component and the harmonics of p.r.f. As $\Delta \tan \theta_N$ is due to all frequencies accepted by the receiver of bandwidth f_o and the a.f. amplifier accepts frequencies up to F_a only, the net audio noise power will be proportional to F_a/f_o .

Audio noise power due to a.m. of zero frequency is proportional to:

$$= Am/2$$

where m = modulation index.

We have amplitude of each sideband of the p.r.f. harmonics

$$(\Delta \tan \theta_{\rm N}) \cdot \sin (\omega_{\rm o} n \alpha_{\rm o}/2)$$

And the total sideband power (including both upper and lower), due to amplitude modulation of the p.r.f. harmonics, is proportional to:

Even though the inclusion of upper and lower sidebands makes the total possible bandwidth equal to $2f_o$, the factor 2 is justified by the fact that the component noisefrequencies above and below the p.r.f. harmonics can produce audio noise.

Hence, combining equations (6) and (7), the total audionoise power accepted by the a.f. amplifier is proportional to:

$$\frac{(\Delta \tan \theta_{\rm N})^2 \cdot F_{\rm o}}{f_{\rm o}} \times \left\{ (a_{\rm o}^2/T_{\rm o}^2) + (2/\pi^2) \sum_{n=1}^{1_{\rm o}/1_{\rm o}} (1/n^2) \left(\sin \frac{\omega_{\rm o} n a_{\rm o}}{2} \right) \right\}$$
(8)

Further calculation on a generalized basis is rather involved. We shall determine the numerical value of the signal-to-noise ratio for a typical p.s.m. system with the following characteristics:

> P.R.F. = $f_0 = 10$ kc/s Pulsewidth $T = 10\mu$ sec

System bandwidth $f_{o} = 1 Mc/s$

Audio bandwidth $F_a = 0$ to 3000c/s

Mean risetime of the pulses $a_0 = 1 \mu \sec \alpha$

$$\omega_{\rm o}=2\pi f_{\rm o}=2\pi\,\times\,10^4$$

 $f_o/f_o = 100 =$ number of p.r.f. harmonics accepted by amplifier.

Since f_c/f_o is equal to the number of p.r.f. harmonics accepted by the receiver and for the typical p.s.m. system, as above:

$$\sum_{n=1}^{c/f_o} (1/n^2) \sin^2 \frac{\omega_o n \alpha_o}{2} = 0.0446$$

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we have, equation (8)

$$= \frac{(\Delta \tan \theta_{\rm N})^2 \cdot F_{\rm a}}{f_{\rm o}} \left\{ a_{\rm o}^2 / T_{\rm o}^2 + (2/\pi^2 \times 0.0446) \right\} (9)$$

... Effective audio noise output voltage

$$= (\Delta \tan \theta_{\rm N})_{\rm eff} \times \sqrt{0.00914} \times \sqrt{(F_{\rm a}/f_{\rm c})}$$
$$= \frac{0.0956}{18.25} \times (\Delta \tan \theta_{\rm N})_{\rm eff} \qquad (10)$$

And r.m.s. audio signal output voltage:

Thus from equations (10) and (11):

R.M.S. audio signal Effective audio noise in the output

If A/N_{peak} is equal to 2; then the effective (signal/noise) ratio in the output is equal to 23.52. Therefore, the effective noise output is 27.41dB below the effective signal level when the input peak signal-to-noise ratio is only 2.

Circuits

The experimental set-up for the determination of the signal-to-noise ratios in the output is given in Fig. 4. The pulse-slope-modulated transmitter and the p.s.m. receiver were of the types reported earlier by the author^{1.6}. The gating pulse was obtained from the transmitter itself. The fluctuation-noise generator had a bandwidth of 1.5Mc/s. The impulse-noise generator was capable of producing 1μ sec pulses of p.r.f. varying from 100c/s to 5kc/s. Provision was made to substitute the impulse-generator in the



place of the noise-generator for the impulse-noise test. The slicer circuits used were of the two types given in Fig. 5, viz: (a) biased double-diode; (b) biased pentode⁸. The design of the slicer circuit was critical, since ordinary slicers produced slope distortion of the p.s.m. pulses and the output was highly distorted. It was, therefore, necessary to maintain the original slope of the pulses and proper highfrequency compensation was provided. The performance



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Fig. 4. Experimental set-up for signal-to-noise ratio determination

of the pentode-slicer circuit was found to be better in this respect.

The transmitter and the receiver systems had the following characteristics:

P.R.F. of the transmitted pulses	=	10kc/s
Pulse width	=	10μs ec
Mean rise-time of the unmodulated	pulses =	1 to 2µsec
A.F. pass-band	=	0 to 3kc/s
System bandwidth	=	1.0Mc/s
A.F. modulating frequency		1kc/s

The input peak signal-to-peak noise ratio was measured with the help of a wideband oscilloscope and attenuator.



 $f_{\rm e} = 1 M c/s; \ a_{\rm o} = 1 \times 10^{-6} sec.$

Initially the signal peak and the noise peak were made to be equal in height (as measured on the oscillograph screen) corresponding to zero dB reading on the attenuator. The noise level was then controlled with the attenuator and the corresponding dB value of the peak signal peak noise ratio was obtained. Attenuator readings were checked with the measurements on the oscillograph screen as well.

The output r.m.s. signal to effective noise ratio was measured with a square-law detector and an output indicator. Proper correction in the results was made for the inherent noise level in the receiver and the audio amplifier, which was of the order of -60dB.

Output r.m.s. signal to effective noise ratio was determined both with the a.f. signal and noise present simultaneously in the transmitted pulses (with a bridge-type distortion-factor meter incorporating a square-law detector), as well as with a.f. signal and noise present alternatively in the transmitted pulses. Both readings were found to give the same signal-to-noise ratios in the output.

Results

It has been shown in equation (12), that the theoretical value of r.m.s. signal to effective noise ratio in the output is given by $11.76 \times (A/N_{peak})$, when the input peak signal to peak noise ratio is 6dB and above. This gives an output signal-to-noise ratio in dB equal to 20 (log A/N_{peak} + log 11.76), i.e. (input peak signal to peak noise ratio in dB + 21.41dB). The curve in Fig. 6 shows the theoretical values of signal-to-noise ratios. The dotted portion of the curve below 6dB input signal-to-noise ratio is approximate and is due to Jelonek⁹.

The curve c of Fig. 7 gives the corresponding signal-tonoise ratios without any slicer circuit in the receiver. The performance is slightly inferior to that of the amplitude-



Fig. 7. Experimental values of signal-to-noise ratios

For curve A: $f_c = 1Mc/s$; $a_o = 1 \times 10^{-6}sec$. For curve B: $f_c = 500kc/s$; $a_0 = 1.8 \times 10^{-6}sec$. For curve C (without slicer circuit): $f_c = 1Mc/s$; $a_o = 1 \times 10^{-u}sec$. Sliced input for curves A and B = 1 per cent of the iransmitted pulses. P and Q are apparent threshold points.

modulated system, especially the output signal-to-noise ratios with input signal-to-noise ratios below 6dB are very poor,

The curve A of Fig. 7 gives the corresponding signal-tonoise ratios with the pentode slicer circuit in the receiver. The bandwidth of the transmission system was 1 Mc/s, the mean rise time α_0 of the transmitted pulses was 1×10^{-6} sec, the sliced input to the receive-amplifier was only 1 per cent of the transmitted pulse. Comparison with the theoretical curve of Fig. 6 shows that above 10dB input signal-to-noise ratio, the experimental values are only 1.4dB below the theoretical values. But the pronounced threshold point at 6dB is not present in curve A. The interpolation of the two portions of curve A gives an apparent threshold point of 7dB input signal-to-noise ratio. The improvement of the output ratio at 6dB is 3.4dB less than theoretical value. All these apparent discrepancies are, perhaps, accounted for by the fact that the sliced pulse was

not of theoretically infinitesimal height and the gating process involved in the pulse lengthener circuit provided an improvement in the output ratios below 6dB input signalto-noise. One interesting point is the improvement of 10dB in the output ratio with zero dB input signal-to-noise ratio. This is specially due to the gating circuit used in the receiver. It can be mentioned here, that the usual signal-to-noise ratios obtained in p.t.m. systems, below the threshold value, are very poor¹⁰

The curve B of Fig. 7 has been obtained with a transmission bandwidth of 500kc/s only and the mean rise-time of the transmitted pulses equal to 1.8×10^{-6} sec. An



Fig. 8. Experimental values of impulse noise tests For curve A: P.R.F. of impulses = 500c/s. For curve B: P.R.F. Impulse width = $1 \times 10^{-6}sec; f_c = 1Mc/s;$ of impulses = $5\ 000c/s$. $a_{o} = 1 \times 10^{-6} sec.$

appropriate filter was used between the transmitter and the receiver of the system. The results are seen to be inferior to that of curve A by about 2.5dB and show an apparent threshold point at approximately 8dB input signal-to-noise ratio. This shows that the signal-to-noise ratio in the output improves with the bandwidth of the system and the improvement at the threshold point is about 4dB.

The results of the impulse-noise tests are shown in Fig. 8. Since in the case of the impulse-noise, the frequency components of the impulses are uniformly distributed over the transmission bandwidth and these components are all in phase, the peak amplitudes of the received impulses are proportional to the bandwidth (as distinct from random noise, the amplitude of which is proportional to the square root of the bandwidth). The noise-power accepted by the a.f. amplifier will then be proportional to $(F_a/f_c)^2$ and the improvement in the output signal-to-noise ratio will be proportional to (f_o/F_a) . Curve A with p.r.f. of impulses equal to 500c/s and curve B with p.r.f. equal to 5kc/s, the pulse width being 1µsec approximately in both cases. show an overall improvement over the curves of Fig. 7.

The apparent threshold is at about 8dB input signal-tonoise ratio.

- It has been seen in equation (1) that:
- $= A/N \times \left(\frac{f_{\circ} (1/2a_{\circ})}{c}\right)$ Maximum signal output Maximum noise output

Since a_0 is dependent on f_c and T, the above ratio can be expressed in terms of independent variables A, N, f_c and T, and we have :

Maximum signal output
Maximum noise output =
$$A/N \times (1/2 - (1/4f_{c}T))$$
...(13)

With the noise peaks equal to half the pulse amplitudes, the output peak signal to peak noise ratio becomes equal to $(1 - (1/2f_cT))$. The curve in Fig. 9 shows the variation of



Input peak signal-to-noise ratio = 6dB. Pulse duration $T = 10 \mu sec.$

the peak signal-to-noise ratio in the output with different transmission bandwidths and the pulse duration equal to 10μ sec. The ratio will be further improved according to equations (8) and (10); but in no case will it show the improvement proportional to the transmission bandwidth as found in the case of p.t.m. systems.

The signal-to-noise characteristics of p.s.m., as seen above, are, of course, inferior to those of the p.t.m. systems. Nevertheless, they are slightly superior to the usual f.m. systems.

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REFERENCES

- DAS, J. Pulse-Slope Modulation—A New Method of Modulating Video Pulses and Its Possible Application on Line Circuits. *Indian J. Phys.* 28, 449 (1954).
- 2. DALORAINE, E., LABIN, E. Pulse-Time Modulation. Elect. Commun. 22, 90 (1944).
- DALDKANES, E., EARIN, E. PUISE THIC INSCILLATION. Elect. Commun. 22, 90 (1944).
 GOLDMAN, S. Frequency Analysis, Modulation and Noise. p. 281 (McGraw-Hill, 1948).
 GOLDMAN, S. Loc-cit. p. 245.
 KRETZMER, E. R. Interference Characteristic of Pulse-Time Modulation. Proc. Inst. Radio Engs. 33, 252 (1950).
 DAS, J. Signal Analysis and Audio Characteristics of Pulse-Slope Modulation. Electronic Engng. 27, 482 (1955).
 KLEENE, S. C. Analysis of Lengthening of Modulated Repetitive Pulses. Proc. Inst. Radio Engrs. 35, 1049 (1947).
 RAO, V. N. Experimental Investigation of Noise Associated with Modulated Pulses. Proc. Indian Acad. Sci. 38, 186 (1953).
 JELONEK, Z. Noise Problems in Pulse Communication. J. Instn. Elect. Engrs. 94, Pt. 3A, 46 (1947).
 MOSKOWITZ, S., GRIEG, D. D. Noise Suppression Characteristics of Pulse-Time Modulation. Proc. Inst. Radio Engrs. 36, 446 (1948).

Some Aspects of the Design of a D.C. Amplifier for Use with a Slow Analogue Computer

By H. Fuchs*, B.Sc.

The design of a d.c. amplifier for use with a "slow" analogue computer is considered with the emphasis on the accuracy to be achieved. The errors introduced as a result of an h.t. resistance common to several amplifiers is calculated, and a criterion for the accuracy of a linear computer stated.

THE amplifier to be described is used in a computer, designed to solve the equations of "ground resonance" occurring in helicopters. These equations were derived by Coleman' who has shown that under certain simplifying assumptions the behaviour of the helicopter can be represented by four second order simultaneous linear differential equations with constant coefficients. The set-up required to solve these equations is basically an electronic differential analyser. In the following an attempt is made to describe the design of a d.c. amplifier for such a computer with the emphasis on the accuracy to be achieved.

Design Criteria

When designing an amplifier for a given computer it is necessary to know to what extent the accuracy and performance of the computer will depend upon the following parameters:

- (1) Gain.
- (2) Bandwidth.
- (3) Drift.
- (4) Grid current.
- (5) Common h.t. resistance.
- (6) Operational impedance errors.

Before going more deeply into the problem of accuracy it is useful to visualize the system to be examined.

By means of the Laplace transformation a set of linear differential equations may be reduced to a similar set of algebraic equations of the same order. The roots are then in general:

$S_n = a + jb$

These roots occur in conjugate pairs. The solution of the original differential equation is then of the form:

$$exp(a \pm jb) t$$

which may also be written:

$\exp(at) \sin(bt) (-\phi)$

The resultant frequencies of the solution are likely to be close to each other, since the equations represent real mechanical systems. The time scale of the equations is adjusted in order that the solutions will occur at the frequency most suitable to the computer. (In the present case this is 1 radian per second.) Let the errors in the real and imaginary parts of the roots be δa and δb respectively. Then the percentage error in the frequency term b is:

100 $\delta b/b$

It is profitless to define the accuracy of the damping term in a similar way. There will be conditions when the system approaches instability, i.e. when at least one of the damping terms tends to zero. It is not possible to maintain any given fractional accuracy in a term which approaches zero. Fortunately such an accuracy is not required in any

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practical case. Once a system approaches instability it is of little interest whether the damping coefficient is say 0.01 or 0.001. For this reason the accuracy of the damping term will be judged by a different criterion.

The solutions of the equation are of the form

 $\exp(a + \delta a) t \sin[(b + \delta b) t + \phi]$

The amplitude after time T seconds is

 $\exp(aT) \exp(\delta aT) \simeq \exp(aT) (1 + \delta aT)$ [if $\delta aT \ll 1$]

This differs from the true amplitude by δaT , and the percentage error is therefore:

$100 \delta aT$

This error in the amplitude may be taken as a measure of the error in the damping term a, since it will be seen that it is independent of a and is finite for a = 0.

If an accuracy of 1 per cent is accepted as sufficient for all envisaged solutions this criterion may be written as.

100	$\delta b / b$	\leq	1			4	•				2	•1'	•	(1)	
100	S.T.	1	1												2	`	

 $100 \quad \delta aT \leq 1 \quad \dots \quad (2)$

These inequalities entail choosing both the frequency at which the computer will operate and the computation time T. This criterion does not immediately determine the accuracy with which the individual mathematical operations have to be performed. However, it can be shown that if the individual mathematical operations are carried out with an error of less than 0.1 per cent, the second order equation will be accurate to 1 per cent.

It is now necessary to show how the design criteria influence the accuracy of computation.



Gain

(a) SUMMING AMPLIFIER

In Fig. 1, the nodal equation is derived by equating the currents at the summing point A, to zero. This includes the current flowing in the effective grid-earth resistance R_{g} .

$$\sum_{0}^{m} (V_{j}/R_{j}) = -(V_{0}/A) \left[\sum_{0}^{m} (1/R_{j}) + (1/R_{g}) \right]$$

The right-hand side of the equation is the error in the output due to the finite gain of the amplifier. (In this analysis, no account is taken of the inaccuracy in the summing resistors.) In order that this error shall be less than 0.1 per cent of the output V_0 :

$$(R_o^{'}/A)\left[\sum_{o}^{m}(1/R_j) + (1/R_g)\right] < \frac{1}{1000} \dots \dots (3)$$

In the present case there are 14 $1M\Omega$ resistors at the input. To achieve the required accuracy:

 $A > 14\,000$

(b) INTEGRATORS

In Fig. 2 the nodal equation is again derived by equating





the currents at the summing point A to zero. (The generalized impedance of a capacitance of value C, is 1/pC).

$$\frac{V_1}{pCR} + V_0 = -(V_0/A) \left[\frac{1}{pCR} + 1 \right]$$
$$= d/dt$$

 $1/p = \int dt$

p

The right-hand side is again the error in the output, V_0 . In order that this error shall be less than 0.1 per cent of the output:

$$(1/A)\left[\frac{1}{pCR}+1\right] < \frac{1}{1\ 000} \dots \dots (4)$$

In the present case $CR = \frac{1}{2}, \frac{1}{p} = \int_{0}^{T} dt, T = \text{the computa-}$

tion time, assumed 30 seconds. To achieve the required accuracy $A > 31\,000$. This analysis again assumes R and C to be perfectly accurate.

Bandwidth

Macnee² has shown that the bandwidth of the amplifiers used in a computer is intimately associated with the accuracy of the roots of the stability polynomial. The stabilization of a feedback amplifier such as is used in an analogue computer, against high frequency oscillations, entails the reduction of the gain with frequency in such a way that a safe phase and gain margin is maintained.

It is well known that about 30dB/decade is the maximum safe average slope at which the gain may be reduced with frequency. Hence, since the frequency at which the stray capacitances shift the phase by 180°, is more or less fixed by the valves used, it will be seen from Fig. 3 that the bandwidth for any given gain is also fixed. Expressed mathematically the maximum safe slope is:

Integrating :

20 log (G) = -30 log (f) + K or $Gf^{3/2} = C$ (a constant) This means that:

 $A = C/B^{3/2} \ldots \ldots \ldots \ldots (6)$

A = the gain of the amplifier

B = the bandwidth

C = a constant, fixed by the frequency at which the valve stray capacities shift the phase by 180° .

This will now be applied to Macnee's result.

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It has been shown that the error in the roots of an equation of the form:

is

$$= -S_n^2 T_1 - \frac{S_n^{m+1} T_2}{F'(S_1)} - 1/T_0 \dots (8)$$

 $\sum_{n=1}^{\infty} A_n p^n x' = 0 = F(p) x' \dots (7)$

 $a_n = -S_n^2 T_1$ where $S_n =$ the *n*th root.,

- T_{1} is the time-constant limiting the h.f. response of the integrator,
- T_2 is the time-constant limiting the h.f. response of the summing amplifier.
- $T_{\rm o}$ is the time-constant limiting the l.f. response of the integrator.

To reduce this error it is desired to keep T_1 and T_2 as small as possible and T_0 as large as possible.

If T is the time-constant of the amplifier operated without feedback, it is easily verified that:

$$T_1 = T/A\beta_1$$
 $T_2 = T/A\beta_2$ $T_0 \simeq ACR$ (9)
 β_1 is the feedback ratio of the integrator,

 β_2 is the feedback ratio of the summing amplifier. T, the time-constant of the amplifier is given by equation (6) as:

$$1/2\pi B = A^{2/3}/2\pi C^{2/3}$$

Hence:

$$T_1 = 1/2\pi C^{2/3} \beta_1 A^{1/3}$$
 $T_2 = 1/2\pi C^{2/3} \beta_2 A^{1/3}$ $T_0 \simeq ACR$
(10)

It is seen from this that the time-constant T_1 and T_2 are inversely proportional to the cube root of the gain and depend on C, the bandwidth of the amplifier before 2OLogG



stabilization. When designing for the minimum error the gain should therefore be as large as possible to reduce T_1 and T_2 and increase T_0 , thus improving the accuracy of summation and of integration. In the present case the value of C is about 1Mc/s and the internal gain of the amplifier of the order of 10⁸.

Drift

The drift in a d.c. amplifier causes voltages to appear at the output terminal which constitute an error. It is necessary to reduce this source of error by careful design even if drift correction is used. The causes of drift are in general:

Changes in the filament supply voltage.

Changes in the h.t. supply voltage. Changes in the resistance values.

Changes in valve characteristics.

To reduce the effect of all these causes special circuits have been devised, but it is necessary in addition to stabilize the h.t. supply and, in some cases, also the filament supply. High stability resistors must be used in all important parts of the circuit.

The circuit in the present amplifier used to compensate for variations in the filament supply voltage is the cascode connected triode pair. This circuit has been chosen in preference to the Miller compensator because the low anode current required in the first stage, in order to reduce the grid current, makes the "Miller" difficult to design. The compensation ratio against filament supply voltage variations achieved with the cascode pair is of the order of 10:1.

To compensate for h.t. variations the second stage, a push-pull long-tail pair, has the h.t. variations applied equally to each grid. This results in their cancellation (to a good approximation) between the two anodes.

The third stage also employs a push-pull long-tail pair, of which one side is a cathode-follower. This converts from a push-pull to a single ended signal and practically maintains the h.t. compensation. This compensation is now from anode to earth.

The final stage is a beam tetrode capable of driving a $10k\Omega$ load through $\pm 100V$.

The $\pm 250V$ and the -400V supplies are both well stabilized. The long term stability of the 250V supply is $\pm 1mV$ and the output impedance is 0.1Ω . The -400V supply which feeds the grid of the last stage has a stability of $\pm 10mV$. This supply does not deliver much current and is near the end of the amplifier. Its requirements are therefore less stringent than those of the 250V supply.

The drift of the amplifier operated with its correct power supplies is of the order of $300\mu V/h$ at the input grid.

Grid Current

The effect of the grid current at the amplifier input may be divided into two parts, viz.:

- (1) The standing current in the absence of any signal, and
- (2) The current which flows in the effective grid-earth resistance, due to the signal.

The latter has already been dealt with when setting the gain of the amplifier. It has the same effect as an equal resistance in parallel with the other input resistors, at the input grid of the amplifier.

The effect of the standing grid current is to produce a voltage drop across the input resistance. If the resulting output voltage is balanced out for one value of the input resistance, rebalancing is necessary should this value be changed. It is, therefore, desirable to balance the amplifiers with the input resistance with which they normally work connected. To avoid the need for frequent balancing, especially when the last stated condition cannot be fulfilled, this standing current should be kept as low as possible by working the valve over the appropriate portion of its characteristics. The grid current in the present case is 10^{-9} A.

Common H.T. Resistance

The effect of an h.t. resistance common to several amplifiers in a computer is to cause intercoupling between these amplifiers. Voltages appearing at the output terminals will be transferred via this resistance, either to the outputs, or to the input stages of all the amplifiers.

Consider the set-up solving the equation :

$$\sum_{o}^{m} A_{n}(1/p^{n})x' = 0 = F(1/p)x' \quad \dots \quad \dots \quad (11)$$

(a) First, the effect of coupling between the outputs will be considered.

In Fig. 4 voltages appearing at points B, C, D, etc., are
$$x'_{2} - (x'/p)$$
, (x'/p^{2}) , etc. $[1/p \equiv \int dt]$

These outputs are added, via the appropriate coefficient potentiometers, at the summing point A, to give equation (11). Suppose that the transfer function from the output terminal of the amplifier to the common h.t. resistance and back to the output is K. The voltage appearing at the point B due to this effect will then be:

$$K \sum_{n=1}^{m} (-1)^{n} (1/p^{n}) x' \equiv tx'$$

This is the sum of all the outputs, transferred via the common h.t. resistance R_{ht} , to the point B. Hence the total voltage at point B is:

and at C is:

$$-x'\left(\frac{1+t}{p}-t\right)$$

x'(1 + t)

and at D is:



Summing all the voltages via the appropriate coefficients, gives at A:

$$\sum_{n=0}^{m} A_{n} (1/p^{n}) x' + xt' \left[\sum_{n=0}^{m} A_{n} (1/p^{n}) - (p \sum_{n=0}^{m} A_{n} (1/p^{n}) - \text{terms in } p^{n}, a > 0) + (p^{2} \sum_{n=0}^{m} A_{n} (1/p^{n}) - \text{terms in } p^{n}, a > 0) + \text{etc.} \right] = 0$$

Hence:

Qn

$$F(1/p) + t [F(1/p) - F_1(1/p) + F_2(1/p) - \dots + (-)^m F_m(1/p)] = 0$$

 $(F_k(1/p) \equiv p^x F(1/p) - \text{ terms in } p^a, a > 0)$

This is the original equation with the roots slightly displaced. Let the roots be $S_n = (1/p)$. Hence, by Taylor's theorem:

 $F(S_n + \alpha_n) \simeq F(S_n) + \alpha_n F'(S_n) \quad \dots \quad \text{[if } \alpha_n \ll S_n]$ Hence:

$$a_{n} F'(S_{n}) = t \sum_{o}^{m} (-)^{k} F_{k} (1/p)$$

$$= \frac{K \left[\sum_{o}^{m} (-1)^{r} S_{n}^{r} \right] \left[\sum_{o}^{m} (-)^{k} F_{k} (S_{n}) \right]}{F'(S_{n})} \dots \dots (12)$$

(b) The effect of coupling to the input stages of the amplifiers will now be considered. If the coupling occurs to the anode of the first stage, the transfer function from the output terminal of the amplifier via the common h.t. resistance to the anode of the first stage may be written as K'. Then if V_1 is the voltage appearing at this anode it is equivalent to a voltage V_1/a (a = the gain of the first stage) at the amplifier input grid. Hence, the voltage at the output due to this effect is $V_1/a\beta$. ($\beta =$ the feedback ratio). As before:

$$V_1 = K' \sum_{0}^{m} (-1)^n (1/p^n) x' \equiv t' x'$$

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Hence the error in the roots is now:

$$u_{n} = (K'/a\beta) \frac{\left[\sum_{\sigma}^{m} (-1)^{r} S_{n}^{r}\right] \left[\sum_{\sigma}^{m} (-)^{k} F_{k} (S_{n})\right]}{F'(S_{n})} \dots (13)$$

In the present amplifier the h.t. variations at the first anode are designed to cancel out at the grids of the second stage. This latter effect may therefore be ignored. The coupling to the outputs may be shown to be negligible in the present case owing to the very low output impedance of an amplifier with such intense negative feedback. If, however, β , the feedback ratio were substantially reduced as is the case with repetitive computers, the coupling effect would not necessarily be negligible.

As an example, the error in the second order equation, having no damping will be calculated. (X'' + X = 0).

$$F(1/p) = 1 + (1/p^2) = 0, \ F_1(1/p) = 1/p, \ F_2(1/p) = 1, F'(1/p) = 2/p$$

$$t = K [1 - (1/p) + (1/p^2)] S_n = \pm$$

Hence:

 $a = -(K/2)(1 \pm j)$ $\therefore S_n' = S_n + a_n = -(K/2) \pm j(1 - (K/2))$ This represents a change in the frequency and the addition



Fig. 5. Transfer from anode to common h.t. resistance

of a damping term -(K/2). In order that the amplitude shall not have changed by more than 0.1 per cent during the computation time of 30 seconds, due to this effect:

$$30\ddot{K}/2 < \frac{1}{1\,000}$$
, $K < \frac{1}{15\,000}$

In Fig. 5 the transfer function from the anode of the last valve of the amplifier, to the common h.t. resistance is:

$$\simeq R_{
m ht}/R_{
m I}$$

 $R_{\rm L}$ = anode load

 $R_{\rm ht} = {\rm h.t.}$ output resistance.

The transfer function back to the anode is:

$$\simeq R_{
m s}/R_{
m L}$$

 R_s = output resistance of the amplifier.

$$R_{\rm s} = R_{\rm out}/A\beta$$

 R_{out} = output resistance of the amplifier without feedback.

These formulæ assume :

$$R_{
m s} \ll R_{
m L} \gg R_{
m ht}$$

Hence:

$$K = \frac{R_{\rm ht}R}{R_{\rm L}^2}$$

In the present case:

$$R_{out} = 5k\Omega$$

$$R_{L} = 10k\Omega$$

$$A = 10^{5}$$

$$\beta = \frac{1}{2}$$

$$K = 5 \times 10^{-10} R_{ht}/\beta$$

This is very much smaller than the limiting value for K as calculated above. No trouble is experienced due to this cause. If, however, no measures are taken to compensate for variations of the h.t. supply, the coupling to the first stage of the amplifier can have an appreciable effect as is shown by the inequality:

$K' [a\beta \gg K$

Care should therefore be taken to ensure that either, R_{ht} is low enough or that some method of h.t. compensation is used, in order that the value of an, the error, is within the design limits. This is especially true in cases where a low value of β , the feedback ratio, is required.

The Coefficient and Operational Impedance Errors

The errors introduced by the coefficient setting device (in the present case, a ten-turn wire-wound potentiometer) and the operational impedances, are of great importance. In Fig. 6 let the amplifier have infinite gain. Then the transfer function of the amplifier is:

$$V_0/V_1 = -(Z_0/Z_1)$$

Let the errors in the impedances Z_0 and Z_1 be δZ_0 and δZ_1 . Then the output is:



Fig. 6. Transfer function of amplifier

and the error in the output V_0 is:

$$V_{0}' - V_{0} = \delta V_{0} = V_{1} \left(Z_{0} / Z_{1} \left[\frac{1 + (\delta Z_{0} / Z_{0})}{1 + (\delta Z_{1} / Z_{1})} \right] \right) - Z_{0} / Z_{0}$$

$$\delta V_0 / V_0 \simeq \delta Z_0 / Z_0 - \delta Z_1 / Z_1 \dots \dots \dots (14)$$

i.e. the percentage error in the output is the sum of the percentage errors in the impedances. Hence to achieve an error of 0.1 per cent in summation the impedances must be selected with an accuracy of 0.05 per cent.

Conclusion

An attempt has been made to show the factors governing the design of a d.c. amplifier for a "slow" analogue computer, bearing in mind the important considerations of the accuracy to be achieved. It may be stated here that, at any rate in "slow" analogue computers, the accuracy is usually limited by operational impedance and coefficient errors. However, it is necessary to know how far one may go in increasing the accuracy of these components before the errors introduced from other sources makes this process profitless.

Acknowledgments

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REFERENCES

- COLEMAN, P. P. Theory of Self-Excited Mechan cal Oscillations of Hinged Rotor Blades. (N.A.C.A., A.R.P. 3229, 1943).
 MACNEE, A. B. Some Limitations on the Accuracy of Electronic Differential Analysers. Proc. Inst. Radio Eners. 40, 303 (1952).
 KORN, G. A., KORN, T. M. Electronic Analogue Computors. (McGraw-Hill, 1951).

The Diode Pump Integrator

By J. B. Earnshaw*, B.Sc.

The characteristics of the diode pump integrator are analysed in detail and the conditions for its operation as a linear or non-linear device are discussed. References are made to the conventional uses of the circuit and a novel application of its non-linear characteristics in a frequency sensitive relay circuit is described.

THE diode pump integrator provides a most convenient way of obtaining an average (integrated) d.c. signal from a train of pulses. The circuit is arranged so that each incoming pulse causes a definite charge to be placed on a reservoir capacitor. The charge on this capacitor leaks away through a high resistance, making the average potential across the capacitor a function of the pulse repetition rate.

It is intended to show that this function can be calculated with reasonable accuracy, and that with certain limitations it can be made to be approximately linear. Under these conditions the circuit can be used as a frequency meter¹, random counting rate meter², or as a device for the demodulation of frequency modulated radio waves³. The utility of the circuit is not, however, limited



Fig. 1. Circuit of diode pump integrator

to linear applications. It has been used for frequency division by Banthorpe⁴ and Clayden⁵, and by the author to form the basis of a frequency-sensitive relay circuit.

Fig. 1 shows the basic circuit diagram and it is assumed that each incoming pulse is a rectangular pulse as depicted.

The amplitude, E, and the duration, T, of these pulses is constant. Where the input is not of this form it must be passed through pulse shaping circuits before introduction to the diode pump integrator.

For a detailed knowledge of the operation of the circuit it is necessary to investigate the actual waveforms of potentials appearing at the points x, y, and z, with respect to earth. These waveforms are most easily understood by developing the integrator from the circuits shown in Figs. 2, 3 and 4.

Development of Integrator

Starting from a basic circuit without diodes and leak resistance, Fig. 2(a) shows the integrator in the form of two series connected capacitors acting as a potential divider across the input pulses. The output, z, is taken from the junction of the capacitors. During a pulse the charges on

 C_1 and C_2 will be the same, viz: $E \frac{C_1 C_2}{C_1 + C_2}$ and the volt-

ages are $E \frac{C_2}{C_1 + C_2}$ across C_1 and $E \frac{C_1}{C_1 + C_2}$ across C_2 .

At the end of each pulse both capacitors are completely discharged. Although in Fig. 2(b) the voltage steps are

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shown to be instantaneous this is never quite so in practice as the pulse shaping circuit will have a finite output impedance. If this impedance is R_1 , the charging and discharg-

ing time-constant is $R_1 \frac{C_1 C_2}{C_1 + C_2}$.

The introduction of diode d_1 , between C_1 and C_2 (see Fig. 3(a)) causes charge to flow on the first pulse only. During this pulse C_2 is charged, the voltage across

it being $E \frac{C_1}{C_1 + C_2}$ as before, if we neglect the forward



resistance of the diode during conduction. At the end of the pulse, the diode ceases to conduct, since its anode is negative with respect to its cathode. Consequently, neither C_1 nor C_2 is discharged, and the potential of y

falls to $-E \frac{C_2}{C_1 + C_2}$. At the advent of the second pulse the potential of Y rises by E to $E \frac{C_1}{C_1 + C_2}$, but this is

insufficient to cause d_1 to conduct again, if no charge has leaked away from C_2 . Successive pulses cause no further deposition of charge on C_2 , therefore the output is a con-

stant voltage $E \frac{C_1}{C_1 + C_2}$ which is independent of the

frequency of the input pulses.

The blocking effect due to the non-discharge of C_1 can be counteracted by providing a discharge path through a second diode d_2 . This diode is connected (see Fig. 4(a)), so that at the end of any pulse it conducts away the whole of the charge on C_1 and restores the point y to earth potential again. (In practice a small negative bias is placed on

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Fig. 3. Introduction of diode into integrator circuit

the anode of d_2 to arrest the small current flow through the diodes due to contact potential differences and the finite velocity of emission of electrons from the cathodes.)

In detail the operation is such that during the first pulse the voltage across C_2 , v_1 , is $E \frac{C_1}{C_1 + C_2}$ as in both of the previous circuits, and at the end of this pulse, since d_1 ceases to conduct, this voltage remains constant. When the second pulse arrives the potential of Y rises to $E \frac{C_1}{C_1 + C_2}$ before d_1 becomes conductive, and charge flows through it. The new charge deposited on each capacitor is

 $\left(E - E \cdot \frac{C_1}{C_1 + C_2}\right) \frac{C_1 C_2}{C_1 + C_2}$ viz $E \left(\frac{C_1 C_2}{C_1 + C_2}\right) \frac{C_2}{C_1 + C_2}$

Fig. 4. Use of second diode



and the total voltage across C_2 is:

$$v_2 = E \cdot \frac{C_1}{C_1 + C_2} + E \cdot \frac{C_1}{C_1 + C_2} + \frac{C_2}{C_1 + C_2}$$

Before charge sharing can take place during the third pulse the potential of y must be $> v_2$ and so on

After p pulses the output voltage is v(p) and: v(p) =

$$E \frac{C_{1}}{C_{1} + C_{2}} \left\{ 1 + \left(\frac{C_{2}}{C_{1} + C_{2}} \right) + \left(\frac{C_{2}}{C_{1} + C_{2}} \right)^{2} + \dots \left(\frac{C_{2}}{C_{1} + C_{2}} \right)^{p-1} \right\}$$

this series is a geometric progression with common factor

$$\frac{C_2}{C_1 + C_2}, \text{ hence:}$$

$$v(p) = E \frac{C_1}{C_1 + C_2} \left\{ \frac{1 - [C_2/(C_1 + C_2)]^p}{1 - [C_2/(C_1 + C_2)]} \right\}$$

$$= E \cdot \left\{ 1 - \left(\frac{C_2}{C_1 + C_2}\right)^p \right\}$$

This expression is dependent solely upon the number of pulses supplied to the input and as the number $p \to \infty$ so $v(p) \to E$. If, on the other hand, p pulses have been supplied.



Fig. 5. Diode d conducting and diode d2 non-conducting

from a source with a pulse repetition frequency $n/\sec in$ a time t, then p = nt, and:

However, as
$$\left(\frac{C_2}{C_1 + C_2}\right)^{n_1} = \exp\left\{-nt \cdot \ln\left(\frac{C_1 + C_2}{C_2}\right)\right\}$$

equation (1) can now be rewritten in the form:

$$v(t) = E\left\{1 - \exp\left[-nt \cdot \ln\left(\frac{C_1 + C_2}{C_2}\right)\right]\right\}$$

From this it can be seen that the voltage across C_2 rises exponentially to E with a time-constant:

$$\left| \int \left[n \ln \left(\frac{C_1 + C_2}{C_2} \right) \right] \right|$$

The rectangular pulse used to operate the circuit can be of either polarity, since the diodes ensure that the correct amount of charge is always transported to C_2 . The polarity and biasing of the diodes may also be reversed, in which case the output voltage will be negative instead of positive.

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Operation of Complete Circuit

The addition of a leak resistor R_2 across C_2 restores the circuit of Fig. 4(a) to that of the complete diode pump integrator. This resistor provides a charging path for C_1 during conduction of d_1 , and a continuous discharging path for C_2 irrespective of the state of d_1 . Neglecting the forward resistances of the diodes, the equivalent circuits of the integrator, both during, and at the end of each pulse, are shown in Figs. 5 and 6.





Tucker⁶ and Brandon' have shown that when supplied with pulses of amplitude $E \frac{C_1}{C_1 + C_2}$, the output from the circuit depicted in Fig. 5(b) is identical with that obtained from the circuit in Fig. 5(a). The output, z_r is simply the differentiated waveform with a decay constant $R_2(C_1+C_2)$, while the voltage across C_1 is the difference between the input, x_r and the output, i.e. x-z. (See Fig. 5(c)).

At the end of a pulse d_1 ceases to conduct, and d_2 conducts away the charge on C_1 . The discharge of C_1 is illustrated in Fig. 6(a), and Fig. 6(b) shows the circuit for the continued discharge of C_2 . The waveforms of points x, y

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and z, are shown in Fig. 6(c) together with the voltage across C_1 , i.e. x-y.

The actual waveforms of the complete circuit during operation with an input consisting of the first three pulses of a pulse train are illustrated in Fig. 7.

For good integration, the time-constant of the filter R_2C_2 must be long compared with the pulse interval 1/n. If this is so the voltage across C_2 may be e, say, when the next pulse arrives. In these circumstances the new charge

given to each capacitor is $(E - e) \frac{C_1 C_2}{C_1 + C_2}$, and the total.

charge on C_2 is increased to:

$$eC_2 + (E-e) \frac{C_1C_2}{C_1 + C_2}$$
 viz. $(eC_2 + EC_1) \cdot \frac{C_2}{C_1 + C_2}$

With the constant pulse repetition rate a quasi-steady state must eventually be reached when the charge conducted by d_1 just replaces that leaking away through R_2 . When this occurs the output voltage waveform at z is a saw-tooth about an average d.c. potential v.

Fig. 8 illustrates the shape of this waveform and the following calculation determines the value of v.



Fig. 8. Integrator output waveform

The charge lost during the pulse interval is :

$$v\left[1 - \exp\left(-\frac{1/n}{R_2C_2}\right)\right]C_2$$

and this must equal the charge gained $(E - v) \frac{C_1}{C_1 + C_2}$

Therefore :

$$(E-\nu)\frac{C_1C_2}{C_1+C_2}=\nu\left[1-\exp\left(-\frac{1/n}{R_2C_2}\right)\right]C_2$$

and since:

$$R_2C_2 \gg 1/n, \ 1 - \exp\left(-\frac{1/n}{R_2C_2}\right) = \left(\frac{1/n}{R_2C_2}\right)$$

therefore :

i.e.:

$$(E - v) \frac{C_1 C_2}{C_1 + C_2} = v \left(\frac{1/n}{R_2 C_2}\right) C_2$$

$$v = nR_2C_1 \cdot (E - v) \cdot \frac{C_2}{C_1 + C_2} \dots \dots \dots (2)$$

The inherent ripple voltage of the filter is $(E-v) \frac{C_1}{C_1 + C_2}$ and in order that this shall be small C_2 is made very much

greater than C_1 . So with $C_2 \gg C_1$, equation (2) reduces to:

$$v = nR_2C_1 (E - v) \qquad (3)$$

which reduces to a non-linear relationship between v and n where:

$$v = \frac{EnC_1R_2}{1 + nC_1R_2}....(4)$$

The transient response of the integrator, that is its response to a suddenly encountered train of pulses, is similar to the response of the R_2C_2 filter to a current step of:

$$f = \frac{EnC_1}{1 + n C_1 R_2}$$

in which case:

$$v(t) = \frac{EnC_1R_2}{1 + nC_1R_2} (1 - \exp(-t/C_2R_2)) \dots (5)$$

Of course this is an "idealized" characteristic, since the voltage v(t) can only rise in steps of $[E - v(t)](C_1/C_2)$ to its equilibrium value.

Linear Characteristics and Counting Rates

The assumption that the circuit will be operated with $v \ll E$ reduces equation (3) to the better known form:

$$v = nC_1R_2E \qquad (6)$$

This is the normally accepted equation showing a linear relationship between v and n, and it is interesting to note the absence of C_2 from this equation.

By first making the assumptions that $E \ge v$, $C_2 \ge C_1$, and T greater than 5 times the initial charging time-constant R_1C_1 , equation (6) can be obtained by much simpler, though less instructive methods^{8,9}.

When using the integrator as a counting rate meter where the input is random in time, a reading taken at any instant may not be a true average value. Due to the "memory" of the filter R_2C_2 , successive observations are not independent of each other, and a special statistical theory is required for the interpretation of the results^{8,9,10}, therefore only the important conclusions need be quoted here.

After the rate-meter has reached an equilibrium condition, the fractional probable error (p.e.) of a single reading is given by:

p.e. =
$$\frac{0.675}{\sqrt{(2nC_2R_2)}}$$
 (7)

When an input is applied to the integrator, it takes some time for the output to rise to the quasi-equilibrium value. If it is assumed that equilibrium has been attained when the theoretical instantaneous voltage does not differ from the ultimate mean value by more than the probable error tion, the fractional probable error (p.e.) of a single reading is given by:

$$t_1 = C_2 R_2 \left(0.394 + \frac{1}{2} \ln 2n C_2 R_2 \right) \dots (8)$$

Random input pulses of varying widths and amplitudes are passed through pulse shaping circuits before introduction to the integrator. The width of the standard pulse, T_i constitutes a "dead time" for the instrument during which some pulses go uncounted. When the "dead time" is short compared with the mean pulse interval, i.e. $T \ll 1/n$, and the apparent mean counting rate is N pulses per second, the true mean counting rate, n is given by:

$$n = \frac{N}{1 - NT} \tag{9}$$

Automatic compensation for the dead time loss is obtained in an instrument designed by Pulsford and described in a recent article¹¹.

Banthorpe⁴ mentions a modification of the integrator, where the diode d_2 does not restore C_1 to earth potential but to a voltage which follows the voltage across C_2 . The charging of C_1 and C_2 then takes place immediately on the advent of a pulse at x, and the charge deposited on C_2 is

always equal to
$$E \frac{C_1 C_2}{C_1 + C_2}$$
. Consequently, the output is a

perfectly linear function of the pulse repetition frequency. This is achieved by returning the anode of d_2 to the output of a cathode-follower whose grid is connected to C_2 (see Fig. 9).

The cathode-follower also represents a convenient circuit arrangement from which the voltage waveform may be extracted for other purposes.



Fig. 9. Circuit for linear relationship between v and n

Non-Linear Characteristics

The non-linear relationship between v and n, formed by equation (4) is shown graphically in Fig. 10. As the values of C_1 and R_2 can be chosen arbitrarily, a family of curves can be drawn with the product R_2C_1 as parameter.

Suppose it is desired to obtain the maximum possible variation in v for a change in the input frequency from n_0 to $n_0 + \delta n$, e.g. what value of R_2C_1 to use in order to obtain the maximum voltage change available between, say, 500p/s and 501p/s.

Since the variable circuit components R_2 and C_1 occur in product form in equation (4) they can be replaced by a single variable τ , where $\tau = R_2C_1$.



Fig. 10. Family of curves showing $v = E \cdot n^{\tau}/1 + n^{\tau}$ for different values of τ

Hence equation (4) becomes (

$$P = E \cdot \frac{n\tau}{1+n\tau} \tag{10}$$

The value of τ which makes $(\partial v / \partial n)$ a maximum for a given value of n, say n_0 , is required and since $(\partial v / \partial n)_{i,j} = g(\tau)$ this maximum will occur when:

$$\partial^2 v / \partial n \partial \tau = 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (11)$$

Let $n\tau = x$, so that $(\partial x/\partial n) = \tau$; $(\partial x/\partial \tau) = n$; and $(\partial^2 x/\partial \tau \partial n) = 1$, then equation (10) can be expressed as

 $\nu = h(n\tau) = f(x)$ $(\partial v/\partial n) = (df/dx) \cdot (\partial x/\partial n) + \dots + \dots + (13)$

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Now

$$(\partial^2 \nu / \partial n \partial \tau) = (d^2 f / dx^2) (\partial x / \partial n) (\partial x / \partial \tau) + (d f / dx) (\partial^2 x / \partial n \partial \tau)$$
(14)

For a maximum of $(\partial v / \partial n)$ the above expression equals zero.

$$\therefore 0 = (d^2f/dx^2) \cdot x + (df/dx) \dots (15)$$

From equation (12):

and

$$(df/dx) = E \frac{1}{(1+x)^2}$$
 and $(d^2f/dx^2) = -\frac{2E}{(1+x)^3}$

and substituting these in equation (15)

Hence when $\tau = 1/n_0$, $(\partial \nu/\partial n)$ has its maximum value for the given input frequency.

From equation (13) we obtain the maximum value of $(\partial v / \partial n)$ as $E/4n_0$, and this occurs according to equation (12) for x = 1 when the equilibrium voltage is equal to half the incoming pulse height.



Application

An application of the diode pump integrator in a frequency sensitive relay circuit utilizes information gained from the above theoretical treatment. The relay circuit closes with an input of 500p/s and opens when the input falls below 497p/s, this corresponds to a frequency variation of less than 1 per cent.

Fig. 11 shows a block diagram of the circuit, which consists essentially of a diode pump integrator followed by a d.c. amplifier. The output from this amplifier operates a Schmitt trigger¹² in which is incorporated the actual relay.

With 70V input pulses the maximum value of $(\partial v / \partial n)$ obtainable, $E/4n_0$ is $70/(4 \times 500)$ volts/cycle when $n_0 = 500 \text{ p/s}$; so that for a frequency change of 3c/s, the integrator output variation is approximately 1/10V. This change in itself is too small to operate any triggering device directly, thus the inclusion of a d.c. amplifier between the integrator and the trigger is necessary.

Most electro-magnetic relays require at least a 30 per cent reduction of the energizing current to become de-energized, however, by including the relay in a Schmitt trigger circuit, where the current through it is either 5mA or zero, "snapaction" is ensured. The trigger design is such that the relay is " on " whenever the input to V_2 (Fig. 12) falls below 100V, and "off" when the input rises above 106V. The "backlash" of the trigger is 6V and for this to correspond to the 1/10V variation available from the integrator the d.c. amplifier gain is 60.

In the practical version of the circuit, the diode pump integrator is connected to the negative h.t. line at P. The potential at P is such that, with the addition of 35V across C_2 (E/2 at 500p/s input), the d.c. amplifier V₁ is biased to amplify in its linear region. The anode voltage of V_1 can be adjusted, to give correct operation of the trigger by means of the potentiometer in the potential dividing chain.

The resistor, R_3 , between V₁ anode and V₂ grid ensures that when V, is cut-off the grid current in V_2 is not excessive. Manual control of the trigger can be achieved by the

selector switch at V_2 grid. The three positions are (1) on, (2) automatic, (3) off.

One feature which has not been considered previously and which requires some discussion is that of the ripple voltage, its effect on triggering, and its suppression.

The value of C_1 (500pF) is determined by the loading on the pulse shaping circuit, and R_2 (4M Ω) is then determined by equation (16) and the switching frequency (500p/s). With $C_3 = 100C_1$, the ripple voltage is 0.35V at z and 20V after amplification, so that it is more than three times the backlash of the trigger. To decrease the ripple by increasing C_2 means increasing the response time R_2C_2 by the same factor. However, as the ripple frequency is 500p/s its magnitude can be reduced to 0.6V at the input to the trigger by C_3 (0.02 μ F) connected between V₂ grid



and earth. The presence of this capacitor introduces another time-constant R_3C_3 (0.01sec) but this is negligible compared with R_2C_2 (0.2sec).

The above analysis, of the detailed operation of the diode pump integrator was developed during the design and construction of a frequency-meter with automatic rangeselection, in which the frequency sensitive relay just described selects between two ranges 0 to 500p/s and 0 to 5000p/s by shunting the leak resistor of a separate and conventional linear integrator. This frequency meter will be described in a separate paper.

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REFERENCES

- **REFERENCES**R. REICH, H. J., UNGVARY, R. L. A Direct-Reading Frequency Meter for the Audio and Supersonic Ranges. *Rev. Sci. Instrum.* 19, 43 (1948).
 ELMORE, W. C. A Counting Rate Meter. *Nucleonics* 43, (April 1948).
 SEELEY, W. S., KIMBALL, C. N., BARCO, A. A. Generation and Detection of Frequency-Modulated Waves. *RCA Rev.* 6, 269 (1942).
 BANTHORPE, C. H. An Electrical Backlash Circuit. *Electronic Engng.* 26, 110 (1954).
 CLAYDEN, R. T. An Improved Frequency Divider. *Electronic Engng.* 20, 196 (1948).
- 196 (1948).
 TUCKER, M. J. A Note on Electronic Analogue Integration and Differentiation. *Electronic Engng. 25*, 35 (1953).
 BRANDON, P. S. Electronic Analogue Integration and Differentiation. *Electronic Engng. 25*, 476 (1953).
 SMITH, G. D. Counting Rate Meters. *Electronic Engng. 24*, 14 (1952).
 ELMORE, W. C., SANDS, M. Electronics (McGraw-Hill, 1949).
 SCHIEF, L. I., EVANS, R. D. Statistical Analysis of the Counting Rate Meter. *Rev. Sci. Instrum. 7*, 456 (1936).
 SCHMITT, O. H. A Thermionic Trigger. J. Sci. Instrum. 15, 24 (1938).

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The Design of Filters Using only RC sections and Gain Stages

(Part 1)

By A. N. Thiele*, B.E. (Syd.), A.M.I.R.E. (Aust.)

A method is described of synthesizing filters, using RC sections within a feedback loop. Design information is given for high- and low-pass filters of 12, 18 and 24dB per octave slope and fixed cut-off frequency, and others of approximately 12 and 18dB per octave slope, whose cut-off frequency is variable continuously by the adjustment of a single element.

Algebraic Expressions for Filter Response

The general expression for the frequency response of a filter may be written:

in which α is normalized frequency, that is (ω/ω_0) , where ω is angular frequency and ω_0 is a reference angular fre-







quency called the cut-off frequency; the coefficients y determine the shape of the curve. Their influence, in the simplest case, is illustrated by Fig. 2.

The curve of equation (1) is asymptotic to two curves:

(1) when a is very small, $|(e_{in}/e_0)|^2 = 1$ (2)

(2) when a is very large, $|(e_{in}/e_0)|^2 = a^{2n}$ (3)

When both $|(e_{in}/e_o)|$ and α are graphed on a logarithmic scale, equations (2) and (3) yield straight lines whose slopes are (a) zero, and (b) 6ndB per octave, and which intersect when $\alpha = 1$, that is, when ω is at the cut-off frequency. When all the coefficients y are zero:

$$|(e_{in}/e_0)|^2 = 1 + a^{2n}$$
 (4)

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This curve, the simplest mathematically, is usually known, as the Butterworth or maximally flat characteristic. When the filter response is not allowed to rise above reference level, that is if $|(e_{in}/e_o)|$ cannot be less than 1, the maximally flat response is that closest to the asymptotic response, its maximum deviation being -3.0dB at the cut-







off frequency, e.g. the curve for y = 0 in Fig. 2. This response may not be the best for all purposes¹, for example, in Fig. 2, the curve for y = -1 has a maximum deviation of only +1.2dB from the asymptotes. However, in the discussion that follows, the maximally flat characteristic will be taken as the ideal to be aimed at.

One- and Three-section RC Networks in a Feedback Loop

When a single section RC network is included in the forward path of a feedback amplifier, the result is the same as connecting the network in cascade with the feedback amplifier, except that its effective cut-off frequency is changed. In Fig. 3, cut-off occurs when $\omega CR = 1$, that is, when $f = (1/2\pi CR)$.

In Fig. 4, if we modify the gain -M of Fig. 4 by the

characteristic of Fig. 3, then:

Thus in Fig. 4, frequency response curve shape is still the same as that of Fig. 3, namely: $(e_{in}/e_0) = 1 + ja$, i.e. $|(e_{in}/e_0)|^2 = 1 + a^2$. The slope of the asymptote is still 6dB per octave. The only change is that cut-off occurs now

when
$$f = \frac{1 + \beta M}{2\pi CR}$$

When a three-section CR network is included in a feedback loop, the circuit becomes the familiar phase shift oscillator when $1 + \beta M$ exceeds 9. By suitable choice of circuit values oscillation can be avoided, but the curve shapes obtained are not suitable for the accurate production of maximally flat filters.

Two-section Low-pass Filter

Now, considering the two-section RC network of Fig. 5: $(e_{\rm in}/e_{\rm o}) = 1 - \omega^2 C_1 R_1 C_2 R_2 + j \omega (C_1 R_1 + C_2 R_2 + C_2 R_1) \quad .. \quad (6)$ Then :

 $(e_{in}/e_0) = 1 - a_2^2 + jx_2a_2 \dots (7)$

where:

$$a_2 = \omega \vee (C_1 R_1 C_2 R_2)$$
 and $x_2 = \frac{C_1 R_1 + C_2 R_2 + C_2 R_1}{\sqrt{(C R C R)}}$



Fig. 5. Two section RC network-low-pass

There is very little advantage to be gained by making the two time-constants different, so making $C_1R_1 = C_2R_2 =$ CR, then $a_2 = \omega CR$ and $x_2 = 2 + (R_1/R_2)$. Note that x_2 can never be less than 2.

When this two-section network is included in the forward path of a feedback amplifier :

$$(e_{in}/e_{o}) = -\left[\frac{1+\beta M}{M}\right] \left[1 - \frac{a_{2}^{2}}{1+\beta M} + \frac{jx_{2}a_{2}}{1+\beta M}\right]$$
$$= -\left[\frac{1+\beta M}{M}\right] \left[1 - a_{3}^{2} + jx_{3}a_{3}\right] \qquad (8)$$
here :

u

$$x_3 = \frac{\alpha_2}{\sqrt{(1+\beta M)}}$$
 and $x_3 = \frac{x_2}{\sqrt{(1+\beta M)}}$

Apart from the change of cut-off frequency due to the change in a, we have now a device for making x as small as we please by suitable adjustment of $1 + \beta M$. This is the crux of the whole method. For if the frequency sensitive part of equation (8) is converted to the form:

 $|(e_{in}/e_o)|^2 = 1 + ya^2 + a^4$

then in this case:

$$y = x_3^2 - 2 = \frac{x_2}{1 + \beta M} - 2 \dots$$
 (9)

and by suitable choice of $1 + \beta M$ and (R_1/R_2) , y can be made any value we please to a lower limit of -2. Without feedback the lowest value that can be achieved by a twosection filter is y = +2. Using the method outlined above, any curve of Fig. 2 can be obtained.

12dB per Octave Slope—Accurate

Thus, if it is desired to obtain a maximally flat curve of

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12dB per octave slope, for which :

hen
$$y = 0$$
 and $1 + \beta M = (x_2^2/2) = \frac{[2 + (R_1/R_2)]^2}{2}$

Having found $1 + \beta M$, we can now calculate the timeconstant of the original network, since :

$$a_3 = (\omega/\omega_0) = \frac{\omega CR}{\sqrt{(1+\beta M)}}$$
. Thus $CR = \frac{\sqrt{(1+\beta M)}}{\omega_0}$. (11)

18dB per Octave Slope—Accurate

For a maximally flat 18dB per octave filter:

 $|(e_{in}/e_0)|^2 = 1 + a^6 = (1 + a^2)(1 - a^2 + a^4) \dots$ (12) Now $|(e_{in}/e_0)|^2 = 1 + a^2$ is the equation of a single RC section and $|(e_{in}/e_0)|^2 = 1 - a^2 + a^4$ is the equation of the type of filter we have been discussing, in which y = -1. We therefore design a filter of this type and connect in cascade with it another single RC section whose cut-off frequency is the same.

24dB per Octave Slope—Approximate

For a filter of 24dB per octave slope, whose equation is: $|(e_{\rm in}/e_{\rm o})|^2 = 1 + a^8 = (1 + \sqrt{2a^2 + a^4})(1 - \sqrt{2a^2 + a^4}) \dots (13)$ we could make two feedback filters having y values of $+ \sqrt{2}$ and $-\sqrt{2}$ respectively and connect them in cascade. However, since this involves two gain stages, it is a rather expensive solution. A simpler method, which gives a close



Fig. 6. Two section RC network-high-pass

approximation to the maximally flat characteristic, is to connect a two-section RC network in cascade with a feedback filter for which:

$$+ \beta M = (x_a^2 x_b^2/2)$$
 (14)

where x_a is the coefficient x of the two-section RC network inside the feedback loop, and x_b the coefficient x of the two-section RC network outside the loop. The greatest deviation of the curve of this filter from maximally flat is only 0.3dB if x_b is 2.0; 0.5dB if x_b is 2.1; and 0.7dB if x_b is 2.2. The time-constant of the RC network within the loop is calculated from $1 + \beta M$ as before. The cut-off frequencies of both the feedback filter and the RC network outside the feedback loop are made the same as the cut-off frequency of the whole filter.

HIGH-PASS FILTER

In the two-section RC network of Fig. 6:

$$(e_{in}/e_{o}) = 1 - \frac{1}{\omega^{2}C_{1}R_{1}C_{2}R_{2}} - (j/\omega)\left(\frac{1}{C_{1}R_{1}} + \frac{1}{C_{2}R_{2}} + \frac{1}{C_{1}R_{2}}\right). \quad (15)$$

and if, as in the low-pass case, we make $a_2 = \omega C_1 R_1 =$ $\omega C_2 R_2$ and $x_2 = 2 + (R_1/R_2)$, then:

$$(e_{in}/e_0) = 1 - (1/a_2^2) - (jx_2/a_2)$$
 (16)

and if this network is included in the forward path of a feedback amplifier, then :

$$(e_{in}/e_{o}) = -\left[\frac{1+\beta M}{M}\right] [1-(1/\alpha_{3}^{2})+(jx_{3}/\alpha_{3})].$$
(17)
re $(1/\alpha_{3}) = \frac{1}{\alpha_{2}\sqrt{(1+\beta M)}}$ and $x_{3} = \frac{x_{2}}{\sqrt{(1+\beta M)}}$

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Thus if we substitute their reciprocals for a_2 and a_3 , the results, including the values for x_2 and x_3 are the same as in the low-pass case. The only point that needs care is that in calculating the time-constants of the network :

$$CR = \frac{1}{\omega_{\rm o} \, \sqrt{(1 + \beta M)}} \, \dots \, (18)$$

which is different from equation (11).

Hence, the constants for 12, 18 and 24dB per octave high-pass filters can be calculated as before.

12dB per Octave Filters Variable by One Element

Cut-off frequencies of the filters already described can be varied by varying simultaneously and in the same proportion one element of each RC section, either the resistance or capacitance. This involves varying two elements for a 12dB per octave, three for an 18dB per octave and four for a 24dB per octave filter. When accurate and repeatable results are required, a ganged switch with fixed components may be the best method, though cumbersome.



Fig. 7. Variation of response of single variable element low-pass 12dB/octave filter with variation of q from q_1 to $1/q_2$ (see Table 1) TABLE 1

q ₁	1	2	3	4	5	6	-8
Maximum response (dB)	0.0	+0.1	+0.3	+0.6	+1.0	+1.3	+ 2 ·0

Ganged potentiometers have been suggested², and the use of a ganged capacitor should not be ruled out. However, the following is another and, to the author's knowledge, novel method by which the cut-off frequency can be varied over a fairly wide range by the variation of only one element.

Suppose that we vary R_2 in Fig. 5. Reference to its characteristic equations (6) and (7) shows that this will not only vary the cut-off frequency (at which $a_2 = 1$) but will vary the curve shape (determined by x_2).

When feedback is applied, the response of the combination varies with R_2 in the manner illustrated by Fig. 7. In

this figure, the parameter $q = \frac{R_{2}var}{R_1 + R_{2}ref}$, where $R_{2}var$ is

the actual value of R_2 , while R_{2ref} is a reference value chosen so that $CR = C_1R_1 = C_2R_{2ref}$.

If the response is adjusted to be maximally flat when q is at its limit of q_1 (and also at $1/q_1$, when the response shape is the same) then at intermediate values of q,

the response tends to peak, reaching a maximum of

$$||e_{\rm in}/e_{\rm o}||^2 = 1 - \left(\frac{q_1 - 1}{q_1 + 1}\right)^4$$
 (19)

on the curve for which q = 1. The values of these maxima are plotted in Table 1 and it turns out that the rise in response is not nearly as serious as might have been thought. Thus even when $q_1 = 5$, the rise of response is still small, so that for a range in cut-off frequencies of 5, and thus a range of 5² or 25 in R_2 , during which q varies from 5 to 1/5, the response does not peak more than 1dB. For this type of filter, we make:

$$1 + \beta M = [1 + (R_1/R_{\text{pref}})] \frac{(q_1 + 1)^2}{2q_1}, \dots \dots (20)$$

Note that the only limit to frequency range is set by the allowable rise in response at the centre of the range.

The above treatment applies equally to high- and lowpass filters except that whereas the cut-off frequency of the low-pass filter:

$$f_{o} = \frac{1}{2\pi C_{1}R_{1}} \sqrt{\left[\frac{1+\beta M}{q(1+(R_{1}/R_{2}ref))}\right]} \dots \dots \dots (21)$$



Fig. 8. Responses of component parts of single-variable element low-pass 18dB/octave filter

the cut-off frequency of the high-pass filter :

$$f_{\rm o} = \frac{1}{2\pi C_1 R_1 \sqrt{[q(1 + (R_1/R_{\rm pref})) (1 + \beta M)]}} \dots (22)$$

18dB per Octave Filters Variable by One Element

To achieve this result, we take first a 12dB per octave variable filter of the kind previously described. A single RC section is then added in cascade. The resulting filter has an ultimate slope of 18dB per octave. Fig. 8 illustrates the separate responses of the two parts. The feedback filter is peaked intentionally in this case so that its rising response compensates for the falling response of the fixed passive filter within the pass-band, while beyond cut-off the two falling responses add to cause more rapid attenuation.

The result is illustrated by Fig. 9. It shows that when the range of q is 8 and the frequency range is thus $\sqrt{8}$ or 2.8 times, the compensation is quite accurate and good curve shape is maintained. Even when the frequency range is four times, as in Fig. 10, the curve shapes will be acceptable for some purposes.

Note that, in Fig. 8, illustrating the performance of the low-pass feedback filter, only the range from q = 1 to $q = q_1$ is used, q being equal to 1 at the extreme (high frequency) end of the range of cut-off frequencies. In the high-pass case, q varies from 1 at the extreme (low frequency) end of the range, to a value which is less



than 1 at the inmost end of its range. If we take this value of q as $1/q_1$, q_1 is numerically the same and the formulæ are similar for both low- and high-pass filters.

$$1 + \beta M = 2 \left[1 + (R_1/R_{2\rm ref}) \right] \frac{(q_1 + 1)^2}{3q_1 + 1} \dots \dots (23)$$

For the low-pass filter (when q = 1) the highest cut-off frequency:

$$f_{o} = \frac{1}{2\pi C_{1}R_{1}} \sqrt{\left[\frac{1+\beta M}{1+(R_{1}/R_{2}r^{e})}\right]} \quad (24)$$

and the time-constant of the associated passive network:

$$C_4 R_4 = \frac{1}{2\pi f_0} \sqrt{\left[\frac{q_1(q_1-1)}{3q_1+1}\right]}.$$
 (25)

For the high-pass filter when q = 1 the lowest cut-off frequency:

$$f_{\rm o} = \frac{1}{2\pi C_1 R_1 \, \sqrt{\left[\left(1 + (R_1/R_{\rm 2ref}) \left(1 + \beta M\right)\right]} \cdots (26)}$$

and the time-constant of the associated passive network



$$C_4 R_4 = \frac{1}{2\pi f_0} \sqrt{\left[\frac{3q_1+1}{q_1(q_1-1)}\right]} \dots \dots \dots (27)$$

While the ultimate slope of these filters is 18dB per octave, the attenuation at the first octave, which is 18dB in the maximally flat case, varies from 15dB at inmost frequencies to 20-23dB at outermost frequencies.

Practical Circuit Details

Having established the methods of calculating the basic design data, namely time-constants of the RC sections, and feedback factors, it remains now to consider the details of the actual circuits used to make a practical filter.

MILLER EFFECT

is:

As is well known, if a capacitance C_2 , as in Fig. 11(a), is placed between the control grid and anode of a valve, the effect on the input circuit is the same as if a capacitance $C_2(M + 1)$ were placed between the control grid and earth. C_2 is usually the unavoidable capacitance due to







Fig. 12. High-pass filter, showing actual and effective circuit components

the construction of the valve and circuit wiring, but if necessary an external capacitance can be added. In lowpass filters this effective capacitance can be used as a circuit element.

In an analogous manner, if a resistance R_2 is connected between control grid and anode (with suitable precautions to block d.c.) its action in the circuit is the same as if a resistance $R_2/(M + 1)$ were connected between the control grid and earth.

The only difficulty in using this equivalent capacitance and resistance is that it varies with the stage gain M, so that if, for example, we use a low-pass section consisting of the resistance R_1 with the effective capacitance $C_2(M+1)$, the effective time-constant will vary with gain changes due to changes in supply voltages or variations between valves.

On the other hand, it will be seen from Fig. 11(b) that if the time-constants C_1R_1 and C_2R_2 are made equal, the effective time-constant of the circuit is C_1R_1 , independent of the gain M. The variation is transferred to the insertion loss of an all-frequency potential divider which stabilizes the gain seen by the external circuit.

If the resistor R_2 is used in a high-pass circuit, as in Fig. 12, to stabilize the overall gain, a good degree of stabilization can be combined with a small change of effective time-constant if R_1 is made somewhat greater than $R_2/(M + 1)$.

EFFECT OF CIRCUIT IMPEDANCES

When negative feedback is applied across a resistance-



Fig. 14(a). Negative feedback assuming zero output impedance and finite input impedance (b). Equivalent circuit

coupled stage, as in Fig. 13(a), the equivalent circuit is that of Fig. 13(b), where R_3 is the output impedance of the anode circuit, consisting of the parallel combination of load resistor R_L , anode resistance r_a (under the actual operating conditions of the circuit), and the following grid resistor R_{E_1} . The equation of this circuit is:

$$(e_{o}/e_{1n}) = \frac{R_{3}}{R_{1} + R_{2} + R_{3}}$$

$$- \frac{R_{1} + R_{2}}{R_{1} + R_{2} + R_{3}} \cdot \frac{R_{2} + R_{3}}{R_{1} + R_{2} + R_{3}} \cdot \frac{M}{1 + \frac{R_{1}}{R_{1} + R_{2} + R_{3}}} \cdot M \dots (28)$$

Note that the ratio taken in this case is (e_0/e_{in}) and not (e_{in}/e_0) as formerly. The ratios which make up the expression each represent the attenuation of a potential divider. The first represents a "feed-through" of signal, regardless of the gain stage. The second represents an attenuation of output voltage, the third an attenuation of input voltage.

When considering the gain reduction due to feedback,

the $M/(1 + \beta M)$ of previous calculations, only the fourth ratio is relevant and β for the purposes of frequencysensitive feedback is simply $R_1/(R_1 + R_2 + R_3)$. In fact, equation (28) can be written down logically by inspection of Fig. 13(b).

Similarly, in the circuit of Fig. 14(a), the expression connecting input and output voltages is

$$(e_{o}/e_{in}) = -\frac{R_{2}}{R_{1} + R_{2}}.$$

$$\frac{MR_{3}/[R_{3} + (R_{1}R_{2})/(R_{1} + R_{2})]}{[+[R_{1}/(R_{1} + R_{2})]] \left\{ MR_{3}/[R_{3} + (R_{1}R_{2})/(R_{1} + R_{2})] \right\}}.$$
(29)

Again this equation can be written down part by part by inspection of Fig. 14(b).

This discussion points the moral that if an attempt were made to measure the effective gain reduction factor $1 + \beta M$ as "the ratio of gain without feedback to gain with feedback", by removing the feedback, for example, by open-circuiting R_2 , alterations would be made to all the ratios in the expressions. The ratios irrelevant to frequency sensitive feedback would be altered, along with the relevant ratios, and the resulting figure would be misleading.

METHOD OF ADJUSTING FILTERS

It is suggested that once the circuit constants have been calculated as accurately as possible, the final adjustment should be made experimentally. First the feedback factor is adjusted to give the correct curve shape regardless of the cut-off frequency. The RC time-constants are then adjusted in equal proportions to correct the cut-off frequency. If the whole filter consists of an active part combined with a passive part, the curve shape and time-constants of the active part should be adjusted first to obtain the correct curve for that part. The time-constant of the passive part should then be adjusted to obtain the correct overall curve.

OUTPUT IMPEDANCE OF THE GAIN STAGE

It is desirable to keep this impedance as small as possible, for two reasons. One is to minimize "feed-through", which sets a limit to the ultimate attenuation of the active filter. From Fig. 13(b) and equation (28), it is clear that reduction of the effective output impedance R_3 will reduce "feedthrough". The other reason is that interaction of the output circuit with either a following amplifying stage or following passive *RC* sections is reduced.

To obtain this low output impedance, it is often desirable to feed the gain stage into a cathode-follower, which has the added advantage that it presents a very high impedance to its input. Then the two RC sections within the feedback loop can be separated, so that $x_2 = 2$.

Alternatively, negative voltage feedback may be applied across the gain stage, as by R_2 in Fig. 11(a). This is specially advantageous if stabilization of the stage gain is desired. In the second design example, both devices are used.

If resort is made to the use of a pentode gain stage to minimize C_{ga} , or if negative current feedback is applied to the gain stage by leaving the cathode resistor unbypassed, to stabilize gain and/or reduce Miller effect capacitance, it is usually desirable to include a cathode-follower.

ORDER OF CIRCUIT ELEMENTS

The positions in the circuit of RC sections before or after the gain stage, or of passive sections relative to the active filter are unimportant. In the latter case, the parts may be separated by several other stages if necessary. However, since the active filter produces a peak near cut-off to compensate for the slower fall of the passive sections,

it is better to place the passive sections first, so that the gain stage cannot be overdriven by signals near the cutoff frequency. When, as often happens, this is impossible, the undistorted output voltage of the filter must be suitably de-rated.

COMBINATION OF LOW-PASS AND HIGH-PASS FILTERS

It is quite feasible to combine a high-pass filter with a low-pass filter in the one feedback loop. However, care must be taken since interaction of the high-pass with lowpass RC sections as well as the combination of the frequency characteristics of the RC sections create an "insertion loss" at centre frequencies. A practical limit is

reached when the cut-off frequencies of the RC sections of the two filters-as distinct from the cut-off frequencies of the resulting active filters-are equal.

REFERENCES

- 1. RODDAM, T. Filters Without Fears, Wireless World 60, 445 (1954).
- 2.
- RODDAM, T. Filters Without Fears, Wireless World 00, 445 (1954).
 BARBER, B. T. Design of a Continuously Variable Low-pass Filter Utilizing Negative Feedback Techniques. Audio Engng. 38, 26 (1954).
 THOMPSON, J. W. A Low-pass Filter with Continuously Variable Cut-off Frequency. Electronic Engng. 26, 120 (1954).
 MILLER, C. H. R.C. Amplifier Filters. Wireless Engr. 27, 26 (1950). 3.
- SHUMARD, C. C. Design of High-pass, Low-pass and Band-pass Filters using R.C. Networks and D.C. Amplifiers with Feedback. RCA Rev. 11, 534 (1950). 5. 6. BAXANDALL, P. J. Gramophone and Microphone Preamplifier. Wireless World 61, 8 (1955).

(To be continued)

A Simple Psychogalvanometer

W. C. R. Withers*

A simple but stable device for recording changes in the electrical resistance of the skin accompanying emotional disturbance of the subject is described

THE psycho-galvanic response (p.g.r.) is the change in resistance of the skin occurring in humans and some other animals when subjected to an emotional disturbance. Some of the earliest experiments in this field were carried out by Vigouroux in 1879.

The earliest psycho-galvanometer consisted of a Wheatstone bridge connected to the palms of the subject's hands by means of metal electrodes. This arrangement has two major disadvantages.

(1) The bridge has to be continually rebalanced and readings noted while a response is actually taking place.

(2) The current through the subject is not constant resulting in varying contact potentials. Since then many designs have been developed^{1,2}. The basic circuit usually consists of a source of constant current which is passed through the subject's palms and a recording valvevoltmeter connected across the subject to record his skin resistance changes graphically. Drift, however, occurs in the valve-voltmeter section, which gives rise to calibration errors. This occurs even when the h.t. is stabilized, and is due to changes in the heater supply.

Present Circuit

The valve-voltmeter used in the present circuit³ (Fig. 1) overcomes this difficulty.

The "patient" current is derived from a high stability resistor of $8.2M\Omega$ connected to an 85A1 neon stabilizer, which itself is supplied from a source stabilized by a VR150. The resistor consists of four high stability resistors in series on a separate tag board so that the risk of shock to the subject is minimized.

The choice of valve for the valve-voltmeter was the subject of some experiment. Finally, the Mullard ECC91 was chosen, since it has a common heater cathode assembly and gives less than 2 per cent change on the record for a change of mains input of 25 per cent.

Five overlapping ranges are used to cover from 0 to 1M Ω . The recorder is returned to zero at the start of each

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range by backing off some of the input potential by means of a grid bias battery and switched potential divider. Since the current drawn is only about 18µA, shelf life or betteris obtained from the battery. The present battery has been in use for eighteen months without change of calibration. The use of a layer type battery here is not recommended as their shelf life is short.



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ELECTRODES

A pair of brass disk electrodes one inch diameter fitted inside plastic bottle tops is attached to the palms of the subject's hands by means of rubber straps and connected to the psychogalvanometer by twin p.v.c. flex.

CALIBRATION

The instrument is calibrated using a resistance box connected to the input jack. A calibration is then prepared for each range, which may be held against the record and the subject's resistance read directly.

SETTING UP

The psychogalvanometer is switched to "calibrate" and allowed to warm up for ten minutes on range A. The potentiometer is then adjusted to set the recording pen to read zero and routine calibration is carried out. The input is switched from "calibrate" to the subject, and the range switch turned until a record is obtained. The range is noted down on the recorder chart and an event

An Electronically Controlled Machine Tool

A copy milling machine installed at Laurence, Scott and Electromotors Ltd of Norwich has recently been modified for fully automatic electronic control by Electric and Musical Industries Ltd.

It is claimed that this is one of the first examples of automation applied to machine tool control actually to be installed in the works of an independent user. The copy milling machine, made by Research Engineers

Ltd, is required for the copying of cams to a high degree of precision from a master cam, a function which it normally precision from a master cam, a function which it normally performs both quickly and cheaply. When a new master is required, however, serious delays can arise due to the length of time involved in making the hand-made master. An electronically controlled machine of this type, on the other hand, can produce new parts immediately the design is com-plete and without the need for a master.

In the present installation, design data in the form of a table giving the dimensions of the cam at specific points is presented to a control unit which deduces the movements to be made by the tool and work table to produce the correct profile and operates the servomotors controlling the feeds of the machine accordingly.

The copy milling machine has a rotating work table and the dimensions of the cam must be in polar co-ordinate form. The

table of dimensions is typed on a perforating machine which codes the information on a plastic tape in which form it can be assimilated by the control unit. The perforator at the same time prints the dimensions on sance time prints the dimensions on another tape for checking and recording purposes. Should an error be made in transcribing, an error code can be in-corporated and the control unit will then ignore the incorrect dimensions. The tape is then run through the

The tape is then run through the reading head of the control unit which transfers the data to the information store. To reduce complication the tape is made to run at a speed which corresponds to the cutting speed of the machine, and the store only retains information referring to points in the immediate vicinity of the tool. In order to reduce to a minimum the number of dimensions to be worked out and listed in the table, the control unit includes an interpolator which permits it to deduce, from a relatively small number of marker points in the table, a large number of other points lying on a smooth curve through the markers. These points are then used to give the profile which the tool is instructed to cut.

An enlarged view of the tape reader unit

The control unit also works out ancillary information, such as the varying speed at which the table must rotate so that the cutting speed is always the optimum regardless of the radius.

The copy-miller to which the E.M.I. control system has been applied is already provided with a servo-control system. The control unit is therefore used only to drive a small servo operating the copying stylus, the movements of which are faithfully followed by the cutter. It is, however, equally able to control the machine feeds directly.

The complete installation of the electronically controlled copy-milling machine. To the right can be seen the console containing the control apparatus with the tape reader unit at the top. On the work table of the machine is the simulator unit operating the servo unit (on the right). At the rear of the machine is the perforator or "encoder" which produces the punched tape



The author wishes to thank Mr. R. Bluffield, who assisted in the development of the psychogalvanometer, and Professor H. J. Eysenck for permission to publish this article. REFERENCES

stimuli.

Conclusion

high stability type.

Acknowledgments

I. MARKUS, J., ZELUFF, V. Handbook of Industrial Electronic Circuits. London : p. 76. (McGraw-Hill, 1948).
2. DAVIS, R. C., SIDDONS, G. F., STOUT, G. L. Apparatus for Recording Autonomic States and Changes. Amer. J. Psychol. 67, 343 (1954).
3. HINTON, W. R. A Simple "Q" Meter. Electronic Engng. 23, 374 (1951).

marker used to record the application of the various

A typical record is shown in Fig. 2 in which the subject

was startled by means of a Klaxon Horn. Immediately before stimulation the subject's resistance was $85k\Omega$ and

about one second afterwards had fallen to $50k\Omega$. For less

All resistors except the VR150 series resistor are of the

violent stimuli the response would naturally be smaller.

Notes from ______ NORTH AMERICA

I.R.E. News

Election of Officers

Arthur V. Loughren, Vice-President in charge of research of the Hazeltine Corp., has been elected President of the Institute of Radio Engineers for 1956.

Herr Rinia, Director of Research of the Philips Research Laboratories in Eindhoven, Holland, will succeed Franz Tank, professor at the Swiss Institute of Technology, Zurich, Switzerland, as I.R.E. vice president.

Elected as Directors for the 1956-1958 term are E. W. Herold, Director of the Electronic Research Laboratory, R.C.A. Laboratories, Princetown, N.J., and J. R. Whinnery, Professor of Electrical Engineering, University of California, Berkeley, Calif.

Election of Fellows

Seventy-five leading radio engineers and scientists from the United States, Canada, and Europe have been elected Fellows of the Institute of Radio Engineers.

Included in the European awards are H. E. M. Barlow, Pender Professor of Electrical Engineering, University College, London, and Leslie C. Jesty, Chief of Television Research Group, Marconi's Wireless Telegraph Company, Ltd.

Annual Awards for 1956

Frank J. Bingley, of the Philco Corp., has been awarded the Vladimir K. Zworykin Television Prize Award for 1956 for his contributions to colourimetric science as applied to television. The award is made annually for outstanding contributions to electronic television.

The Browder J. Thompson Memorial Prize for 1956 was awarded to Jack E. Bridges, of the Zenith Radio Corp., for his paper entitled, "Detection of Television Signals in Thermal Noise." The award is made annually to an author under thirty years of age at date of submission of manuscript for a paper recently published by the I.R.E. which constitutes the best combination of technical contribution and presentation of the subject.

Television Conference

The Tenth Annual Spring Television Conference, sponsored by the Cincinnati Section of the Institute of Radio Engineers, will be held this season on Friday and Saturday, April 13 and 14, 1956, at the Engineering Society of Cincinnati Building, 1349 E. McMillan Street, Cincinnati, Ohio.

Information concerning advance registration or hotel reservations can be obtained from Mr. Reuben Nathan, Crosley Division, Avco Mfg. Corporation, Glendale-Milford Road, Evendale, Ohio.

New RCA Valves

Two new 7-pin miniature type valves have been announced by the RCA. They are the 3DT6 and 6DT6 and are intended particularly for use as f.m. detectors in television receivers. In suitable circuits, these valves will provide high sensitivity and high output voltage at low cost. The 3DT6 is like the 6DT6 except that it utilizes a heater having controlled heating time to insure dependable performance in television receivers employing seriesheater string arrangement. Separate base-pin terminals are provided for grid No. 1 and grid No. 3. Each of these grids has a sharp-cut-off characteristic and can be used independently as a control electrode.

Because of the sharp-cut-off characteristic of grid No. 3, the 3DT6 and 6DT6 are especially suitable for use in locked-oscillator, quadrature-grid f.m. circuits. In such circuits, they can perform the combined functions of detector and limiter and are capable of providing a high audio output voltage adequate to drive a medium-power output valve such as the 6AQ5.

In a typical locked-oscillator, quadrature-grid f.m. detector circuit, the 3DT6 and 6DT6 can provide a sensitivity of 5 millivolts r.m.s. with ± 7.6 kc/s deviation and 15 millivolts r.m.s. with ± 25 kc/s deviation. Furthermore, under conditions involving a signal input of 200 millivolts r.m.s. to the grid of the driver tube, they can provide a.m. rejection in the order of 30dB with ± 25 kc/s deviation and can supply an audio output voltage of 21 volts r.m.s. at a total harmonic distortion of about 3 per cent.

A New Multi-Trace Oscilloscope

A 24-trace cathode-ray oscilloscope has just been developed by the Southwestern Industrial Electronics Company of 2831 Post Oak Road, Houston, Texas, for the direct presentation on a 21in tube of multi-channel information.

Information is presented by intensity modulation of a 5000c/s raster. The raster method results in frequency response useful to 500c/s on all 24 traces with only a single electron-gun. Each trace may cross over other traces, and is limited in amplitude only by the size of the cathode-ray tube. A position control is provided for each trace. A special "window" control permits a portion of the sweep to be accelerated for detailed examination.

An auxiliary unit, the Event Pre-Selector, puts 0.1 and 0.01 second timing lines over the scope, and triggers the sweep after an adjustable delay period following the initial pulse. This feature is especially useful when the oscilloscope is used as a monitor for 24-channel magnetic tape recordings, such as those used in seismograph work.

The illustration is a photograph taken from the oscilloscope showing 24 simultaneous traces.



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Short News Items

Associated Electrical Industries Ltd have been appointed to act as consultants in a major project to create a heavy electrical manufacturing industry in India. It is the intention of the Indian Government to build up an independent industry capable of meeting a substantial proportion of the country's requirements for hydro electric plant, electric traction equipment, generating plant and distribution gear. A.E.I. have been appointed for a basic period of fifteen years. The company will act as the Government's consultants in the design, layout and construction of the factory or factories, the general arrangements and procurement of plant, machines and equipment, and the preparation of a suit-able training scheme. The company will supply designs for equipment and assist in the establishment of the engineering design department at the factory. A.E.I. will provide in India a resident engineer and the necessary technical specialists. A number of A.E.I. experts will be leaving for India shortly to survey suitable factory sites.

The British Standards Institution announces the publication of a standard "Glossary of terms relating to Automatic Digital Computers" The development Digital Computers". The development of high-speed digital computers had produced a need for the standardization of the terms used in this technique. This standard contains about one hundred entries, covering the following subjects: fundamental terms, coding systems storage techniques, programming and routines. A few definitions relating to programming and number representation and number scales are included. Copies may be obtained from the British Standards Institution, Sales Branch, 2 Park Street, London, W.1. Free 3s.

A Public Schools Convention of Science Teaching was recently organized by British Insulated Callender's Cables Ltd, Automatic Telephone & Electric Co Ltd and British Telecommunications Research Ltd, primarily with the object of strengthening the contacts between the science sections of universities, public schools and the respective companies. Sir George Thomson, Master of Corpus Christi College, Cambridge, "addressed headmasters and careers masters of public schools at the Convention which was held in Liverpool. The programme included visits to the BICC Works in Lancashire and Cheshire, a tour of ATE's main Liverpool Factory and recently established School of Electronics.

The International Atomic Exposition in Cleveland, Ohio, held from 10-16 December contained a United Kingdom display. Fourteen United Kingdom in-strument firms specializing in the field of nuclear energy exhibited on an official stand taken by the Board of Trade. The display was organized with the co-opera-tion of the Scientific Instrument Manufacturers' Association, whose 140 mem-bers cover every branch of instrumentation.

The Radio Industry Council announces that exports of British radio equipment reached their highest level in October last, the value, for the first time, exceeding £3m.

Decca Radar Ltd announce that the total number of ships of all classes for which their radar has been ordered now exceeds 4 500 for more than 1 000 shipowners and authorities throughout the world, including the Royal Navy and the navies of twenty-six Commonwealth and foreign countries. Orders for the and foreign countries. Orders for the company's Decca 45 continue, especially for larger vessels. One of the latest ships to be equipped is the new "Empress of Britain", now being completed by the Fairfield Shipbuilding and Engineering Co Ltd. The Decca 212, which the company introduced in February last as the world's lowest priced full performance set, has been ordered for over 600 ships, bringing the benefits of radar to a much wider range of shipping.

A delegation from the United Kingdom Atomic Energy Authority recently paid a six-day visit to the Soviet Union. Among the delegation were Dr. Willis Jackson, representing the Royal Society and the British Nuclear Energy Conference, and Mr. B. D. Goodlet, Head of Engineering Research and Development Division. A.E.R.E. The delegation was led by Dr. B. F. J. Schonland.

BINDING OF VOLUMES

- Arrangements for the binding service are being continued this year, and the 1955 volume can be bound at an inclusive charge of £1.
- Copies will be bound, complete with index and with advertising pages removed, in a good quality red cloth covered case blocked in gold on the spine.
- Home and Overseas readers who wish to have their copies bound are asked to comply with the following instructions :---
- (1) Tie the twelve issues (January to December, 1955) securely together before
- parcelling.
- (2) Enclose a remittance for £1 and a gummed label bearing the sender's name and address.
- (3) Enclose the copies, remittance and label in a closed parcel and address to :-The Circulation Dept. (E.E. Binding), 28, Essex Street, Strand, London, W.C.2. (No other correspondence s necessary.)

* * * *

The following are also available from our Circulation Dept. :---A limited number of Bound Volumes for 1954. Price, Two Guineas, post free. Binding Cases for twelve issues. Price 5s., postage 6d.

The Index for Volume XXVII (1955) free.

Sir Godfrey Ince has been appointed Chairman of Cable and Wireless Ltd and Chalfman of Cable and wheress Ltd and its associated companies in succession to Major-General Sir Leslie Nicholls. The management of Cable and Wireless Ltd was strengthened in March last year by the appointment of Mr. H. H. Eggers as an additional Managing Director, work-ing jointly with Mr. N. C. Chapling. Sir Godfrey Ince will not, therefore, be called upon to devote his whole time to the chairmanship.

Sir Cyril Hinshelwood, Dr. Lee's Professor of Chemistry in the University of Oxford since 1937 and Fellow of Exeter College, Oxford, was elected President of the Royal Society at a recent meeting.

Dr. Mervin J. Kelly and Sir Gordon Radley, Director-General of the British Post Office, recently received in Genoa the first Christopher Columbus Inter-national Communication Prize. They received the prize in recognition of "the planning, now being placed into practice, of the submarine telephone cable which will make it possible to establish thirtysix telephone circuits across the Atlantic between Scotland and Canada with extension to New York, intending further-more to reward hereby the numerous scientists, research workers and engineers who have contributed in the planning, production and placing in operation of the intercontinental submarine telephone line". The award will be conferred each year in one of four categories of communications; maritime, air and land communications, and telecommunications.

South East London Technical College announce a short course of six lectures each Tuesday, commencing on 10 Janu-ary at 7 p.m., on Transmission System Protection, fee 10s. A course of six lec-tures will also be held each Tuesday at 7 p.m., commencing on 21 February, on Colculations of Symmetrical and Uneach Tuesday, commencing on 10 Janu-Calculations of Symmetrical and symmetrical Short Circuits, fee 10s. Applications for these courses should be made as soon as possible to the Head of the Electrical Engineering Department, South East London Technical College, Lewisham, S.E.4.

Southall Technical College are holding short courses on Colour Television and Experimental Servo-Mechanisms. The Experimental Servo-Mechanisms. The former will be given at 7 p.m. on Wed-nesdays, commencing on 25 January, by members of the scientific staff of E.M.I. Research Laboratories. The course fee is £1. In the course on Experimental Servo-Mechanisms there will be six meetings under the direction of Mr. H. H. Ogilvy at 7 p.m. on Thursdays, com-mencing on 2 February. The course fee is 10s. Further particulars may be obtained on application to the College, Beaconsfield Road, Southall, Middlesex.

LETTERS TO THE EDITOR

(We do not hold ourselves responsible for the opinions of our correspondents)

Analysis of the Common-Base **Transistor Circuit**

DEAR SIR,-In the March 1955 issue of ELECTRONIC ENGINEERING Mr. Francis Oakes has described a method of calculating circuit characteristics of the grounded-base transistor amplifier by using a hybrid inverted- π equivalent circuit This circuit contains the four parameters r, γ , s, and a introduced (in slightly different form) by Knight et al

It may be useful to note that these four parameters are in different notation the series-parallel, or h, parameters that have been used in linear four-pole circuit analysis for many years³. Thus

 $r=h_{11}$, $\gamma = h_{12}$, $s=1/h_{22}$ and $a = -h_{21}$. However, the main purpose of this letter is to point out an error in several of the relations derived by Mr. Oakes in his calculation of the modification of the transistor parameters due to the addition of an external base resistance (modified parameters were denoted by primes). Although the calculations for γ' and s' Although the calculations for γ' and s'are correct, in calculating r' and α' the effect of the external base resistance R has not been included in all resistance R has not been included in all places where has not occur included in places where r_{ac} is given the starting equation for α' is given as $\alpha' = (r_b + R + r_m)/s$, while R is neglected in the denominator. If R is added there, the expression for α' becomes.

$$= (r_{\rm b} + R + r_{\rm m})/(r_{\rm b} + r_{\rm c} + R)$$

= $a_{\rm c} + R)/(s + R)$, (1)

which is the correct result, rather than equation (50) of Mr. Oakes, as can be verified by direct calculation of the shortcircuit output current due to an applied input current in the circuit of Mr. Oakes Fig. 8(b).

a'

In a similar manner the correct result for r' should be

$$r' = r + \frac{R(1 - \alpha)(1 - \gamma)}{(1 + R/s)} \dots$$
 (2)

rather than the result given in equation (46).

These results (1) and (2) above and the results given by Mr. Oakes for s' and γ' also have been given previously by the writers⁵, using the h_{11} notation. If $R/s \ll 1$ the above results do not

differ appreciably from the results given by Mr. Oakes. On the other hand, for the case of R = s which Mr. Oakes considers in detail, a fairly significant error is introduced. Thus, in the calculation of a' and r' for R = s, Mr. Oakes' re-sults⁶ are in error by a factor of 2, e.g. in his numerical example⁷ with a = 2. a'=3 was obtained for R=s=20000, where the error that the second secon whereas the correct result should be a' = 1.5. Similary, $s' = -10\,000\Omega$ rather than the $-20\,000\Omega$ quoted for his example.

Yours faithfully,

R. L. PRITCHARD. Research Laboratory, General Electric Company. Schenectady, New York.

REFERENCES

1. OAKES, FRANCIS. Analysis of the Common-Base Transistor Circuit. Electronic Engng. 27, 120 1955)

- KNIGHT, G. JR., JOHNSON, R. A., HOLT, R. B. Measurement of the Small Signal Parameters of Transistors. *Proc. Inst. Radio Engrs.* 41, 983 (1953).
- (1953).
 (a) GUILLEMIN, E. A. Communication Networks.
 2, Ch. 4 (John Wiley and Sons, 1935).
 (b) FELDTKELLER, R. Einfuhrung in die Vierpol-theorie der Electrischen Nachrichten Technik.
 Ch. 3 (S. Hirzel, Leipzig, 1948).
 (c) DEARDS, S. R. Matric Theory Applied to Thermionic Valve Circuits. Electronic Engng. 24, 264 (1952).
 Coaks. FRANCIS. As above. Environment of the second second
- 5.
- 264 (1952). OAKES, FRANCIS. As above. Equation following (49), p. 123. PRITCHARD, R. L. Frequency Variations of Junction-Transistor Parameters. *Proc. Inst. Radio Engrs.* 42, 787, Fig. 2 and Eq. (2) (1954). OAKES, FRANCIS. As above. Table 2, p. 124. OAKES, FRANCIS. As above. Table 4, p. 125.

DEAR SIR,-In your March 1955 issue Mr. Oakes has derived erroneous values of the parameters r' and a' which pertain to a transistor in whose base lead a resistance R has been included.

The correct expressions for these parameters should be

$$= r + \gamma s(\alpha - \alpha') - R(\alpha' - 1)$$

$$x' = \frac{\alpha + R/s}{1 + R/s}$$

Use of the correct values of these para-meters affects the detailed considerations of the various input and output resis-tances treated in the latter part of the paper.

It is evident that a' does not necessarily increase as R is increased: in fact for R = s and a = 1 as for a point type transistor a' decreases with increasing R.

Yours faithfully, F. J. HYDE,

Radio Research Station, Slough,

Bucks.

The author replies :

DEAR SIR,-I am obliged to Mr. F. J. Hyde and Mr. R. L. Pritchard for point-ing out the error in equations (46) and (50) of my article in the March issue of ELECTRONIC ENGINEERING. I should like to add that this article was written in February 1954, i.e. before the publica-tion of Mr. Pritchard's paper, and that it is for this reason that the latter was not included in the bibliography. The article by Knight, Johnson and Holt, on the other hand, was duly mentioned among other bibliographical references.

Yours faithfully,

FRANCIS OAKES,

Ferguson Radio Corporation Ltd,

Enfield.

Transistor Voltmeters

DEAR SIR,-I was interested to read Mr. Slater's letter in the October, 1955, issue and Mr. Potok's reply.

I have made some investigations of temperature effects on the circuit des-cribed in the original article, and they do not appear to be nearly so great as might be expected.

The circuit may be regarded as a type of bridge in which the batteries form one pair of arms and the transistor and the zero setting resistor R form the other pair of arms (Fig. 1).

When the zero is disturbed by a temperature change, balance is restored by adjustment of R. When the collector current is changed by the application of a voltage to the input terminals of the circuit the value of R determines what proportions of the collector current pass through the meter and through R itself. In other words, R determines the meter sensitivity. When the current gain of the transistor is changed by temperature changes, the change in value of R to restore zero is in the required sense to change the sensitivity of the meter to compensate for the change in current gain.



Fig. 1. Transistor voltmeter showing bridge arrangement

Thus the action of setting zero gives partial temperature compensation. Mr. Potok and his colleague do not appear to have appreciated this property of their circuit.

Yours faithfully, V. F. ARNOLD, St. Albans, Herts.

The correspondent replies :

DEAR SIR,-Mr. Arnold's analysis is very interesting and I am glad to see that he confirms our own results showing the basic transistor voltmeter circuit quite stable under normal working conditions.

Yours faithfully, M. H. N. POTOK. Department of Electrical Engineering, The Royal Technical College, Glasgow, C.1.

A Shift Control with no Effect on Gain

DEAR SIR,-In designing direct-coupled amplifiers I have frequently made use of a principle which does not seem to be widely known, and may in fact be novel. It is frequently necessary to provide a control which will shift the d.c. level of the output of an amplifier, and it is desir-able that adjustment of this control should not affect the gain. A convenient way of achieving the result is shown in where the following relation holds Fig. 1, where the following relation holds among the resistance values:

$$R_1/R_2 = R_4/R_3$$
 (1)

but it is not necessary that $R_1 = R_4$ nor that $R_2 = R_3$. The output is connected to a valve grid, so is not appreciably loaded.

To show that there is no change in gain when potentiometer P is adjusted. suppose first that potentiometer P and suppose inter that perform a point of C are omitted. Then since equation (1) holds, the fluctuations in potential of point λ due to the signal on the valve anode are equal to the fluctua-tions in potential of point B. So far as the fluctuations are concerned, therefore, Thévenin's theorem shows that the potentiometer is, in effect, connected to a generator of zero e.m.f. Hence, no fluctuating current flows in the potentiometer. There is, however, a steady potential difference between its ends. The meter. potentiometer therefore provides a shift control with no effect on gain.



Fig. 1. The basic circuit

If P has a high resistance value, it may be advantageous to include the capacitor C to ensure that the high-frequency response is not unduly impaired by stray capacitance when the wiper of P is not near one end. The circuit can be used with a cathode-follower instead of with a conventional stage as in Fig. 1, and it is not necessary for R_1 and R_4 to go to the h.t. line and to earth; they may go to any two fixed potentials, so long as



Fig. 2. The principle applied to a push-pull. circuit

suitable operating conditions are provided for the valve. In the form in which it is used with a cathode-follower, the circuit has already appeared in a publication by Andrew and Roberts', but was not discussed at much length.

The principle can be applied to push-pull circuits, either by the use of a ganged potentiometer as in Fig. 2, or by making the adjustment on one side only, as in Fig. 3. These circuits provide a further way of obtaining static balance of a pair of values in addition to the methods valves, in addition to the methods of

Fig. 3. An alternative arrangement of Fig. 2.



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listed by Mr. Aitchison² in your May, 1955, issue, with the advantage that in these circuits the adjustment of static balance does not affect the other adjustments which may have to be made.

Yours faithfully,

A. M. ANDREW, Institute of Physiology, The University, Glasgow. REFERENCES

- ANDREW, A. M., ROBERTS, T. D. M. A Pulse-Interval Meter, Pt. 1. *Electronic Engng.* 26, 469 (1954). (The circuit appears in Fig. 5, between V₂₃ and V₈₄.)
 AITCHISON, R. E. A New Circuit for Balancing the Characteristics of Pairs of Valves. *Electronic Engng.* 27, 224 (1955).

Saturated Diodes

DEAR SIR,—In their articles "Saturated Diodes" (August, 1955) and "Characteristics of the Temperature Limited Diode Type 29C1" (November, 1953), Dr. Benson and Mr. Seaman state that the saturated diode current and filament voltage are related by the law $I_n = K V_f^n$ and that over the filament voltage range of 3.3 to 4.1 volts *n* varies from 8.4 to 7.8 for the AV33 and from 9.3 to 8.4 for the 29C1.

It is obscure how these values for n were determined as they do not appear to fit the experimental curves of (August, 1955, issue) which we find fitted by a constant *n* of 8.32 for both diodes. A plot of I_a/V_r on log-log paper re-sults in almost straight lines of slope 8.32.

The intercepts of these projected lines on the I_a axis gives values for K of 4.8 $\times 10^{-5}$ for the AV33 and 2.3 $\times 10^{-5}$ for the 29C1.

The table given below shows the following values of I_{a} .

(a) I_{s} taken from the experimental curves of Fig. 1.

(b) I_n calculated for n = 8.32 K = 4.8× 10^{-5} (AV33) n - 8.32 K = 2.3 × 10^{-5} (29C1)

in which we intended it to be used, which is over a very small range of filament voltage only. If a constant value of Kis assumed, and the desired value of the filament voltage and the appropriate value of the index n are then inserted in the formula, we are not surprised at the inaccurate answers which they have ob-tained. Also, the nature of the curves in Fig. 1 of the article, "Saturated Diodes" does not allow very accurate values of the anode current to be obtained from it.

The graph below is constructed from the data used to draw the characteristics Figs. 3 and 4 of the later article and from this we find it difficult to see how a straight line relationship can be justi-



Fig. A. Graph constructed from data in article discussed

fied (the dotted line is a straight line through the first and last points, and has been drawn in for comparison purposes only). Also, we cannot see how the same value of n (which is the slope of the curve) is obtained for both valves.

			AV33		29C1							
		(a)	(b)	(c)	(a)	(b)	(c)					
3·3 3·7 4·1	•••	1.0 2.8 6.3	0·99 2·46 6·02	1.00 1.55 2.6	0·46 1·2 2·9	0·47 1·22 2·88	0·46 0·79 0·97					

(c) I_n calculated for the values of *n* given in table 2 of the August issue (assum-ing K = 4.412 for the AV33 and 0.69 for the 29C1 which makes the values of (a) and (c) equal at $V_t = 3.3V$).

It is seen that good agreement with the experimental curves is obtained if n is a constant value of 8.32 whereas the values of n given in the article result in very large discrepancies.

Yours faithfully, H. J. FRASER, V. C. ANTHONY, Amalgamated Wireless Valve Co. Pty. Ltd, Sydney, Australia.

The authors' reply :

DEAR SIR,-It would appear that Messrs. Fraser and Anthony are not using the formula $I_n = K \cdot V_f^n$ in the manner

differentiating the formula On = $K.V_t^n$, we obtain $dI_n/I_n = n. dV_t/V_t$, which we have found very useful in circuit calculations concerning these diodes. The values of n given in the Tables in the articles have been calculated to fit this differential equation, and have also been verified by experimental observa-tions in a practical circuit. Even though the filament voltage was measured with a potentiometer and a first grade instrument was used to measure the anode current, these values of n can, in fact, be in error by nearly 0.1. A similar treatment was used to obtain the index m in the relationship $V_t = k I_t^m$. Generally speaking, the values of m and n published by other workers in this field have been obtained in a manner similar to that outlined above

> Yours faithfully, F. A. BENSON, M. S. SEAMAN,

Department of Electrical Engineering, University of Sheffield.

BOOK REVIEWS

Schaltungstheorie Und Messtechnik **Des Dezimeter Und** Zentimeterwellengebietes

By Albert Weissfloch. 308 pp., 288 figs. Medium 8vo. Birkhäuser Verlag, Basel and Stuttgart. 1954. Price Sfr.(DM)33-50.

'HIS well produced book written in a This well produced book that the first lucid and readable style forms the first volume of a new series of textbooks and monographs in the field of physics. It sets a high standard for the volumes to follow it and author and publishers may be congratulated upon this accomplishment. It is particularly welcome at a time when v.h.f. engineering is in a state of rapid development. While practically no use is made of Maxwell's equations the reason for this is given in a footnote on page 116/117-circular geometry or geometry of linear fractional func-tions and quadripole theory are given prominence in this treatise. It should be remarked that symbols of elementary group theory are occasionally applied.

After an introduction in which the task of the book and the special conditions of ultra-high frequencies are briefly stated Section I, starting from measurements, deals with the derivation of the most im-portant quadripole theorems. Graphical methods of calculation are given primary consideration as these have proved to be generally most perspicuous in many fields of application. It is shown how circular of application. It is shown how enclarating geometry is a very useful tool for dealing with many of the problems investigated. This is the most detailed section and occupies about one third of the book.

Section II treats the simpler kinds of connexions in which homogeneous lines with or without return conductors are used. While the first are mainly used for power transmission and long or medium waves or-in concentric design-for short waves as used, for example in long distance transmission of television, the latter play their most important part in the shape of waveguides in the micro-wave range. Although the direct measurement of voltage and current in waveguides is not possible it is shown how these values may be determined by measuring impedances and active power. Use is made of what is called the transformer theorem which states that on any complicated loss-free quadripole connected to homogeneous lines in the pass range two definite positions may be found such that the quadripole has the properties of an ideal transformer. The applications of this theorem are discussed in detail.

Section III, which like the preceding section occupies about one fifth of the book, deals with more complicated cir-cuits, branchings like hexapoles or, more generally speaking. 2n-poles, filters and the like and particularly discusses the transformation properties of such circuits as they depend on frequency. Also cavity resonators are briefly dealt with in this section. this section.

Section IV occupying about one fourth of the book is devoted to the problem of matching. While in the preceding sec-tions the properties of the component

parts were dealt with, here the total circuit is taken into consideration and it is investigated when and to what amount power is transmitted from the generator to the consumer of ultra-high frequency. The generator may be a transmitting valve and the consumer a transmitting aerial or the generator may be a receiving aerial and the consumer the electron path of a receiving diode or the rectifier layer of a detector. A distinction is made here between a generator with constant inner impedance and constant e.m.f., as for example a receiving aerial, and a more complicated generator like the self-excited "matching ratio", i.e. the ratio between actual power transmitted and its optimal value is used as a quality figure. The resonance method of measurement of transformation properties and of impedances in valves and waveguides, matching with fixed and variable adjustment and wideband matching, are discussed in detail.

A combined name and subject matter index concludes the book which forms a valuable addition to the literature of theory and practice of microwave engineering.

R. NEUMANN

Advanced Mathematics for Engineers

By H. W. Reddick and F. H. Miller. 548 pp. 131 figs. Demy 8vo. 3rd Edition. Revised by F. H. Miller. John Wiley and Sons, Inc., New York. Chapman and Hall Ltd., London. 1955.

A there has emerged from engineering science during the subject which may properly phase of the subject which may properly be called mathematical engineering. Its analogue in physics is mathematical phy-The mathematical engineer seeks to sics. establish a mathematical formulation of engineering problems; that is, in the place of the physical system he substitutes an idealized mathematical system and hence appeals to mathematical method in his search for new engineering knowledge. There can be little doubt that the future progress of engineering science will de-pend largely upon the further development of mathematical engineering.

The techniques of mathematical engineering make considerable demands on the resources of mathematics. Among the first of the text-books which have appeared during recent years and which endeavour to bring advanced mathematical processes within the reach of the mathematical engineer was the original edition of this book by Professors H. W. Reddick and F. H. Miller. Evolved from courses given by the authors at the Cooper Union Institute of Technology in New York the book was first published in 1938 and again in 1946 with modifications designed to increase its usefulness such as the addition of an appendix dealing with units and dimensional analysis. A further revision has been made by Professor F. H. Miller and a third edi-tion is now available.

Among the changes which appear in the latest edition are the addition of a theorem on reversion of power series to Chapter IV and the addition of an intro-ductory account of Legendre's equation to Chapter VI. Chapter IX has been re-organized and now includes two articles on numerical methods for solving ordinary and partial differential equations. The final chapter, which deals with the Cauchy-Heaviside operational calculus, now contains an introduction to the Laplace transformation calculus. In a book which touches on so many topics it would seem to have been more desirable in this new edition to discontinue the discussion on the operational calculus and to devote the chapter exclusively to the tranformation calculus.

The book is well presented and beautifully produced. As is true of its prede-cessors, this edition will be of inestimable value to those who are concerned with bringing engineering into closer relation-ship with pure science.

S. R. DEARDS,

Practical Electroacoustics

By M. Rettinger. 264 pp. 178 figs. Demy 8vo Thames & Hudson Ltd. 1955. Price 63s.

THE writer of this book is a well-known authority on studio acoustics and a member of the RCA staff. Consequently the contents are a review of RCA practice and all other designs are virtually excluded. The subject matter is princi-pally of interest to the recording, tele-vision or broadcast engineer, and the production is on excellent paper with very clear illustrations. Once again, the price of 63s. appears excessive for a book of 264 pages, and it is difficult to see why the cost of printing in the U.S.A. should be as high as \$10.00.

With the above reservations, Mr. Rettinger's work is a most valuable contribution to the art. Every problem likely to confront the recording or public address practitioner is fully and lucidly presented, with emphasis on the practical means of solution. This is possible by virtue of the author's remarkable ability to condense the essence of his point, and by the complete absence of general matter which can be found in other publications.

Only a staff member of such large or-ganizations can write this kind of book, which draws on experience representing the expenditure of millions of dollars, and this makes it difficult to single out any section for particular mention. The many tabulated measurements, response curves, tables and specially prepared data for deriving design requirements are more comprehensive—and more generally useful-than any to be found in other books. Perhaps the excellent chapter on magnetic recording can be cited as a model of concise expression.

Microphones, loudspeaker enclosures, mixing and separating networks; magnets public address systems and architectural acoustics all receive adequate attention and the book is packed with the results of the author's experience in every phase of audio engineering. However, it is not especially written for the constructor: no amateur equipment is described, nor are there any amplifiers, disk reproducers, or radio circuits.

It is obviously difficult to choose a title for a book of this kind, but if it is understood that the contents are intended for the practising professional man, then this entirely up to date, authoritative and excellently written book must rank as one of the best treatises so far produced on practical, electro-acoustics. The bibliography could have been much more extensive, and the author is perhaps a little optimistic in saying "gone are the 'condensor' loudspeakers"; but as a summary of first-class modern practice the book can be regarded as indispensable. ALAN DOUGLAS

An Introduction To Electronic Analogue Computers

By C. A. A. Wass. 227 pp., 149 figs. Demy 8vo. Pergamon Press Ltd. 1955. Price 40s.

THIS book falls naturally into two unlargest is devoted to d.c. analogue computing techniques with particular reference to simulators and design of the basic computing elements. A large number of practical examples is included. The second describes the use of simulators and is also informative though fewer examples from real life are given. The author has been for many years closely associated with simulator work carried out at the Royal Aircraft Establishment and the book, which is readily understood without great knowledge of electronics or mathematics, fulfils its purpose of serving as an introduction to the subject extremely well.

After a brief introductory comparison of the digital and analogue techniques, the ideal basic analogue computing elements are described and their use discussed. The essential difference in approach between "differential analyser" and "simulator" methods is made clear and the latter principles lucidly extended stage by stage from very simple cases to a complex ballistics problem including over 60 elements. The design of real drift corrected d.c. amplifiers and various other computing units including nonlinear and electromechanical devices is next described and much information regarding the magnitude and relative importance of errors resulting from imperfections in such systems is given which is particularly valuable since it is evidently based on a great deal of experience.

The chapter entitled "Using Simulators" deals mainly with scale factors and the setting of "initial conditions" in integrators and is not on the whole so instructive as the foregoing from the practical aspect. The author admits that many of the examples of initial conditions setting techniques are of academic interest only and, strangely, one of the simplest practical methods is neglected while some of the statements concerning the restriction on the integrator working range due to the initial signal are actually misleading. A useful account is given of apparatus which is commercially available or can be easily constructed for provision of the usual simulator input functions and for recording output signals, and there is a discussion on power supplies. The book concludes with an analysis of the relative merits of types of simulators other than the d.c. analogue machine and gives a brief account of several existing computers in which some interesting automatic monitoring and control schemes are described.

J. C. NUTTER





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Foreword by

RT. HON. LORD JUSTICE DENNING

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An indispensable book on the

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NOISE

A. van der Ziel

Illustrated

An important book concerned

with noise in electronic devices.

37 ESSEX STREET, LONDON, W.C.2

(Professor of Electrical Engineeri University of Minnesota)

Books on radio theory, practice and maintenance for the beginner and books on new developments in circuit design, new components, methods of application, and the established text books can be obtained through your local Smith's shop or bookstall. Books not in stock at the branch can be quickly obtained from Head Office.

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ELECTRONIC EQUIPMENT

A description, compiled from information supplied by the manufacturers, of new components, accessories and test instruments.

Precision Pulse Oscilloscope (Illustrated below)

THIS instrument has been designed primarily for quantitative measure-ments on pulsed or transient waveforms. It is fitted with a short persistence c.r.t. giving a blue trace which is clearly visible giving a blue trace which is clearly visible under full daylight conditions. The stabilized e.h.t. supply is variable from 1.8kV to 3.6kV in four steps. Direct access is provided to the X and Y plates. The Y amplifier is d.c. coupled throughout and has a gain of approxi-mately 1 000 times. The rise-time of output voltage for a step function input (10 per cent to 90 per cent) is 0.04/wsec

(10 per cent to 90 per cent) is 0.04µsec.

The time-base circuits are d.c. coupled and produce an 11cm trace at high e.h.t.



Nine calibrated sweeps are available with durations of 0.5μ sec to lmsec. The Y shift control is calibrated directly in Direct, amplified and Nine calibrated sweeps are available with microseconds. Direct, amplified and delayed triggering facilities are provided.

Newport Instruments (Scientific & Mobile) Ltd, Newport Pagnell, Buckinghamshire.

Noise Cancelling Microphone

THE Lustraphone noise cancelling "Velodyne" microphone described below is designed as a close-talking microphone for use under conditions of high ambient noise encountered in aircraft, ships and all other mechanically pro-pelled vehicles, for conference work, in crowded surroundings and under noisy conditions in general.

The Velodyne microphone is a departure from conventional practice in differ-ential microphones and is characterized by a steep decline of output with increased distance from the speaker's mouth, due to operation on the pressuregradient principle and the uni-directional field configuration with a high front-toback ratio, resulting in the virtual exclusion of background noises.

The differential characteristic of the Velodyne is maintained at frequencies considerably higher than is the case with conventional differential microphones which become increasingly ineffective with rising frequency, owing to the finite distance between two diaphragms or apertures.

Due to its light weight and small size, this microphone may be conveniently fitted to a headset boom, in addition to the more conventional hand-held model. The instrument is of rugged construc-

tion and can be used at extremes of temperature and humidity. The design is such that the unit is unaffected by rapid changes of air pressure.

> Lustraphone Ltd, St. George's Works, Regents Park Road, London, N.W.1.

V.L.F. Signal Generator (Illustrated below)

THE Airmec signal generator type 852 has a continuously controllable sinusoidal output with small distortion over the very low frequency range 0.03 to 30c/s.

A specially designed rotating capacitor is employed to modulate a high frequency This capacitor is driven by a signal. variable speed motor via a three ratio gearbox at shaft speeds continuously variable from 0.03 to 30 revolutions per The modulated signal is rectisecond. fied and then amplified by a direct coupled negative feedback amplifier to provide the very low frequency output. The use of a modulation system fol-

lowed by a stable amplifier enables the output level to be set accurately by means of the high frequency signal. An imme-diate reading of output level is therefore obtained and the difficulties inherent in attempting to measure the voltage of the actual output signal are completely avoided.

Since the frequency of the output signal does not depend upon the use of very high value resistors or capacitors, a good long term stability relatively independent of temperature and atmospheric conditions is obtained. Moreover, accurate monitoring of the output frequency is simply effected by measuring the motor speed, the result being indicated on a 6in meter calibrated directly in frequency.



The circuit is designed so that the output in the absence of a signal is at earth potential and means are provided for adjusting the balance of the output signal to ensure that it contains no d.c. component.

Calibrated attenuators enable the output level to be continuously varied from 500 microvolts to 50 volts peak. output impedance is normally The $10k\Omega$ but a low impedance position on the attenuator switch enables a direct connexion from a cathode-follower to be made.

> Airmec Ltd, High Wycombe, Buckinghamshire.

Miniature Transformers

(Illustrated below)

TWO new Miniature types of trans-formers have recently been added to the Partridge range. This miniature the Partridge range.



potted range, which is available in five sizes, is being manufactured in two alternative constructions, either hermetically sealed, oil or compound filled, or embodied in synthetic resin (resin cast construction).

A standard range of high fidelity line and microphone transformers is available in this resin cast construction and these In this resin cast construction and these standard items, which will be available from stock, will cover primary imped-ances from 6.25Ω to 600Ω with ratios from 80:1 to 20:1; the frequency charac-teristics being flat \pm 1dB from 20 to 20 000c/s.

Partridge Transformers Ltd, Roebuck Road, Tolworth, Surrey.

Dynamic Balancing Machine (Illustrated in next column)

THIS machine is designed to indicate out-of-balance on rotating components in a weight range from a few ounces to about a hundredweight. The perform-ance of the machine is independent of supply voltage and frequency variations.

The machine is comprised principally of a framework arranged to carry the rotating component to be balanced or "work" and a cabinet which houses the electronic apparatus. A meter vindicat-ing out of balance and a stroboscopic lamp for illuminating the "work" may

be mounted in the most convenient positions on the tie bars which form part of the framework. A synchronous driving motor is also mounted on the framework.

The work is placed in two half bearings free to move in a horizontal plane and is rotated by a belt drive from the synchronous motor. To facilitate load-ing and unloading the belt may be lifted off the work by the movement of a hand lever. Tests may be carried out over a wide range of running speeds.

> Small Electric Motors Ltd. Eagle Works, Churchfields Road, Beckenham. Kent.





Mercury Switches (Illustrated above)

THIS hand-operated switch combines the advantages of mercury switching with the convenience of multiple circuit switching by a single rotary knob. The unit illustrated mounts up to four

mercury switches of 15A rating. The operating shaft is of square section and quarter segment cams are slipped on to this to provide the action required. The movement is four position and is provided with spring clip stops to give positive location in each position.

Engel & Gibbs Ltd, Warwick Road, Boreham Wood, Hertfordshire.

Miniature Heavy Duty Relay (Illustrated above right)

"HIS relay, the type J.O4 is of comparatively small size, measuring 31in $1\frac{7}{8}$ in \times $2\frac{1}{2}$ in, but has contacts having a continuous rating of 20A at 250V a.c.



The contact arrangement is double pole changeover. The coil consumption is under 5VA on a.c. and 2.5W on d.c. This relay is also available in a weatherproof cast aluminium box having

conduit entries at top and bottom.

Besson & Robinson Ltd. East Industrial Estate, Harlow Essex.

Flameproof Electronic Flash Equipment (Illustrated below)

DESIGNED to give single flashes of white light of high intensity and short duration, the Ernest Turner model graphic application with conventional cameras under conditions where the pre-



sence of inflammable gases constitutes a hazard, as for example in coal mines and petrol refineries. The flash energy is 200 joules. A specially developed noninductive triggering circuit, with a short-circuit energy of less than 2 microjoules, permits the flash to be synchronized with the shutter of any normal camera, no special precautions being necessary.

Ernest Turner Electrical Instruments Ltd, Chiltern Works, High Wycombe, Buckinghamshire.

Constant Voltage Amplifier and Oscillator

(Illustrated below) HIS instrument is manufactured to a design by Dr. G. N. Patchett¹ and provides a source of power suitable for



energizing a.c. bridges, potentiometers and similar apparatus where a stable

and similar apparatus where a stable sinusoidal voltage is required. The stability is obtained by using a feedback circuit incorporating a therm-istor bridge. This bridge provides nega-tive or positive feedback depending on the magnitude of the input voltage.

When the input voltage decreases below a certain level the amplifier acts as an oscillator, the frequency being determined by the tuned circuit of the first stage.

These amplifiers can be designed to operate at any fixed frequency in the range 400c/s to 2kc/s and can be arranged to give various output power readings according to individual requirements.

1. PATCHETT, G. N. A Constant Voltage Ampli-fier and Oscillator. Electronic Engng. 27, 536 (1955)

> A. Tinsley & Co. Ltd, Werndee Hall, South Norwood, London, S.E.25.

Fractional Horsepower A.C. Motor (Illustrated below)

THIS squirrel cage a.c. motor has a nominal driving speed of 2 700 rev/min and is rated at approximately $1/70^{\rm th}$



h.p. Measuring $2in \times 3in$ with a $\frac{3}{4}in$ long shaft the motor weighs 8oz. It is precision built with a ground shaft mounted in sintered bearings and requires no maintenance.

> Jones & Stevens Ltd, Long Lane, Littlemore, Oxford.

Infra-red Relays

A RANGE of electronic relays, using a lead sulphide photo-conductive cell as the sensitive element, has recently been introduced by B.T.-H. The relays are capable of controlling up to 1A at 440V a.c

One of them, the IR2 is capable of detecting a body four feet away at 440°C; another, the IR3, is more sensitive and will detect a hot plate at 80°C (e.g. a kettle of hot water) under-ideal conditions at the same distance.

The detecting head, incorporating the lead sulphide cell, is normally supplied as a water cooled housing.

Both relays are of the robust con-struction necessary for industrial applications.

The British Thomson-Houston Co. Ltd, Rugby, Warwickshire.

Meetings this Month

THE BRITISH INSTITUTION OF RADIO ENGINEERS

Date: 5 January. Time: 6.30 p.m. Held at: The London School of Hygiene and Tropical Medicine, Keppel Street, Gower Street, London, W.C.1. Lecture: Domestic Tape Recording Applications with Special Reference to Stereophonic Reproduction. By: M. B. Martin and D. L. A. Smith. Date: 25 January. (Time and place as above.) Symposium on Electronic Methods of Pictorial Reproduction.

Reproduction.

Merseyside Section

Date: 4 January. Time: 7 p.m. Held at: The Council Room, Chamber of Commerce, 1 Old Hall Street, Liverpool 3. Lecture: Electronic Instrumentation for Nuclear Power. By: R. J. Cox.

North-Western Section

Date: 5 January. Time: 6.30 p.m. Held at: The Reynolds Hall, College of Tech-nology, Sackville Street, Manchester. Lecture: Radio and Television Interference, its Growth and Effects. By: M. Smith.

North-Eastern Section

Norm-Eastern Section Date: 11 January. Time: 6 p.m. Held at: The Institution of Mining and Mechani-cal Engineers, Neville Hall, Westgate Road, Newcastle upon Tyne. Lecture: Some Interference Problems Associated with the Television Service. By: J. C. Belcher.

West Midlands Section

Date: 11 January. Time: 7.15 p.m. Held at: Wolverhampton and Staffordshire Tech-nical College, Wulfruna Street, Wolverhampton. Lecture: Computer Control of Machine Tools. Lecture: Comp By: H. Ogden.

Scottish Section

Date: 12 January. Time: 7 p.m. Held at: The Institution of Engineers and Ship-builders, 39 Elmbank Crescent, Glasgow. Lecture: Peaceful uses of Atomic Energy. By: K. G. McNeill.

South Wales Section

Date: 25 January. Time: 6.30 p.m. Held at: The University of College of Si Wales and Monmouthshire, Cardiff. Lecture: Magnetic Amplifiers. By: O. I. Butler. South

THE INSTITUTION OF ELECTRICAL ENGINEERS

All London meetings, unless otherwise stated. will be held at the Institution, commencing at 5.30 p.m. Discussion: The Efficient use of Technical Personnel. Opened by the President.

Radio and Telecommunications Section

Radio and retrebuilded for the second second

Date: 23 January. Lecture: Particle Accelerators. By: E. L. Wiblin.

Joint Meeting

Date: 16 January. Held at: The Institution of Mechanical Engineers, 1 Birdcage Walk, Westminster, London, S.W.1. Papers on: Advanced Courses for Engineers in Industry. By: Willis Jackson, H. D. Morgan and G. F. Muscher

By: Willis Mucklow. meet

int meeting with the Institution of Civil Engineers and the Institution of Mechanical Engineers.

East Anglian Sub-Centre

Date: 2 January. Time: 6.30 p.m. Held at: The Crown and Anchor Hotel, Ipswich. Lecture: The Industrial Application of X-Rays. By: F. L. Veale.

ELECTRONIC ENGINEERING

North-Eastern Radio and Measurements Group

North-Eastern Radio and Measurements Group Date: 16 January. Time: 6.15 p.m. Held at: King's College, Newcastle upon Tyne. Lectures: The Application of the Hall Effect in a Semi-Conductor to the Measurement of Power in an Electromagnetic Field. The Design of Semi-Conductor Wattmeters for Power-Frequency and Audio-Frequency Applications

Applications. By: H. E. M. Barlow.

By: H. E. M. Barlow.
South Midland Radio and Telecommunication Group
Date: 23 January. Time: 6 p.m.
Held at: The James Watt Memorial Institute, Great Charles Street, Birmingham.
Lecture: Maintenance Principles for Automatic Telephone Exchange Plant.
By: R. W. Palmer.
(Joint meeting with the Birmingham Centre of the Institution of Post Office Electrical Engineers.)

the Institu Engineers.)

Southern Centre

Date: 4 January. Time: 6.30 p.m. Lecture: Receiving Aerials for British Television. By: F. R W. Strafford. Date: 11 January. Time: 7.30 p.m. Held at: The R.A.E. Technical College, Farn-biorough.

borough. Discussion: The Applications and Limitations of Electronic and other Computers. Opened by: L. G. Brazier.

THE INSTITUTION OF POST OFFICE ELECTRICAL ENGINEERS

Date: 3 January. Time: 5 p.m. Held at: The Institution of Electrical Engineers, Savoy Place, London, W.C.2. Lecture: Power Plant for Telecommunications. By: W. J. Marshall.

RADIO SOCIETY OF GREAT BRITAIN

Date: 27 January. Held at: The Institution of Electrical Engineers. Savoy Place, London, W.C.2.

THE TELEVISION SOCIETY

Date: 19 January. Time: 7 p.m.
Held at: The Cinematograph Exhibitors' Association, 164 Shaftesbury Avenue, London, W.C.2.
The Fleming Memorial Lecture: Non-Entertainment Aspects of Television.
By: J. D. McGee.

PUBLICATIONS

RECEIVED

A BEGINNER'S GUIDE TO RADIO is an elementary first course in radio transmission and reception specially planned for teachers and students. The student is taught how to build simple receivers and the theory and function of each part is explained. George Newnes Ltd. Tower House, Southampton Street, Strand, London, W.C.2. Price 7s. 6d.

THE PRESENT THE FUTURE covers many aspects of the present state and future out-look for the electrical and electronic industries. It illustrates the theme by a series of short surveys of topics of urgent importance in which the General Electric Co Ltd has a specialist interest. The General Electric Co Ltd, Publicity Department, Magnet House, Kingsway, London, W.C.2.

TRU-PROCESS CASTINGS describes a new system of producing castings. Tru-process is an investment technique employing a high grade refractory material which will resist the attack of steels and other special alloys of high melting point. It has been used with success for the production of dies and moulds for the plastic and other industries. Darwins Ltd, Tinsley, Sheffield 9.

THE ELECTRICAL INDUSTRIES BENEVO-LENT ASSOCIATION YEAR BOOK 1955 has recently been issued. This includes the Annual Report and Accounts for the year 1954. The Electrical Industries Benevolent Association, 10 Buckingham Palace Gardens, London, S.W.1.

RADIO AND TELEVISION SERVICING is a careers booklet by the Central Youth Employ-ment Executive which describes the training, qualities required, opportunities of employment and avenues of promotion in this field. Separate sections are devoted to basic training, training in systematic testing, apprenticeship, technical studies and wages and working conditions. The booklet is intended primarily for boys who are about to decide what to do on leaving school, but it will also be of interest to parents, teachers and others. Her Majesty's Stationery Office, Kingsway, London, W.C.2. Price 1s. 6d.

PATENT LAW IN THE RESEARCH LABORA-TORY by J. K. Wise is a book which is in-tended to give the industrial research worker a practical insight into the workings of the patent system and to show him clearly the impact of that system on his day-to-day research activities. Reinhold Publishing Cpn, 430 Park Avenue, New York 22, N.Y. Chapman & Hall Ltd, 37 Essex Street, London, W.C.2. Price 24s.

VISIBILITY OF NOISE IN TELEVISION is number 3 in the BBC Engineering Division series of monographs. About six of these monographs are to be produced each year. Individual copies price 5s. post free, annual subscription 20s. post free. Orders can be placed with newsagents and booksellers, or BBC Publications, 35 Marylebone High Street, London, W.1.

THE ANNUAL REPORT OF THE BRITISH STANDARDS INSTITUTION 1954-55 has recently been published. It contains in its 240 pages a review of the wide range of activity which the Institution now embraces. British Standards Institution, British Standards House, 2 Park Street, London, W.1.

WESTOOL LTD, St. Helen's Auckland, Co. Durham, have recently issued a series of tech-nical data sheets dealing with their standard range of transformers. These data sheets and loose leaf binders are available on request.

BASIC SYNCHROS AND SERVOMECHAN-ISMS by Van Valkenburgh, Nooger & Neville, Inc, covers synchros and servo fundamentals, servo error detectors, motors and control systems as currently taught at U.S. Navy specialty schools, and now released for civilian use. John F. Rider Publisher, Inc, 480 Canal Street, New York 13, N.Y., U.S.A. Price \$5.50 for two volumes.

GOOD PRESENTATION discusses and makes recommendations on the subjects of training for projectionists, the maintenance of films and equip-ment, and operating conditions and requirements. It is an authoritative guide for all users of l6mm. films and equipment. The British Kine-matograph Society, 164 Shaftesbury Avenue, London, W.C.2. Price 1s. 6d.

FM EXPLAINED by E. A. W. Spreadbury is a reprint of articles from Wireless and Electrical Trader explaining in simple terms the principles of the f.m. system of transmission as used in the BBC v.h.f. radio services. Its effect on receiver design and servicing problems is ex-plained, together with methods of circuit align-ment. Trader Publishing Co. Ltd., Dorset House, Stamford Street, London, S.E.I. Price 2s. 6d.

RADIO TUBE VADE-MECUM 1955 is the 12th edition of this now well-known reference book. P. H. Brans, Ltd., Antwerp. Bailey Bros. & Swinfen Ltd., 46 St. Giles High Street, London, W.C.2. Price 27s. 6d.

THE A.R.R.L. ANTENNA BOOK has two main divisions. The first five chapters deal with the principles of antennas and transmission lines. wave propagation and its relationship to antenna design, and the performance characteristics of directive antenna systems. Beginning with the sixth chapter, there is a series of chapters in which complete data are given on specific designs for the various amateur bands. The American Radio Relay League, Inc., West Hartford, Con-necticut, U.S.A. Price \$2.00.